AST3220 - Project 3:
Inflation without approximation

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1. PROBLEM A)

Using the convention of represent As H_i neccesarily has the same dimensions as H it is clear that h must be dimensionless:

$$[h] = [H][H_i]^{-1} = 1 \tag{1.1}$$

The hubble parameter has dimension velocity per distance, which can be written

$$[H] = (\mathsf{L}\mathsf{T}^{-1})\mathsf{L}^{-1} = \mathsf{T}^{-1} \tag{1.2}$$

so that τ is dimensionless as well:

$$[\tau] = [H][t] = \mathsf{T}^{-1}\mathsf{T} = 1 \tag{1.3}$$

Both ϕ and the Planck energy has units of energy, making ψ dimensionless as well:

$$[\psi] = [\phi][E_p]^{-1} = \mathsf{M}\mathsf{L}^2\mathsf{T}^{-2}(\mathsf{M}\mathsf{L}^2\mathsf{T}^{-2})^{-1} = 1 \tag{1.4}$$

Lastly, we rewrite the potential v using the definiton $E_p^2 = \hbar c^5/G$, so that

$$v = \frac{\hbar c^3}{H_i^2 E_p^2} = \frac{1}{H_i^2}\frac{G}{c^2}V \tag{1.5}$$

Using the definiton of H_i, we then see that

$$[v] = [H_i]^{-1}[Gc^{-2}V] = [H_i]^{-1}[H_i] = 1 \tag{1.6}$$

so v is dimensionless as well

2. PROBLEM B)

A. Hubble parameter and scale factor

Using the definitons of the dimensionless variables and applying the chain rule, we see that

$$\frac{d}{d\tau}\left(\ln\frac{a}{a_i}\right) = \frac{dt}{d\tau}\frac{d}{dt}\left(\ln\frac{a}{a_i}\right) \tag{2.1}$$

$$= \frac{1}{H_i}\frac{\dot{a}}{a} \tag{2.2}$$

$$= \frac{H}{H_i} \tag{2.3}$$

$$= h \tag{2.4}$$

B. Continuity equation (name???)

Similarly we continue using the chain rule to rewrite $\dot{\phi}$, $\ddot{\phi}$ and V' in terms of τ and ψ:

$$\frac{d\phi}{dt} = \frac{d\tau}{dt}\frac{d\phi}{d\psi}\frac{d\psi}{d\tau} = H_i E_p \frac{d\psi}{d\tau} \tag{2.5}$$

$$\frac{d^2\phi}{dt^2} = \frac{d\tau}{dt}\frac{d}{d\tau}\frac{d\phi}{dt} = H_i^2 E_p \frac{d^2\psi}{d\tau^2} \tag{2.6}$$

$$\frac{dV}{d\phi} = \frac{d\psi}{d\phi}\frac{dV}{dv}\frac{dv}{d\psi} = \frac{1}{E_p}\frac{H_i^2 E_p^2}{\hbar c^3}\frac{dv}{d\psi} \tag{2.7}$$

Which can be inserted to the (????) equation, giving

$$H_i^2 E_p \frac{d^2\psi}{d\tau^2} + 3HH_iE_p\frac{d\psi}{d\tau} + H_i^2 E_p \frac{dv}{d\psi} = 0 \tag{2.8}$$

Dividing both sides by $H_i^2 E_p$ and using the definition $h = H/H_i$, this reduces to

$$\frac{d^2\psi}{d\tau^2} + 3h\frac{d\psi}{d\tau} + \frac{dv}{d\psi} = 0 \tag{2.9}$$

1. *Hubble parameter*

afgasfas

3. PROBLEM C)

Read up on this shit

4. PROBLEM D)

During slow-roll, the number of remaining e-folds until inflation ends can be calculated as

$$N(t) = \frac{8\pi}{E_p^2}\int_{\phi_{\text{end}}}^{\phi}\frac{V}{V'}d\phi \tag{4.1}$$

$$\tag{4.2}$$

Where we easily find

$$\frac{V}{V'} = \frac{1}{2}\phi \tag{4.3}$$

and ψ_{end} is given by requiring $\epsilon(\phi_{\text{end}}) = 1$:

$$\epsilon(\phi_{\text{end}}) = \frac{E_p^2}{16\pi}\left(\frac{V'}{V}\right)^2 \tag{4.4}$$

$$= \frac{E_p^2}{4\pi\phi_{\text{end}}^2} \tag{4.5}$$

$$= 1 \tag{4.6}$$

$$\Rightarrow \qquad \phi_{\text{end}} = \frac{E_p}{\sqrt{4\pi}} \tag{4.7}$$

We can then calculate the integral:

$$N(t) = \frac{8\pi}{E_p^2}\int_{\phi_{\text{end}}}^{\phi} \frac{1}{2}\phi d\phi \tag{4.8}$$

$$= \frac{2\pi}{E_p^2}\left(\phi - \frac{E_p^2}{4\pi}\right) \tag{4.9}$$

$$= \frac{2\pi}{E_p^2}\phi^2 - \frac{1}{2} \tag{4.10}$$

By definition, the number of remaining e-folds at the initial time $t = t_i$ is the total number of e-folds N_tot. Evaluating $N(t)$ at t_i and solving for the initial field value ϕ_i we then get:

$$\phi_i = \frac{E_p}{\sqrt{2\pi}}\sqrt{N_{tot} + \frac{1}{2}} \tag{4.11}$$

or in terms of the dimensionless field:

$$\psi_i = \sqrt{\frac{1}{2\pi}\left(N_{tot} + \frac{1}{2}\right)} \tag{4.12}$$

Which evaluates to $\psi_i \approx 8.925$ for 500 e-folds of inflation.

5. PROBLEM E)

6. PROBLEM F)

7. PROBLEM G)

8. PROBLEM H)

9. PROBLEM I)

10. PROBLEM J)

11. PROBLEM K)

12. PROBLEM L)

13. PROBLEM M)

14. PROBLEM N)

15. PROBLEM O)

16. PROBLEM P)

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