MATH 152

Midterm Exam

February 7, 2013

NAME (please print legibly): _____

Your University ID Number: _____

- **No calculators are allowed on this exam.**
- **Answers such as $\frac{23\cdot 5}{30} - \frac{2^5}{3\cdot 34}$ are perfectly fine!! However you MUST simplify expressions such as $\sin(\pi/3)$.**
- **Please show all your work. You may use back pages if necessary. You may not receive full credit for a correct answer if there is no work shown.**
- **Please include all information about u-substitutions, and use correct mathematical grammar in the presentation of your solution.**

Part A		
QUESTION	VALUE	SCORE
1	16	
2	16	
3	16	
4	16	
5	36	
TOTAL	100	

Part A

$\sin^2 x = \frac{1}{2}\big(1 - \cos(2x)\big)$	$\cos^2 x = \frac{1}{2}\big(1 + \cos(2x)\big)$	$\sin(2x) = 2\sin(x)\cos(x)$

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$$

$$\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$$

$$\sin(\alpha)\cos(\beta) = \frac{1}{2}\left[\sin(\alpha - \beta) + \sin(\alpha + \beta)\right]$$

$$\sin(\alpha)\sin(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) + \cos(\alpha + \beta)\right]$$

$$\cos(\alpha)\cos(\beta) = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$$

$t = \tan(\frac{x}{2}), \quad \sin(x) = \frac{2t}{t^2+1}, \quad \cos(x) = \frac{1-t^2}{1+t^2}, \quad dx = \frac{2}{1+t^2}dt$

Expression	Substitution	
$\sqrt{a^2 - x^2}$	$x = a\sin\theta,$	$-\pi/2 \le \theta \le \pi/2$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta,$	$-\pi/2 < \theta < \pi/2$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta,$	$0 \le \theta < \pi/2$ or $\pi \le \theta < 3\pi/2$

$$\int_{t_0}^{t_1} \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\,dt \qquad \int_{\alpha}^{\beta} \frac{1}{2}r^2 d\theta$$

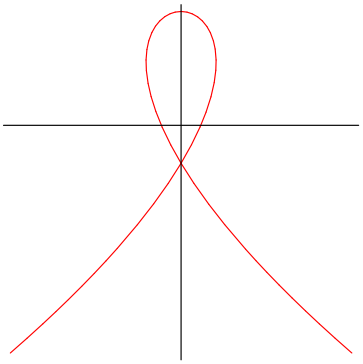
$$\sum_{i=1}^{n} a = a \cdot n \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

1. (16 pts) The graph of the parametric curve $\begin{cases} x &= -t^2+3 \\ y &= 4t-t^3 \end{cases}$ forms a loop.

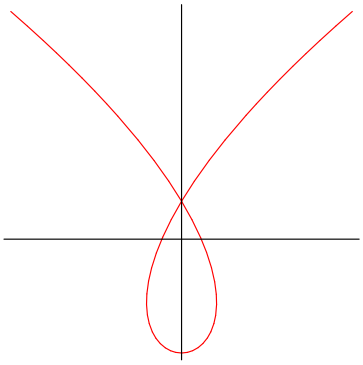
(1) Find $\frac{dy}{dx}$ for this parametric curve.

(2) Circle the correct graph of the loop from the four graphs below and use $\frac{dy}{dx}$ to **explain** why your answer is correct (or why the other answers are wrong).

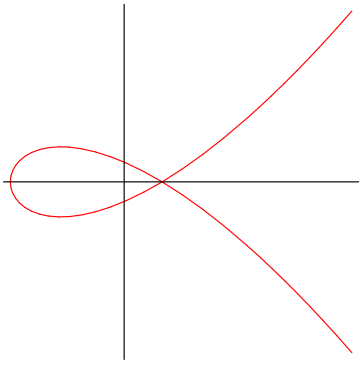
Explanation:



1



2



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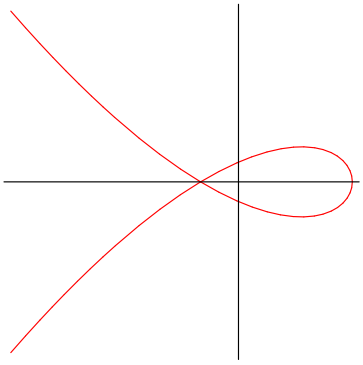


4

2. (16 pts) Do exactly ONE of the following two problems. Clearly mark the problem you do not want graded.

Problem 1. An object moves clockwise from the point $(-10, 0)$ around a circle of radius 10 centered at the origin and completes one revolution in the time interval $0 \leq t \leq 2\pi$ seconds. **Model the motion** of this object with a parametric equation. To earn partial credit, please show your work and explain your thinking.

