

Домашняя работа №2.

$$\text{№ 1. } f(x) = x_1^3 - x_1 x_2 + x_2^2 - 2x_1 + 3x_2 - 4 \rightarrow \min$$

$$x^0 = (0; 0)^T, \quad M = 10$$

$$\nabla f(x) = \begin{pmatrix} 3x_1^2 - x_2 - 2 \\ -x_1 + 2x_2 + 3 \end{pmatrix}$$

$$\varepsilon_1 = 0,41, \quad \varepsilon_2 = 2,592$$

МНГ

$$x_0 = (0; 0), \quad k = 0$$

$$\nabla f(x_0) = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$$

$$\|\nabla f(x_0)\| = \|(-2; 3)\| = \sqrt{4+9} = \sqrt{13} \approx 3,61 \nless \varepsilon_1 \Rightarrow k \nless M$$

$$\varphi(t_0) = f(x_0 - t_0 \nabla f(x_0)) \rightarrow \min_{t_0}$$

$$w = (0; 0) - t_0(-2; 3) = (2t_0; -3t_0) \Rightarrow \begin{cases} x_1 = 2t_0 \\ x_2 = -3t_0 \end{cases}$$

$$f(w) = (2t_0)^3 + 6t_0^2 + 9t_0^2 - 4t_0 - 9t_0 - 4 = 15t_0^2 - 13t_0 - 4 + 8t_0^3$$

$$\cancel{(f(w))'_{t_0} = 30t_0 - 13 = 0 \Rightarrow t_0 = 1/6} \quad (f(w))'_{t_0} = 24t_0^2 + 30t_0 - 13 \Rightarrow t_0 = \frac{30 \pm \sqrt{30^2 - 4 \cdot 24 \cdot (-13)}}{2 \cdot 24} = 1,591$$

$$x_1 = x_0 - t_0 \nabla f(x_0) = (0; 0) - \frac{1}{6} \cdot (-2; 3) = \left(\frac{1}{3}; -0,5\right)$$

Проверка.

$$\|x_1 - x_0\| < \varepsilon_2 \Rightarrow \left\| \left(\frac{1}{3}; -0,5\right) \right\| = \sqrt{1/9 + 0,25} = 0,601 < \varepsilon_2$$

$$|f(x_1) - f(x_0)| \leq \varepsilon_2 \Rightarrow \left| \left(\frac{1}{3}\right)^3 - \frac{1}{3} \cdot \left(-\frac{1}{2}\right) + \left(-\frac{1}{2}\right)^2 - 2 \cdot \frac{1}{3} + 3 \cdot \left(-\frac{1}{2}\right) - 4 + 4 \right| =$$

$$= 1,713 < \varepsilon_2 \Rightarrow x^* = \left(\frac{1}{3}; -0,5\right)$$

МГС

$$x_0 = (0; 0), \quad k = 0, \quad t_0 = 0,9$$

$$\nabla f(x_0) = \begin{pmatrix} -2 \\ 3 \end{pmatrix} \Rightarrow \|\nabla f(x_0)\| = 3,61 \nless \varepsilon_1 \Rightarrow k \nless M$$

$$x_1 = x_0 - 0,9 \cdot (-2; 3) = (1,8; -2,7)$$

$$f(x_1) - f(x_0) < 0 \Rightarrow 1,8^3 + 1,8 \cdot 2,7 + 2,7^2 - 2 \cdot 1,8 - 3 \cdot 2,7 - 4 + 4 = 6,282 < 0$$

$$t_0 = 0,45$$

$$x_1 = (0; 0) - 0,45 \cdot (-2; 3) = (0,9; -1,35)$$

$$f(x_1) - f(x_0) \leq 0 \Rightarrow 0,9^3 + 0,9 \cdot 1,35 + 1,35^2 - 2 \cdot 0,9 - 3 \cdot 1,35 - 4 + 4 = -2,084 < 0$$

Следующий шаг:

$$\|x_1 - x_0\| < \varepsilon_2 \Rightarrow \|(0,9; -1,35)\| = 1,623 < \varepsilon_2$$

$$|f(x_1) - f(x_0)| < \varepsilon_2 \Rightarrow |0,9^3 + 0,9 \cdot 1,35 + 1,35^2 - 2 \cdot 0,9 - 3 \cdot 1,35 - 4 + 4| = 2,084 < \varepsilon_2 \Rightarrow x^* = (0,9; -1,35)$$

