Dinâmica de Fluidos

$K_B = 1,373 \cdot 10^{-23}\ J/K$

Ficha 1 / 7

1 $\langle v^2 \rangle = \int_0^\infty v^2 f(v)\, dv = \frac{3 K_B \cdot T}{m}$

$m = M \cdot n \quad \frac{1}{6,023 \cdot 10^{23}} \to 1$ partícula

$m_{H_2} = 3,35381 \cdot 10^{-27}\ kg$

$m_{He} = 6,64536 \cdot 10^{-27}\ kg$

$m_{O_2} = 5,31297 \cdot 10^{-26}\ kg$

$\langle v^2 \rangle_{H_2} = 3,63718 \cdot 10^6\ m^2/s^2 \Rightarrow V_{Rms} = 1907\ m/s$

$\langle v^2 \rangle_{He} = 1,83563 \cdot 10^6\ \text{"} \Rightarrow V_{Rms} = 1355\ m/s$

$\langle v^2 \rangle_{O_2} = 229597\ m^2/s^2 \Rightarrow V_{Rms} = 479\ m/s$

$\langle KE \rangle = \frac{1}{2} m \langle v^2 \rangle = \frac{3 K_B T}{2}$

$PE = -G \cdot \frac{mM}{R}$

$\Delta Em = 0 \Rightarrow PE_{inicial} + \langle KE \rangle_{escape} = \underset{R \to +\infty}{PE} + \underset{no\ escape}{KE} \Rightarrow$

mínimo

$\Rightarrow -\frac{G \cancel{m} M_{terra}}{r_{terra}} + \frac{1}{2} \cancel{m} \langle v_{esc}^2 \rangle = 0 + 0 \Rightarrow$

$\Rightarrow \langle v_{esc}^2 \rangle = \frac{2 G M_{terra}}{r_{terra}} = \frac{2 \cdot 6,67 \cdot 10^{-11} \cdot 5,9776 \cdot 10^{24}}{6,378 \cdot 10^6} \Rightarrow$

$\Rightarrow \langle v_{esc} \rangle^2 = 1,25025 \cdot 10^8 \Rightarrow \langle v_{esc} \rangle = \underline{11181\ m/s}$

2 Num gás de Boltzmann: $\langle v \rangle = \int_0^\infty v f(v)\, dv = \sqrt{\frac{8 K_B T}{\pi m}}$

$V_{Rms} = \sqrt{\frac{3 K_B T}{m}}$ $\quad$ erro $= \frac{|\langle v \rangle - V_{Rms}|}{\langle v \rangle} = \frac{\left| \sqrt{\frac{8}{\pi}} - \sqrt{3} \right|}{\sqrt{\frac{8}{\pi}}} = \underline{0,0854}$

3 a) $pV = N \cdot k_B \cdot T \Rightarrow p = \dfrac{N}{V} k_B T \xRightarrow{\text{1 par}} \dfrac{1}{V} \cdot k_B T$

$\langle KE \rangle = \dfrac{1}{2} m \langle v^2 \rangle = \dfrac{1}{2} \cancel{m} \cdot 3 \dfrac{k_B T}{\cancel{m}} = \dfrac{3}{2} k_B T$ → para 1 partícula

$\dfrac{\langle KE \rangle}{V}$ = densidade de energia cinética = u

$$\boxed{u = \frac{3}{2} p}$$

b) $u = \dfrac{3}{2} \cdot \overset{\approx 10^5}{p_{atm}} = \boxed{150 \cdot 10^3 \ J/m^3}$

4 → Temperatura aumenta → + agitação corpuscular
→ Saída de partículas da sala

} Energia mantém-se $\simeq$ constante

5 a) $pV = N k_B \cdot T \Rightarrow$

$\Rightarrow p = \dfrac{N}{V} \cdot k_B \cdot T = 500 \cdot (10^3)^2 \cdot 1{,}373 \cdot 10^{-23} \cdot 50 =$

$= 3{,}4325 \cdot 10^{-13}$ Pa

b) $m_{Nuvem} = 100$ mol $\qquad M_H = 1{,}01$ g/mol

$m_{Nuvem} = 1{,}99 \cdot 10^{32}$ kg

$n = \dfrac{m}{M} = \dfrac{1{,}99 \cdot 10^{32}}{1{,}01 \cdot 10^{-3}} = 1{,}9703 \cdot 10^{35}$ mol

$pV = nR^* T \Rightarrow V = \dfrac{1{,}9703 \cdot 10^{35} \cdot 8{,}314 \cdot 50}{3{,}4325 \cdot 10^{-13}} = 2{,}38617 \cdot 10^{50} \ m^3 =$

$= \dfrac{2{,}38617 \cdot 10^{50}}{8{,}468 \cdot 10^{47}}$ ano-luz cúbico =

1 ano luz cúbico $= 8{,}468 \cdot 10^{47} \ m^3$

= 282 ano-luz cúbico

Escanear com AltaScanner

Dinâmica de Fluidos - Ficha 2 / 8

1

$$\frac{10^{12}\text{ moléculas}}{1\, mm^3}$$

$T_{ambiente}$

$p_{ar} = 101325$ Pa

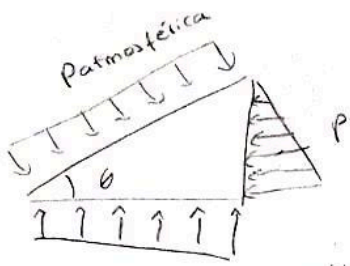
$\rho_{ar} = 1{,}29$ kg/m^3

$$pV = N \cdot k_B \cdot T \Rightarrow p = \frac{N}{V} \cdot k_B \cdot T \Rightarrow$$

$$\Rightarrow p = \frac{10^{12}}{(10^{-3})^3} \cdot 1{,}38 \cdot 10^{-23} \cdot 293{,}15 = \underline{4{,}045 \text{ Pa}}$$

