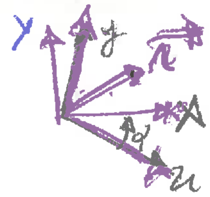
1.º DEFINIR REFERENCIAL INERCIAL (OXYZ)

2.º SE NECESSÁRIO:

i) DEFINIR EM OXYZ:

- $\vec{F}_R = \sum_i \vec{F}_{i\,ext}$
- $\vec{a}_G$ USANDO MRR OU MDTRR



$\vec{r} = \underset{\sim}{R}^T \vec{\pi}$

$\vec{\pi} = \underset{\sim}{R}\, \vec{r}$

$$\underset{\sim}{R}^T = \begin{bmatrix} \cos\alpha & -\operatorname{sen}\alpha & 0 \\ \operatorname{sen}\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

ii) SELECCIONAR G OU A E DEFINIR EM OXYZ:

G

- $\vec{M}_{R/G} = \sum \vec{M}_{BIN} + \sum_i \vec{GP}_i \times \vec{F}_{i\,ext}$
- $\boxed{\dot{\vec{H}}_G}$?

A

- $\vec{M}_{R/A} = \sum \vec{M}_{BIN} + \sum_i \vec{AP}_i \times \vec{F}_{i\,ext}$
- $\boxed{\dot{\vec{H}}_A}$?

3.º PARA DEFINIR $\vec{H}_G$ OU $\vec{H}_A$, SELECCIONAR REFERENCIAL FIXO NO CORPO RÍGIDO (NÃO-INERCIAL) E OBTER:

- $\vec{\omega} = \underset{\sim}{R}^T \vec{\omega}$
- $\vec{\alpha} = \underset{\sim}{R}^T \vec{\alpha}$
- $[I_G] \longrightarrow$ SE $\begin{cases} \text{R N-I, N-P: TEOREMA DE STEINER} \\ \text{R N-I, P: TABELAS} \end{cases}$

G

- $\dot{\vec{H}}_G = [I_G]\vec{\alpha} + \vec{\omega} \times ([I_G]\vec{\omega})$
- $\dot{\vec{H}}_G = \underset{\sim}{R}\, \dot{\vec{H}}_G$

$\boxed{\text{NOTA: } \vec{H}_G = [I_G]\vec{\omega};\ \vec{H}_G = \underset{\sim}{R}\vec{H}_G}$

A-I

- $\dot{\vec{H}}_G = [I_G]\vec{\alpha} + \vec{\omega} \times ([I_G]\vec{\omega})$

~~$\dot{\vec{H}}_A = \dot{\vec{H}}_G + \vec{AG} \times m\vec{a}_G$ (TEOREMA DE KÖNIG)~~

- $\dot{\vec{H}}_G = \underset{\sim}{R}\, \dot{\vec{H}}_G$
- $\dot{\vec{H}}_A = \dot{\vec{H}}_G + \vec{AG} \times m\vec{a}_G$

2.º TEOREMA DE KÖNIG

$\boxed{\text{NOTA: } \vec{H}_G = [I_G]\vec{\omega};\ \vec{H}_G = \underset{\sim}{R}\vec{H}_G;\ \vec{H}_A = \vec{H}_G + \vec{AG} \times m\vec{v}_G \text{ (1.º T. KÖNIG)}}$

A-II

- $[I_G] \to [I_A]$ PELO TEOREMA DE STEINER E
- $\dot{\vec{H}}_A = [I_A]\vec{\alpha} + \vec{\omega} \times ([I_A]\vec{\omega})$
- $\dot{\vec{H}}_A = \underset{\sim}{R}\, \dot{\vec{H}}_A$

$\boxed{\text{NOTA: } \vec{H}_A = [I_A]\vec{\omega};\ \vec{H}_A = \underset{\sim}{R}\vec{H}_A}$

4.º RESOLVER

$$\vec{F}_R = m\vec{a}_G$$

G

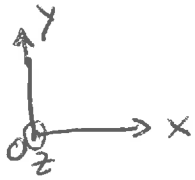
$$\vec{M}_{R/G} = \dot{\vec{H}}_G$$

A

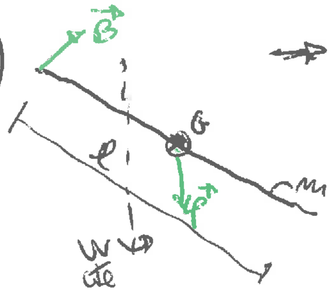
$$\vec{M}_{R/A} = \dot{\vec{H}}_A$$

EX 1

1.°



2.° i)



en OXYZ

$\Rightarrow \begin{cases} \vec{B} = B_x\hat{I} + B_y\hat{J} + B_z\hat{K} \\ \vec{P} = -mg\hat{J} \end{cases} \Rightarrow \vec{F_R} = \vec{B} + \vec{P}$

$\Rightarrow \boxed{\vec{F_R} = B_x\hat{I} + (B_y - mg)\hat{J} + B_z\hat{K}}$

$\vec{a_G}$?

