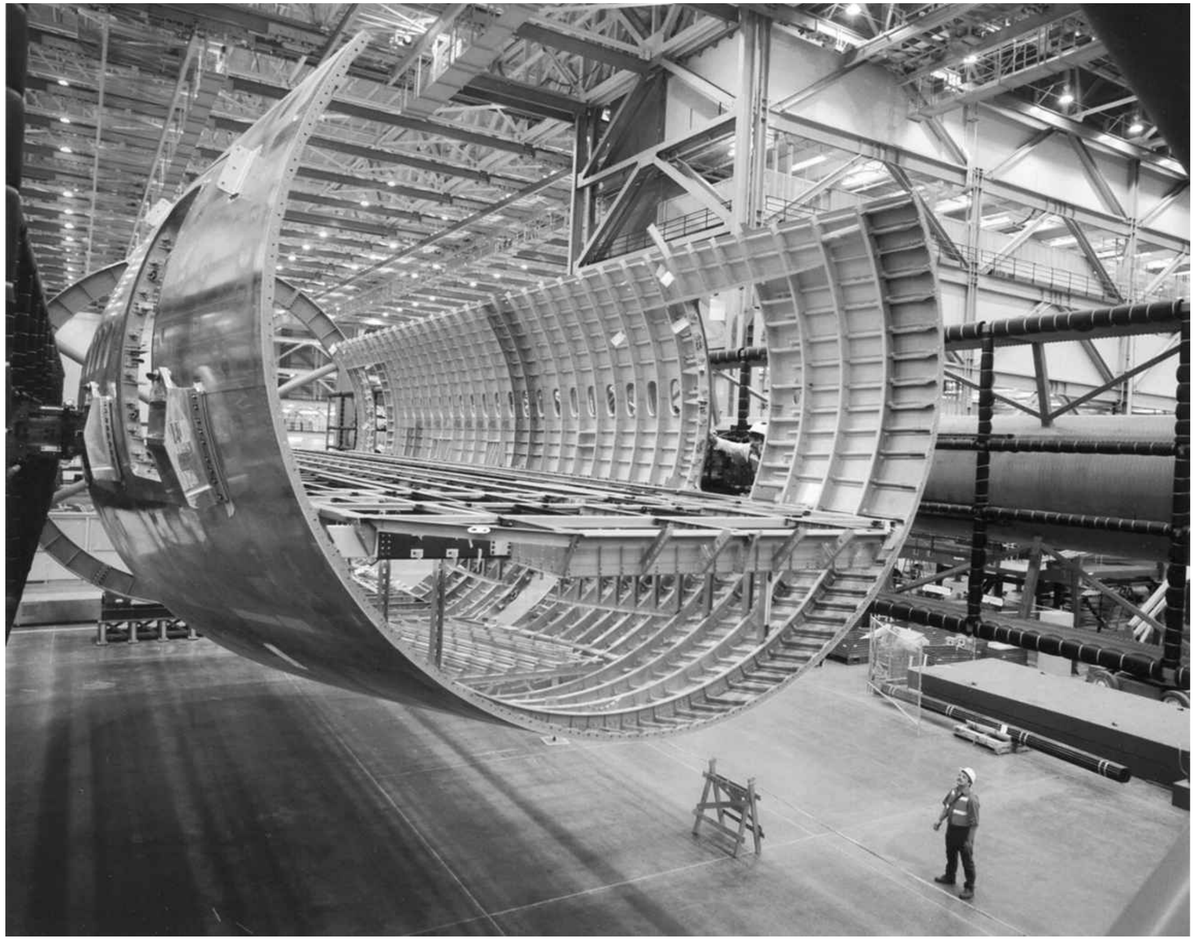
Aircraft Structural Analysis

Master Course in Aerospace Engineering

Session #01 — 08th October, 2024

Motivation



Fuselage of a Boeing 777 under construction.

Source: Courtesy of the Boeing Company.

Motivation

Reality:
continuous system
subjected to continuous
phenomena

STRUCTURAL IDEALIZATION

Idealization:
discrete system

Simplicity is the ultimate sophistication.

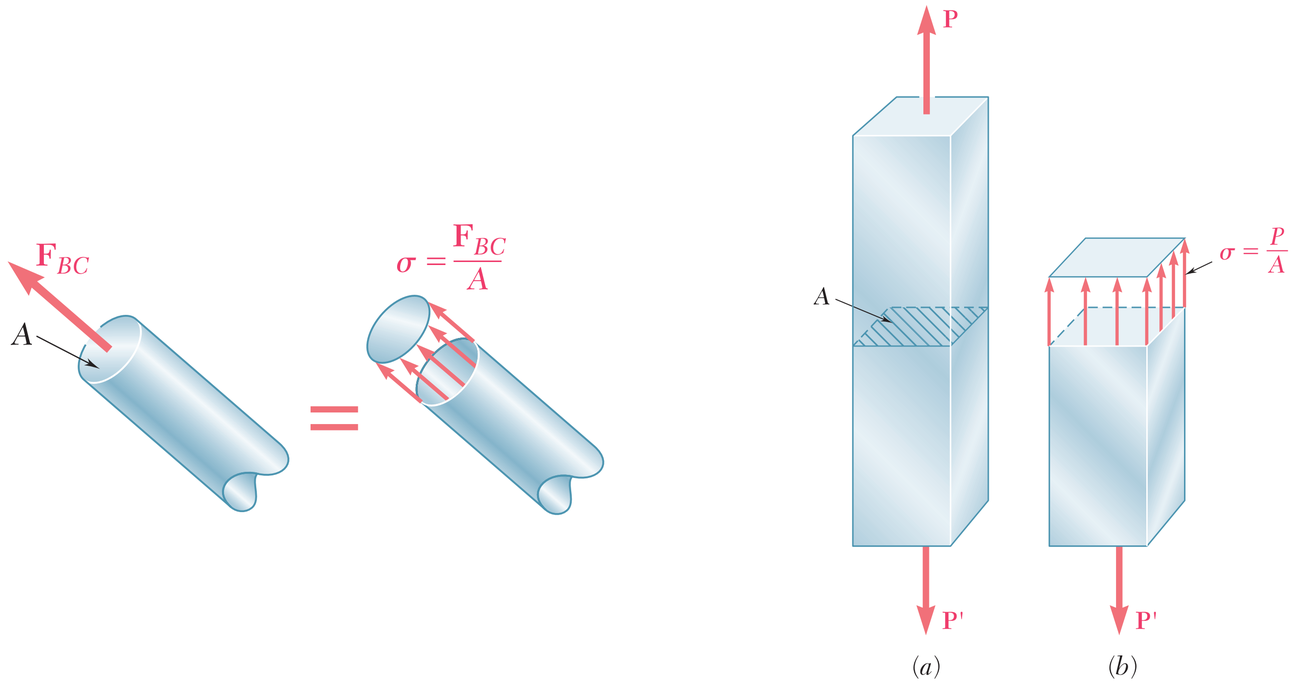
Leonardo da Vinci

Methodology (starting point)

- Mechanics of Materials
- Structural behaviour under simple and deterministic loads
- Elastic behaviour as a starting point for more complex situations
- Modelling and numerical tools as a complementary and powerful resource
- (…)

