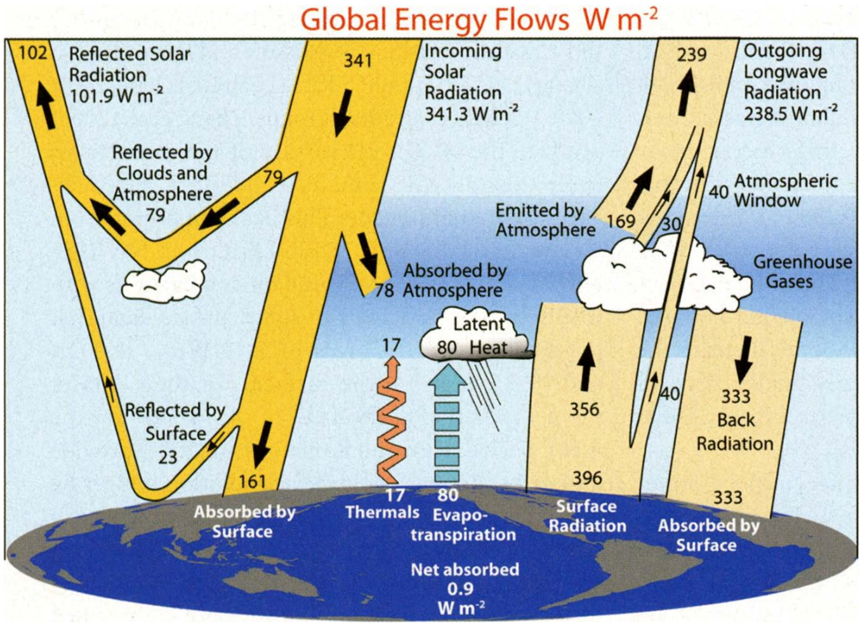
1 Simplified Energy Transfer Model

Model by energy transfer coefficients to mimic



from [1].

$$\varepsilon_E \sigma T_E^4 = R_0^E \overset{\text{equil.}}{=} P_{\text{in}}^E + R_{\text{in}}^E - P_{\text{latent}} - P_{\text{E}\to\text{A}}, \tag{1}$$

$$\varepsilon_A \sigma T_A^4 = R_0^A \overset{\text{equil.}}{=} P_{\text{in}}^A + R_{\text{in}}^A + P_{\text{latent}} + P_{\text{E}\to\text{A}}. \tag{2}$$

$$P_{\text{latent}} + P_{\text{E}\to\text{E}} = (\alpha + \beta)(T_E - T_A).$$

Let

$$P_{\text{in}}^E = \gamma_0 P_0^S,$$
$$P_{\text{in}}^A = \gamma_1 P_0^S.$$

and

$$R_{\text{in}}^E = \lambda_0 R_0^E + \mu_0 R_0^A,$$
$$R_{\text{in}}^A = \lambda_1 R_0^E + \mu_1 R_0^A,$$

We have a system on the form

$$a_0 T_E^4 + b_0 T_A^4 + c_0 T_E + d_0 T_A = e_0, \tag{3}$$

$$a_1 T_E^4 + b_1 T_A^4 + c_1 T_E + d_1 T_A = e_1, \tag{4}$$

where the coefficients are given by

$$
\begin{aligned}
a_0 &= \sigma\varepsilon_{\mathrm{E}}(1-\lambda_0),\\
b_0 &= -\sigma\varepsilon_{\mathrm{A}}\mu_0,\\
c_0 &= \alpha+\beta,\\
d_0 &= -(\alpha+\beta),\\
e_0 &= \gamma_0 P_0^S,
\end{aligned}
$$

and

$$
\begin{aligned}
a_1 &= -\sigma\varepsilon_{\mathrm{E}},\\
b_1 &= \varepsilon_{\mathrm{A}}\sigma(1-\mu_1),\\
c_1 &= -(\alpha+\beta),\\
d_1 &= \alpha+\beta,\\
e_1 &= \gamma_1 P_0^S.
\end{aligned}
$$