MPR c/ 1 Nível
E $\vec{\mu} = \vec{\pi} = \vec{0}$

$\Rightarrow \begin{cases} \vec{v_G} = \underbrace{\cancel{\vec{v_c}}}_{\vec{0}} + \underbrace{\vec{\Omega}}_{\vec{w}} \times \underbrace{\vec{\rho}_{/R}}_{\vec{CG}} + \underbrace{\cancel{\vec{v}_{/R}}}_{\vec{0}} \\ \vec{a_G} = \underbrace{\cancel{\vec{a_c}}}_{\vec{0}} + \underbrace{\cancel{\dot{\vec{\Omega}}}}_{\vec{0}} \times \vec{\rho}_{/R} + \underbrace{\vec{\Omega}}_{\vec{w}} \times (\vec{\Omega} \times \underbrace{\vec{\rho}_{/R}}_{\vec{CG}}) + 2\vec{\Omega} \times \underbrace{\cancel{\vec{v}_{/R}}}_{\vec{0}} + \underbrace{\cancel{\vec{a}_{/R}}}_{\vec{0}} \end{cases}$

$\Rightarrow \vec{a_G} = \vec{w} \times (\vec{w} \times \vec{CG})$

c/ $\begin{cases} \vec{w} = w\hat{J} \\ \vec{CG} = (\frac{\ell}{2}\cos\theta - \ell/3)\hat{I} - \frac{\ell}{2}\sin\theta\hat{J} \end{cases} \Rightarrow \boxed{\vec{a_G} = -w^2(\frac{\ell}{2}\cos\theta - \ell/3)\hat{I}}$

ii) Apoio $\underline{B}$

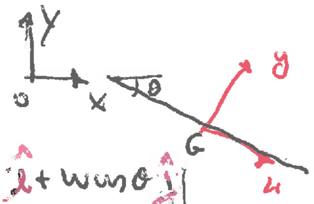
$\vec{M}_{R/B} = \underbrace{\cancel{\Sigma \vec{M}_{B\,IN}}}_{\vec{0}} + \sum_{i=2} \vec{BP_i} \times \vec{F}_{i\,ext} = \underbrace{\cancel{\vec{BB}}}_{\vec{0}} \times \vec{B} + \vec{BG} \times \vec{P}$

c/ $\begin{cases} \vec{BG} = \frac{\ell}{2}\cos\theta\hat{I} - \frac{\ell}{2}\sin\theta\hat{J} \\ \vec{P} = -mg\hat{J} \end{cases}$

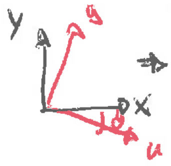
$\Rightarrow \boxed{\vec{M}_{R/B} = -mg\frac{\ell}{2}\cos\theta\,\hat{K}}$

3.° $\dot{\vec{H}}_B$?

Seleccionando o R N.I, P ...



$\Rightarrow$



$\Rightarrow \underset{\sim}{R}^T = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\begin{cases} \vec{w} = w\hat{J} \\ \vec{\alpha} = \vec{0} \end{cases} \Rightarrow \begin{cases} \vec{w} = \underset{\sim}{R}^T\vec{w} = -w\sin\theta\,\hat{u} + w\cos\theta\,\hat{v} \\ \vec{\alpha} = \underset{\sim}{R}^T\vec{\alpha} = \vec{0} \end{cases}$

$[I_G] \underset{\text{TABELA}}{=} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{m\ell^2}{12} & 0 \\ 0 & 0 & \frac{m\ell^2}{12} \end{bmatrix}$

$\Rightarrow \dot{\vec{H}}_G = [I_G]\vec{\alpha} + \vec{w} \times ([I_G]\vec{w}) = -\frac{m\ell^2}{12}\sin\theta\cos\theta\,\hat{k}$

$\dot{\vec{H}}_G = \underset{\sim}{R}\,\dot{\vec{H}}_G = -\frac{m\ell^2}{12}\sin\theta\cos\theta\,\hat{K}$

