Team Name: [redacted]

Chosen theme: [redacted]

Organisation name: [redacted]

Country: [redacted]

Introduction

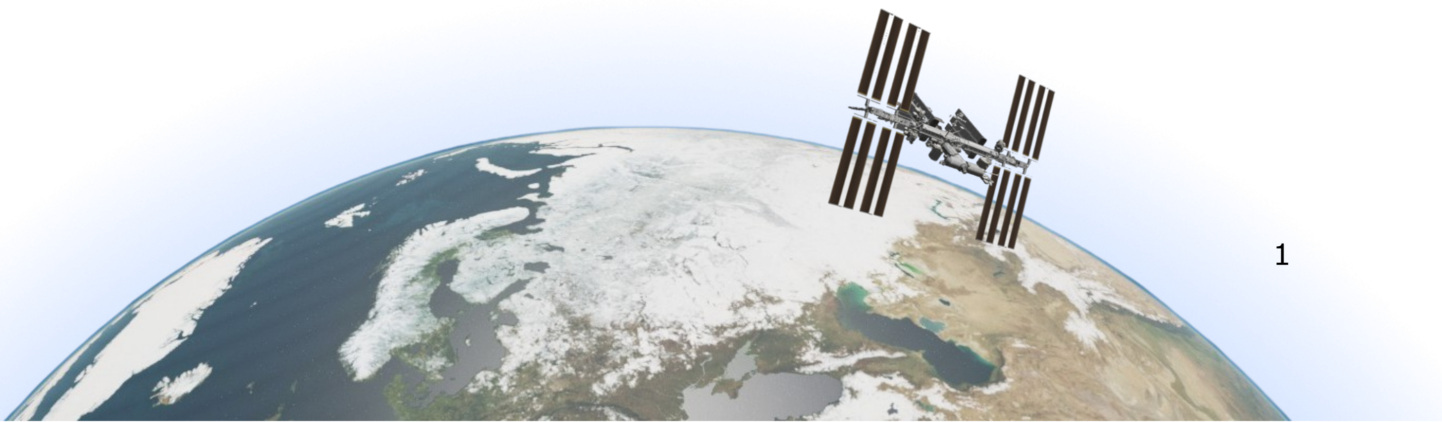
The Earth's magnetic field is very important. It serves as a shield against dangerous radiation from deep space, and as a navigation tool through the use of a compass, helping us find where the North is. Because of this, it is natural that humans have tried to describe and predict its behaviour. The International Geomagnetic Reference Field (IGRF) is one of these models and it is widely used in scientific studies.

Our goal with this experiment was to develop a model of a stellar body's magnetic field (if it has one) and locate its magnetic poles. In this case, our study object was Earth, but procedures and calculations were made so that they could be applied in other cases too.

This way, we aimed to have an equation that describes the Earth's magnetic field up to a certain degree of detail. This could be useful for many purposes, including the study of the geology of planets or other celestial bodies, and even interstellar exploration (for example, discovering if a planet is suitable for life).

Methods

To achieve this, we need to know the magnetic field intensity and direction at different points in space. Hence, we used the magnetometer and gyroscope included in the Raspberry Pi SenseHat. After running our code, we received the data in the form of a spreadsheet, containing coordinates, orientation, magnetometer and gyroscope raw data, and the time at which the measurements were taken, the latter used only for checking the integrity of the data.



A magnetic field can be described as the sum of simpler components in different proportions. If we know what these values are, we can get an accurate approximation of the field. With this in mind, we decided to go with two different models:

A dipolar one (the simplest possible):

$$\vec{B}\,(\vec{r}\,) = \frac{\mu_0}{4\,\pi}\left[\frac{3\,(\vec{m}\cdot\vec{s}\,)\,\vec{s}}{s^5} - \frac{\vec{m}}{s^3}\right]\ with\ \vec{s} = \vec{r} - \vec{R}$$

μ_0 – Vacuum permeability (constant, $T^2 m^3 J^{-1}$); $\vec{r}$ – position vector of the ISS (m); $\vec{R}$ – position vector of the magnetic moment (m); $\vec{m}$ – magnetic moment ($J\,T^{-1}$).

