TECNOLOGIAS PARA SISTEMAS DE ENERGIA ESPACIAIS

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MAGNETISM

Aula 3

MAGNETIC FLUX DENSITY

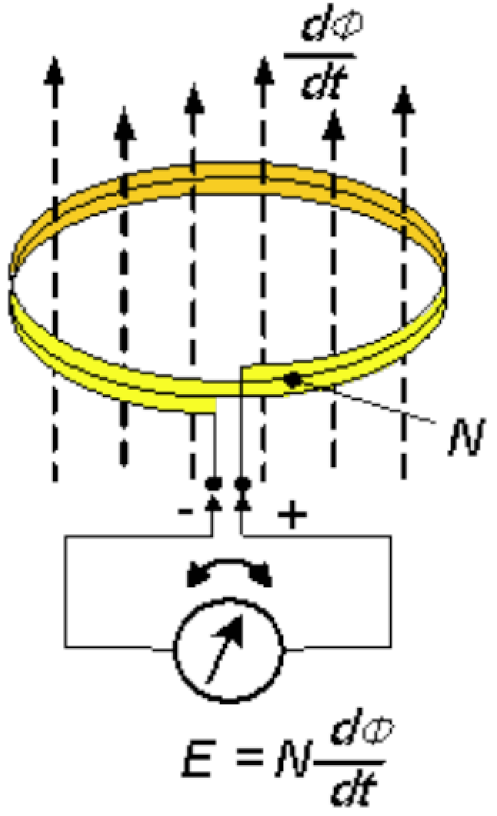
Magnetic Flux can be expressed by

$$Magnetic\ flux\ density = \frac{magnetic\ flux}{area}$$

the magnetic flux unity is called Henry x Ampere=Weber (Wb), and the magnetic Flux density, B, Wb/m^2 which is called Tesla.

$$B = \frac{\phi}{A}$$

$$B/_H = \mu_0\mu_r$$



E – Electric Field
H – Magnetic Field
B – Magnet Flux Density
D – Electric Flux Density

MAGNETIC FIELD

Table 28-1

Some Approximate Magnetic Fields

At surface of neutron star	10^8 T
Near big electromagnet	1.5 T
Near small bar magnet	10^{-2} T
At Earth's surface	10^{-4} T
In interstellar space	10^{-10} T
Smallest value in magnetically shielded room	10^{-14} T

The Si units for the magnetic field density B, is newton per coulomb-meter per second, and this is normally called T (Tesla).

$$1\, tesla = 1T = 1\frac{newton}{(coulomb)(\frac{meter}{second})} =$$

$$= \frac{newton}{\left(\frac{coulomb}{second}\right)(meter)} = \frac{N}{A\, m}$$

Sometimes the magnetic field can also be expressed in Gauss which is 1 Tesla = 10_4 Gauss

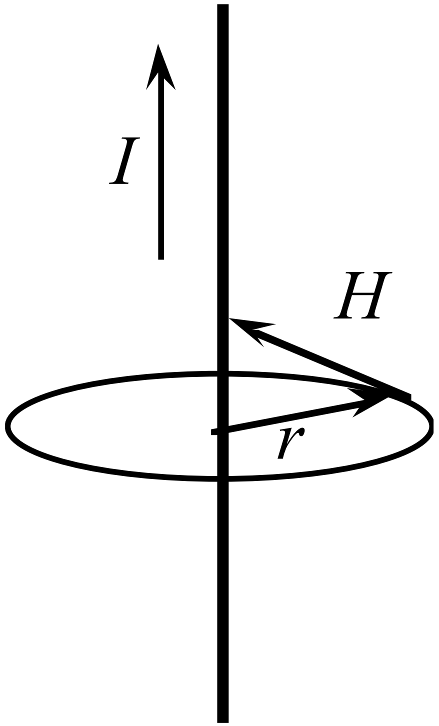
$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

Slide 23, class 2

SLOWLY VARYING MAGNETIC FIELD

Magnetic fields in can be described by ampere's law:

$$\oint \vec{H}\overrightarrow{dl} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \rightarrow \oint \vec{H}\overrightarrow{dl} = N\vec{I} \qquad \left(\frac{\partial \vec{D}}{\partial t} = 0\right)$$



$$\oint \vec{H}\overrightarrow{dl} = 2\pi r H = N\vec{I}$$

$$H = \frac{NI}{2\pi r}$$

$$B/_H = \mu_0 \mu_r$$

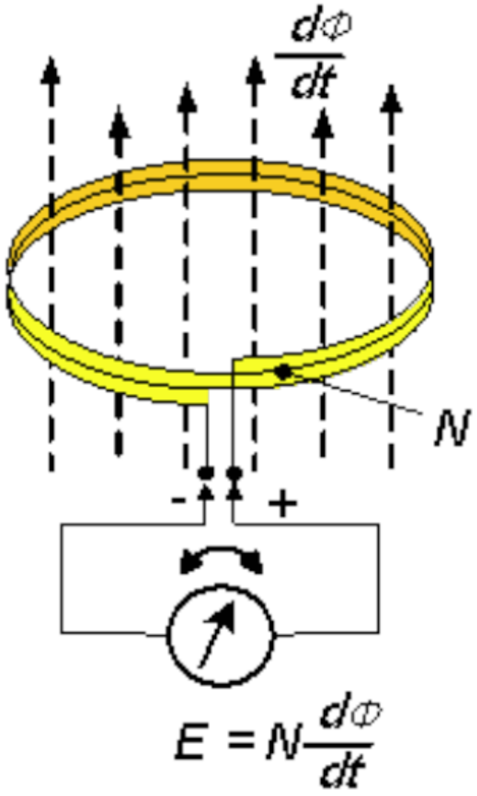
NI is normally called magneto-motive force

Força magnetromotriz

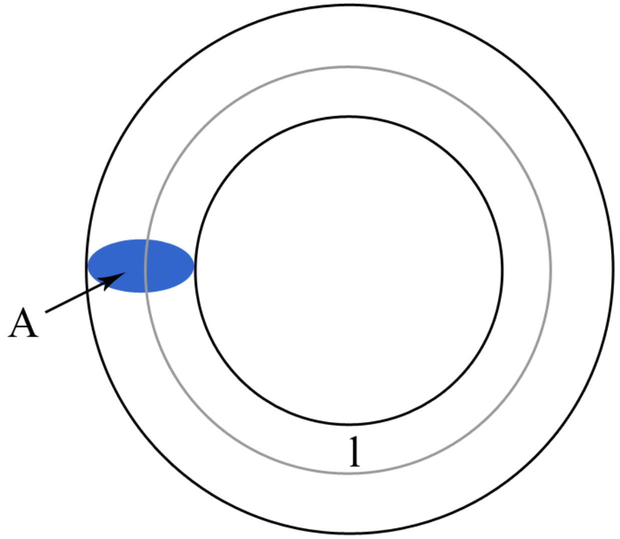
RELUCTANCE

Reluctance is equivalent to the "magnetic resistance" to the presence of the magnetic flux:

$$Reluctance, S = \frac{F_M}{\phi} = \frac{NI}{\phi} = \frac{Hl}{BA} = \frac{l}{\mu_0 \mu_r A}$$