Mechanics of Materials

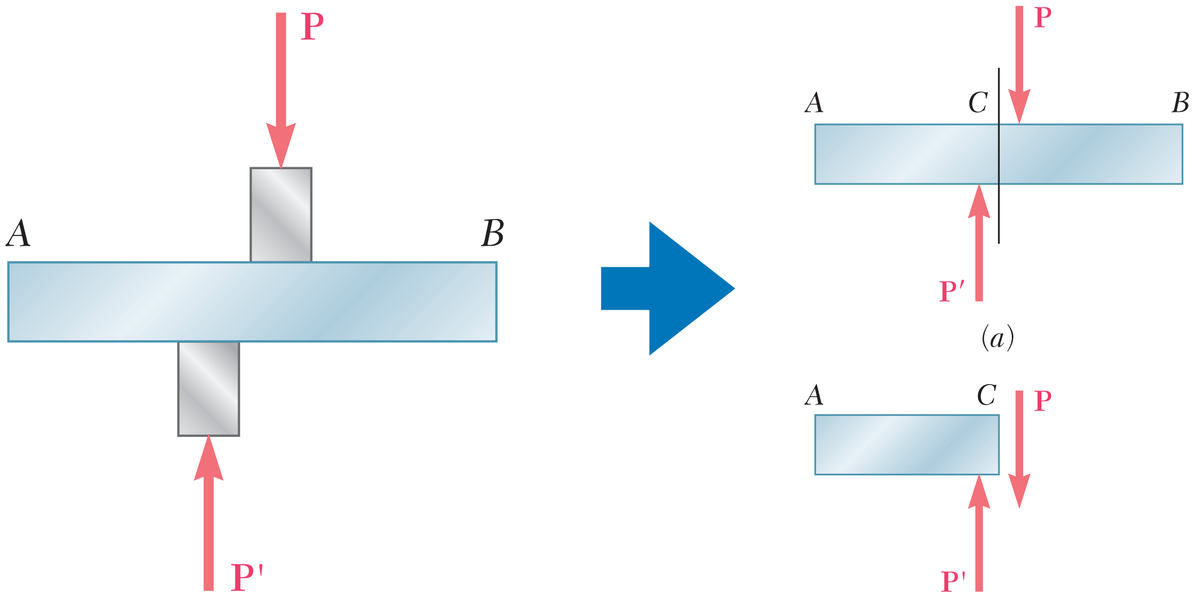
Initial steps: normal stresses



$$\sigma = \frac{P}{A}$$

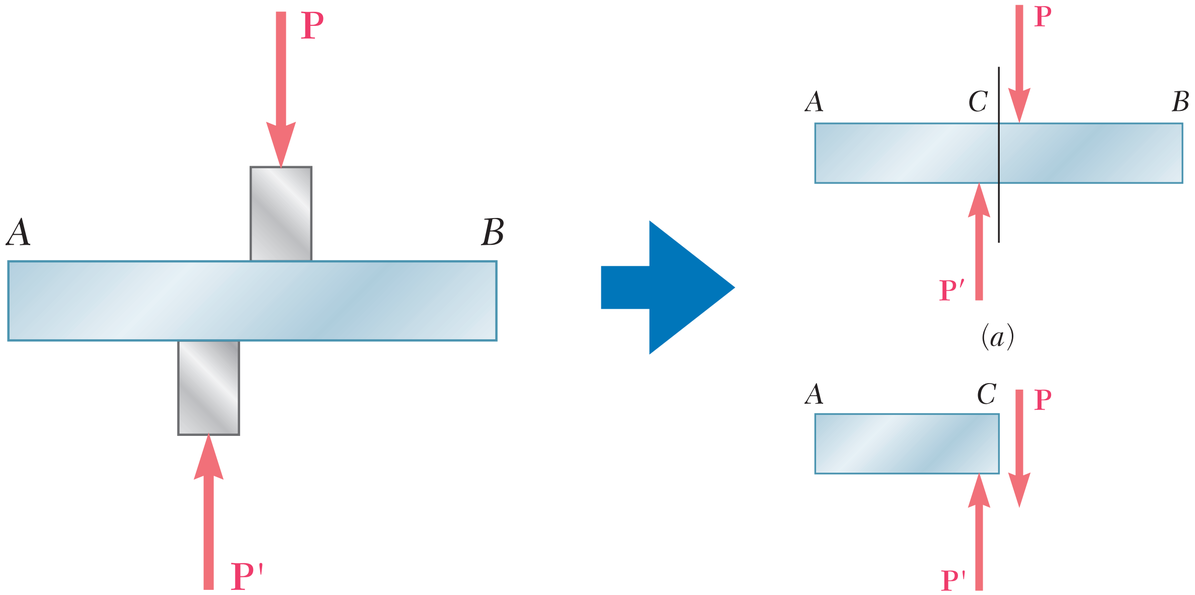
Mechanics of Materials

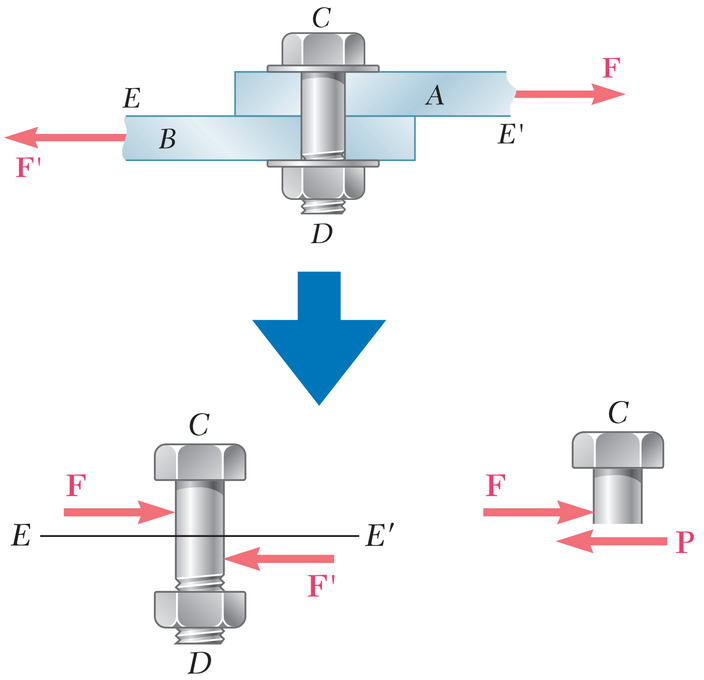
Initial steps: shear stresses



Mechanics of Materials

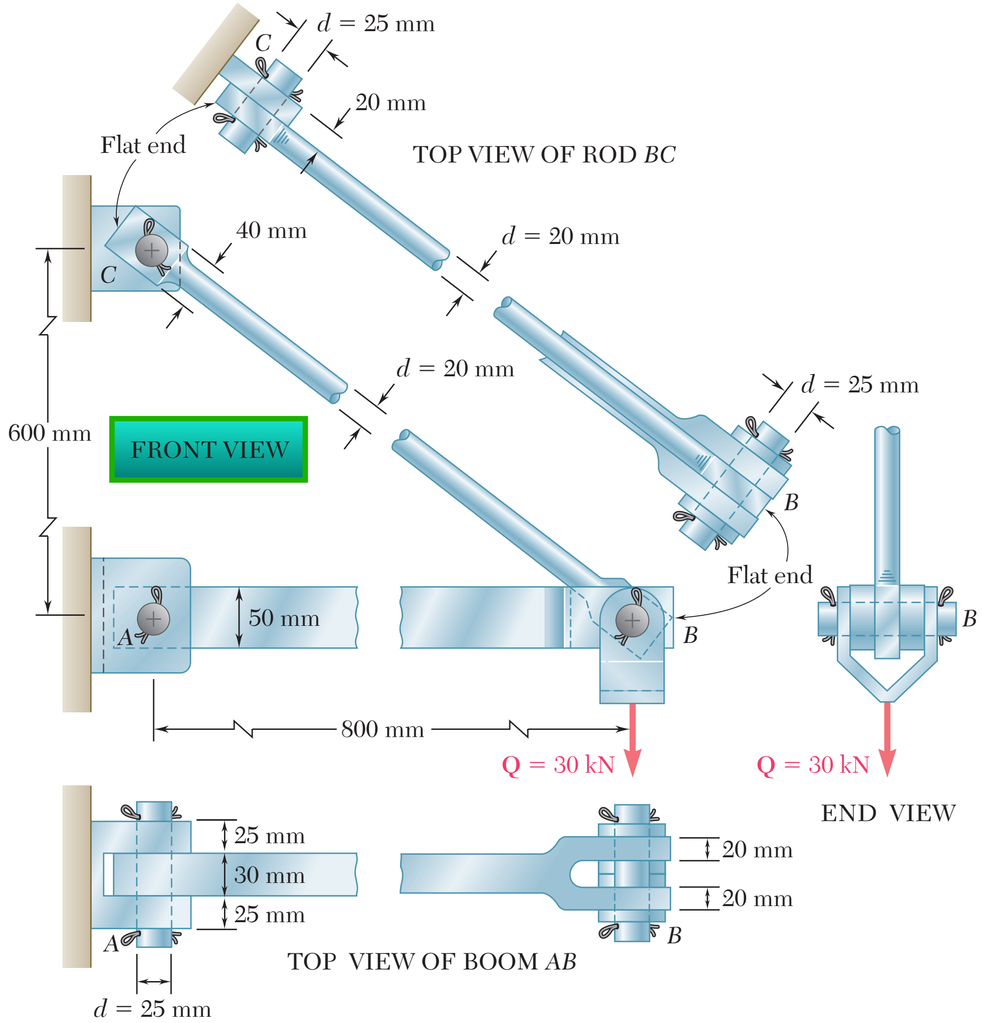
