

# Analysis and modelling of stress concentration on riveted zones of thin structures

AEA 2024/2025 – Presentation 2



Group 4

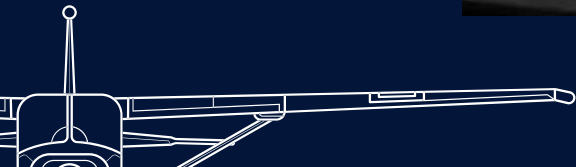
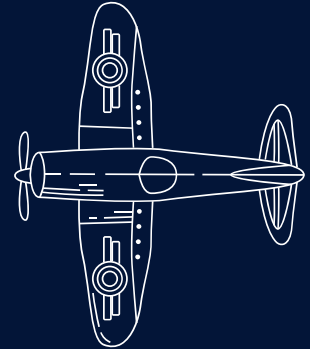
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# Introdução



<https://boltflight.com/why-do-planes-use-rivets-instead-of-welds/>

# Casos de Falha

## Rivet shear

$$P = \frac{\pi d^2 \tau_1}{4b}$$

## Bearing pressure

$$P = \frac{p_b t d}{b}$$

## Shear failure in a plate

$$P = \frac{2at\tau_2}{b}$$

## Plate failure in tension

$$P = \frac{\sigma_{ult} t (b - d)}{b}$$

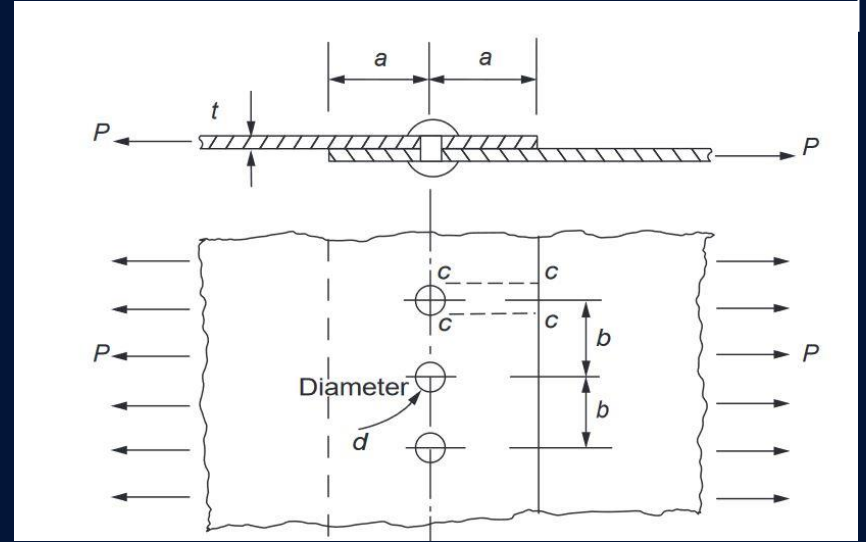
$\tau_1$ - Tensão de Corte no rebite

$p_b$ - Tensão de Cedência

$\sigma_{ult}$ - Tensão Última na placa

$\tau_2$ - Tensão de corte na placa

$P$ - Força por Unidade de comprimento[N/M]



Megson, T. H. G. (2021). Aircraft Structures for Engineering Students. In *Aircraft Structures for Engineering Students*. Elsevier.

# Casos de Falha

## Rivet shear

$$P = \frac{\pi d^2 \tau_1}{4b} \Rightarrow P = 159 \text{ kN}$$

## Bearing pressure

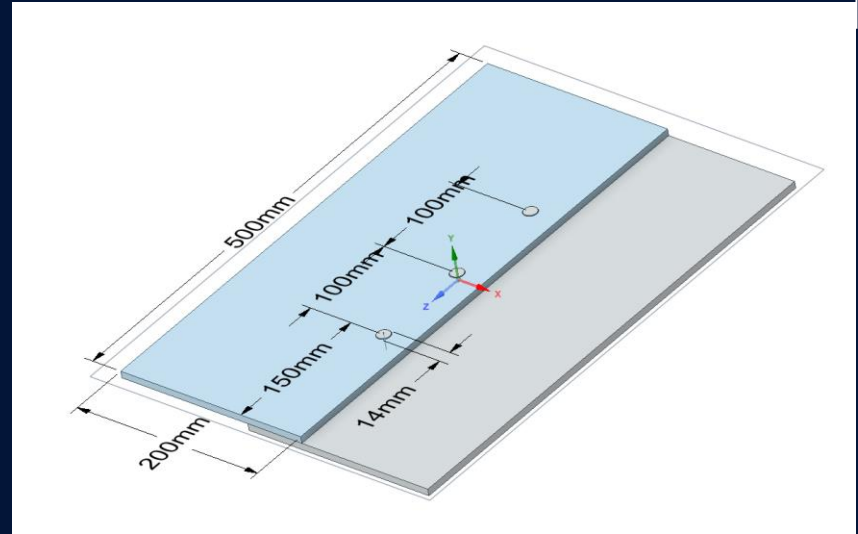
$$P = \frac{p_b t d}{b} \Rightarrow P = 189 \text{ kN}$$

## Shear failure in a plate

$$P = \frac{2at\tau_2}{b} \Rightarrow P = 435 \text{ kN}$$

## Plate failure in tension

$$P = \frac{\sigma_{ult} t (b - d)}{b} \Rightarrow P = 933 \text{ kN}$$



AL6061 T6

$$\tau_1 = \tau_2 = 207 \text{ MPa}$$

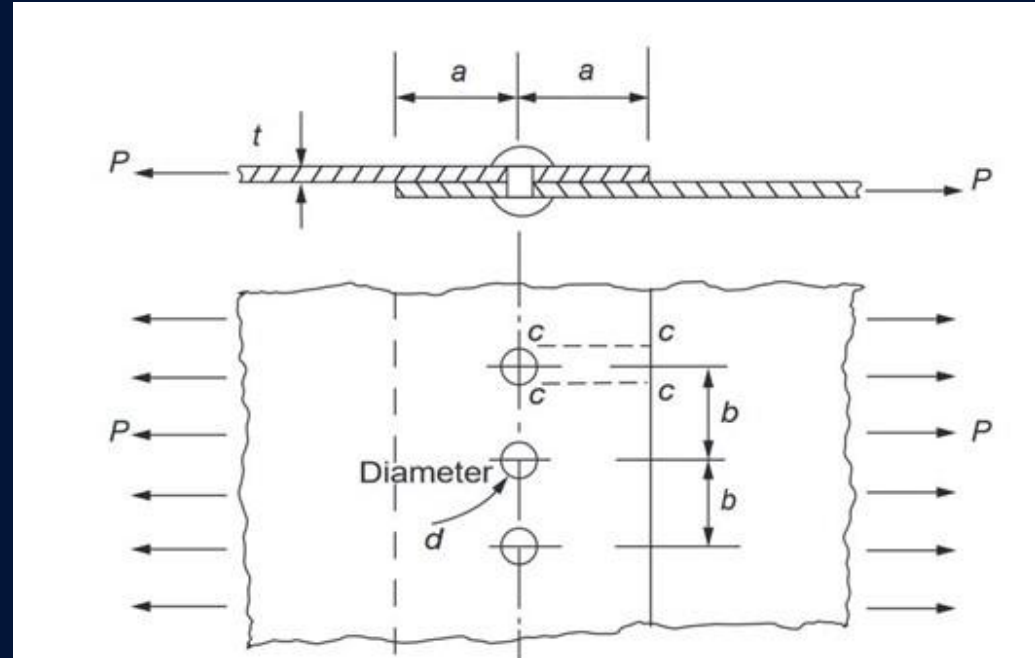
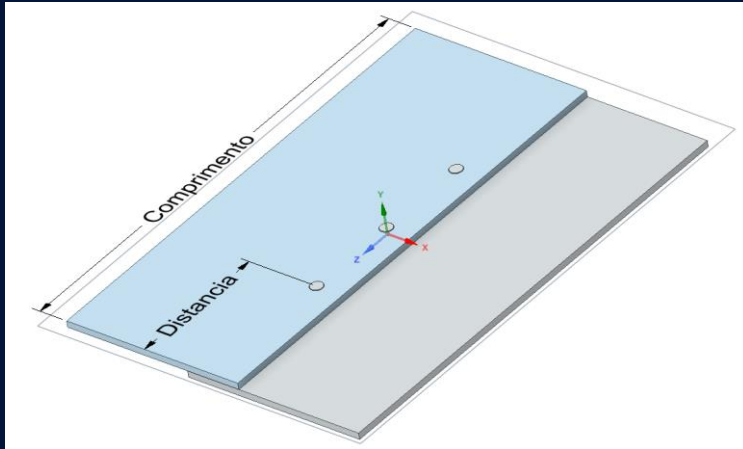
$$p_b = 276 \text{ MPa}$$

$$\sigma_{ult} = 310 \text{ MPa}$$

# Casos de Falha

## Aproximação placa infinita

- Aumento do comprimento e distância do rebite a extremidade da placa
- Aplicação de força limite
- Validação da distância mínima para aproximar a uma placa infinita



Megson, T. H. G. (2021). Aircraft Structures for Engineering Students. In *Aircraft Structures for Engineering Students*. Elsevier.

