In [4]:

```
%matplotlib notebook
import numpy as np
import matplotlib.pyplot as plt
from matplotlib.animation import FuncAnimation
from mpl_toolkits import mplot3d
import sys
```

Task 1

a)

In [5]:

```
ux0 = 0
u0t = 1
def FTBS(r=0.5, a=1, h =1/160):
    k = r*h
    N = int(2/k -1)
    M = int(3/h -1)

    U = np.zeros((M+1,N+1))

    for i in range(N):
        U[0,i+1] = u0t


    for i in range(N):
        n = i
        for j in range(M):
            m = j+1
            U[m,n+1] = U[m,n] - r*(U[m,n] - U[m-1,n])
    return U

def LaxWendrof(r=0.5, a=1, h =1/160):
    k = r*h
    N = int(2/k -1)
    M = int(3/h -1)

    U = np.zeros((M+1,N+1))

    for i in range(N):
        U[0,i+1] = u0t

    for i in range(N):
        n = i
        for j in range(M-1):
            m = j+1
            U[m,n+1] = U[m,n] - r/2*(U[m+1,n] - U[m-1,n]) + r**2/2*(U[m+1,n]-2*U[m,n] + U[m

    return U

def Wendrof(r=0.5, a=1, h =1/160):
    k = r*h
    N = int(2/k -1)
    M = int(3/h -1)

    U = np.zeros((M+1,N+1))

    for i in range(N):
        U[0,i+1] = u0t

    A = np.zeros((M,M))
    for i in range(M):
        A[i,i] = 1
        if M > 0:
            A[i,i-1] = (1-r)/(1+r)

    for i in range(N):
        n = i
        b = np.zeros(M)

        b = U[:-1,n] + (1-r)/(1+r)*U[1:,n]
```

```python
        b[0] += -(1-r)/(1+r)*u0t

        U[1:,n+1] = np.linalg.solve(A,b)

    return U
```

In [ ]:

```python

```

In [6]:

```python
def pltftbs(r=0.5, a=1, h =1/160):
    k = r*h
    N = int(2/k -1)
    M = int(3/h -1)
    UFTBS = FTBS(r,a,h)
    fig = plt.figure()
    ax = plt.axes(xlim=(0, 3), ylim=(-2, 2))
    line, = ax.plot([], [], lw=2)


    def init():
        line.set_data([], [])
        return line,

    def animate(i):
        titl = "Numerical solution for t = " + str(round((k*i),2)) + ", r = " + str(r)
        ax.set_title(titl)
        x = np.linspace(0, 3, M+1)
        y = UFTBS[:,i]
        line.set_data(x, y)
        return line,

    anim = FuncAnimation(fig, animate, init_func=init,
                               frames=N+1, interval=10, blit=True)
    plt.show()
    return anim
```

In [7]:

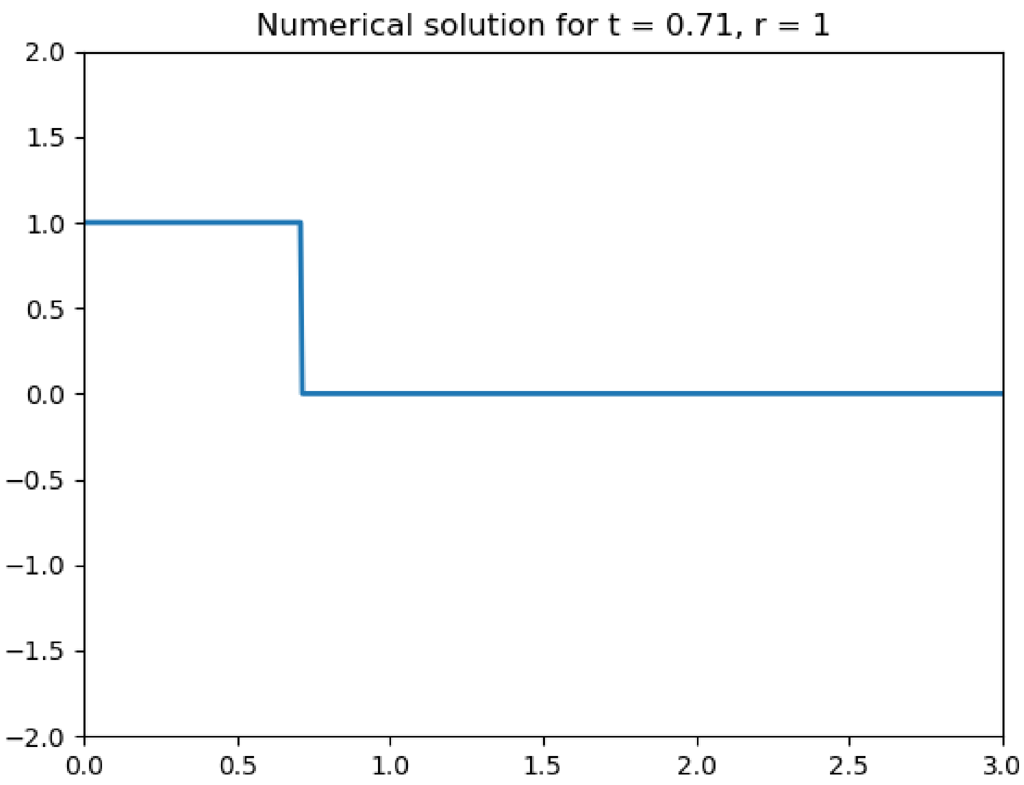
```
def pltlw(r=0.5, a=1, h =1/160):
    k = r*h
    N = int(2/k -1)
    M = int(3/h -1)
    ULW = LaxWendrof(r,a,h)
    fig = plt.figure()
    ax = plt.axes(xlim=(0, 3), ylim=(-2, 2))
    line, = ax.plot([], [], lw=2)


    def init():
        line.set_data([], [])
        return line,

    def animate(i):
        titl = "Numerical solution for t = " + str(round((k*i),2)) + ", r = " + str(r)
        ax.set_title(titl)
        x = np.linspace(0, 3, M+1)
        y = ULW[:,i]
        line.set_data(x, y)
        return line,

    anim = FuncAnimation(fig, animate, init_func=init,
                                   frames=N+1, interval=10, blit=True)
    plt.show()
    return anim
```

In [8]:

```
def pltuw(r=0.5, a=1, h =1/160):
    k = r*h
    N = int(2/k -1)
    M = int(3/h -1)
    UW = Wendrof(r,a,h)
    fig = plt.figure()
    ax = plt.axes(xlim=(0, 3), ylim=(-2, 2))
    line, = ax.plot([], [], lw=2)


    def init():
        line.set_data([], [])
        return line,

    def animate(i):
        titl = "Numerical solution for t = " + str(round((k*i),2)) + ", r = " + str(r)
        ax.set_title(titl)
        x = np.linspace(0, 3, M+1)
        y = UW[:,i]
        line.set_data(x, y)
        return line,

    anim = FuncAnimation(fig, animate, init_func=init,
                                   frames=N+1, interval=10, blit=True)
    plt.show()
    return anim
```