Initial steps: shear stresses



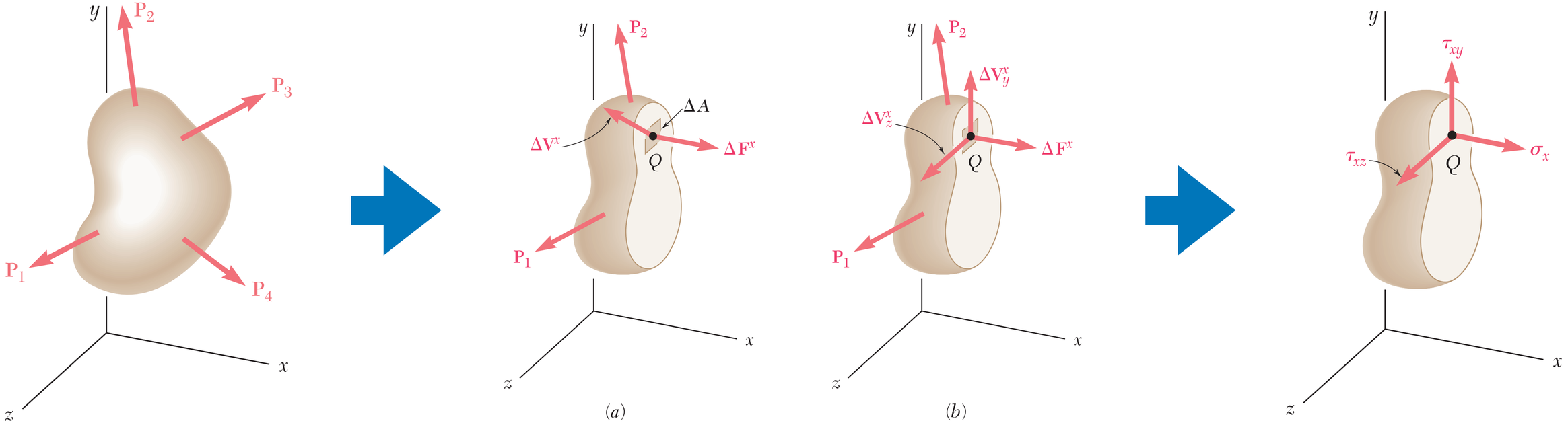


Mechanics of Materials

Initial steps: normal and shear stresses, macroscopic analysis



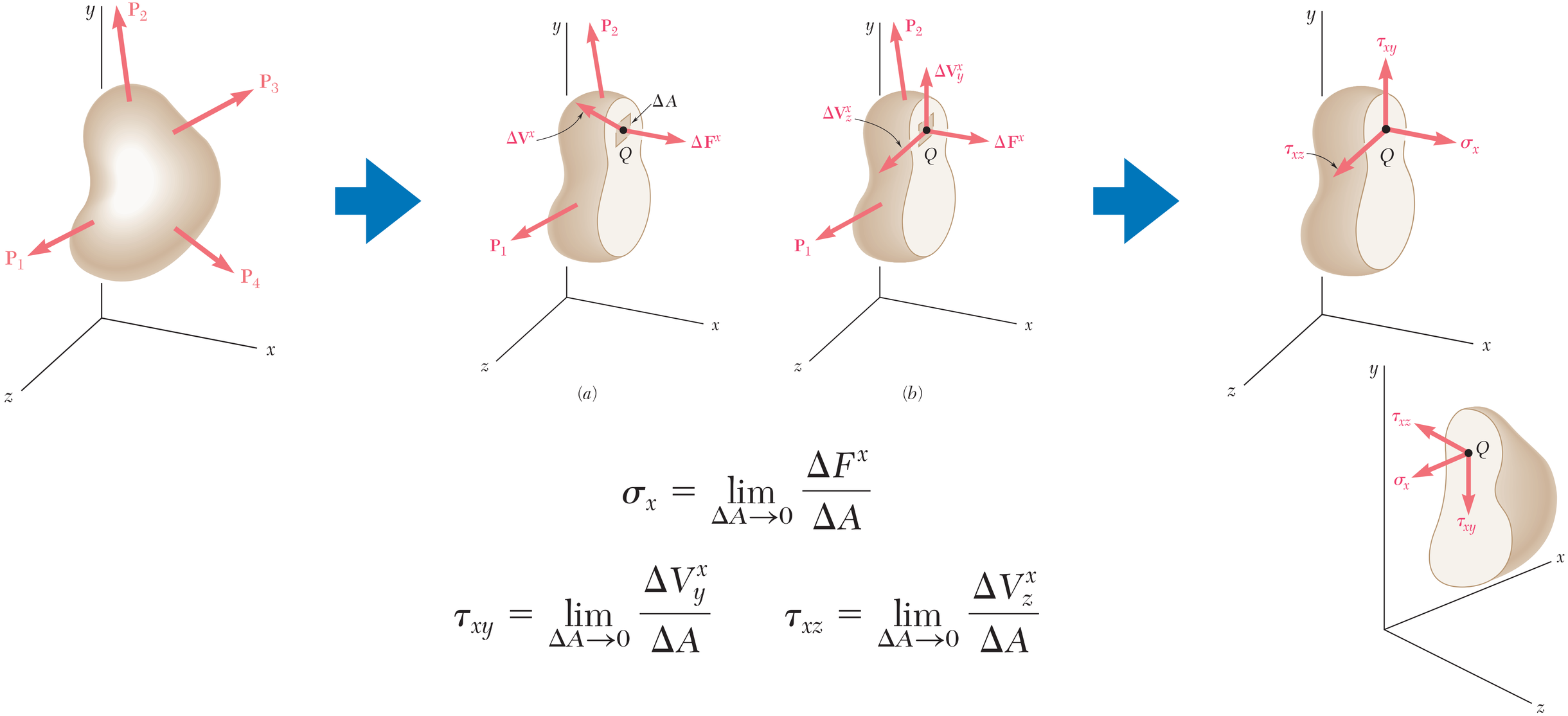
Stress under general loading conditions



$$\sigma_x = \lim_{\Delta A \to 0} \frac{\Delta F^x}{\Delta A}$$