And a multipolar one[1] (up to order 3), which includes more components and, therefore, is more accurate:

$$\vec{B}\,(r\ ,\ \theta\ ,\ \phi) = -\vec{\nabla}\ V = -\vec{\nabla}\left(a\sum_{n=1}^{N}\left(\frac{a}{r}\right)^{n+1}\sum_{m=0}^{n}\left[g_n^m\cos\,(m\ \theta) + h_n^m\sin\,(m\ \theta)\right]P_n^m\,(\sin\ \phi)\right)$$

$\vec{\nabla}$ – gradient (spherical coordinates); V – magnetic potential (A); N – order; a – Earth's radius (m); g_n^m, h_n^m – parameters to be found (μT), P_n^m – associated Legendre polynomials; (r, θ, ϕ) – ISS's spherical coordinates.

To find the best approximation, we used Wolfram Mathematica to minimize the difference between the measured and real field. This involves some coordinate conversions and other calculations, which are explained further in our repository.

Results

Once we got the spreadsheet, we realized that the gyroscope data was not very reliable, as the values we got didn't match the ISS's angular velocity. We also didn't use the orientation values because they are calculated using this sensor. As a result, we now had three extra parameters to find: the Euler angles of the Raspberry Pi in relation to the ISS.

After our calculations, we found that this is the dipolar magnetic moment that approximates better the field we measured:

$$\vec{m} = -2.861\times10^{20}\ \vec{e}_x + 6.950\times10^{22}\ \vec{e}_y - 1.448\times10^{23}\ \vec{e}_z\ (J\ T^{-1})$$
$$with\ an\ offset\ of\ \vec{R} = -6.185\times10^{5}\ \vec{e}_x - 1.307\times10^{6}\ \vec{e}_y + 2.413\times10^{6}\ \vec{e}_z\ (m)$$

We represented the expected magnetic field intensity on a map in the next page. There appears to be a high range of values across the globe, reaching as high as 650μT.

[1] P. Alken et all, International Geomagnetic Reference Field: the thirteenth generation, Earth Planets and Space, 73, 49 (2021)

To determine the position of the magnetic poles, we used an algorithm to minimize the angle between the field and a vector normal to the Earth's surface. This showed us where the field is perpendicular to the surface, which defines the location of the *dip* poles. The magnetic North is predicted to be at 37.20°S 172.57°E, close to New Zealand, and the magnetic South at 61.12°N 108.66°W, on the North of Canada.

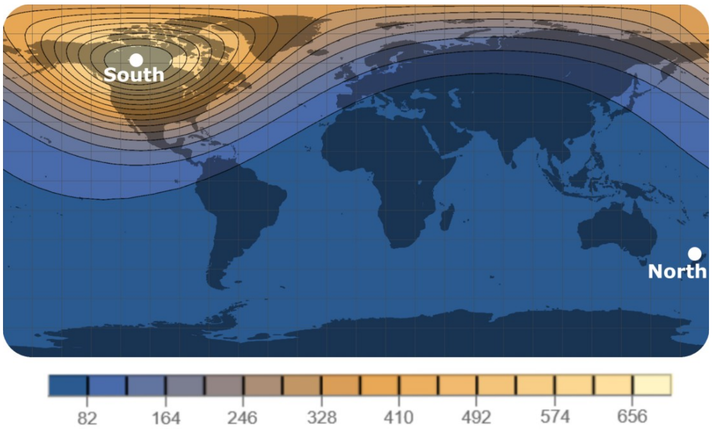


***Fig.1 -** Dipolar field magnitude in microteslas (μT) and location of dip poles*

In comparison to the IGRF's reference values[1], our calculated moment is 95.3% bigger. If we define a relative error function as follows...

$$\%Error = \frac{\sum |\vec{B}_{measured} - \vec{B}_{real}|}{\sum |\vec{B}_{real}|}$$

...our results are off by about 20.12%.

