Robótica Espacial

Rover Locomotion and Relative Positioning

Dead reckoning

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Summary

Perseverance Path 2021-2024

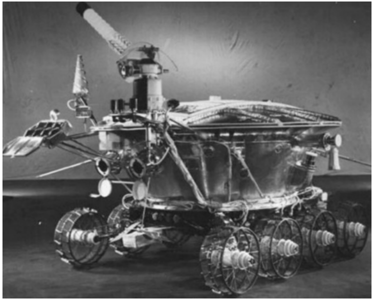
- Route taken by Perseverance Mars rover since its Feb-2021 landing at Jezero Crater to Jul-2024.
- As of October 2024, the rover has driven over 30 kilometers.

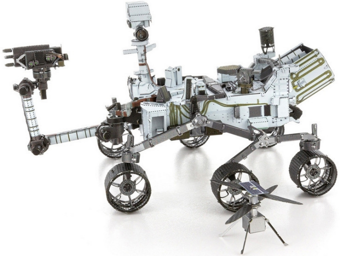
https://youtu.be/JWJfiYCo8ao?si=49e-VKgZcuH7bE-B

Basics

Challenges from yesterday and tomorrow

- Mobile robotics is complex because:
  - Perception is absolutely necessary
  - There are many sources of uncertainty
  - Many interactions among agents and the environment
- Planetary rovers have the additional challenges:
  - Higly unstructured terrain
  - Unexisting or very limited maps
  - Communication delay
  - Autonomy





Rovers Lunakhod, 1973 (top) and Perseverance, 2020 (bottom).

Locomotion

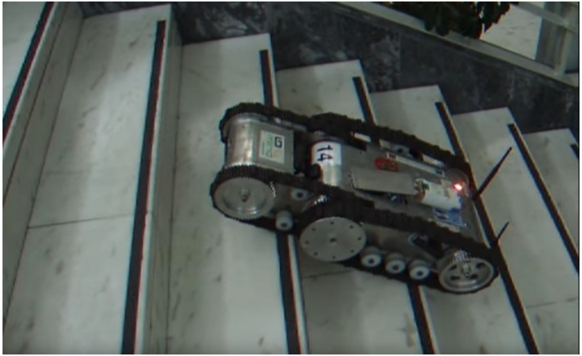
- Locomotion is the deliberate physical process that makes the robot move in its environment from one location to another
- Several solutions available:
  - Tracked locomotion
  - Legged locomotion
  - Wheeled locomotion
- Besides the land-oriented solutions there are also
  - Airborne robots (also known as drones, UAVs, etc.)
  - Marine robots (both surface and underwater)

Tracked locomotion

- Locomotion system well suited for robots that evolve in very rough terrain (e.g., in natural disaster situations)
- Great traction power - the track contact area with the ground is greater than the one provided by a wheel
- Change of direction is achieved by sliding the tracks, which makes it very difficult to use odometry as a method of localization
- Requires a large amount of power to turn
- The robots that use this type of movement are typically tele-operated

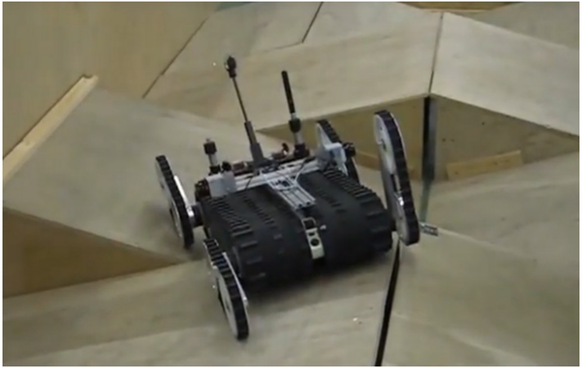
Tracked locomotion - two examples

Raposa (IST - Lisbon)



`https://youtu.be/4UxPvNu5x5s`

Quince Robot - Japan



`https://youtu.be/AkIWxFD2k5Y`
`https://youtu.be/J7GkNF5gLAk`

Legged locomotion

- Locomotion with legs is often based on living beings (as those that move in difficult environments)
- The implementations of this type of locomotion system in robots is complex:
  - Mechanical complexity
  - Stability
  - Power consumption
- Besides mechanical complexity, also kinematics and dynamics are very complex.
  - Their study deserves its own course!

Legged locomotion (some Boston Dynamics robots)

https://youtu.be/fn3KWM1kuAw

Wheeled locomotion

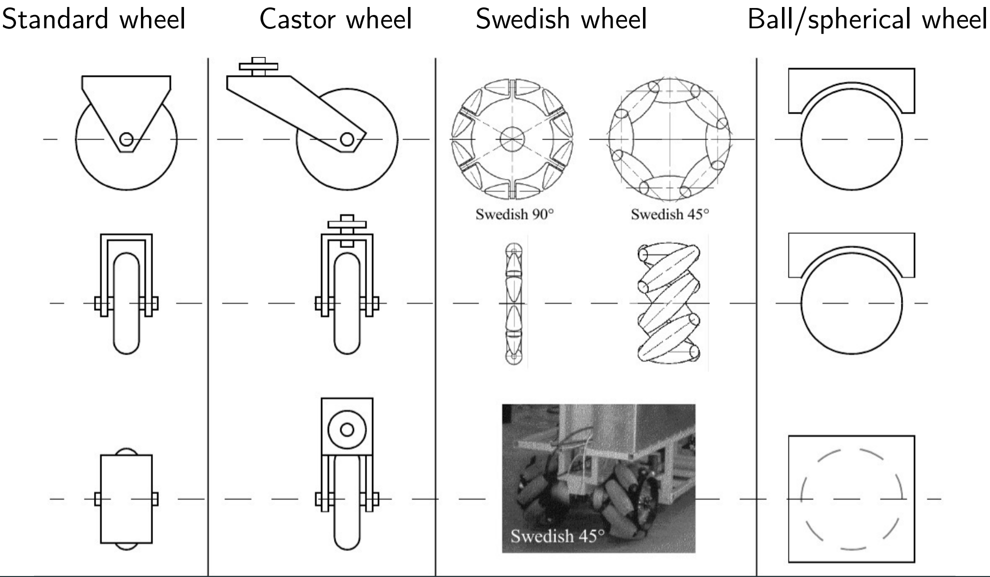
- The wheeled locomotion solution is the most suitable for common applications
  - ... because rolling is very efficient!
- The configuration and type of wheel to use is dependent on the application
- Main constraint: requires flat terrain (or at most slightly irregular)
- Bigger wheels allow the robot to overcome bigger obstacles. However:
  - Motors with larger torque are needed (or gearboxes with bigger reduction ratios, i.e., lower output speed for the same motor)
- Planetary Rovers have even more requirements.

