1 Derivadas

Função	Derivada
u^r	$r\,u^{r-1}u'$
$\ln\lvert u\rvert$	$\frac{u'}{u}$
e^u	$u'\,e^u$
a^u	$u'\,a^u\,\ln a\ (a>0, a\neq 1)$
$\operatorname{sen} u$	$u'\cos u$
$\cos u$	$-u'\,\operatorname{sen} u$
$\operatorname{senh} u$	$u'\cosh u$
$\cosh u$	$u'\operatorname{senh} u$

Função	Derivada
$\operatorname{tg} u$	$u'\,\sec^2 u$
$\operatorname{cotg} u$	$-u'\,\operatorname{cosec}^2 u$
$\sec u$	$u'\,\sec u\,\operatorname{tg} u$
$\operatorname{cosec} u$	$-u'\,\operatorname{cosec} u\,\operatorname{cotg} u$
$\operatorname{arcsen} u$	$\frac{u'}{\sqrt{1-u^2}}$
$\operatorname{arctg} u$	$\frac{u'}{1+u^2}$
$\arccos u$	$-\frac{u'}{\sqrt{1-u^2}}$
$\operatorname{arccotg} u$	$-\frac{u'}{1+u^2}$

2 Primitivas

Função	Primitiva
$u^r\,u'$	$\frac{u^{r+1}}{r+1}\ (r\neq -1)$
$\frac{u'}{u}$	$\ln\lvert u\rvert$
$u'\,e^u$	e^u
$u'\,a^u$	$\frac{a^u}{\ln a}\ (a>0, a\neq 1)$
$u'\cos u$	$\operatorname{sen} u$
$u'\,\operatorname{sen} u$	$-\cos u$
$u'\cosh u$	$\operatorname{senh} u$
$u'\operatorname{senh} u$	$\cosh u$

Função	Primitiva
$u'\,\sec^2 u$	$\operatorname{tg} u$
$u'\,\operatorname{cosec}^2 u$	$-\operatorname{cotg} u$
$u'\,\sec u\,\operatorname{tg} u$	$\sec u$
$u'\,\operatorname{cosec} u\,\operatorname{cotg} u$	$-\operatorname{cosec} u$
$\frac{u'}{\sqrt{1-u^2}}$	$\operatorname{arcsen} u$
$\frac{u'}{1+u^2}$	$\operatorname{arctg} u$
$u'\,\sec u$	$\ln\lvert\sec u+\operatorname{tg} u\rvert$
$u'\,\operatorname{cosec} u$	$-\ln\lvert\operatorname{cosec} u+\operatorname{cotg} u\rvert$

3 Primitivas quase imediatas: exercícios

a) $\displaystyle\int x(1+x^2)^9\,dx$ b) $\displaystyle\int \operatorname{sen} x\cos^5 x\,dx$ c) $\displaystyle\int \frac{x^5}{1+x^6}\,dx$ d) $\displaystyle\int \operatorname{tg} x\,dx$

e) $\displaystyle\int \frac{1}{1+4x^2}\,dx$ f) $\displaystyle\int e^{\operatorname{tg} x}\sec^2 x\,dx$ g) $\displaystyle\int x7^{x^2}\,dx$ h) $\displaystyle\int \operatorname{tg}^2 x\,dx$

i) $\displaystyle\int \frac{x}{x^2+9}\,dx$ j) $\displaystyle\int \frac{1}{x^2+9}\,dx$ k) $\displaystyle\int \frac{1}{(x+9)^2}\,dx$ l) $\displaystyle\int \frac{x^2}{x^2+9}\,dx$

m) $\displaystyle\int x^3\sqrt{1-x^4}\,dx$ n) $\displaystyle\int \frac{x^3}{\sqrt{1-x^4}}\,dx$ o) $\displaystyle\int \frac{3x}{\sqrt{1-x^4}}\,dx$ p) $\displaystyle\int \frac{e^x}{1+e^{2x}}\,dx$

q) $\displaystyle\int \frac{\ln x}{x}\,dx$ r) $\displaystyle\int \frac{5}{x\ln^3 x}\,dx$ s) $\displaystyle\int \frac{1}{x\ln x}\,dx$ t) $\displaystyle\int \frac{e^x}{1+e^x}\,dx$

u) $\displaystyle\int \operatorname{sen}^3 x\cos^5 x\,dx$ v) $\displaystyle\int \frac{1}{\sec x-\cos x}\,dx$ w) $\displaystyle\int \frac{1}{\sqrt{x-x^2}}\,dx$ x) $\displaystyle\int \frac{1}{1+e^x}\,dx$

4 Umas fórmulas de trigonometria

$$\boxed{\operatorname{sen}(\alpha\pm\beta)=\operatorname{sen}\alpha\cos\beta\pm\cos\alpha\operatorname{sen}\beta \qquad \cos(\alpha\pm\beta)=\cos\alpha\cos\beta\mp\operatorname{sen}\alpha\operatorname{sen}\beta}$$

(donde $\operatorname{sen}(2x)=2\operatorname{sen}x\cos x$ e $\cos(2x)=\cos^2 x-\operatorname{sen}^2 x=2\cos^2 x-1=1-2\operatorname{sen}^2 x$)

$$(A)\begin{cases}\operatorname{sen}\alpha\cos\beta=\frac12\big(\operatorname{sen}(\alpha-\beta)+\operatorname{sen}(\alpha+\beta)\big)\\ \cos\alpha\cos\beta=\frac12\big(\cos(\alpha-\beta)+\cos(\alpha+\beta)\big)\\ \operatorname{sen}\alpha\operatorname{sen}\beta=\frac12\big(\cos(\alpha-\beta)-\cos(\alpha+\beta)\big)\end{cases} \Rightarrow \quad (B)\begin{cases}\operatorname{sen}x\cos x=\frac12\operatorname{sen}(2x)\\ \cos^2 x=\frac12\big(1+\cos(2x)\big)\\ \operatorname{sen}^2 x=\frac12\big(1-\cos(2x)\big)\end{cases}$$

