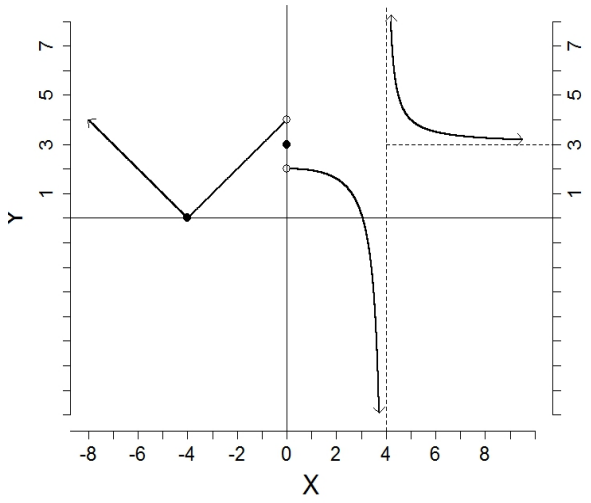
Math 151	Final Exam A	March 7, 2016

NAME:

Read the questions carefully and be sure to justify all your responeses.

1. **(2 points each)** Using the graph of f given below, answer the following questions.



(a) $\lim_{x \to 0^-} f(x) =$

(b) $\lim_{x \to 4^-} f(x) =$

(c) $\lim_{x \to \infty} f(x) =$

(d) Give a value of x where f is NOT differentiable.

(e) $f'(-2)$

(f) $f''(-2)$

2. **(3 points each)** Evaluate the following limits. Justify your responses.

(a) $\lim_{x\to\infty} \dfrac{3\ln x}{e^x} =$

(b) $\lim_{x\to 1} \dfrac{\ln x}{x^2-1} =$

3. **(5 points)** Let

$$f(x) = \frac{2}{x+1}$$

Compute $f'(2)$ using the **limit definition** of the derivative. Show all intermediate steps.

4. **(5 points)** Use implicit differentiation to find the equation of the tangent line to the curve $xy^3 + xy = 10$ at the point $(5, 1)$.

5. Consider the function $f(x) = \dfrac{1}{\sqrt{16 - x}}$.

   (a) **(4 points)** Find the linearization $L(x)$ of $f(x)$ at $x = 0$.

   (b) **(2 points)** Use the linearization $L(x)$ to approximate the value of $\dfrac{1}{\sqrt{16.1}}$.

6. For the function $f(x) = 2x^5 - 6x^4$, determine the following. Be sure to show your work.

(a) **(4 points)** Find the critical numbers of f.

(b) **(3 points)** Find the intervals of increase and decrease for f.

(c) **(2 points)** Use the First Derivative Test to find any local maximum(s)/minimum(s).

(d) **(4 points)** Find the intervals over which f is concave up and concave down.

(e) **(2 points)** Identify any inflection points of f.

7. **(4 points each)** Compute the derivative of the following functions. First, simplify if helpful and consider what differentiation technique is appropriate/required.

(a) $f(t) = te^{-t^5}$

(b) $g(t) = \ln\left(\sqrt{\dfrac{6t-9}{9t+8}}\right)$

(c) $f(x) = x^{6x}$

(d) $h(x) = \sin\left(e^{\cos x}\right)$

8. **(5 points each)** Solve TWO of the following three optimization problems. Show your work neatly!

   (a) Find two positive integers such that the sum of the first number and four times the second number is 1000 and the product of the numbers is as large as possible.

   (b) A rectangle has one side on the x-axis, one side on the y-axis, one corner at the origin and one on the curve $y = e^{-2x}$ for $x \geq 0$. Find the maximum area of the rectangle.

   (c) A square-bottomed box with no top has a fixed volume of 50. What are the dimensions of the box that minimize the surface area?