2. $f(x) = (x_2^2 + x_1^2 - 1)^2 + (x_1 + x_2 - 1)^2 \rightarrow \min$ $x^0 = (0; 3)^T, x^1 = (3; 0)^T$

$$\nabla f(x) = \begin{pmatrix} 2(x_1^2 + x_2^2 - 1) \cdot 2x_1 + 2(x_1 + x_2 - 1) \\ 2(x_1^2 + x_2^2 - 1) \cdot 2x_2 + 2(x_1 + x_2 - 1) \end{pmatrix}$$

$\varepsilon_1 = 101,97$
 $\varepsilon_2 = 40,38$
 $M = 10$

1) $x_0 = (0; 3)$

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$$\nabla f(x_0) = \begin{pmatrix} 4 \\ 100 \end{pmatrix} \Rightarrow \|\nabla f(x_0)\| = \|(4; 100)\| = 100,08 < \varepsilon_1$$

$$x^* = (0; 3)$$

МГС

$$\|\nabla f(x_0)\| = 100,08 < \varepsilon_1$$

2) $x_0 = (3; 0)$

МНГС

$$\nabla f(x_0) = \begin{pmatrix} 100 \\ 4 \end{pmatrix} \Rightarrow \|\nabla f(x_0)\| = 100,08 < \varepsilon_1$$

МГС

$$\|\nabla f(x_0)\| = 100,08 < \varepsilon_1$$

$$x^* = (3; 0)$$

3. $f(x) = 4(x_1 - 5)^2 + (x_2 - 6)^2 \rightarrow \min$

$$x^0 = (8; 9)^T; \varepsilon_1 = 0,1;$$

$$\varepsilon_2 = 0,1; M = 10; t = 0,1$$

$$\nabla f(x) = \begin{pmatrix} 8(x_1 - 5) \\ 2(x_2 - 6) \end{pmatrix}$$

МНГС

$$k=0 \quad \nabla f(x_0) = \begin{pmatrix} 24 \\ 6 \end{pmatrix} \Rightarrow \|\nabla f(x_0)\| = \|(24; 6)\| = 6\sqrt{17} = 24,739 < \varepsilon_1$$

к.т.д.

$$\varphi(t_0) = f(x_0 - t_0 \cdot \nabla f(x_0)) \rightarrow \min$$

$$w = (8; 9) - t_0 \cdot (24; 6) = (8 - 24t_0; 9 - 6t_0) \Rightarrow \begin{cases} x_1 = 8 - 24t_0 \\ x_2 = 9 - 6t_0 \end{cases}$$

$$f(w) = 4(3 - 24t_0)^2 + (3 - 6t_0)^2$$

$$(f(w))'_{t_0} = -192(3 - 24t_0) - 12(3 - 6t_0) \Rightarrow t_0 = + \frac{612}{4680} = +0,131$$

$$x_1 = (8; 9) - 0,131 \cdot (24; 6) = (\cancel{11,144}; \cancel{9,786}) (4,856; 8,214)$$

$$\|x_1 - x_0\| < \varepsilon_2 \Rightarrow \|(3,144; 0,786)\| = 3,241 < \varepsilon_2 \Rightarrow$$

$$\Rightarrow k=1.$$

$$x_1 = (11,144; 9,786)$$

$$\nabla f(x_1) = \begin{pmatrix} 49,152 \\ 7,572 \end{pmatrix} \Rightarrow \|\nabla f(x_1)\| = \|(49,152; 7,572)\| = 49,732 < \varepsilon_2$$

$$w = (11,144; 9,786) - t_1 \cdot (49,152; 7,572) = (11,144 - 49,152 \cdot t_1; 9,786 - 7,572 \cdot t_1)$$

$$(f(w))'_{t_1} = 8 \cdot (-49,152) \cdot (-49,152 \cdot t_1 + 6,144) - 2 \cdot 7,572 \cdot (-7,572 \cdot t_1 + 3,786) = 0 \Rightarrow t_1 = 0,127$$

$$x_2 = (11,144; 9,786) - 0,127$$

$$\|x_1 - x_0\| < \varepsilon_2 \Rightarrow \|(4,856; 8,214) - (8; 9)\| = \|(-3,144; -0,786)\| = \sqrt{3,144^2 + 0,786^2} = 3,241 < \varepsilon_2$$

$$k=1, x_1 = (4,856; 8,214)$$

$$\nabla f(x_1) = \begin{pmatrix} -1,152 \\ 4,428 \end{pmatrix} \Rightarrow \|\nabla f(x_1)\| = \|(-1,152; 4,428)\| = 4,575 < \varepsilon_2$$

