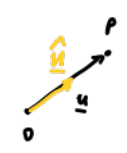
Geometria 20231019

⇦ eq. parametriche

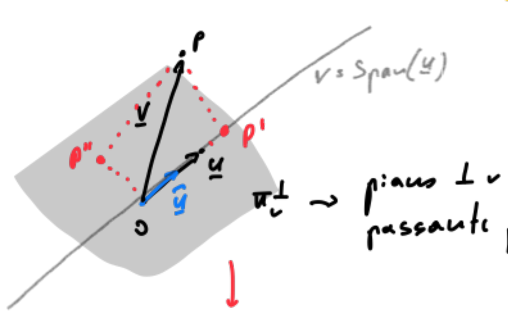
Proiezione ortogonale

Fisso $\underline{u} \in E^3_O \neq \underline{0}$

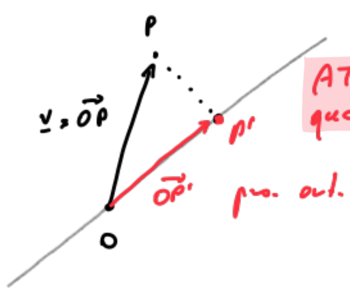
$\hat{\underline{u}}$ = versore associato a $\underline{u}$ $= \frac{\underline{u}}{\|\underline{u}\|}$



$\|\hat{\underline{u}}\| = \left\|\frac{\underline{u}}{\|\underline{u}\|}\right\| = \left\|\frac{1}{\|\underline{u}\|}\underline{u}\right\| = \frac{1}{\cancel{\|\underline{u}\|}}\cancel{\|\underline{u}\|} = 1$ versore unitario da vettore



$\pi_r^\perp$ → piano ⊥ r passante per O

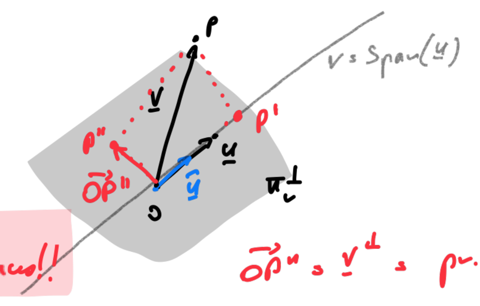


ATTENZIONE! questa retta è ⊥ al piano!!

$\vec{OP'}$ pro. ort. di $\underline{v}$ su $\underline{u}$
su $\hat{\underline{u}}$
su Span($\underline{u}$)
su r
lungo $\underline{u}$
lungo r

=

pr. parallela $\underline{v}^{\parallel} = \vec{OP'}$



$\vec{OP''} = \underline{v}^\perp$ = pr. perpendicolare di $\underline{v}$ su $\pi_r^\perp$ ortogonale a $\underline{u}$

$\underline{v} = \vec{OP} = \vec{OP'} + \vec{OP''}$ → decomp. unica
$= \underline{v}^{\parallel} + \underline{v}^\perp$

Ossv.: $\forall \vec{OP}, \vec{OQ} \in E^3_O \quad \forall \alpha \in \mathbb{R}$ → $\vec{OP} + \vec{OQ} = \vec{OR}$

$(\vec{OP} + \vec{OQ})^{\parallel} = (\underbrace{\vec{OP'}}_{\parallel} + \underbrace{\vec{OP''}}_{\perp} + \underbrace{\vec{OQ'}}_{\parallel} + \underbrace{\vec{OQ''}}_{\perp})^{\parallel} = \vec{OP'} + \vec{OQ'}$
$= \vec{OR'}$
$= \vec{OR}^{\parallel}$

stessa cosa con $()^\perp$

Analog.: $(\alpha\vec{OP}^{\parallel}) = \alpha(\vec{OP}^{\parallel})$ omogeneità
dim a casa? $(\alpha\vec{OP}^\perp) = \alpha(\vec{OP}^\perp)$

ortogonale ⟺ perpendicolare ?

