Контрольная работа №2

Вариант 16.

№1

$$\sum_{n=1}^{\infty} \frac{2}{4n^2-9}$$

а) Интегральный признак

$$\int_1^{\infty} \frac{2dx}{4x^2-9} = \int_1^{\infty} \frac{1}{3(2x-3)}dx + \int_1^{\infty} \frac{-1}{3(2x-3)}dx = \frac{1}{6}\int_1^{\infty} \frac{d(2x-3)}{2x-3} -$$

$$- \frac{1}{6}\int_1^{\infty} \frac{d(2x+3)}{2x+3} = \lim_{n\to\infty}\left(\frac{1}{6}\ln(2n-3)\right)\Big|_1^n - \lim_{n\to\infty}\left(\frac{1}{6}\right.$$

$$\left.\cdot \ln(2n+3)\right)\Big|_1^n = \frac{1}{6}\lim_{n\to\infty}(\ln(2n-3)-\ln 1) - \frac{1}{6}\cdot$$

$$\cdot \lim_{n\to\infty}(\ln(2n+3)-\ln 1) = \frac{1}{6}\lim_{n\to\infty}(\ln(2n-3)) -$$

$$- \frac{1}{6}\lim_{n\to\infty}(\ln(2n+3)) = \frac{1}{6}\lim_{n\to\infty}\left(\ln(2n-3)-\ln(2n+3)\right) =$$

$$= \frac{1}{6}\lim_{n\to\infty}\left[\ln\left(\frac{2n-3}{2n+3}\right)\right] = 0$$

Данный ряд сходится

в) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ сходится $\Rightarrow \sum_{n=1}^{\infty} \frac{2}{n^2} = 2\sum_{n=1}^{\infty} \frac{1}{n^2}$ сходится

$$\lim_{n\to\infty} \frac{n^2}{4n^2+9} = \lim_{n\to\infty} \frac{1}{\frac{1}{n^2}(4n^2+9)} = \lim_{n\to\infty} \frac{1}{4+\frac{9}{n}} = \frac{1}{4}$$

По придельному признаку сравнения данный ряд сходится

с) Признак сравнения

$\sum_{n=1}^{\infty} \frac{2}{n^2}$ сходится

$$\frac{2}{4n^2+9} \leq \frac{2}{n^2} \Rightarrow \sum_{n=1}^{\infty} \frac{2}{4n^2+9} \text{ сходится.}$$

№2. $\sum_{n=1}^{\infty} (-1)^n \left(\frac{1}{(6n+1)(3n+1)}\right)^n$ – знакочередующийся ряд.

Признак Лейбница

$$a_1 = \frac{1}{28} \geq a_2 = \left(\frac{1}{91}\right)^2 \geq a_3 = \left(\frac{1}{190}\right)^3$$

$$\lim_{n\to\infty} \left(\frac{1}{(6n+1)(3n+1)}\right)^n = 0$$

Теперь признак Коши.

$$\lim_{n\to\infty} \sqrt[n]{\left(\frac{1}{(6n+1)(3n+1)}\right)^{n}} = \lim_{n\to\infty} \frac{1}{(6n+1)(3n+1)} = 0$$

Данный ряд абсолютно сходится.

№3. $\sum\limits_{n=1}^{\infty}\left(\frac{1}{\sin(1/n)\cdot n}-\cos\left(\frac{1}{n}\right)\right)\cdot\cos(\pi n)$

а) Признак Коши:

$$\lim_{n\to\infty}\sqrt[n]{\left(\frac{1}{n\sin(1/n)}-\cos(1/n)\right)\cdot\cos(\pi n)}=\lim_{n\to\infty}\sqrt[n]{\left(\frac{1}{n\cdot\frac{1}{n}}-\right.}$$

$$\overline{\left.-\left(1-\frac{(1/n)^2}{2}\right)\right)\cdot\left(1-\frac{(\pi n)^2}{2}\right)}=\lim_{n\to\infty}\sqrt[n]{\left(1-1+\frac{1}{2n^2}\right)\left(1-\frac{\pi^2n^2}{2}\right)}=$$

$$=\lim_{n\to\infty}\sqrt[n]{\frac{1}{2n^2}-\frac{\pi^2n^2}{4n^2}}=\lim_{n\to\infty}\left(\frac{\pi^2}{4}\right)^{\frac{1}{n}}=1 \text{ Неиз.}$$

б) Признак Даламбера.