Wheeled locomotion – static stability

- Two wheels
  - Minimum number of wheels to achieve stability
  - Center of gravity must be below the axle that links the wheels
- Three wheels
  - Stable configuration
  - Center of gravity must be inside the triangle formed by the wheels
- Four wheels
  - Stable configuration
  - Requires a suspension system to compensate for irregularities in the environment where the robot has to move
- More than four wheels
  - Configuration dependent
- Most planetary rovers have 6 or more wheels mainly for redundancy

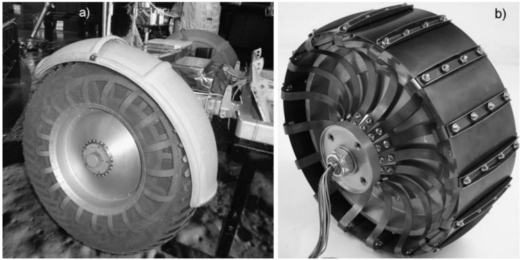
Wheel types

The four wheel types according to Siegwart et al.



Wheels for Rovers

- Planetary Rovers wheels have specific requirements. Follow some:
  - High traction, Slip minimization, Lightweight;
  - Non-pneumatic design, Shock absorption;
  - Wide temperature range, Radiation and UV resistance;
  - No maintenance, Resistance to wear, etc.



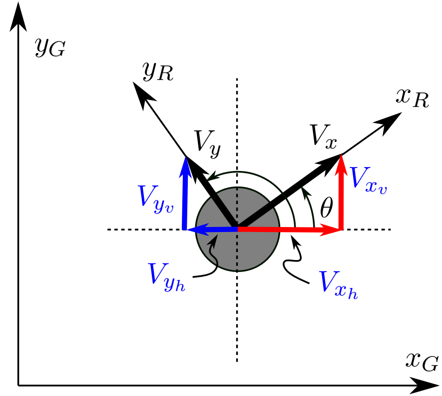
Kinematics

Kinematics

- Locomotion
  - The process that causes the movement of the robot from one location to another
  - In order to produce a motion, forces must be applied to the robot
- Dynamics
  - The study of motion, in which forces, masses and inertias are taken into account
- Kinematics
  - Modeling the motion without considering the forces that affect the motion

Local and global reference frames

- The **orthogonal rotation matrix** to relate velocities between frames
- R is a local reference frame on the robot and G is the global frame
- Generic coordinates on either frame are given by $\xi = \begin{bmatrix} x & y & \theta \end{bmatrix}^\mathsf{T}$
- $\dot\theta_G = \dot\theta_R = \dot\theta$ and the other velocities relate as shown:

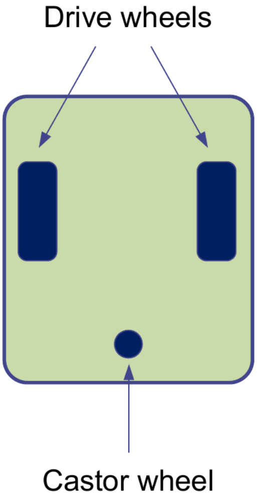


- $\dot x_G = V_{x_h} + V_{y_h} = V_x \cos\theta - V_y \sin\theta$
- $\dot y_G = V_{x_v} + V_{y_v} = V_x \sin\theta + V_y \cos\theta$
- $\begin{bmatrix} \dot x_G \\ \dot y_G \\ \dot\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot x_R \\ \dot y_R \\ \dot\theta \end{bmatrix}$
- $\begin{bmatrix} \dot x_R \\ \dot y_R \\ \dot\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot x_G \\ \dot y_G \\ \dot\theta \end{bmatrix}$

Differential drive

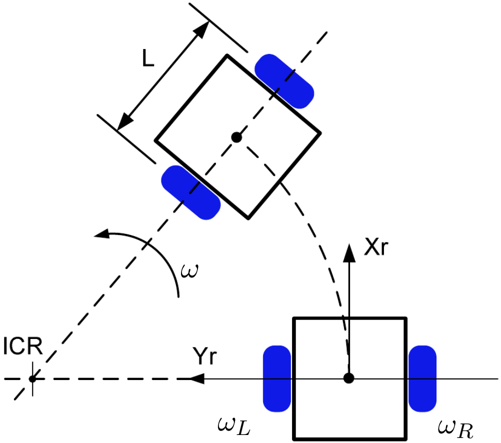
Differential drive

- Common configuration:
  - 2 active independent drive wheels
  - 1 or 2 passive castor wheels
- Robot follows a trajectory which is defined by the speed of each wheel
- Trajectory is sensitive to differences in the relative velocity of the two wheels
  - caused by asymmetries in motors and/or wheels
  - a small error results in a path different from that intended
- Easy mechanical implementation



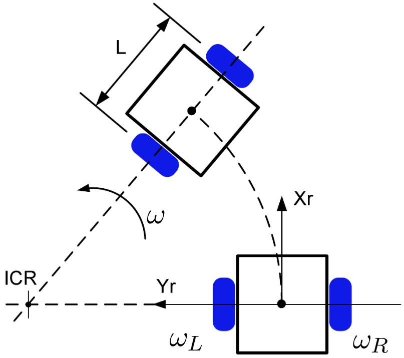
Differential drive – kinematics

- ω_R – angular velocity, right wheel
- ω_L – angular velocity, left wheel
- V_R – linear velocity, right wheel
- V_L – linear velocity, left wheel
- ω – angular velocity of the robot about ICR
- r – wheel radius
- L – distance between wheels



Differential drive – kinematics

- Kinematic model in **local** robot frame
  - $V_R(t) = \omega_R(t) \times r$
  - $V_L(t) = \omega_L(t) \times r$
  - $V_X(t) = \frac{V_R(t)+V_L(t)}{2} = \frac{r}{2}\left(\omega_L(t) + \omega_R(t)\right)$
  - $V_Y(t) = 0$
  - $\omega(t) = \frac{V_R(t)-V_L(t)}{L} = \frac{r}{L}\left(\omega_R(t) - \omega_L(t)\right)$



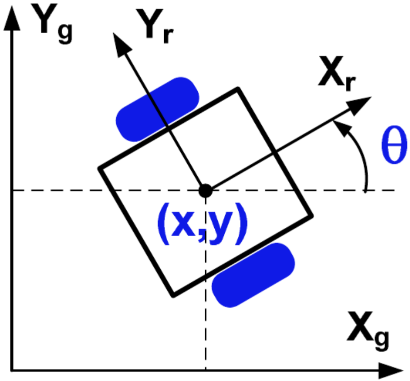
$$\begin{bmatrix} V_X(t) \\ V_Y(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ -r/L & r/L \end{bmatrix} \begin{bmatrix} \omega_L(t) \\ \omega_R(t) \end{bmatrix}$$

