TECNOLOGIAS PARA SISTEMAS DE ENERGIA ESPACIAIS

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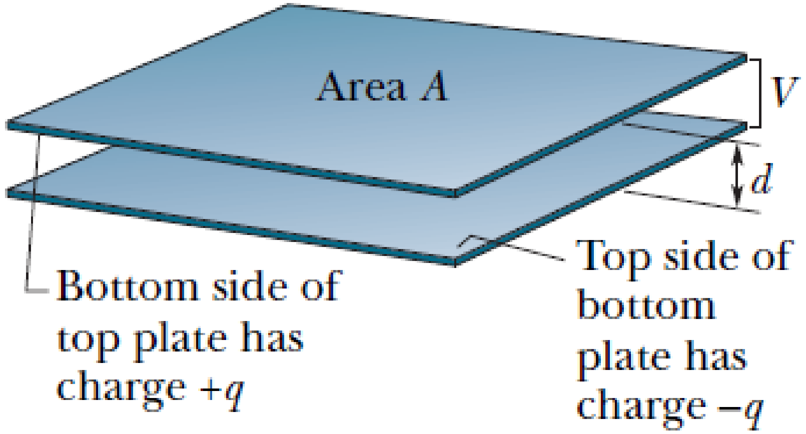
CAPACITORS

CAPACITANCE



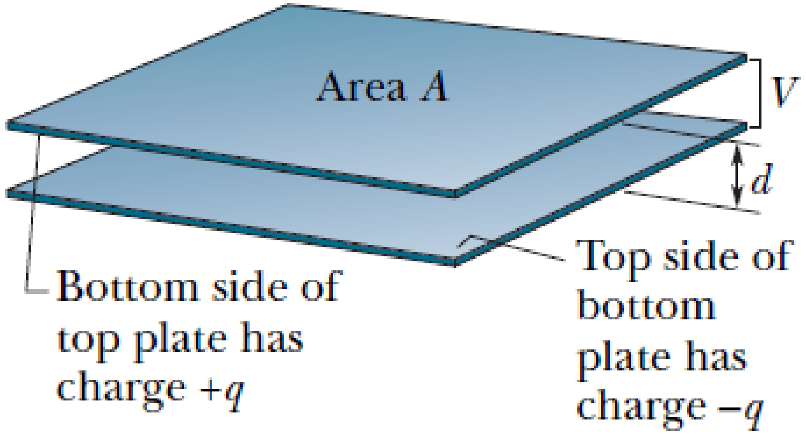
It is easy to assume that there is a relationship between the charges accumulated in capacitor plates and the voltage between them. This proportionality can be expressed as:

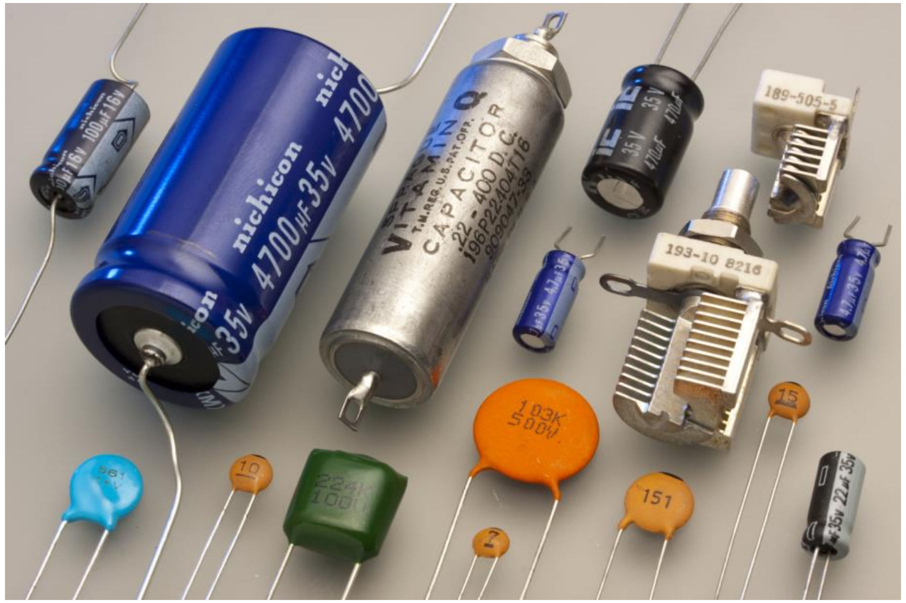
$$\boxed{Q = CV}$$

C is the proportionality constant and is called the capacitance of the capacitor. Its value is strongly dependent only on the geometry and not on the charge quantity or the potential difference.