$$\lim_{n\to\infty}\frac{\left(\frac{1}{\sin\left(\frac{1}{n+1}\right)(n+1)}-\cos\left(\frac{1}{n+1}\right)\right)\cos(\pi(n+1))}{\left(\frac{1}{\sin\left(\frac{1}{n}\right)\cdot n}-\cos\left(\frac{1}{n}\right)\right)\cos(\pi n)}=\lim_{n\to\infty}\frac{\left(\frac{n+1}{n+1}-\right.}{\left(\frac{n}{n}-\right.}$$

$$\frac{\left.-1+\frac{1}{(n+1)^2\cdot 2}\right)\cdot\cancel{\cos(\pi n+\pi)}\cdot\left(1-\frac{\pi^2(n+1)^2}{2}\right)}{\left.-1+\frac{1}{2n^2}\right)\;\cancel{\cos\pi}\left(1-\frac{(\pi n)^2}{2}\right)}=$$

$$=\lim_{n\to\infty}\frac{\left(1-1+\frac{1}{2\cdot(n+1)^2}\right)\left(1-\frac{\pi^2(n+1)^2}{2}\right)}{\left(1-1+\frac{1}{2n^2}\right)\left(1-\frac{\pi^2n^2}{2}\right)}=\lim_{n\to\infty}\frac{\frac{1}{2(n+1)^2}-}{\frac{1}{2n^2}-}$$

$$\frac{-\frac{\pi^2(n+1)^2}{4(n+1)^2}}{-\frac{\pi^2n^2}{4n^2}}=\frac{0-\frac{\pi^2}{4}}{0-\frac{\pi^2}{4}}=1 \text{ Неиз.}$$

№4. $f(x) = e^{-\frac{1}{4}x} \qquad x \in [-\pi; \pi]$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-\frac{1}{4}x}\,dx = -\left.\frac{4e^{-\frac{1}{4}x}}{\pi}\right|_{-\pi}^{\pi} = -\frac{4e^{-\frac{\pi}{4}}}{\pi} + $$

$$+\frac{4e^{\frac{\pi}{4}}}{\pi} = \frac{4e^{-\frac{\pi}{4}}\left(e^{\frac{\pi}{2}}-1\right)}{\pi}$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-\frac{1}{4}x}\cos(nx)\,dx = \left.\frac{e^{-\frac{1}{4}x}\left(n\sin(nx) - \frac{1}{4}\cos(nx)\right)}{\left(n^2+\frac{1}{16}\right)\pi}\right|_{-\pi}^{\pi} =$$

$$= \frac{e^{-\frac{\pi}{4}}\left(n\sin(n\pi) - \frac{1}{4}\cos(\pi n)\right) - e^{\frac{\pi}{4}}\left(n\sin(-n\pi) - \frac{1}{4}\cos(-n\pi)\right)}{\left(n^2+\frac{1}{16}\right)\pi} =$$

$$= \frac{\cancel{4}e^{-\frac{1}{4}\pi}\left(\left(e^{\frac{\pi}{2}}+1\right)n\cdot\sin(\pi n) + \left(\frac{1}{4}e^{\frac{\pi}{2}} - \frac{1}{4}\right)\cdot\cos(\pi n)\right)}{\left(n^2+\frac{1}{16}\right)\pi} =$$

$$= \frac{\cancel{4}\left(e^{\frac{\pi}{4}} - e^{-\frac{\pi}{4}}\right)(-1)^n}{\left(4n^2+\frac{1}{4}\right)\pi}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} e^{-\frac{\pi}{4}}\sin(nx)\,dx = -\left.\frac{e^{-\frac{\pi}{4}}\left(\frac{1}{4}\sin(nx) + n\cos(nx)\right)}{\left(n^2+\frac{1}{16}\right)\pi}\right|_{-\pi}^{\pi} =$$

$$= -\frac{e^{-\frac{\pi}{4}}\left(\left(\frac{1}{4}e^{\frac{\pi}{2}}+\frac{1}{4}\right)\sin(\pi n)+\left(1-e^{\frac{\pi}{2}}\right)n\cos(\pi n)\right)}{\left(n^2+\frac{1}{16}\right)\pi} =$$

$$= \frac{\left(e^{\frac{\pi}{4}}-e^{-\frac{\pi}{4}}\right)\cdot n(-1)^n}{\left(n^2+\frac{1}{16}\right)\pi}$$

$$f(x) \approx \frac{2e^{-\frac{\pi}{4}}\left(e^{\frac{\pi}{2}}-1\right)}{\pi} + \sum_{n=1}^{\infty}\left[\frac{\left(e^{\frac{\pi}{4}}-e^{-\frac{\pi}{4}}\right)(-1)^n\cos(nx)}{\left(4n^2+\frac{1}{4}\right)\pi} + \frac{\left(e^{\frac{\pi}{4}}-e^{-\frac{\pi}{4}}\right)\cdot n(-1)^n}{\left(n^2+\frac{1}{16}\right)\pi}\cdot\sin(nx)\right]$$

