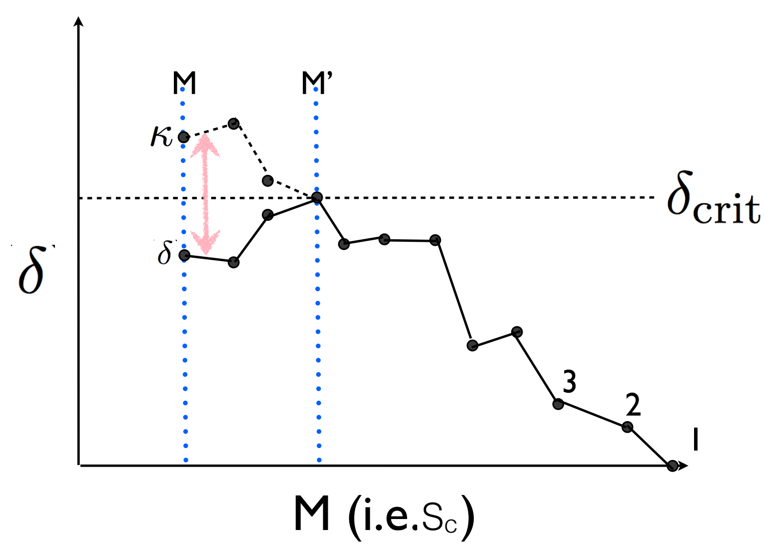
ASSIGNMENT 2

AST4320

1. Exercises to Support Lecture 6-10.

Exercise 1. Consider the top-hat smoothing function (in 1D), i.e. $W(x) = 1$ if $|x| < R$ and $x = 0$ otherwise. Compute and plot the Fourier conjugate $\tilde{W}(k)$ of $W(x)$. Compute what the 'full width at half maximum' (FWHM) of $\tilde{W}(k)$ is (i.e. how wide is this function at half its maximum?).

Exercise 2. This numerical exercise aims to give you a bit more intuitive insight into the random walk process we talked about in the lecture. It involves generating random numbers from Gaussian random distributions. If you have trouble doing this, check with Max for tips.



Consider a power spectrum of the form $P(k) = k$. The variance as a function of scale $S_c = 2\pi/k$ is given by $\sigma^2(S_c) = \pi/S_c^4$. We are going to generate $N = 10^5$ random walks. For each random walk:

(1) Start at a radius such that $\sigma^2(S_c) = \sigma^2(S_1) < 10^{-4}$. Draw a random δ from the Gaussian PDF (with variance $\sigma^2(S_1)$. This is your first point of the random walk (S_1, δ)

(2) Reduce S_c by some factor ϵ (play with values of ϵ: your random walk should not terminate after < 5 steps for example, it also should not take too long to finish a single random walk). Compute the new $\sigma^2(S_2)$. Draw a random β from the Gaussian PDF with variance $\sigma_{12}^2 \equiv \sigma^2(S_2) - \sigma^2(S_1)$. Your new $\delta_2 = \delta + \beta$, and will give you your second point (S_2, δ_2).

(3) Reduce S_c by another factor of ϵ, and repeat until $S_c = 1.0$. The sequence of steps until reaching S_c is a single 'realization' of a random walk.

(4) For a Gaussian random field, the overdensity δ -smoothed on comoving scale S_c, which corresponds to mass scale M - is Gaussian distributed with a variance $\sigma^2(M)$:

$$P(\delta|M) = \frac{1}{\sqrt{2\pi}\sigma(M)} \exp\Big[-\frac{\delta^2}{2\sigma^2(M)}\Big].$$

Make a histogram for the values of δ you get at the end of the chain ($S_c = 1.0$). Compare with the analytic expression.

(5) Make another histogram of δ, but only consider those chains which never crosses the threshold $\delta_{\rm crit} = 1$. Compare the histogram you get to the expression:

$$P_{\rm nc}(\delta|M) = P(\delta|M) - P([2\delta_{\rm crit} - \delta]|M) = \frac{1}{\sqrt{2\pi}\sigma(M)}\Big(\exp\Big[-\frac{\delta^2}{2\sigma^2(M)}\Big] - \exp\Big[-\frac{[2\delta_{\rm crit}-\delta]^2}{2\sigma^2(M)}\Big]\Big).$$

Exercise 3. In the previous exercise we showed that the PDF for δ - provided that δ was never larger than $\delta_{\rm crit}$ at some scale $M' > M$ - was given by

$$P_{\rm nc}(\delta|M) = P(\delta|M) - P([2\delta_{\rm crit} - \delta]|M) = \frac{1}{\sqrt{2\pi}\sigma(M)}\Big(\exp\Big[-\frac{\delta^2}{2\sigma^2(M)}\Big] - \exp\Big[-\frac{[2\delta_{\rm crit}-\delta]^2}{2\sigma^2(M)}\Big]\Big).$$

(1) Argue that the probability that mass at $\mathbf{x}$ is therefore embedded within a collapsed object of mass $> M$ is then

$$P(> M) = 1 - \int_{-\infty}^{\delta_{\rm crit}} d\delta \; P_{\rm nc}(\delta|M)$$

(2) Show that this naturally gives rise to the infamous factor of 2 in the Press-Schechter formalism, i.e. show that :

$$P(> M) = 2P(\delta > \delta_{\rm crit}|M) = 1 - {\rm erf}(\nu/\sqrt{2}).$$