Coefficients are given by

$$\gamma_0 = (1-r_{\mathrm{SM}})(1-a_{\mathrm{SW}})(1-a_{\mathrm{O_3}})(1-C_{\mathrm{C}}r_{\mathrm{SC}})(1-C_{\mathrm{C}}a_{\mathrm{SC}})(1-r_{\mathrm{SE}})$$

$$
\begin{aligned}
&P_{\mathrm{in}}^E = (1-r_{\mathrm{SM}})(1-a_{\mathrm{SW}})(1-a_{\mathrm{O_3}})(1-C_{\mathrm{C}}r_{\mathrm{SC}})(1-C_{\mathrm{C}}a_{\mathrm{SC}})(1-r_{\mathrm{SE}})P_0^S,\\
&P_{\mathrm{in}}^A = \big((1-r_{\mathrm{SM}})a_{\mathrm{SW}} + (1-r_{\mathrm{SM}})(1-a_{\mathrm{SW}})a_{\mathrm{O_3}}\\
&+ (1-r_{\mathrm{SM}})(1-a_{\mathrm{SW}})(1-a_{\mathrm{O_3}})(1-C_{\mathrm{C}}r_{\mathrm{SC}})C_{\mathrm{C}}a_{\mathrm{SC}}\big)P_0^S
\end{aligned}
$$

$$
\begin{aligned}
R_{\mathrm{in}}^E &= f_A(1-r_{\mathrm{LE}})R_0^A + C_C r_{\mathrm{LC}}(1-r_{\mathrm{LE}})R_0^E,\\
R_{\mathrm{in}}^A &= \big((1-C_{\mathrm{C}}r_{\mathrm{LC}})C_{\mathrm{C}}a_{\mathrm{LC}} + (1-C_{\mathrm{C}}r_{\mathrm{LC}})(1-C_{\mathrm{C}}a_{\mathrm{LC}})a_{\mathrm{LW}}\big)R_0^E.
\end{aligned}
$$

Then

$$
\begin{aligned}
&\varepsilon_E\sigma T_E^4 = (1-r_{\mathrm{SM}})(1-a_{\mathrm{SW}})(1-a_{\mathrm{O_3}})(1-C_{\mathrm{C}}r_{\mathrm{SC}})(1-C_{\mathrm{C}}a_{\mathrm{SC}})(1-r_{\mathrm{SE}})P_0^S\\
&+ f_A(1-r_{\mathrm{LE}})\varepsilon_A\sigma T_A^4 + C_C r_{\mathrm{LC}}(1-r_{\mathrm{LE}})\varepsilon_E\sigma T_E^4 - (\alpha+\beta)(T_E-T_A),
\end{aligned}
$$

and

$$
\begin{aligned}
&\varepsilon_A\sigma T_A^4 = \big((1-r_{\mathrm{SM}})a_{\mathrm{SW}} + (1-r_{\mathrm{SM}})(1-a_{\mathrm{SW}})a_{\mathrm{O_3}}\\
&+ (1-r_{\mathrm{SM}})(1-a_{\mathrm{SW}})(1-a_{\mathrm{O_3}})(1-C_{\mathrm{C}}r_{\mathrm{SC}})C_{\mathrm{C}}a_{\mathrm{SC}}\big)P_0^S\\
&+ \big((1-C_{\mathrm{C}}r_{\mathrm{LC}})C_{\mathrm{C}}a_{\mathrm{LC}} + (1-C_{\mathrm{C}}r_{\mathrm{LC}})(1-C_{\mathrm{C}}a_{\mathrm{LC}})a_{\mathrm{LW}}\big)\varepsilon_E\sigma T_E^4\\
&+ (\alpha+\beta)(T_E-T_A),
\end{aligned}
$$

System has solution

$$T_E = 14.9\ C^\circ, \qquad T_A = -23.5\ C^\circ,$$

found numerically by Newtons method.

Need to assume clouds and atmosphere has same temperature and emits radiation together. Compute all flows to double check at the end.

References

[1] Kevin E Trenberth, John T Fasullo, and Jeffrey Kiehl. *Earth's global energy budget*. Bulletin of the American Meteorological Society 90.3 (2009), pages 311–324.