Differential drive – kinematics

- Kinematic model in **world** frame

$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & -\sin\theta(t) & 0 \\ \sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_X(t) \\ V_Y(t) \\ \omega(t) \end{bmatrix}$$

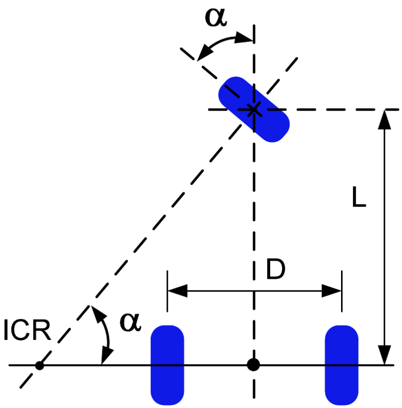
$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & 0 \\ \sin\theta(t) & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V(t) \\ \omega(t) \end{bmatrix}$$



Tricycle drive

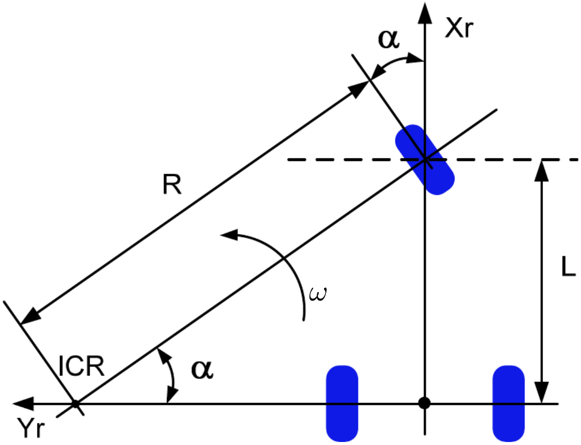
Tricycle drive

- Three wheels: two rear wheels and one (steering) front wheel
- Two possible configurations of traction:
  - Driving wheel on the front (rear wheels are passive) – easier to implement
  - Front wheel is passive - the two rear wheels are driving wheels (must use differential gear)
- Main problems of the front wheel drive configuration:
  - When going uphill, the driving wheel may loose traction due to the displacement of the center of mass
  - The traction contact area with the ground is half of the rear wheel drive configuration



Tricycle drive – kinematics

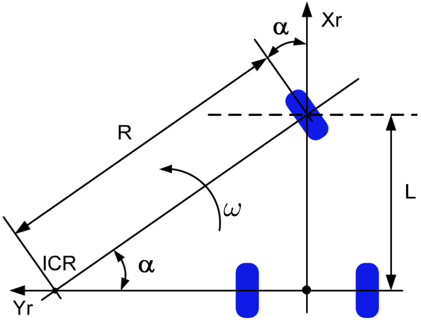
- V_S – linear velocity of the steering wheel
- ω_S – angular velocity of the steering wheel
- r – steering wheel radius
- α – steering angle
- ω – angular velocity of the robot about ICR



Tricycle drive – kinematics

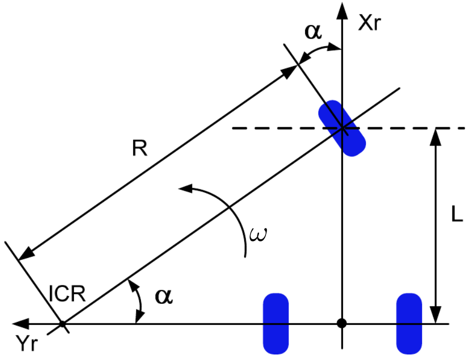
- $V_S = \omega_S \times r \rightarrow$ linear velocity of the steering wheel
- $\omega_S = \frac{V_S}{r} \rightarrow$ angular velocity of the steering wheel
- $R = \frac{L}{\sin\alpha}$
- $\omega = \frac{V_S}{R} = \dfrac{V_S \sin\alpha}{L} \rightarrow$ angular velocity of the robot about ICR

- Kinematic model in the **local** frame
  - $V_X(t) = V_S(t)\cos\alpha(t)$
  - $V_Y(t) = 0$
    - because the origin of the coordinate frame does not slide along Y_r!
  - $\omega(t) = \dfrac{V_S(t)}{L}\sin\alpha(t)$



Tricycle drive – kinematics

- Kinematic model in the **world** frame

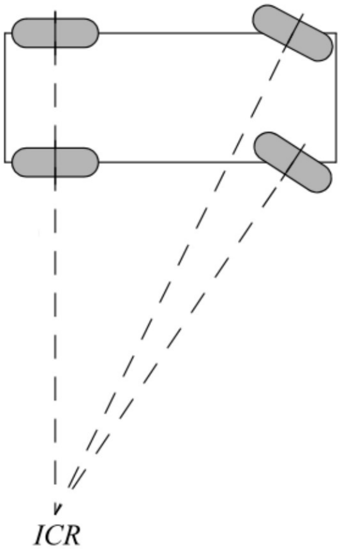


$$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta(t) & -\sin\theta(t) & 0 \\ \sin\theta(t) & \cos\theta(t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_S(t)\cos\alpha(t) \\ 0 \\ \frac{V_S(t)}{L}\sin\alpha(t) \end{bmatrix}$$

Ackermann steering

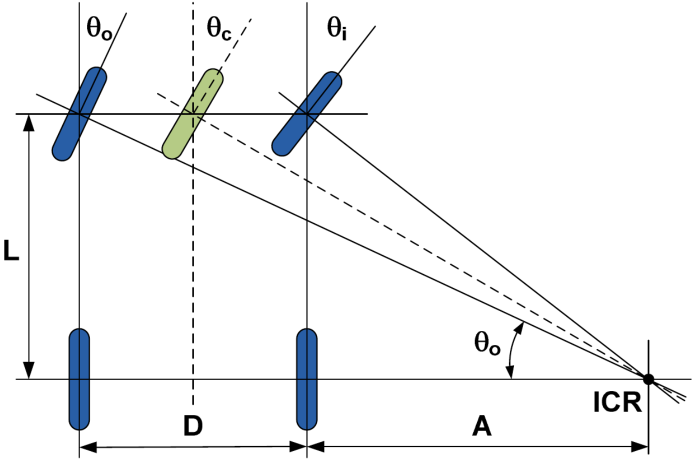
Ackermann steering

- Generally the method of choice for outdoor autonomous vehicles
- The inside front wheel is rotated slightly more than the outside wheel (reduces tire slippage)
- The extension of the axis of the two wheels intersects a common point - ICR
- 4 or 3 wheel system support rear and/or front traction
- A differential gear must be used in the traction axel (unless a single motorized wheel is used in that axel)



