Problem 1

If $\{x_1, x_2, ..., x_n\}$ satisfy all the equations in the original system, then they will satisfy the sum of any two of them because the sum of two true equations is still a true equation.

Problem 2

1. Let $x = \alpha$. Then from the second equation, $y = 4 + z - 2\alpha \to \alpha - 2(4 + z - 2\alpha) + 4z = 12 \to z = -\frac{5}{2}\alpha + 10$. Plugging this back in, $y = 4 + (-\frac{5}{2}\alpha + 10) - 2\alpha = -\frac{9}{2}\alpha + 14$.

2. A redundant equation is $2x - 4y + 8z = 24$.

3. An inconsistent equation is $x - 2y + 4z = 13$.

4. A third equation which produces a unique solution is $x + y + z = 0$. We can substitute in $z = 2x + y - 4$ into the first and third equations to get

$$\begin{aligned} 9x + 2y &= 28 \\ 3x + 2y &= 4 \end{aligned}$$

so $6x = 24 \to x = 4$. Plugging this into either of the above equations, $y = -4$ and $z = 0$.

Problem 3

1. Sometimes

2. Never - the span can change at most 4 times when adding the 4-tuples one by one

3. Never - there are not enough equations

4. Sometimes

5. Never - it always has a solution

6. Sometimes - there can also be infinite solutions

Problem 4

The agumented matrix is

$$\begin{bmatrix} 1 & 1 & 2 & | & 1 \\ 2 & 1 & -1 & | & 4 \\ 1 & -1 & 1 & | & 0 \end{bmatrix}$$

$-2R1 + R2$, $-R1 + R3$, and $R2 \cdot (-1)$:

$$\begin{bmatrix} 1 & 1 & 2 & | & 1 \\ 0 & 1 & 5 & | & -2 \\ 0 & -2 & -1 & | & -1 \end{bmatrix}$$

$-R2 + R1$, $-2R2 + R3$, $R3/9$:

$$\begin{bmatrix} 1 & 0 & -3 & | & 3 \\ 0 & 1 & 5 & | & -2 \\ 0 & 0 & 1 & | & -\frac{5}{9} \end{bmatrix}$$

$3R3 + R1$, $-5R3 + R2$:

$$\begin{bmatrix} 1 & 0 & 0 & | & \frac{4}{3} \\ 0 & 1 & 0 & | & \frac{7}{9} \\ 0 & 0 & 1 & | & -\frac{5}{9} \end{bmatrix}$$

The planes are only parallel to the x, y, and x axes at the very last step, when all the other entries in their respective rows are 0.

Problem 5

The row-echelon form of this matrix has 3 pivots, so dim(span(u1, u2, u3, u4)) = 3. A quadruplet that is not in the given span is $[1, 0, 0, 0]$ beacuse it cannot be formed from a linear combination of the four vectors (i.e. $au_1 + bu_2 + cu_3 + du_4$).

Problem 6

1. After normalizing the vectors and subtracting the projection of each previous vector from the next one, we get the basis

$$[0.183, 0.365, 0.548, 0.730],$$

$$[-0.750, -0.542, 0.144, 0.351],$$

$$[0.514, -0.521, -0.475, 0.488],$$

$$[0.374, 0. - 0.549, 0.674, -0.324].$$

2. We can find the component of v in each basis by dotting it with them one by one: $v \cdot u_1 = 14.242$, $v \cdot u_2 = 0.868$, $v \cdot u_3 = 1.144$, and $v \cdot u_4 = 3.898$.

Problem 7

Let the expansion be written as $(1, 0, 1) = a(5, -1, 2) + b(2, 0, 1) + c(1, -1, 1)$. Then we have a system of equations:

$$\begin{aligned} 5a + 2b + c &= 1 \\ -a - c &= -1 \\ 2a + b + c &= 1 \end{aligned}$$

Solving this system, we get $a = -\frac{1}{2}, b = \frac{3}{2}, c = \frac{1}{2}$. So $(1, 0, 1) = -\frac{1}{2}(5, -1, 2) + \frac{3}{2}(2, 0, 1) + \frac{1}{2}(1, -1, 1)$

Problem 8

The weight limitation can be expressed as $5x + 3y = 25$. Since there cannot be fractional drones, the only possible integer solution pair is $x = 2$ and $y = 5$. The total flight time logged by all drones is then $\frac{5 \cdot 100}{10} + \frac{3 \cdot 100}{15} = 70$ drone-time-units.