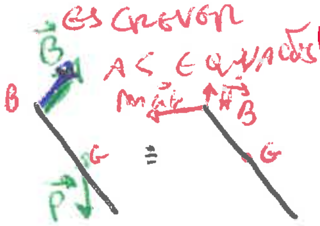
$\dot{\vec{H}}_B = \dot{\vec{H}}_G + \vec{BG} \times m\vec{a_G}$ c/ $\vec{BG}$ já definidos $\Rightarrow$
2.° Teor. König $\vec{a_G}$

$\boxed{\dot{\vec{H}}_B = \left(-\frac{m\ell^2}{3}w^2\sin\theta\cos\theta + \frac{m\ell^2}{6}\sin\theta\, w^2\right)\hat{K}}$

4.° Resolver

$\vec{F_R} = m\vec{a_G}$

$\vec{M}_{R/B} = \dot{\vec{H}}_B$

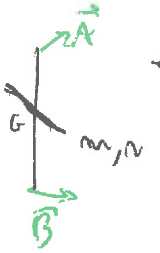


EX 2

1°

O≡G

2°



⇒ NESTE PROBLEMA NÃO NOS PEDEM AS REACÇÕES, SÓ $\vec{H}_G \to \measuredangle(\vec{H}_G, \vec{w})$ e $\dot{\vec{H}}_G$!!

ASSIM, BASTA PASSAR PARA 3° PONTO! SELECCIONANDO R!

3° SELECCIONANDO O R, N=I, P=..

$\to R^T = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{cases} w = w\hat{J} \\ \alpha = \alpha\hat{J} \end{cases} \Rightarrow \boxed{\begin{cases} \vec{w} = R^T\vec{W} = -w\sin\beta\,\hat{i} + w\cos\beta\,\hat{j} \\ \vec{\alpha} = R^T\vec{\alpha} = -\alpha\sin\beta\,\hat{i} + \alpha\cos\beta\,\hat{j} \end{cases}}$$

$[I_G] =$ TABELA $\begin{bmatrix} \frac{mR^2}{4} & 0 & 0 \\ 0 & \frac{mR^2}{2} & 0 \\ 0 & 0 & \frac{mR^2}{4} \end{bmatrix}$ $\Rightarrow \vec{H}_G = [I_G]\vec{w} = \frac{mR^2}{4}(-\sin\beta\,\hat{i} + 2\cos\beta\,\hat{j})$

Sendo $\vec{w} = -w\sin\beta\,\hat{i} + w\cos\,\hat{j}$

a) $\cos\measuredangle(\vec{H}_G, \vec{w}) = \cos\theta = \dfrac{\vec{H}_G \cdot \vec{w}}{\|\vec{H}_G\|\,\|\vec{w}\|}$

b) $\dot{\vec{H}}_G = [I_G]\vec{\alpha} + \vec{w} \times ([I_G]\vec{w}) = \frac{mR^2}{4}(-\alpha\sin\beta\,\hat{i} + 2\alpha\cos\beta\,\hat{j} - w^2\sin\beta\cos\beta\,\hat{k})$

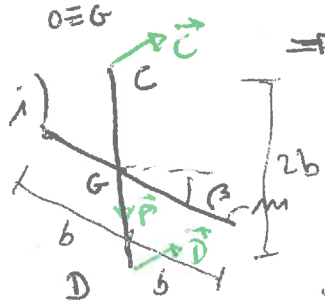
$\dot{\vec{H}}_G = R\,\dot{\vec{H}}_G = \frac{mR^2}{4}(\alpha\sin\beta\cos\beta\,\hat{I} + \alpha(1+\cos^2\beta)\,\hat{J} - w^2\sin\beta\cos\beta\,\hat{K})$

EX3

1.º 

$O \equiv G$

Em $Oxyz$

2.º 

0, p.h., para não ser hiperestático!

$$\Rightarrow \begin{cases} \vec{C} = C_x\hat{I} + \cancel{C_y}\hat{J} + C_z\hat{K} \\ \vec{D} = D_x\hat{I} + D_y\hat{J} + D_z\hat{K} \\ \vec{P} = -mg\hat{J} \end{cases} \Rightarrow \vec{F_R} = \vec{C} + \vec{D} + \vec{P} \Rightarrow$$

$$\boxed{\vec{F_R} = (C_x + D_x)\hat{I} + (D_y - mg)\hat{J} + (C_z + D_z)\hat{K}}$$