# Equações de Kirsch

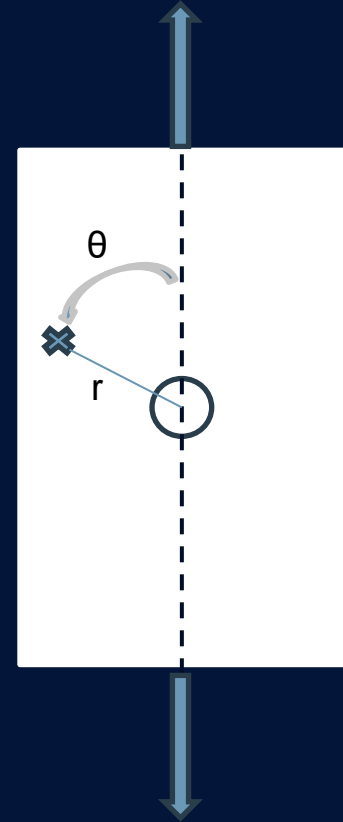


- Capazes de aproximar os fatores de concentração de tensões em zonas furadas.
- Usadas quando temos apenas um único furo circular numa placa infinita.

$$\sigma_{rr} = \frac{\sigma}{2} \left( 1 - \frac{a^2}{r^2} \right) + \frac{\sigma}{2} \left( 1 + 3 \frac{a^4}{r^4} - 4 \frac{a^2}{r^2} \right) \cos(2\theta)$$

$$\sigma_{\theta\theta} = \frac{\sigma}{2} \left( 1 + \frac{a^2}{r^2} \right) - \frac{\sigma}{2} \left( 1 + 3 \frac{a^4}{r^4} \right) \cos(2\theta)$$

$$\sigma_{r\theta} = -\frac{\sigma}{2} \left( 1 - 3 \frac{a^4}{r^4} + 2 \frac{a^2}{r^2} \right) \sin\theta$$



$$\begin{aligned} F &= 1000\text{N} \\ L &= 200\text{mm} \\ t &= 2\text{mm} \end{aligned} \longrightarrow \sigma_i = 2,5 \text{ MPa}$$

$$\sigma_{result} = \sqrt{\sigma_{rr}^2 + \sigma_{\theta\theta}^2 + 2\sigma_{r\theta}^2}$$

<u>P1</u> a = 5mm r = 100mm θ = 0°	{	$\sigma_{rr} = 2,484 \text{ MPa}$ $\sigma_{\theta\theta} = 0,0031 \text{ MPa}$ $\sigma_{r\theta} = 0 \text{ MPa}$	→	$\sigma = 2,484 \text{ MPa}$
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<u>P2</u> a = 5mm r = 50mm θ = 60°	{	$\sigma_{rr} = 0,6373 \text{ MPa}$ $\sigma_{\theta\theta} = 1,8877 \text{ MPa}$ $\sigma_{r\theta} = -1,1039 \text{ MPa}$	→	$\sigma = 2,5311 \text{ MPa}$
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<u>P3</u> a = 5mm r = 20mm θ = 80°	{	$\sigma_{rr} = 0,2771 \text{ MPa}$ $\sigma_{\theta\theta} = 2,5165 \text{ MPa}$ $\sigma_{r\theta} = -1,3705 \text{ MPa}$	→	$\sigma = 3,1884 \text{ MPa}$
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<u>P4</u> a = 5mm r = 20mm θ = 230°	{	$\sigma_{rr} = 1,0218 \text{ MPa}$ $\sigma_{\theta\theta} = 1,5023 \text{ MPa}$ $\sigma_{r\theta} = 1,0085 \text{ MPa}$	→	$\sigma = 3,8937 \text{ MPa}$
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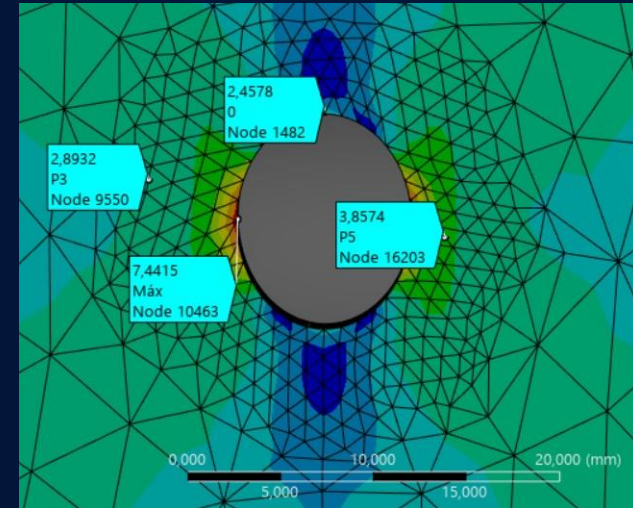
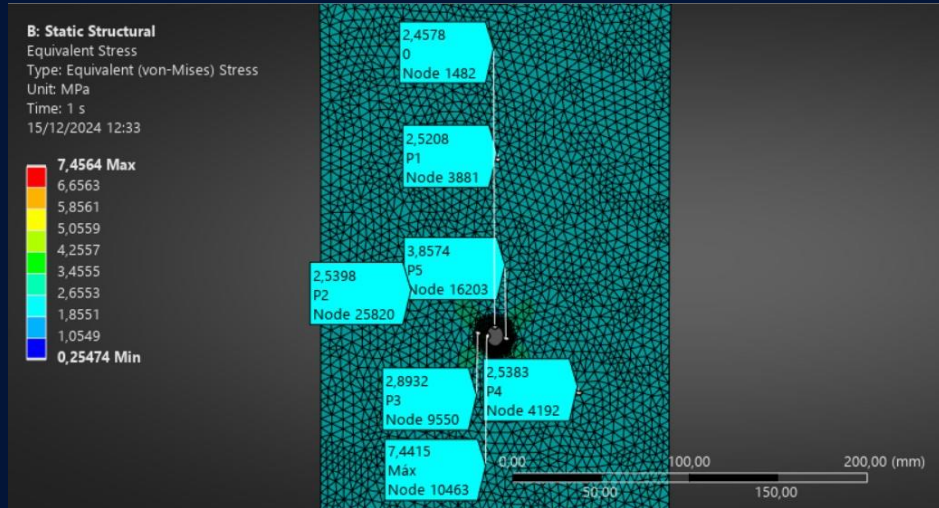
$$\begin{aligned} &\underline{r = a} \\ &\left( \begin{array}{l} \sigma_{rr} = 0 \\ \sigma_{\theta\theta} = \sigma(1 - 2 \cos 2\theta) \\ \sigma_{r\theta} = 0 \end{array} \right) \end{aligned}$$



$$\theta = 0^\circ \longrightarrow \sigma_{\theta\theta} = \sigma_i$$

$$\theta = 90^\circ \longrightarrow \sigma_{\theta\theta} = 3\sigma_i$$

# Análise Computacional - Ansys



	Analítico (MPa)	Ansys (MPa)
P1	2,484	2,5208
P2	2,5311	2,5398
P3	3,1884	2,8932
P4	2,3097	2,5383
P5	3,8937	3,8574

$$\theta = 0^\circ \longrightarrow \sigma_{\theta\theta} = \sigma_i = 2,4578 \text{ MPa}$$

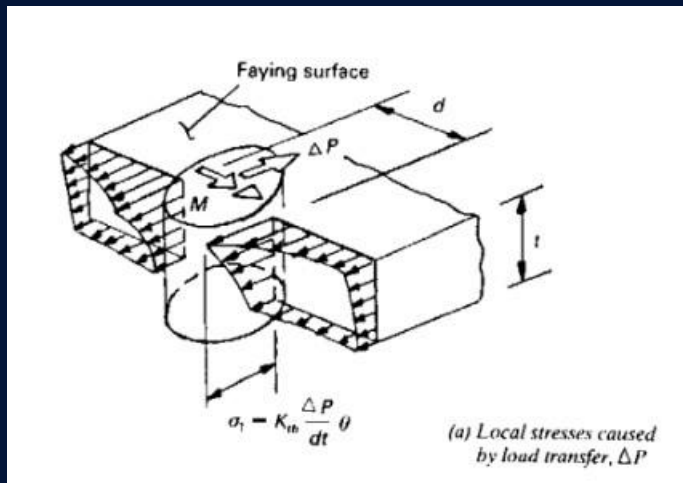
$$\theta = 90^\circ \longrightarrow \sigma_{\theta\theta} = 3\sigma_i = 7,4415 \text{ MPa}$$



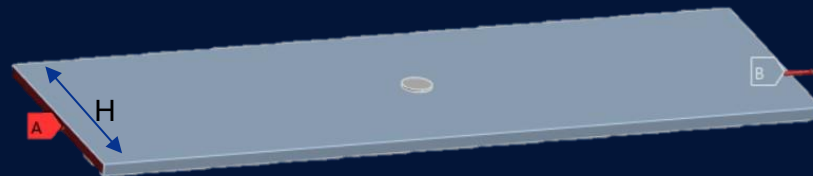
# Tensão num Rebite

$$K_{in} = 0,288 - 8,820 \left( \frac{d}{H} \right) - 23,169 \left( \frac{d}{H} \right)^2 + 29,167 \left( \frac{d}{H} \right)^3$$

$$\sigma_{m\acute{a}x} = K_{in} \cdot \sigma_{nom}$$



GREMEN, Tom A.W. Stress concentrations in riveted plate girders due to fatigue loading: defining a new stress range parameter to accurately describe the life prediction of riveted bridge girders. 2021.