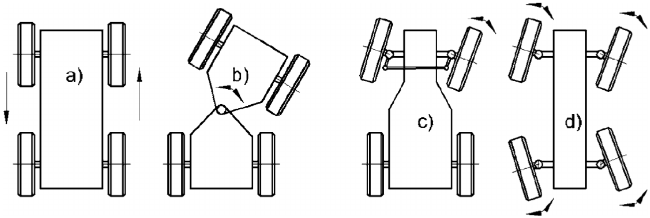
Ackermann steering

- $\cot\theta_i = \frac{A}{L}$
- $\cot\theta_o = \frac{A+D}{L}$
- $L\cot\theta_o = L\cot\theta_i + D$
- $\cot\theta_c = \cot\theta_o - \frac{D}{2L}$
- $\cot\theta_o - \cot\theta_i = \frac{D}{L}$



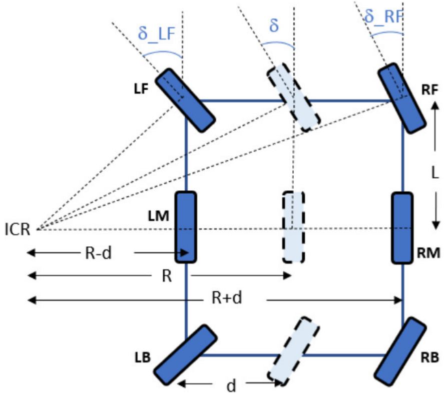
Rover steerings

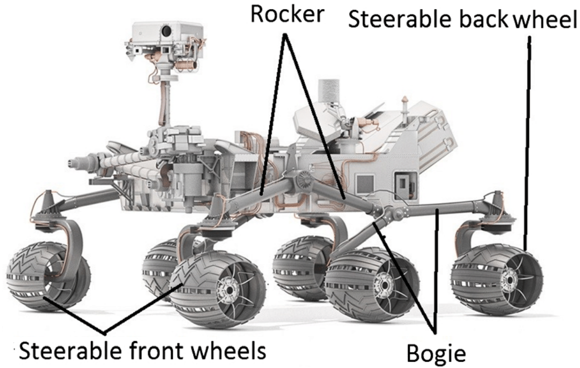
Potentially usable steerings in rovers



- a) Slip steering (very basic - wheels slip to change direction)
- b) Articulated steering
- c) Coordinated steering (similar to Ackermann)
- d) Independent steering (all 4 wheels are directional)

Six wheel Perseverance/Curiosity kinematics





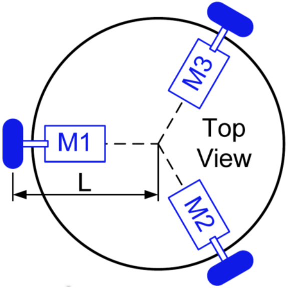
- The solution is similar to Ackermann except that 4 wheels can steer virtually allowing rotation in place
- More details and formulas can be found here:

  `https://www.mathworks.com/help/sm/ug/mars_rover.html`

Omnidirectional drive

Omnidirectional drive

- Uses Swedish wheels
- Each wheel has one independent drive motor
- Allows movement in any direction by setting appropriate speeds in each of the three motors
- Allows complex movements (for instance translation combined with rotation)
- Three wheels configuration:
  - the wheels are spaced 120°

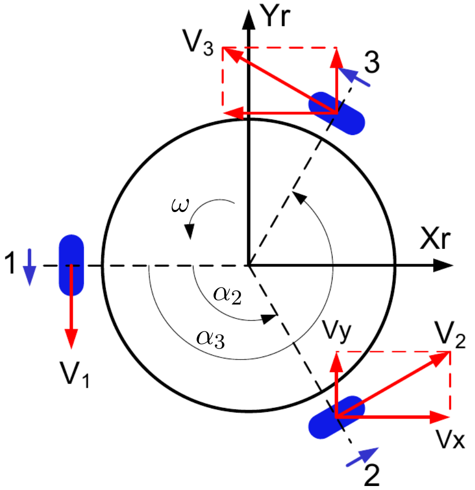


Omnidirectional drive – kinematics

- Knowing that
  - $\omega_1 = \frac{1}{r}\left(-V_y + L\omega\right)$
  - $\omega_2 = \frac{1}{r}\left(\frac{\sqrt{3}}{2}V_x + 0.5V_y + L\omega\right)$
  - $\omega_3 = \frac{1}{r}\left(-\frac{\sqrt{3}}{2}V_x + 0.5V_y + L\omega\right)$
- Solving for V_x, V_y and ω gives the robot velocity after the 3 wheels velocities:

$$\begin{bmatrix} V_x \\ V_y \\ \omega \end{bmatrix} = \begin{bmatrix} 0 & \frac{r}{\sqrt{3}} & -\frac{r}{\sqrt{3}} \\ -\frac{2r}{3} & \frac{r}{3} & \frac{r}{3} \\ \frac{r}{3L} & \frac{r}{3L} & \frac{r}{3L} \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$

- This is the kinematic model in the **local** frame



Omnidirectional drive (Example of CAMBADA robots)

http://y2u.be/PXq89EONEz0

Direct and Inverse Kinematics

Direct and Inverse Kinematics in Mobile Robots

- Direct kinematics provides the velocity (and ultimately the position by integration) of the robot
- based on the internal actuation (wheel velocities and steering) and mechanical parameters (wheel radius, wheel separation or distance to local coordinate frame origin, etc.)
  - $\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \mathbf{f}(\omega_1(t), \omega_2(t), \cdots, \alpha(t))$
- Inverse kinematics concerns the determination of the robot actuated variables (wheel velocities and steering) to accomplish a given path/trajectory/posture.
  - $\begin{bmatrix} \omega_1(t) \\ \omega_2(t) \\ \vdots \\ \alpha(t) \end{bmatrix} = \mathbf{g}(x(t), y(t), \theta(t))$

Reminder of Local and Global Robot Kinematics

- A robot has its local kinematics (3 velocity expressions) that depend on individual wheel angular velocity, wheel radius and steering orientation (when applicable):
  - V_x
  - V_y
  - ω
- The velocity seen in the global reference frame is given by:
  - $\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ \omega \end{bmatrix} = \mathbf{R}^{-1}(\theta) \begin{bmatrix} V_x \\ V_y \\ \omega \end{bmatrix}$
- And conversely, velocity in local frame relates to velocity seen from gobal frame as:
  - $\begin{bmatrix} V_x \\ V_y \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \omega \end{bmatrix} = \mathbf{R}(\theta) \begin{bmatrix} \dot{x} \\ \dot{y} \\ \omega \end{bmatrix}$
- It must be noticed that all these quantities $(V_x, V_y, \theta, x, y, \omega)$ depend on time t.