MAGNETIC CIRCUITS SUMMARY



In summary we can say, that the magnetic field strength H can be written as:

$$\int H dl = NI$$

N is the number of turns and I the current trough the solenoid. H can be related to the magnetic flux density by:

$$B = \mu_0 \mu_r H$$

Since the magnetic flux can be calculated by:

$$\Phi = BA$$

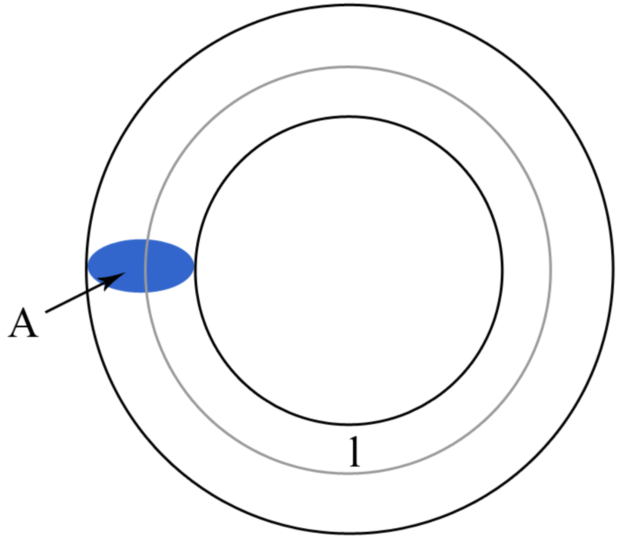
We can write:

$$\frac{\Phi}{A} = \mu_0 \mu_r H$$

And thus

$$H = \frac{\Phi}{\mu_0 \mu_r A}$$

MAGNETIC CIRCUITS SUMMARY



Assuming H as uniform we have:

$$Hl = NI$$

And combining with the last equations we have:

$$NI = \frac{\Phi l}{\mu_0 \mu_r A}$$

thus:

$$\Phi = NI / \frac{l}{\mu_0 \mu_r A}$$

We can thus write write:

$$\Re = \frac{l}{\mu_0 \mu_r A}$$

Lenz's Law - seeing that the magnetic field induced by a current induced by a change in magnetic flux (Faraday's Law) counteracts the change in flux.

INDUCTANCE

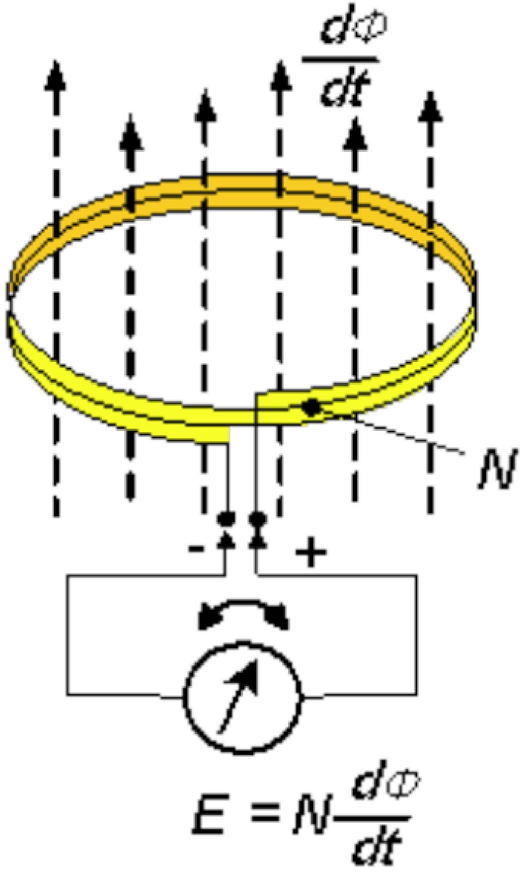
Looking back at Faradays law, we can write it as:

$$e = -\frac{\partial \Phi}{\partial t} \text{ (Lenz law)}$$

There is a relationship between the magnetic flux and the current, and that can be related by a constant, called inductance, L:

$$\Phi = BA = Li,$$

L is expressed in Henry’s [H],



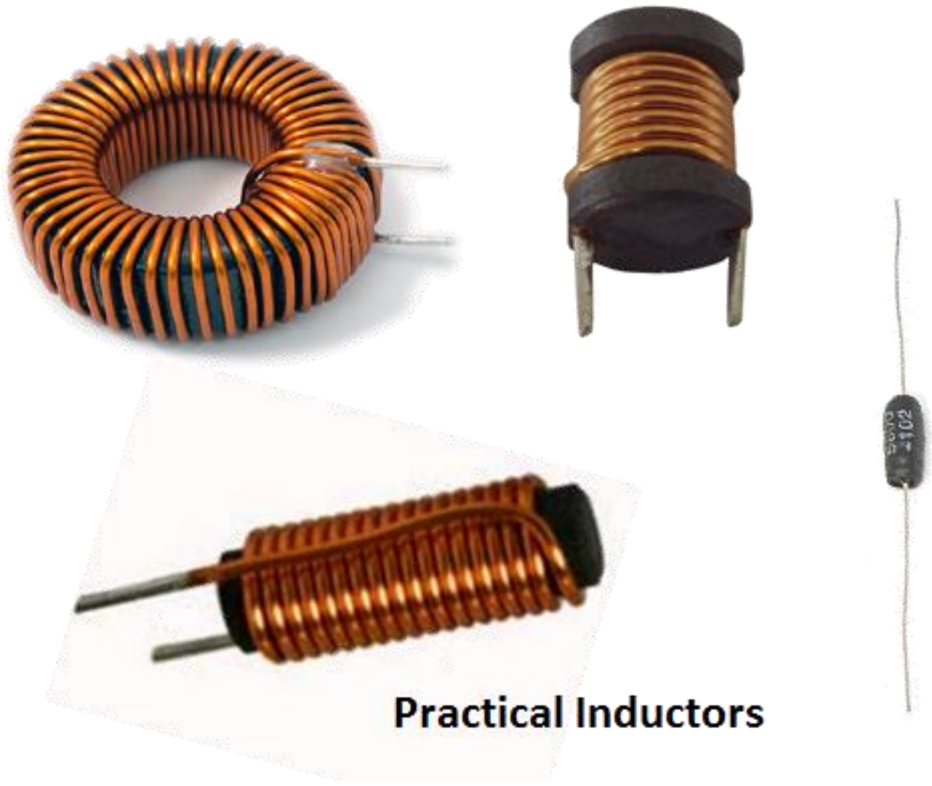
INDUCTANCE

By deriving each part of the equation we have:

$$\frac{\partial \Phi}{\partial t} = L\frac{\partial i}{\partial t} \text{ and thus } e = -L\frac{\partial i}{\partial t}$$

