Soluções do 2ºTeste de TDF 12/01/2024

Note que as soluções apresentadas podem corresponder a dados diferentes dos apresentados no Teste.

Problema I

The test section wall area is $(\pi)(0.8\text{ m})(4\text{ m}) = 10.053\text{ m}^2$, hence the total number of holes is $(1200)(10.053) = 12064$ holes. The total suction flow leaving is

$$Q_{suction} = NQ_{hole} = (12064)(\pi/4)(0.005\text{ m})^2(8\text{ m/s}) \approx 1.895\text{ m}^3/\text{s}$$

(a) Find V_o: $\quad Q_o = Q_1 \quad$ or $\quad V_o\dfrac{\pi}{4}(2.5)^2 = (35)\dfrac{\pi}{4}(0.8)^2,$

solve for $\quad V_o \approx \mathbf{3.58}\ \dfrac{\mathbf{m}}{\mathbf{s}} \quad$ *Ans.* (a)

(b) $\quad Q_2 = Q_1 - Q_{suction} = (35)\dfrac{\pi}{4}(0.8)^2 - 1.895 = V_2\dfrac{\pi}{4}(0.8)^2,$

or: $\quad V_2 \approx \mathbf{31.2}\ \dfrac{\mathbf{m}}{\mathbf{s}} \quad$ *Ans.* (b)

(c) Find V_f: $\quad Q_f = Q_2 \quad$ or $\quad V_f\dfrac{\pi}{4}(2.2)^2 = (31.2)\dfrac{\pi}{4}(0.8)^2,$

solve for $\quad V_f \approx \mathbf{4.13}\ \dfrac{\mathbf{m}}{\mathbf{s}} \quad$ *Ans.* (c)

Problema II

A componente vertical da força resultante a actuar no sistema é $F_y = N_y - Mg = 36$ N, onde M é a massa da água dentro da caixa mais a da caixa e N_y é a componente vertical da reacção da superfície.

. Continuity requires that $\dot{m}_1 = \dot{m}_2 + \dot{m}_3$, and then, with a control volume around the entire system, steady vertical momentum requires that

$$\sum F_y = \dot{m}_2 v_2 + \dot{m}_3 v_3 - \dot{m}_1 v_1 = \dot{m}_2 V \sin 40° + \dot{m}_3 V \sin 40° - \dot{m}_1(0) =$$
$$(\dot{m}_2 + \dot{m}_3) V \sin 40° = \dot{m}_1 (7 m/s) \sin 40° = \dot{m}_1 (4.50 m/s) = 36 N$$

Solve for $\dot{m}_1 = \mathbf{8.0\,kg/s}$ *Ans.*

$$\dot{m} = \rho A V$$

Problema III

A densidade do ar deve ser $\rho = 1.189\ \text{kg/m}^3$.

For sea-level, take $\rho_{air} = 1.2255\ \text{kg/m}^3$. Section 2 must be less than atmospheric. How much less? Determine the pressure change for 10 cm of water:

$$\Delta p = p_3 - p_2 = \rho_{water}\, g\, h = (998\, kg/m^3)(9.81\, m/s^2)(0.1\ m) = 979\ Pa$$

This must be the pressure difference between sections 2 and 3. From Bernoulli's equation,

$$p_3 - p_2 = \frac{\rho}{2}(V_2^2 - V_3^2) \quad \text{plus continuity: } V_3 = \frac{A_2}{A_3} V_2 = \frac{D_2^2}{D_3^2} V_2 = \frac{9}{25} V_2$$

$$\text{or: } 979\, Pa = \frac{1.2255}{2}(V_2^2)[1 - (\frac{9}{25})^2]; \; Solve\ for\ V_2 = 42.8\ \frac{m}{s},\ V_3 = 15.4\ \frac{m}{s}$$

$$\text{Finally, } \dot{m}_{air} = \rho A_3 V_3 = (1.2255 \frac{kg}{m^3}) \frac{\pi}{4} (0.05 m)^2 (15.4 \frac{m}{s}) = \mathbf{0.037}\ \frac{kg}{s}\ \ Ans.$$

Problema IV

(a) For two-dimensional steady flow, the acceleration components are

$$\frac{du}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \left(U_o\frac{x}{L}\right)\left(\frac{U_o}{L}\right) + \left(-U_o\frac{y}{L}\right)(0) = \frac{U_o^2}{L^2}x$$

$$\frac{dv}{dt} = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \left(U_o\frac{x}{L}\right)(0) + \left(-U_o\frac{y}{L}\right)\left(-\frac{U_o}{L}\right) = \frac{U_o^2}{L^2}y$$

Therefore the resultant $\mathbf{a} = (U_o^2/L^2)(x\mathbf{i} + y\mathbf{j}) = (U_o^2/L^2)\mathbf{r}$ (purely radial) *Ans.* (a)

(b) For the given resultant acceleration of 25 m/s^2 at (x, y) = (1 m, 1 m), we obtain

$$|a| = 25\ \frac{m}{s^2} = \frac{U_o^2}{L^2}|r| = \frac{U_o^2}{(1.5\ m)^2}\sqrt{2}\ m,\quad \text{solve for } U_o = \mathbf{6.3\ \frac{m}{s}}\quad \textit{Ans. (b)}$$

Problema V

a) A equação de Navier-Stokes para a componente x do movimento reduz-se a

$$0 = \rho g \sin\theta + \mu\frac{d^2u}{dy^2}$$

Integrando esta equação, obtém-se

$$u = -\frac{\rho g \sin\theta}{2\mu}y^2 + Ay + B$$

onde *A* e *B* são duas constantes, que podem ser determinadas pelas condições de fronteira:

$$u(0) = 0 \qquad e \qquad \tau_{yx}(h) = \mu\left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right) = \mu\frac{du}{dy} = 0$$

$$u = \frac{\rho g \sin\theta}{2\mu}y(2h - y)$$

b) A tensão τ_{yx} é dada por

$$\tau_{yx}(y) = \mu\frac{du}{dy} = \frac{\rho g \sin\theta}{2}(2h - 2y) = \rho g \sin\theta\,(h - y)$$

Assim, a força por unidade de área da placa é

$$\tau_{yx}(0) = \rho g h \sin\theta$$

E a força por unidade de comprimento será

$$\frac{F}{L} = \rho g h D \sin\theta$$

onde D é a largura da placa.