Kinematics Matrix of a mobile robot

Summary of direct kinematics of common robots

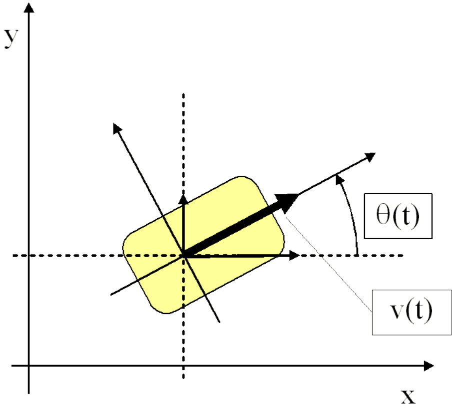
- In each type of robot, **local** velocity depends on wheel parameters:
  - $\begin{bmatrix} V_x & V_y & \omega \end{bmatrix}^\intercal = \mathbf{f}(\omega_i, r_i, \alpha_i, L_i)$, $i = \{1, 2, ...\}$
    - $\omega_i \rightarrow$ angular velocity of wheel i
    - $r_i \rightarrow$ radius of wheel i
    - $\alpha_i \rightarrow$ Steering direction of wheel i (when applicable)
    - $L_i \rightarrow$ Wheel distance in local frame (meaning varies with topologies)
- Or in general matrix form:
  - $\begin{bmatrix} V_x \\ V_y \\ \omega \end{bmatrix} = \mathbf{M} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \end{bmatrix}$
- Where $\mathbf{M}$ has values that depend on individual robot tipology like:
  - Differential drive, Tricycle, Omnidirectional, etc.
  - $\mathbf{M}$ depends on fixed parameters, but may also depend on variable parameters (like steering angles).
  - We may call $\mathbf{M}$ the **kinematics matrix** of the robot (though unusual!)

Examples of $\mathbf{M}$ for several robots

- Differential drive
  - $$\begin{bmatrix} V_x(t) \\ V_y(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} r/2 & r/2 \\ 0 & 0 \\ -r/L & r/L \end{bmatrix} \begin{bmatrix} \omega_L(t) \\ \omega_R(t) \end{bmatrix}$$
- Tricycle
  - $$\begin{bmatrix} V_x(t) \\ V_y(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} r\cos\alpha(t) \\ 0 \\ r/L\sin\alpha(t) \end{bmatrix} \omega_S(t)$$
- Omnidirectional
  - $$\begin{bmatrix} V_x(t) \\ V_y(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} 0 & \frac{r}{\sqrt{3}} & -\frac{r}{\sqrt{3}} \\ -\frac{2r}{3} & \frac{r}{3} & \frac{r}{3} \\ \frac{r}{3L} & \frac{r}{3L} & \frac{r}{3L} \end{bmatrix} \begin{bmatrix} \omega_1(t) \\ \omega_2(t) \\ \omega_3(t) \end{bmatrix}$$

Generic expression of direct kinematics for Diferencial Drive and similars

- Obtain global coordinates of robot after the controlled variables (velocities):
  - $\dot{x}(t) = V(t)\cos\theta(t)$
  - $\dot{y}(t) = V(t)\sin\theta(t)$
  - $\dot{\theta}(t) = \omega(t)$
- By integration we obtain:
  - $x(t) = \int_0^t V(\tau)\cos\theta(\tau)\mathrm{d}\tau + x_0$
  - $y(t) = \int_0^t V(\tau)\sin\theta(\tau)\mathrm{d}\tau + y_0$
  - $\theta(t) = \int_0^t \omega(\tau)\mathrm{d}\tau + \theta_0$
    - Where (x_0, y_0, θ_0) is the initial posture.



Implementing Inverse Kinematics

How to perform inverse kinematics?

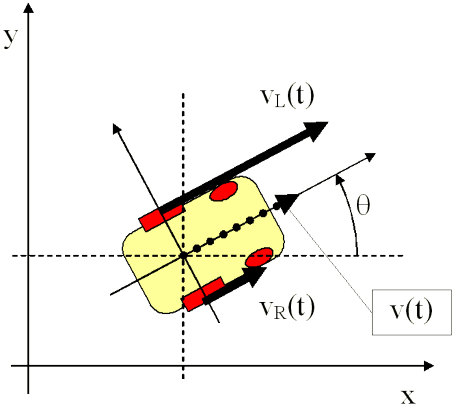
- Given a trajectory $(x(t), y(t), \theta(t))$ the challenge is then to solve or manipulate the previous equations and obtain:
  - $V(t)$
  - $\omega(t)$
- and then obtain the individual wheel angular velocities (and steering angles when applicable)
  - $\omega_1(t)$
  - $\omega_2(t)$
  - etc.

Challenge in performing inverse kinematics

- Problem in determining which wheel velocities (like $V_L(t)$ and $V_R(t)$ in differential drive robots) are required to obtain desired paths for $x(t)$, $y(t)$ and $\theta(t)$
- Complex issue for several reasons:
  - Cases of coupled inputs
    - Three inputs (x, y, θ) for only two independent controlled variables, (like V_L and V_R in differential drive or V_S and α in tricycle), although in ominiriectional robot this is not a problem!
  - Kinematic functions are not integrable in the general case.
  - There is an infinite number of solutions in many cases.
- Only a few simple cases can be solved, namely if the speeds are constant like these examples in differential drive:
  - Different velocities (circular movement, appropriate x and y)
  - Equal velocities (straight line movement)
  - Symmetrical velocities (rotating and turning itself in the case of the differential robot)

Particular case of differential drive robot

- Following the generic expression presented earlier we have:
  - $x(t) = \frac{1}{2}\int_0^t [V_R(\tau) + V_L(\tau)] \cos\theta(\tau) \mathrm{d}\tau$
  - $y(t) = \frac{1}{2}\int_0^t [V_R(\tau) + V_L(\tau)] \sin\theta(\tau) \mathrm{d}\tau$
  - $\theta(t) = \frac{1}{L}\int_0^t [V_R(\tau) - V_L(\tau)] \mathrm{d}\tau$



Simple case of inverse kinematics in differential drive

- Consider a differential drive robot with wheel velocities V_L and V_R and wheel separation L, where $\omega = \frac{V_R - V_L}{L}$ and $V = \frac{V_L + V_R}{2}$;
- If we restrict to: $V_L(t) = V_L$ (const.) and $V_R(t) = V_R$ (const.) and $V_L \neq V_R$ we have the following:
  - $\theta(t) = \int_0^t \omega \mathrm{d}\tau = \omega \int_0^t \mathrm{d}\tau = \omega t$
  - $x(t) = V \int_0^t \cos(\omega\tau)\mathrm{d}\tau = \frac{V}{\omega}\sin(\omega t)$
  - $y(t) = V \int_0^t \sin(\omega\tau)\mathrm{d}\tau = -\frac{V}{\omega}\left[\cos(\omega\tau)\right]_0^t = \frac{V}{\omega}\left(1 - \cos(\omega t)\right)$
- We assumed that $(x_0, y_0, \theta_0) = (0, 0, 0)$
- Note: In the previous expressions, the three coordinates $x(t), y(t), \theta(t)$ cannot be specified independently. Only two of them can be set independently and the third will be derived from them:
  - $x(t), y(t) \rightarrow \theta(t)$
  - $\theta(t), x(t) \rightarrow y(t)$
  - $\theta(t), y(t) \rightarrow x(t)$