$$w = (4,856; 8,214) - t_1 \cdot (-1,152; 4,428) = (4,856 + 1,152 \cdot t_1; 8,214 - 4,428 \cdot t_1)$$

$$(f(w))'_{t_1} = 8 \cdot 1,152 \cdot (4,856 + 1,152 \cdot t_1 - 5) - 2 \cdot 4,428 (8,214 - 4,428 \cdot t_1 - 6) = 0 \Rightarrow t_1 = 0,420$$

$$x_2 = (4,856; 8,214) - 0,42 (-1,152; 4,428) = (5,34; 6,354)$$

$$\|x_2 - x_1\| < \varepsilon_2 = \|(0,484; -1,86)\| = 1,922 < \varepsilon_2$$

$$k=2, x_2 = (5,34; 6,354)$$

$$\nabla f(x_2) = \begin{pmatrix} 2,72 \\ 0,708 \end{pmatrix} \Rightarrow \|\nabla f(x_2)\| = 2,811 \notin \varepsilon_1$$

$$u = x_2 - t_2 \cdot \nabla f(x_2) = (5,34; 6,354) - t_2(2,72; 0,708) = (5,34 - 2,72 \cdot t_2; 6,354 - 0,708 \cdot t_2)$$

$$f(u) = 4(0,34 - 2,72 \cdot t_2)^2 + (0,354 - 0,708 \cdot t_2)^2$$

$$(f(u))'_{t_2} = -8 \cdot 2,72 \cdot (0,34 - 2,72 \cdot t_2) - 0,708 \cdot 2(0,354 - 0,708 \cdot t_2) = 0 \Rightarrow t_2 = 0,131$$

$$x_3 = x_2 - t_2 \nabla f(x_2) = (5,34; 6,354) - 0,131(2,72; 0,708) = (4,984; 6,261)$$

$$\|x_3 - x_2\| = \|(4,984; 6,261) - (5,34; 6,354)\| = \|(-0,356; -0,093)\| = 0,368 \notin \varepsilon_2$$

$$k=3, x_3 = (4,984; 6,261) \Rightarrow \nabla f(x_3) = \begin{pmatrix} -0,128 \\ 0,522 \end{pmatrix} \Rightarrow \|\nabla f(x_3)\| = 0,537 \notin \varepsilon_1$$

$$u = (4,984; 6,261) - t_3 \cdot (-0,128; 0,522) = (4,984 + 0,128 t_3; 6,261 - 0,522 t_3)$$

$$(f(u))'_{t_3} = 8 \cdot 0,128(0,128 t_3 - 0,016) - 2 \cdot 0,522(0,261 - 0,522 t_3) = 0 \Rightarrow$$

$$\Rightarrow t_3 = 0,427$$

$$x_4 = (4,984; 6,261) - 0,427 \cdot (-0,128; 0,522) = (5,039; 6,038)$$

$$\|x_4 - x_3\| = \|(0,055; -0,223)\| = 0,23 \notin \varepsilon_2$$

$$k=4, x_4 = \begin{pmatrix} 5,039 \\ 6,038 \end{pmatrix} \Rightarrow \nabla f(x_4) = \begin{pmatrix} 0,312 \\ 0,076 \end{pmatrix} \Rightarrow \|\nabla f(x_4)\| = 0,321 \notin \varepsilon_1$$

$$u = (5,039; 6,038) - t_4(0,312; 0,076) = (5,039 - 0,312 t_4; 6,038 - 0,076 t_4)$$

$$(f(u))'_{t_4} = -8 \cdot 0,312(0,039 - 0,312 t_4) + 2 \cdot 0,076(0,038 - 0,076 t_4) = 0 \Rightarrow$$

$$\Rightarrow t_4 = 0,119$$

$$x_5 = (5,039; 6,038) - 0,119(0,312; 0,076) = (5,002; 6,029)$$

$$\|x_5 - x_4\| = \|(-0,037; -0,009)\| = 0,038 < \varepsilon_2$$

$$|f(x_5) - f(x_4)| < \varepsilon_2 \Rightarrow 4 \cdot 0,002^2 + 0,029^2 = 857 \cdot 10^{-6} < \varepsilon_2$$

$$x^* = x_5 = (5,002; 6,029)$$

$$\|x - x_0\| = 100, 875 < \varepsilon_1 \Rightarrow x = (3, 0)$$

$$\text{3. } f(x) = 4(x_1 - 5)^2 + (x_2 - 6)^2 \rightarrow \min$$

$$x^0 = (8; 9)^T, \varepsilon_1 = 0,1; \varepsilon_2 = 0,1;$$

Программирование задачи:

$$M = 10; t_0 = 0,1.$$

$$\text{К } \nabla f(x_0) = \begin{pmatrix} 8(x_1 - 5) \\ 2(x_2 - 6) \end{pmatrix} \Rightarrow f(x_0) = \begin{pmatrix} 24 \\ 6 \end{pmatrix} \Rightarrow \|f(x_0)\| = \|(24; 6)\| = 24,739 < \varepsilon_1$$