H = 100mm    d = 10mm    t = 5mm    F = 1000N

Método Analítico:     $K_{in} = 0,7965$      $\sigma_{m\acute{a}x} = 1,593 \text{ MPa}$

Método Computacional:



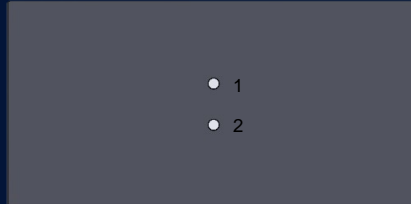
# Efeito da distribuição dos rebites

→ Considerou-se 5 casos

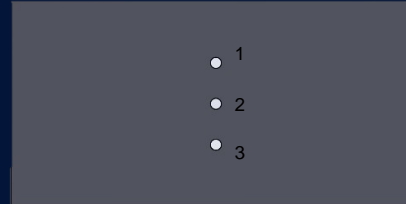
Caso1



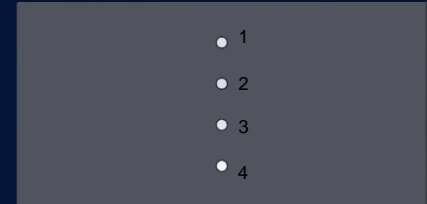
Caso2



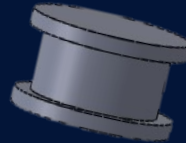
Caso3



Caso4



Caso5



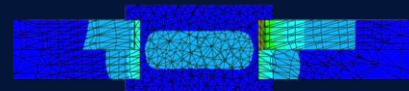
- 2 placas iguais: 300x150 mm
- Diâmetro dos furos: 8mm
- Distância entre furos: 15mm
- Fixação nas faces do rebite
- Força aplicada: 2000N

# Resultados



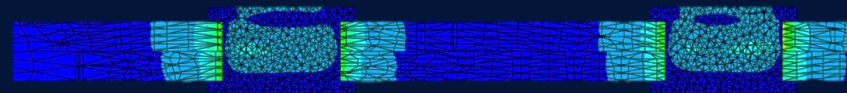
## Caso1

Tensão Máx (MPa)	242,15
Rebite T.Máx (MPa)	130,15
Rebite D.Máx (mm)	0,0036196



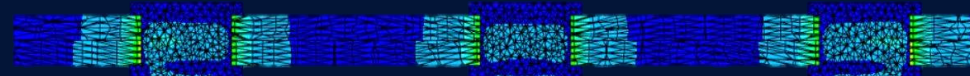
## Caso2

Tensão Máx (MPa)	104,3	
Rebite T.Máx (MPa)	(1) 68,362	(2) 66,336
Rebite D.Máx (mm)	(1) 0,0018511	(2) 0,0018511



## Caso3

Tensão Máx (MPa)	70,335		
Rebite T.Máx (MPa)	(1) 41,892	(2) 37,192	(3) 43,349
Rebite D.Máx (mm)	(1) 0,0012465	(2) 0,0011354	(3) 0,0012640



## Caso4

Tensão Máx (MPa)	48,38			
Rebite T.Máx (MPa)	(1) 30,769	(2) 28,032	(3) 29,489	(4) 29,639
Rebite D.Máx (mm)	(1) 0,00099413	(2) 0,00088067	(3) 0,00092543	(4) 0,00097243



## Caso5

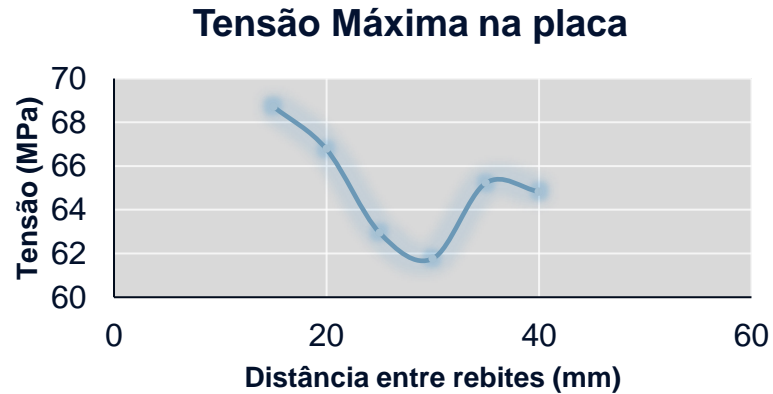
Tensão Máx (MPa)	83,485			
Rebite T.Máx (MPa)	(1) 41,094	(2) 27,889	(3) 27,133	(4) 42,365
Rebite D.Máx (mm)	(1) 0,001355	(2) 0,00091267	(3) 0,00089161	(4) 0,0013499



# Efeito do espaçamento entre rebites

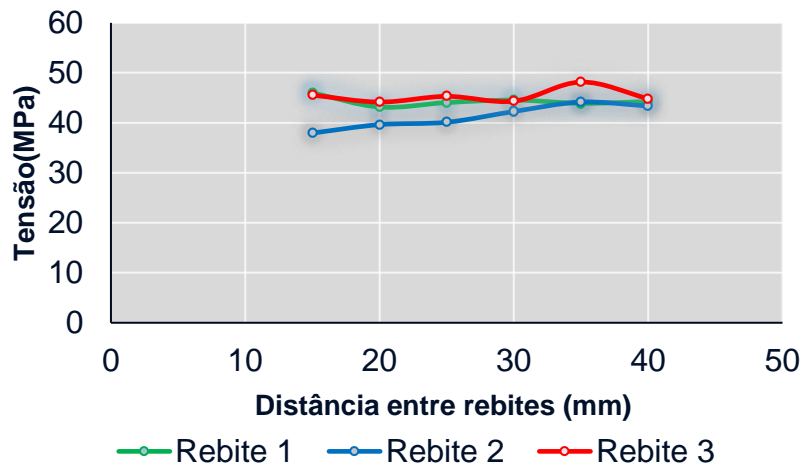
- 1
- 2
- 3

- 2 placas iguais: 300x150 mm
- Diâmetro dos furos: 8mm
- Fixação nas faces do rebite
- Força aplicada: 2000N

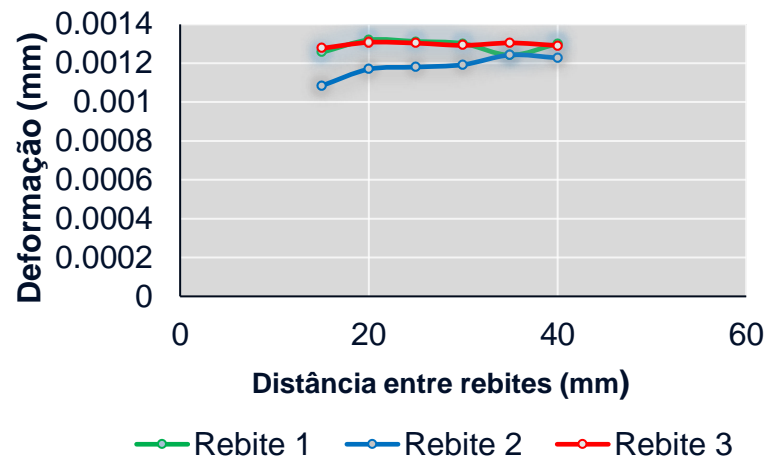




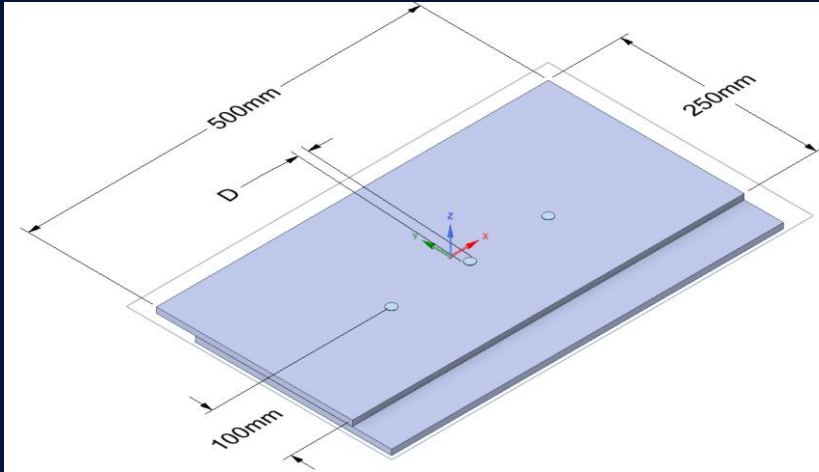
### Tensão Máxima no rebite



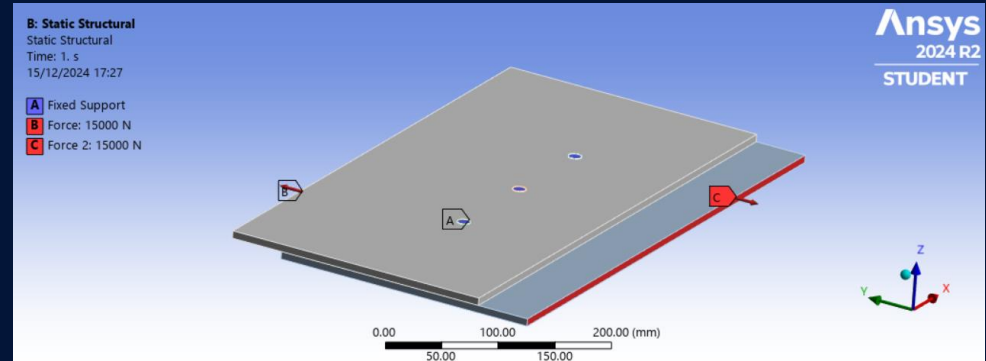
### Deformação Máxima no rebite



# Efeito do Diâmetro na concentração de tensões



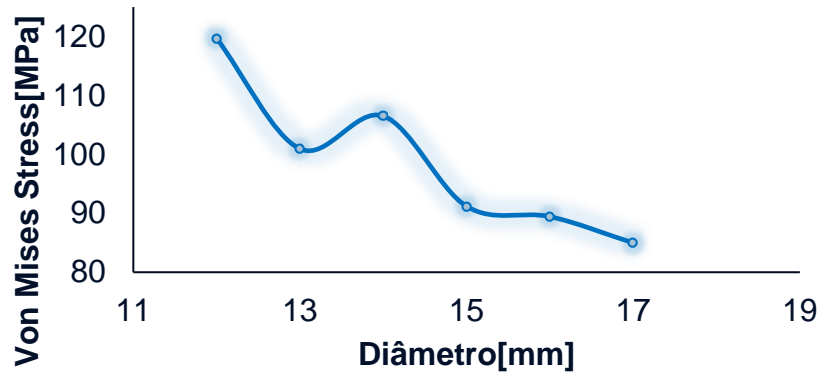
- 2 placas iguais: 500x250 mm
- Diâmetro dos furos:  $12 < D < 18$  mm
- Distância entre furos: 100mm
- Força aplicada: 15000N