Example exercise of a specific calculation

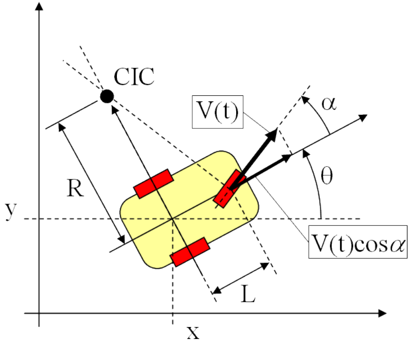
- It is intended that a differential drive robot, with $L = 50\,\text{cm}$ between wheels, starting at (0,0,0) arrives in $t = 5\,\text{s}$, $2\,\text{m}$ further ahead (x) with a final orientation (θ) of $+30°$. What should be the constant speeds of the two wheels, V_L and V_R?
- Base expressions for differential drive in these particular conditions:
  - $\theta(t) = \omega t$
  - $x(t) = \frac{V}{\omega}\sin(\omega t)$
  - $y(t) = \frac{V}{\omega}(1 - \cos(\omega t))$
  - $\omega = \frac{V_R - V_L}{L}$
  - $V = \frac{V_L + V_R}{2}$
- Calculations are:
  - $\theta(5) = 5\omega \Leftrightarrow \frac{\pi}{6} = 5\omega \Leftrightarrow \omega = \frac{\pi}{30}$
  - $x(5) = 2 \Leftrightarrow 2 = \frac{V}{\omega}\sin(5\omega) \Leftrightarrow V = 2\pi/15$
  - $\begin{cases} \frac{V_R + V_L}{2} = \frac{2\pi}{15} \\ \frac{V_R - V_L}{0.5} = \frac{\pi}{30} \end{cases}, \begin{cases} V_R + V_L = \frac{4\pi}{15} \\ V_R - V_L = \frac{\pi}{60} \end{cases}, \begin{cases} 2V_R = \frac{17\pi}{60} \\ V_L = V_R - \frac{\pi}{60} \end{cases}$
- And finally: $V_R \approx 0.445\,\text{m/s}$, $V_L \approx 0.393\,\text{m/s}$.
- Notice that, as previously mentioned, we can not impose simultaneously $y(t)$ because variables are coupled!

Variant of the previous example

- What if the geometric specification would be in $y(t)$ and not in $x(t)$? For example, $y(5) = 1\,\text{m}$?
- Using the equations presented before ($V = \frac{V_R+V_L}{2}$, $\omega = \frac{V_R-V_L}{0.5}$), the calculations would be as follows:
  - $\theta(5) = 5\omega \Leftrightarrow \frac{\pi}{6} = 5\omega \Leftrightarrow \omega = \frac{\pi}{30}$ (the same as before)
  - $y(5) = \frac{V}{\omega}\left(1 - \cos(5\omega)\right) = 1 \Leftrightarrow \omega = V\left(1 - \frac{\sqrt{3}}{2}\right) \approx 0.13397V$
  - $\begin{cases} V_R + V_L \approx 1.5632 \\ V_R - V_L \approx 0.0524 \end{cases}$
- And finally: $V_R \approx 0.8078\,\text{m/s}$, $V_L \approx 0.7554\,\text{m/s}$.
  - These velocities are slightly different from the earlier case, which is expectable because the final position is different, although the orientation is the same.

Inverse kinematics of a tricycle

- The direct kinematics of the tricycle is conceptually simpler than the differential drive:
  - $$\begin{bmatrix} \dot{x}(t) \\ \dot{y}(t) \\ \dot{\theta}(t) \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} r\cos\alpha(t) \\ 0 \\ r/L\sin\alpha(t) \end{bmatrix} \omega_S(t)$$
- Or, expanding [being $V(t) = \omega_S(t)r$]:
  - $$\begin{cases} \dot{x}(t) = V(t)\cos\alpha(t)\cos\theta(t) \\ \dot{y}(t) = V(t)\cos\alpha(t)\sin\theta(t) \\ \dot{\theta}(t) = \frac{V(t)}{L}\sin\alpha(t) \end{cases}$$
- Giving the following inverse kinematics
  - $$\begin{cases} \theta(t) = \frac{1}{L}\int_0^t V(\tau)\sin\alpha(\tau)\mathrm{d}\tau \\ x(t) = \int_0^t V(\tau)\cos\alpha(\tau)\cos\theta(\tau)\mathrm{d}\tau \\ y(t) = \int_0^t V(\tau)\cos\alpha(\tau)\sin\theta(\tau)\mathrm{d}\tau \end{cases}$$



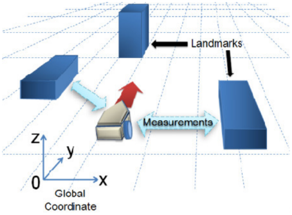
Inverse kinematics of tricycle

- It is clear that these equations are unsolvable analytically for the general case
  - $$\begin{cases} \theta(t) = \frac{1}{L}\int\limits_0^t V(\tau)\sin\alpha(\tau)\mathrm{d}\tau \\ x(t) = \int\limits_0^t V(\tau)\cos\alpha(\tau)\cos\theta(\tau)\mathrm{d}\tau \\ y(t) = \int\limits_0^t V(\tau)\cos\alpha(\tau)\sin\theta(\tau)\mathrm{d}\tau \end{cases}$$
- But if velocity and steering angle are made **constant, some solutions are possible (but of course, it is still impossible to impose all three variables x, y and θ at the same time to calculate V and α!).**
  - $$\begin{cases} \theta(t) = \frac{V}{L}\sin\alpha\int\limits_0^t \mathrm{d}\tau = t\frac{V}{L}\sin\alpha \\ x(t) = V\cos\alpha\int\limits_0^t \cos\theta(\tau)\mathrm{d}\tau \\ y(t) = V\cos\alpha\int\limits_0^t \sin\theta(\tau)\mathrm{d}\tau \end{cases}$$

Robot Localization

Introduction

- Robot localization is the process of determining where a mobile robot is located with respect to its environment.
- More specifically, determine the location (position and orientation) of the mobile robot in a map or a coordinate system.
- In a typical robot localization scenario, a map of the environment is available and the robot is equipped with sensors that observe the environment as well as monitor its own motion.
- The localization problem consists of estimating the robot position and orientation within the map using information gathered from the sensors.
- Robot localization techniques need to be able to deal with noisy observations and generate not only an estimate of the robot location but also a measure of the uncertainty of the location estimate.