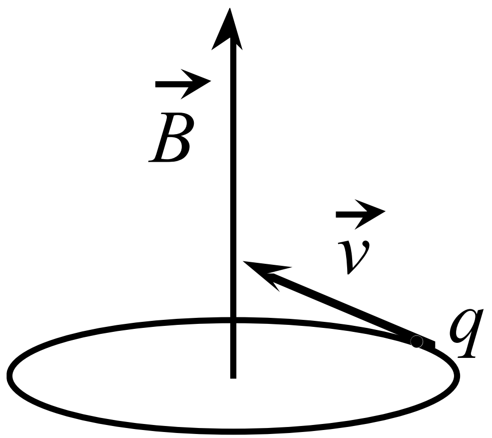
This equations give rise to the well known value of:

$$v = L\frac{\partial i}{\partial t}$$

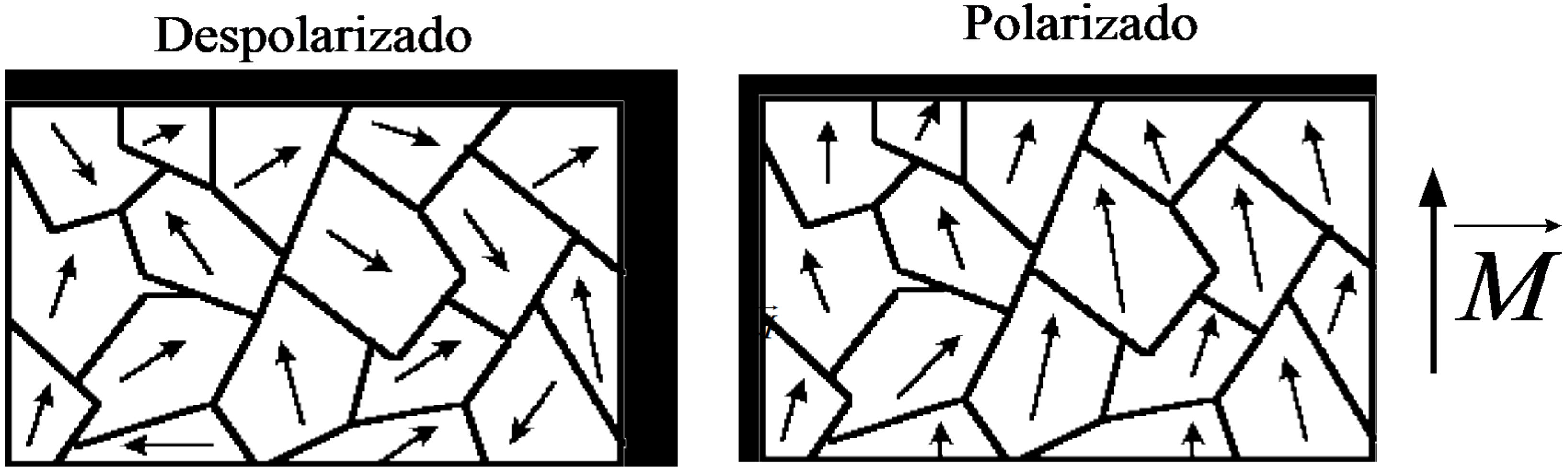


Practical Inductors

FERROMAGNETIC MATERIALS



There are materials where its relative permeability, μ_r, is several orders of magnitude higher than vacuum, and normally they are non-linear, these materials are called ferromagnetic, one of the most important materials is Iron.



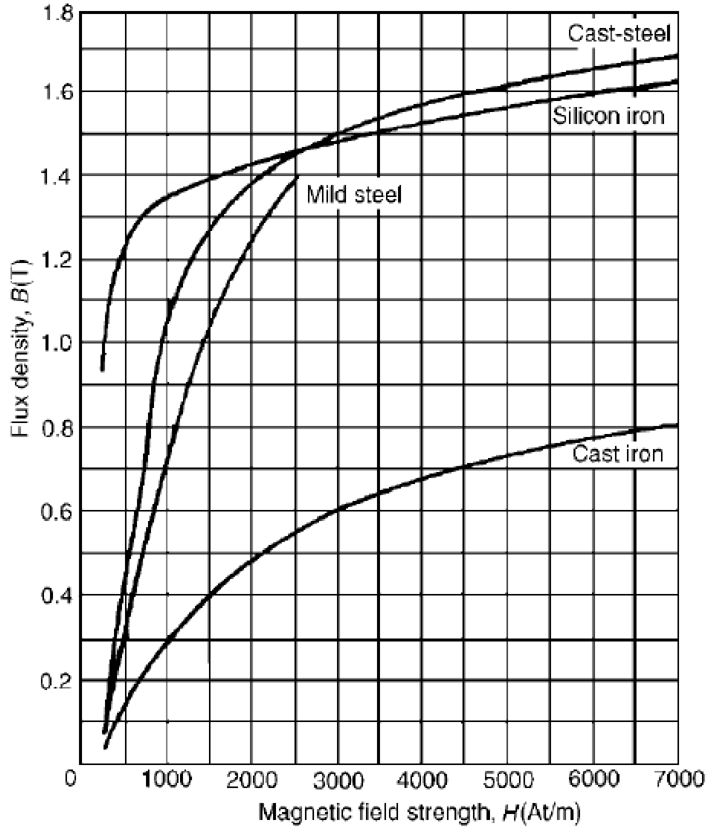
This phenomena changes for a certain temperature (called Curie point)

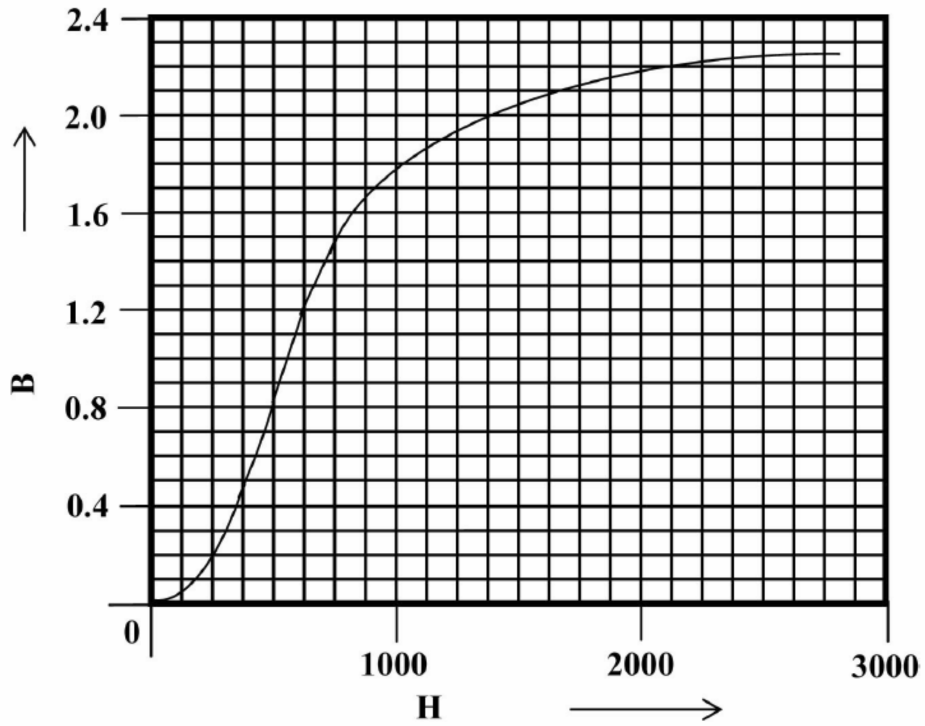
FERROMAGNETIC MATERIALS

Magnetic flux density

$$\vec{B} = \mu \vec{H}$$

The magnetization curve in most material is:





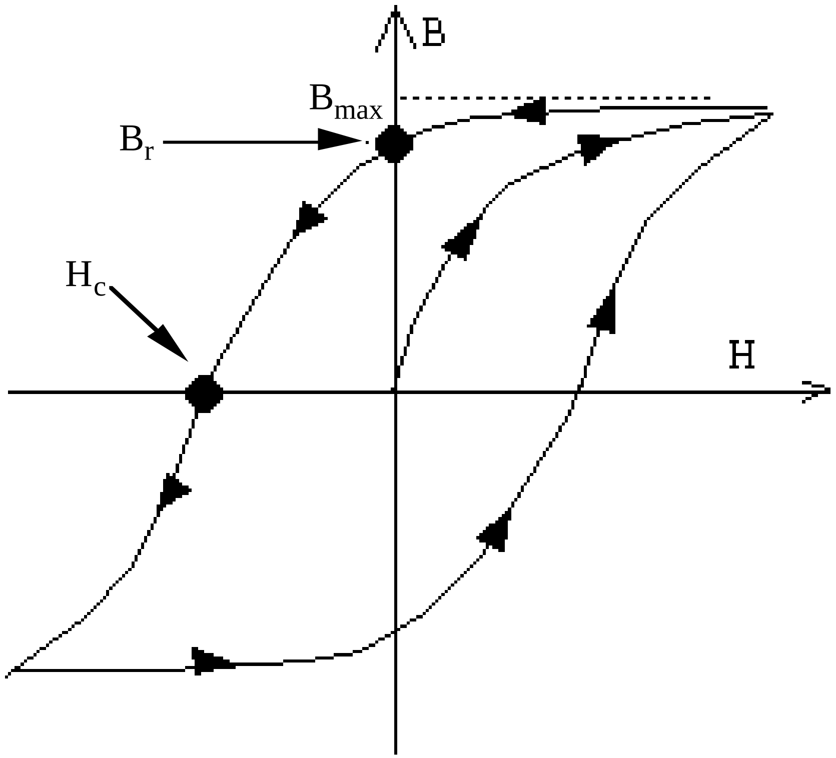
In this case in the linear part of the curve we have: $3.2mT/Am$, and thus the permeability is

$$\mu_r = \frac{1}{\mu_0}\frac{dB}{dH} \approx 2546$$

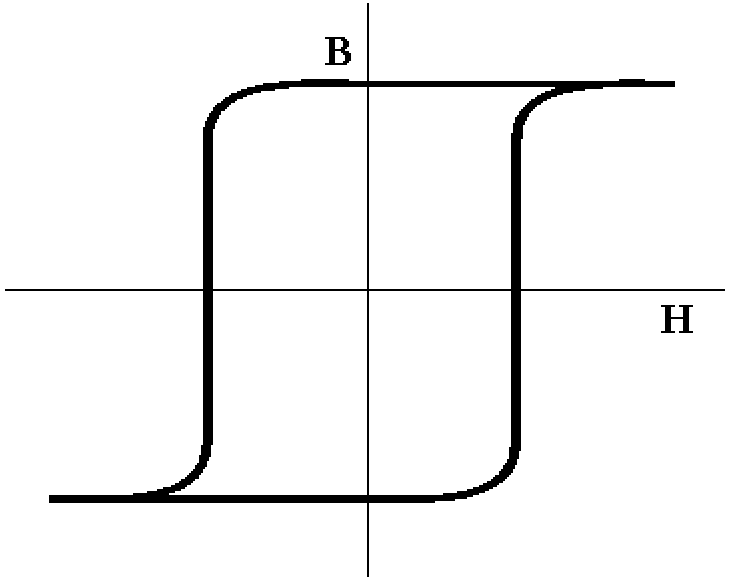
permeabilidade

HYSTERESIS

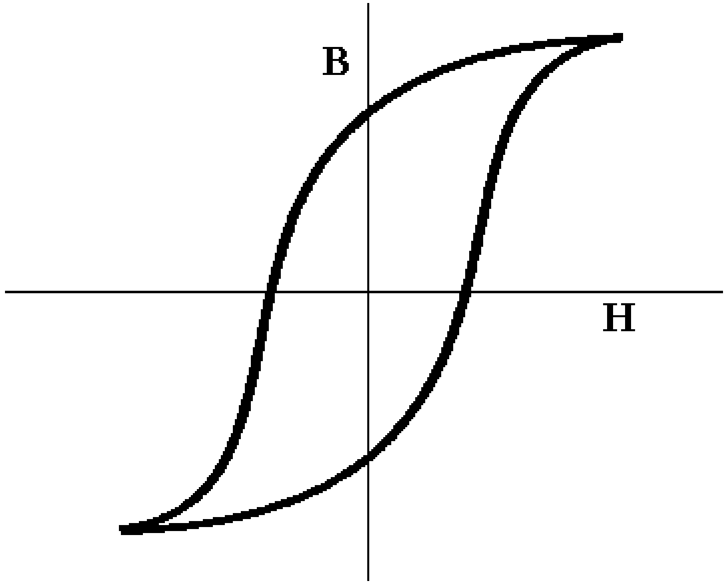
It is normal to find hysteresis in these materials, with different approaches.



HYSTERESIS



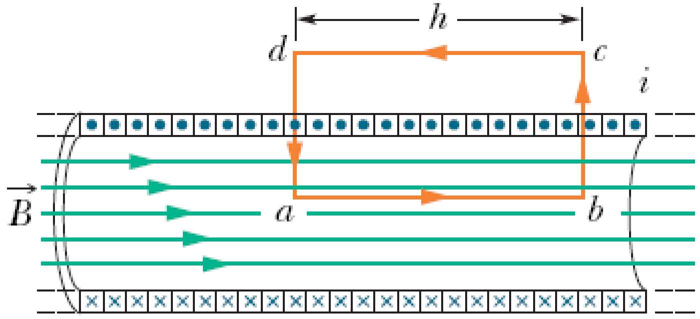
Hard material



Soft material

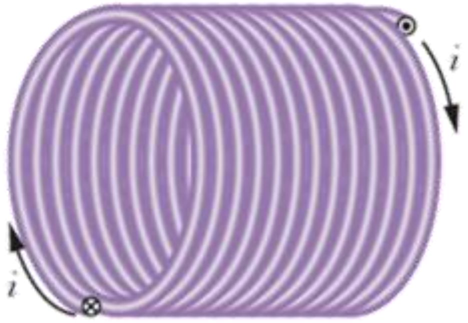
SOLENOIDS

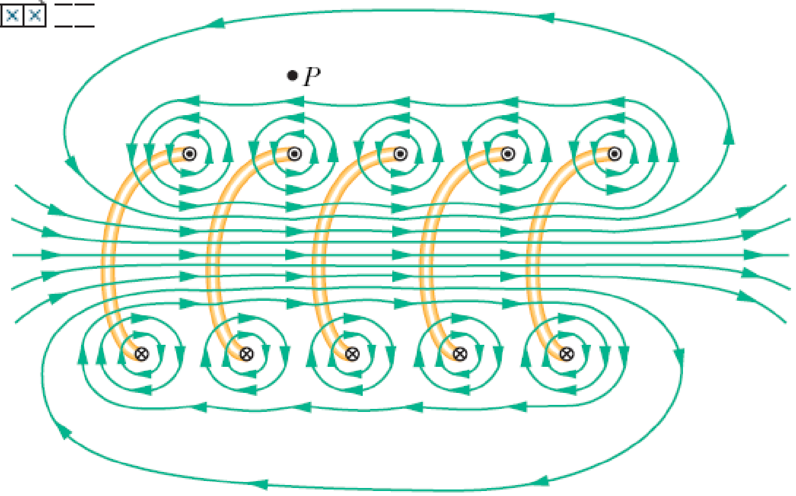
$$H = \frac{B}{\mu}$$
$$\Phi = BA$$



$$\oint \vec{B} d\vec{s} = \mu_0 i_{enc}$$

$$\oint \vec{B} d\vec{s} = \int_a^b \vec{B} d\vec{s} + \int_b^c \vec{B} d\vec{s} + \int_c^d \vec{B} d\vec{s} + \int_d^e \vec{B} d\vec{s}$$