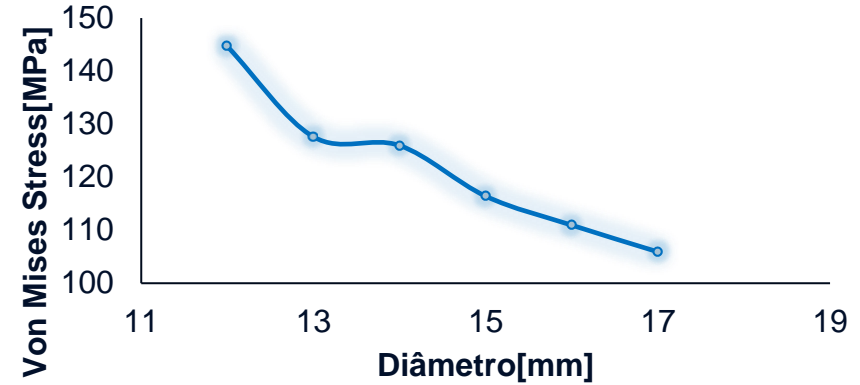
# Efeito do Diâmetro na concentração de tensões



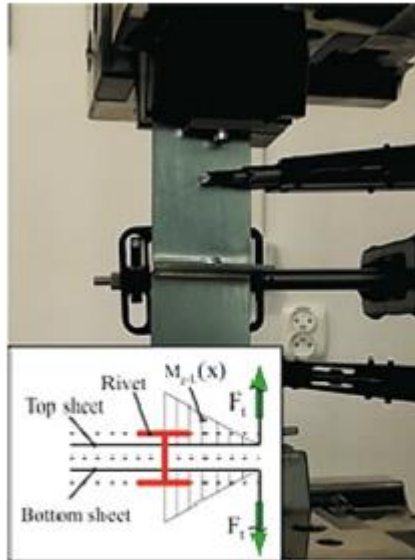
## Placas



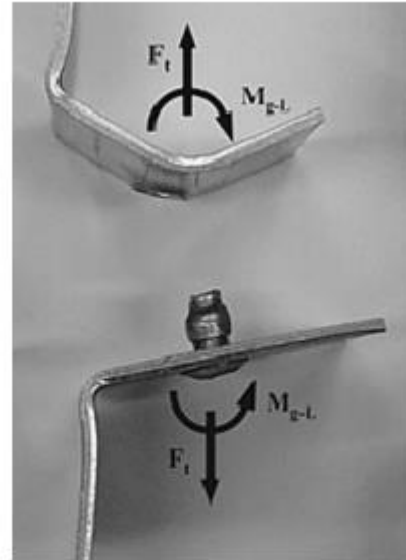
## Rebites



# Outro teste realizado



a

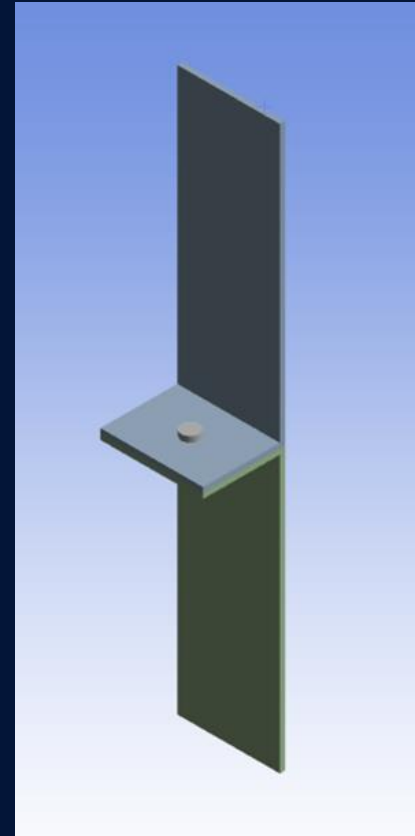
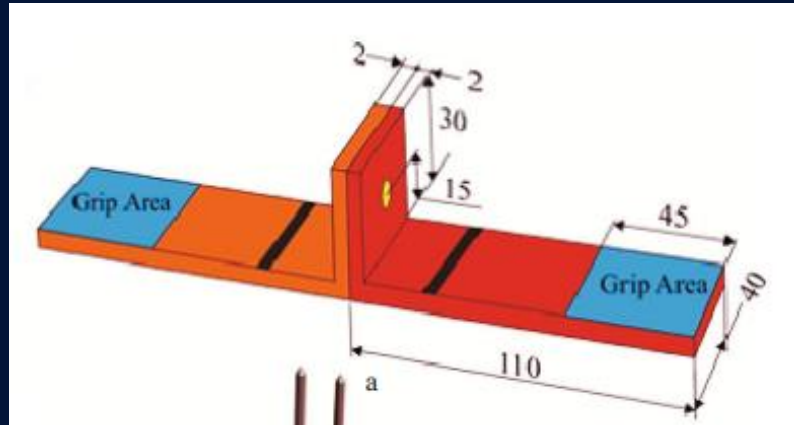


b

Fig. 4. The tensile test of T-specimens and the loading conditions: (a) T-specimen in the holder; (b) joint after tensile test.



# Geometria

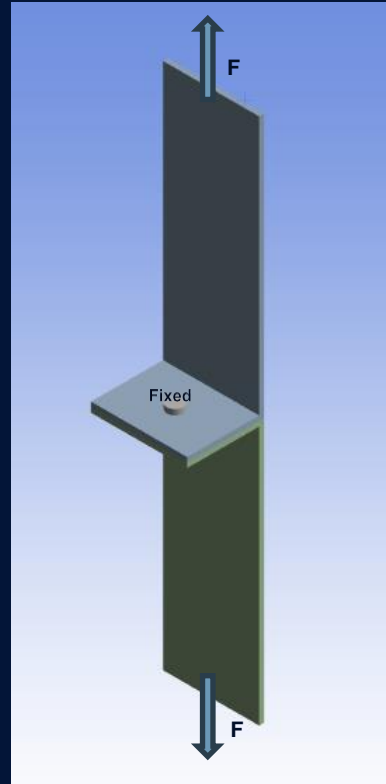


# Condições Fronteira

## 1º Caso

Caso usado no estudo

Rebite fixo  
F aplicado em cada lado  
com sentidos diferentes

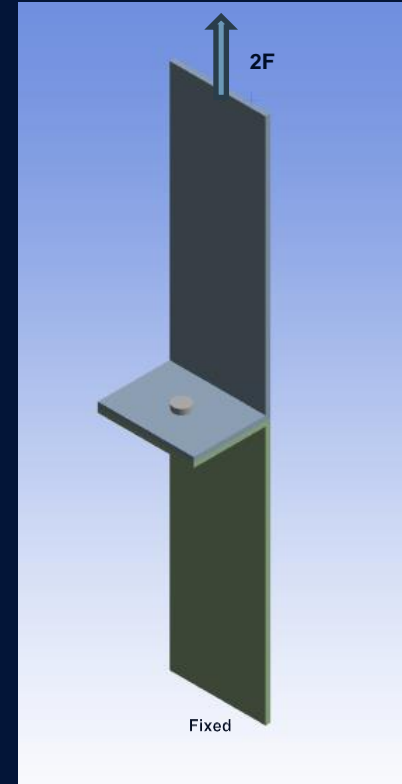


## 2º Caso

Caso usado na simulação

Rebite livre  
Face debaixo fixa  
2F aplicado na face superior

Foi escolhido por ser  
mais estável e porque  
energeticamente e  
mecanicamente é similar  
ao outro caso



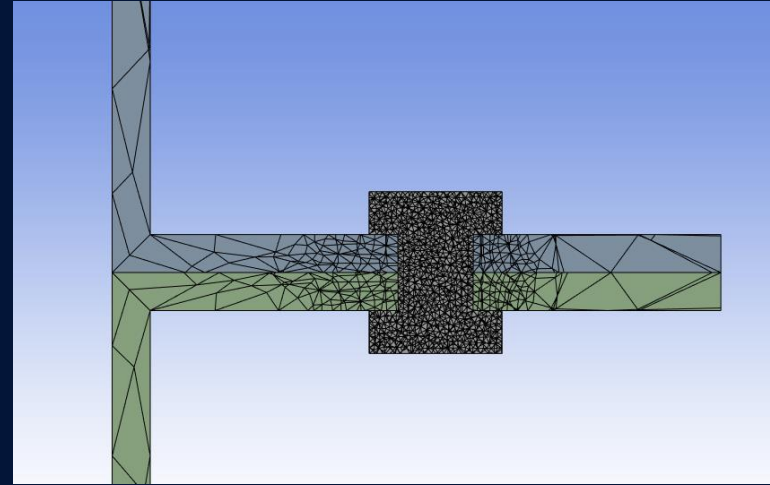
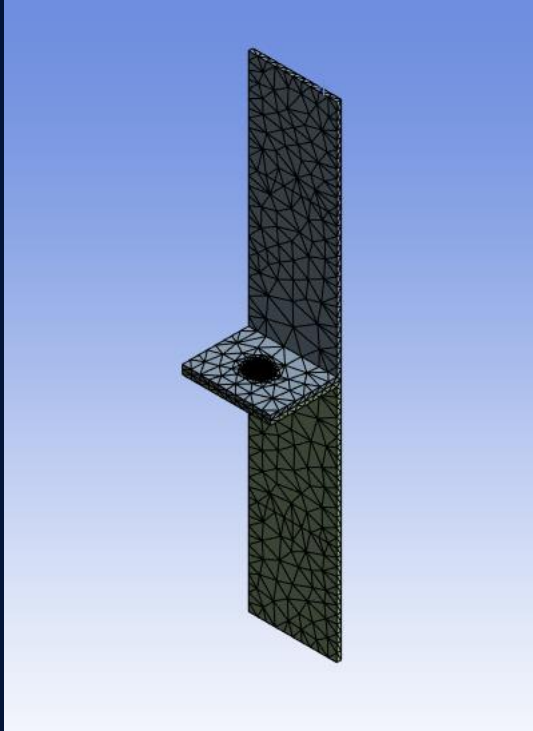
# Análise de convergência de malha

