1 Simplified Energy Transfer Model

$$\varepsilon_E \sigma T_E^4 = R_0^E \stackrel{\text{equil.}}{=} P_{\text{in}}^E + R_{\text{in}}^E - P_{\text{latent}} - P_{\text{E}\to\text{A}}, \tag{1}$$

$$\varepsilon_A \sigma T_A^4 = R_0^A \stackrel{\text{equil.}}{=} P_{\text{in}}^A + R_{\text{in}}^A + P_{\text{latent}} + P_{\text{E}\to\text{A}}. \tag{2}$$

$$P_{\text{latent}} + P_{\text{E}\to\text{E}} = (\alpha+\beta)(T_E - T_A).$$

Let

$$\begin{aligned} P_{\text{in}}^E &= \gamma_0 P_0^S, \\ P_{\text{in}}^A &= \gamma_1 P_0^S. \end{aligned}$$

and

$$\begin{aligned} R_{\text{in}}^E &= \lambda_0 R_0^E + \mu_0 R_0^A, \\ R_{\text{in}}^A &= \lambda_1 R_0^E + \mu_1 R_0^A, \end{aligned}$$

We have a system on the form

$$a_0 T_E^4 + b_0 T_A^4 + c_0 T_E + d_0 T_A = e_0, \tag{3}$$

$$a_1 T_E^4 + b_1 T_A^4 + c_1 T_E + d_1 T_A = e_1, \tag{4}$$

where the coefficients are given by

$$\begin{aligned} a_0 &= \sigma\varepsilon_{\text{E}}(1-\lambda_0), \\ b_0 &= -\sigma\varepsilon_{\text{A}}\mu_0, \\ c_0 &= \alpha+\beta, \\ d_0 &= -(\alpha+\beta), \\ e_0 &= \gamma_0 P_0^S, \end{aligned}$$

and

$$\begin{aligned} a_1 &= -\sigma\varepsilon_{\text{E}}, \\ b_1 &= \varepsilon_{\text{A}}\sigma(1-\mu_1), \\ c_1 &= -(\alpha+\beta), \\ d_1 &= \alpha+\beta, \\ e_1 &= \gamma_1 P_0^S. \end{aligned}$$

Coefficients are given by

$$\gamma_0 = (1-r_{\text{SM}})(1-a_{\text{SW}})(1-a_{\text{O}_3})(1-C_{\text{C}} r_{\text{SC}})(1-C_{\text{C}} a_{\text{SC}})(1-r_{\text{SE}})$$

$$\begin{aligned} &P_{\text{in}}^E = (1-r_{\text{SM}})(1-a_{\text{SW}})(1-a_{\text{O}_3})(1-C_{\text{C}} r_{\text{SC}})(1-C_{\text{C}} a_{\text{SC}})(1-r_{\text{SE}})P_0^S, \\ &P_{\text{in}}^A = \big((1-r_{\text{SM}})a_{\text{SW}} + (1-r_{\text{SM}})(1-a_{\text{SW}})a_{\text{O}_3} \\ &+ (1-r_{\text{SM}})(1-a_{\text{SW}})(1-a_{\text{O}_3})(1-C_{\text{C}} r_{\text{SC}})C_{\text{C}} a_{\text{SC}}\big)P_0^S \end{aligned}$$

$$R_{\text{in}}^E = f_A(1-r_{\text{LE}})R_0^A + C_C r_{\text{LC}}(1-r_{\text{LE}})R_0^E,$$
$$R_{\text{in}}^A = \big((1-C_{\text{C}}r_{\text{LC}})C_{\text{C}}a_{\text{LC}} + (1-C_{\text{C}}r_{\text{LC}})(1-C_{\text{C}}a_{\text{LC}})a_{\text{LW}}\big)R_0^E.$$

Then

$$\varepsilon_E\sigma T_E^4 = (1-r_{\text{SM}})(1-a_{\text{SW}})(1-a_{\text{O}_3})(1-C_{\text{C}}r_{\text{SC}})(1-C_{\text{C}}a_{\text{SC}})(1-r_{\text{SE}})P_0^S$$
$$+ f_A(1-r_{\text{LE}})\varepsilon_A\sigma T_A^4 + C_C r_{\text{LC}}(1-r_{\text{LE}})\varepsilon_E\sigma T_E^4 - (\alpha+\beta)(T_E-T_A),$$

and

$$\varepsilon_A\sigma T_A^4 = \big((1-r_{\text{SM}})a_{\text{SW}} + (1-r_{\text{SM}})(1-a_{\text{SW}})a_{\text{O}_3}$$
$$+ (1-r_{\text{SM}})(1-a_{\text{SW}})(1-a_{\text{O}_3})(1-C_{\text{C}}r_{\text{SC}})C_{\text{C}}a_{\text{SC}}\big)P_0^S$$
$$+ \big((1-C_{\text{C}}r_{\text{LC}})C_{\text{C}}a_{\text{LC}} + (1-C_{\text{C}}r_{\text{LC}})(1-C_{\text{C}}a_{\text{LC}})a_{\text{LW}}\big)\varepsilon_E\sigma T_E^4$$
$$+ (\alpha+\beta)(T_E-T_A),$$

System has solution

$$T_E = 14.9\ C^\circ, \qquad T_A = -23.5\ C^\circ,$$

found numerically by Newtons method.

Need to assume clouds and atmosphere has same temperature and emits radiation together. Compute all flows to double check at the end.