The SI unit is called the farad (F): 1 farad (1 F)= 1 coulomb per volt =1 C/V.

CAPACITANCE





By differentiation we can find that:

$$i = \frac{dq}{dt} = C\frac{dv}{dt}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\oint \vec{E} \cdot d\boldsymbol{A} = \frac{q}{\epsilon_0}$$

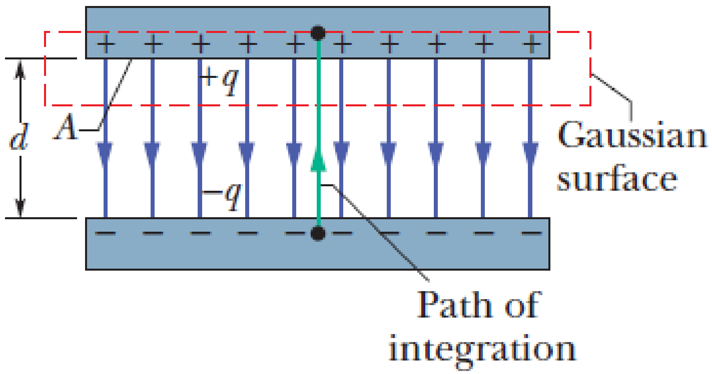
THE PARALLEL-PLATE CAPACITOR

Assuming that the field between the two plates is perpendicular to them, and the Gauss law:

$$\varepsilon \oint \vec{E} d\vec{A} = q$$

$$\varepsilon E A = q$$

We use Gauss' law to relate q and E. Then we integrate the E to get the potential difference.



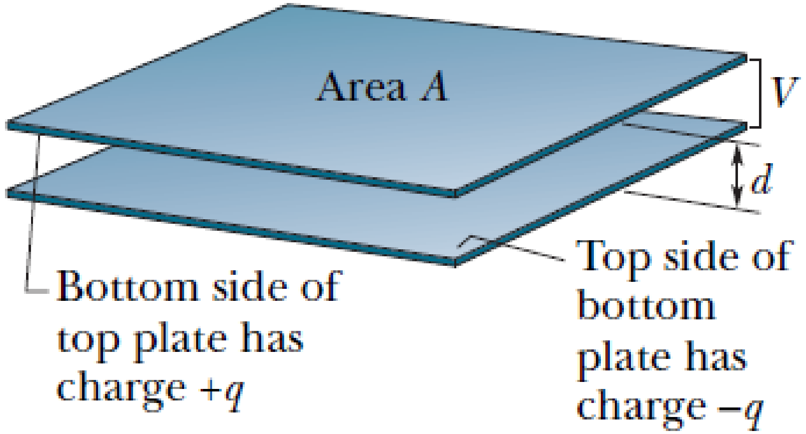
The voltage between the plates will then be:

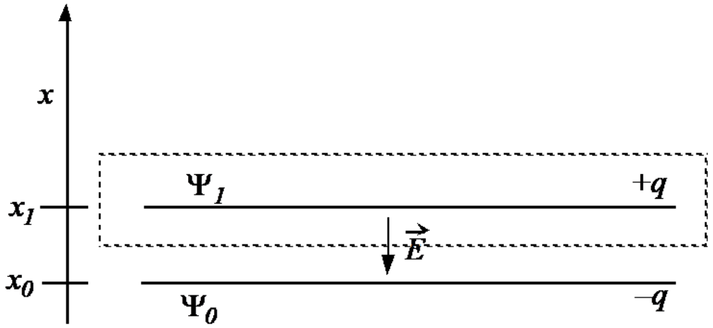
$$V_f - V_i = -\int_i^f \vec{E} d\vec{s}$$

$$v = \int_-^+ E ds = E \int_0^d ds = Ed$$

$$C = \frac{q}{v} = \frac{\varepsilon E A}{E d} = \varepsilon \frac{A}{d}$$

THE PARALLEL-PLATE CAPACITOR





$$\frac{q}{v} = C = e\frac{A}{d}$$

CYLINDRICAL CAPACITANCE

Assuming that the field between the two cylindrical surfaces is perpendicular to them, and the Gauss law:

