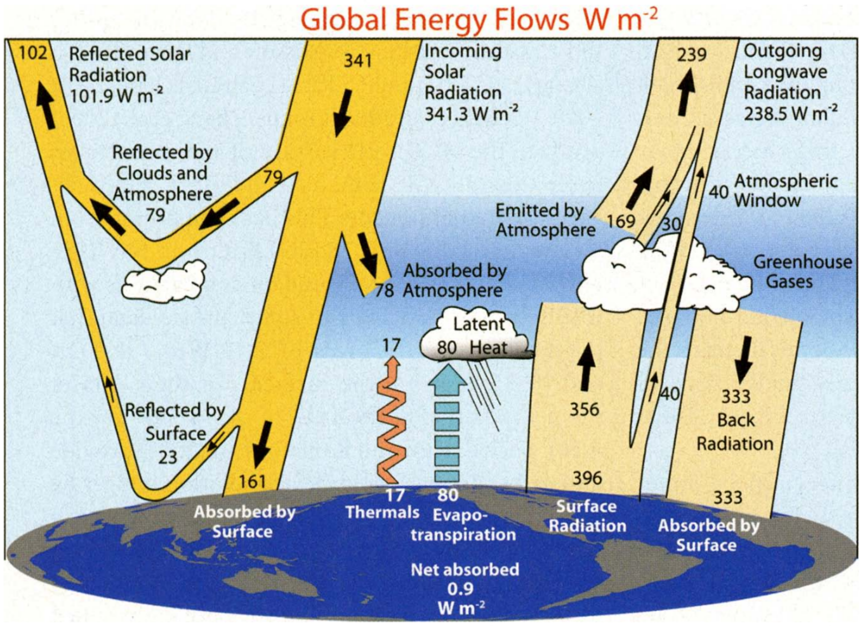
# Simplified Energy Transfer Model

Model by energy transfer coefficients to mimic




from [1].

By the Stefan–Boltzmann law, the total energy radiated from a body is given by $\varepsilon\sigma T^4$, where $\varepsilon$ is the emissivity of the body relative to a black body and $\sigma$ is the Stefan–Boltzmann constant. We assume energy-equilibrium in the earth-atmosphere system, so that the total energy radiated has to equal the incoming energy for both the earth and the atmosphere. This yields the system

$$\varepsilon_E \sigma T_E^4 = R_0^E \overset{\text{equil.}}{=} P_{\text{in}}^E + R_{\text{in}}^E - P_{\text{latent}} - P_{\text{E}\to\text{A}}, \tag{1}$$

$$\varepsilon_A \sigma T_A^4 = R_0^A \overset{\text{equil.}}{=} P_{\text{in}}^A + R_{\text{in}}^A + P_{\text{latent}} + P_{\text{E}\to\text{A}}. \tag{2}$$

where

$$P_{\text{latent}} + P_{\text{E}\to\text{E}} = (\alpha + \beta)(T_E - T_A).$$

To simplify the testing of different approaches and different transfer coefficients, we formulate a canonical model. Let

$$\begin{aligned} P_{\text{in}}^E &= \gamma_0 P_0^S, \\ P_{\text{in}}^A &= \gamma_1 P_0^S. \end{aligned}$$

and

$$\begin{aligned} R_{\text{in}}^E &= \lambda_0 R_0^E + \mu_0 R_0^A, \\ R_{\text{in}}^A &= \lambda_1 R_0^E + \mu_1 R_0^A, \end{aligned} \tag{3}$$

Equations (1) and (2) can be written as a system for the temperatures as follows.

$$a_0 T_E^4 + b_0 T_A^4 + c_0 T_E + d_0 T_A = e_0, \tag{4}$$

$$a_1 T_E^4 + b_1 T_A^4 + c_1 T_E + d_1 T_A = e_1. \tag{5}$$

In the above notation, the coefficients are given by

$$\begin{array}{cc} a_0 = \sigma\varepsilon_{\mathrm{E}}(1-\lambda_0) & a_1 = -\sigma\varepsilon_{\mathrm{E}}, \\ b_0 = -\sigma\varepsilon_{\mathrm{A}}\mu_0 & b_1 = \varepsilon_{\mathrm{A}}\sigma(1-\mu_1), \\ c_0 = \alpha+\beta & c_1 = -(\alpha+\beta), \\ d_0 = -(\alpha+\beta) & d_1 = \alpha+\beta, \\ e_0 = \gamma_0 P_0^S & e_1 = \gamma_1 P_0^S \end{array}$$

In order to solve this system we need to express the coefficients $\gamma_0$, $\gamma_1$, $\lambda_0$, $\lambda_1$, $\mu_0$, $\mu_1$ in terms of the transfer coefficients provided in the problem description.