In [10]:

```
pltftbs(r=1)
```

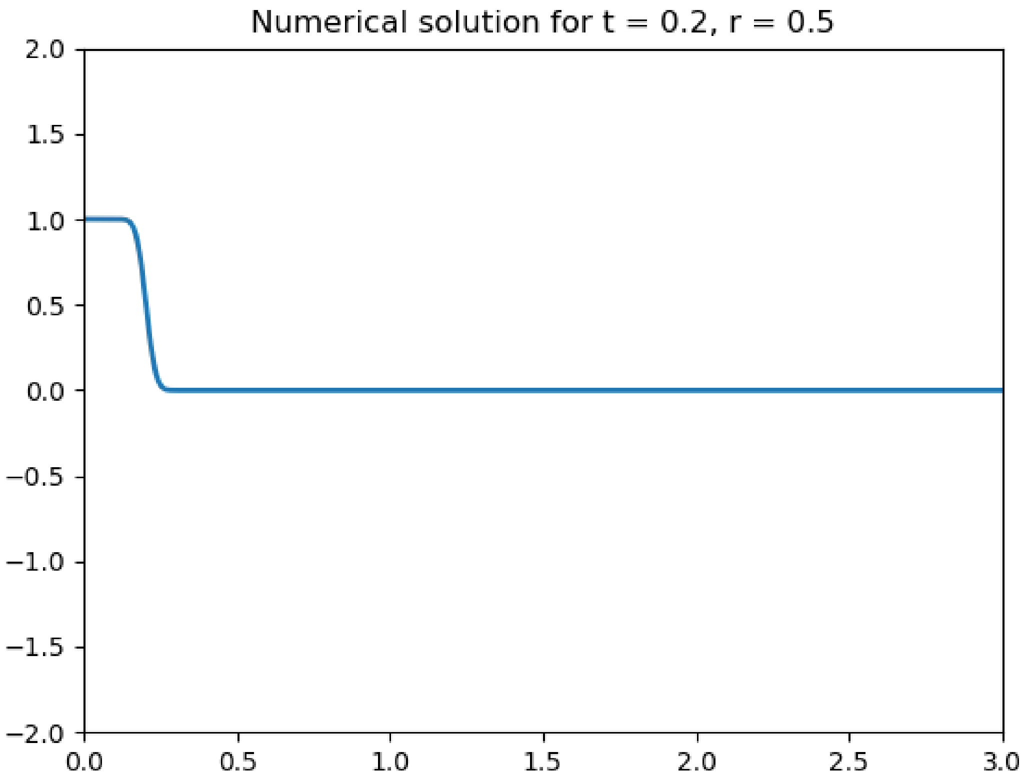


Out[10]:

```
<matplotlib.animation.FuncAnimation at 0x1f23f5b6828>
```

In [18]:

```
pltftbs(r=0.5)
```

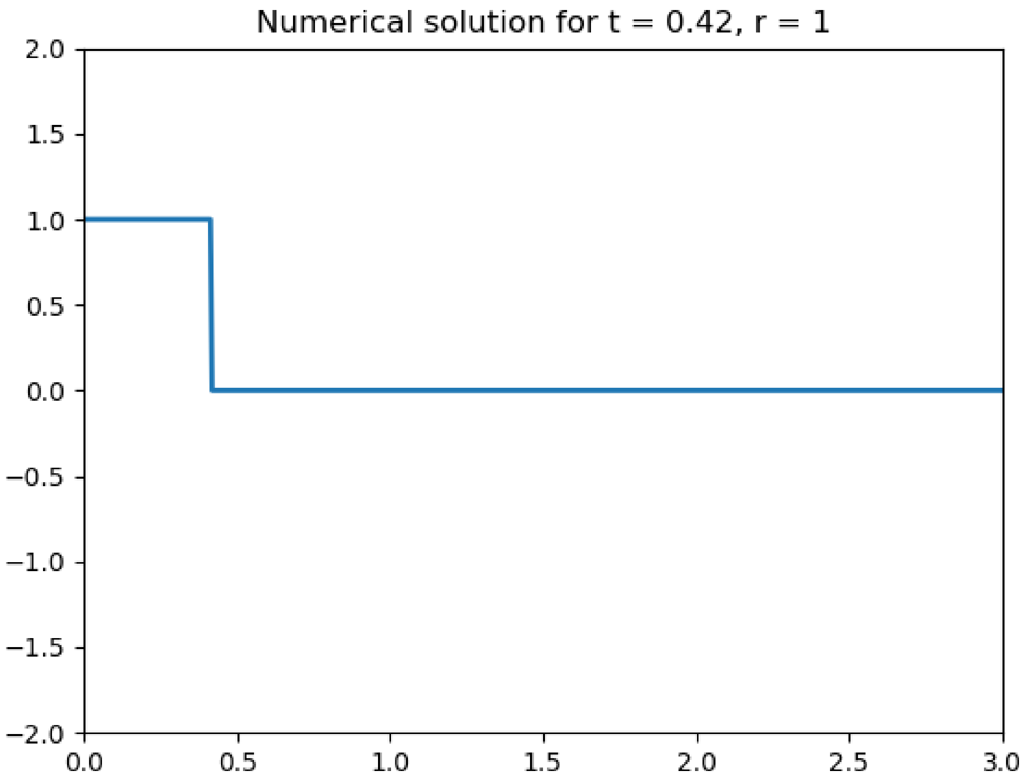


Out[18]:

```
<matplotlib.animation.FuncAnimation at 0x1f241d8d080>
```

In [19]:

```
pltlw(r=1)
```

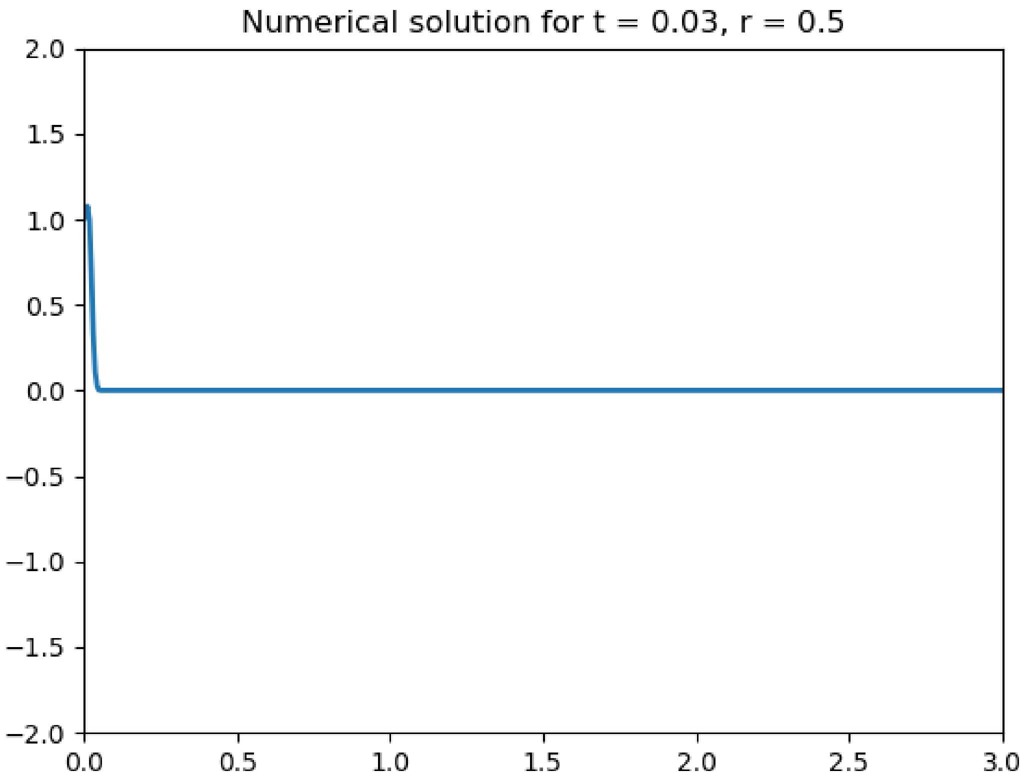


Out[19]:

```
<matplotlib.animation.FuncAnimation at 0x1f242415b70>
```

In [25]:

```
pltlw(r=0.5)
```

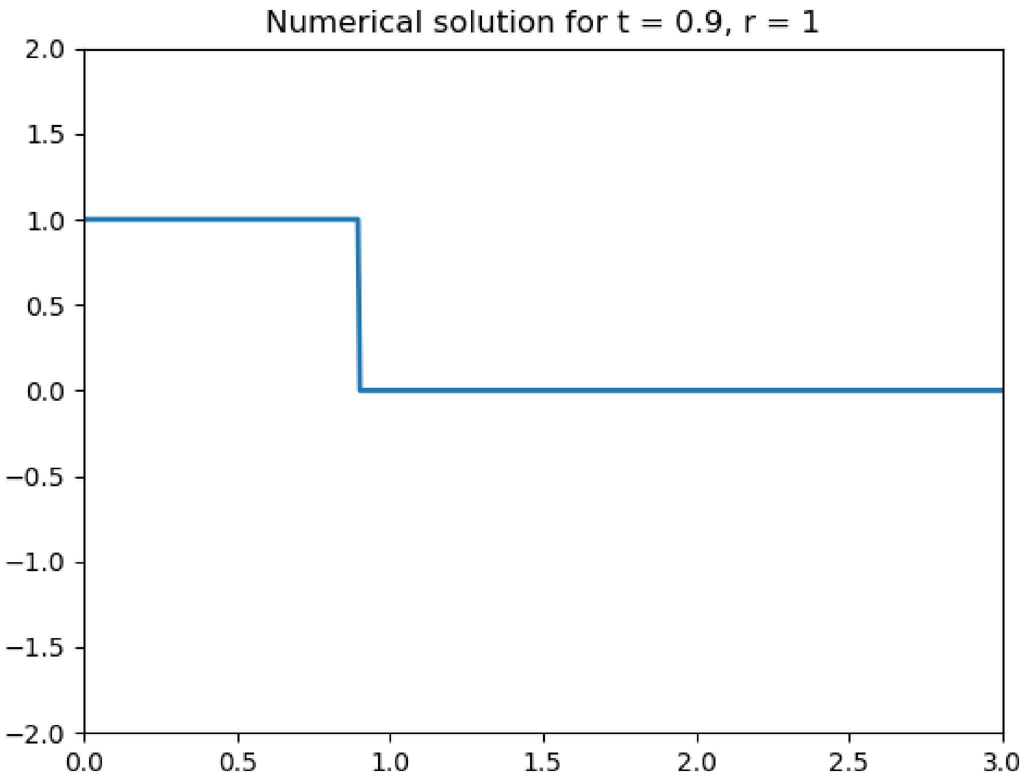


Out[25]:

<matplotlib.animation.FuncAnimation at 0x1f9502514e0>

In [26]:

```
pltuw(r=1)
```

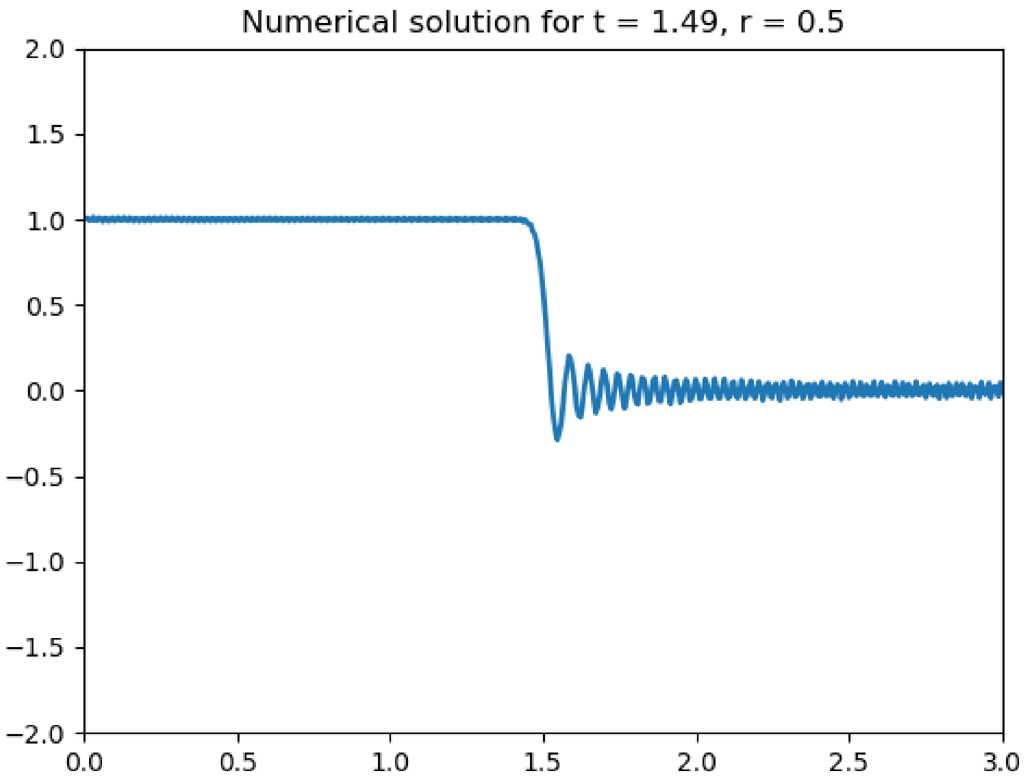


Out[26]:

```
<matplotlib.animation.FuncAnimation at 0x1f9507b5080>
```

In [27]:

```
pltuw(r=0.5)
```



Out[27]:

```
<matplotlib.animation.FuncAnimation at 0x1f950bb9898>
```

Above are plots of numerical solutions of the transport problem

$$u_t + au_x = 0, \quad 0 \le x \le 3, \quad u(x,0) = f(x), \quad u(0,t) = g(t)$$

produced with respectively forward-time backwards-space, Lax-Wendroffs method and Wendroffs method with Courant number $r = ak/h = 1$ and $r = 0.5$ on each. We observe that all of the methods produces the exact solution whith $r = 1$ as expected.

For the different methods above we observe that with $r = 0.5$ they each produces an unique numerical solution- Forward time backwards space can seem to have increasing dispertion, which can be seen as the solution takes on a gentler slope as time progresses. We can see the same result for Wendroffs method and Lax-wendroffs method. Lax-Wendroff does in addition have visible increasing amplitude something that can suggest dissipation.

b)

By applying $U_m^n = \xi^n e^{i\beta x_m}$ to Wendroffs method

$$\xi^{n+1} e^{i\beta x_m} = \xi^n e^{i\beta x_{m-1}} - \frac{1-r}{1+r}(\xi^{n+1} e^{i\beta x_{m-1}} - \xi^n e^{i\beta x_m})$$

one can get the expression

$$\xi = e^{i\beta h} \frac{\frac{1-r}{1+r} + e^{-i\beta h}}{\frac{1-r}{1+r} + e^{i\beta h}}.$$

Because the numerator in the fraction is the complex conjugate of the denominator and $|e^{i\beta h}| = 1$ we get $|\xi| = 1$ we can then conclude that the scheme does not dissipate. To analyze the dispersion we can use

$$\phi = \alpha\beta k = arctan\left(\frac{-Im(\xi)}{Re(\xi)}\right)$$

and check for α's dependency on β.

In [20]:

```
def xiLaxWen(Bh, r = 0.5,):

    return np.sqrt(1-4*r**2*(1-r**2)*(np.sin(Bh/2))**4)

def ImWen(bh,g):
    return g**2*np.sin(bh) - np.sin(bh)*(np.cos(bh))**2 - (np.sin(bh))**3

def ReWen(bh,g):
    return g**3 + 2*g*(np.cos(bh))**2 + np.cos(bh)*g**2 + np.cos(bh)*(np.sin(bh))**2 + 2*g*


def alfaWen(bh,r,a = 1):

    gamma = (1-r)/(1+r)

    return a * 1/(r*bh)*np.arctan((-ImWen(bh,gamma))/(ReWen(bh,gamma)))

def laxwendrofal(bh,r, a=1):
    alf = a * (1/(r * bh)) * np.arctan2((r * np.sin(bh)),(1-(r**2)*(1-np.cos(bh))))
    return alf

def wendrofxi(bh, r , a=1):
    return 1

def wendrofal(bh):
    r = 0.5
    a = 1
    gam = (r-1)/(r+1)
    c = np.cos(bh)
    s = np.sin(bh)
    #alf =  a * (1/(r * bh)) *np.arctan2(-(-2*gam*np.sin(bh)*np.cos(bh) -2*np.cos(bh)**2 *
    alf = a * (1/(r*bh)) * np.arctan(-(-3*c**2 * s - 4*c*gam*s-gam**2 * s + s**3)/(c**3+2*c
    return alf
```

In [21]:

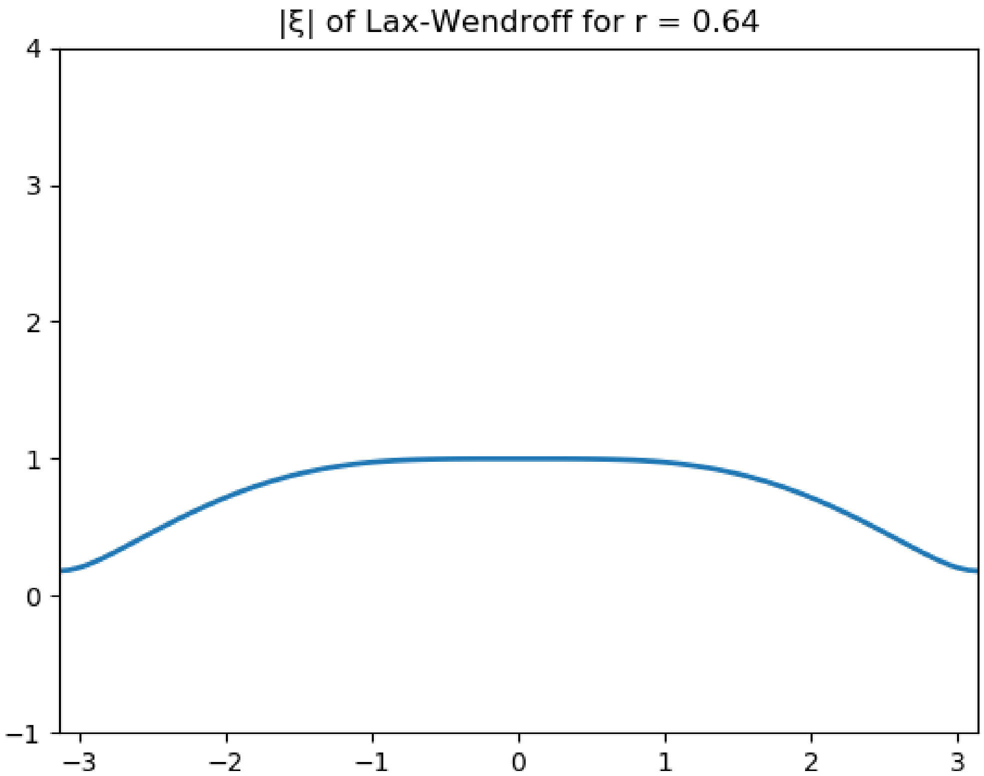
```
def pltxilaxwen():
    rmin = 0
    rmax = 1.5

    xmin = -np.pi
    xmax = np.pi

    M = 100

    k = 0.01
    N = int((rmax-rmin)/k)
    fig = plt.figure()
    ax = plt.axes(xlim=(xmin, xmax), ylim=(-1, 4))
    line, = ax.plot([], [], lw=2)

    def init():
        line.set_data([], [])
        return line,

    def animate(i):
        titl = "|ξ| of Lax-Wendroff for r = " + str(round((rmin+ k*i),2))
        ax.set_title(titl)
        x = np.linspace(xmin,xmax,M)
        y = xiLaxWen(x,rmin+ k*i)
        line.set_data(x, y)


        return line,

    anim = FuncAnimation(fig, animate, init_func=init,
                                   frames=N, interval=20, blit=True)
    plt.show()
    return anim

def pltxiwen():
    rmin = 0
    rmax = 1.5

    xmin = -np.pi
    xmax = np.pi

    M = 100

    k = 0.01
    N = int((rmax-rmin)/k)
    fig = plt.figure()
    ax = plt.axes(xlim=(xmin, xmax), ylim=(-1, 4))
    line, = ax.plot([], [], lw=2)

    def init():
        line.set_data([], [])
        return line,

    def animate(i):
        titl = "|ξ| of Wendroffs method for r = " + str(round((rmin+ k*i),2))
        ax.set_title(titl)
        x = np.linspace(xmin,xmax,M)
        y = wendrofxi(x,rmin+ k*i)
        line.set_data(x, y)
```

```
        return line,

    anim = FuncAnimation(fig, animate, init_func=init,
                               frames=N, interval=20, blit=True)
    plt.show()
    return anim



def pltalflaxwen():
    rmin = 0
    rmax = 1.5

    xmin = -np.pi
    xmax = np.pi

    M = 100

    k = 0.01
    N = int((rmax-rmin)/k)
    fig = plt.figure()
    ax = plt.axes(xlim=(xmin, xmax), ylim=(-1, 4))
    line, = ax.plot([], [], lw=2)

    def init():
        line.set_data([], [])
        return line,

    def animate(i):
        titl = "Plot of α for r = " + str(round((rmin+ k*i),2))
        ax.set_title(titl)
        x = np.linspace(xmin,xmax,M)
        y = laxwendrofal(x,rmin+ k*i)
        line.set_data(x, y)


        return line,

    anim = FuncAnimation(fig, animate, init_func=init,
                               frames=N, interval=20, blit=True)
    plt.show()
    return anim

def pltalfwen():
    rmin = 0
    rmax = 1.5

    xmin = -np.pi
    xmax = np.pi

    M = 100

    k = 0.01
    N = int((rmax-rmin)/k)
    fig = plt.figure()
    ax = plt.axes(xlim=(xmin, xmax), ylim=(-1, 4))
    line, = ax.plot([], [], lw=2)

    def init():
```

```python
        line.set_data([], [])
        return line,

    def animate(i):
        titl = "Plot of α for r = " + str(round((rmin+ k*i),2))
        ax.set_title(titl)
        x = np.linspace(xmin,xmax,M)
        y = alfaWen(x,rmin+ k*i)
        line.set_data(x, y)


        return line,

    anim = FuncAnimation(fig, animate, init_func=init,
                                   frames=N, interval=20, blit=True)
    plt.show()
    return anim
```

In [24]:

```python
pltxilaxwen()
```



Out[24]:

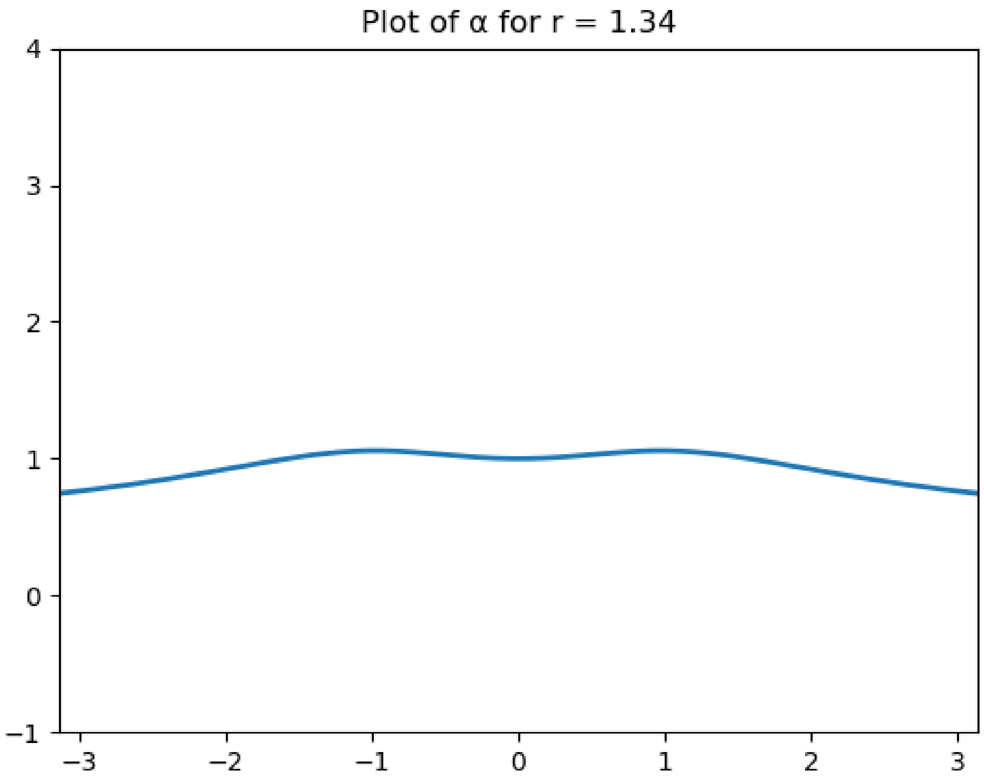
```
<matplotlib.animation.FuncAnimation at 0x1f242cee940>
```