Resposta para uma força de 1000N

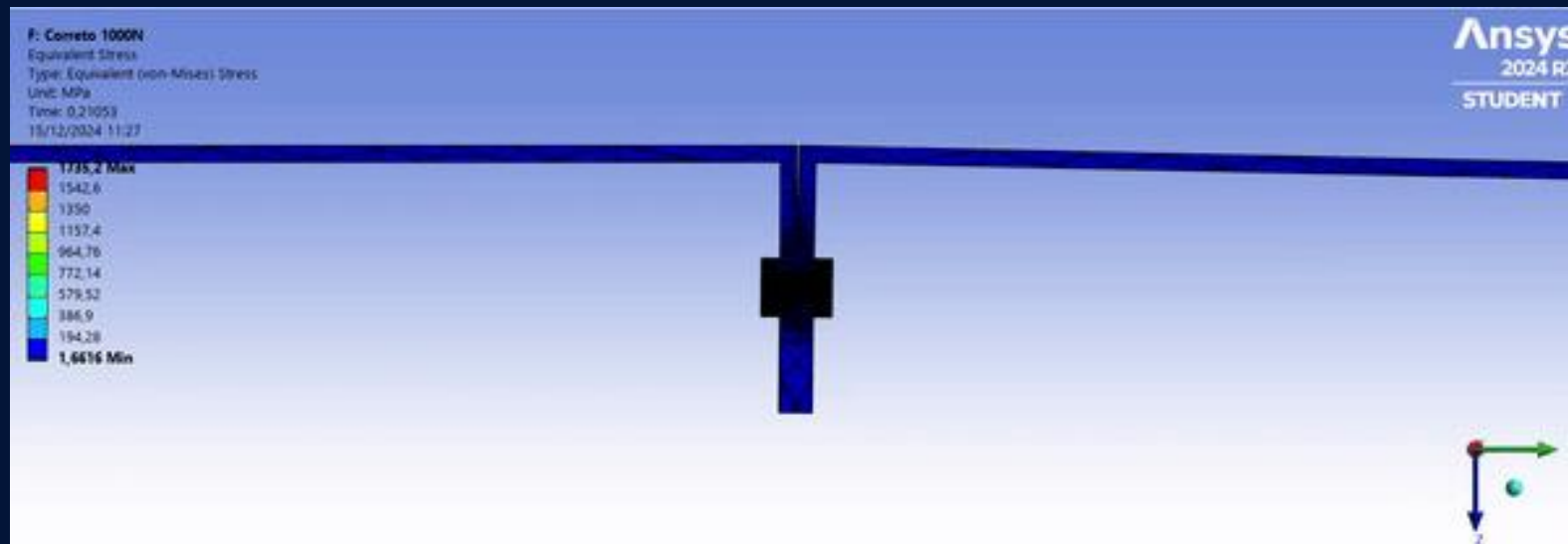
	1º Caso	2ºCaso	3ºCaso
Nº de nós	36969	83796	122422
Nº de elementos	23075	55965	83176
Tensão Máx de Von Mises	1580MPa	1735MPa	1800MPa

Foi escolhida a malha do caso 2, por ter um resultado similar (diferença de 3%) e ser mais eficiente

# Malha Final



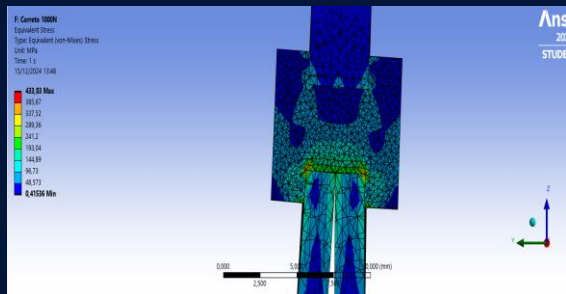
# Exemplo de simulação



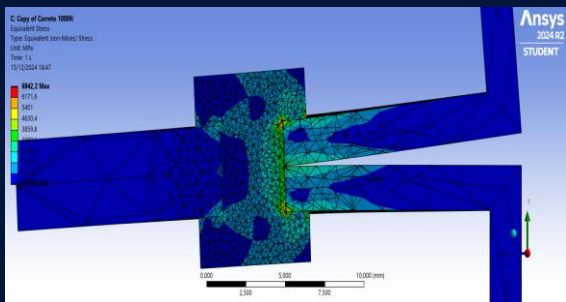
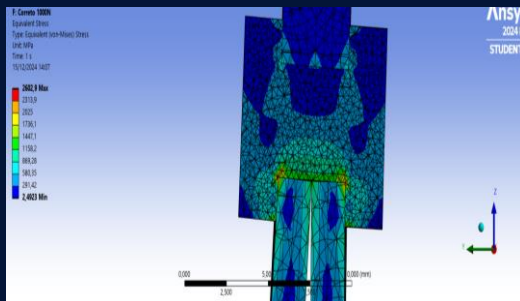
# Resultados - Stress



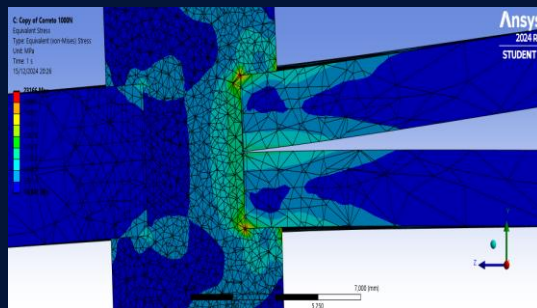
2F=250N



2F=1500N



2F=4000N



2F=12000N

2F (N)	$\sigma_{max}$ (Mpa)
250	433
500	867
1000	1735
1500	2602
2000	3470
4000	6940
8000	14500
12000	23166

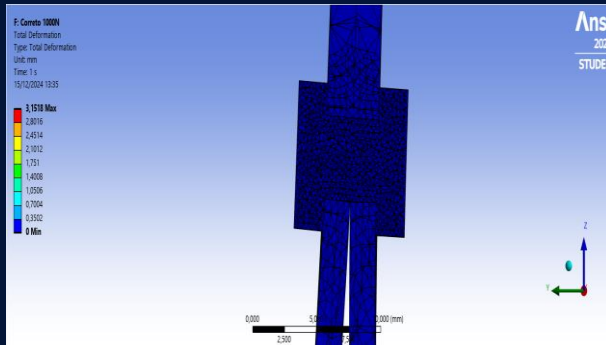
Aço estrutural

$$\sigma_y = 250MPa$$

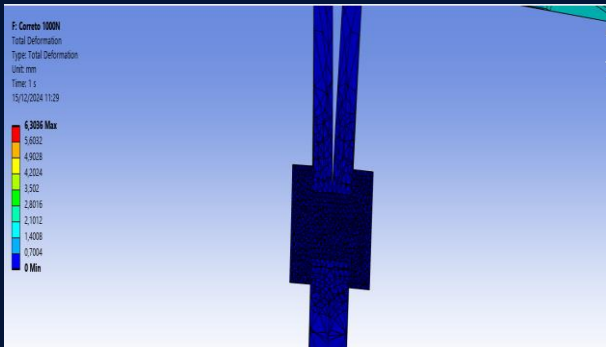


# Resultados - Displacement

$2F=500N$



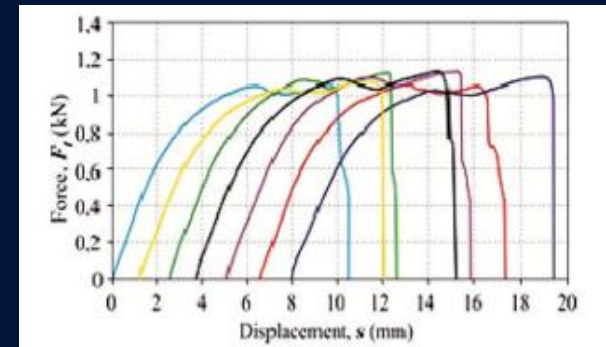
Apenas alguns  
exemplos



$2F=1000N$

## Observações

Em nenhuma das forças apresentadas  
o sistema rompeu  
Muito contrário ao que é encontrado na  
literatura para esta geometria

