Oblig09 — Fys2160 — 2015

Exercise 0.14. Varied questions

a) Explain why the multiplicity of an Einstein-crystal with N oscillators and a total energy $E = q\Delta\varepsilon$ is

$$\Omega(q,N) = \binom{q+N-1}{q} \, . \tag{0.148}$$

b) What is an adiabatic process and what is an isothermal process? Sketch adiabatic and isothermal processes for an ideal gas in a p-V-diagram.

c) The heat capactiy of a crystal is $C_V = 3Nk$. Find an expression for the entropy, $S = S(N,T)$, as a function of temperature T.

Exercise 0.15. Spin-system in external magnetic field

In this project we will study the behavior of a spin-system in an external magnetic field. We will address the behavior of N spins that are localized to specific positions in space. Each spin, i, can be in two possible states, $S_i = \pm 1$. The energy of spin i is $\varepsilon_i = S_i mB$, where m and B are constants ($mB > 0$).

a) Find the partition function for a single spin.

b) Find an expression for the partition function for N spins. Explain your reasoning.

c) Show that Helmholtz free energy for the spin system is

$$F = -NkT \ln\cosh\left(\frac{mB}{kT}\right) - NkT\ln 2 \, , \tag{0.149}$$

where we remember that $2\cosh x = e^x + e^{-x}$ og $2\sinh x = e^x - e^{-x}$.

d) Find the entropy, $S(T,V,N)$, of the system.

e) Find the average value, $\overline{S}_i$, of S_i for spin i.

f) What happens to $\overline{S}_i$ when B is large? What happens with $\overline{S}_i$ when T is large?

g) We describe a state of a system with N spins spinn as $(S_1, S_2, \ldots, S_N)$. Are all such states equally probable? (You must provide an argument for your answer).

h) We want to simulate the spin system and we would like to use the simulation to estimate $\overline{S}_i(T)$. The simulations should produce a sequence of states $(S_1, S_2, \ldots, S_N)$ and use this sequence to find $\overline{S}_i$. Provide a sketch of such as program and how you can use the data generated to estimate $\overline{S}_i$. Comment on strengths and weaknesses with the method you use to generate the sequence.

i) We now want to study a spin-system with interactions between nearest-neighbor spins. The energy for such a system is

$$E(S_1, S_2, \ldots, S_N) = \sum_{i=1}^{N} S_i mB + \sum_{i=1}^{N-1} J S_i \cdot S_{i+1} \, , \tag{0.150}$$

where J is a given constant. Sketch a new algorithm to find $\overline{S}_i$ in this system.