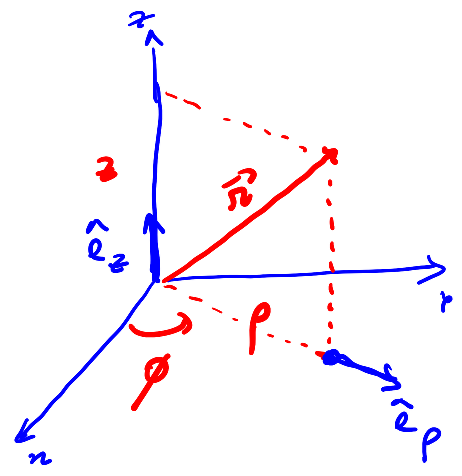
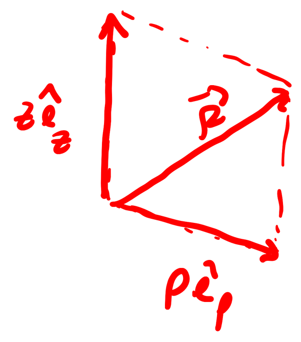
Vetor posição na base das coord. cilíndricas



ρ, ϕ, z



$$\vec{r} = \rho\,\hat{e}_\rho + 0\,\hat{e}_\phi + z\,\hat{e}_z$$

$$\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k}$$

$$\vec{A}\cdot\hat{\imath} = A_x\,\underbrace{\hat{\imath}\cdot\hat{\imath}}_{1} + A_y\,\underbrace{\hat{\jmath}\cdot\hat{\imath}}_{0} + A_z\,\underbrace{\hat{k}\cdot\hat{\imath}}_{0}$$

$$\vec{A}\cdot\hat{\imath} = A_x\,; \quad \vec{A}\cdot\hat{\jmath} = A_y\,; \quad \vec{A}\cdot\hat{k} = A_z$$

$$\vec{A} = A_x\,\hat{\imath} + A_y\,\hat{\jmath} + A_z\,\hat{k}$$
$$= (\vec{A}\cdot\hat{\imath})\,\hat{\imath} + (\vec{A}\cdot\hat{\jmath})\,\hat{\jmath} + (\vec{A}\cdot\hat{k})\,\hat{k}$$

$$\vec{A} = A_\rho\,\hat{e}_\rho + A_\phi\,\hat{e}_\phi + A_z\,\hat{e}_z$$
$$= (\vec{A}\cdot\hat{e}_\rho)\,\hat{e}_\rho + (\vec{A}\cdot\hat{e}_\phi)\,\hat{e}_\phi + (\vec{A}\cdot\hat{e}_z)\,\hat{e}_z$$

$$\hat{e}_\rho = \cos\phi\,\hat{\imath} + \sin\phi\,\hat{\jmath} + 0\,\hat{k}$$
$$= (\hat{e}_\rho\cdot\hat{\imath})\,\hat{\imath} + (\hat{e}_\rho\cdot\hat{\jmath})\,\hat{\jmath} + (\hat{e}_\rho\cdot\hat{k})\,\hat{k}$$

Campo vetorial: $\vec{v} = (x+1)\,\hat{\imath} + \hat{\jmath} + y\,\hat{k}$

Base das coord. esféricas:

$$x = r\sin\theta\cos\phi$$
$$y = r\sin\theta\sin\phi$$
$$z = r\cos\theta$$

$$\begin{cases} \hat{e}_r = \sin\theta\cos\phi\,\hat{\imath} + \sin\theta\sin\phi\,\hat{\jmath} + \cos\theta\,\hat{k} \\ \hat{e}_\theta = \cos\theta\cos\phi\,\hat{\imath} + \cos\theta\sin\phi\,\hat{\jmath} - \sin\theta\,\hat{k} \\ \hat{e}_\phi = -\sin\phi\,\hat{\imath} + \cos\phi\,\hat{\jmath} + 0\,\hat{k} \end{cases}$$

$$\begin{cases} \hat{\imath} = \sin\theta\cos\phi\,\hat{e}_r + \cos\theta\cos\phi\,\hat{e}_\theta - \sin\phi\,\hat{e}_\phi \\ \hat{\jmath} = \sin\theta\sin\phi\,\hat{e}_r + \cos\theta\sin\phi\,\hat{e}_\theta + \cos\phi\,\hat{e}_\phi \\ \hat{k} = \cos\theta\,\hat{e}_r - \sin\theta\,\hat{e}_\theta + 0\,\hat{e}_\phi \end{cases}$$

Gustão:

1º passo: $\vec{v} = (r\sin\theta\cos\phi + 1)\,\hat{\imath} + \hat{\jmath} + r\sin\theta\sin\phi\,\hat{k}$

2º passo: $= (r\sin\theta\cos\phi + 1)\,(\sin\theta\cos\phi\,\hat{e}_r + \cos\theta\cos\phi\,\hat{e}_\theta - \sin\phi\,\hat{e}_\phi)$

$+ \quad 1 \quad (\sin\theta\sin\phi\,\hat{e}_r + \cos\theta\sin\phi\,\hat{e}_\theta + \cos\phi\,\hat{e}_\phi)$

$+ r\sin\theta\sin\phi\,(\cos\theta\,\hat{e}_r - \sin\theta\,\hat{e}_\theta + 0\,\hat{e}_\phi)$

3º passo:
$$= \left[r\sin^2\theta\cos^2\phi + \sin\theta\cos\phi + \sin\theta\sin\phi + r\sin\theta\cos\theta\sin\phi\right]\hat{e}_r$$
$$\hat{e}_\theta$$
$$+$$
$$+ \left[-r\sin\theta\cos\phi\sin\phi + \cos\phi\right]\hat{e}_\phi$$