Fair Allocation with Special Externalities

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Resource Allocation

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Fair Division

Objective: Divide items among agents **fairly** and **efficiently**

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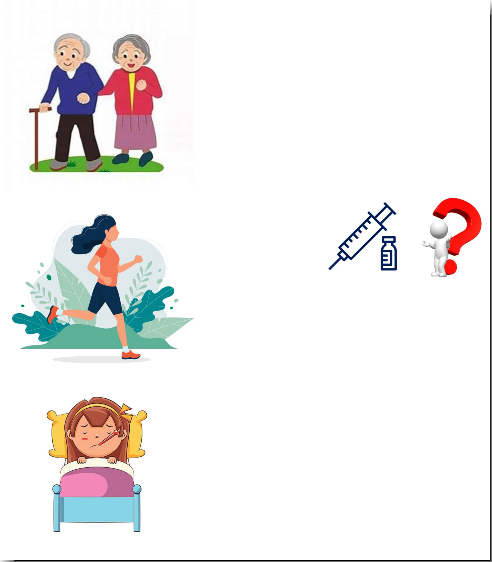
- Envy free
- Proportionality
- Equitability
- MMS

							total
Agent 1	1	6	1	2	1	1	12
Agent 2	4	1	2	1	1	3	12
Agent 3	1	1	3	3	3	1	12

Envy Free (EF)

Figure: Envy Free (EF) : No agent envies the other

Motivation



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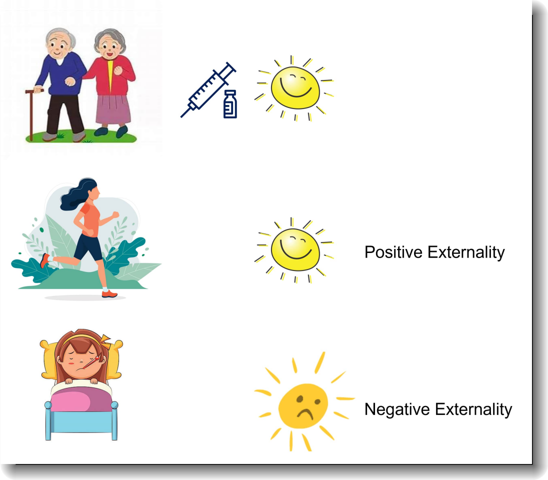

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Achieving fairness in the presence of externalities

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- Do existing definition generalizes? Or new definitions?
- When can we leverage existing algorithms?

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Contributions

- **PROP-E**: introduce proportionality for general valuations with full externalities
- $\mathfrak{T}$: Transformation on 2-D valuations that retains fairness and efficiency notions – One can adapt the existing algorithms for these settings
- **Shifted α-MMS**: α-MMS may not exist in 2-D. We propose Shifted α-MMS, a novel way of approximating MMS in 2-D

State of the Art

Key Results without Externalities

- Utilities/Valuations only if agent receives goods - we call it **1-D valuations**

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Items	Valuation	Fairness	Results
any	general	EF1	✓[Bhaskar et al., 2020]
goods	identical general	EFX	✓[Plaut and Roughgarden, 2020]
chores	IDO general	EFX	✓[Li et al., 2021]
goods	additive	PROPX	✗[Aziz et al., 2022]
goods	additive	PROPM	✓[Baklanov et al., 2021]
chores	additive	PROPX	✓[Li et al., 2021]
goods	additive	$3/4 + 1/12n$-MMS	✓[Garg et al., 2019]
chores	additive	$11/9$-MMS	✓[Huang and Lu, 2021]
any	additive	α-MMS	✗[Kulkarni et al., 2021]

Table: Existing results for 1-D valuations

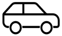
Key Results for Externalities

Items	Valuation	Externalities	Fairness	Results	Paper
goods	additive	positive	MMS	✓Proposed extended-maximin-share (EMMS)	[Seddighin et al., 2019]
				✓Explored relaxation of EMMS	
			Proportionality	✓Proposed Average Share (Proportionality)	
any	additive	any	EF	✓EF1/EFX for two agents	[Aziz et al., 2021]
				✗EF1 for three agents	
			Proportionality	✓Proposed General Fair Share	

Table: Existing results for indivisible items with Externalities

Fairness with Special Externalities

Challenges with Externalities

						
Agent 1	(1,..,..)	(6,..,..)	(1,..,..)	(2,..,..)	(1,..,..)	(1,..,..)
Agent 2	(4,..,..)	(1,..,..)	(2,..,..)	(1,..,..)	(1,..,..)	(3,..,..)
Agent 3	(1,..,..)	(1,..,..)	(3,..,..)	(3,..,..)	(3,..,..)	(1,..,..)

EF ??

Agent 1 might not value pen much, but on not receiving it they might incur a high negative utility.