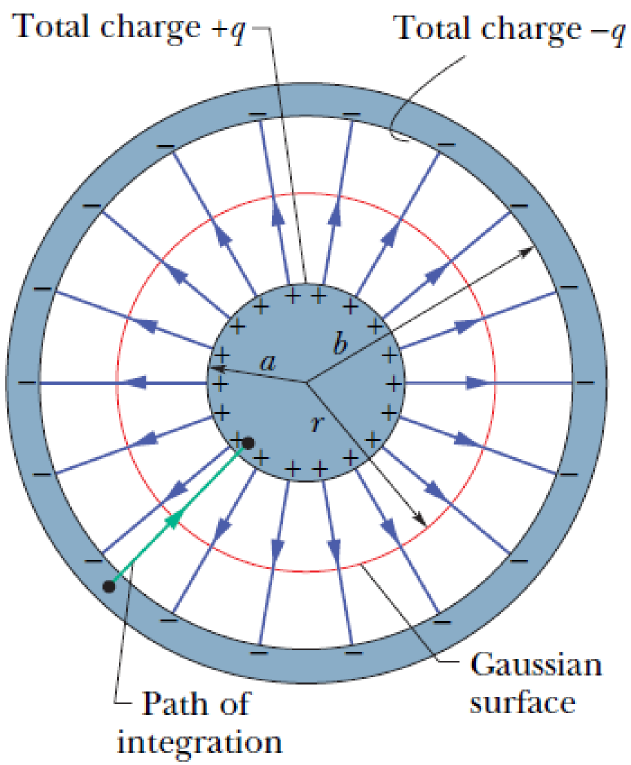
$$q = \varepsilon E A = \varepsilon E(2\pi r L)$$
$$\Rightarrow E = \frac{q}{2\pi\varepsilon L r}$$

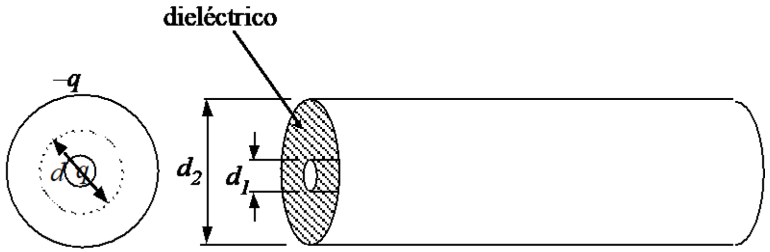
The voltage will then be:

$$v = \int_{-}^{+} E ds = -\frac{q}{2\pi\varepsilon L}\int_{b}^{a}\frac{dr}{r} = \frac{q}{2\pi\varepsilon L}\ln\left(\frac{b}{a}\right)$$

Capacity will then be:

$$C = 2\pi\varepsilon\frac{L}{\ln\left(\frac{b}{a}\right)}$$





SPHERICAL CAPACITOR

Assuming that the field between the two cylindrical surfaces is perpendicular to them, and the Gauss law:

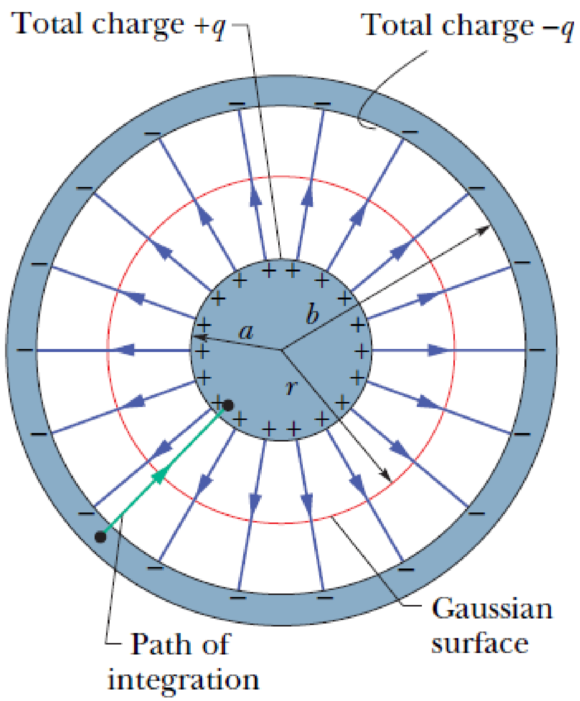
$$q = \varepsilon E A = \varepsilon E(4\pi r^2)$$
$$\Rightarrow E = \frac{1}{4\pi\varepsilon}\frac{q}{r^2}$$

