1.) $\int_{-\pi}^{\pi} x^2 \cos x \, dx = 2\int_0^{\pi} x^2 \cos x \, dx = 2\left[\left[x^2 \sin x\right]_0^{\pi} - \int_0^{\pi} 2x \sin x \, dx\right] = -4\int_0^{\pi} x \sin x \, dx = -4\left[\left[-x\cos x\right]_0^{\pi} + \int 1 \cos x \, dx\right] = -4\left[\pi + \left[\sin x\right]_0^{\pi}\right] = \boxed{-4\pi}$

$\cos x = (\sin x)'$
$(x^2)' = 2x$

$\sin x = (-\cos x)'$
$1 = (x)'$?

2) $\int_{\frac{1}{2}}^{2} |\ln x| \, dx$

$|\ln x| = \begin{cases} \ln x & x \geq 1 \\ -\ln x & 0 < x < 1 \end{cases}$

$\Rightarrow = -\int_{\frac{1}{2}}^{1} \ln x \, dx + \int_1^2 \ln x \, dx = \left[x \ln x - x\right]_{\frac{1}{2}}^{1} - \left[x \ln x - x\right]_1^2$

$= \dots = + 2 - \frac{2}{e}$

3) $\int_0^{\pi} e^x \cos 3x \, dx = \left[e^x \cos 3x\right] - \int_0^{\pi} e^x (-3\sin 3x) \, dx = -e^{\pi} - 1 + 3\left\{\left[e^x \sin 3x\right]_0^{\pi} - \int_0^{\pi} e^x \, 3\cos 3x \, dx\right\}$

$e^x = (e^x)'$
$-3\sin 3x = (\cos 3x)'$

$= -e^{\pi} - 1 - 9\int_0^{\pi} e^x \cos 3x \, dx$

trovata incognita

$\Rightarrow \left[\int_0^{\pi} e^x \cos 3x \, dx\right] + 9\left[\int_0^{\pi} e^x \cos 3x \, dx\right] = -e^{\pi} - 1$

$\Leftrightarrow \int_0^{\pi} e^x \cos 3x \, dx = \frac{-e^{\pi} - 1}{10}$ ✓

4) $\int_{-2}^{0} x \sin x^2 \cdot e^{2x^2} \, dx = -\frac{1}{2}\int_{(-2)^2 = 4}^{(0)^2 = 0} \sin t \cdot e^{2t} \, dt = -\frac{1}{2}\left\{\left[\frac{1}{2} e^{2t} \sin t\right]_0^4 - \int_0^4 \frac{e^{2t}}{2} \cos t \, dt\right\}$

cambio var: $t = x^2$

$\Rightarrow dt = 2x\,dx$

$\Uparrow$

$(t)'dt = (x^2)'dx$

per $\int_a^b$ scambio indici $\int_4^0 \to \int_0^4$

$= -\frac{1}{4} e^8 \sin 4 + \frac{1}{4}\left\{\frac{e^8 \cos 4}{2} - \frac{1}{2} + \frac{1}{2}\int_0^4 e^{2t} \sin t \, dt\right\}$

$= -\frac{1}{4} e^8 \sin 4 + \frac{1}{8} e^8 \cos 4 - \frac{1}{8} + \frac{1}{8}\int_0^4 e^{2t} \sin t \, dt$

trovata incogn.

$\Rightarrow \left(-\frac{1}{2} - \frac{1}{8}\right)\int_0^4 e^{2t} \sin t \, dt = -\frac{1}{4} e^8 \sin 4 + \frac{1}{8} e^8 \cos 4 - \frac{1}{8}$

$= -\frac{1}{4} e^8 \sin 4 + \frac{1}{8} e^8 \cos 4 - \frac{1}{8}$

⚠ In caso di $\int$ definito:

NEI CAMBI DI VARIABILE GLI ESTREMI IN $x \Rightarrow t$!

$\int_0^2 \Rightarrow \int_{t(0)}^{t(2)}$

$t = x^2 \Rightarrow \int_0^4$

INTEGR. FN. RAZIONALI

Fn. razionale: $\frac{P(x)}{Q(x)}$ P, Q polinomi

Se P ha grado $\geq$ Q allora procedo con divisione tra polinomi

altrimenti: quindi P grado < Q

assumo grado Q ≤ 2:

grado Q = 1 $\Rightarrow \int \frac{P}{Q} = \ln \dots$

grado Q = 2 $\Rightarrow$ ⚠

Es.: $\int \frac{2}{3x+5} \, dx = \frac{2}{3}\int \frac{3}{3x+5} = \frac{2}{3} \ln|3x+5| + c$

Q grado 1

$\int \frac{f'(x)}{f(x)} dx$

ricostruisco f' per poter fare $\int$

Q grado 2

$Q(x) = ax^2 + bx + c \quad a, b, c \in \mathbb{R} \quad a \neq 0$

$\Delta = b^2 - 4ac \begin{cases} \Delta > 0 \\ \Delta = 0 \\ \Delta < 0 \end{cases}$

$\Delta > 0$

$\int \frac{x+2}{x^2+x-6} \, dx$

$x^2 + x - 6 = 0$

$x_{1,2} < {}^{-3}_{2}$ a $-x_1$ $-x_2$

$\Rightarrow Q(x) = 1(x+3)(x-2)$

Facciamo: DECOMP. IN FRATTI SEMPLICI

$\frac{x+2}{x^2+x-6} = \frac{A}{x-2} + \frac{B}{x+3} = \frac{A(x+3) + B(x-2)}{(x-2)(x+3)} = \frac{(A+B)x + 3A - 2B}{x^2+x-6}$

