

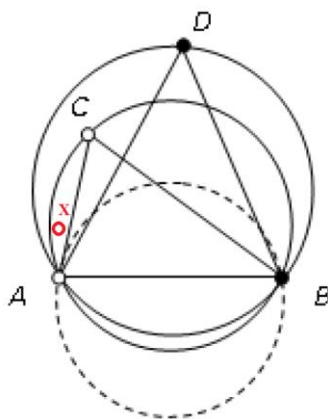
Paper: Natalia F. Dyshkant, Leonid M. Mestetskiy, Elena Tsarik, “Surface Comparison Based on $O(n)$ -Merging Overlapping Delaunay Triangulations”

The authors suggest a novel algorithm for combining Delaunay triangulations of two planar point sets, the convex hulls of which are allowed to overlap, and mention the problem of surface comparison as a potential practical application for their approach.

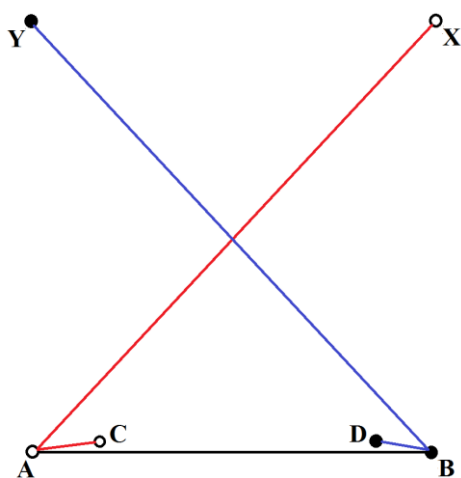
This manuscript summarizes results of a long-term thorough and dedicated research. However, there are a few issues to be addressed before this work may be considered for publication.

MAJOR ISSUES

There are two major flaws. The first one is in the proof of Lemma 2: page 10, lines 1-2, “To the left of AB , one of these circles is also empty. This circle corresponds to the triangle that has a larger angle opposite to AB ”. A counterexample is easy to obtain from Figure 6 by introducing one more white point lying slightly above A (point X in the below figure).



The second one is the assumption (underlying the reasoning in Section 7.2) that it is sufficient to test for correctness only the one-color edges adjacent to a newly formed two-color edge: the below figure provides an example when the two one-color edges adjacent to the two-color edge AB do not intersect, while the other two edges incident to A and B , respectively, do intersect.



The above-mentioned flaws violate the logic of the proposed method along with the statements about its correctness and time complexity (Section 7.2 and Theorem 4). In order to fix these problems, substantial changes should be introduced in the algorithm, and the paper will likely

become much different afterwards. At the same time, since in its present form, the algorithm is incorrect, I cannot recommend acceptance of the present paper even upon a major revision.

GENERAL COMMENTS

The title of the paper suggests its main focus is on surface comparison. However, this question is addressed only in the second-last section (Section 8, which immediately precedes the Conclusion) less than 1-page long, which contains no new results but rather briefly cites those presented in previous papers (by the same authors) published in Russian, and thus, not accessible for IJCGA readers. In reality, this section can be considered only as an attempt to give motivation for the present research (which apparently should be rather given in the Introduction), but at the same time, the description of earlier results is too superficial for such motivation to be convincing. Moreover, the term “interpolated” (line 9 from below) bears a non-standard meaning in this context, and should better be replaced with some other term (most likely, a new one should be introduced to this end).

An overall recommendation in this respect is thus to rename the paper so that the title would appropriately reflect what is discussed inside.

The figures mostly are hard to analyze. It might be a good idea to provide simpler figures illustrating individual steps of the method, and to turn the existing ones into color figures (from those in black-and-white).

Several times certain terms are introduced, or simply used, the meaning of which deviates from the standard one (improper, measure, meet, etc.). Widely applied mathematical terms and notions should be better not overloaded in this way.

A number of simple and widely known observations are stated as Lemmas followed with detailed proofs; at the same time, proofs of less obvious facts often lack some details (e.g. explicit references to the Lemmas proved earlier), which sometimes embarrasses reading of the manuscript.

Overall, grammatical mistakes and typos abound, and the manuscript needs expensive editing.

DETAILED COMMENTS

p 1., abstract, lines 8-9: “It is assumed that the given triangulations have overlapping convex hulls and their vertices (“sites”) are “well mixed”.

In this context, “well mixed” sounds like a term that can be formally defined, and this phrase gives an impression the vertices *must* possess this property for the proposed approach to work. However, this is not the case, and the word combination “well mixed” never appears in the text again. Therefore, it should be better removed from the abstract.

(At the same time, Section 3 contains a word combination “completely mixed” (paragraph 2, line 2); similar criticisms apply to it as well.)

p.1, abstract, lines 3-4 from below: “The proposed algorithm can be used for merging Voronoi diagrams of overlapping site sets”.

If such possibility is considered essential, it should be also explicitly discussed in the paper itself (so far, this is not mentioned in the main text at all).

p.2, Section 2, the beginning: what is the difference between “the grid” and “the nodes of the grids”? Here, these terms are used as interchangeable, which is inconsistent.

p. 7, line 6: “Seams are grey”. On page 5 (line 1), seams were defines as sets of edges, while here, they are discussed as planar regions. This issue arises several times later on.

p. 7:

The proposed algorithm is illustrated on a rather complex example, and the workflow description can be fully understood only upon reading of the entire paper (e. g. why the staters are selected in the suggested way is explained only in Section 6). It would make more sense to first explain the needed details, and provide an example afterwards.

p.7, last paragraph (step 8 of the algorithm): it should be formally justified that if the bridge list becomes empty, the work is over.

p. 8, proof of Lemma 1, line 1: it would be better to say “Consider an empty circle passing through the sites A and B...” rather than saying at the end of this paragraph “if the circle is empty...”.

p. 9, paragraphs 3 and 4: one of those must have been supposed to address “bunch overflow” (while now, the one on “bunch intersection” is duplicated twice).

p. 11, step (5) of the algorithm: there is no need in such a step, especially since each other one represents an action (“create”, “correct”, etc.). Step (5) is referred only at the end of the description of step (3); instead, it can be simply said the algorithm terminates at this point.

p. 12, proof of Lemma 4, last sentence: “an incident empty circle” does not satisfy the conditions of the Lemma, which require a *semi-empty* circle to exist (and by definition, a semi-empty circle is non-empty). This reasoning needs to be made consistent with the definition.

p. 12, last paragraph: the circle constructed as described may happen to be empty, and if so, will not satisfy conditions of Lemma 4. This needs to be stated more accurately.

p. 13, the paragraph before Lemma 5: why not defining “a circle of influence” for any edge of a DT (rather than for an EMST edge)?

p. 15, proof of Lemma 8: it looks like this proof was copied from some other place by mistake: neither the notation nor the content corresponds to the Lemma statement.

p. 16, paragraph 2, line 1 and step (i) of the algorithm: better to say explicitly we look for a face containing the free site rather than for “a face whose circumcircle contains the free site”, especially since this may be the unbounded face loosely referred to as “half-plane”.

p. 18, line 4 from below: the term “interface triangles” is unclear from the context – it should be explicitly defined if mentioned here.