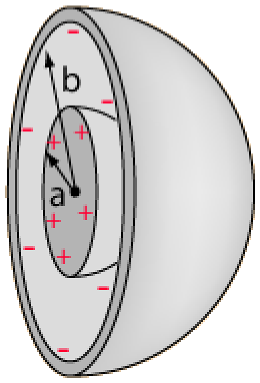
The voltage will then be:

$$v = \int_{-}^{+} E ds = -\frac{q}{4\pi\varepsilon}\int_{b}^{a}\frac{dr}{r^2} = \frac{q}{4\pi\varepsilon} = \frac{q}{4\pi\varepsilon}\left(\frac{1}{a}-\frac{1}{b}\right) = \frac{q}{4\pi\varepsilon}\frac{b-a}{ab}$$

Capacity will then be:

$$C = 4\pi\varepsilon\frac{ab}{b-a}$$





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DIELECTRICS – RELATIVE PERMITTIVITY

A dielectric, is an insulating material such as mineral oil or plastic, and is characterized by a numerical factor ε_r, called the relative dielectric constant of the material or permittivity.

$$\text{dielectric constant} = \frac{\text{absolute permittivity}}{\text{permittivity in a vacuum}}$$

Table 25-1

Some Properties of Dielectrics[a]

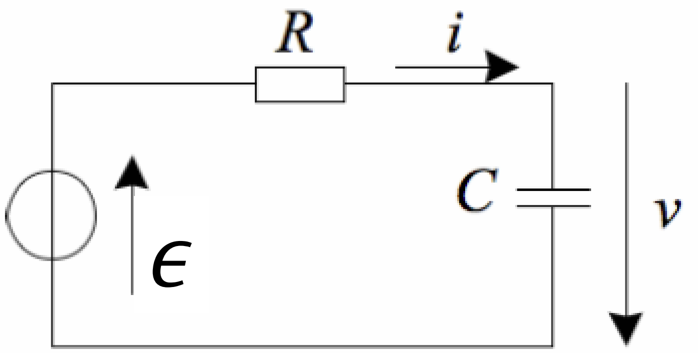
Material	Dielectric Constant κ	Dielectric Strength (kV/mm)
Air (1 atm)	1.00054	3
Polystyrene	2.6	24
Paper	3.5	16
Transformer oil	4.5	
Pyrex	4.7	14
Ruby mica	5.4	
Porcelain	6.5	
Silicon	12	
Germanium	16	
Ethanol	25	
Water (20°C)	80.4	
Water (25°C)	78.5	
Titania ceramic	130	
Strontium titanate	310	8

For a vacuum, κ = unity.

[a]Measured at room temperature, except for the water.

ENERGY STORED IN A CAPACITOR

The energy stored in a capacitor is related to the electrical field between its plates.



$$\epsilon = Ri + v \quad and \quad i = C\frac{dv}{dt}$$

$$\epsilon i dt = Ri^2 dt + vidt = Ri^2 dt + vC\frac{dv}{dt}dt$$

$Ri^2 dt$ is the dissipated energy in the resistor, and the second term is the energy in the capacitor

