Ricerca della mat. inversa in una singola entrata

$$Q = \begin{pmatrix} 2 & 4 & 3 & 5 & 1 & 2 \\ 3 & -2 & 6 & 2 & 1 & 0 \\ -1 & 5 & 2 & 8 & 7 & 4 \\ 5 & 0 & 6 & 1 & -1 & 1 \\ 2 & 2 & 1 & 0 & 3 & 3 \\ 1 & 3 & 0 & 2 & 7 & 1 \end{pmatrix} \qquad (Q^{-1})_{46} = ?$$

Per calcolare il $|Q|$ usiamo un mix di:

i. Scambio tra colonne
ii. Estrazione di scalare da colonna
iii. C.L. su una colonna dalle altre
iv. Mosse di Gauss

Evidenziato in:
rosso: Laplace
verde: Gauss

$$|Q| = \begin{vmatrix} 2 & 4 & 3 & 5 & 1 & 2 \\ 3 & -2 & 6 & 2 & 1 & 0 \\ -1 & 5 & 2 & 8 & 7 & 4 \\ 5 & 0 & 6 & 1 & -1 & 1 \\ 2 & 2 & 1 & 0 & 3 & 3 \\ 1 & 3 & 0 & 2 & 7 & 1 \end{vmatrix} = \begin{vmatrix} 5 & 0 & 6 & 1 & -1 & 1 \\ 3 & -2 & 6 & 2 & 1 & 0 \\ -1 & 5 & 2 & 8 & 7 & 4 \\ 2 & 4 & 3 & 5 & 1 & 2 \\ 2 & 2 & 1 & 0 & 3 & 3 \\ 1 & 3 & 0 & 2 & 7 & 1 \end{vmatrix} (-1) = \begin{array}{r} \\ 0T_1 \\ -4T_1 \\ -2T_1 \\ -3T_1 \\ -T_1 \end{array} \begin{vmatrix} 5 & 0 & 6 & 1 & -1 & 1 \\ 3 & -2 & 6 & 2 & 1 & 0 \\ -21 & 5 & -22 & 4 & 11 & 0 \\ -8 & 4 & -9 & 3 & 3 & 0 \\ -13 & 2 & -17 & -3 & 6 & 0 \\ -4 & 3 & -6 & 1 & 8 & 0 \end{vmatrix} (-1) =$$

$$= \begin{vmatrix} 3 & -2 & 6 & 2 & 1 \\ -21 & 5 & -22 & 4 & 11 \\ -8 & 4 & -9 & 3 & 3 \\ -13 & 2 & -17 & -3 & 6 \\ -4 & 3 & -6 & 1 & 8 \end{vmatrix} \cancel{(-1)} =$$

$-1T^1$ $-1T^2$

$$= \begin{array}{r} \\ \\ \\ \\ T_5 \end{array} \begin{vmatrix} 3 & -2 & 3 & 2 & 3 \\ -21 & 5 & -1 & 4 & 6 \\ -8 & 4 & -1 & 3 & -1 \\ -13 & 2 & -4 & -3 & 4 \\ -4 & 3 & -2 & 1 & 5 \end{vmatrix} = \begin{array}{r} -2T_5 \\ -4T_5 \\ -3T_5 \\ +3T_5 \\ \\ \end{array} \begin{vmatrix} 11 & -8 & 7 & 0 & -7 \\ -5 & -7 & 7 & 0 & -14 \\ 4 & -5 & 5 & 0 & -16 \\ -25 & 11 & -10 & 0 & 19 \\ -4 & 3 & -2 & 1 & 5 \end{vmatrix} = \begin{vmatrix} 11 & -8 & 7 & -7 \\ -5 & -7 & 7 & -14 \\ 4 & -5 & 5 & -16 \\ -25 & 11 & -10 & 19 \end{vmatrix} (-1) = \begin{array}{r} \\ \\ \\ T_4 \end{array} \begin{vmatrix} 11 & -1 & 7 & 4 \\ -5 & 0 & 7 & -19 \\ 4 & 0 & 5 & -12 \\ -25 & 1 & -10 & -6 \end{vmatrix} (-1) =$$

$+1T^3$ $+1T^1$

$$= \begin{array}{r} +T_4 \\ 0T_4 \\ 0T_4 \\ \\ \end{array} \begin{vmatrix} -14 & 0 & -3 & -2 \\ -5 & 0 & 7 & -19 \\ 4 & 0 & 5 & -12 \\ -25 & 1 & -10 & -6 \end{vmatrix} (-1) = {}_{+T_3} \begin{vmatrix} -14 & -3 & -2 \\ -5 & 7 & -19 \\ 4 & 5 & -12 \end{vmatrix} (-1) = \begin{vmatrix} -14 & -3 & -2 \\ -1 & 12 & -31 \\ 4 & 5 & -12 \end{vmatrix} (-1) = \begin{vmatrix} -14 & -3 & -5 \\ -1 & 12 & -19 \\ 4 & 5 & -7 \end{vmatrix} (-1)$$

