UiO : Institute of Theoretical Astrophysics
University of Oslo

Fys3150/4150 - Computational Physics

5. Continuation on Astronomical project - N-body simulation of an open galactic cluster

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Magnus Christopher Bareid
un: magnucb

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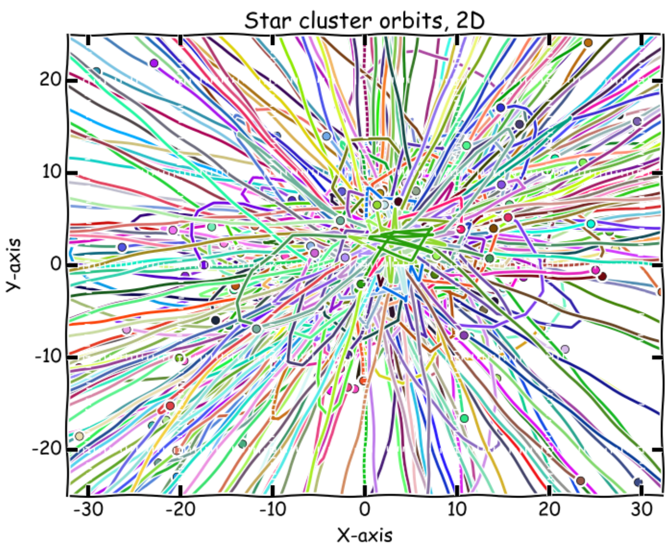

Abstract

Empty void

Contents

N-body simulation of an open galactic cluster

1 Introduction, or picking up where we left off

This project is a continuation on a previous project given to the students this semester, in which the students were to gravitationally and numerically simulate the Solar System.

For the current project, the students are to simulate an N-body system, unrelated to the Solar system, and investigate properties of the system as it is simulated to reflect on the model's physical validity.

For this purpose, we may review the mathematical equations related to Newtonian gravitation, and other relevant equations.

It is worth noting that the differential integration method we will now utilize, will be the Verlet method, as opposed to the Euler or RungeKutta4 method.

2 Underlying math

The most important mathematical expression for an assignment investigating properties of a gravitational system, will be the equations gravitational force exchange. The force equation between two objects, will be as per Newton's law of gravity:

$$\vec{F}_{G_{i,j}} = -G\frac{m_i m_j}{r_{i,j}^3}\vec{r}_{i,j} \tag{1}$$

where i and j denotes which objects' force, in the system of objects' forces, are being measured, the ms represents their masses, r is the displacement between the two objects, and G is the Newton's gravitational constant.

Furthermore, Newton's third law of motion comes into place here, as objects are being iterated over, such that

$$\vec{F}_{G_{j,i}} = -\vec{F}_{G_{i,j}} \tag{2}$$

and the force experienced by the second object of the two equals the same force that the first object experiences, only in the opposite direction.

At a later point in the project, there is introduced a smoothing factor that helps deal with numerical round-off problems, modifying the force equation thusly:

$$\vec{F}_{G_{i,j}} = -G\frac{m_i m_j}{r_{i,j}^2 + \varepsilon^2}\frac{\vec{r}_{i,j}}{r_{i,j}} \tag{3}$$

wherein the new quantity ε represents the aforementioned smoothing factor, with which different magnitudes of values will be experimented.

Then it is the matter of the system's time scale. It seems arbitrarily that we should measure time in years for a problem that doesn't connect to years in any way. Instead, time is defined through, and normalized by, an analytical expression for an N-body system's duration of collapse τ_{crunch} (or τ_c for short), from which we then pull the gravitational constant, dependent on the system's density ρ_0, and by extension the cluster's limiting radius R_0.

$$\begin{aligned}
\tau_c &= \sqrt{\frac{3\pi}{32G\rho_0}} \\
\Rightarrow G &= \frac{3\pi}{32\tau_c^2\rho_0}\ , \\
\rho_0 &= \frac{\sum_i m_i}{\frac{4}{3}\pi R_0^3}\ , \\
\Rightarrow G &= \frac{1}{8}\frac{\pi^2 R_0^3}{\tau_c^2 \sum_i m_i}
\end{aligned}$$

however, as we measure time as an iterated increment of τ_c, its value is normalized to 1, yielding:

$$G = \frac{1}{8}\frac{\pi^2 R_0^3}{\sum_i m_i} \tag{4}$$

3 Major changes in the code

The previous project utilized some small-scale general relativity, which at the scales of this current project is obsolete. Instead, the force equation is rewritten according to equation 1 and 3.

Command line functionality is modified to take in the system's number of bodies **N**, a time step size $\mathbf{dt}$, correction factor $\varepsilon\mathbf{dt}$, and duration of the simulation.

4 Appendix

4.1 Code and GitHub

All my code is located at this address:
`https://github.com/magnucb/p5`

References

[1] Jensen, M 2016, `CodeName`,

viewed *something*th of December 2016, `URL`.

[2] Jensen, M 2016, Computational Physics Lecture Notes Fall 2015:

13.2.2: Canonical Ensemble &

13.3.1: Ising Model and Phase Transitions in Magnetic Systems,

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