$$i_{enc} = IN$$

$$Bh = \mu_0 IN$$

$$B = \frac{\mu_0 IN}{h}$$

$$\Phi = \mu_0 \frac{NIA}{h}$$

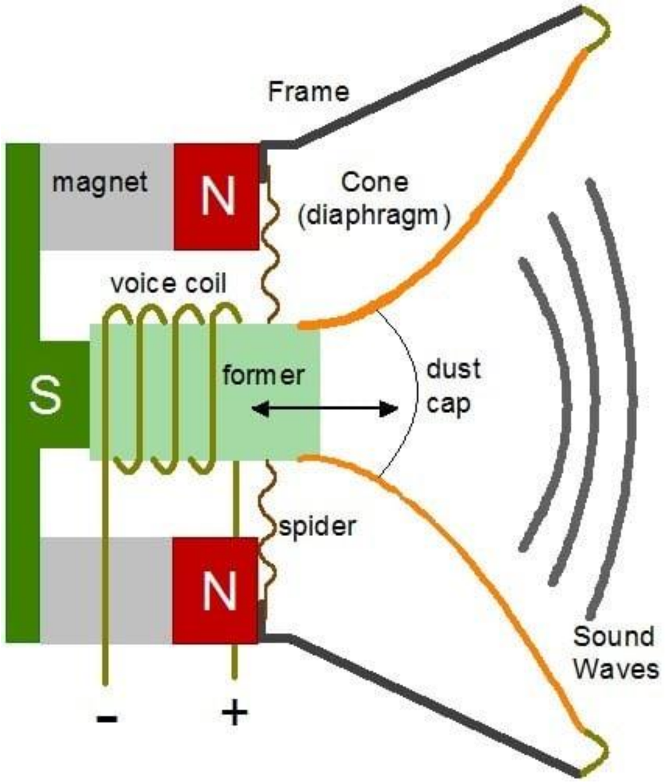
$$\Phi = Li = L\frac{I}{N}$$

$$\boxed{L = \mu_0 \frac{N^2 A}{h}}$$

Here $n=N/h$ is the number of turns per unit length of the solenoid

SOLENOIDS

$$H = \frac{B}{\mu}$$
$$\Phi = BA$$



The force between magnetic field and current is:

$$\overrightarrow{F_B} = q\vec{v} \times \vec{B}$$

$$Force\ F = BIl\ newtons$$

MAGNETIC CIRCUITS SUMMARY

Consider a toroid with the mean length of 20cm, the cross section of 2 cm^2, and the relative magnetic permeability of 6700. What is the magnetic flux and the magnetic flux density if the coil has 10 turns and the current is 2 amperes ?

$$\Re = \frac{l}{\mu_0 \mu_r A} = \frac{0,2}{4\pi 10^{-7} 6700 \cdot 2x10^{-4}} = 1,19 \times 10^5 At/Wb$$

Since

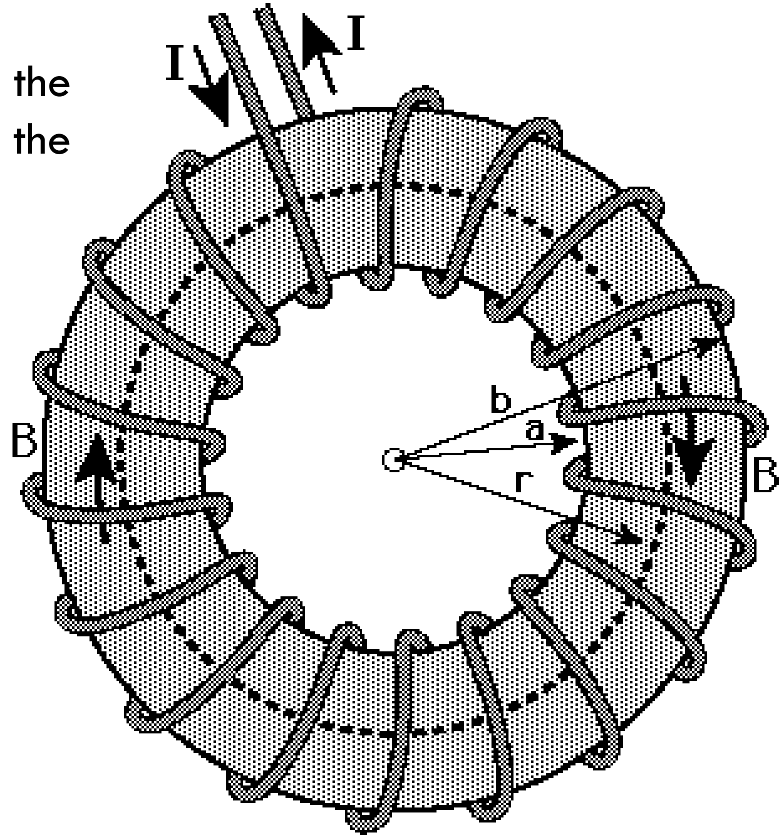
$$\Re = \frac{NI}{\Phi}$$

We can then calculate the flux as:

$$\Phi = \frac{NI}{\Re} = \frac{10 \times 2}{1,19 \times 10^5} = 1,68 \times 10^{-4} Wb$$

The magnetic flux density will then be:

$$B = \frac{\Phi}{A} = 0,84 Wb/m^2$$



At-Ampere turns

STORED ENERGY IN INDUCTORS

Energy can be calculated by first consider the power in the component terminals:

$$p = \frac{dW_L}{dt} = vi$$

But in an inductor we have:

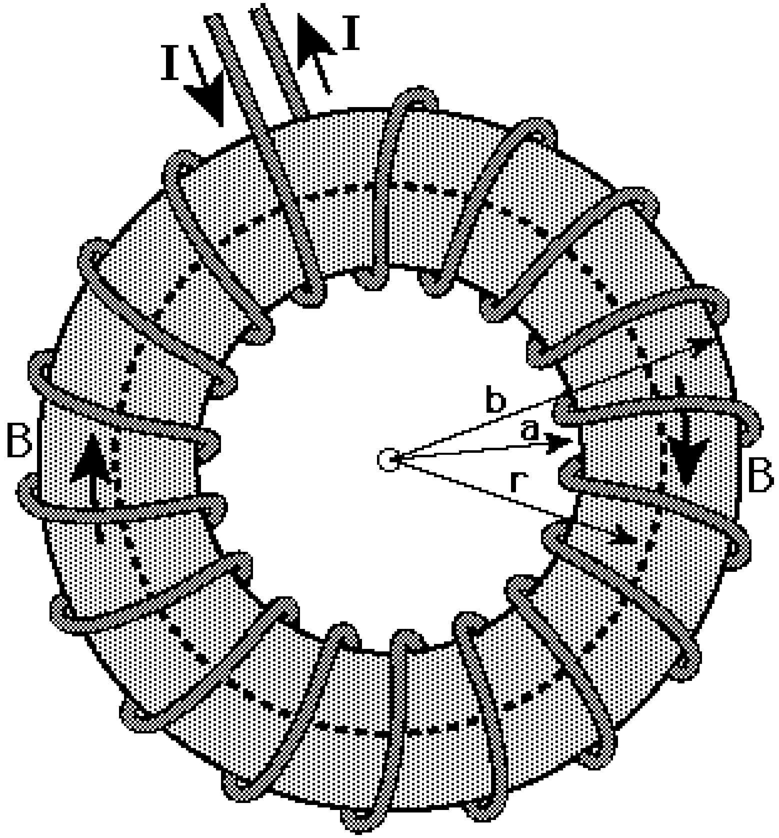
$$v = L\frac{di}{dt}$$

In this case we can thus calculate:

$$dW_L = Li\frac{di}{dt}dt = Lidi$$

By integration we can thus calculate the stored energy as:

$$W_L = \frac{1}{2}\int_0^I \frac{d}{dt}(Li^2)dt = \frac{1}{2}[Li^2]_0^I = \frac{1}{2}Li^2$$