$\Updownarrow$

$\frac{x+2}{x^2+x-6} = \frac{(A+B)x + 3A - 2B}{x^2+x-6}$

quindi: $\begin{cases} A + B = 1 \\ 3A - 2B = 2 \end{cases} \quad \begin{cases} A = \frac{4}{5} \\ B = \frac{1}{5} \end{cases}$

$\Downarrow$

come da prima

$\frac{x+2}{x^2+x-6} = \frac{A}{x-2} + \frac{B}{x+3} = \frac{4}{5}\frac{1}{x-2} + \frac{1}{5}\frac{1}{x+3}$

uso questo nel $\int$

$\Delta = 0$

$ax^2 + bx + c$ = quadrato di binomio

$\int \frac{x+1}{(3x+2)^2} dx = \frac{1}{3}\int \frac{\frac{t-2}{3} + 1}{t^2} = \frac{1}{3}\int \frac{t-2+3}{3t^2} = \frac{1}{9}\int \frac{t+1}{t^2} \, dt = \frac{1}{9}\left[\int \frac{1}{t} \, dt + \int \frac{1}{t^2} \, dt\right]$

$t = 3x+2$
$dt = 3dx$
$(t)'dt = (3x+2)'dx$

$= \frac{1}{9}\left[\ln|t| - \frac{1}{t} + c\right] = \frac{1}{9}\ln|3x+2| - \frac{1}{9}\left(\frac{1}{3x+2}\right) + c$

torno ad x

$\Delta < 0$

$\int \frac{dx}{x^2+3}$

riconduco all'arctan $\Rightarrow \int \frac{dt}{1+t^2} = \arctan t + c$

$\Downarrow$

$\int \frac{dx}{3\left(1+\frac{x^2}{3}\right)} = \frac{1}{3}\int \frac{dx}{1+\left(\frac{x^2}{\sqrt{3}}\right)^2} = \frac{1}{3}\int \frac{dt\sqrt{3}}{1+t^2} = \frac{\sqrt{3}}{3} \arctan \frac{x}{\sqrt{3}} + c$

$t = \frac{x^2}{\sqrt{3}}$

$dx = \left(\frac{x^2}{\sqrt{3}}\right)' dt$?

$= \sqrt{3} \, dt$?

questo, ma come?

in un trin. completo:

$\int \frac{dx}{x^2+2x+4}$

$x^2 + 2x + 4 = (Ax+B)^2 + C^2$ avanza una costante positiva

voglio avere $x^2 + 2x + (1)$

$\Rightarrow = (x+1)^2 + 3$ gli aggiunto da (1) al valore necessario: 4

$\int \frac{dx}{x^2+2x+4} = \int \frac{dx}{(x+1)^2+3} =$ come prima cambio var e procedo verso arctan

$t = \frac{x+1}{\sqrt{3}}$

Se in P(x) abbiamo grado > 0 e Q(x) ha grado 2 e $\Delta < 0$:

$\int \frac{x+2}{x^2+2x+4} dx = \frac{1}{2}\int \frac{2x+4}{\dots} = \frac{1}{2}\int \frac{2x+2+2}{x^2+2x+4} = \frac{1}{2}\int \left(\frac{2x+2}{x^2+2x+4} + \frac{2}{x^2+2x+4}\right) dx$

$(x^2+2x+4)' = 2x+2$ Icerco la f'

$(x^2+2x+4) = (x+1)^2 + 3 \Rightarrow$ quindi $\frac{f'(x)}{f(x)}$

$\frac{1}{2} \ln(x^2+2x+4) \dots$

Ex.: $\int \frac{x^3+2}{x^2+1} \, dx \Rightarrow$ divisione tra polinomi (P grado > Q)

x^3	0	0	+2	x^2+1
//		$-x$	+2	x

$\Rightarrow \frac{x^3+2}{x^2+1} = \frac{(x)(x^2+1) - x + 2}{x^2+1} = \frac{x}{x^2+1} - \frac{x-2}{x^2+1}$

$\Rightarrow \int \frac{x}{x^2+1} \, dx - \int \frac{x-2}{x^2+1} \, dx = \frac{1}{2}x^2 + \int \frac{2dx}{1+x^2} - \int \frac{x}{x^2+1} \, dx = \frac{1}{2}x^2 + 2\arctan x - \frac{1}{2}\ln(x^2+1) + c$

$t = e^x$
$dt = e^x dx \Rightarrow dx = \frac{dt}{t}$

Es.: $\int \frac{1+e^x}{\operatorname{Ch} x} = 2\int \frac{1+e^x}{e^x + e^{-x}} = 2\int \frac{1+t}{t+t^{-1}} \frac{dt}{t} = 2\int \frac{1+t}{t^2+1} \, dt = 2 \operatorname{atan} e^x + \ln(1+e^{2x}) + c$

Es.: $\int \frac{2x+3}{x^2+x-2} dx$

Se $\Delta Q \geq 0$ si può decomporre subito

altrimenti servono le operaz. per ridurre Q.

NEI CAMBI DI VARIABILE GLI ESTREMI IN $x \Rightarrow t$!

$\int_0^2 \Rightarrow \int_{t(0)}^{t(2)}$

$t = x^2 \Rightarrow \int_0^4$