The localization challenge

- Localization (positioning) can be done by
  - Internal incremental means (**dead-reckoning**)
  - External absolute references
- Sensors are required in both cases
  - Motion measurements, which are **integrated**
  - Detection of **features with known locations** and calculate the **relative position** to them
  - Reception of **absolute coordinates** from an externally **known** referenced system (like GPS)
- Integration of motion measurements exhibits incremental errors and uncertainties, but detection of external features also has uncertainties!
- Probabilistic techniques are used to improve the estimation of the localization by using models of motion and perception
  - Kalman filtering, Particle filter (or Monte Carlo) localization, Markov localization, etc.
- In this context we will only address relative localization or "dead-reckoning"

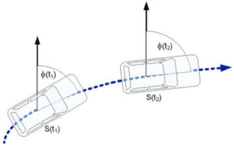
Models

- To perform localization estimation two types of models are required:
  - Robot and motion models
  - Sensor model
- Motion models are the **kinematics equations** specific to each typology of robot
- Sensor model
  - The relationship between the observations from the sensors and the location of the robot in the map.
  - The sensor model is dependent on the characteristics of the sensor mounted on the robot as well as on the way the map of the environment is represented
- The map of the environment is typically defined by:
  - **Coordinates** of known landmarks or features
  - **Occupancy grid** where the status of each grid cell defines whether the area represented by the cell is occupied or free space.

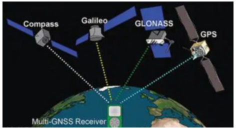
General approaches for localization (positioning)

- Predetermined path (wire guided, line on the pavement, etc. . .)
  - Advantage: the entire route is already delimited.
  - Limitations: low flexibility, precarious installation.
  - Following this path is mainly a technological issue and the problem of localization is simpler!

- Relative positioning systems
  - Determination relative to the last known position based on the measured movement.



- Absolute positioning systems
  - Determination in relation to a fixed coordinate system



- Mixed systems
  - Combine both types

Relative Positioning

Relative Positioning Systems (Dead-reckoning)

- Principle
  - Obtain the current position (location) based on the knowledge of previous position and the physical evolution of quantities associated with the movement of the mobile system.
  - These are actually incremental positioning systems.
- Examples
  - Odometry
  - Inertial navigation

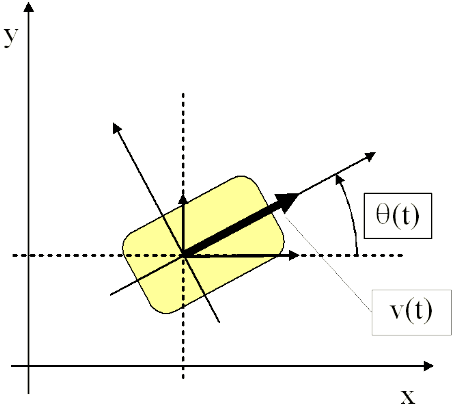
Odometry

Odometry

- Count the wheel angular displacements
- Fully self-contained system
- Errors can grow without limit and without control
- Sensors for odometry
  - Encoders
    - Optical (absolute and incremental)
    - Magnetic
    - Capacitive
  - Potentiometers
    - They present a resistance proportional to an angular displacement. Usually limited to systems with few revolutions (less than one sometimes)
  - Tachometers
    - Devices that measure an angular velocity and, through appropriate transformations, a linear velocity.

Odometry

- Start with basic kinematics equations
  - $\dot{x}(t) = V(t)\cos\theta(t)$
  - $\dot{y}(t) = V(t)\sin\theta(t)$
  - $\dot{\theta}(t) = \omega(t)$
- Derive the inverse kinematics continuous model
  - $x(t) = \int_0^t V(\tau)\cos\theta(\tau)\mathrm{d}\tau$
  - $y(t) = \int_0^t V(\tau)\sin\theta(\tau)\mathrm{d}\tau$
  - $\theta(t) = \int_0^t \omega(\tau)\mathrm{d}\tau$



- Establish the discrete version assuming $\Delta\theta_i$ is very small in the measuring interval Δt
  - $\theta_i = \theta_{i-1} + \Delta\theta_i$
  - $x_i = x_{i-1} + \Delta l_i \cos\theta_i$
  - $y_i = y_{i-1} + \Delta l_i \sin\theta_i$
    - Where $\Delta l_i \approx V_i \Delta t$

Sensors for inertial based positioning

Sensors for Inertial Navigation I

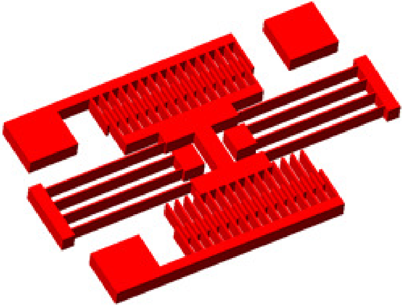
- Accelerometers
  - Indicate acceleration along one direction
  - Basic principle: measurement of a force $F = ma$ through its effects (resistive, piezoelectric, etc.)
  - The acceleration must be integrated twice to obtain the position.
  - In practice, the discrete variant of integration is used: a_i is the acceleration measured at the end of the interval Δt_i and which is normally assumed to be constant during this time interval

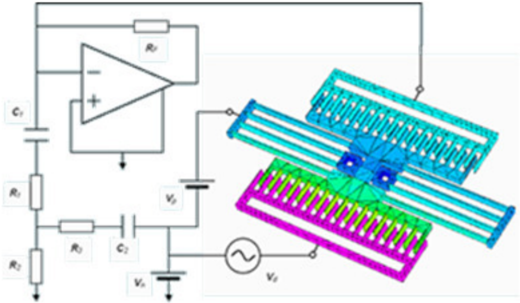
$$s = \iint a\,(t)\mathrm{d}t\mathrm{d}t \qquad s = \sum_{i=1}^{N}\left[\left(\sum_{i=1}^{N} a_i \Delta t_i\right)\Delta t_i\right]$$

- Uncertainties (systematic) in $a(t)$ accumulate and can result in great uncertainty in the final result.