In [ ]:

```python
pltxiwen()
```

In [23]:

```
pltalflaxwen()
```

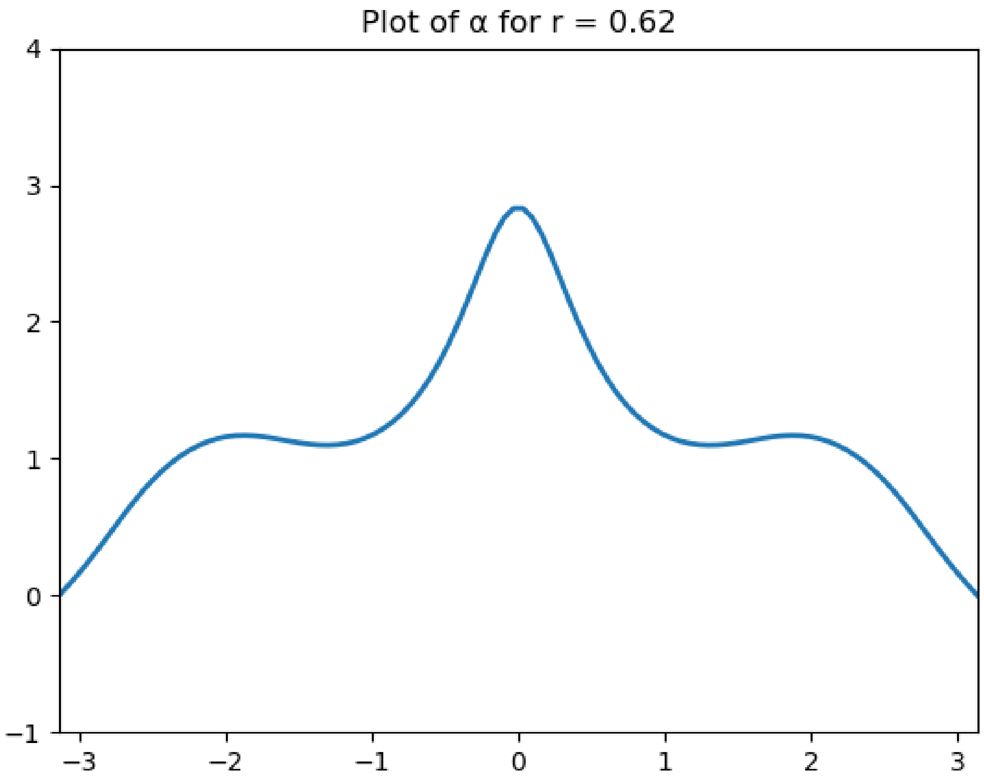


Out[23]:

```
<matplotlib.animation.FuncAnimation at 0x1f242a0f518>
```

In [106]:

```
pltalfwen()
```



Out[106]:

```
<matplotlib.animation.FuncAnimation at 0x1f95b9702b0>
```

For the plot of $|\xi|$ of Lax-Wendroff we see that when $r = 1$ it doesn't dissipate or disperse. We can also see that it becomes unstable when $r > 1$ something confirmed by the Neumann- analysis. While the Wendroff method doesn't dissipate it still disperses a bit.

c)

In [107]:

```
a = 1
h = 1/200
#r = 0.5
k = 1/400
r = a*k/h

xlimit = 3
tlimit = 2

N = int(tlimit/k -1)
M = int(xlimit/h -1)

ux0 = 0
u0t = 0

def f(x):
    return np.exp(-64*(x-0.5)**2)*np.sin(32*np.pi*x)

def FTBSc():
    U = np.zeros((M+1,N+1))

    for i in range(N):
        U[0,i+1] = u0t

    for i in range(M+1):
        U[i,0] = f(i*h)


    for i in range(N):
        n = i
        for j in range(M):
            m = j+1
            U[m,n+1] = U[m,n] - r*(U[m,n] - U[m-1,n])
    return U

def LaxWendrofc():
    U = np.zeros((M+1,N+1))

    for i in range(N):
        U[0,i+1] = u0t

    for i in range(M+1):
        U[i,0] = f(i*h)

    for i in range(N):
        n = i
        for j in range(M-1):
            m = j+1
            U[m,n+1] = U[m,n] - r/2*(U[m+1,n] - U[m-1,n]) + r**2/2*(U[m+1,n]-2*U[m,n] + U[m

    return U

def Wendrofc():
    U = np.zeros((M+1,N+1))

    for i in range(N):
        U[0,i+1] = u0t

    for i in range(M+1):
```

```python
        U[i,0] = f(i*h)


    A = np.zeros((M,M))
    for i in range(M):
        A[i,i] = 1
        if M > 0:
            A[i,i-1] = (1-r)/(1+r)

    for i in range(N):
        n = i
        b = np.zeros(M)

        b = U[:-1,n] + (1-r)/(1+r)*U[1:,n]
        b[0] += -(1-r)/(1+r)*u0t

        U[1:,n+1] = np.linalg.solve(A,b)

    return U
```

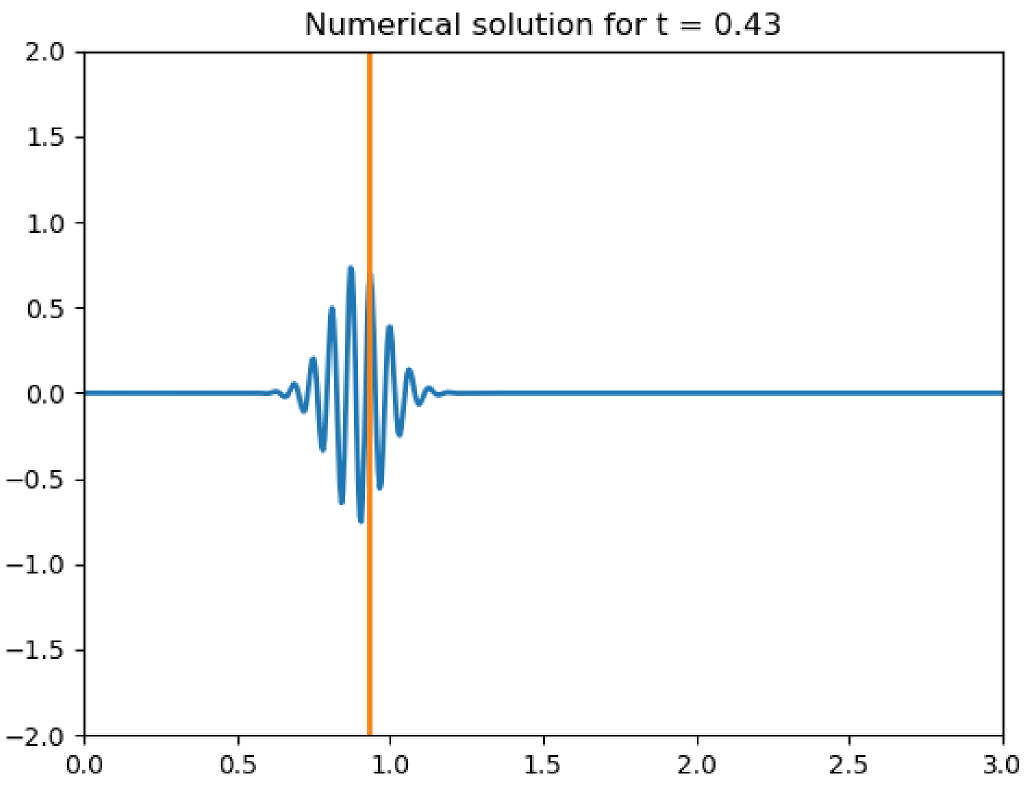
In [108]:

```python
UFTBS = FTBSc()
UW = Wendrofc()
ULW = LaxWendrofc()
```

Lax - Wendrof

In [109]:

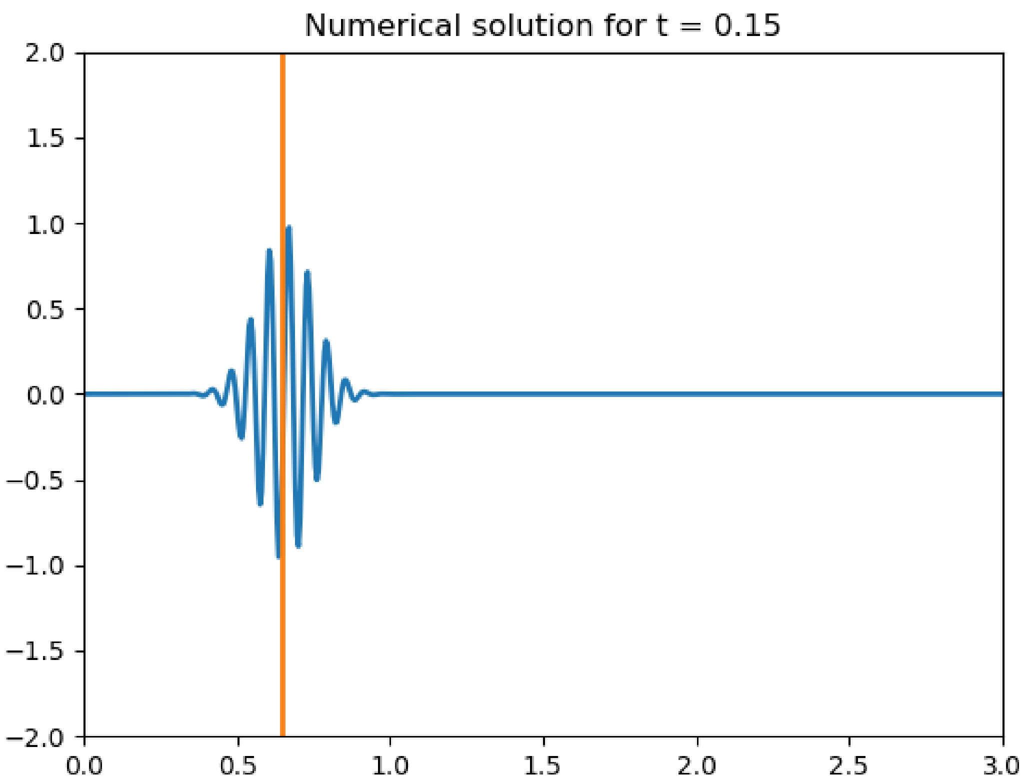
```
fig = plt.figure()
ax = plt.axes(xlim=(0, xlimit), ylim=(-2, 2))
line, = ax.plot([], [], lw=2)
line2, = ax.plot([], [], lw=2)
#line3, = ax.plot([], [], lw=2)


def init():
    line.set_data([], [])
    line2.set_data([], [])
    #line3.set_data([], [])
    return line,

def animate(i):
    titl = "Numerical solution for t = " + str(round((k*i),2))
    ax.set_title(titl)
    x = np.linspace(0, xlimit, M+1)
    y = ULW[:,i]
    line.set_data(x, y)
    line2.set_data(a*k*i+0.5,[-2,2])
    #line3.set_data(k*i*1.05+0.75,[-2,2])


    return line, line2, #line3,

anim = FuncAnimation(fig, animate, init_func=init,
                               frames=N+1, interval=10, blit=True)
plt.show()
```