2-D Valuations for Special Externalities

2-D Valuation

$v = (v_{ik}, v'_{ik}) \in \mathbb{R}^2$

v_{ik}: valuation that agent i obtains if **receives** an item k

v'_{ik}: valuation that agent i obtains if **does not receive** an item k

$v'_{ik} < 0$ indicates loss for not receiving item k

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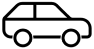
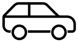
		
Agent 1	(5,-20)	(6,-10)
Agent 2	(6,-19)	(5,-20)

Our Key Results for 2-D Valuations

Items	Valuation	1-D Fairness	2-D Fairness
any	general	EF1	EF1
goods	identical general	EFX	EFX
chores	IDO general	EFX	EFX
goods	additive	PROPM	~~PROPM~~ **PROPM-E**
chores	additive	PROPX	~~PROPX~~ **PROPX-E**
goods	additive	$3/4 + 1/12n$-MMS	~~$3/4 + 1/12n$-MMS~~
chores	additive	$11/9$-MMS	~~$11/9$-MMS~~

Table: Fairness: 1-D vs 2-D

Fairness: Proportionality

		
Agent 1	(5,-20)	(6,-10)
Agent 2	(6,-19)	(5,-20)

~~Proportionality~~

Proportionality in 2-D

Definition (Proportionality (PROP))

An allocation A satisfies PROP if, $\forall i \in N, u_i(A_i) \geq \frac{1}{n} \cdot v_i(M)$

Definition (Proportionality with externality (PROP-E))

An allocation A satisfies PROP-E if,
$\forall i \in N, u_i(A_i) \geq \frac{1}{n} \cdot \sum_{j \in N} u_i(A_j)$

Transformation from 2-D $\mathcal{V}$ to 1-D $\mathcal{W}$

Definition (Transformation $\mathfrak{T}$)

Given a resource allocation problem $(N, M, \mathcal{V})$, we obtain an equivalent problem in 1-D valuations with valuations denoted by $\mathcal{W} = \mathfrak{T}(\mathcal{V}(\cdot))$ as follows,

$$\forall i \in N, w_i(A_i) = \mathfrak{T}(V_i(A_i)) = v_i(A_i) + v_i'(A_{-i}) - v_i'(M)$$

Illustration of $\mathfrak{T}$

			
Agent 1	(5,-20)	(6,-10)	(16,-40)
Agent 2	(6,-19)	(5,-20)	(16,-40)

			
Agent 1	35	26	56
Agent 2	26	29	56

Theorem

An allocation A is $\mathfrak{F}$-Fair and $\mathfrak{E}$-Efficient in $\mathcal{V}$ iff A is $\mathfrak{F}$-Fair and $\mathfrak{E}$-Efficient in the transformed 1-D, $\mathcal{W}$, where

$\mathfrak{F} \in \{EF, EF1, EFX, PROP\text{-}E, PROP1\text{-}E, PROPX\text{-}E, MMS\}$ *&*
$\mathfrak{E} \in \{PO, MUW\}$.

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- **Implication** – We can adapt the existing algorithms in 2-D for the above properties

Existing algorithms cannot be directly applied

- In 2-D, MNW cannot be defined. MNW allocation implies EF1 and PO [Caragiannis et al., 2019] doesn't extend to 2-D

Existing algorithms cannot be directly applied

- Modified **leximin** algorithm gives PROP1 and PO for chores for 3 or 4 agents in [Chen and Liu, 2020], but it is not PROP1-E (or PROP1) and PO in 2-D when applied on utilities

	c1	c2	c3	c4
Agent 1	(-30,1)	(-20,1)	(-30,1)	(-30,1)
Agent 2	(-30,1)	(-20,1)	(-30,1)	(-30,1)
Agent 3	(-1,40)	(-1,40)	(-1,40)	(-1,40)

leximin

PROP1-E + PO

	c1	c2	c3	c4
Agent 1	-31	-21	-31	-31
Agent 2	-31	-21	-31	-31
Agent 3	-41	-41	(-41)	(-41)

leximin

PROP1-E + PO

α-MMS

Another important fairness notion studied in the literature: MMS and its approximations

α-MMS

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Theorem

There may not exist α-MMS for any $\alpha \in [0, 1]$ for $\mu_i > 0$ or $1/\alpha$-MMS allocation for any $\alpha \in (0, 1]$ for $\mu_i < 0$ in the presence of externalities

Shifted α-MMS

Definition (Shifted α-MMS)

An allocation A guarantees **shifted α-MMS** if $\forall i \in N, \alpha \in (0, 1]$

$$u_i(A_i) \geq \alpha\mu_i + (1-\alpha)v_i'(M)\} \qquad \text{for goods}$$

$$u_i(A_i) \geq \frac{1}{\alpha}\mu_i + \frac{\alpha-1}{\alpha}v_i'(M)\} \qquad \text{for chores}$$

Shifted α-MMS

Theorem

An allocation A is shifted α-MMS in $\mathcal{V}$ iff A is α-MMS in $\mathcal{W}$

Future Directions

- Better approximation for MMS
- Explore MMSX
- Explore Proportionality Adaptation
- Increase complexity in Externalities

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Questions?

Thank you

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