$$dW = \frac{1}{2}Cdv^2$$

$$W = \frac{1}{2}Cdv^2]_0^V = \frac{1}{2}CV^2$$

FILM CAPACITOR

Fig. 1

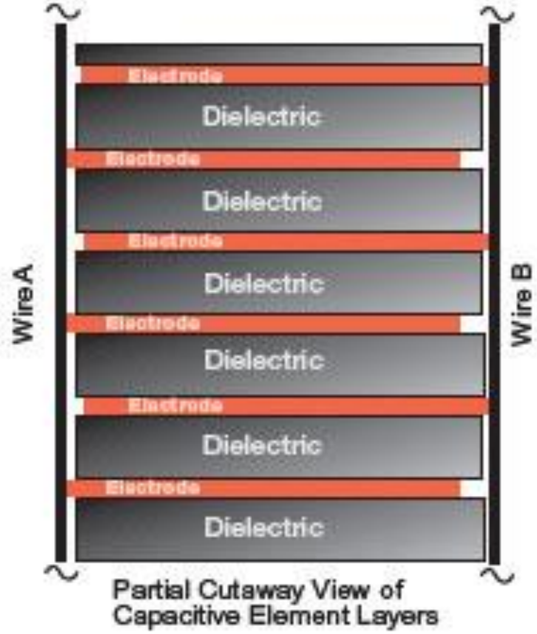
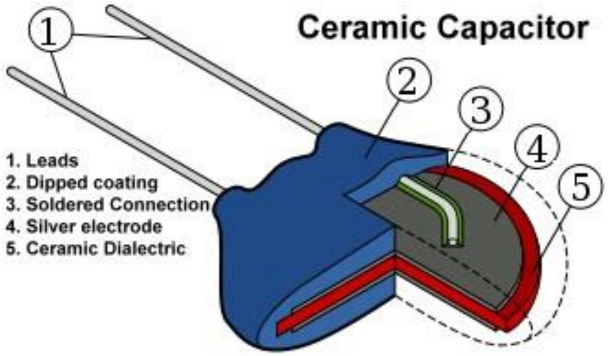
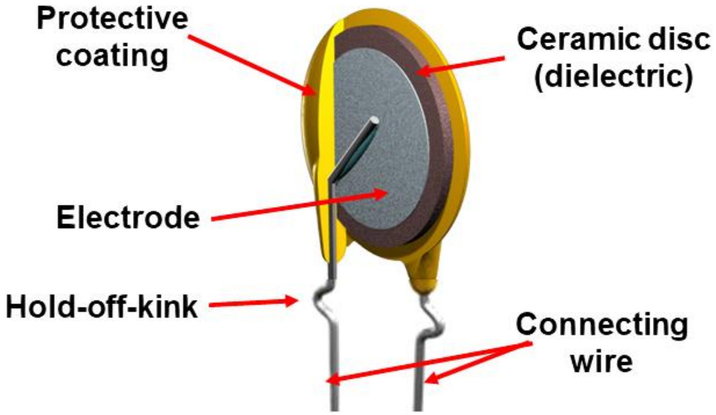