Wendrof

In [110]:

```
fig = plt.figure()
ax = plt.axes(xlim=(0, xlimit), ylim=(-2, 2))
line, = ax.plot([], [], lw=2)
line2, = ax.plot([], [], lw=2)
#line3, = ax.plot([], [], lw=2)


def init():
    line.set_data([], [])
    line2.set_data([], [])
    #line3.set_data([], [])
    return line,

def animate(i):
    titl = "Numerical solution for t = " + str(round((k*i),2))
    ax.set_title(titl)
    x = np.linspace(0, xlimit, M+1)
    y = UW[:,i]
    line.set_data(x, y)
    line2.set_data(a*k*i+0.5,[-2,2])
    #line3.set_data(k*i*1.05+0.75,[-2,2])


    return line, line2, #line3,

anim = FuncAnimation(fig, animate, init_func=init,
                               frames=N+1, interval=10, blit=True)
plt.show()
```



In Lax-Wendroff we see dissipation and we see that the phase speed \alfa is different from the group speed.

The group speed is also different from the analytical speed a. On the Wendroff method we notice all the same things except dissipation. From the results in b it was unexpected to see no dispersion on neither Lax-Wendroff or Wendroff.

Task 2

The numerical solution for $U^n_{m,p}$

$$u_{tt} = u_{xx} + u_{yy} + f(x, y, t)$$

using central difference becomes

$$\frac{U^{n+1}_{m,p} - U^n_{m,p} + U^{n-1}_{m,p}}{k^2} = \frac{U^n_{m+1,p} - 2U^n_{m,p} + U^n_{m-1,p}}{h^2} + \frac{U^n_{m,p+1} - 2U^n_{m,p} + U^n_{m,p-1}}{h^2} + f(x, y, t) \qquad (1)$$

Aolving for $U^{n+1}_{m,p}$ we get:

$$U^{n+1}_{m,p} = 2U^n_{m,p} + \frac{k^2}{h}(U^n_{m+1,p} + U^n_{m,p+1} - 4U^n_{m,p} + U^n_{m,p-1} + U^n_{m-1,p}) - U^{n-1}_{m,p} + k^2 f(x, y, t)$$

To find the first time tep we approximate $u_t(x, y, 0)$ with a central difference:

$$u_t(x, y, 0) = +\frac{U^1_{m,p} - U^{-1}_{m,p}}{2k} = v_0(x, y)$$
$$U^{-1}_{m,p} = U^1_{m,p} - 2kv_0(x, y) \qquad (2)$$

By inserting the equation for $U^{-1}_{m,p}$ (2) in (1) with $n = 0$, we find

$$U^1_{m,p} = (2U^0_{m,p} + \frac{k^2}{h^2}(U^n_{m+1,p} + U^n_{m,p+1} - 4U^n_{m,p} + U^n_{m,p-1} + U^n_{m-1,p}) + 2kv_0(x, y) + k^2 f(x, y, t))/2$$

Reflecting boundary:

The Neumann condition:

$$\frac{\delta u}{\delta \boldsymbol{n}} = 0$$

at the boundary:

$x = -1 \rightarrow u_x = 0$

$x = 1 \rightarrow -u_x = 0$

$y = -1 \rightarrow u_y = 0$

$y = 1 \rightarrow -u_y = 0$

Using a central difference we get

$$\frac{U^n_{m+1,p} - U^n_{m-1,p}}{2h} = 0$$

$$U^n_{m-1,p} = U^n_{m+1,p} \qquad (3.1)$$

for the x-boundaries, and likewise for the y-boundaries:

$$U^n_{m,p-1} = U^n_{m,p+1} \qquad (3.2)$$

We find $U^{n+1}_{m,p}$ at the boundaries by inserting equations (3) in equation (1).

Absorbing boundary:

By using the central difference on the approximation, on the boundary $x = -1$: $u_t - u_x = 0$. We get:

$$\frac{U_{m,p}^{n+1} - U_{m,p}^{n-1}}{2k} - \frac{U_{m+1,p}^{n} - U_{m-1,p}^{n}}{2h} = 0$$

Solving for $U_{m-1,p}^{n}$:

$$U_{m-1,p}^{n} = U_{m+1,p}^{n} - \frac{h}{k}(U_{m,p}^{n+1} - U_{m,p}^{n-1}) \qquad (4)$$

By inserting equation (4) into equation (1) we get:

$$U_{m,p}^{n+1} = 2U_{m,p}^{n} + \frac{k^2}{h^2}(U_{m+1,p}^{n} + U_{m,p+1}^{n} - 4U_{m,p}^{n} + U_{m,p-1}^{n}U_{m-1,p}^{n}) + (U_{m+1,p}^{n} - \frac{h}{k}(U_{m,p}^{n+1} - U_{m,p}^{n-1})) - U_{m,p}^{n-1} +$$

Solving for $U_{m,p}^{n+1}$ we get:

$$U_{m,p}^{n+1} = (2U_{m,p}^{n} + \frac{k^2}{h^2}(2U_{m+1,p}^{n} + U_{m,p+1}^{n} - 4U_{m,p}^{n} + U_{m,p-1}^{n}) + \frac{k}{h}U_{m,p}^{n-1} - U_{m,p}^{n-1} + kf(x,y,t))/(1 + \frac{k}{h})$$

By symmetry the other boundaries will be similar, only changing what spacial term $U_{m\pm1,p\pm1}^{n}$ is doubled. Due to the corners having two boundaries, the k/h -terms will be doubled there aswell.