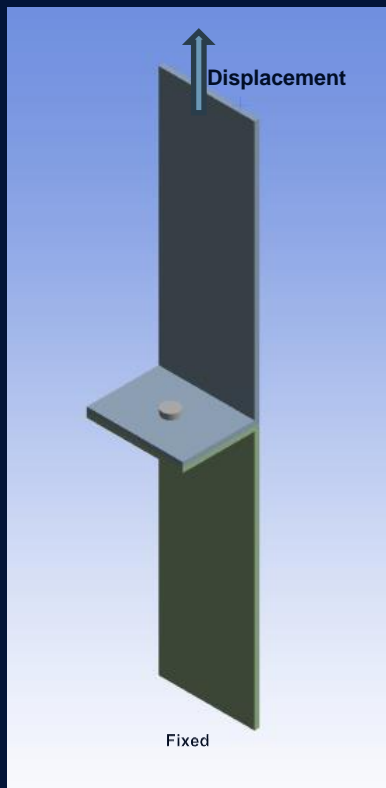


Mucha, J., & Witkowski, W. (2015). Mechanical behavior and failure of riveting joints in tensile and shear tests. *Strength of Materials*, 47(5), 1–9. <https://doi.org/10.1007/s11223-015-9712-5>

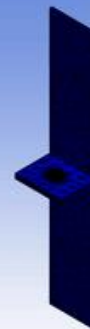
# Ensaio de tração

Similar ao anterior

Aplicado displacement,  
invés

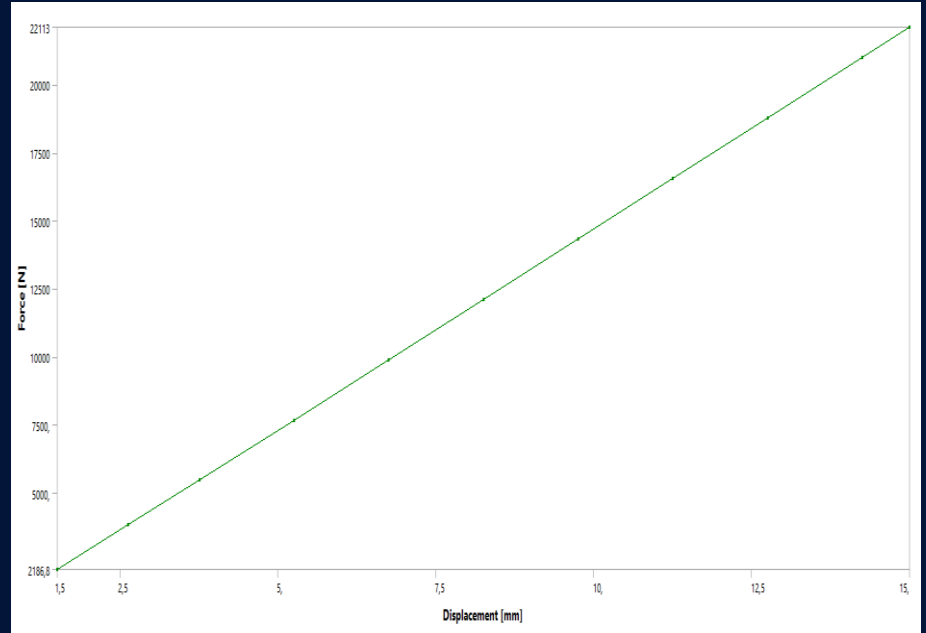
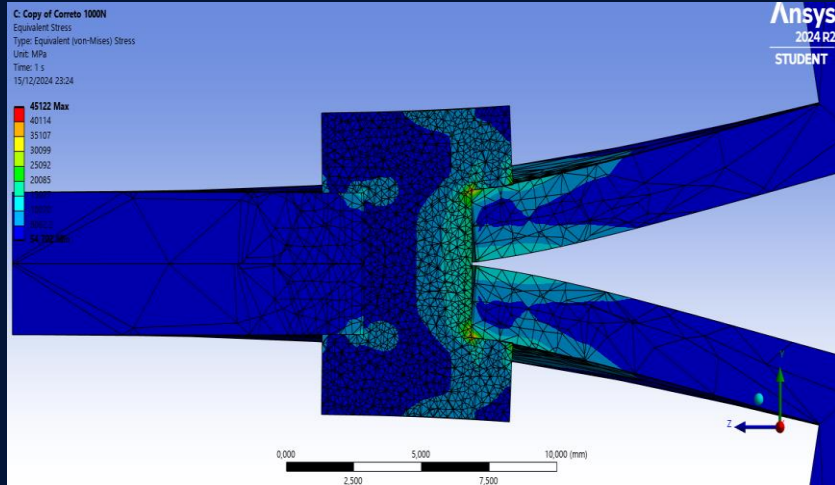


C: Copy of Correo 100001  
Equivalent Stress  
Type: Equivalent (von Mises) Stress  
Unit: MPa  
Time: 1.0101e-002  
15/12/2024 20:48  
20562 Max  
16277  
15993  
13709  
11424  
9140.2  
6855.9  
4571.6  
2287.9  
3,0468 Min

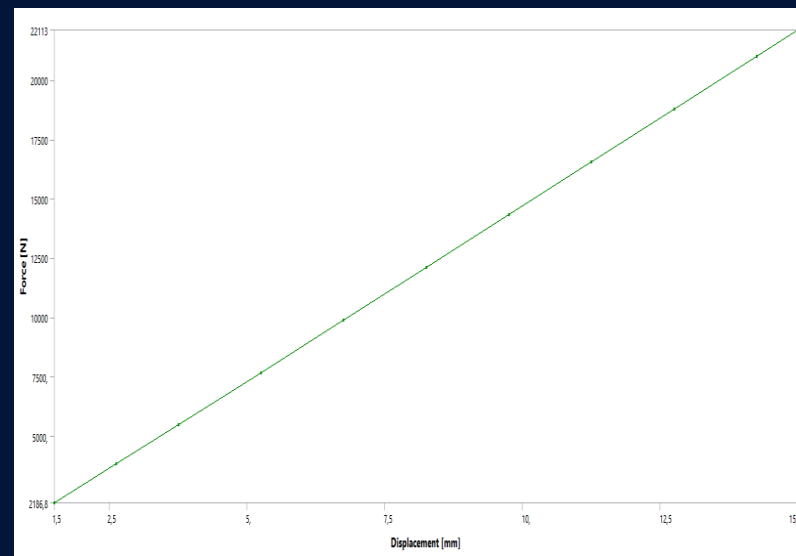
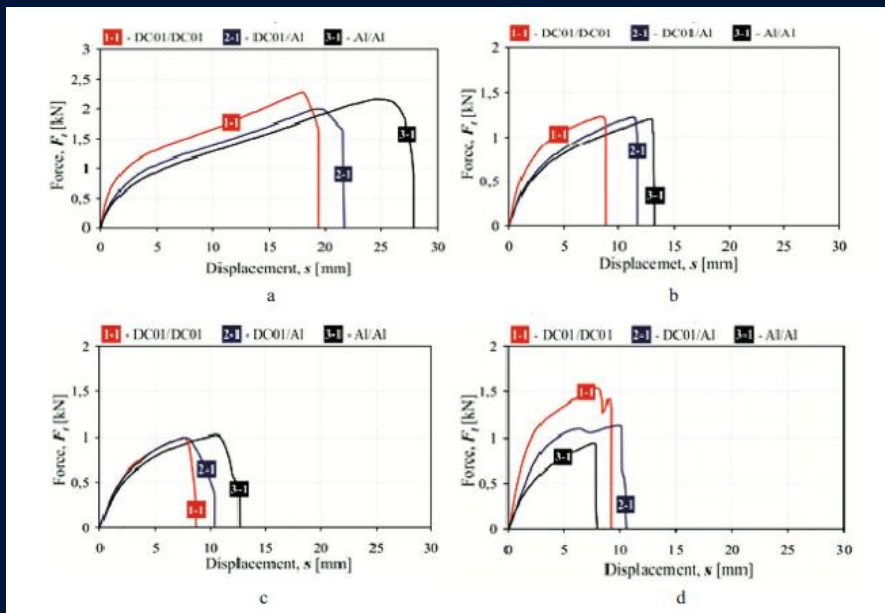




# Ensaio de tração



# Comparação de Resultados



Mucha, J., & Witkowski, W. (2015). Mechanical behavior and failure of riveting joints in tensile and shear tests. Strength of Materials, 47(5), 1–9. <https://doi.org/10.1007/s11223-015-9712-5>

Impossível ir depois dos 16mm de displacement, o computador não tinha RAM e apareciam erros

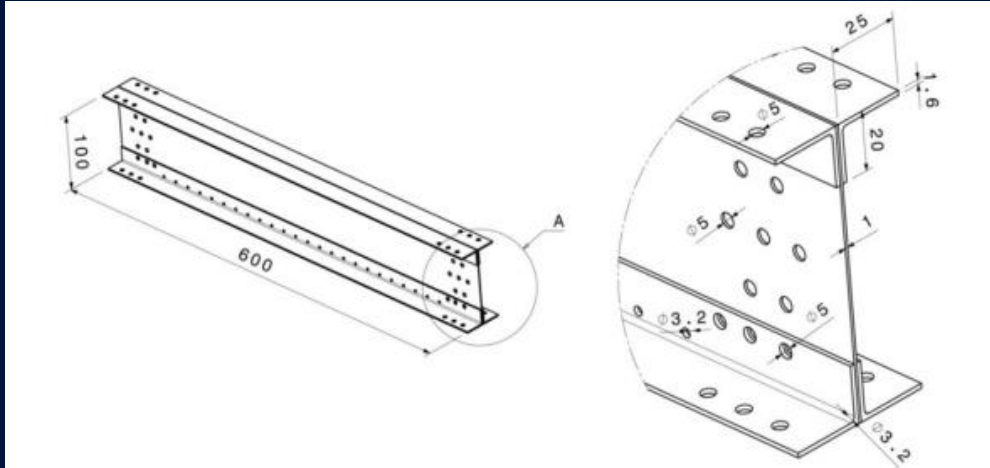
# Observações



- Em nenhum dos casos o sistema rompeu
- Apesar de realizar um bom trabalho os resultados obtidos não foram correspondentes com os obtidos na literatura
- Uma das razões para isso ter acontecido, foi do material utilizado não ter sido o mesmo que no teste experimental
- A geometria dos rebites usadas no teste experimental não totalmente iguais às usadas na simulação no Ansys porque estas não foram encontradas