$\vec{a_G}$? MPR c/ 1 nível c/ $\vec{\omega} = \dot{\vec{\omega}} = \vec{0}$ $\Rightarrow \vec{r_G} = \vec{a_G} = \vec{0} \Rightarrow \boxed{\vec{a_G} = \vec{0}}$

ii) Apoio D

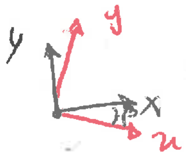
$$\vec{M_R}_{/D} = \underbrace{\cancel{\Sigma \vec{M}_{BIN}}}_{\vec{0}} + \sum_{i=3} \vec{DP_i} \times \vec{F_{i\,ext}} = \underbrace{\vec{DD} \times \vec{D}}_{\vec{0}} + \underbrace{\vec{DG} \times \vec{P}}_{\vec{0},\ \text{Pois} //} + \vec{DC} \times \vec{C}$$

c/ $\vec{DC} = 2b\hat{J}$

$\vec{C}$ já definido

$$\Rightarrow \boxed{\vec{M_R}_{/D} = +2bC_z\hat{I} - 2bC_x\hat{K}}$$

3.º $\dot{\vec{H_D}}$?

Seleccionando o R.N-J, β ∴  $\Rightarrow R^T = \begin{bmatrix} \cos\beta & -\sin\beta & 0 \\ \sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$\begin{cases} \vec{\omega} = \omega\hat{J} \\ \vec{\dot\omega} = \vec{0} \end{cases} \Rightarrow \begin{cases} \vec{w} = R^T\vec{\omega} = -\omega\sin\beta\,\hat{i} + \omega\cos\beta\,\hat{j} \\ \vec{\dot w} = R^T\vec{\dot\omega} = \vec{0} \end{cases}$$

$[I_G] \underset{\text{TABELA}}{=} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{mb^2}{3} & 0 \\ 0 & 0 & \frac{mb^2}{3} \end{bmatrix}$ NOTA $\frac{ml^2}{12} = \frac{m(2b)^2}{12} = \frac{mb^2}{3}$

$$\Rightarrow \dot{\vec{h_G}} = [I_G]\vec{\dot w} + \vec{w} \times ([I_G]\vec{w}) = -\frac{mb^2}{3}\omega^2 \sin\beta\cos\beta\,\hat{k}$$

$$\dot{\vec{H_G}} = R\,\dot{\vec{h_G}} = -\frac{mb^2}{3}\omega^2 \sin\beta\cos\beta\,\hat{K}$$

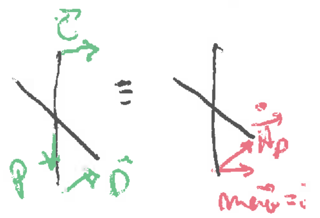
∴ $\dot{\vec{H_D}} = \dot{\vec{H_G}} + \vec{DG} \times m\vec{a_G}$ c/ $\vec{DG}$, $\vec{a_G}$ já definidos $\Rightarrow$

$$\dot{\vec{H_D}} = \dot{\vec{H_G}} + \vec{DG} \times m\cancel{\vec{a_G}}^{\vec{0}} \Rightarrow \dot{\vec{H_D}} = \dot{\vec{H_G}}$$

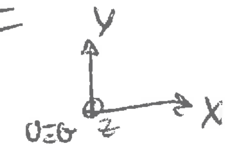
$$\Rightarrow \boxed{\dot{\vec{H_D}} = -\frac{mb^2}{3}\omega^2 \sin\beta\cos\beta\,\hat{K}}$$

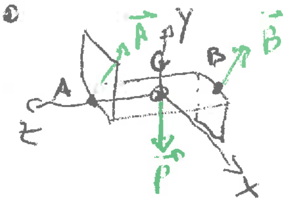
4.º Resolver $\vec{F_R} = m\vec{a_G}$ os cálculos as equações!