In [94]:

```
h = 1/50
k = 1/100

omega = 32
tmark = np.pi/omega

a = 100

xmin = -1
xmax = 1
ymin = -1
ymax = 1

T = 3

N = int(T/k)
M = int((xmax-xmin)/h)
P = int((ymax-ymin)/h)


def u0(x,y):
    return 0

def v0(x,y):
    return 0

def f1(x,y,t):
    if 0 <= t and t <= tmark:
        return np.exp(-a*((x+0)**2+(y+0)**2))*np.sin(omega*t) #+ np.exp(-a*((x-0.5)**2+(y-0
    else:
        return 0

def f2(x,y,t):
    if 0 <= t and t <= tmark:
        return np.exp(-a*((x+0.5)**2+(y+0.5)**2))*np.sin(omega*t) #+ np.exp(-a*((x-0.5)**2+
    else:
        return 0

def f3(x,y,t):
    return np.exp(-a*((x+0)**2+(y+0)**2))*np.sin(omega*t) #+ np.exp(-a*((x-0.5)**2+(y-0.5)*

def f4(x,y,t):
    return np.exp(-a*((x+0.5)**2+(y+0.5)**2))*np.sin(omega*t) + np.exp(-a*((x-0.5)**2+(y-0.

def reflect(f):

    U = np.zeros((M,P,N))


    for m in range(M):
        for p in range(P):
            U[m,p,0] = u0(xmin + m*h, ymin + p*h)

    for m in range(M):
        for p in range(P):
            if (0 < m and m < M-1) and (0 < p and p < P-1):
                U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m+1,p,0] + U[m,p+1,0] - 4*U[m,p,0] +
            else:
                if (m == 0 and p == 0):
```

```
                U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m+1,p,0] + U[m,p+1,0] - 4*U[m,p,0

            elif (m == 0 and p == P-1):
                U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m+1,p,0] + U[m,p-1,0] - 4*U[m,p,0

            elif (m == M-1 and p == 0):
                U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m-1,p,0] + U[m,p+1,0] - 4*U[m,p,0

            elif (m == M-1 and p == P-1):
                U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m-1,p,0] + U[m,p-1,0] - 4*U[m,p,0

            elif (m == 0):
                U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m+1,p,0] + U[m,p+1,0] - 4*U[m,p,0

            elif (p == 0):
                U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m+1,p,0] + U[m,p+1,0] - 4*U[m,p,0

            elif (m == M-1):
                U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m-1,p,0] + U[m,p+1,0] - 4*U[m,p,0

            elif (p == P-1):
                U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m+1,p,0] + U[m,p-1,0] - 4*U[m,p,0

            else:
                print ("error")




for i in range(N-2):
    n = i+1
    print('\r', 'Iteration', n, '/', N, end='')
    for m in range(M):
        for p in range(P):
            if (0 < m and m < M-1) and (0 < p and p < P-1):
                U[m,p,n+1] = 2*U[m,p,n] + k**2/h**2*(U[m+1,p,n] + U[m,p+1,n] - 4*U[m,p,
            else:
                if (m == 0 and p == 0):
                    U[m,p,n+1] = 2*U[m,p,n] + k**2/h**2*(U[m+1,p,n] + U[m,p+1,n] - 4*U[

                elif (m == 0 and p == P-1):
                    U[m,p,n+1] = 2*U[m,p,n] + k**2/h**2*(U[m+1,p,n] + U[m,p-1,n] - 4*U[

                elif (m == M-1 and p == 0):
                    U[m,p,n+1] = 2*U[m,p,n] + k**2/h**2*(U[m-1,p,n] + U[m,p+1,n] - 4*U[

                elif (m == M-1 and p == P-1):
                    U[m,p,n+1] = 2*U[m,p,n] + k**2/h**2*(U[m-1,p,n] + U[m,p-1,n] - 4*U[

                elif (m == 0):
                    U[m,p,n+1] = 2*U[m,p,n] + k**2/h**2*(U[m+1,p,n] + U[m,p+1,n] - 4*U[

                elif (p == 0):
                    U[m,p,n+1] = 2*U[m,p,n] + k**2/h**2*(U[m+1,p,n] + U[m,p+1,n] - 4*U[

                elif (m == M-1):
                    U[m,p,n+1] = 2*U[m,p,n] + k**2/h**2*(U[m-1,p,n] + U[m,p+1,n] - 4*U[

                elif (p == P-1):
                    U[m,p,n+1] = 2*U[m,p,n] + k**2/h**2*(U[m+1,p,n] + U[m,p-1,n] - 4*U[
```

```
                else:
                    print ("error")

    return U


def absorb(f):

    U = np.zeros((M,P,N))


    for m in range(M):
        for p in range(P):
            U[m,p,0] = u0(xmin + m*h, ymin + p*h)

    for m in range(M):
        for p in range(P):
            if (0 < m and m < M-1) and (0 < p and p < P-1):
                U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m+1,p,0] + U[m,p+1,0] - 4*U[m,p,0] +
            else:
                if (m == 0 and p == 0):
                    U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m+1,p,0] + U[m,p+1,0] - 4*U[m,p,0

                elif (m == 0 and p == P-1):
                    U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m+1,p,0] + U[m,p-1,0] - 4*U[m,p,0

                elif (m == M-1 and p == 0):
                    U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m-1,p,0] + U[m,p+1,0] - 4*U[m,p,0

                elif (m == M-1 and p == P-1):
                    U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m-1,p,0] + U[m,p-1,0] - 4*U[m,p,0

                elif (m == 0):
                    U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m+1,p,0] + U[m,p+1,0] - 4*U[m,p,0

                elif (p == 0):
                    U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m+1,p,0] + U[m,p+1,0] - 4*U[m,p,0

                elif (m == M-1):
                    U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m-1,p,0] + U[m,p+1,0] - 4*U[m,p,0

                elif (p == P-1):
                    U[m,p,1] = (2*U[m,p,0] + k**2/h**2*(U[m+1,p,0] + U[m,p-1,0] - 4*U[m,p,0

                else:
                    print ("error")


    for i in range(N-2):
        n = i+1
        #sys.stdout.write(str(n) + "/"+ str(N))
        #sys.stdout.flush()
        print('\r', 'Iteration', n, '/', N, end='')
        for m in range(M):
            for p in range(P):
                if (0 < m and m < M-1) and (0 < p and p < P-1):
                    U[m,p,n+1] = 2*U[m,p,n] + k**2/h**2*(U[m+1,p,n] + U[m,p+1,n] - 4*U[m,p,
                else:
                    if (m == 0 and p == 0):
                        U[m,p,n+1] = (2*U[m,p,n] + k**2/h**2*(U[m+1,p,n] + U[m,p+1,n] - 4*U
```

```
                elif (m == 0 and p == P-1):
                    U[m,p,n+1] = (2*U[m,p,n] + k**2/h**2*(U[m+1,p,n] + U[m,p-1,n] - 4*U

                elif (m == M-1 and p == 0):
                    U[m,p,n+1] = (2*U[m,p,n] + k**2/h**2*(U[m-1,p,n] + U[m,p+1,n] - 4*U

                elif (m == M-1 and p == P-1):
                    U[m,p,n+1] = (2*U[m,p,n] + k**2/h**2*(U[m-1,p,n] + U[m,p-1,n] - 4*U

                elif (m == 0):
                    U[m,p,n+1] = (2*U[m,p,n] + k**2/h**2*(U[m+1,p,n] + U[m,p+1,n] - 4*U

                elif (p == 0):
                    U[m,p,n+1] = (2*U[m,p,n] + k**2/h**2*(U[m+1,p,n] + U[m,p+1,n] - 4*U

                elif (m == M-1):
                    U[m,p,n+1] = (2*U[m,p,n] + k**2/h**2*(U[m-1,p,n] + U[m,p+1,n] - 4*U

                elif (p == P-1):
                    U[m,p,n+1] = (2*U[m,p,n] + k**2/h**2*(U[m+1,p,n] + U[m,p-1,n] - 4*U

                else:
                    print ("error")

    return U
```

In [89]:

```
#UR = reflect()
```

In [90]:

```
#UA = absorb()
```

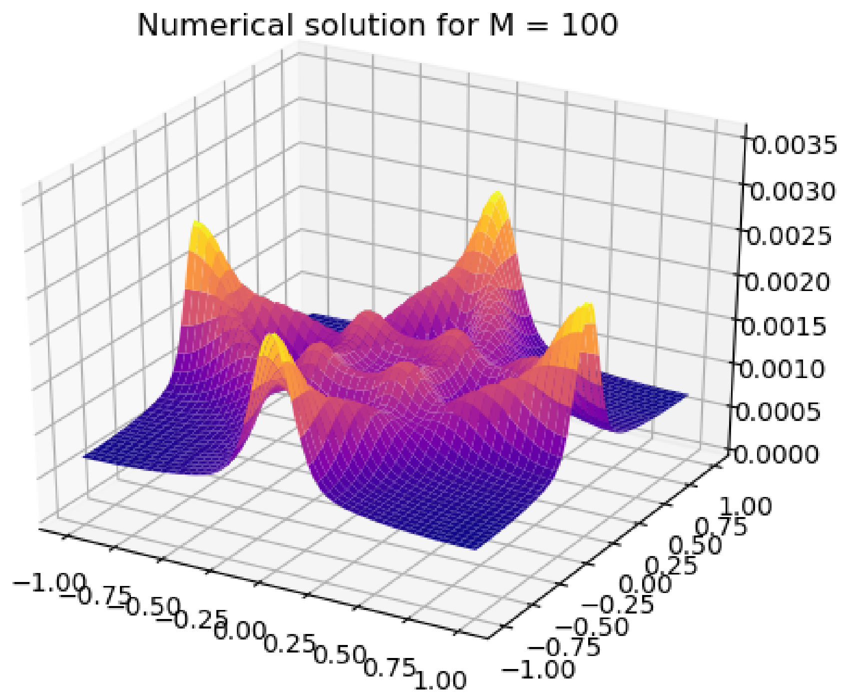
In [91]:

```
def plotWave(bcf, f, save = False):


    U = bcf(f)

    fig = plt.figure()

    ax = plt.axes(projection='3d')
    x = np.linspace(xmin,xmax,M)
    y = np.linspace(ymin,ymax,P)
    X, Y = np.meshgrid(x, y)

    def animateI(i):
        ax.clear()
        Z  = U[:,:,i]
        titl = "Numerical solution for M = " + str(M)
        ax.set_title(titl)
        ax.set_zlim3d(np.amin(U),np.amax(U)*1.1)
        surfI = ax.plot_surface(X, Y, Z, cmap='plasma')
        return surfI

    animI = FuncAnimation(fig, animateI,
                                  frames=N, interval=10, blit=True)
    if save == True:
        animI.save('wave.gif')

    return animI
```