Primitivas

- para produtos de $\operatorname{sen}(nx)$ e $\cos(mx)$, $m,n\in\mathbb{N}$, basta usar (A)
- para $\operatorname{sen}^n x\cos^m x$, $m,n\in\mathbb{N}$, um deles ímpar, usar $\cos^2 x+\operatorname{sen}^2 x=1$
- para $\operatorname{sen}^n x\cos^m x$, $m,n\in\mathbb{N}$, ambos pares, usar (B)

Calcular: (a) $\displaystyle\int \operatorname{sen}(3x)\operatorname{sen}(5x)\,dx$ (b) $\displaystyle\int \cos^2 x\,dx$ (c) $\displaystyle\int \operatorname{sen}^2 x\cos^4 x\,dx$

5 Integração por partes

(a) $\displaystyle\int x\operatorname{sen}x\,dx$ (b) $\displaystyle\int xe^{-x}\,dx$ (c) $\displaystyle\int \ln x\,dx$ (d) $\displaystyle\int \operatorname{arctg}x\,dx$

(e) $\displaystyle\int \operatorname{sen}^2 x\,dx$ (f) $\displaystyle\int e^x\cos x\,dx$ (g) $\displaystyle\int x^3e^{x^2}\,dx$ (h) $\displaystyle\int x\arcsin x^2\,dx$

Mais primitivas trigonométricas ...

- (co)tg$^n x$, $n\in\mathbb{N}$: destaca-se, substituindo, (co)tg$^2 x=$ (co)sec$^2 x-1$
- (co)sec^n, n par: destaca-se (co)sec$^2 x$ e, noutro fator, (co)sec$^2 x=$ (co)tg$^2 x+1$
- (co)sec^n, n ímpar: por partes, *primitivando* o fator (co)sec$^2 x$

Calcular: (i) $\displaystyle\int \operatorname{tg}^3 x\,dx$ (j) $\displaystyle\int \operatorname{cosec}^4 x\,dx$ (k) $\displaystyle\int \sec^3 x\,dx$

6 Primitivação de funções racionais

Primitivas elementares, com $a,\alpha,\beta,\gamma\in\mathbb{R}$, $\beta\neq 0$, $n\in\mathbb{N}$ e $C\in\mathbb{R}$ (verificar!)

$$\int\frac{1}{(x+a)^n}\,dx=\begin{cases}\ln|x+a|+C & n=1\\ \frac{1}{(1-n)(x+a)^{n-1}}+C & n>1\end{cases}$$

$$\int\frac{x+\gamma}{(x+\alpha)^2+\beta^2}\,dx=\frac12\ln\big((x+\alpha)^2+\beta^2\big)+\frac{\gamma-\alpha}{\beta}\operatorname{arctg}\frac{x+\alpha}{\beta}+C$$

- Calcula (a) $\displaystyle\int\frac{x^4-6}{x^3-2x^2}\,dx$ (b) $\displaystyle\int\frac{3x}{x^3-3x^2+4}\,dx$ (c) $\displaystyle\int\frac{x^2-1}{2x^3+6x^2+5x}\,dx$

Potências de fatores irredutíveis de grau 2? Método de Hermite-Ostrogradski!

Dada a função racional própria $\frac{n(x)}{d(x)}$, sejam

- $d_s(x)$ o produto dos fatores de $d(x)$ *sem considerar a multiplicidade* e
- $d_r(x)=d(x)/d_s(x)$;

então existem, e são únicas, as frações próprias $\frac{n_s(x)}{d_s(x)}$ e $\frac{n_r(x)}{d_r(x)}$ tais que

$$\frac{n(x)}{d(x)}=\frac{n_s(x)}{d_s(x)}+\left(\frac{n_r(x)}{d_r(x)}\right)'$$

7 Decomposições e primitivas

Comparação de diferentes decomposições: por exemplo,

$$\begin{aligned}
\frac{1}{(x+5)^3(x^2+1)^2} &= \frac{\alpha}{x+5} + \frac{\beta}{(x+5)^2} + \frac{\gamma}{(x+5)^3} + \frac{\delta x+\epsilon}{x^2+1} + \frac{\varphi x+\nu}{(x^2+1)^2} \\
&\quad \text{(frações simples)} \\
&= \frac{ax^2+bx+c}{(x+5)(x^2+1)} + \left(\frac{dx^3+ex^2+fx+g}{(x+5)^2(x^2+1)}\right)' \\
&\quad \text{(Hermite-Ostrogradski)} \\
&= \frac{A}{x+5} + \frac{B}{(x+5)^2} + \frac{C}{(x+5)^3} + \frac{Dx+E}{x^2+1} + \left(\frac{Fx+G}{x^2+1}\right)' \\
&\quad \text{(f.s.+H.O. para potências de fatores de grau 2)}
\end{aligned}$$

$$\Rightarrow \int \frac{1}{(x+5)^3(x^2+1)^2}\,dx = \int \frac{A}{x+5} + \frac{B}{(x+5)^2} + \frac{C}{(x+5)^3} + \frac{Dx+E}{x^2+1}\,dx + \frac{Fx+G}{x^2+1}$$

- Calcula (a) $\displaystyle\int \frac{x-1}{(x^2+1)^2}\,dx$ (b) $\displaystyle\int \frac{x+1}{x(x^2-2x+2)^2}\,dx$