Integrated Accelerometers - MEMS Technology

- Micro-Electro-Mechanical Systems (MEMS)
- The accelerometer is micro-machined
- Based on capacitive or resistive transduction
  - Moving parts affect geometries by changing electrical capacities and/or electrical resistances





Sensors for Inertial Navigation II

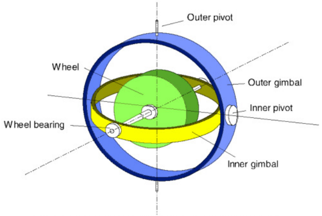
- The Gyroscopes (Gyros)
  - Indicate an angular velocity
  - Integration of these measurements results in absolute heading (orientation)
- Main types of Gyros
  - Mechanics
    - Delicate; expensive; ...
  - Optics
    - No mechanical parts.
    - Based on optical interference from laser beams.
    - More economical with very reasonable accuracies
    - Drift problem
  - Piezoelectric
- Some systems combine accelerometers and gyroscopes - the INS (Inertial Navigation Systems)

Types of Gyroscopes

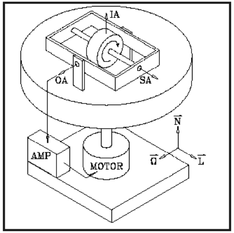
- Mechanical
    - Principle of conservation of angular momentum
    - Precession effect
    - Experience of rotating bicycle wheel when axle position is changed
    - Force measurement / angular deviation
- Optical (Fiber Optical Gyroscope - FOG)
    - Based on Sagnac effect
        - Active or passive resonator
        - Open or closed loop fiber optic interferometer (IFOG)
        - Fiber optic resonator (RFOG)
    - Measurement of phase differences (propagation time)
- Piezoelectric
    - Based on Coriolis force
    - Vibrating material
    - Piezoelectric measurement

Mechanical Gyroscope

- A mass is kept in constant rotation (eg with a precise servomotor)
- The mass rotation axis is supported by a system that can rotate orthogonally
- The entire system is supported by an external structure to which the rotation is applied (on an axis orthogonal to the other two)
- The angular deviation of the second axis indicates the angular velocity of the outer system
- There are other variants but the principle is the same...



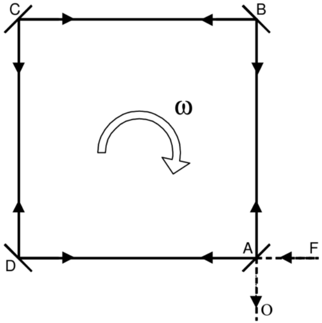
Typical two-axis mechanical gyroscope configuration [Everett, 1995].

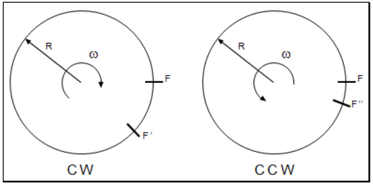


Sagnac Effect

- Phenomenon observed by Georges Sagnac (1896-1926) in 1913
- Propagate light along a closed path
- Different propagation times depending on the direction of rotation of the path (clockwise or counterclockwise)

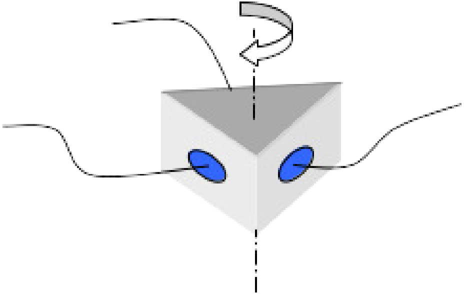
$$\Delta t = \frac{4\pi R v}{c^2 - v^2} \approx \frac{4\pi R^2}{c^2}\omega$$

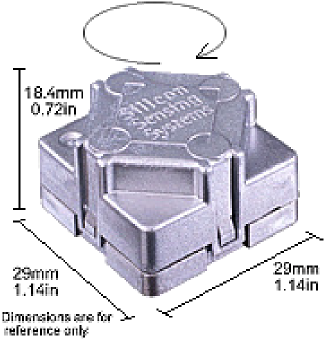




Piezoelectric gyros

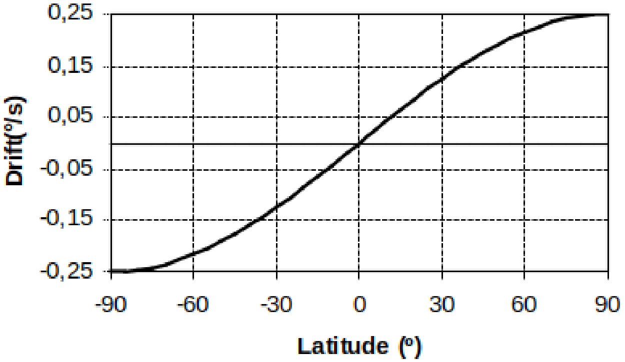
- Typically a triangular quartz prism
- Excitations on one side at resonant frequency
- The other two faces have transducers that measure vibrations affected by device rotation
- Coriolis forces affect the propagation of vibrations between faces





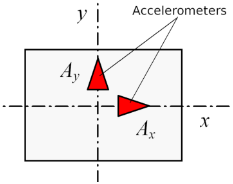
The Drift Question

- Residual indication of angular velocity when it is actually immobilized!
- In high precision applications it is necessary to take into account the earth rotation which is an apparent drift!
    - Its value depends on latitude and is in the worst case $360°$/24h or $0.25°$/min



An exercise with accelerometers

- Robot with two orthogonal accelerometers with readings every $100\,\text{ms}$
- Determine the velocities and position knowing that the robot starts from a stopped position.
- $s = \sum_{i=1}^{N} \left[\left(\sum_{i=1}^{N} a_i \Delta t_i \right) \Delta t_i \right]$



			Velocity			Space run		
t	A_x	A_y	In x $\sum A_x$	In y $\sum A_y$	Ampl $\lvert \sum A \rvert$	In x $\sum \sum A_x$	In y $\sum \sum A_y$	Linear $\lvert \sum \sum A \rvert$
0.0	0.10	0.00						
0.1	0.05	0.00	0.010	0.000	0.0100	0.0010	0.0000	0.0010
0.2	0.05	0.00	0.015	0.000	0.0150	0.0025	0.0000	0.0025
0.3	0.04	0.05	0.020	0.000	0.0200	0.0045	0.0000	0.0045
0.4	0.04	0.04	0.024	0.005	0.0245	0.0069	0.0005	0.0069
0.5	0.03	0.04	0.028	0.009	0.0294	0.0097	0.0014	0.0098
0.6	0.00	0.03	0.031	0.013	0.0336	0.0128	0.0027	0.0131
0.7	0.00	0.01	0.031	0.016	0.0349	0.0159	0.0043	0.0165
0.8	-0.05	-0.07	0.031	0.017	0.0354	0.0190	0.0060	0.0199
0.9	-0.10	-0.10	0.026	0.010	0.0279	0.0216	0.0070	0.0227
1.0			**0.016**	**0.000**	**0.0160**	**0.0232**	**0.0070**	**0.0242**
			0.018	0.000	0.0176	0.0255	0.0070	0.0265

- Red values would occur if the measurements of A_x had 10% error!