$\underline{v} = \underline{v}^{\parallel} + \underline{v}^\perp$

non abbiamo una $\vec{OR}'''$ perché = 0 → piano ⊥ alla Span($\underline{v}$)!

Pr. scalare in E^3_O (detto)

per $\underline{u}, \underline{v} \in E^3_O$

$\underline{u} \cdot \underline{v} = \|\underline{u}\| \|\underline{v}\| \cos\theta$ $\quad \theta \in \mathbb{R}$ (scalare)

"u scalar v"
"p. s."

$\theta < \pi$ quindi non comprende i prolungamenti delle rette di $\underline{u}, \underline{v}$

Def.: $\underline{u}, \underline{v} \in E^3_O$
sono ⊥ se $\underline{u} \cdot \underline{v} = \underline{0}$

Ossv.:
$\underline{u} \cdot \underline{v} = \underline{0}$
$\to \underline{u} = \underline{0} \vee \underline{v} = \underline{0} \vee \overbrace{\theta = \frac{\pi}{2}}^{\text{rett. } \perp}$

proprietà:

- $\forall \underline{u}, \underline{v} \in E^3_O$
  $\underline{u} \cdot \underline{v} = \underline{v} \cdot \underline{u}$ commutativo o simmetrico
- $\forall \underline{u}, \underline{v} \in E^3_O, \quad \forall \alpha \in \mathbb{R}$
  $(\alpha\underline{u}) \cdot \underline{v} = \underline{u} \cdot (\alpha\underline{v}) = \alpha(\underline{u} \cdot \underline{v})$ omogeneo
- $\forall \underline{u}, \underline{v}, \underline{w} \in E^3_O$
  $\underline{u} \cdot (\underline{v} + \underline{w}) = \underline{u} \cdot \underline{v} + \underline{u} \cdot \underline{w}$ distributiva
- $\forall \underline{u} \in E^3_O$
  $\underline{u} \cdot \underline{u} \geq 0$; inoltre, $\underline{u} \cdot \underline{u} = 0 \iff \underline{u} = 0$ positività
  sempre ≥ 0 quando $\underline{u} \cdot \underline{u}$

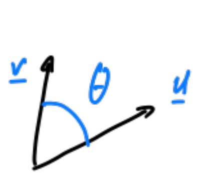
$(\underline{u} \cdot \underline{u} = \|\underline{u}\| \|\underline{u}\| \cos 0 = \|\underline{u}\|^2 \cdot 1 = \|\underline{u}\|^2)$

$\|\underline{u}\|^2 = 0$
⇕
$\|\underline{u}\| = 0$
⇕
$\underline{u} = 0$

Dimos. 2ª proprietà

dimostrare che $(\alpha\underline{u}) \cdot \underline{v} = \alpha(\underline{u} \cdot \underline{v})$

$(\alpha\underline{u}) \cdot \underline{v} = \alpha(\underline{u} \cdot \underline{v})$



banale per: $\underline{u} = 0 \vee \underline{v} = 0 \vee$
$\theta = \frac{\pi}{2}$
$\alpha = 0$

→ $\underbrace{(\alpha\underline{u}) \cdot \underline{v}}_{\text{partenza}} = \underbrace{\alpha(\underline{u} \cdot \underline{v})}_{\text{obiettivo}}$
$\|\alpha\underline{u}\| \|\underline{v}\| \cos\theta$
$|\alpha| \|\underline{u}\| \|\underline{v}\| \cos\theta$
↳ se $\alpha > 0$
$\alpha \|\underline{u}\| \|\underline{v}\| \cos\theta$
$\alpha(\underline{u} \cdot \underline{v})$ raggiunto l'obiettivo