2 Nop

3

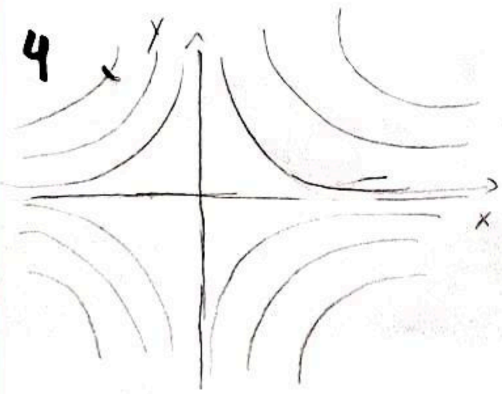


p atmosférica → Varia linearmente

pressão chão - equilibra f verticais por causa do peso

∴ forças horizontais ñ equilibradas → Resultam tensões de corte

4



$$\begin{cases} \partial x = kx \\ \partial y = -ky \end{cases} \Rightarrow \begin{cases} \frac{\partial x}{x} = -\frac{\partial y}{y} \end{cases}$$

$$\Rightarrow \ln(x) = -\ln(y) + C \Rightarrow$$

$$\Rightarrow \ln(xy) = C \Rightarrow$$

$$\Rightarrow \underline{xy = n,\ n \in \mathbb{R}}$$

5

$$\frac{dx}{u} = \frac{dy}{v} \Rightarrow \frac{dx}{x(1+2t)} = \frac{dy}{y} \Rightarrow \frac{\ln(x(1+2t))}{1+2t} = \ln(y) + C \Rightarrow$$

$$\Rightarrow [x \cdot (1+2t)]^{\frac{1}{1+2t}} = y \cdot e^C \qquad e^C = \frac{[x_0(1+2t)]^{\frac{1}{1+2t}}}{y_0}$$

Escanear com AltaScanner

$$\therefore \quad \left[x \cdot (1+2t) \right]^{\frac{1}{1+2t}} = y \cdot \frac{\left[x_0 \, (1+2t) \right]^{\frac{1}{1+2t}}}{y_0} \Rightarrow$$

$$\Rightarrow \quad y = \left(\frac{x}{x_0} \right)^{\frac{1}{1+2t}} \cdot y_0$$

Dinâmica de Fluídos - Ficha 3/8

$u = C\,(R^2 - r^2)$

$R \to$ raio do tubo

$r \to$ distância ao centro

1

$$Q = \int_S \vec{V} \cdot \hat{n}\, dA =$$

$$= \int_S C(R^2 - r^2)\, dA =$$

$$= \int_0^{2\pi} \int_0^{R} C(R^2 - r^2) \cdot r \; dr\, d\theta =$$

$$= 2\pi \cdot C \cdot \left[\frac{R^2 r^2}{2} - \frac{r^4}{4} \right]_0^R = 2\pi C \cdot \frac{R^4}{4} = \boxed{\frac{C \pi R^4}{2}}$$

2

$T = 20\,°C$

$R = \frac{12{,}7}{2}$ cm

Esc. turbulento

$u = U_0 \cdot \left(\frac{r}{R}\right)^{1/8}$

$U_0 = 7{,}62$ m/s

$r = R - x$

$$Q = \int_S \vec{v} \cdot \hat{n}\, dA = \int_S U_0 \left(\frac{r}{R}\right)^{1/8} dA =$$

$$= \frac{U_0}{R^{1/8}} \int_0^{2\pi} \int_0^{R} (R-x)^{1/8} \cdot x \; dr\, d\theta \qquad \underline{(*)}$$

integração por partes

$$= \frac{U_0}{R^{1/8}} \cdot 2\pi \cdot \left[-x(R-x)^{9/8} \cdot \frac{8}{9} - (R-x)^{17/8} \cdot \frac{64}{153} \right]_0^R =$$

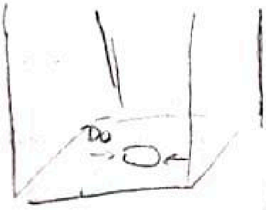
$$= \frac{U_0\, 2\pi}{R^{1/8}} \cdot \left[+ R^{17/8} \cdot \frac{64}{153} \right] = \frac{U_0 \cdot 2\pi \cdot 64}{153} R^2 =$$

$$= \frac{7{,}62 \cdot 2\pi \cdot 64}{153} \cdot \left(\frac{12{,}7 \cdot 10^{-2}}{2} \right)^2 = 0{,}080755 \text{ m}^3/\text{s} \overset{\times 60 \cdot 10^3}{=} \underline{4845 \text{ l/min}}$$

(*) $\int (R-x)^{1/8}\, x\, dx = -x(R-x)^{9/8} \cdot \frac{8}{9} + \int \frac{8}{9} \cdot (R-x)^{9/8}\, dx =$

$$= -x(R-x)^{9/8} \cdot \frac{8}{9} - (R-x)^{17/8} \cdot \left(\frac{8}{17}\right) \cdot \frac{8}{9}$$

3

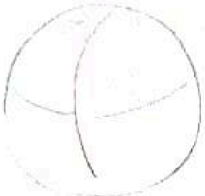


$$\frac{\partial m_{sis}}{\partial t} = \int_{cv} \frac{\partial \rho}{\partial t}\, \partial V + \dot{m}_{out} - \underset{0}{\dot{m}_{in}} \Rightarrow$$

$$\Rightarrow 0 = \frac{\partial}{\partial t} \rho \cdot \mathcal{S} \cdot h + \rho \cdot A \cdot V \Rightarrow$$

$$\Rightarrow -\frac{\partial h}{\partial t} = \frac{A \cdot V}{\underbrace{\mathcal{S}}_{\text{Área da base}}} = \frac{V \cdot \left(\frac{d_0}{2}\right)^2 \pi}{\left(\frac{\underset{\text{diâmetro do tanque}}{D}}{2}\right)^2 \pi} = \boxed{V \cdot \frac{D_0^2}{D^2}}$$

4



$D = 35 \cdot 10^{-2}\ m$

$D_0 = 5 \cdot 10^{-3}\ m$

$V_{saída} = 360\ m/s$

$\rho_0 = 2,5\ kg/m^3$

$$\frac{\partial m_{sis}}{\partial t} = \int_{cv} \frac{\partial \rho}{\partial t}\, \partial V + \dot{m}_{out} - \underset{0}{\dot{m}_{in}} \Rightarrow$$

$$\Rightarrow 0 = \frac{\partial \rho}{\partial t} \cdot \frac{4}{3}\left(\frac{D}{2}\right)^3 \pi + \rho \cdot V_{saída} \cdot \left(\frac{D_0}{2}\right)^2 \pi$$

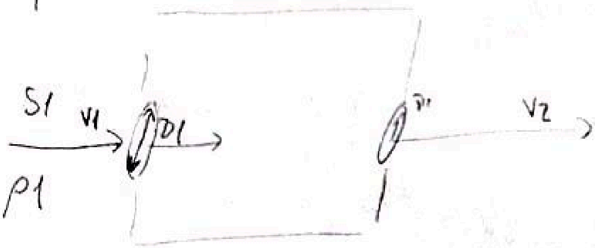
$$\Rightarrow -\frac{\partial \rho}{\partial t} = \frac{3 \cdot D_0^2 \cdot V_{saída} \cdot \rho_0}{4 D^3} \div 2$$

$$\Rightarrow -\frac{\partial \rho}{\partial t} = 0,787 \Rightarrow$$

$$\Rightarrow \frac{\partial \rho}{\partial t} = -0,787\ kg/(m^3 \cdot s)$$

5 Salinidade S

Densidade ρ



$$\frac{\partial m_{sis}}{\partial t} = \int_{CV} \frac{\partial \rho}{\partial t}\, \partial \forall + \dot{m}_{out} - \dot{m}_{in} \Rightarrow$$

$$\Rightarrow 0 = + S \rho v_2 \cdot \left(\frac{D_2}{2}\right)^2 \pi - S_1 \cdot \rho_1 \cdot v_1 \cdot \left(\frac{D_1}{2}\right)^2 \pi$$