For the multipolar expansion, we get an array of parameters (as opposed to just one vector), which are represented in the table below, along with the IGRF's values for comparison. Again, we mapped the expected field intensity and calculated the poles' location: the magnetic North is located at 26.71°S 92.13°E, on the Indic Ocean, just left of Australia, and the magnetic South at 38.64°N 96.17°W, on Kansas, USA.

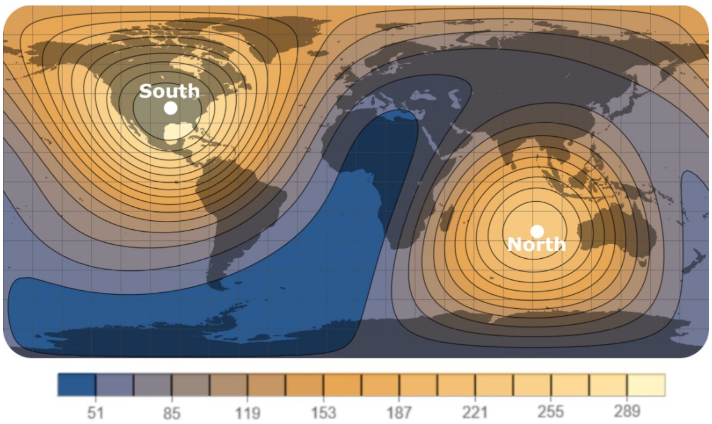
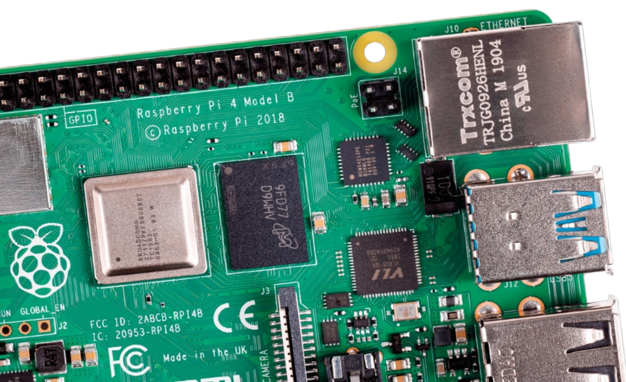


***Fig.2 -** Multipolar field magnitude in microteslas (μT) and location of dip poles*

The error of this approximation is around 14.83%.

	IGRF	Our results
g_1^0	-29.4048	-56.0633
g_1^1	-1.4509	0.1406
g_2^0	-2.4996	-13.3966
g_2^1	2.9820	6.0843
g_2^2	1.6770	4.3187
g_3^0	1.3632	11.0517
g_3^1	-2.3812	1.4564
g_3^2	1.2362	18.1936
g_3^3	0.5257	-0.7269
h_1^1	4.6525	61.4330
h_2^1	-2.9916	12.8321
h_2^2	-0.7346	-6.1966
h_3^1	-0.0821	2.2583
h_3^2	0.2419	-3.8870
h_3^3	-0.5434	-15.3229

***Fig.3 -** Multipolar field expansion parameters in microteslas (μT)*



[1] P. Alken et all, International Geomagnetic Reference Field: the thirteenth generation, Earth Planets and Space, 73, 49 (2021)

Conclusion

On both representations, while the magnitude of the field we modelled is considerably bigger than the real one, the poles' locations aren't as bad as we thought.

Although the calculated dipole momentum is relatively similar in value to the actual one, it is important to note that it is very off-centre (with a distance of about a third of an Earth radius), causing the irregularities in the field. Furthermore, the multipolar expansion parameters are severely different than the IGRF's.