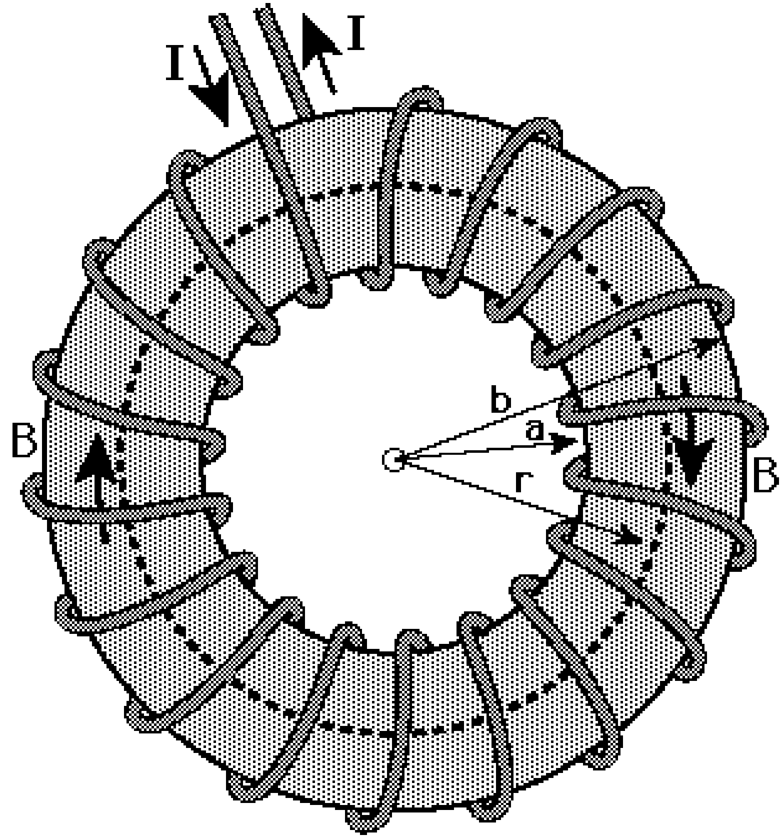
STORED ENERGY IN INDUCTORS

Using the concept presented before we can also say that the stored energy per volume can be calculated as:

$$\frac{W}{Vol} = \frac{1}{2} BH$$

For a typical 1T field we can calculate the stored energy per volume which will be:

$$5000 \frac{J}{m^3}$$

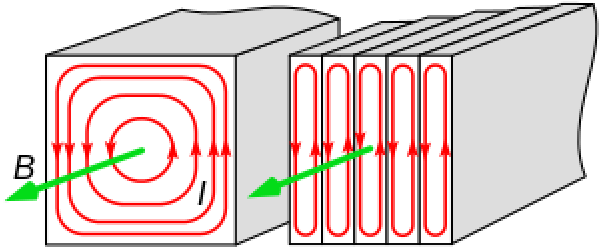


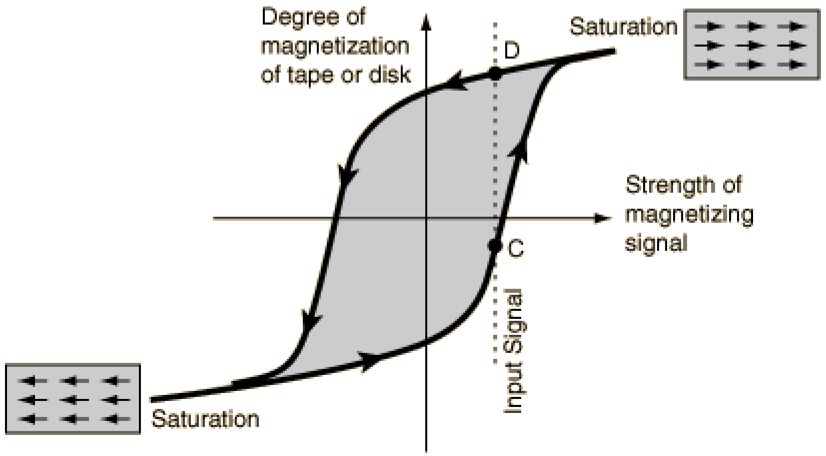
MAGNETIC LOSSES

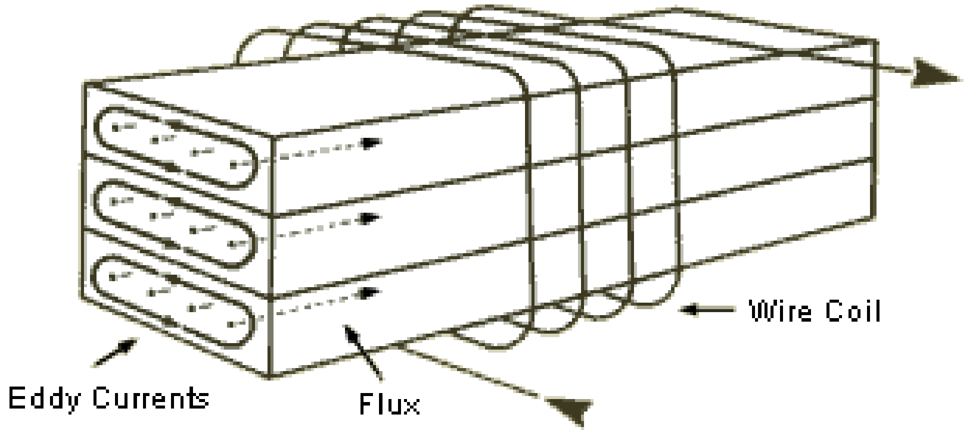
Losses in magnetic components can happen due to two main sources:

1. Hysteresis Losses
2. Inductive Losses (Eddy Currents)

Correntes de Foucault



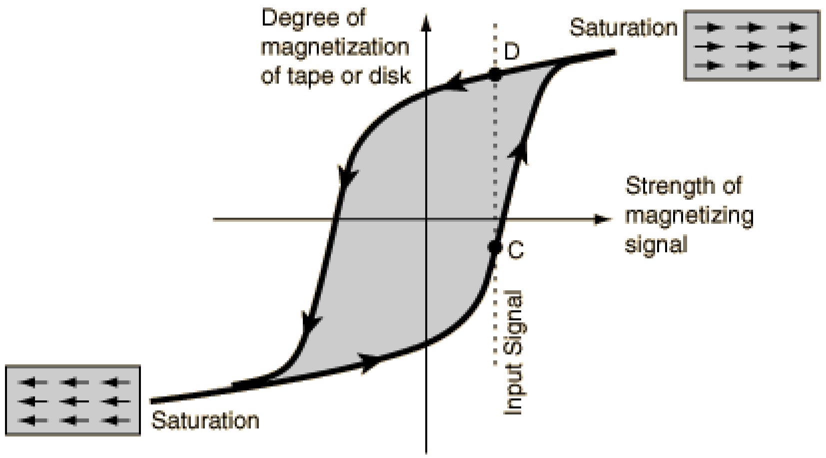




MAGNETIC LOSSES

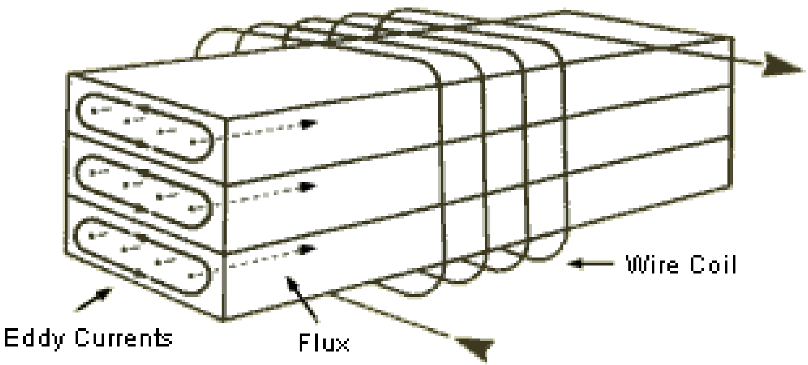
Hysteresis losses can be calculated as the difference between the stored energy when the hysteresis cycle is driven in both directions, this difference can be expressed by:

$$p_H = k_1 f$$



Eddy Current Losses are due to the magnetomotive force been induced in the ferromagnetic core, these currents are called eddy currents. This magneto force can be calculated by:

$$e = kf\Phi$$



The power losses can then be calculated by:

$$p_i = k_i f^2 \Phi^2$$

$$p_m = p_h + p_i$$

MAGNETIC CIRCUITS

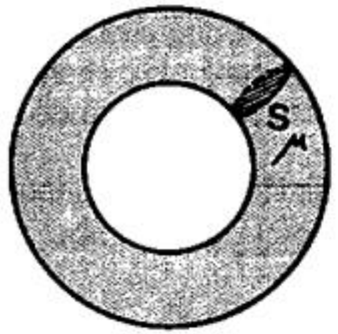


Fig. 63 — *Circuito magnético homogéneo.*

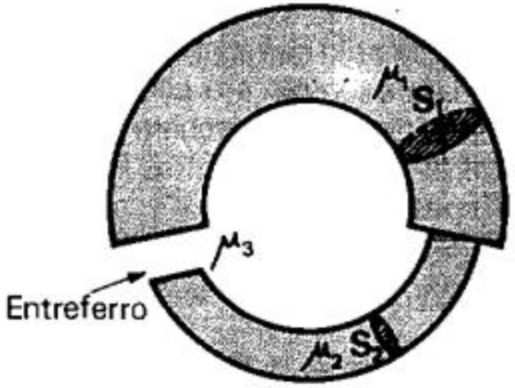
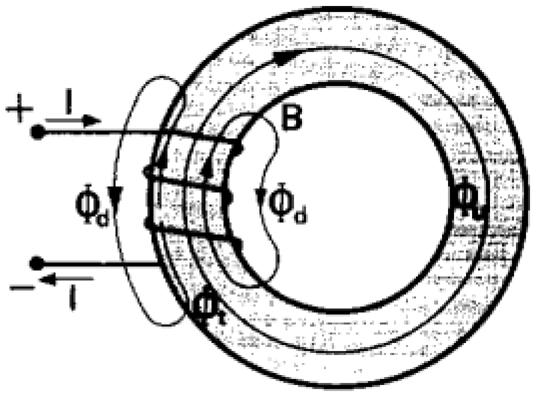


Fig. 64 — *Circuito heterogéneo.*



Dispersão magnética

MAGNETIC CIRCUITS

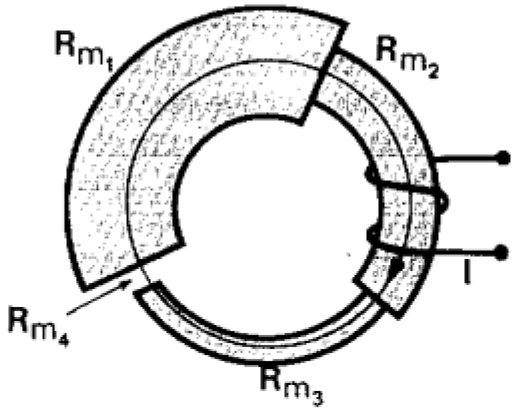


Fig. 69 — *Circuitos magnéticos em série.*

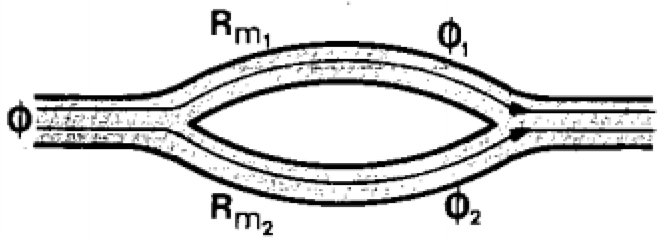


Fig. 70 — *Circuitos magnéticos em paralelo.*

MAGNETIC CIRCUITS

1 — Considere o seguinte circuito magnético constituído por um núcleo de ferro rectangular e de secção quadrangular e que tem uma bobina de 300 espiras colocada num dos seus troços.

Determine o valor da corrente eléctrica que deve percorrer a bobina de modo que se obtenha uma indução de 1 T no ferro, sendo dada a curva de magnetização do ferro utilizado (curva dada em 2-3°) e as dimensões do núcleo que estão dadas em milímetros.