$+T^2$ T'

$$= \begin{vmatrix} -14 & -171 & 261 \\ -1 & 0 & 0 \\ 4 & 53 & -83 \end{vmatrix} = \begin{vmatrix} -171 & 261 \\ 53 & -83 \end{vmatrix} (-1) = \boxed{-360} = |Q|$$

$+12T' \; -19T'$

verificato

14 · 12 = 28, 14, 168
19 · 14 = 76, 19, 266
171 · 83 = 513, 1368, 14193
261 · 53 = 783, 1305, 13833
14193 − 13833 = 00360

Ora che conosciamo $|Q|$, troviamo Q^{-1} in indice 4, 6 usando Cramer:

$$Q = \begin{pmatrix} 2 & 4 & 3 & 5 & 1 & 2 \\ 3 & -2 & 6 & 2 & 1 & 0 \\ -1 & 5 & 2 & 8 & 7 & 4 \\ 5 & 0 & 6 & 1 & -1 & 1 \\ 2 & 2 & 1 & 0 & 3 & 3 \\ 1 & 3 & 0 & 2 & 7 & 1 \end{pmatrix} \qquad |Q| = -360$$

$$(Q^{-1})_{46} = \frac{1}{|Q|} (-1)^{10} \left| (Q^T)_{[4,6]} \right|$$

$$Q^T = \begin{pmatrix} 2 & 3 & -1 & 5 & 2 & 1 \\ 4 & -2 & 5 & 0 & 2 & 3 \\ 3 & 6 & 2 & 6 & 1 & 0 \\ 5 & 2 & 8 & 1 & 0 & 2 \\ 1 & 1 & 7 & -1 & 3 & 7 \\ 2 & 0 & 4 & 1 & 3 & 1 \end{pmatrix}$$

$$(Q^{-1})_{46} = -\frac{1}{360} \begin{vmatrix} 2 & 3 & -1 & 5 & 2 \\ 4 & -2 & 5 & 0 & 2 \\ 3 & 6 & 2 & 6 & 1 \\ 1 & 1 & 7 & -1 & 3 \\ 2 & 0 & 4 & 1 & 3 \end{vmatrix} = -\frac{1}{360} \begin{vmatrix} -4 & -9 & -5 & -7 & 0 \\ -2 & -14 & 1 & -12 & 0 \\ 3 & 6 & 2 & 6 & 1^{+} \\ -8 & -17 & 1 & -19 & 0 \\ -7 & -18 & -2 & -17 & 0 \end{vmatrix}$$

$-T^2$

$$= -\frac{1}{360} \begin{vmatrix} -4 & -9 & -5 & -7 \\ -2 & -14 & 1 & -12 \\ -8 & -17 & 1 & -19 \\ -7 & -18 & -2 & -17 \end{vmatrix} = -\frac{1}{360} \begin{vmatrix} -4 & -9 & -5 & 2 \\ -2 & -14 & 1 & 2 \\ -8 & -17 & 1 & -2 \\ -7 & -18 & -2 & 1 \end{vmatrix} -T_1$$

$$= -\frac{1}{360} \begin{vmatrix} -4 & -9 & -5 & 2 \\ 2 & -5 & 6 & 0 \\ -4 & -8 & 6 & -4 \\ -3 & -9 & 3 & -1 \end{vmatrix} = -\frac{1}{360} \begin{vmatrix} -10 & -27 & 1 & 2 \\ 2 & -5 & 6 & 0 \\ 8 & 28 & -6 & -4 \\ 0 & 0 & 0 & -1^{+} \end{vmatrix} =$$

$$= \frac{1}{360} \begin{vmatrix} -10 & -27 & 1 \\ 2 & -5 & 6 \\ 8 & 28 & -6 \end{vmatrix} = \frac{1}{360} \left(\begin{vmatrix} 2 & -5 \\ 8 & 28 \end{vmatrix} - 6 \begin{vmatrix} -10 & -27 \\ 8 & 28 \end{vmatrix} - 6 \begin{vmatrix} -10 & -27 \\ 2 & -5 \end{vmatrix} \right)$$

$$= \frac{1}{360} \left[56 + 40 - 6 (-280 + 376) - 6 (50 + 54) \right] = \frac{1}{360} \left[96 - 6(96) - 6(104) \right]$$

$$= \frac{1}{360} (480 - 624) = -\frac{144}{360} = -\frac{3^2 \cdot 2^4}{5 \cdot 3^2 \cdot 2^3} = \boxed{-\frac{2}{5}} = (Q^{-1})_{46}$$

verificato

In posizione 4, 6 la matrice inversa Q^{-1} conterrà l'entrata $\boxed{(Q^{-1})_{46} = -\frac{2}{5}}$.

verificato