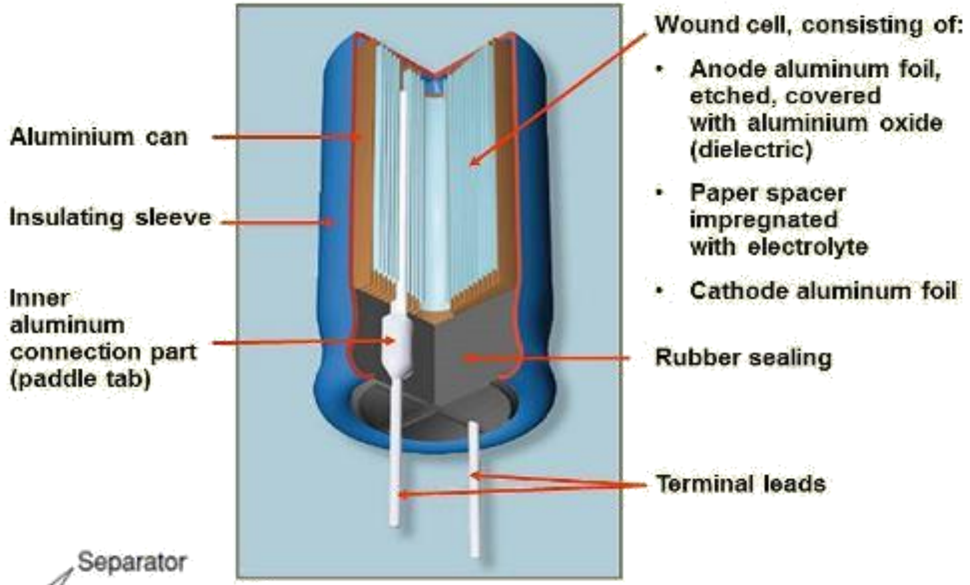
Fig.2

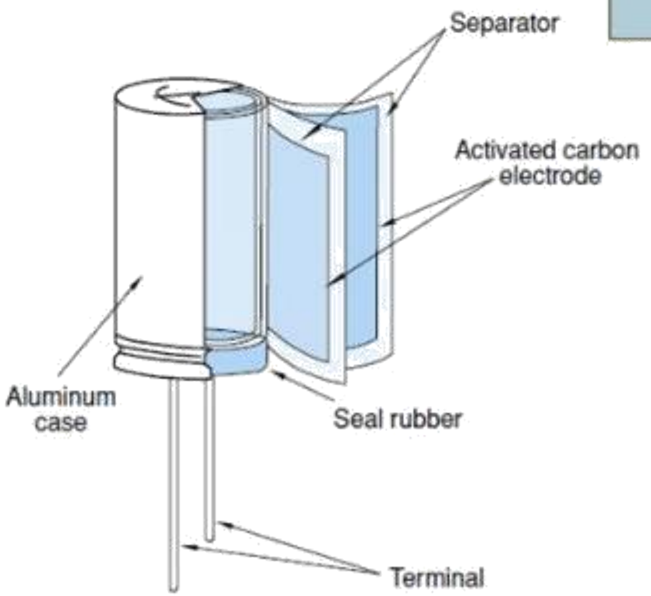
CERAMIC CAPACITOR



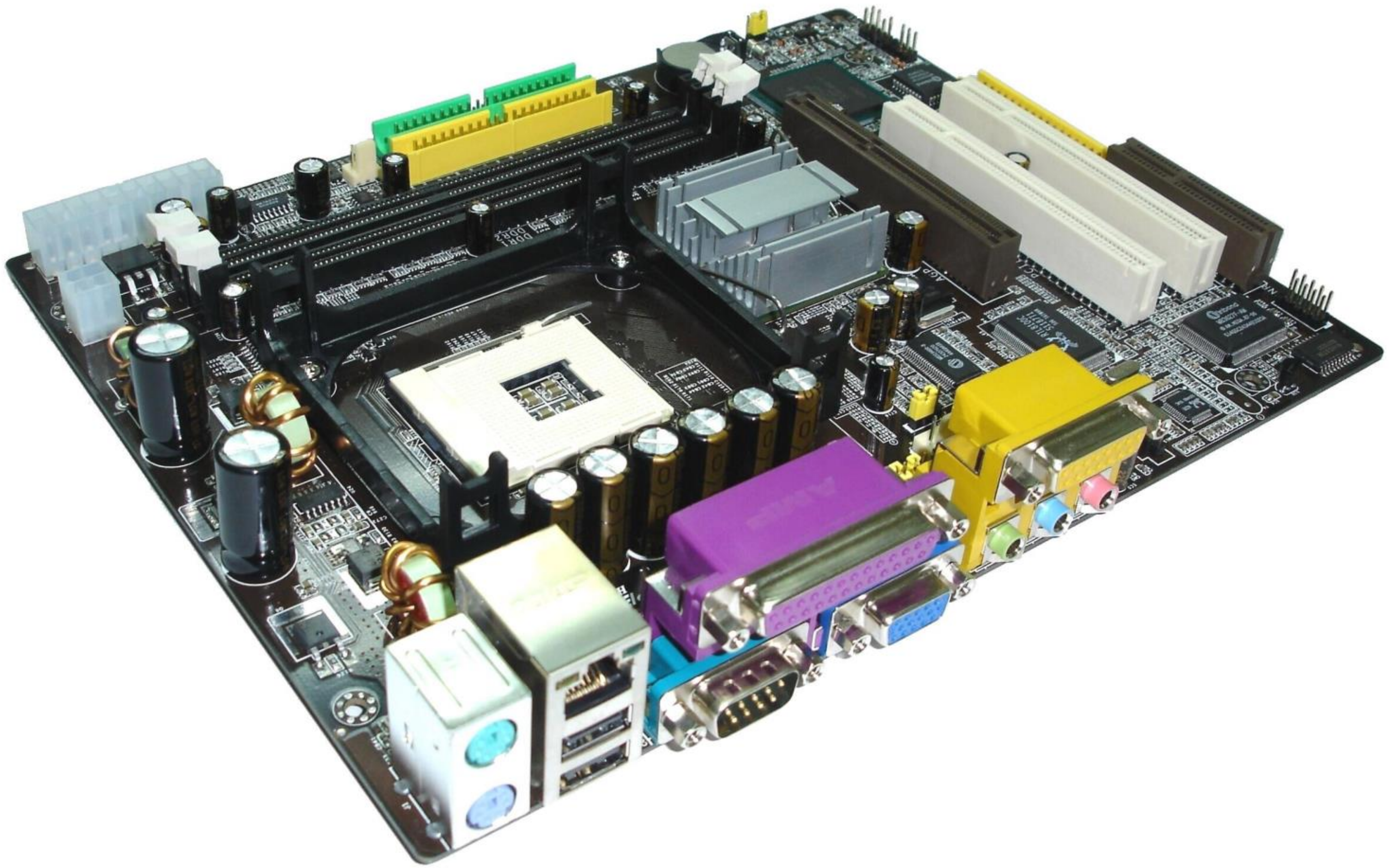


ELECTROLYTIC CAPACITOR





USE OF CAPACITORS



CAPACITORS IN CIRCUITS

In parallel Capacitors, the voltage across them is equal so:

$$q_1 = C_1 V,\ q_2 = C_2 V \text{ and } q_3 = C_3 V$$

So: $$q = q_1 + q_2 + q_3 = (C_1 + C_2 + C_3)V$$

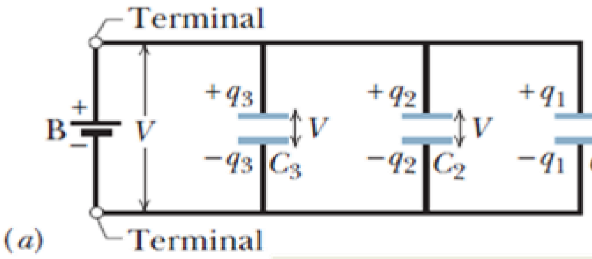
$$C_{eq} = \frac{q}{V} = C_1 + C_2 + C_3$$

In series capacitors, the summation of voltage is equal to the applied voltage so:

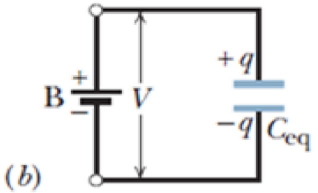
$$V_1 = \frac{q}{C_1}, V_2 = \frac{q}{C_2}, \text{and } V_3 = \frac{q}{C_3}$$

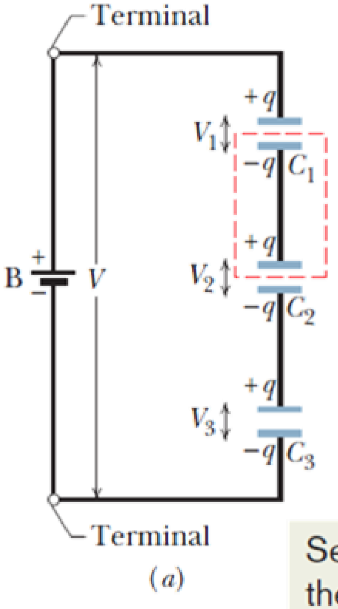
$$V = V_1 + V_2 + V_3 = q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$C_{eq} = \frac{q}{V} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$



Parallel capacitors and their equivalent have the same V ("par-V").





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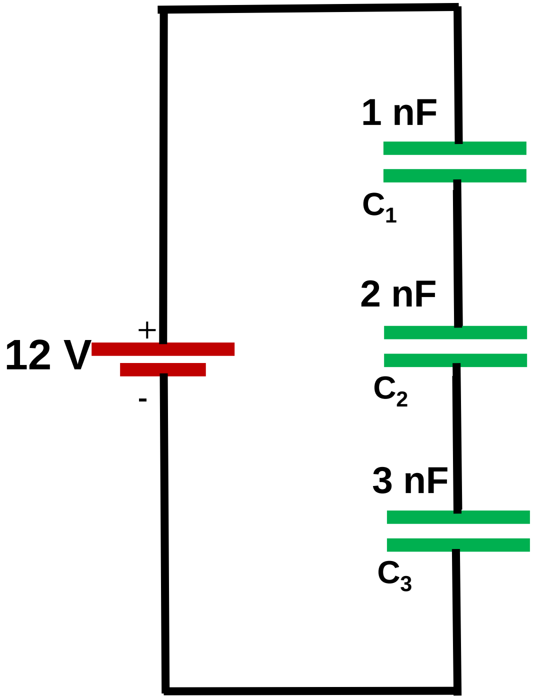
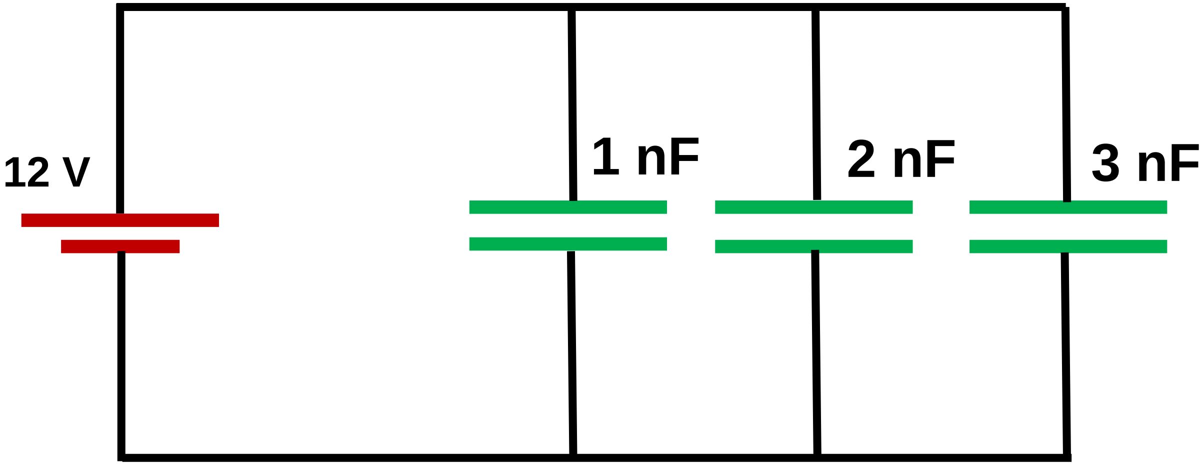
CAPACITORS IN CIRCUITS