$\Rightarrow$

6

entrada
$u = U_0$

saída
$u = U_0 \left(\frac{3\eta - \eta^3}{2}\right)$

$\eta = \frac{y}{\delta}$

$$\frac{\partial m_{sis}}{\partial t} = \int_{CV} \frac{\partial \rho}{\partial t}\, \partial \forall + \dot{m}_{out} - \dot{m}_{in} \Rightarrow$$

$$\Rightarrow 0 = \int_{CV} \frac{\partial \rho}{\partial t}\, \partial \forall + U_0 \cdot \delta \cdot \text{largura} -$$

$$- \frac{U_0}{2} \int_0^\delta \left(3\frac{y}{\delta} - \frac{y^3}{\delta^3}\right) \cdot \text{largura}\ \partial y \Rightarrow$$

$$\Rightarrow 0 = \int_{CV} \frac{\partial \rho}{\partial t}\, \partial \forall + U_0\, \delta\ \text{largura} - \frac{U_0}{2} \text{largura} \cdot \left[\frac{3y^2}{2\delta} - \frac{y^4}{4\delta^3}\right]_0^\delta \Rightarrow$$

$$\Rightarrow 0 = \int_{CV} \frac{\partial \rho}{\partial t}\, \partial \forall + U_0\, \delta \cdot \text{largura} \left[1 - \frac{1}{2} \cdot \left(\frac{3}{2} - \frac{1}{4}\right)\right] \Rightarrow$$

$$\Rightarrow \int_{CV} \frac{\partial \rho}{\partial t}\, \partial \forall = U_0 \cdot \delta \cdot \underbrace{\text{largura}}_{b} \cdot \left(-\frac{3}{8}\right)$$

$$Q = \frac{3}{8}\, U_0 \cdot \delta \cdot b$$

7 $\frac{\partial m_{sis}}{\partial t} = \int_{cv} \frac{\partial \rho}{\partial t} \, \partial \forall + \dot m_{out} - \dot m_{in} \Rightarrow$

volume do cone

$\Rightarrow 0 = \rho \cdot \frac{h \cdot (h \tan\theta)^2 \pi}{3} + 0 - \int V \cdot \rho \cdot \left(\frac{d}{2}\right)^2 \pi \, dt$

$\Rightarrow 0 = \rho \frac{h^3 \tan\theta^2 \pi}{3} - \frac{Kt^2}{2} \cdot \rho \cdot \frac{d^2}{4}\pi \Rightarrow \boxed{h^3 = \frac{3}{8} \cdot \frac{Kt^2 d^2}{\tan\theta^2}}$

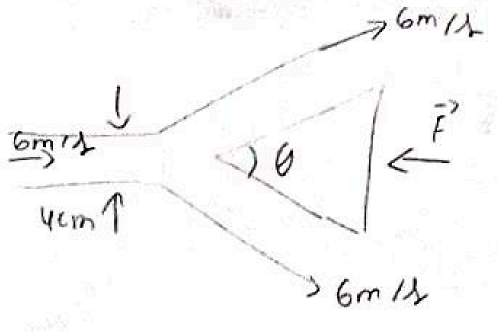
8 $h(t) = L \tan(\theta(t))$

$\frac{\partial m_{sis}}{\partial t} = \int_{cv} \frac{\partial \rho}{\partial t} \, \partial \forall + \dot m_{out} - \underbrace{\dot m_{in}}_{0} \Rightarrow$

$\Rightarrow 0 = \frac{\partial \rho}{\partial t} \cdot b \cdot L \cdot L \tan\theta_t + \dot m_0 \Rightarrow$

$\Rightarrow \dot m_0 = -\frac{\rho \cdot b \cdot L^2 \tan\theta(t)}{dt} = -\rho b L^2 \cdot \left[\frac{1}{\cos\theta_t^2} \cdot \dot\theta(t)\right]$

9



$\frac{\partial \vec V_{sis}}{\partial t} = \underbrace{\frac{\partial}{\partial t} \int_{cv} \rho \vec v \, \partial \forall}_{0} + \sum \dot m_{out} \vec V_{out} - \sum \dot m_{in} \vec V_{in} \Rightarrow$

$\Rightarrow \vec F = -\dot m \cdot (6;0) + \frac{\dot m}{2}\left(6\cos\frac{\theta}{2}; 6\sin\frac{\theta}{2}\right) + \frac{\dot m}{2}\left(6\cos\frac{\theta}{2}; -6\sin\frac{\theta}{2}\right)$

$\Rightarrow \begin{cases} F_x = \dot m \left(-6 + 6\cos\frac{\theta}{2}\right) \\ 0 = 0 \end{cases} \Rightarrow -124 = \rho_{H_2O} \cdot (4 \cdot 10^{-2} b) \cdot 6 \cdot \left(-6 + 6\cos\frac{\theta}{2}\right)$

$$\Rightarrow \frac{-1124 \cdot b}{6 \cdot 997 \cdot (4 \cdot 10^{2} \cdot b)} = 6\cos\frac{\theta}{2} - 6 \Rightarrow$$

$$\Rightarrow \frac{-775 \cdot b}{8973\, b} = \cos\frac{\theta}{2} - 1 \Rightarrow$$

$$\Rightarrow \cos\frac{\theta}{2} = \frac{8198}{8973} \Rightarrow \frac{\theta}{2} = 23,988° \Rightarrow \theta = 48°$$