$$\tau_{xy} = \lim_{\Delta A \to 0} \frac{\Delta V_y^x}{\Delta A} \qquad \tau_{xz} = \lim_{\Delta A \to 0} \frac{\Delta V_z^x}{\Delta A}$$

Stress under general loading conditions



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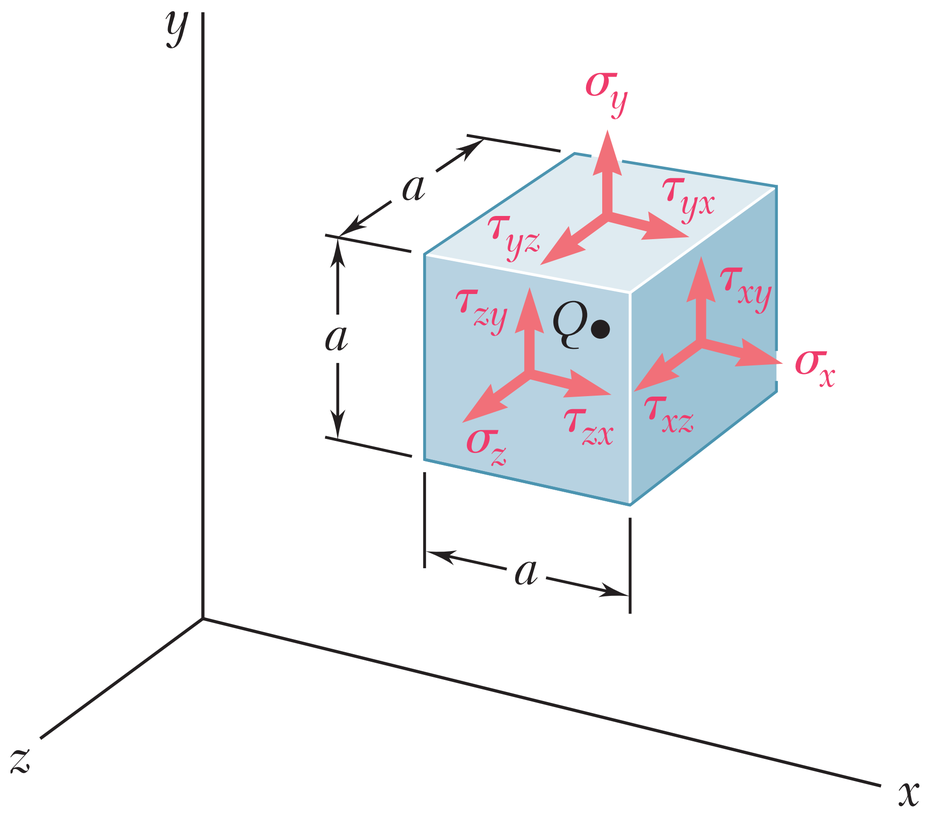
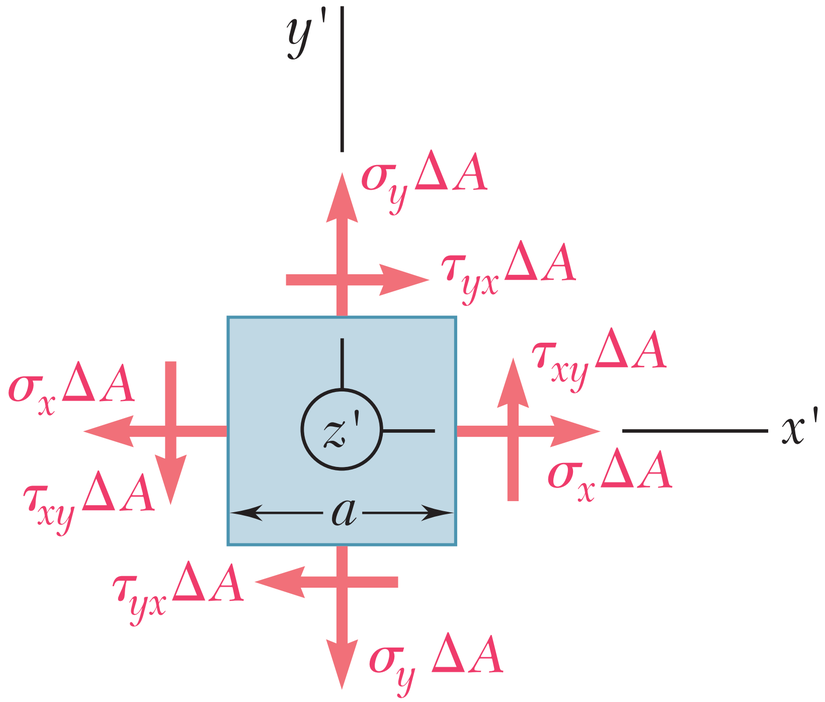
Stress under general loading conditions

