# Fairness of Exposure in Online Restless Multi-armed Bandits



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## **Healthcare Intervention**

Need to provide medical intervention to patients, but only have a limited budget to do so (number of doctors, number of available rooms, etc.). There are many things to consider for any potential solution.

- The patient's condition may change.
- The doctor may not have a good estimate of how a patient's condition might evolve.
- The limited budget.

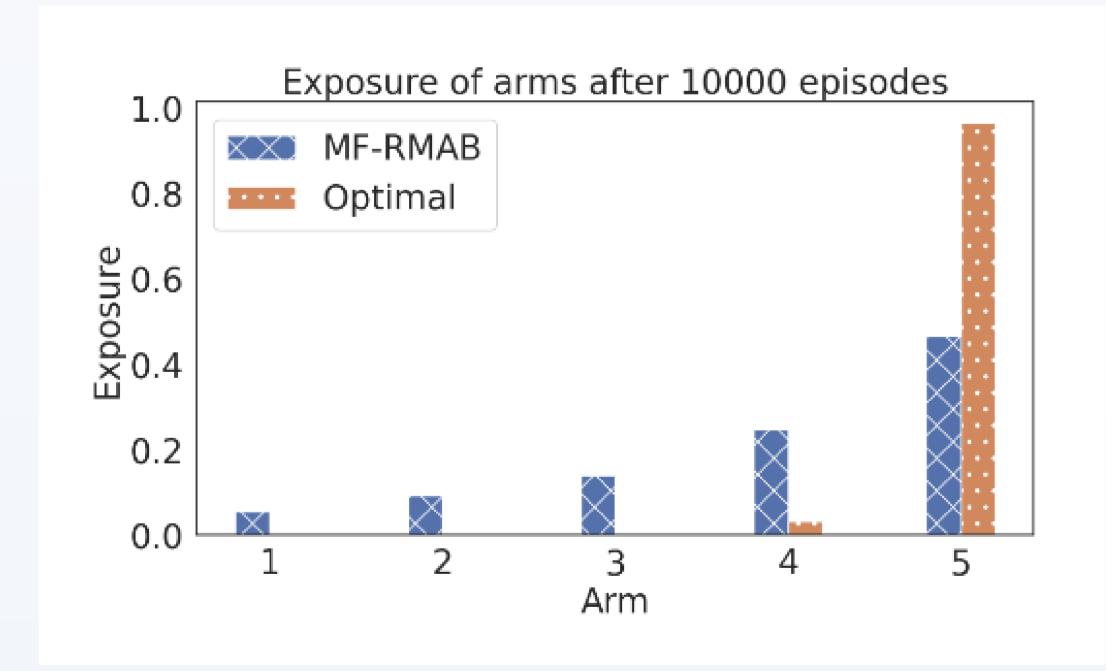
#### **Restless Multi-armed Bandit**

- Each patient is modelled as an 'arm'. Total number of arms = N
- Providing intervention to a patient is called 'pulling' that arm.
- Each arm has two states: 'good' and 'bad'.
- Arms transition from one state to another based on their 'transition probabilities' P.
- We run our algorithm (policy) for total T episodes.
- We can only pull K (< N) arms at one-time step.</li>
- Ideally, we want to pull the arm which will go from bad to good state due to our intervention.
- We can define the 'reward' of an arm as the benefit the arm receives from getting pulled.

The Optimal Policy need not be fair!

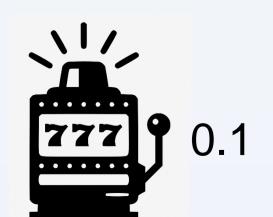
# **Meritocratic Fairness**

- Pulling arms with the highest reward leads to some arms getting starved of attention.
- In healthcare, this would imply that some patients barely receive medical help!
- There is a need for policies that provide fair exposure to each arm.



Pull each arm with probability proportional to their reward.

# **Motivation**







Optimal Allocation: Arm 3 with reward 0.9

Fair Allocation: Pulling probabilities of 0.055, 0.44, and 0.5 respectively

# **Proposed Algorithm: MF-RMAB**

For each episode *t*,

- 1. Learn the transition probabilities  $P_i^t$  for each arm i via Upper Confidence Bound approach [1].
- 2. Find out steady state probability  $f_i(P_i^t, p_i)$  of arm being in 'good' state (when hypothetically pulled with probability  $p_i$ ) [2].
- 3. Estimate reward  $\mu_{i}^{t} = f_{i}(P_{i}^{t}, 1) f_{i}(P_{i}^{t}, 0)$ .
- 4. Define meritocratic fair policy  $\pi$ , where  $\pi_i$  is the probability of arm i being pulled.  $\pi_i^t = \frac{g(\mu_i^t)}{\sum_i g(\mu_i^t)}$ , where g(.) is a non-decreasing positive Lipschitz-continuous function [3].
- 5. Sample K arms from  $\pi^t$ .

## **Fairness Regret**

Suppose we already know the true transition probabilities  $P^*$  of all the arms. The subsequent policy according to MF-RMAB is denoted by  $\pi^*$ .