These discrepancies are likely the result of lack of data. Our experiment ran for only two orbits, which provided information of a small portion of the Earth. This is also why the approximations work better on the places we measured the field, but get worse the further away we get, as seen on the map. Not knowing the Raspberry's orientation inside the ISS also added uncertainty.

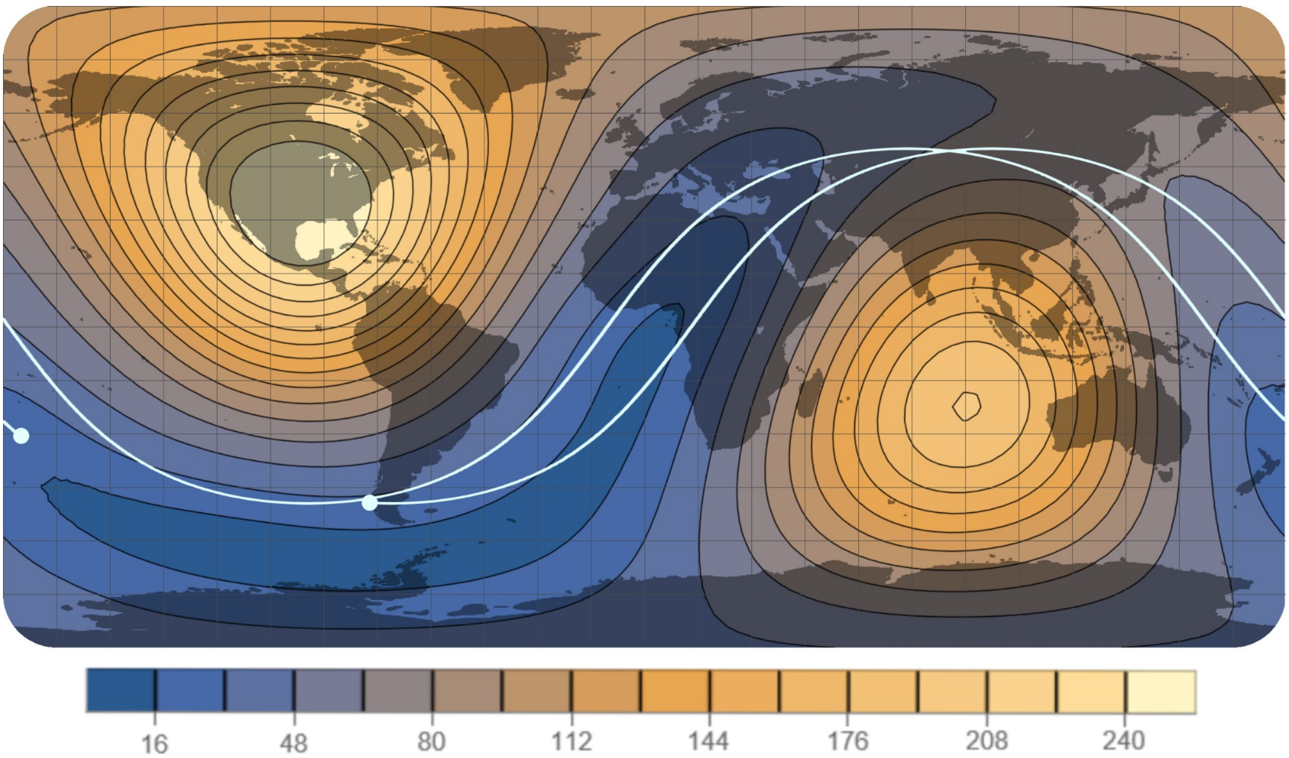


Fig.4 *– Trajectory of the ISS and magnitude of the difference between the predicted (multipolar) and real field in microteslas (μT)*

All in all, to conclude if these methods are reliable for magnetic field studies, we would need more data, either by running the program for a longer time, or by covering a bigger area. Even so, the theoretical foundation of the models appears to be solid and accurate.

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