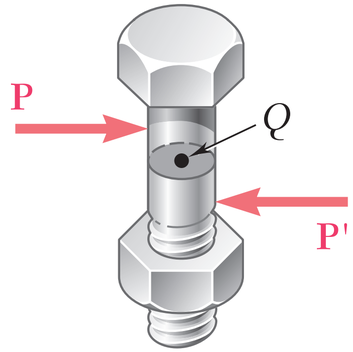




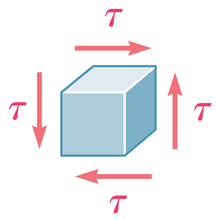
$$\tau_{xy} = \tau_{yx}$$

$$\tau_{yz} = \tau_{zy}$$

$$\tau_{zx} = \tau_{xz}$$

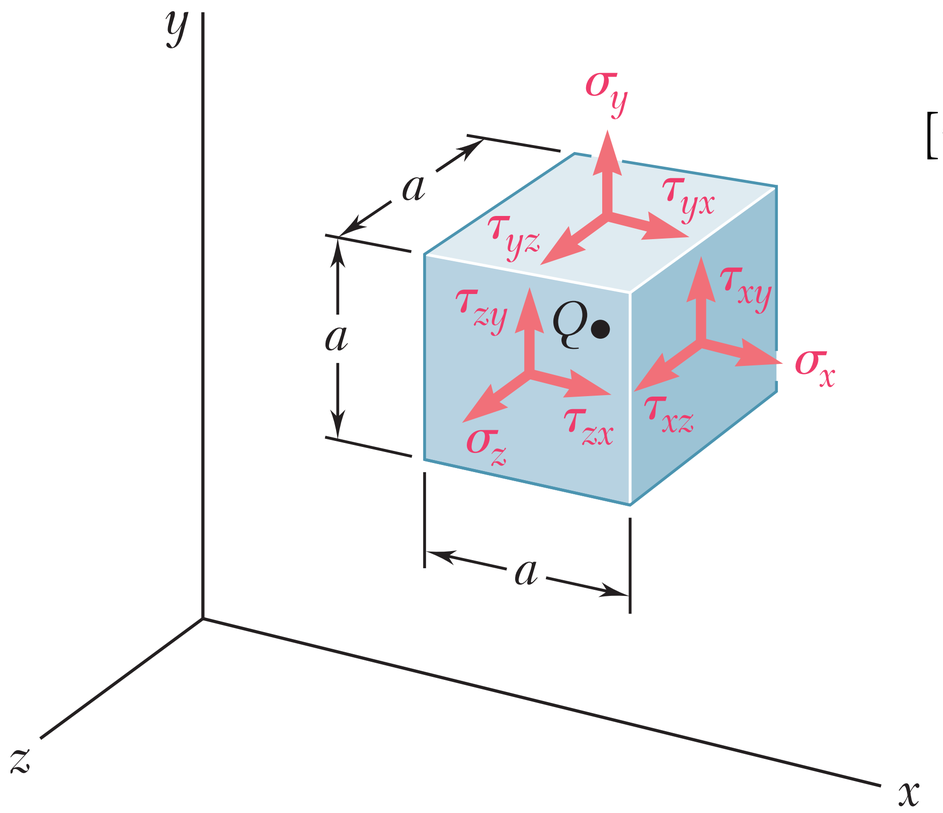


(a)



(b)

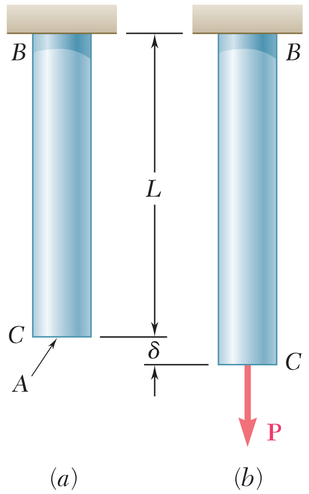
Stress under general loading conditions

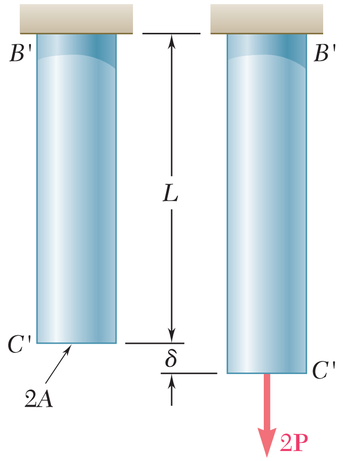


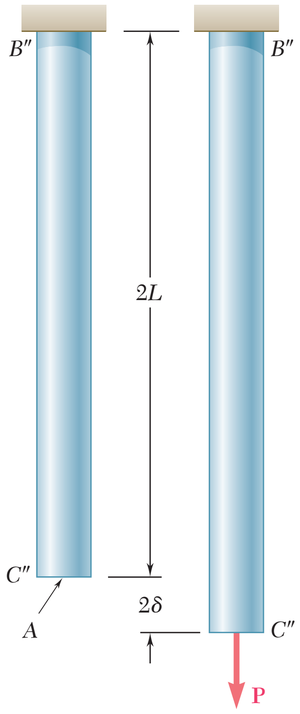
$$[\tau_{ij}] = \begin{bmatrix} \tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \tau_{zz} \end{bmatrix} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}.$$

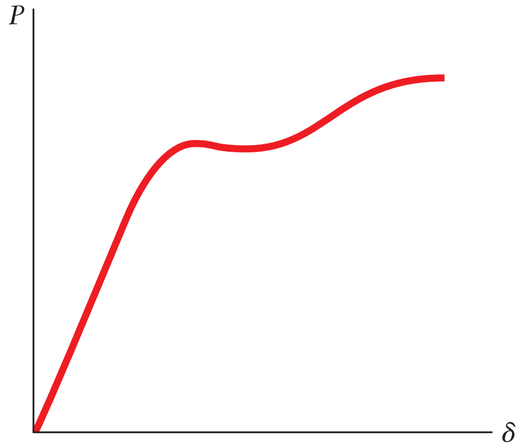
Stress and Strain relations

Axial loadings







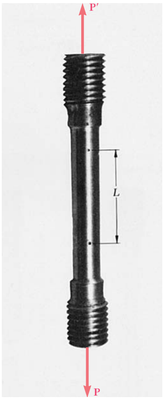


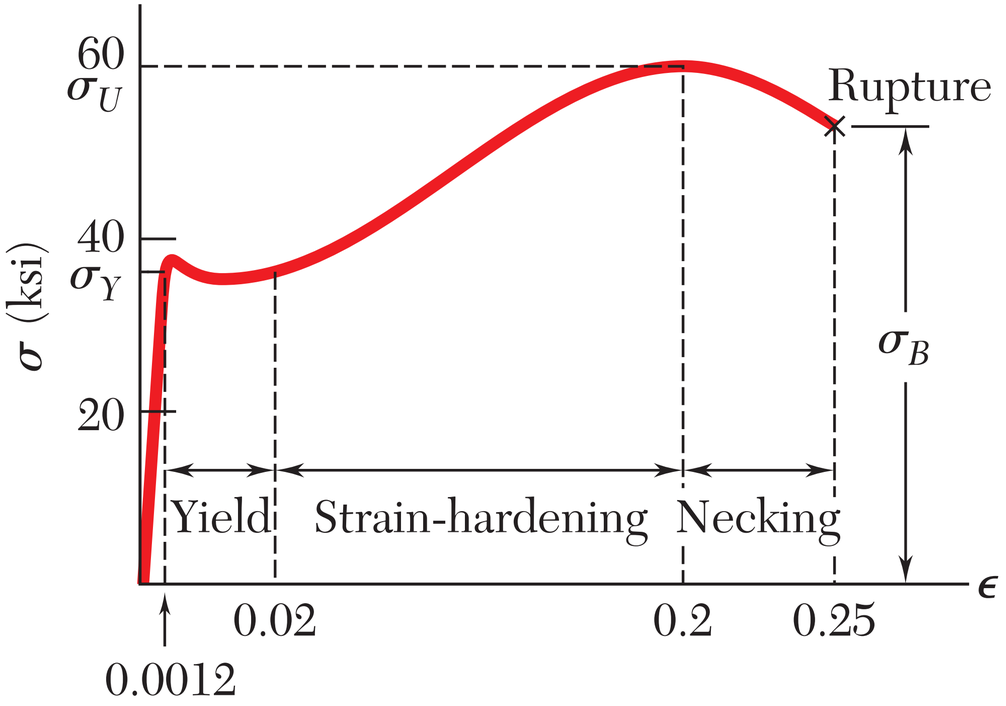
Load-deformation diagram.