In [84]:

```
plotWave(reflect,f1)
```

```
Iteration 298 / 300
```



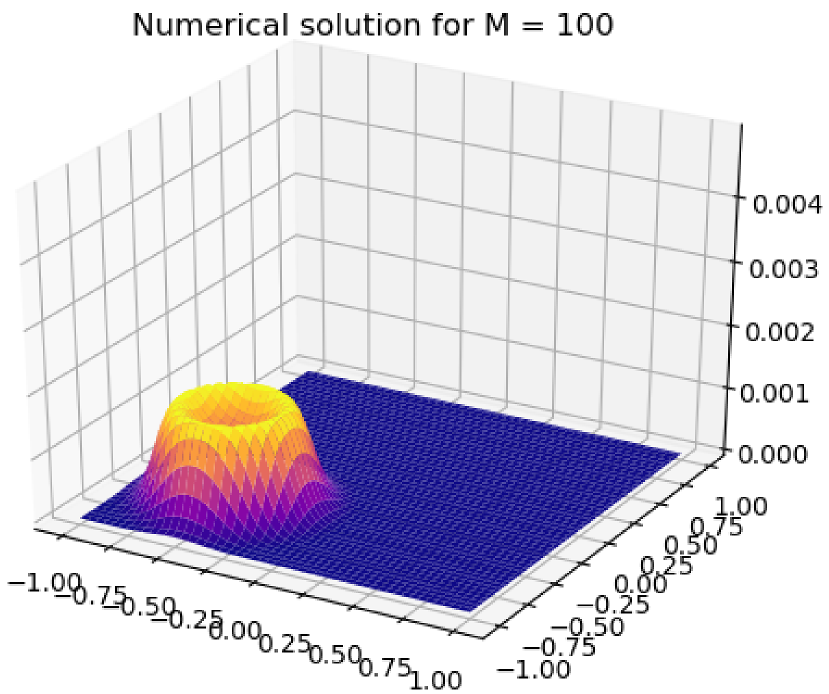
Out[84]:

```
<matplotlib.animation.FuncAnimation at 0x1f9569ad9b0>
```

In [87]:

```
plotWave(reflect,f2)
```

```
Iteration 298 / 30000129 / 300300
```



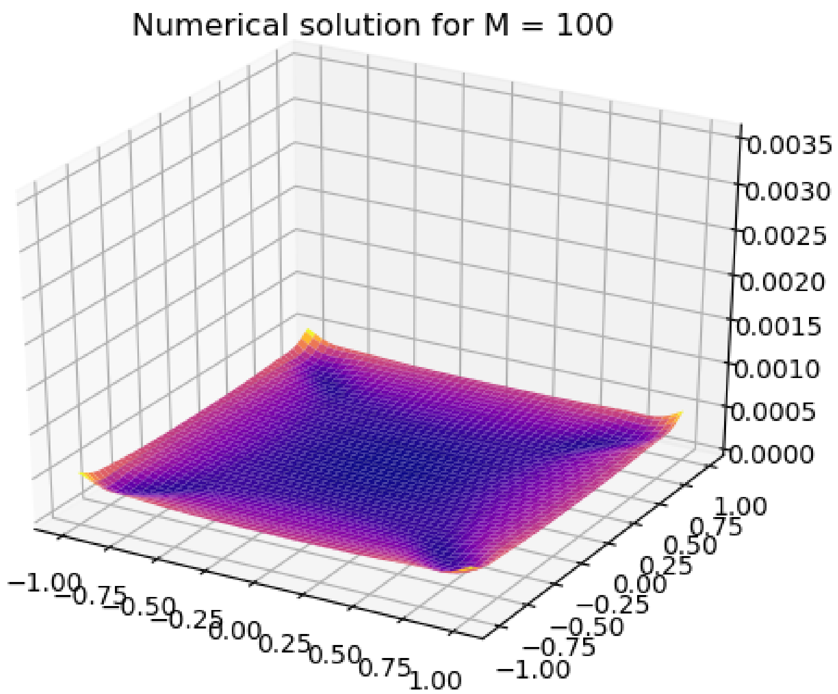
Out[87]:

```
<matplotlib.animation.FuncAnimation at 0x1f956aa8a58>
```

In [85]:

```
plotWave(absorb,f1)
```

```
 Iteration 298 / 300 175 / 300
```



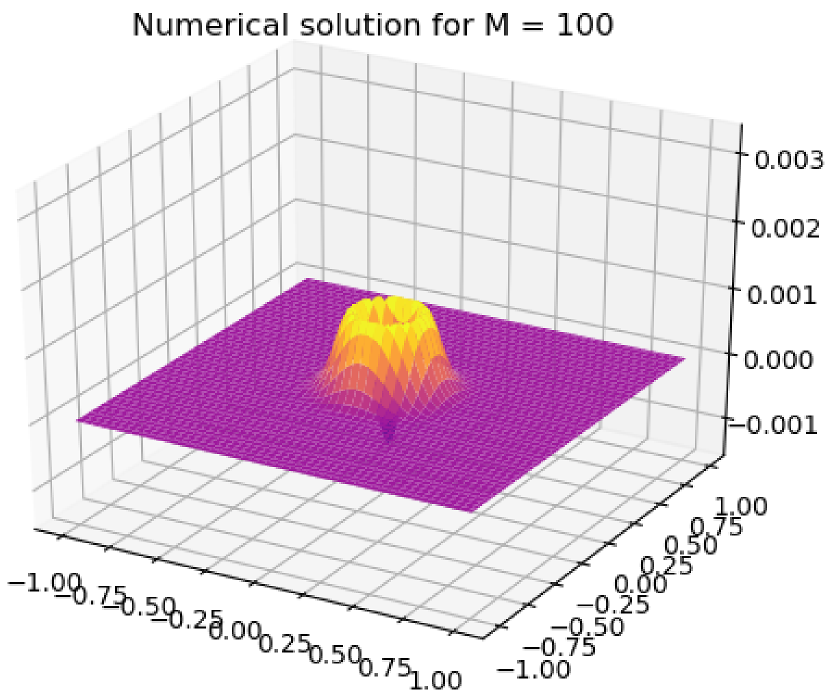
Out[85]:

```
&lt;matplotlib.animation.FuncAnimation at 0x1f9569119b0&gt;
```

In [92]:

```
plotWave(absorb,f3)
```

```
 Iteration 298 / 300
```



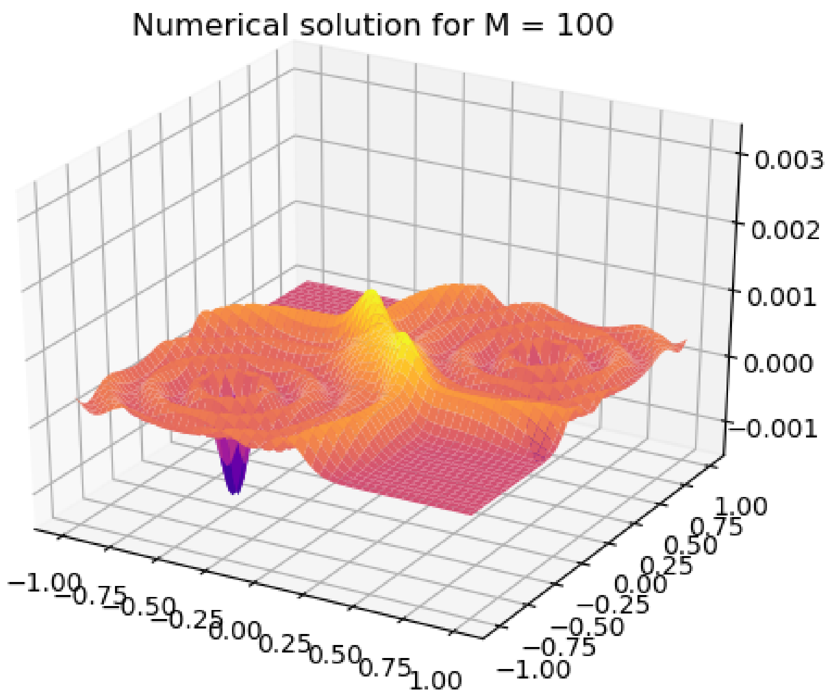
Out[92]:

```
<matplotlib.animation.FuncAnimation at 0x1f95b8b7860>
```

In [95]:

```
plotWave(absorb,f4)
```

```
Iteration 298 / 300 214 / 300
```



Out[95]:

```
&lt;matplotlib.animation.FuncAnimation at 0x1f9565023c8&gt;
```

the numerical solution for $\boxed{U\_}$

In [ ]: