Assignment # 1

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1.

Theorem. *For integers a and b, if a is even or b is even then ab is even.*

Proof. We prove the theorem by direct proof. Assume a is even and b is odd. Therefore there exists an integer l such that $a = 2l$ and there exists an integer j such that $b = 2j+1$. Substituting we get,

$$ab = (2l)(2j+1) = 4lj + 2l = 2(2lj + l).$$

Therefore since $k = 2(2lj+l)$ is an integer, ab is even.

□

2.

Theorem. *For integers a and b, if ab is even then a is even or b is even.*

Proof. We prove the theorem by contrapositive. Assume a is an odd integer and b is and odd integer. Therefore there exists an integer j such that $a = 2j+1$ and there exists an integer h such that $b = 2h+1$. Substituting we get,

$$ab = (2j+1)(2h+1) = (4jh + 2j + 2h) + 1.$$

Therefore since $k = (4jh)+2j+2h)+1$ is an integer, ab is odd. By contrapositve if ab is even, then a is even or b is even. □

3.

Theorem. *For real numbers x and y, if x is rational and y is rational then $x+y$ is rational.*

Proof. We prove the theorem by direct proof. Assume x and y are rational. Therefore there exists a rational number $\frac{a}{b}$ such that $x = \frac{a}{b}$ and there exists a rational number $\frac{c}{d}$ such that $y = \frac{c}{d}$. Substituting we get,

$$x + y = \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}.$$

Therefore since the sum of $ad + bc$ is a real number and the product of bd is a real number, $x + y$ is rational. $\square$

4.

Theorem. *If x is an integer then $x^2 \geq x$*

Proof. We proof the theorem by contradiction. Assume x is an integer and $x^2 \leq x$. Therefore there exists an integer 2 such that $x = 2$. Substituting we get,

$$4 \leq 2.$$

Since $4 \nleq 2$, this is a contradiction. Therefore if x is an integer, then $x^2 \geq x$. $\square$