$$\epsilon = \frac{\delta}{L}$$

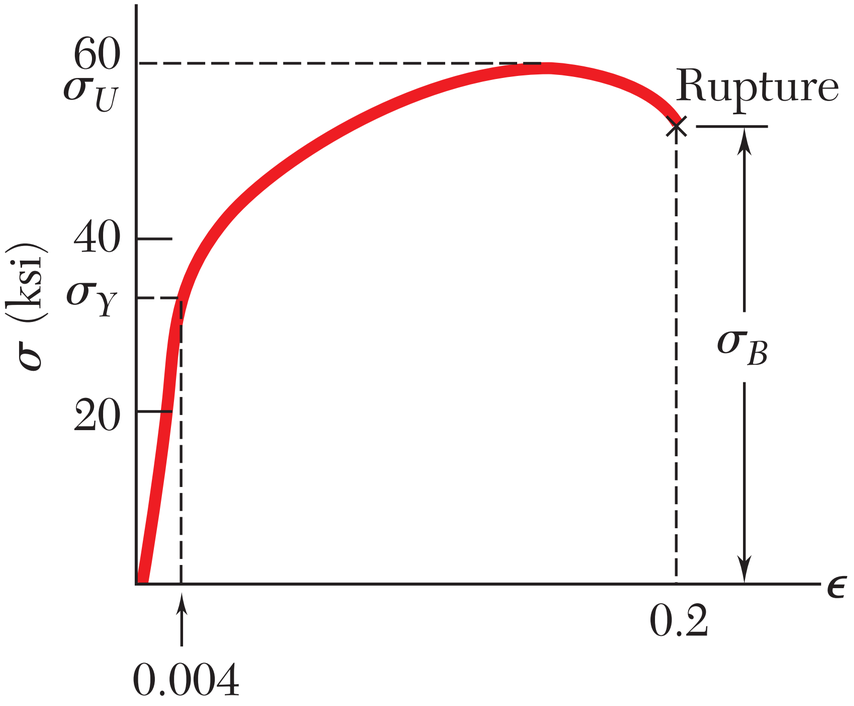
Stress and Strain relations

Stress vs. Strain diagrams (the tensile test)