10

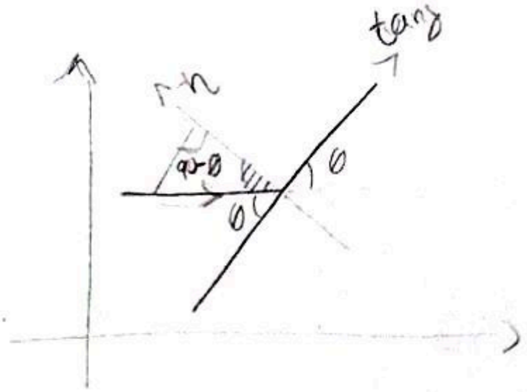
Escanear com AltaScanner

11

$$\frac{\partial \vec{V}m}{\partial t} = \underbrace{\int_{CV} \frac{\partial \rho \vec{V}}{\partial t} \partial \forall}_{0} + \dot{m}\vec{V}_{out} - \dot{m}\vec{V}_{in} \Rightarrow$$

$$\Rightarrow \vec{F} = \alpha Q \cdot V (\overset{\text{tangencial}}{1} ; \overset{\text{normal}}{0}) + (1-\alpha) Q \cdot V \cdot$$

$$\cdot (-1 ; 0) - Q \cdot V (\underbrace{\sin_{(90-\theta)}}_{\cos\theta} ; \underbrace{-\cos_{(90-\theta)}}_{-\sin\theta}) \Rightarrow$$

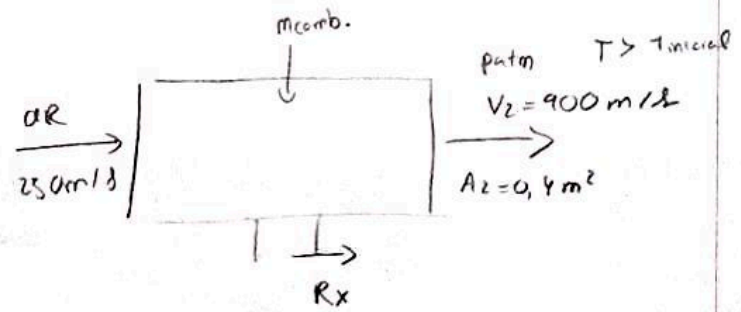


$$\begin{cases} F_n = Q \cdot V \sin\theta \\ \overset{F_t}{0} = \alpha Q \cdot V + (-1+\alpha) Q \cdot V - \\ \quad - Q V \cos\theta \end{cases}$$

$$\Rightarrow \left\{ (2\alpha - 1) = \cos\theta \right. \Rightarrow \left\{ \alpha = \frac{\cos\theta + 1}{2} \right.$$

12

$T = 20°C$
$p = 1 atm$
$V_1 = 250 m/s$
$A_1 = 0,5 m^2$
$\dot{m}_{comb} = \frac{\dot{m}_{ar}}{30}$



$$\frac{\partial m\vec{V}}{\partial t} = \int_{CV} \frac{\partial \rho \vec{V}}{\partial t} \partial \forall + \dot{m} \cdot V_{out} - \dot{m} V_{in} \Rightarrow$$

$$\Rightarrow R_x = 0 - \dot{m}_{ar} \cdot 250 + (\dot{m}_{ar} + \dot{m}_{comb}) \cdot 900 \Rightarrow$$

$$\Rightarrow R_x = \dot{m}_{ar} \left(-250 + \left(1 + \frac{1}{30}\right) \cdot 900\right) \Rightarrow$$

$$\Rightarrow R_x = 250 \cdot 0,5 \cdot \underbrace{\rho_{ar\,atm}}_{1,19\ kg/m^3} \left(-250 + \frac{31}{30} \cdot 900\right) = \underline{101\ 150\ N}$$

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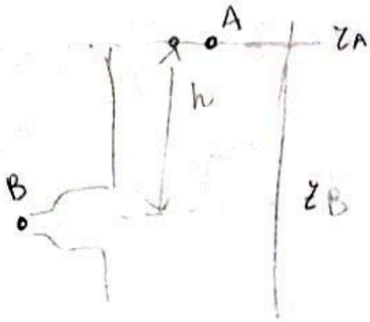
13 a) $\frac{1}{2}v^2 + \frac{P}{\rho} + gz = \text{constante}$ $\Leftarrow$ Linha de corrente



$$\frac{1}{2}v_A^2 + \frac{p_A}{\rho} + g z_A = \frac{1}{2}v_B^2 + \frac{p_B}{\rho} + g z_B \Rightarrow$$

$$\Rightarrow 0 + \frac{p_A - p_B}{\rho} \xrightarrow{} 0 + g(z_A - z_B) = \frac{1}{2}v_B^2 \Rightarrow$$

$$\Rightarrow gh = \frac{1}{2}v_B^2 \Rightarrow v_B^2 = 2gh$$

$$\vec{F} = m\vec{a} \Rightarrow \vec{F} = \rho A\, \partial x \cdot \frac{\vec{V_B}}{\partial t} \Rightarrow$$

$$\Rightarrow \vec{F} = 997 \cdot (2\cdot 10^{-2})^2 \pi \cdot \vec{V_B} \cdot \underbrace{\frac{\partial x}{\partial t}}_{\to V_B} \Rightarrow$$

$$\Rightarrow F = 997 \cdot (2\cdot 10^{-2})^2 \pi \cdot V_B^2 \Rightarrow V_B^2 = \frac{40}{997 \cdot (2\cdot 10^{-2})^2 \pi}$$

$$\Rightarrow 2gh = 31{,}9268 \Rightarrow h = \frac{31{,}9268}{2\cdot 9{,}81} = \underline{1{,}63\,m}$$

b) $\frac{\partial m\vec{V}}{\partial t} = \underbrace{\int_{CV} \frac{\partial \rho}{\partial t}\, \partial V}_{0} + \dot{m}_{out}\vec{V}_{out} \underbrace{- \dot{m}_{in}\vec{V}_{in}}_{0} \Rightarrow$

$$\Rightarrow F = (2{,}5\cdot 10^{-2}) \cdot A \cdot \rho_{água} \cdot \vec{V}_{out}$$

$$\Rightarrow F = \frac{(2{,}5\cdot 10^{-2})^2}{16\cdot 10^{-4}} D^2 \cdot \pi \cdot \frac{D^2}{4} \cdot 997 \Rightarrow$$

$$\Rightarrow D^4 = \frac{40}{305{,}876} \Rightarrow D = \sqrt[4]{0{,}13077}$$

$$\Rightarrow \underline{D = 0{,}60\,m}$$

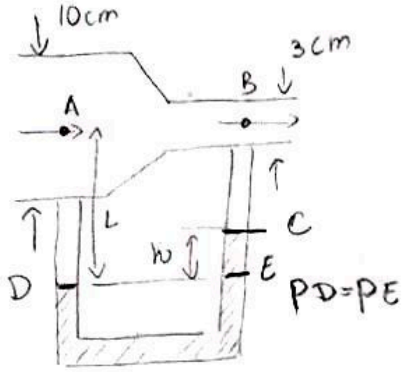
Conservação massa

$A_{tanque} \cdot 2{,}5 = A_{tubo} \cdot V_{saída}$

$$V_{saída} = \frac{\frac{D^2}{4}\pi \cdot 2{,}5}{\frac{d^2}{4}\pi} = \frac{D^2 \cdot 2{,}5\cdot 10^{-2}}{(4\cdot 10^{-2})^2}$$