К * M

$$x_1 = (8; 9) - 0,1(24; 6) = (5,6; 8,4)$$

$$f(x_1) - f(x_0) < 0 \Rightarrow 4 \cdot 0,6^2 + 2,4^2 - 4 \cdot 3^2 - 3^2 = -37,8 < 0.$$

$$\|x_1 - x_0\| < \varepsilon_2 \Rightarrow \|(-2,4; -0,6)\| = 2,474 < \varepsilon_2$$

$$k=1, x_1 = (5,6; 8,4)$$

$$\nabla f(x_1) = \begin{pmatrix} 4,8 \\ 4,8 \end{pmatrix} \Rightarrow \|f(x_1)\| = \|(4,8; 4,8)\| = 6,788 < \varepsilon_1$$

К * M

$$x_2 = (5,6; 8,4) - 0,1(4,8; 4,8) = (5,12; 7,92)$$

$$f(x_2) - f(x_1) < 0 \Rightarrow 4 \cdot 0,12^2 + 1,92^2 - 4 \cdot 0,6^2 - 2,4^2 = -3,456 < 0.$$

$$\|x_2 - x_1\| < \varepsilon_2 \Rightarrow \|(-0,48; -0,48)\| = 0,679 < \varepsilon_2$$

$$k=2, x_2 = (5,12; 7,92)$$

$$\nabla f(x_2) = \begin{pmatrix} 0,96 \\ 3,84 \end{pmatrix} \Rightarrow \|f(x_2)\| = \|(0,96; 3,84)\| = 3,958 < \varepsilon_1$$

К * M

$$x_3 = (5,12; 7,92) - 0,1(0,96; 3,84) = (5,024; 7,536)$$

$$f(x_3) - f(x_2) < 0 \Rightarrow 4 \cdot 0,024^2 + 1,536^2 - 4 \cdot 0,12^2 - 1,92^2 = -1,382 < 0$$

$$\|x_3 - x_2\| = \|(-0,096; -0,384)\| = 0,396 < \varepsilon_2$$

$$k=3, x_3 = (5,024; 7,536)$$

$$\nabla f(x_3) = \begin{pmatrix} 0,192 \\ 3,072 \end{pmatrix} \Rightarrow \|f(x_3)\| = \|(0,192; 3,072)\| = 3,078 < \varepsilon_1$$

К * M

$$x_4 = (5,024; 7,536) - 0,1(0,192; 3,072) = (5,005; 7,229)$$

$$f(x_4) - f(x_3) = 4 \cdot 0,005^2 + 1,229^2 - 4 \cdot 0,024^2 - 1,536^2 = 0,851 < 0$$

$$\|x_4 - x_3\| = \|(-0,019; -0,307)\| = 0,308 < \varepsilon_2$$

$$k=4, x_4 = (5,005; 7,229)$$

$$\nabla f(x_4) = \begin{pmatrix} 0,04 \\ 2,458 \end{pmatrix} \Rightarrow \|f(x_4)\| = \|(0,04; 2,458)\| = 2,458 \notin \varepsilon_1$$

$k \neq M$

$$x_5 = (5,005; 7,229) - 0,1 \cdot (0,04; 2,458) = (5,001; 6,983)$$

$$f(x_5) - f(x_4) = 4 \cdot 0,001^2 + 0,983^2 - 4 \cdot 0,005^2 - 1,229^2 = -0,544 < 0$$

$$\|x_5 - x_4\| = \|(5,001; 6,983) - (5,005; 7,229)\| = \|(-0,004; -0,246)\| = 0,246 \notin \varepsilon_2$$

$$k=5, x_5 = (5,001; 6,983)$$

$$\nabla f(x_5) = \begin{pmatrix} 0,008 \\ 1,966 \end{pmatrix} \Rightarrow \|f(x_5)\| = \|(0,008; 1,966)\| = 1,966 \notin \varepsilon_1$$

$k \neq M$

$$x_6 = (5,001; 6,983) - 0,1 \cdot (0,008; 1,966) =$$