$\vec{M_R}_{/D} = \dot{\vec{H_D}}$



Ex 4

1.° 

2.° 

EM OXYZ

O, P.H., PARA NÃO SER HIPERESTÁTICO!

$$\begin{cases}\vec{A} = A_x\hat{I} + A_y\hat{J} + \cancel{A_z\hat{K}} \\ \vec{B} = B_x\hat{I} + B_y\hat{J} + B_z\hat{K} \\ \vec{P} = -mg\,\hat{J}\end{cases} \Rightarrow \vec{F}_R = \vec{A} + \vec{B} + \vec{P} \Rightarrow$$

$$\boxed{\vec{F}_R = (A_x + B_x)\hat{I} + (A_y + B_y - mg)\hat{J} + B_z\hat{K}}$$

$\vec{a}_G$? MRR c/ 1 NÍVEL c/ $\vec{\mu} = \dot{\vec{\mu}} = \vec{0}$ $\Rightarrow \vec{v}_G = \vec{a}_G = \vec{0} \Rightarrow \boxed{\vec{a}_G = \vec{0}}$

ii) APOIO B

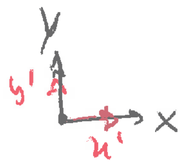
$$\vec{M}_{R/B} = \Sigma \underbrace{\vec{M}_{B\,IN}}_{M_0\hat{K}} + \sum_{i=3} \vec{BP_i} \times \vec{F}_{i\,ext.} = \underbrace{\vec{BB}}_{\vec{0}} \times \vec{B} + \vec{BG} \times \vec{P} + \vec{BA} \times \vec{A}$$

c/ $\begin{cases}\vec{BG} = a\hat{K} \\ \vec{BA} = 2a\hat{K}\end{cases}$

$$\Rightarrow \boxed{\vec{M}_{R/B} = (a\,m_T g - 2aA_y)\hat{I} + 2aA_x\hat{J} + M_0\hat{K}}$$

3.° $\dot{\vec{H}}_B$?

NÃO CONHECEMOS R NEM P ⇒ ESCOLHEMOS OR NI, NP ⇒ USAR TEOREMA DE STEINER →



$\Rightarrow \underset{\sim}{R}^T = \begin{bmatrix}1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1\end{bmatrix}$

$$\begin{cases}\vec{w} = \vec{0} \\ \vec{\alpha} = \alpha\hat{K}\end{cases} \Rightarrow \begin{cases}\vec{w} = \underset{\sim}{R}^T\vec{w} = \vec{0} \\ \vec{\alpha} = \underset{\sim}{R}^T\vec{\alpha} = \alpha\hat{k}\end{cases}$$

T. STEINER

$$\underset{\text{T. STEINER}}{[I_G]} = \begin{bmatrix}I_{xx} & -I_{xy} & \boxed{-I_{x3}} \\ -I_{xy} & I_{yy} & -I_{y3} \\ -I_{x3} & -I_{y3} & I_{33}\end{bmatrix}$$

COMO SÓ PODE HAVER ROTAÇÃO EM OZ ≡ G3, (w_3 ou α_3) SÓ CONSIDERAMOS A 3.ª COLUNA

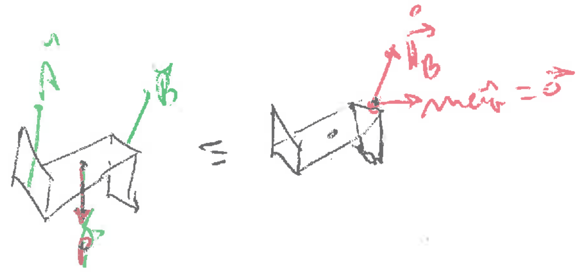
$$\begin{bmatrix}0 \\ -\dfrac{m_T a^2}{4} \\ \dfrac{m_T a^2}{4}\end{bmatrix}$$

$$\Rightarrow \dot{\vec{H}}_G = [I_G]\,\vec{\alpha} + \underbrace{\vec{w}}_{\vec{0}} \times ([I_G]\underbrace{\vec{w}}_{\vec{0}}) = [I_G]\begin{Bmatrix}0 \\ 0 \\ \alpha\end{Bmatrix} = \frac{\alpha\, m_T a^2}{4}(-\hat{\jmath} + \hat{k})$$

$$\hookrightarrow \dot{\vec{H}}_G = \underset{\sim}{R}\,\dot{\vec{H}}_G = \frac{\alpha\, m_T a^2}{4}(-\hat{I} + \hat{K})$$

$$\therefore \dot{\vec{H}}_B = \dot{\vec{H}}_G + \vec{BG} \times m\cancel{\vec{a}_G}^{\,\vec{0}} \Rightarrow \dot{\vec{H}}_B = \dot{\vec{H}}_G$$

$$\Rightarrow \boxed{\dot{\vec{H}}_B = \frac{\alpha\, m_T a^2}{4}(-\hat{I} + \hat{K})}$$



4.° RESOLVER $\vec{F}_R = m\vec{a}_G$ ESCREVER AS EQUAÇÕES

$\vec{M}_{R/B} = \dot{\vec{H}}_B$