Dinâmica de Fluidos - Ficha 4 / 10

1 a) c b) Eq de Bernoulli $\frac{1}{2}v_A^2 + \frac{p_A}{\rho} + g z_A = \frac{1}{2}v_B^2 + \frac{p_B}{\rho} + g z_B \Rightarrow$



$p_D = p_E$

$p_A + L\rho_{água} g = p_B + (L-h)\rho_{água} g + h\rho_{Hg} g \Rightarrow$

$\Rightarrow \boxed{p_A - p_B = h \cdot (\rho_{Hg} - \rho_{água}) g}$

$\Rightarrow \frac{1}{2}(v_A^2 - v_B^2) + \frac{p_A - p_B}{\rho} + g(\overset{0}{z_A - z_B}) = 0 \Rightarrow$

$\Rightarrow \frac{1}{2}(v_A^2 - v_B^2) + \frac{h(\rho_{Hg} - \rho_{água}) g}{\rho} = 0 \Rightarrow$

$\Rightarrow \frac{1}{2}(v_A^2 - v_B^2) = h \cdot \frac{(13{,}534 \cdot 10^3 - 997) \cdot 9{,}81}{997}$

$\Rightarrow v_A^2 - v_B^2 = 246{,}416 \cdot h$ (*)

$$\vec{F} = m\vec{a} \Rightarrow \vec{F} = \rho A \partial x \cdot \frac{\partial \vec{v}}{\partial t} \Rightarrow$$

$$\Rightarrow F = \rho A v^2 \Rightarrow 70 = 997 \cdot (1{,}5 \cdot 10^{-2})^2 \pi \cdot v^2 \Rightarrow$$

$$\Rightarrow v_B^2 = 99{,}3277 \Rightarrow \underline{v_B = 9{,}967 \text{ m/s}}$$

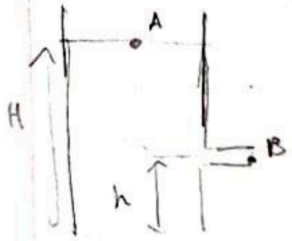
Conservação da massa:

$$\cancel{\rho_A} \cdot A_A \cdot v_A = \cancel{\rho_B} \cdot A_B \cdot v_B \Rightarrow \left(\frac{D_1}{2}\right)^2 \pi \cdot v_A = \left(\frac{D_2}{2}\right)^2 \pi\, v_B \Rightarrow$$

$$\Rightarrow v_A = v_B \cdot \frac{D_2^2}{D_1^2} = 9{,}967 \cdot \frac{3^2}{10^2} = \underline{0{,}897 \text{ m/s}}$$

(*) $h = \frac{99{,}3277 - 0{,}897^2}{246{,}416} = \underline{0{,}40 \text{ m}}$

2 Eq. de Bernoullie $\frac{1}{2}v_A^2 + \frac{p_A}{\rho} + gz_A = \frac{1}{2}v_B^2 + \frac{p_B}{\rho} + gz_B$



$$\frac{1}{2}(v_A^2 - v_B^2) + \frac{p_A - p_B}{\rho} + g(z_A - z_B) = 0 \Rightarrow$$

$$\Rightarrow -\frac{1}{2}v_B^2 + 0 + g(H-h) = 0 \Rightarrow$$

$$\Rightarrow v_B = \sqrt{2g(H-h)}$$

$\vec{V}_B = v_B\,\hat{\imath}$

A partir de B $\Rightarrow$ queda livre

Para 1 partícula

$\vec{V}\,(v_B\,;\,-gt)$ $\quad \begin{cases} x = v_B t \\ y = -g\frac{t^2}{2} \\ v_y = -gt \end{cases}$

$y = -h \Rightarrow +h = +g\frac{t^2}{2} \Rightarrow t = \sqrt{\frac{2h}{g}}$

$x = v_B\sqrt{\frac{2h}{g}} \Rightarrow$

$\Rightarrow \boxed{x = 2\cdot\sqrt{Hh - h^2}}$

$\Rightarrow x = 2H\sqrt{\frac{h}{H} - \left(\frac{h}{H}\right)^2}$

seja $\frac{h}{H} = a$ $\quad x = 2H\sqrt{a - a^2}$

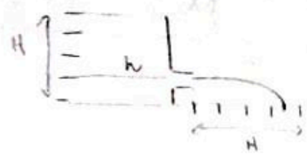
$\frac{dx}{da} = \frac{2H}{2}\cdot(1-2a)\cdot(a-a^2)^{-1/2}$

$\frac{dx}{da} = 0 \Rightarrow 2a = 1 \vee a = a^2 \Rightarrow$

$\Rightarrow \boxed{a = 0,5} \vee a = 0 \vee a = 1$

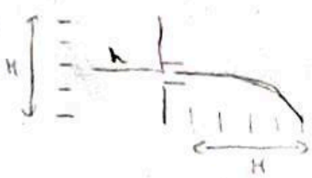
$\boxed{\frac{h}{H} = 0,5 \text{ maximiza } X}$

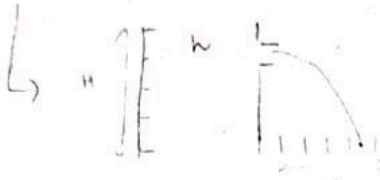
$a = 0,25 \quad X = 0,866\,H \longrightarrow$



$a = 0,5 \quad X = H$

$a = 0,75 \quad X = 0,866\,H$





3 a)

b) $\Delta p = 10342\,Pa$

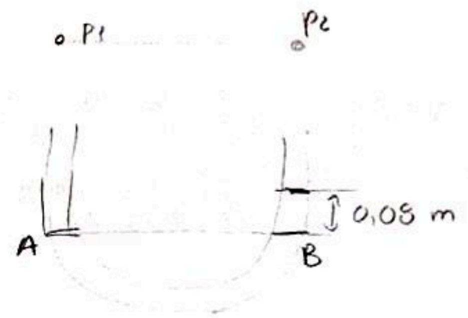
$$\frac{1}{2} v_A^2 + \frac{p_A}{\rho} + g z_A = \frac{1}{2} v_B^2 + \frac{p_B}{\rho} + g z_B \Rightarrow$$

$$\Rightarrow \frac{1}{2}(v_A^2 - v_B^2) + \frac{p_A - p_B}{\rho} + g\underbrace{(z_A - z_B)}_{0} = 0 \Rightarrow$$