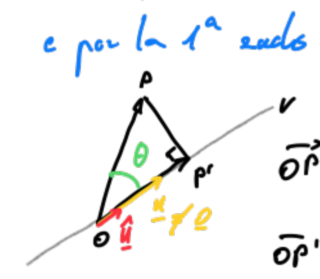
↳ se $\alpha < 0$ ×

per $\alpha < 0$ $\underline{u}$ va indietro

$\|\alpha\underline{u}\| \|\underline{v}\| \cos\varphi$
$|\alpha| \|\underline{u}\| \|\underline{v}\| \cos(\pi - \theta)$
inverte il segno in uscita!
$-|\alpha| \|\underline{u}\| \|\underline{v}\| \cos\theta$ per $\alpha < 0$
$\alpha \|\underline{u}\| \|\underline{v}\| \cos\theta$
$\alpha(\underline{u} \cdot \underline{v})$ ob. raggiunto

Dimos. 3ª:

$(\underline{u} + \underline{v}) \cdot \underline{w} = \underline{u} \cdot \underline{w} + \underline{v} \cdot \underline{w}$
e per la 1ª anche $\underline{u} \cdot (\underline{v} + \underline{w}) = \underline{u} \cdot \underline{v} + \underline{u} \cdot \underline{w}$

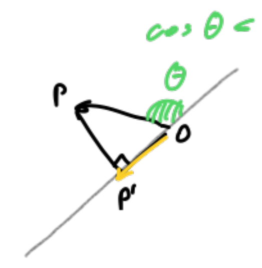


$\vec{OP} = \underline{v}$
$\vec{OP'} = \underline{v}^{\parallel}$

$\vec{OP'} = \vec{OP} \cos\theta$
$\|\vec{OP'}\| = \|\underline{v}\| \cos\theta$

$\underline{u} \cdot \underline{v} = \|\underline{u}\| \|\underline{v}\| \cos\theta \qquad \underline{u} \neq \underline{0}$
$\hat{\underline{u}} \cdot \underline{v} = \|\underline{v}\| \cos\theta = \|\underline{v}^{\parallel}\|$
$\vec{OP'} = \underline{v}^{\parallel} = (\hat{\underline{u}} \cdot \underline{v}) \cdot \hat{\underline{u}} = \left(\frac{\underline{u}}{\|\underline{u}\|} \cdot \underline{v}\right) \cdot \frac{\underline{u}}{\|\underline{u}\|} = \frac{\underline{u} \cdot \underline{v}}{\|\underline{u}\|^2} \underline{u}$

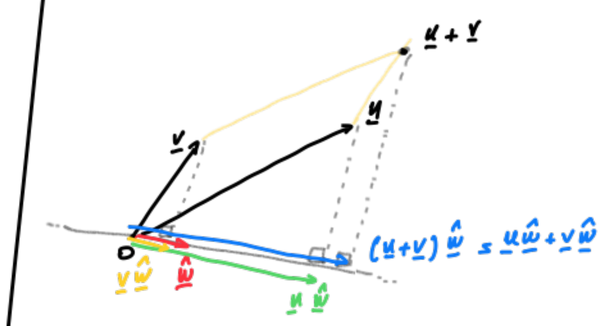
$\cos\theta < 0$



$(\hat{\underline{u}} \cdot \underline{v}) \hat{\underline{u}}$ ancora valido
formula della proiezione ∥ per versore

caso $\underline{w} \neq \underline{0}$
$\hat{\underline{w}} = \frac{\underline{w}}{\|\underline{w}\|} \qquad \underline{w} = \|\underline{w}\| \hat{\underline{w}}$

→ $(\underline{u} \cdot \underline{v})(\|\underline{w}\| \hat{\underline{w}}) = \|\underline{w}\| [(\underline{u} + \underline{v}) \hat{\underline{w}}]$ → $(\underline{u} + \underline{v}) \underline{w} = \underline{u}\underline{w} + \underline{v}\underline{w}$?
$\underline{u} \cdot \underline{w} = \underline{u} \cdot (\|\underline{w}\| \hat{\underline{w}}) = \|\underline{w}\| (\underline{u} \cdot \hat{\underline{w}})$
$\underline{u} \cdot \underline{v} = \underline{v} \cdot (\|\underline{w}\| \hat{\underline{w}}) = \|\underline{w}\| (\underline{v} \cdot \hat{\underline{w}})$



⚠ $\hat{\underline{k}} \cdot \hat{\underline{k}} = \|\hat{\underline{k}}\|^2 = 1$

Google

$[\underline{u}] = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix}$ equivale a $\underline{u} = u_x\hat{\underline{i}} + u_y\hat{\underline{j}} + u_z\hat{\underline{k}}$

$[\underline{v}] = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$ ...