N5. $f(x) = 3x^3+4x^2+x \qquad x \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$

$$\cancel{a_0 = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(3x^3+4x^2+x)\,dx = \frac{1}{\pi}(9x^2+8x+1)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} =}$$

$$\cancel{= \frac{9\left(\frac{\pi}{2}\right)^2+8\left(\frac{\pi}{2}\right)+\frac{\pi}{2}}{\pi}}$$

$$a_0 = \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}(3x^3+4x^2+x)\,dx = \frac{2}{\pi}\left(\frac{3}{4}x^4+\frac{4}{3}x^3+\frac{x^2}{2}\right)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} =$$

$$= \frac{2}{\pi}\left(\frac{3}{4}\cdot\left(\frac{\pi}{2}\right)^4+\frac{4}{3}\left(\frac{\pi}{2}\right)^3+\frac{1}{2}\left(\frac{\pi}{2}\right)^2 - \frac{3}{4}\left(\frac{-\pi}{2}\right)^4 - \frac{4}{3}\left(\frac{-\pi}{2}\right)^3 - \frac{1}{2}\left(\frac{-\pi}{2}\right)^2\right) = \frac{2}{\pi}\cdot\frac{8}{3}\cdot\frac{\pi^3}{8} = \frac{2\pi^2}{3}$$

$$a_n = \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (3x^3+4x^2+x)\cdot\cos(nx)\,dx =$$

Интегрирование по частям: $\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$

$\cancel{u = 3x^3+4x^2+x \Rightarrow du = \left(\frac{3}{4}x^4+\frac{4}{3}x^3+\frac{1}{2}x^2\right)dx}$

$$dv = \cos(nx)dx \Rightarrow v = \int\cos(nx)dx = \frac{1}{n}\int\cos(nx)\,d(nx) =$$

$$= \frac{1}{n}\sin(nx)$$

$$u = 3x^3+4x^2+x \Rightarrow du = (9x^2+8x+1)dx$$

$$a_n = \frac{2}{\pi}\left[(3x^3+4x^2+x)\left(\frac{1}{n}\cdot\sin(nx)\right)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{n}\sin(nx)\cdot\right.$$

$$\left.\cdot(9x^2+8x+1)dx\right] = \frac{2}{\pi}\left[\underbrace{\left(\frac{3\pi^3}{8}+\frac{4\pi^2}{4}+\frac{\pi}{2}\right)\cdot\frac{1}{n}\cdot\sin\left(\frac{\pi n}{2}\right)}_{a} - \right.$$

$$-\underbrace{\left(\frac{-3\pi^3}{8}+\frac{4\pi^2}{4}+\frac{-\pi}{2}\right)\cdot\frac{1}{n}\sin\left(\frac{-\pi n}{2}\right)}_{b} - \frac{1}{n}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin(nx)\cdot$$

$$\left.\cdot(9x^2+8x+1)dx\right] = \frac{2}{\pi}\left[a-b-\frac{1}{n}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\sin(nx)\cdot\right.$$

$$\left.\cdot(9x^2+8x+1)dx\right] = \left|\begin{array}{l} u = 9x^2+8x+1 \Rightarrow du = (18x+8)dx \\ dv = \sin(nx)dx \Rightarrow v = \int\sin(nx)dx = \\ \quad = \frac{1}{n}\int\sin(nx)\,d(nx) = -\frac{1}{n}\cos(nx) \end{array}\right| =$$

$$= \frac{2}{\pi}\left[a-b-\frac{1}{n}\left(\left(-\frac{1}{n}\right)\cdot\cos(nx)\cdot(9x^2+8x+1)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -\frac{1}{n}\cos(nx)\cdot\right.\right.$$

$$\left.(18x+8)dx\right)\Big] = \frac{2}{\pi}\left[\frac{3\pi^3+8\pi^2+4\pi}{8n}\cdot\sin\left(\frac{\pi n}{2}\right)+\right.$$

$$+\underbrace{\frac{-3\pi^3+8\pi^2-4\pi}{8n}\sin\left(\frac{\pi n}{2}\right)}_{} + \frac{1}{n}\left(\cos\left(\frac{\pi n}{2}\right)\cdot\right.$$

