LAB # 12

04/08/2025

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1 Introduction

This lab is centered around *Adaptive Quadrature*. Throughout this lab, we will be investigating the performance of adaptive quadrature and its relation to the underlying quadrature used in each sub-interval.

2 Pre-Lab

The Pre-Lab for this lab requires us to develop code for the following methods in *Python*.

1. **Composite trapezoidal on an interval:** Takes the inputs a, b, $f(x)$ and N number of points
2. **Composite Simpsons rule on an interval:** Takes the inputs a, b, $f(x)$ and N number of points

The code that is developed in python will be avalaible on the GitHub repository under `Lab_12`. In addition to the above, we are to watch the posted video on *Adaptive Quadrature* in preparation for lab.

3 Lab Day: Adaptive Quadrature

In this portion of the lab, we are asked to review the posted code, `adaptive_quad.py` as well as the Pre-Lab video. The provided code is an implementation of the methods described in the video which uses *Gaussian Quadrature* for each of the underlying sub-intervals. We are to review both and discuss the results with the TA.

4 Exercises

4.1 Different Adaptive Quadratures

In this exercise, we are to develop three different Adaptive Quadrature sub-routines where each uses a different Quadrature method on each sub-interval. We are asked to develop a routine that uses *Composite Trapezoidal*, *Composite Simpsons*, and *Gaussian Quadrature* on the sub-intervals. The first two methods will use the sub-routines from the Pre-Lab while the last is already implemented in the provided code, `adaptive_quad.py`. All of the produced code can be found in the GitHub repository in the `Lab_12` directory.

4.2

For this question, we will be approximating the following function in (1) using each of the methods from the last question. For the approximations, we are to let $n = 5$ denote the number of nodes within each sub-interval. For each approximation, we are to approximate to within 10^{-3}.

$$f(x) = \int_{0.1}^{2} \sin\left(\frac{1}{x}\right) dx \tag{1}$$

Using the code produced in the Pre-Lab and for the last question, the following plots were produced. The first figures use the *Gaussian Quadrature*.

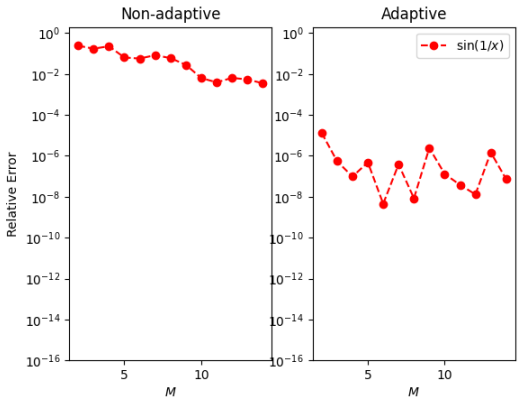

Figure 1: Gaussian Quadrature Error

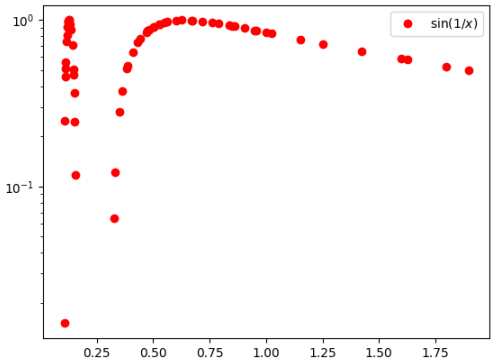

Figure 2: Gauss Quadrature

The following plots were created using the code for *Composite Trapezoidal* method.

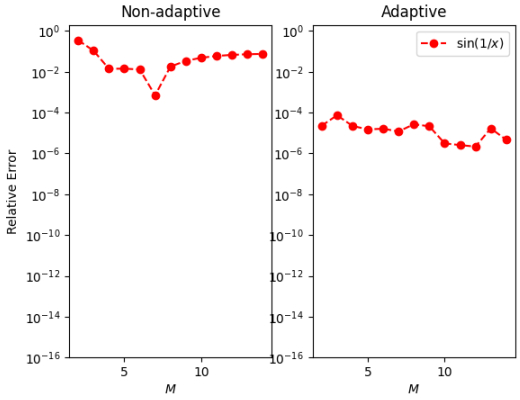

Figure 3: Composite Trapezoidal Error

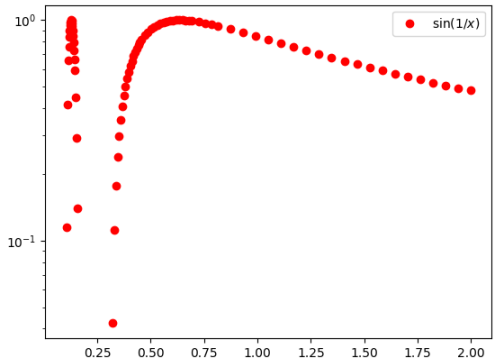

Figure 4: Composite Trapezoidal

The following plots were created using the code for the *Composite Simpsons* method.

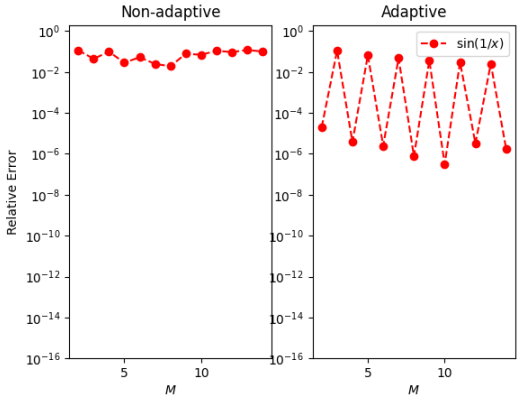

Figure 5: Composite Simpsons Error

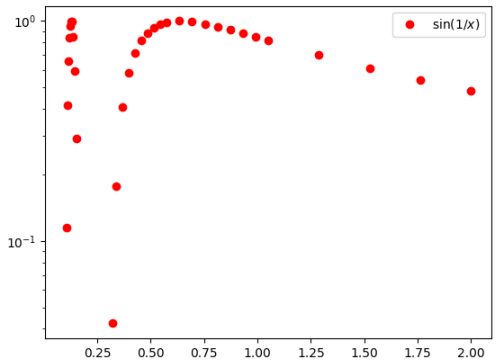

Figure 6: Composite Simpsons