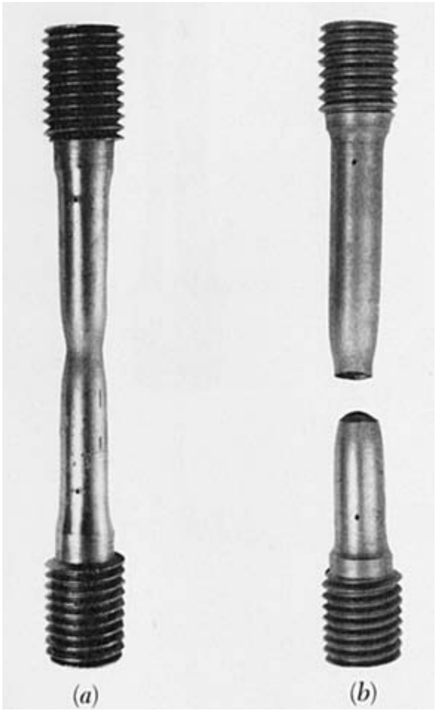




(*a*) Low-carbon steel

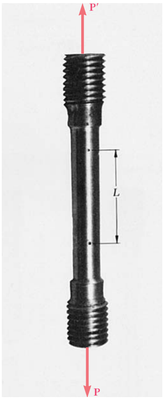


(*b*) Aluminum alloy

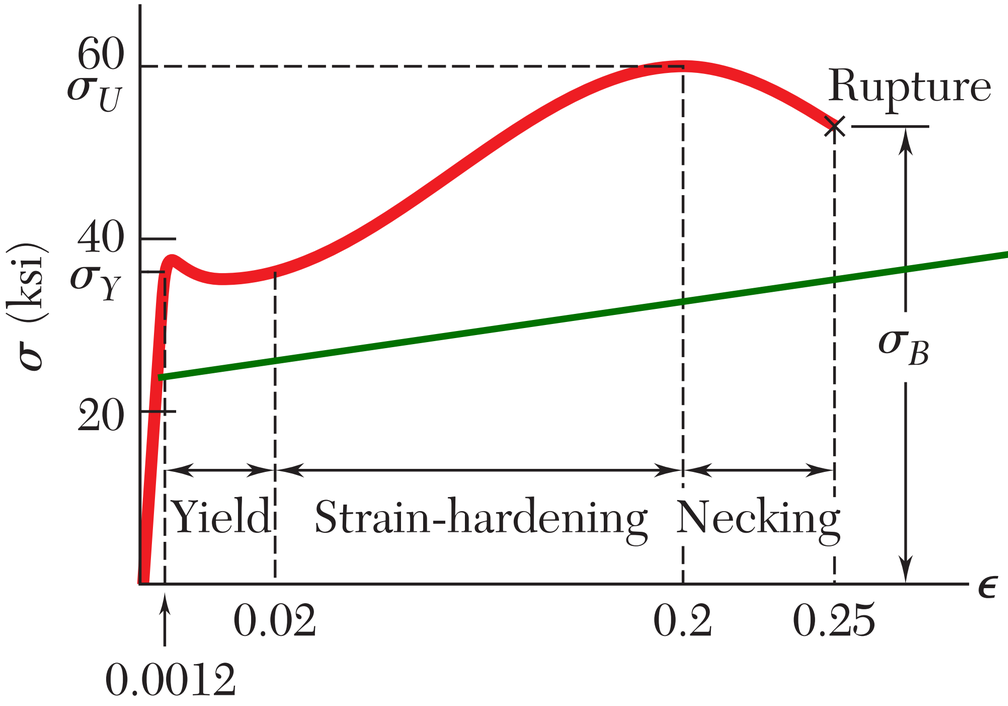


Stress and Strain relations

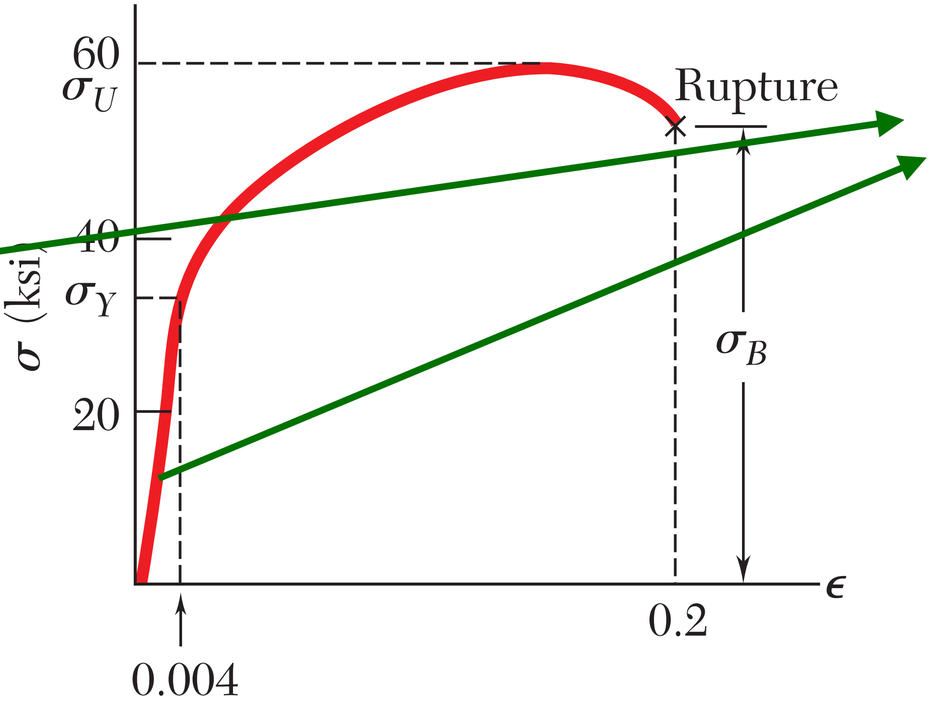
Stress vs. Strain diagrams (the tensile test)



$$\sigma = E\epsilon$$



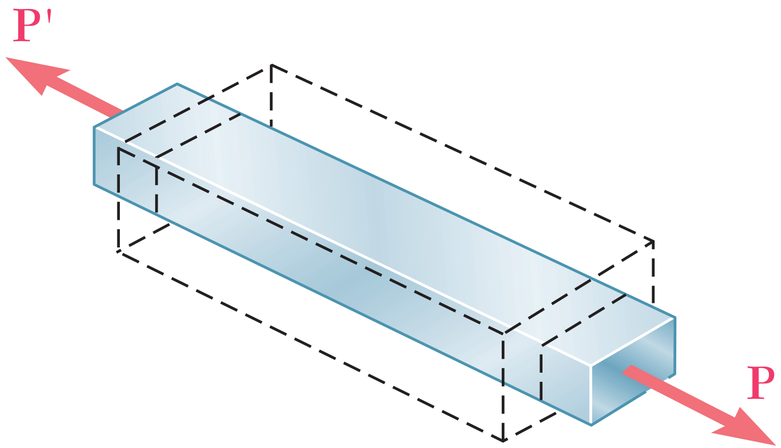
(*a*) Low-carbon steel



(*b*) Aluminum alloy

Stress and Strain relations

Elastic behaviour (uniaxial stress states)



$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}}$$

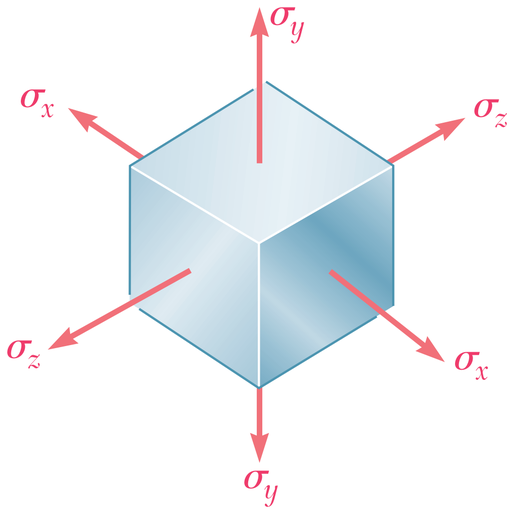
$$\nu = -\frac{\epsilon_y}{\epsilon_x} = -\frac{\epsilon_z}{\epsilon_x}$$

$$\epsilon_x = \frac{\sigma_x}{E}$$

$$\epsilon_y = \epsilon_z = -\frac{\nu\sigma_x}{E}$$

Stress and Strain relations

Elastic behaviour (triaxial stress states)



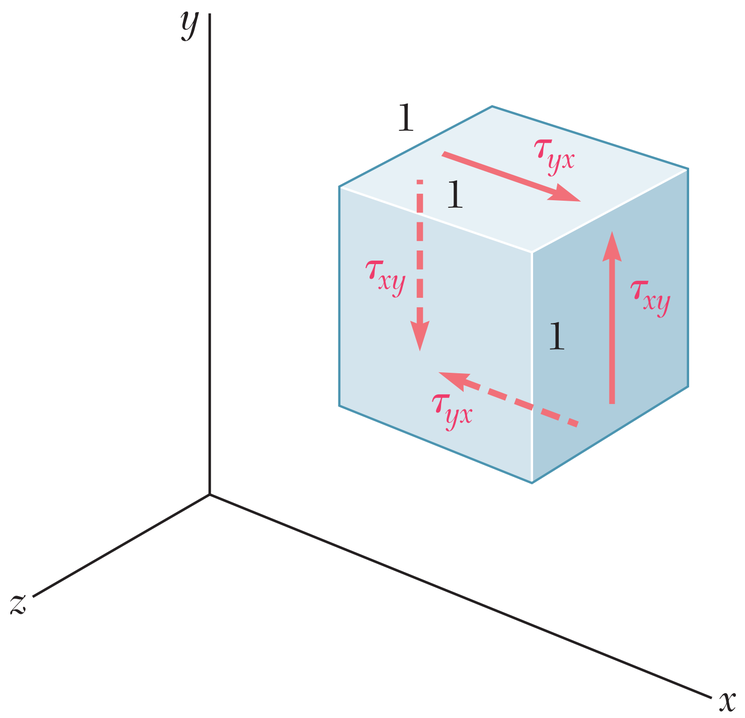
$$\epsilon_x = +\frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\epsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$\epsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