$$\Rightarrow \frac{1}{2}(v_A^2) - \frac{10342}{1,2} \Rightarrow v_A = \sqrt{\frac{2 \cdot 10342}{1,29}} = 126,626\,m/s =$$

$$= \frac{126,626 \cdot 60^2}{10^3}\,Km/h \Rightarrow 455,85\,Km/h$$

4 a)



$$p_1 = p_A + h \cdot \rho_{CO_2} g$$

$$p_2 = \underset{p_A}{p_B} + 0,08\,\rho_{óleo}\, g + \rho_{CO_2} g\,(h - 0,08)$$

$$p_1 - p_2 = 0,08g\,(\rho_{CO_2} - \rho_{óleo}) \Rightarrow$$

$$\Rightarrow 170 \cdot 10^3 - p_2 = 0,08 \cdot 9,81 \cdot (\underbrace{1,8474}_{\rho_{CO_2}} - 998 \cdot 0,827) \Rightarrow$$

$$\Rightarrow p_2 = 169\,354\,Pa.$$

b) $$\frac{1}{2} v_1^2 + \frac{p_1}{\rho} + z_1 g = \frac{1}{2} v_B^2 + \frac{p_2}{\rho} + z_2 g \Rightarrow$$

$$\Rightarrow \frac{1}{2}\left(v_1^2 - \left(\frac{25}{9} v_1\right)^2\right) + \frac{p_1 - p_2}{\rho_{CO_2}} = 0 \Rightarrow$$

$$\Rightarrow \frac{1}{2} \cdot \frac{544}{81} v_1^2 = \frac{646}{1,8474} \Rightarrow v_1 = 10,2045\,m/s$$

Conservação de massa

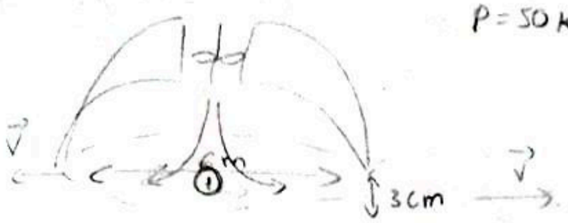
$$v_1 \cdot \left(\frac{D_1}{2}\right)^2 \pi = v_2 \left(\frac{D_2}{2}\right)^2 \pi \Rightarrow$$

$$\Rightarrow v_2 = v_1 \frac{D_1^2}{D_2^2} = v_1 \cdot \frac{25}{9}$$

$Vazão = V_1 \cdot A_1 = 10{,}2045 \cdot 0{,}05^2 \pi = 0{,}080146 \, m^3/s \cdot 3600 =$

$= 288{,}53 \, m^3/h$??

5 a)



$P = 50 \, kN$

$\frac{1}{2} v_1^2 + \frac{p_1}{\rho} + z_1 g = \frac{1}{2} v_2^2 + \frac{p_2}{\rho} + z_2 g$ (0, iguais)

$\Rightarrow \frac{p_1}{\rho} = \frac{1}{2} v_2^2 + \frac{p_2}{\rho}$

cancela-se ao longo das superfícies

$\vec{F_R} = 0_{cv} \Rightarrow \vec{F_P} + \vec{F}_{patm} + \vec{F}_{ps} = 0 \Rightarrow$

$\Rightarrow 50 \cdot 10^3 \hat{k} + \oint_{ps} (p - p_a) \hat{n} \, dS \Rightarrow$

$\Rightarrow \int_A ps \, \hat{k} \, dS = +50 \cdot 10^3 \hat{k} \Rightarrow ps \cdot \pi \cdot 3^2 \hat{k} = +50 \cdot 10^3 \hat{k}$

$\Rightarrow \boxed{ps = +1768{,}39 \, Pa}$

$p_1 = ps + patm$

$p_2 = patm = 10^5$

$\frac{1}{2} v_2^2 = \frac{p_1 - p_2}{\rho} \Rightarrow V_2 = \sqrt{2 \cdot \frac{1768{,}39}{1{,}25}} = 53{,}2 \, m/s$

$Q = \int \vec{v} \cdot \hat{n} \, dA =$

$= V \cdot \underset{\text{Perímetro} \cdot h}{A} = 53{,}2 \cdot 6\pi \cdot 0{,}03 = \underline{30{,}1 \, m^3/s}$

b) $W = F \cdot deslocamento = p \cdot A \cdot deslocamento = p \cdot \partial V$

potência $= \frac{\partial W}{\partial t} = p \frac{\partial V}{\partial t} = p \cdot Q = 1768{,}39 \cdot 30{,}1 = 53{,}228 \cdot 10^3 W =$

$= \boxed{53{,}2 \, kW}$

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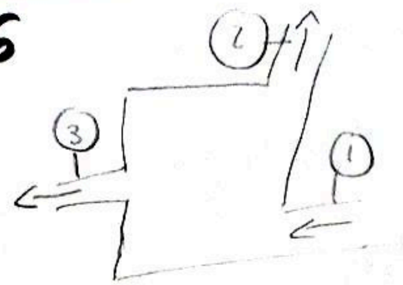
6



$\Delta U \simeq 0$

$Q = 0$

$V = \frac{Q}{A}$

$Q_1 = Q_2 + Q_3 \Rightarrow$

$\Rightarrow 220 = 100 + Q_3 \Rightarrow Q_3 = 120$

$$\dot{m}_{in}\left(\frac{P_1}{\rho g} + \frac{V_1^2}{2g}\right)_{in} = \left[\dot{m}\left(\frac{P_2}{\rho g} + \frac{V_2^2}{2g}\right)\right]_{out} - \frac{\dot{W}_{eixo}}{g}$$

$$\Rightarrow Q_1 \cdot \left[\frac{P_1}{\rho g} + \left(\frac{Q_1}{A_1}\right)^2 \cdot \frac{1}{2g}\right] = Q_2 \cdot \left[\frac{P_2}{\rho g} + \left(\frac{Q_2}{A_2}\right)^2 \cdot \frac{1}{2g}\right] +$$

$$+ Q_3\left[\frac{P_3}{\rho g} + \left(\frac{Q_3}{A_3}\right)^2 \cdot \frac{1}{2g}\right] \overset{+}{-} \frac{\dot{W}_{eixo}}{\rho} \Rightarrow$$

$$\Rightarrow \frac{220}{60^2} \cdot \left[\frac{150 \cdot 10^3}{998} + \left(\frac{220}{60^2 \cdot (4{,}5 \cdot 10^{-2})^2 \pi}\right)^2 \cdot \frac{1}{2}\right] =$$

$$= \frac{100}{60^2} \cdot \left[\frac{225 \cdot 10^3}{998} + \left(\frac{100}{60^2 \cdot (3{,}5 \cdot 10^{-2})^2 \pi}\right)^2 \cdot \frac{1}{2}\right] +$$

$$+ \frac{120}{60^2} \cdot \left[\frac{265 \cdot 10^3}{998} + \left(\frac{120}{60^2 \cdot (2 \cdot 10^{-2})^2 \pi}\right)^2 \cdot \frac{1}{2}\right] - \frac{\dot{W}_{eixo}}{\rho} \Rightarrow$$

$$\Rightarrow 12{,}0046 = 6{,}986 + 20{,}578 - \frac{\dot{W}_{eixo}}{\rho} \Rightarrow$$

$$\Rightarrow \dot{W}_{eixo} = 15{,}5594 \cdot 998 \Rightarrow \dot{W}_{eixo} = 15528{,}3\ W$$

(+) Como o sinal é − o trabalho é realizado no fluido.