Problem 2. An object is located at the point $P = (1, 2)$ at time $t = 1$ second, at the point $Q = (5, -4)$ at time $t = 3$ seconds, and moves along the line between these two points at a constant speed. **Model the motion** of this object with a parametric equation. To earn partial credit, please show your work and explain your thinking.

3. (16 pts) The definition of the definite integral for a continuous function $f(x)$ on the interval $[a, b]$ using right hand endpoints in the Riemann sum is given below.

$$\int_a^b f(x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

For each question below, put a check mark, ✓, next to **every correct/valid answer** to that question. Some questions may have more than one correct answer.

(a) What does Δx represent?

1. _____ The width $\Delta x = \frac{n}{b-a}$.
2. _____ The distance between x_i and x_{i+1}.
3. _____ The height of the function f at x_0.
4. _____ The width obtained when $[a, b]$ is cut up into n pieces of equal width.

(b) What does x_i represent?

1. _____ The number given by $x_i = a - i(\Delta x)$.
2. _____ All the numbers between a and b.
3. _____ The number given by $x_i = a + i(\Delta x)$.
4. _____ The x-value of the i^{th} tick mark if the interval $[a, b]$ is cut into n pieces of equal width.

(c) What is $f(x_i)$?

1. _____ Assuming $f(x) \geq 0$, $f(x_i)$ is the height of the rectangle that approximates the area of the strip under $f(x)$ over the interval $[x_{i-1}, x_i]$.
2. _____ For all values of i, $f(x_i) = f(i)$.
3. _____ Assuming $f(x) \geq 0$, $f(x_i)$ is the width of the rectangle that approximates the area of the strip under $f(x)$ over the interval $[x_{i-1}, x_i]$.

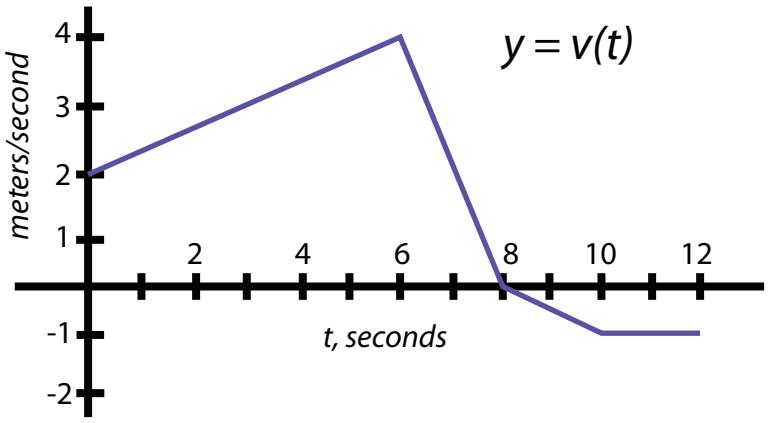
(d) What does the symbol $\sum_{i=1}^{n}$ mean in the definition of the integral?

1. _____ The notation $\sum_{i=1}^{n}$ means multiply over all n approximating rectangles.
2. _____ This notation represents that the rate of change in $f(x)$ has been divided up into n pieces.
3. _____ The notation $\sum_{i=1}^{n}$ means the summation from the first to the n^{th} approximating rectangles.

(e) What does $\lim_{n\to\infty}$ mean? Why is this part of the definition?

1. _____ The number of approximating rectangles increases to infinity.
2. _____ The quantity $\sum_{i=1}^{n} f(x_i)\Delta x$ will always increase to infinity as $n \to \infty$.
3. _____ The quantity $\sum_{i=1}^{n} f(x_i)\Delta x$, without the $\lim_{n\to\infty}$, is only an approximation to the net area under $f(x)$ and over $[a, b]$. Evaluating $\lim_{n\to\infty}$ makes the error in the approximation decrease to zero.

4. (16 pts) The velocity of an object at time t (in *seconds*) is given by the function $v(t)$ graphed below.



Find the **total distance traveled** by the object over the time interval $[0, 12]$ *seconds*. For full credit you must show your work and explain your answer.

5. (36 pts) Evaluate any **THREE** of the following four integrals. Do not do all four integrals. You will not receive extra credit for doing all four. You must show your work for full credit.

(a) $\displaystyle\int_2^5 \frac{2}{x^3} - \frac{1}{\sqrt[3]{x}} + \frac{1+e^x}{e^x}\, dx$

(b) $\displaystyle\int_0^{\pi/16} \cos^3(4x)\sin(4x)\, dx$

(c) $\displaystyle\int \frac{x^3}{\sqrt{x^2+1}}\, dx$

(d) $\displaystyle\int r^2 \ln(r)\, dr$