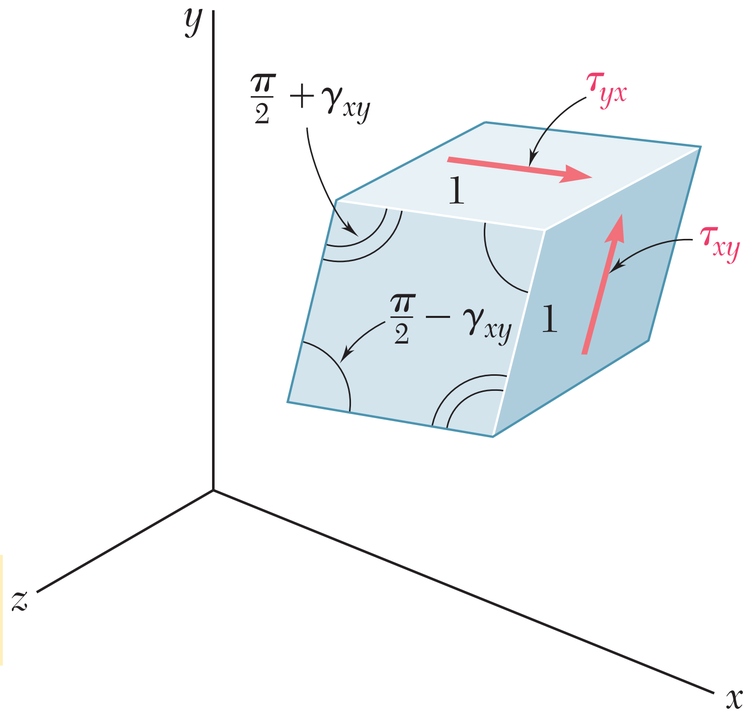
Shear stress and Shear strain relations

Elastic behaviour



$$\tau_{xy} = G\gamma_{xy}$$

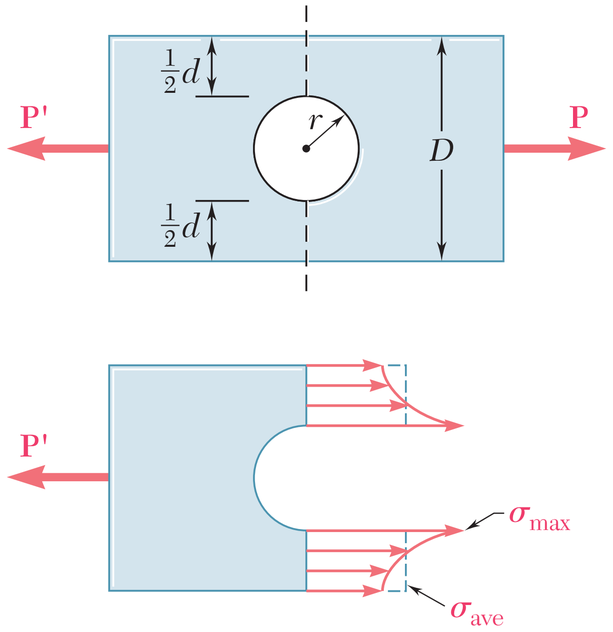
$$\tau_{yz} = G\gamma_{yz} \qquad \tau_{zx} = G\gamma_{zx}$$

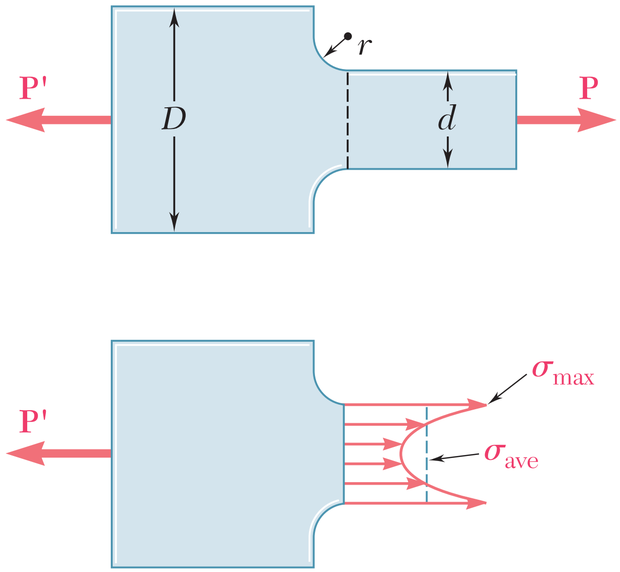


$$G = \frac{E}{2(1 + \nu)}$$

Extrapolation / adjustment from the theory

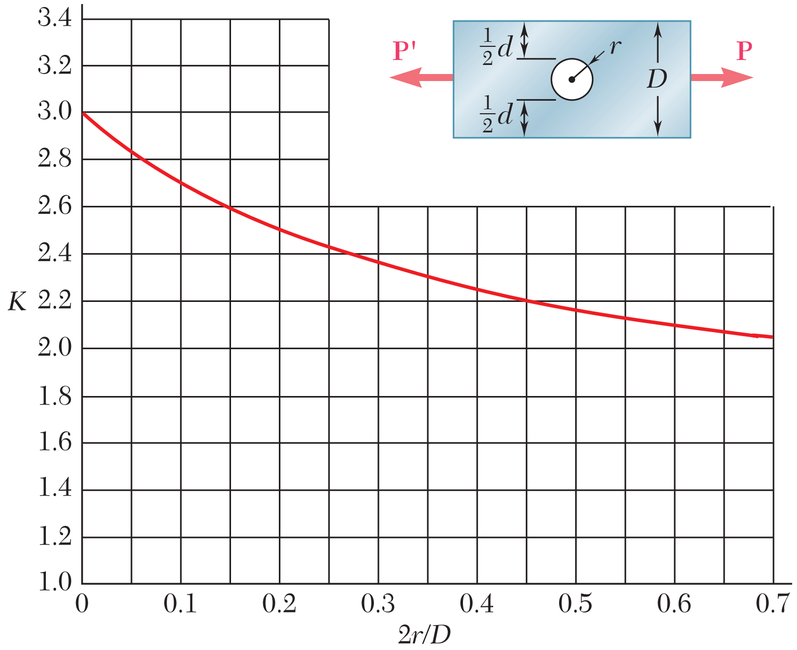
Example: Stress concentration factors





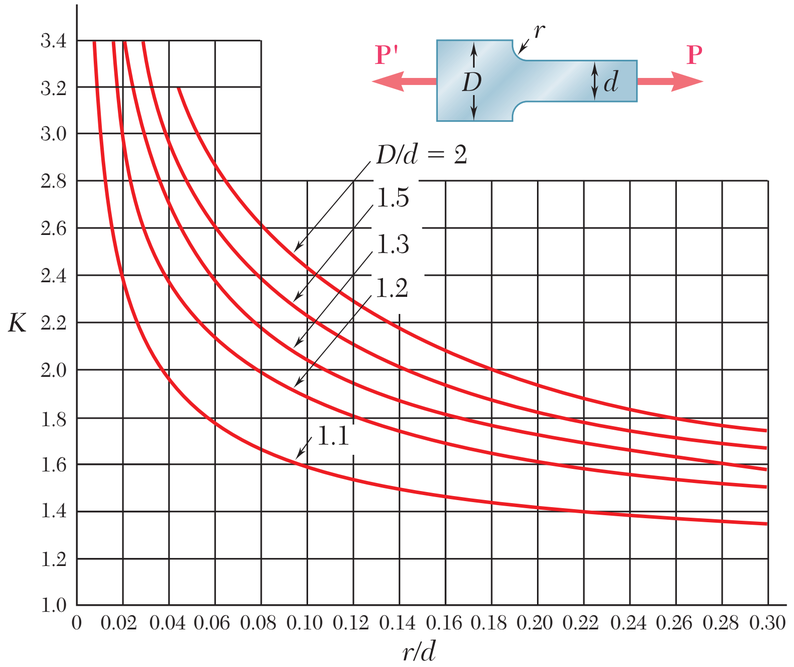
Extrapolation / adjustment from the theory

Example: Stress concentration factors (from experiments)



(a) Flat bars with holes

$$K = \frac{\sigma_{\max}}{\sigma_{\text{ave}}}$$



(b) Flat bars with fillets

Next topics

- Torsion
- Bending
- Combined effects
- Thin plates
- Thin shells
- Reinforced plates
- Reinforced shells
- (…)

References

- “Mechanics of Materials”, Beer, Johnston, DeWolf, Mazurek
- “Introduction to Aircraft Structural Analysis”, Megson
- “Stresses in Beams, Plates, and Shells”, Ugural

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