# Longarina de uma asa

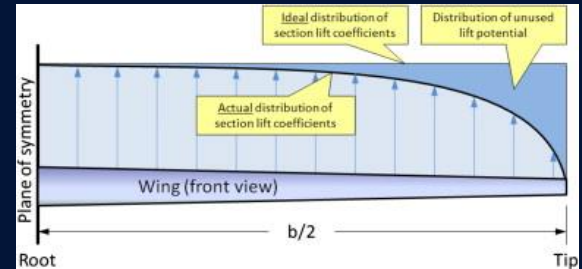


Dimensões utilizadas sem furos (light aircraft / UAV)

[doi.org/10.1016/j.ijfatigue.2019.02.013](https://doi.org/10.1016/j.ijfatigue.2019.02.013)

No nosso caso:

- Viga em I com alma ligada aos banzos por uma fila de rebites ( $d=5\text{mm}$ )
- Sem *Fillets* visto não haver concentração de tensões nestas áreas aquando da simulação

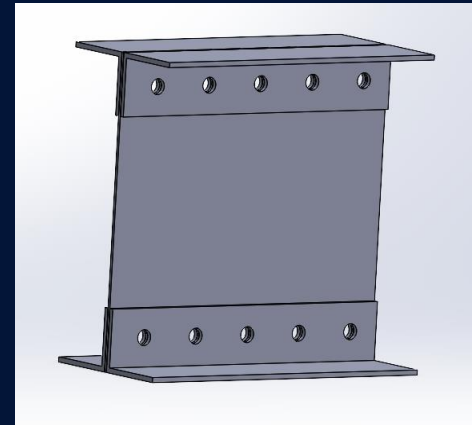
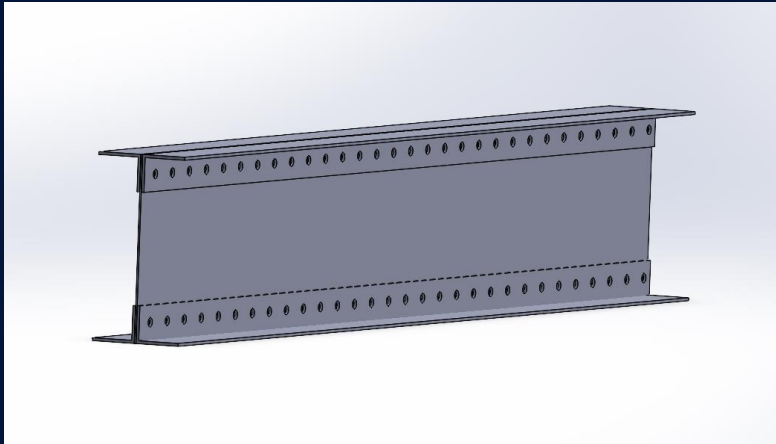


# Longarina de uma asa

Simulação de todo o conjunto não foi possível por limite de elementos (depois de aplicado plano de simetria)

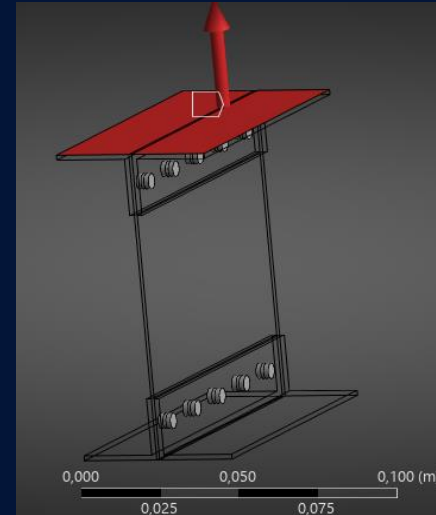
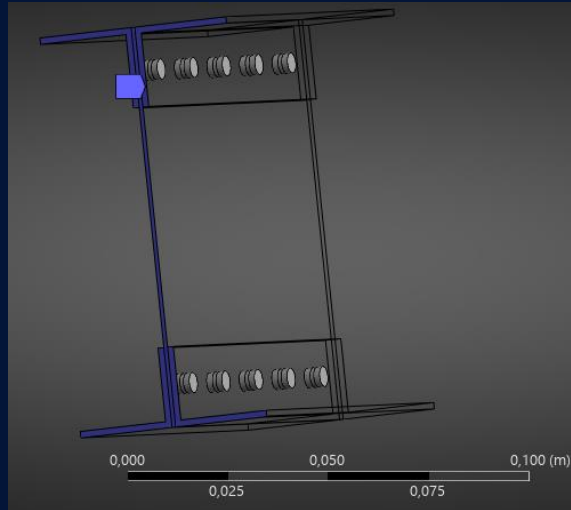


Simulação da secção inicial com 10 rebites



# Definição do problema

- Material: Aluminum Alloy (Ansys)
- Viga encastrada num lado
- Força de 3000 N nas faces superiores e inferiores da viga
- Contactos entre sólidos do tipo frictionless
- Elementos viga nos furos para simular os rebites



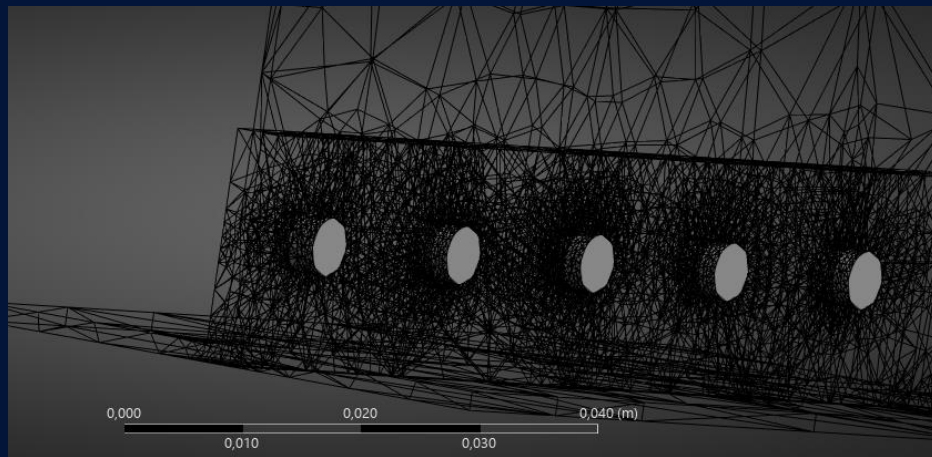
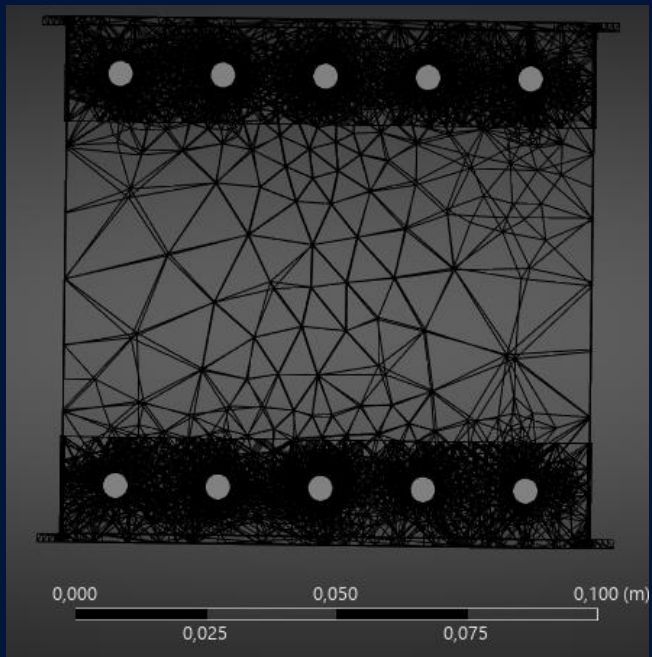
# Análise de Convergência de malha

	1º Caso	2ºCaso	3ºCaso	4ºcaso
Nº de nós	25433	56903	102060	110360
Nº de elementos	12276	30366	56187	61360
Tensão Máx de Von Mises	0,91 GPa	1,02 GPa	1,616 GPa	1,617 GPa

Selecionada a malha do 3ºcaso

# Malha

Nível de refinement máximo (3) à volta dos rebites





# Resultados da simulação

## C: Static Structural

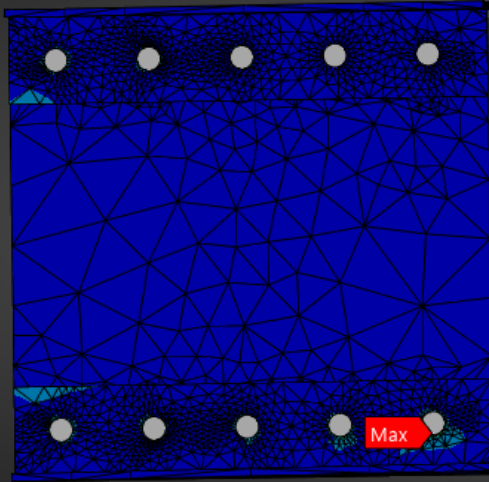
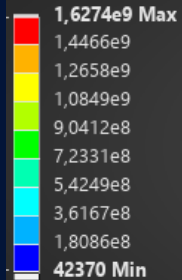
Equivalent Stress

