Analisi 20231011 - p2

Formule di De Moivre

→ con due n. complessi in f. trig.ca

z_1, z_2

→ la loro molt.:

$$z_1 z_2 = \rho_1\rho_2(\cos\theta_1 + j\sin\theta_1)(\cos\theta_2 + j\sin\theta_2)$$

$$= \rho_1\rho_2\left[\cos\theta_1\cos\theta_2 + j^2\sin\theta_1\sin\theta_2 + j\cos\theta_1\sin\theta_2 + j\cos\theta_2\sin\theta_1\right]$$

$$= \rho_1\rho_2\left[(\underbrace{\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2}_{=\cos(\theta_1+\theta_2)}) + j(\underbrace{\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2}_{=\sin(\theta_1+\theta_2)})\right]$$

$$= \rho_1\rho_2\left[\cos(\theta_1+\theta_2) + j\sin(\theta_1+\theta_2)\right]$$

sembra + fosse giusto

→ $|z_1 z_2| = |z_1|\cdot|z_2|$

→ $\arg(z_1 z_2) = \arg(z_1) + \arg(z_2)$

I formula di D.M.

<u>notazione per argomento</u>

→ $z_1 z_2 =$

finiscono sommate all'interno di sin() e cos()

Siano $z_1 = \rho_1(\cos\theta_1 + j\sin\theta_1)$

$z_2 = \rho_2(\cos\theta_2 + j\sin\theta_2)$

scriviamo $\frac{z_1}{z_2}$ in f. trig.ca

raz. per il coni.

$$\frac{z_1}{z_2} = \frac{\rho_1}{\rho_2}\frac{(\cos\theta_1 + j\sin\theta_1)}{(\cos\theta_2 + j\sin\theta_2)}\frac{(\cos\theta_2 - j\sin\theta_2)}{(\cos\theta_2 - j\sin\theta_2)}$$

$$= \frac{\rho_1}{\rho_2}\frac{(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2) + j(\sin\theta_1\cos\theta_2 - \sin\theta_2\cos\theta_1)}{\cos^2\theta_2 - j^2\sin^2\theta_2}$$

$$= \frac{\rho_1}{\rho_2}\frac{\cos(\theta_1-\theta_2) + j\sin(\theta_1-\theta_2)}{\underbrace{\cos^2\theta_2 + \sin^2\theta_2}_{=1 \text{ per } \cos^2\alpha + \sin^2\alpha = 1}}$$

Formula ↑
Formula ↑ } da ripassare formule sin cos

$$\frac{z_1}{z_2} = \frac{\rho_1}{\rho_2}\left[\cos(\theta_1-\theta_2) + j\sin(\theta_1-\theta_2)\right]$$

$$\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$$

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$$

II formula D.M.

Caso particolare della I f.: potenze

$z = \rho(\cos\theta + j\sin\theta) \quad n \in \mathbb{N} \setminus \{0\}$

$$z^n = z\cdot z\cdot\ldots\cdot z = \rho^n\left[\cos(n\theta) + j\sin(n\theta)\right]$$

I f. per le potenze

<u>Esercizio:</u> $(1+j)^{20}$ → modulo e arg.

$|1+j| = \sqrt{2}$

$\theta = \frac{\pi}{4}$

why?

$$(1+j)^{20} = (\sqrt{2})^{20}\left[\cos\left(20\frac{\pi}{4}\right) + j\sin\left(20\frac{\pi}{4}\right)\right]$$

$$= 2^{10}(\underbrace{\cos\pi}_{=-1} + j\underbrace{\sin\pi}_{=0})$$

$$= -2^{10}$$

<u>Esempio</u> come prima, con:

$(1-j\sqrt{3})^8$

→ $|1-j\sqrt{3}| = \sqrt{1+3} = 2$

→ $\theta = -\frac{\pi}{3}$

qui usa $\begin{cases}\cos\theta\\ \sin\theta\end{cases}$

$$(1-j\sqrt{3})^8 = 2^8\left[\cos\left(-\frac{8\pi}{3}\right) + j\sin\left(-\frac{8\pi}{3}\right)\right]$$

↓ $-\frac{2}{3}\pi$ ↓ $-\frac{2}{3}\pi$ → da $-\frac{8}{3}\pi = \underbrace{-\frac{6}{3}\pi}_{-2\pi \text{ si annulla}} - \frac{2}{3}\pi$

$$= 2^8\left[-\frac{1}{2} - \frac{\sqrt{3}}{2}\right]$$

$$= 256\left(-\frac{1}{2}\right) + j\,256\left(-\frac{\sqrt{3}}{2}\right)$$

$$= -128 - j128\sqrt{3}$$

$$= -128(1+j\sqrt{3})$$

<u>Es.:</u> mod. arg:

$$\frac{(1+j)^4}{\underbrace{(\sqrt{2}-\sqrt{2}j)^3}_{(\sqrt{2})^3(1-j)^3}}$$

→ $|1+j| = \sqrt{1+1} = \sqrt{2}$; $\arg(1+j) = \frac{\pi}{4}$ θ

→ $|1-j| = \sqrt{2}$; $\arg(1-j) = -\frac{\pi}{4}$

$$z = \frac{(\sqrt{2})^4(\cos\pi + j\sin\pi)}{2\sqrt{2}(\sqrt{2})^3\left[\cos\left(-\frac{3}{4}\pi\right) + j\sin\left(-\frac{3}{4}\pi\right)\right]}$$

applicato D.M. II (sul quoziente)

$$= \frac{4}{2\sqrt{2}\cdot2\sqrt{2}}\left[\cos\left(\pi+\frac{3}{4}\pi\right) + j\sin\left(\pi+\frac{3}{4}\pi\right)\right]$$

$$= \frac{1}{2}\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) = \frac{\sqrt{2}}{4} - j\frac{\sqrt{2}}{4}$$

<u>Forma esp. di prodotti ed esponenti</u>

$z_1 = \rho_1 e^{j\theta_1}$

$z_2 = \rho_2 e^{j\theta_2}$

$$z_1 z_2 = \rho_1\rho_2\, e^{j\theta_1} e^{j\theta_2} = \rho_1\rho_2\, e^{j(\theta_1+\theta_2)}$$

$$\frac{z_1}{z_2} = \frac{\rho_1 e^{j\theta_1}}{\rho_2 e^{j\theta_2}} = \frac{\rho_1}{\rho_2} e^{j(\theta_1-\theta_2)}$$

esp. velocizzano tutte le operazioni