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$$\left.\cdot\frac{9\pi^2+16\pi+4}{4n} - \cos\left(\frac{-\pi n}{2}\right)\frac{9\pi^2-16\pi+4}{4n}\right) +$$

$$\left.\frac{1}{n}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} -\frac{1}{n}\cos(nx)(18x+8)dx\right] = \frac{2}{\pi}\left[\frac{\cancel{16}\pi^2\cdot\sin\left(\frac{\pi n}{2}\right)}{8n}+\right.$$

$$\left.+\frac{32\pi\cdot\cos\left(\frac{\pi n}{2}\right)}{4n^2} - \frac{1}{n^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos(nx)\cdot(18x+8)dx\right] =$$

$$= \left| \begin{array}{l} u = 18x+8 \Rightarrow du = 18\,dx \\ dv = \cos(nx)dx \Rightarrow v = \int\cos(nx)dx = \frac{1}{n}\int\cos(nx)\,d(nx) = \frac{1}{n}\sin(nx) \end{array}\right| =$$

$$= \frac{2}{\pi}\left[\frac{2\pi\sin\left(\frac{\pi n}{2}\right)}{n} + \frac{8\pi\cos\left(\frac{\pi n}{2}\right)}{n^2} \cancel{+} \frac{1}{n^3}\sin(nx)\cdot\right.$$

$$\left.\cdot(18x+8)\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{1}{n^3}\sin(nx)\cdot 18\,dx\right] = \frac{2}{\pi}\left[\frac{2\pi n\sin\left(\frac{\pi n}{2}\right)+}{n^2}\right.$$

$$\left.\frac{+8\pi\cos\left(\frac{\pi n}{2}\right)}{} \cancel{+} \frac{18\pi\sin(nx)}{n^3}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} - \frac{18\cos(nx)}{n^4}\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\right] =$$

$$= \frac{2}{\pi}\left[\frac{4\left(\pi^2n^2\sin\left(\frac{\pi n}{2}\right) - 8\sin\left(\frac{\pi n}{2}\right) + 4\pi n\cos\left(\frac{\pi n}{2}\right)\right)}{\pi n^3}\right.$$

$$b_n = \frac{2}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (3x^3+4x^2+x)\cdot \sin(nx)\,dx = \frac{2}{\pi}\left[\frac{-3x^3\cos(nx) - 4x\cos(nx) - x\cos(nx)}{n} + \frac{9(x^2n^2\sin(nx)+2nx\cos(nx)-\sin(nx))}{n^4} + \frac{8(nx\sin(nx)+\cos(nx))}{n^3} + \frac{\sin(nx)}{n^2}\right]\Bigg|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} = \frac{4\sin\left(\frac{\pi n}{2}\right)}{\pi n^2} +$$

$$+ \frac{36\cos\left(\frac{\pi n}{2}\right)}{n^3} + \frac{9\pi\sin\left(\frac{\pi n}{2}\right)}{n^2} - \frac{3\pi^2\cdot\cos\left(\frac{\pi n}{2}\right)}{2n} - \frac{72\sin\left(\frac{\pi n}{2}\right)}{\pi n^4} - \frac{2\cos\left(\frac{\pi n}{2}\right)}{n}$$

$$f(x) \sim \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \left[\frac{4\left(\pi^2 n^2 \sin\left(\frac{\pi n}{2}\right) - 8 \sin\left(\frac{\pi n}{2}\right) + 4\pi n \cdot \cos\left(\frac{\pi n}{2}\right)\right.}{\pi n^3} \cdot \cos(nx) + \left(\frac{4 \sin\left(\frac{\pi n}{2}\right)}{\pi n^2} + \frac{36 \cos\left(\frac{\pi n}{2}\right)}{n^3} + \frac{9\pi \sin\left(\frac{\pi n}{2}\right)}{n^2} - \frac{3\pi^2 \cdot \cos\left(\frac{\pi n}{2}\right)}{2n} - \frac{72 \sin\left(\frac{\pi n}{2}\right)}{\pi n^4} - \frac{2\cos\left(\frac{\pi n}{2}\right)}{n} \right) \cdot \sin(nx) \right]$$