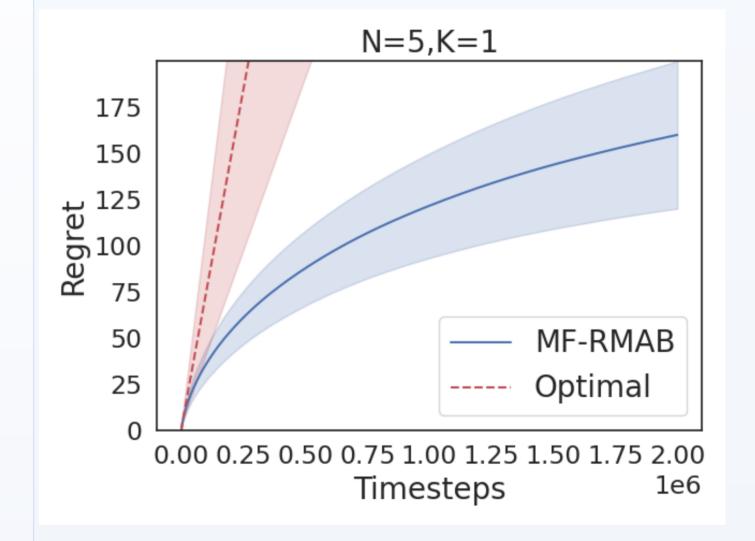
The Fairness Regret (FR) is defined as the difference between the policies when all information is known vs when we have to estimate the transition probabilities.

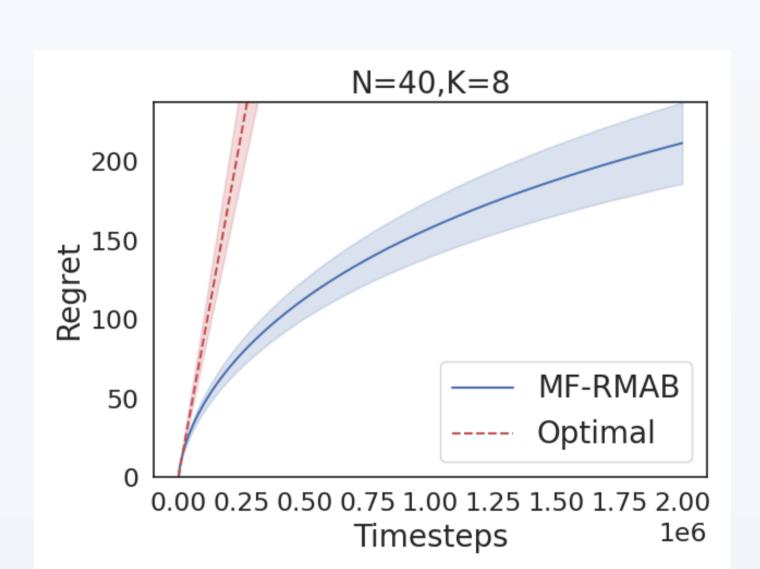
$$FR^{T} = \sum_{t=1}^{T} \sum_{i \in [N]} |\pi_{i}^{*} - \pi_{i}^{t}|$$

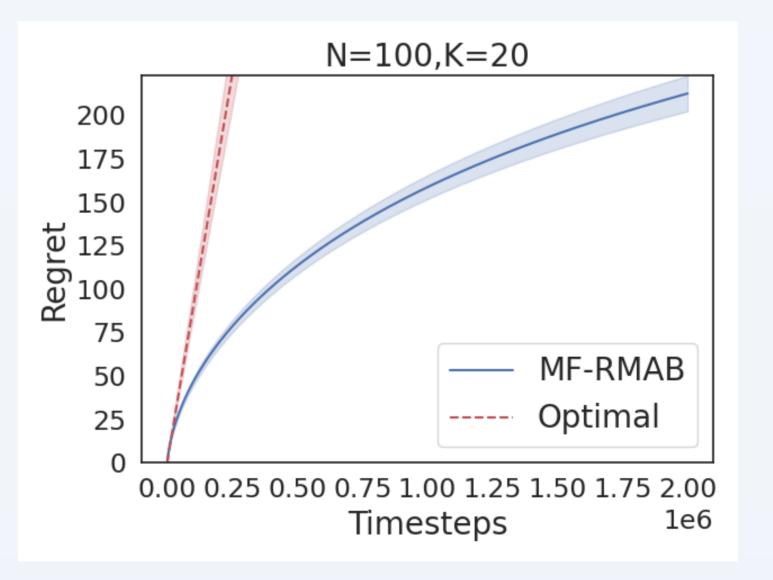
### **Theoretical Results**

**Theorem**: MF-RMAB incurs  $O(\sqrt{T \ln T})$  fairness regret for sufficiently large T and K = 1.

## **Experimental Results**







## References

- [1] Kai Wang, Lily Xu, Aparna Taneja, and Milind Tambe. Optimistic whittle index policy: Online learning for restless bandits. (AAAI 2023) [2] Christine Herlihy, Aviva Prins, Aravind Srinivasan, and John P Dickerson. Planning to fairly allocate: Probabilistic fairness in the restless bandit
- setting. (ACM SIGKDD 2023)
- [3] Legun Wang, Yiwei Bai, Wen Sun, and Thorsten Joachims. Fairness of exposure in stochastic bandits. (ICML 2021)