We use the energy transfer approach. Ideally, we should be able to reproduce the empirical energy flows as given in Figure 1. in the problem description, and to find an equilibrium temperature for the earth that is close to some measured mean global surface temperature.

We assume that the atmosphere is a single layer. For incoming high-frequency solar radiation, the earth and the atmosphere absorbs energy according to

$$\begin{aligned} \gamma_0 =& (1-r_{\mathrm{SM}})(1-a_{\mathrm{SW}})(1-a_{\mathrm{O_3}})(1-C_{\mathrm{C}}r_{\mathrm{SC}})(1-C_{\mathrm{C}}a_{\mathrm{SC}})(1-r_{\mathrm{SE}}), \\ \gamma_1 =& \big((1-r_{\mathrm{SM}})a_{\mathrm{SW}} + (1-r_{\mathrm{SM}})(1-a_{\mathrm{SW}})a_{\mathrm{O_3}} \\ &+ (1-r_{\mathrm{SM}})(1-a_{\mathrm{SW}})(1-a_{\mathrm{O_3}})(1-C_{\mathrm{C}}r_{\mathrm{SC}})C_{\mathrm{C}}a_{\mathrm{SC}}\big). \end{aligned}$$

Using $P_0^S = 341\ Wm^{-2}$, we get absorbed high-frequency radiation equal to 156 $Wm^{-2}$ and 82 $Wm^{-2}$ for the earth and the atmosphere, respectively.

The real challenge lies in determining appropriate coefficients for the low-frequency dynamics (3). A given fraction of the atmosphere is covered by clouds, which absorb and reflect low-frequency radiation, but no emissivity is given in the problem description. Moreover, it is unclear exactly how to model clouds using transfer coefficients since we assumed the atmosphere to be a single, one-dimensional layer. It is evident that much of the uncertainty in modelling energy dynamics of the climate is connected to the role of clouds.

To overcome this difficulty, we propose new aggregate reflection and absorption coefficients for the fraction of the atmosphere where clouds are present. Assume for the moment that the atmosphere has a vertical height, and that the clouds are on average in the middle of the atmosphere. If we assume

$$\begin{aligned} a_{\mathrm{LAC}} =& \frac{1}{2}a_{\mathrm{LW}} + (1-\frac{1}{2}a_{\mathrm{LW}})(1-r_{\mathrm{LC}})a_{\mathrm{LC}} + (1-\frac{1}{2}a_{\mathrm{LW}})(1-r_{\mathrm{LC}})(1-a_{\mathrm{LC}})a_{\mathrm{LW}}, \\ r_{\mathrm{LAC}} =& (1-\frac{1}{2}a_{\mathrm{LW}})^2 r_{\mathrm{LC}}, \end{aligned}$$

then it follows that

$$\begin{aligned}
\lambda_0 =& C_C(1 - r_{\mathrm{LE}})r_{\mathrm{LAC}}, \\
\mu_0 =& f_A(1 - r_{\mathrm{LE}}), \\
\lambda_1 =& C_C(1 - r_{\mathrm{LAC}})a_{\mathrm{LAC}} + (1 - C_C)a_{\mathrm{LW}}, \\
\mu_1 =& 0.
\end{aligned}$$

This system has solution

$$T_E = 8.5\ C^\circ, \qquad T_A = -8.2\ C^\circ,$$

found numerically by Newtons method. The low-frequency flows at equilibrium are $R_{\mathrm{in}}^E = 318\ Wm^{-2}$ and $R_{\mathrm{in}}^A = 290\ Wm^{-2}$.

To find the sensitivity of the equilibrium temperature to changes in the transfer coefficients

## References


[1] Kevin E Trenberth, John T Fasullo, and Jeffrey Kiehl. *Earth's global energy budget*. Bulletin of the American Meteorological Society 90.3 (2009), pages 311–324.