$\underline{u} \cdot \underline{v} = (u_x\hat{\underline{i}} + u_y\hat{\underline{j}} + u_z\hat{\underline{k}}) \cdot (v_x\hat{\underline{i}} + v_y\hat{\underline{j}} + v_z\hat{\underline{k}})$
$= u_xv_x(\hat{\underline{i}}\hat{\underline{i}}) + u_xv_y(\hat{\underline{i}}\hat{\underline{j}}) + u_xv_z(\hat{\underline{i}}\hat{\underline{k}}) +$
$+ u_yv_x(\hat{\underline{j}}\hat{\underline{i}}) + u_yv_y(\hat{\underline{j}}\hat{\underline{j}}) + u_yv_z(\hat{\underline{j}}\hat{\underline{k}}) +$
$+ u_zv_x(\hat{\underline{k}}\hat{\underline{i}}) + u_zv_y(\hat{\underline{k}}\hat{\underline{j}}) + u_zv_z(\hat{\underline{k}}\hat{\underline{k}})$
$= u_xv_x\|\hat{\underline{i}}\|^2 + u_yv_y\|\hat{\underline{j}}\|^2 + u_zv_z\|\hat{\underline{k}}\|^2$

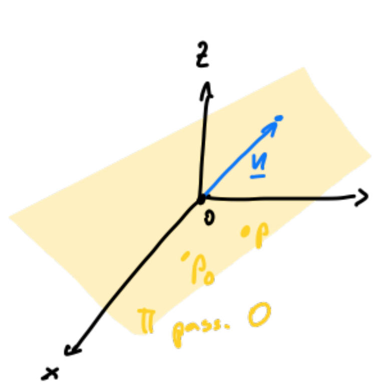
↳ equivale a → $\begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$ se = 0 allora ⊥
$[\underline{u}] \cdot [\underline{v}]$
ma non con angolo

Ossv.: $\underline{u}, \underline{v} \neq \underline{0}$

$\underline{u} \cdot \underline{v} = \|\underline{u}\| \|\underline{v}\| \cos\theta$

$\underline{u}$: $[\underline{u}] = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ $\quad \underline{u} \perp \underline{v}$?
$\underline{v}$: $[\underline{v}] = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

$\underline{u} \cdot \underline{v} = 0 + 2 + 1 = 3$
$\|\underline{u}\| = \sqrt{\underline{u} \cdot \underline{u}} = \sqrt{u_x^2 + u_y^2 + u_z^2} = \sqrt{2}\sqrt{3}$
$\|\underline{v}\| = \sqrt{\underline{v} \cdot \underline{v}} = \sqrt{2}$
$\cos\theta = \frac{\underline{u} \cdot \underline{v}}{\|\underline{u}\| \|\underline{v}\|} = \frac{3}{\sqrt{2}\sqrt{2}\sqrt{3}} = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{6}$



$\underline{u} \perp \pi$
$[\underline{u}] = \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix}$

$P \in \pi$ → $\vec{OP} \cdot \underline{u} = 0$ perché ⊥

$[\vec{OP}] = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$ → $\alpha x + \beta y + \gamma z = 0$

tutti i vettori ∈ π avranno sempre p.s. = 0 con $\underline{u}$ per ⊥!

altrimenti: $\underbrace{\vec{P_0P}}_{\text{segm. orientato}} = \underbrace{\vec{OP}}_{v.} - \underbrace{\vec{OP_0}}_{v.}$ → $(\vec{OP} - \vec{OP_0}) \cdot \underline{u} = 0$

→ $\alpha x + \beta y + \gamma z = \underbrace{\alpha x_0 + \beta y_0 + \gamma z_0}_{\delta \text{ detto } \neq 0}$
↳ solo se ... ?