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Домашняя работа №5.

№1. $\int_0^1 \dot{x}^2 dt \to extr \quad x(0)=1, \; x(1)=0.$

Экстремаль – ?

$\frac{\partial f}{\partial x} = 0 \quad \frac{df}{d\dot{x}} = 2\dot{x}$

$\frac{\partial f}{\partial x} - \frac{d}{dt} \cdot \frac{\partial f}{\partial \dot{x}} = 0 \Rightarrow 0 - 2\ddot{x} = 0 \Rightarrow \ddot{x} = 0 \Rightarrow \dot{x} = C_1 \Rightarrow$

$\Rightarrow x = C_1 t + C_2$

Подставляем и проверяем краевые условия.

$x(t) = x(0) = 1 \Rightarrow C_2 = 1.$

$x(t) = x(1) = 0 \Rightarrow C_1 = -1.$

$x^1 = -t + 1.$ – Экстремаль.

№2. $\int_0^{T_0} \dot{x}^2 dt \to extr \quad x(0)=0, \; x(T_0)=\xi$

$2\ddot{x} = 0 \Rightarrow x = C_1 t + C_2$

$\left.\begin{array}{l} x(t)=x(0)=0 \Rightarrow C_2 = 0 \\ x(t)=x(T_0)=\xi \Rightarrow C_1 = \frac{\xi}{T_0} \end{array}\right\} \Rightarrow x^1 = \frac{\xi}{T_0} \cdot t_1$ – Экстремаль.

№3. $\int_0^1 (x - \dot{x}^2) dt \to extr. \quad x(0) = x(1) = 0.$

$\frac{\partial f}{\partial x} - \frac{d}{dt} \cdot \frac{\partial f}{\partial \dot{x}} = 0 \Rightarrow 1 + 2\ddot{x} = 0 \Rightarrow \ddot{x} = -0{,}5 \Rightarrow \dot{x} = -0{,}5t + C_1$

$x = -0{,}25t^2 + C_1 t + C_2$

$x(0) = 0 \Rightarrow C_2 = 0$

$x(1) = 0 \Rightarrow C_1 = 0{,}25.$

$x^1 = -0{,}25 t^2 + 0{,}25 \cdot t_1$ – Экстремаль.

№4 $\int_0^{T_0} (\dot{x}^2 - x) dt \to extr. \quad x(0)=0, \; x(T_0)=\xi$

$\frac{\partial f}{\partial x} - \frac{d}{dt} \cdot \frac{\partial f}{\partial \dot{x}} = 0 \Rightarrow -1 - 2\ddot{x} = 0 \Rightarrow \ddot{x} = -0{,}5 \Rightarrow x = -0{,}25 \cdot$

$\cdot t^2 + C_1 t + C_2$

$x(0)=0 \Rightarrow C_2=0$

$x(T_0)=\xi \Rightarrow C_1=\dfrac{\xi+0{,}25T_0^2}{T_0}$

$x^1=-0{,}25t^2+\dfrac{\xi+0{,}25T_0^2}{T_0}\cdot t$ — Экстремаль

№5. $\int_0^1(\dot{x}^2+tx)dt \to extr.\ x(0)=x(1)=0$

$\dfrac{\partial f}{\partial x}-\dfrac{d}{dt}\cdot\dfrac{\partial f}{\partial \dot{x}}=0 \Rightarrow t-2\ddot{x}=0 \Rightarrow \dot{x}=\dfrac{1}{4}\cdot t^2+C_1 \Rightarrow x=\dfrac{t^3}{12}+$

$+C_1t+C_2$

$x(0)=0 \Rightarrow C_2=0$

$x(1)=0 \Rightarrow C_1=-\dfrac{1}{12}$

$x^1=\dfrac{t^3}{12}-\dfrac{1}{12}t$ — Экстремаль

№6. $\int_0^1(t^2x-\dot{x}^2)dt \to extr.$ $x(0)=x(1)=0$

$\dfrac{\partial f}{\partial x}-\dfrac{d}{dt}\cdot\dfrac{\partial f}{\partial \dot{x}}=0 \Rightarrow t^2+2\ddot{x}=0 \Rightarrow \ddot{x}=-\dfrac{t^2}{2} \Rightarrow \dot{x}=-\dfrac{t^3}{6}+$

$+C_1 \Rightarrow x=-\dfrac{t^4}{24}+C_1t+C_2$

$x(0)=0 \Rightarrow C_2=0$

$x(1)=0 \Rightarrow C_1=1/24$

$x^1=-\dfrac{t^4}{24}+\dfrac{t}{24}$ — экстремаль

№7. $\int_0^{T_0}\dot{x}^3dt \to extr.$ $x(0)=0,\ x(T_0)=\xi$

$\dfrac{\partial f}{\partial x}-\dfrac{d}{dt}\cdot\dfrac{\partial f}{\partial \dot{x}}=0 \Rightarrow 0-3\ddot{x}^2=0 \Rightarrow x=C_1t+C_2$

$x(0)=0 \Rightarrow C_2=0$

$x(T_0)=\xi \Rightarrow C_1=\xi/T_0$

$x^1=\dfrac{\xi}{T_0}\cdot t$