Series:

$$C_{eq} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)^{-1} = \left(\frac{1}{1} + \frac{1}{2} + \frac{1}{3} \right)^{-1} = \left(\frac{11}{6} \right)^{-1} = \frac{6}{11} nF$$

Parallel:

$$C_{eq} = C_1 + C_2 + C_3 = 1 + 2 + 3 = 6nF$$

DIELECTRIC LOSSES

In vacuum we have $\vec{D} = \varepsilon_0 \vec{E}$, in a material we will have: $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}_e$

In this case $\vec{P}_e = \varepsilon_0 \chi_e \vec{E}$, and so we can define $\varepsilon = \varepsilon_0 (1 + \chi_e) = \varepsilon' - j\varepsilon''$

χ_e is called the dielectric medium susceptibility.

In this case $\vec{D} = \varepsilon_0 \vec{E} + \vec{P}_e = \varepsilon_0 (1 + \chi_e) \vec{E} = \varepsilon \vec{E}$

$$\tan \delta_\varepsilon = \frac{\varepsilon''}{\varepsilon'}$$

A complex dielectric has the same effect as a loss.

DIELECTRIC LOSSES

In a capacitor the losses comes from dielectric losses and Joule losses.
For the dielectric losses we should assume that the permittivity in this case will be complex:

$$\varepsilon = \varepsilon' + j\varepsilon'' = |\varepsilon|e^{-j\delta}$$

δ is the loss angle.
In this case we can also assume C=C'-jC'' and define a relationship as:

$$\tan\delta = \frac{C''}{C'}$$

Substrate	tan δ @ 10 GHz	ε_r
GaAs	0.0016	13.1
Si	0.015	11.9
Alumina 99.5%	0.0003	9.6
Sapphire	0.0001	9.4

Table 1: Loss tangents and relative dielectric constants for common semiconductor and microwave substrates

DIELECTRIC LOSSES

$$i = C\frac{dv}{dt} \Rightarrow I = j\omega VC$$

Complex Power in a capacitor can be calculated as:

$$\vec{S} = \vec{V}\vec{I}^* = -jV^2\omega\vec{C} = -jV^2(\epsilon' - j\epsilon'')\frac{\omega A}{d}$$
$$= V^2\varepsilon_0\varepsilon_r\frac{\omega A}{d}(-j + \tan\delta)$$

Using the real part of the capacity as: $C = \varepsilon_0\varepsilon_r\frac{A}{d}$, then $\vec{S} = \omega CV^2(-j + \tan\delta)$, so the losses in the capacitor will be equal to the real part of the complex power and thus:

$$P = \omega CV^2 \tan\delta$$

This power will be dissipated as heat in the dielectric.

FERROELECTRIC MATERIALS

Some materials are special in the sense that they present a deviation from linear behaviour, and so $\vec{D} = \vec{\varepsilon}\vec{E}$, will present what is called hysteresis, and behave non-linearly.