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7 Calor$_{in}$ = 55 MW

Num segundo 55 MJ entram para 2,5 m³ de água

A 18 °C $\Rightarrow u = 75,57$ kJ/kg , $v = 1,0014 \cdot 10^{-3}$ m³/kg

$$U = 75,57 \cdot \frac{2,5}{1,0014 \cdot 10^{-3}} = 188\,661 \text{ kJ}$$

$$U_{final} = 188\,661 + 55 \cdot 10^{3} = \underline{243\,661 \text{ kJ}}$$

$$u_{final} = \frac{U}{m} = \frac{243\,661}{2,5} \cdot 1,0014 \cdot 10^{-3} = 97,6008 \text{ kJ/kg}$$

$u_f\,(23°C) = 96,51$

$u_f\,(24°C) = 100,70$

$$97,6008 = 96,51 + (100,70 - 96,51) \cdot (T - 23)$$

$$\Rightarrow T - 23 = 0,26 \Rightarrow \boxed{T_{final} = 23,26 °C}$$

$$v_f\,(23,26 °C) = 1,0024 + (1,0027 - 1,0024) \cdot 0,26 = 1,00248 \cdot 10^{-3} \text{ m}^3/\text{kg}$$

$$Q_{a\ sair\ por\ s} = \frac{2,5}{1,0014 \cdot 10^{-3}} \cdot 1,00248 \cdot 10^{-3} = \boxed{2,5027 \approx Q_i}$$

8

$$\left(\frac{p_1}{g\rho} + \frac{v^2}{2g}^{\approx 0} + z\right)_{in} = \left(\frac{p}{\rho g} + \frac{v^2}{2g} + z\right)_{out} + h_{atr.} + h_{turb.} - h_{bomba}$$

$$\Rightarrow h_{bomba} = \underbrace{\frac{p_2 - p_1}{g\rho}}_{0} + (z_2 - z_1) + \frac{v_2^2}{2g} + 5$$

$$\Rightarrow h_{bomba} = 2 + \frac{31,1236^2}{2 \cdot 9,81} + 5 \Rightarrow$$

$$\Rightarrow h_{bomba} = 56,3721 \Rightarrow$$

$$\Rightarrow W_{bomba} = 56,3721 \cdot 9,81 = 553 \text{ W/kg}$$

$$Q = A \cdot V \Rightarrow V_e = \frac{Q}{\left(\frac{D_e}{2}\right)^2 \pi} \Rightarrow$$

$$\Rightarrow V_e = \frac{220}{(2,5 \cdot 10^{-2})^2 \pi} = 112045 \text{ m/h}$$

$$= 31,1236 \text{ m/s}$$

$553\ W/kg = 553 \cdot 60{,}99\ W =$

$= 33727\ W = 33{,}73\ kW$

$\frac{220\ m^3}{h} = \frac{220 \cdot 998}{60^2}\ kg/s =$

$= 60{,}99\ kg/s$

9 a) Desprezando altura e atrito

$$\frac{p_1}{\rho g} + \frac{\hat{u}_1}{g} + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + \frac{\hat{u}_2}{g} + \frac{v_2^2}{2g} + h_{turbina} \Rightarrow$$

$$\Rightarrow \underbrace{\frac{p_1}{\rho} + \hat{u}_1}_{} + \frac{v_1^2}{2} = \frac{p_2}{\rho} + \hat{u}_2 + \frac{v_2^2}{2} + W_{turbina}$$

$$\Rightarrow h_1 + \frac{v_1^2}{2} = h_2 + \frac{v_2^2}{2} + w_{turbina} \Rightarrow$$

$$\Rightarrow 3383 \cdot 10^3 + \frac{40^2}{2} = 2628 \cdot 10^3 + \frac{225^2}{2} + w$$

$$\Rightarrow w = 7{,}30488 \cdot 10^5\ J/kg = 730{,}5\ kJ/kg$$

2,5 MPa e 450 °C

$h = 3383 \times 10^3\ J/kg$

↓

tabela

↑

22 kPa 70 °C

$h = 2628 \cdot 10^3\ J/kg$

b) $Pot = w \cdot \dot{m} = 7305\ kJ/s = 7305\ kW$

c) Para 70 °C press. sat $= 31{,}19 \cdot 10^3$ J

$p <$ press sat $\Rightarrow$ permanece gasoso

Ficha 11 Dinâmica de Fluidos

Nota: todos os resultados (1) estão errados: ficha c/ erro $u = x^2 - y^2 + x$ dá soluções corretas

1 a) $a_x = \frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial u}{\partial y} \cdot \frac{\partial y}{\partial t} =$

$= (2x) \cdot (x^2 - y^2) + (-2y) \cdot (-2xy - y) =$

$= 2x^3 - 2xy^2 + 4xy^2 + 2y^2 = 2x^3 + 2xy^2 + 2y^2$

$a_x(1,2) = 2 + 8 + 8 = 18$

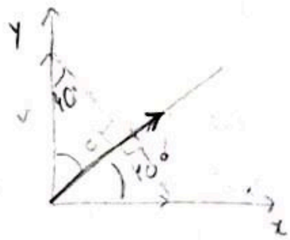
$a_y = \frac{\partial v}{\partial t} = \frac{\partial v}{\partial x} \cdot \frac{dx}{dt} + \frac{dv}{dy} \cdot \frac{dy}{dt} = -2y \cdot (x^2 - y^2) +$

$+ (-2x - 1) \cdot (-2xy - y) = -2yx^2 + 2y^3 + 4x^2y + 2xy + 2xy + y =$

$= 2y^3 + 2x^2y + 4xy + y$

$a_y(1,2) = 16 + 4 + 8 + 2 = 30$

b)



$V_{(40°)} = u \cdot \cos 40° + v \sin 40° =$

$\underset{\text{em } (1,2)}{\Longrightarrow} -3 \cos 40° + (-6) \sin 40° = -6{,}15 \text{ m/s}$

c) $\vec{a} \cdot \vec{v} = \|\vec{a}\| \cdot \|\vec{v}\| \cos\theta \Rightarrow$