Type: Equivalent (von-Mises) Stress

Unit: Pa

Time: 1 s

16/12/2024 00:16



## C: Static Structural

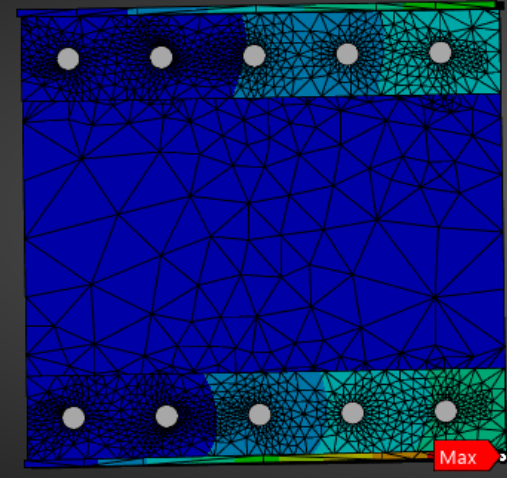
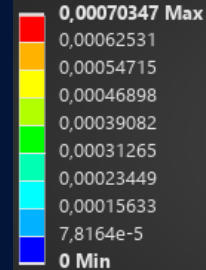
Total Deformation

Type: Total Deformation

Unit: m

Time: 1 s

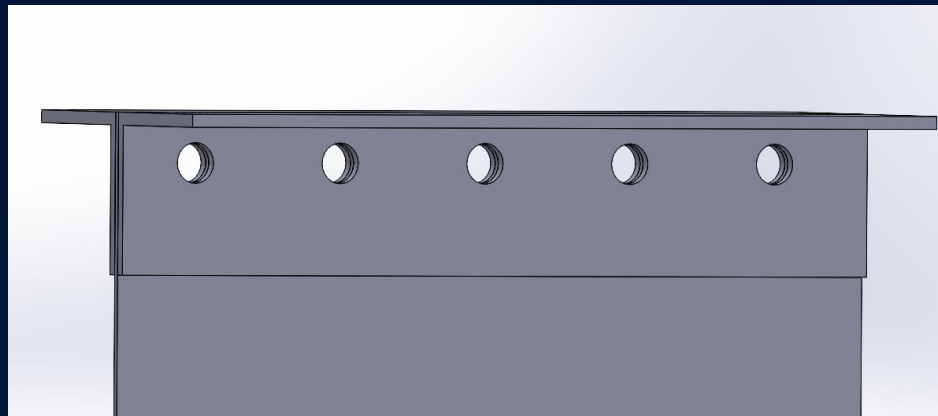
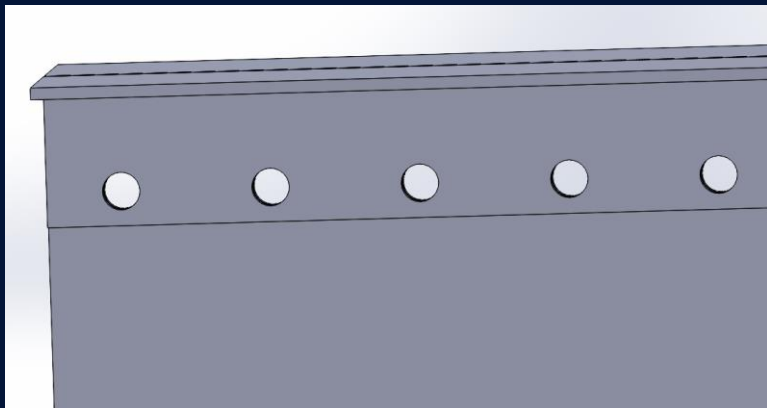
16/12/2024 00:18



# Variação da posição dos rebites

Diâmetro, espaçamento e posição longitudinal constante

Variação da posição transversal

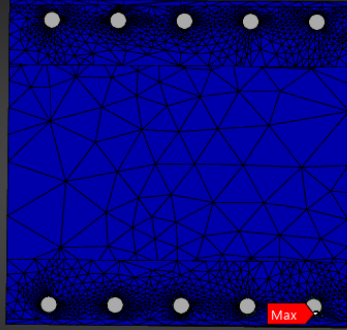


# Variação da posição dos rebites

## E: Static Structural

Equivalent Stress  
Type: Equivalent (von-Mises) Stress  
Unit: Pa  
Time: 1 s  
15/12/2024 23:48

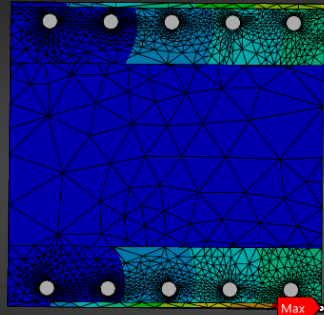
2,2768e9 Max  
2,0238e9  
1,7708e9  
1,5179e9  
1,2649e9  
1,0119e9  
7,5895e8  
5,0597e8  
2,53e8  
32423 Min



## E: Static Structural

Total Deformation  
Type: Total Deformation  
Unit: m  
Time: 1 s  
15/12/2024 23:48

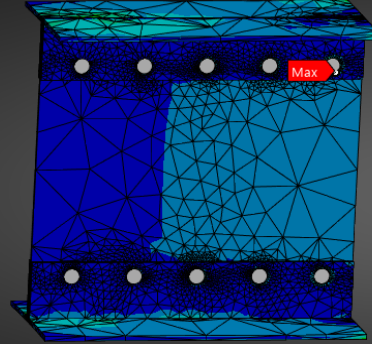
0,00055317 Max  
0,00049171  
0,00043024  
0,00036878  
0,00030732  
0,00024585  
0,00018439  
0,00012293  
6,1463e-5  
0 Min



## G: Copy of Static Structural

Equivalent Stress  
Type: Equivalent (von-Mises) Stress  
Unit: Pa  
Time: 1 s  
16/12/2024 08:39

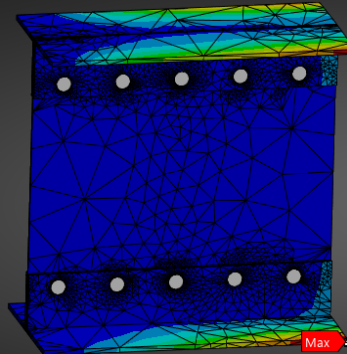
2,0815e8 Max  
1,8503e8  
1,6191e8  
1,3879e8  
1,1566e8  
9,2545e7  
6,9424e7  
4,6304e7  
2,3184e7  
63391 Min



## G: Copy of Static Structural

Total Deformation  
Type: Total Deformation  
Unit: m  
Time: 1 s  
16/12/2024 08:41

0,00047404 Max  
0,00042137  
0,0003687  
0,00031603  
0,00026336  
0,00021068  
0,00015801  
0,00010534  
5,2671e-5  
0 Min



# Comparação

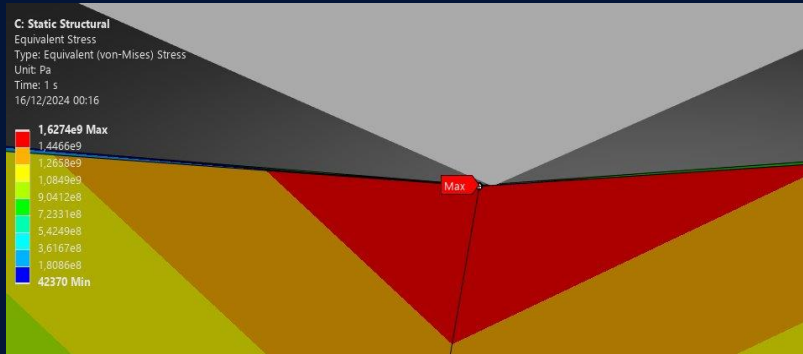
	Cima	Centro	Baixo
Deslocamento máximo	0,55 mm	0,70 mm	0,47 mm
Tensão Máx de Von Mises	2,28 GPa	1,63 GPa	0,21 GPa

**Rebites colocados mais próximos do centro da viga demonstraram menores tensões máximas**

**É o melhor local?**

# Observações/Pontos de melhoria

Elementos viga como rebites



Contemplar o esforço provocado pelo restante da spar

Outras variações nos rebites

Frequências naturais

Tabular Data		
	Mode	<input checked="" type="checkbox"/> Frequency [Hz]
1	1.	549.94
2	2.	930.57
3	3.	1435.6
4	4.	1437.4
5	5.	1549.2
6	6.	1783.
7	7.	2131.8
8	8.	2270.8
9	9.	2403.7
10	10.	2727.6
11	11.	2929.9

# Conclusão





# Obrigado!

Questões?





# Additional Bibliography

Mucha, J., & Witkowski, W. (2015). Mechanical behavior and failure of riveting joints in tensile and shear tests. *Strength of Materials*, 47(5), 1–9. <https://doi.org/10.1007/s11223-015-9712-5>

GREMEN, Tom A.W. Stress concentrations in riveted plate girders due to fatigue loading: defining a new stress range parameter to accurately describe the life prediction of riveted bridge girders. 2021.

Aleksandar Grbović, Gordana Kastratović, Aleksandar Sedmak, Igor Balać, Mihajlo D. Popović, Fatigue crack paths in light aircraft wing spars, *International Journal of Fatigue*, Volume 123, 2019, Pages 96-104, ISSN 0142-1123, [doi.org/10.1016/j.ijfatigue.2019.02.013](https://doi.org/10.1016/j.ijfatigue.2019.02.013).

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