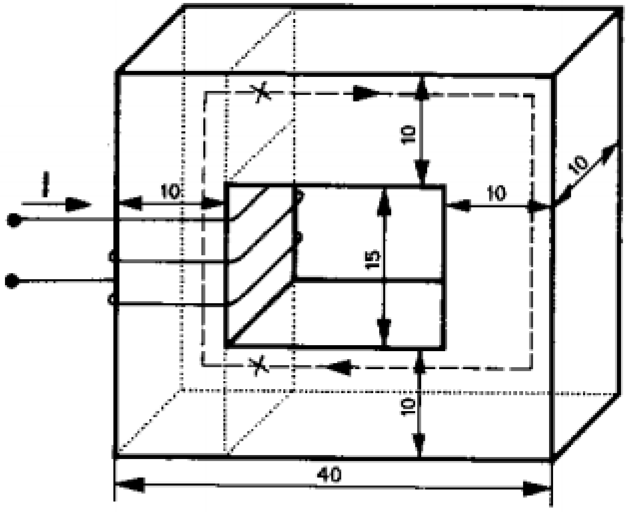


Fig. 71

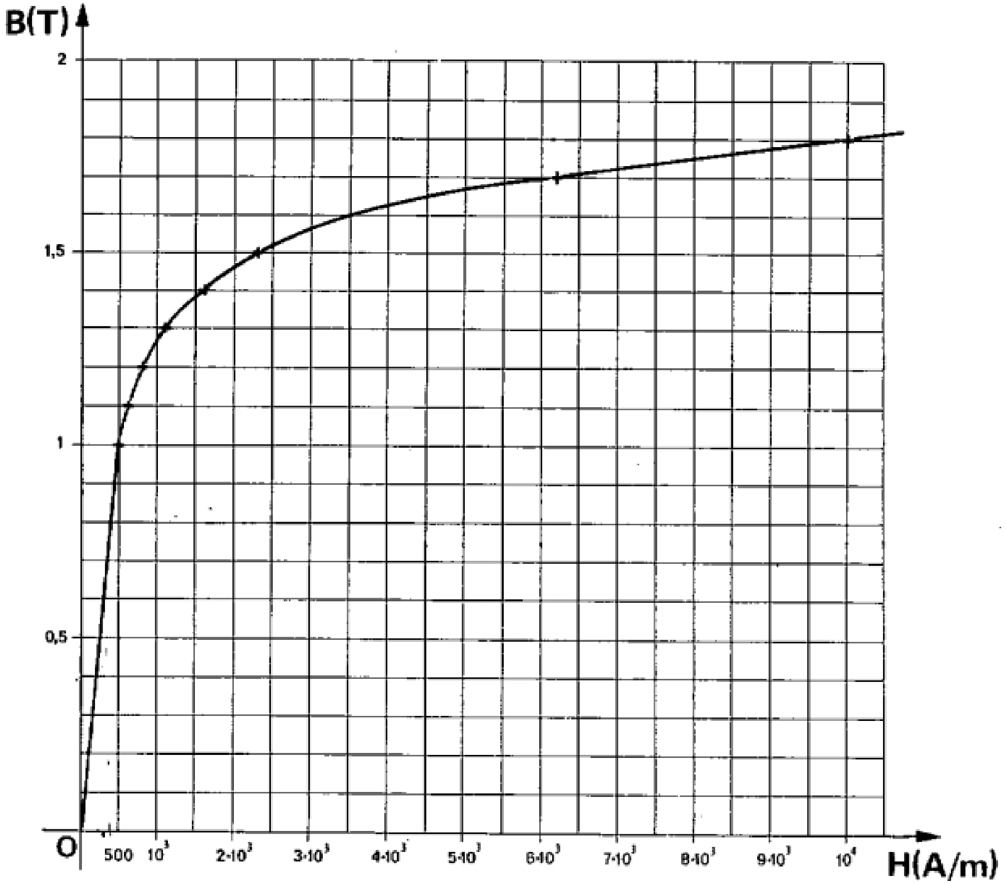


Fig. 54 — *Curva de magnetização.*

MAGNETIC CIRCUITS

1 — Considere o seguinte circuito magnético constituído por um núcleo de ferro rectangular e de secção quadrangular e que tem uma bobina de 300 espiras colocada num dos seus troços.

Determine o valor da corrente eléctrica que deve percorrer a bobina de modo que se obtenha uma indução de 1 T no ferro, sendo dada a curva de magnetização do ferro utilizado (curva dada em 2-3°) e as dimensões do núcleo que estão dadas em milímetros.

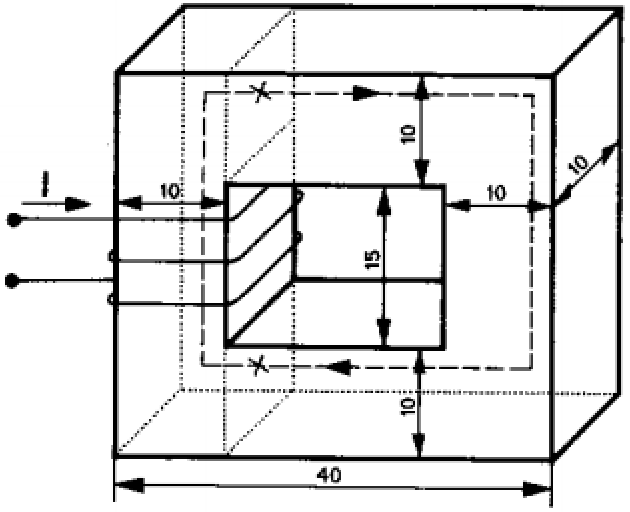


Fig. 71

Assuming H as uniform we have:

$$Hl = NI$$

For B=1T this implies H=500Ae/m

$$l = (40 - 10) + (40 - 10) + (15 + 10) + (15 + 10) = 110mm = 0{,}11m$$

thus:

$$Hl = 500 * 0{,}11 = 55Ae$$
$$NI = 300I$$
$$Hl = NI \Rightarrow 55 = 300I$$
$$I = 0{,}18A$$

MAGNETIC CIRCUITS

2 — Considere o circuito magnético representado cujas dimensões estão expressas em milímetros. As duas bobinas estão ligadas em série tendo cada uma delas 500 espiras.

Assim, sendo dada a curva de magnetização do ferro (a mesma do caso anterior) e a permeabilidade do ar, calcule a corrente que deve percorrer as bobinas para criar uma indução de 1,2 T no entreferro.

$$\mu_o = \frac{1}{8 \cdot 10^5}$$

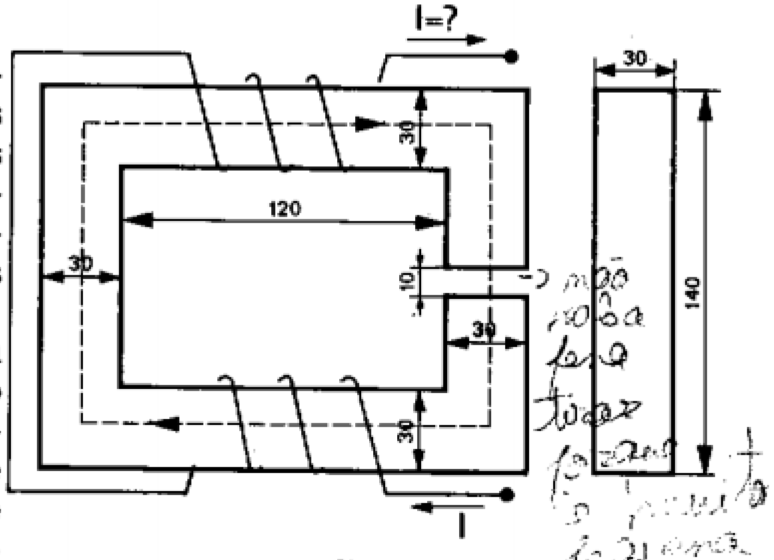
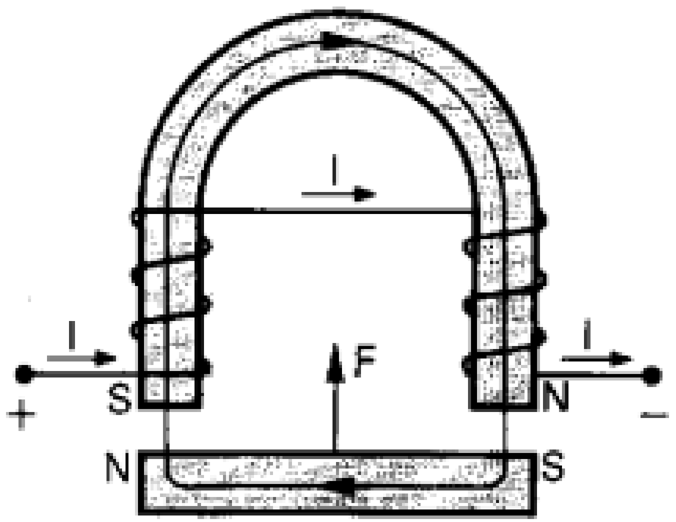


Fig. 72

MAGNETIC CIRCUITS



The force in this case will be:

$$F = \frac{B^2 A}{2\mu_0} = 400000 B^2 A$$