$\Rightarrow (18; 30) \cdot (-3; -6) = \sqrt{18^2 + 30^2} \cdot \sqrt{9 + 36} \cos\theta \Rightarrow$

$\Rightarrow \cos\theta = \frac{-234}{\sqrt{55080}} \Rightarrow \theta = 175{,}6°$

2 a) $a = \frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} \cdot \frac{\partial x}{\partial t} = \boxed{\frac{2V_0}{L} \cdot u} = \frac{2V_0^2}{L}\left(1 + \frac{2x}{L}\right)$

b) $a(0) = \frac{2 \cdot 3^2}{0{,}15} = 120 \text{ m/s}^2$

$a(0{,}15) = 120 \cdot \left(1 + \frac{0{,}30}{0{,}15}\right) =$

$= 120 \cdot 3 = 360 \text{ m/s}^2$

3 Eq. de Continuidade

$$\underbrace{\frac{\partial \rho}{\partial t}}_{\substack{0 \\ \text{incomp.}}} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0 \Rightarrow$$

$$\Rightarrow 4y^2 + \frac{\partial f(y)}{\partial y} - y^2 = 0 \Rightarrow \frac{\partial f(y)}{\partial y} = -3y^2 \Rightarrow$$

$$\Rightarrow f(y) = -y^3 + C, \quad C \in \mathbb{R}$$

4 Eq. de Continuidade (coord. cilíndricas)

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \cdot \frac{\partial}{\partial r}(r \rho v_r) + \frac{1}{r} \cdot \frac{\partial}{\partial \theta}(\rho v_\theta) + \frac{\partial}{\partial z}(\rho v_z) = 0 \Rightarrow$$

0

$$\Rightarrow \frac{1}{r} \cdot \underbrace{\frac{\partial}{\partial r}(\rho \cdot C)}_{0} + \frac{1}{r} \underbrace{\frac{\partial}{\partial \theta}\left(\rho \cdot \frac{k}{r}\right)}_{0} + \frac{\partial}{\partial z} \cdot 0 = 0 \Rightarrow$$

$$\Rightarrow 0 + 0 + 0 = 0$$

5

$$L(t) = L_0 - \text{deslocamento} = L_0 - x = L_0 - V \cdot t$$

$$\rho_0 = \frac{m}{A \cdot L_0}$$

$$\rho = \frac{m}{A \cdot L(t)} = \frac{\rho_0 \cdot L(0)}{L(t)} = \frac{\rho_0 \cdot L_0}{L_0 - V \cdot t}$$

6 a) entrada de fluido

$\rho u \, \partial y \, \partial z + \rho v \, \partial x \, \partial z$

saída de fluido

$\left[\rho u + \frac{\partial}{\partial x}(\rho u)\, dx\right] \partial y \, dz + \left[\rho v + \frac{\partial}{\partial y}(\rho v)\, dy\right] \partial x \, \partial z$

∴ eq da continuidade expime a conservação da massa

$$\underbrace{\frac{\partial \rho}{\partial t}}_{=0} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \Rightarrow -2y + 2y = 0 \quad ✓$$

b) $$\begin{cases} \rho \frac{Du}{Dt} = \rho g_x - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \\ \rho \frac{Dv}{Dt} = \rho g_y - \frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \rho\left[\frac{Du}{dx}\cdot\frac{\partial x}{\partial t} + \frac{du}{\partial y}\cdot\frac{\partial y}{dt}\right] = 0 - \frac{\partial p}{\partial x} + \mu\left(\frac{\partial(-2y)}{\partial x} + \frac{\partial(-2x)}{\partial y}\right) \\ \rho\left[\frac{\partial v}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial v}{\partial y}\cdot\frac{\partial y}{\partial t}\right] = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial(-2x)}{\partial x} + \frac{\partial(2y)}{\partial y}\right) \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} [-2y \cdot u + (-2x)\cdot v]\rho = -\frac{\partial p}{\partial x} \\ [-2x \cdot u + 2y\, v]\rho = -\frac{\partial p}{\partial y} \end{cases} \Rightarrow \begin{cases} -\frac{\partial p}{\partial x} = [4xy^2 + 2x^3 - 2xy^2]\rho \\ -\frac{\partial p}{\partial y} = [4x^2 y + 2y^3 - 2yx^2]\rho \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \frac{\partial p}{\partial x} = [-2x^3 - 2xy^2]\rho \\ -\frac{\partial p}{\partial y} = [2y^3 + 2yx^2]\rho \end{cases} \Rightarrow \begin{cases} p = \left[-\frac{x^4}{2} - x^2y^2\right]\cdot\rho + C \\ p = \left[-\frac{y^4}{2} - x^2y^2\right]\cdot\rho + C \end{cases}, \; C \in \mathbb{R}$$

$$p = p_a + \left[-\frac{x^4}{2} - \frac{y^4}{2} - x^2y^2\right]\cdot\rho$$

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7

Eq. Navier-Stokes

$$\rho \frac{Dw}{Dt} = \rho g_z + \mu \left(\frac{\partial^2 w}{\partial x^2} \right) \Rightarrow$$

$$\Rightarrow \rho \frac{\partial w}{\partial x} \cdot \underbrace{\frac{\partial x}{\partial t}}_{0} = \rho g_z + \mu \left(\frac{\partial^2 w}{\partial x^2} \right) \Rightarrow$$

$$\Rightarrow \frac{\partial^2 w}{\partial x^2} = -\frac{\rho}{\mu} g_z \Rightarrow \frac{\partial w}{\partial x} = -\frac{\rho}{\mu} g x + C_1 \Rightarrow$$

$$\Rightarrow w = -\frac{\rho g}{2\mu} x^2 + C_1 x + C_2 \quad , \quad C_1 \text{ e } C_2 \in \mathbb{R}$$

$$\begin{cases} w(-h) = 0 \\ w(h) = 0 \end{cases} \Rightarrow \begin{cases} -\frac{\rho g}{2\mu} h^2 - C_1 h + C_2 = 0 \\ -\frac{\rho g}{2\mu} h^2 + C_1 h + C_2 = 0 \end{cases} \Rightarrow \begin{cases} -C_1 h = C_1 h \\ - \end{cases}$$

$$\Rightarrow \begin{cases} C_1 = 0 \\ C_2 = \frac{\rho g}{2\mu} h^2 \end{cases}$$

$$\boxed{\therefore\ w = -\frac{\rho g}{2\mu} x^2 + \frac{\rho g}{2\mu} h^2}$$