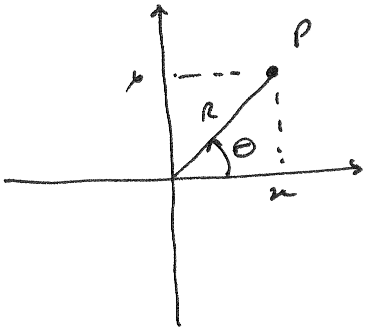
Movimento de uma partícula na base das coordenadas polares:



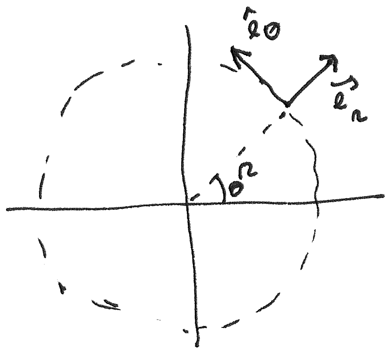
$$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$$

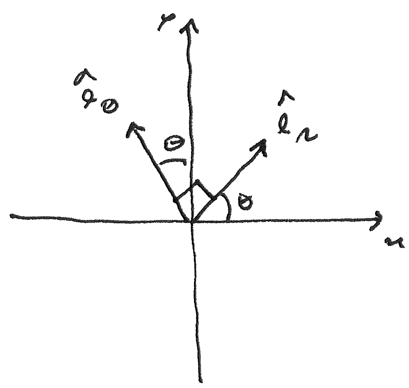
Vetor posição:

$$\vec{r} = (x, y) = (r\cos\theta, r\sin\theta) = r\,\overbrace{(\cos\theta, \sin\theta)}^{\text{vetor unitário } \hat{e}_r}$$

$$= r\,\hat{e}_r$$

Base das coordenadas polares





$$\hat{e}_r = (\cos\theta, \sin\theta)$$

$$\hat{e}_\theta = (-\sin\theta, \cos\theta)$$

Derivada em ordem ao tempo:

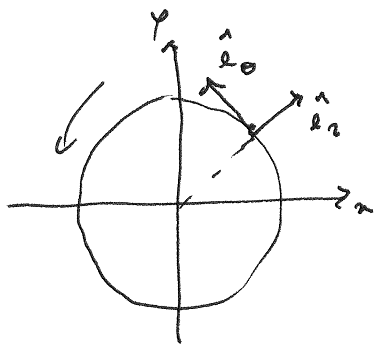
$$\frac{d\hat{e}_r}{dt} = (-\dot{\theta}\sin\theta, \dot{\theta}\cos\theta) = \dot{\theta}\,(-\sin\theta, \cos\theta)$$

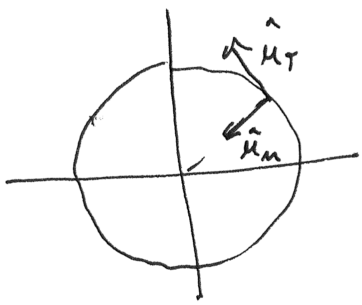
$$= \dot{\theta}\,\hat{e}_\theta$$

$$\frac{d\hat{e}_\theta}{dt} = (-\dot{\theta}\cos\theta, -\dot{\theta}\sin\theta) = -\dot{\theta}\,(\cos\theta, \sin\theta)$$

$$= -\dot{\theta}\,\hat{e}_r$$

Relação entre $\hat{e}_r$ e $\hat{e}_\theta$ e $\hat{u}_n$ e $\hat{u}_t$ no movimento circular

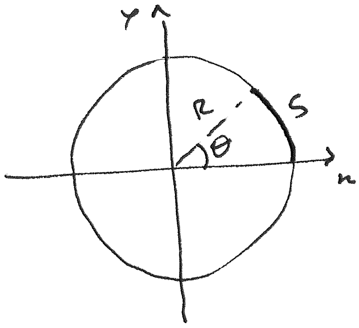




$\hat{e}_\theta = \hat{u}_t$ (atenção aos sentidos)

$\hat{e}_r = -\hat{u}_n$

Relação entre quantidades lineares e angulares no movimento circular:



linear	angular
$s = R\theta$	θ
$v = \frac{ds}{dt}$ $= R\dot{\theta}$ $= R\omega$	$\omega = \dot{\theta}$
$a_t = \frac{dv}{dt}$ $= R\ddot{\theta}$ $= R\alpha$ $a_n = \frac{v^2}{R}$ $= \omega^2 R$	$\alpha = \ddot{\theta}$

Velocidade na base das coordenadas polares (movimento geral):

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}(r\hat{e}_r) = \dot{r}\hat{e}_r + r\frac{d\hat{e}_r}{dt} = \underbrace{\dot{r}\hat{e}_r}_{\text{compon. radial}} + \underbrace{r\dot{\theta}\hat{e}_\theta}_{\text{comp. azimutal}}$$

Aceleração na base das coord. polares (movimento geral):

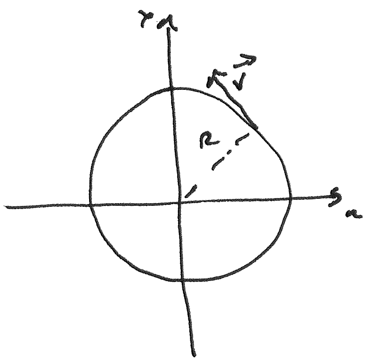
$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\dot{r}\hat{e}_r + r\dot{\theta}\hat{e}_\theta)$$

$$= \ddot{r}\hat{e}_r + \dot{r}\dot{\theta}\hat{e}_\theta + \dot{r}\dot{\theta}\hat{e}_\theta + r\ddot{\theta}\hat{e}_\theta - r\dot{\theta}^2\hat{e}_r$$

$$= (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\hat{e}_\theta$$

Caso particular do movimento circular:

$$r = \text{const} \Rightarrow \begin{cases} \dot{r} = 0 \\ \ddot{r} = 0 \end{cases}$$



velocidade: $\vec{v} = r\dot{\theta}\hat{e}_\theta$

aceleração: $\vec{a} = -r\dot{\theta}^2\hat{e}_r + r\ddot{\theta}\hat{e}_\theta$

$$= r\dot{\theta}^2\hat{u}_n + r\ddot{\theta}\hat{u}_t$$

$$= r\omega^2\hat{u}_n + r\alpha\hat{u}_t$$

$$= a_n\hat{u}_n + a_t\hat{u}_t$$