Using short-run restrictions for the effect of a monetary policy shock

Advanced Macroeconomics

Accademic Year 2021-2022

1 Research Question

In this projects I will make use of a structural Vector Autoregression model to consider the effect of a policy rate shock on output and inflation in Italy, considering observations from 1999 until 2021.

2 Importance of SVAR models

Structural vector autoregression (SVAR) models have become a widely used tool in the analysis of the monetary policy trasmission mechanism and source of business cycle fluctuations [1]. The implementation of this particular model is due to the fact that an issue in the estimation of a structural model based on dynamic simulataneous equation model is the identification of the empirical model [2]. SVAR methodology allows to avoid this issue by achieving identification while focusing on the role of shocks for the dynanimics of the model.

SVAR models are used in macroeconomics to study the two main policy considerations, that is, fiscal policy and monetary policy. In this project I consider only the latter case. Adding a structure to the restrictions which essentially creates an a-priori-timeline by which our model behaves, SVAR models prevents us from having erroneous conclusion on the effects of the monetary policy.

3 Data and Methodology

Our dataset contains quarterly italian data for the following variables: real GDP (in millions of chained 2010 Euro, seasonally adjusted, obtained from St. Louis Fed website using the mnemonics CLVMNACSCAB1GQIT), the output gap (in percentage, from the Istat website), the EONIA 1-day interbank interest rate for the Euro zone (in percentage, from the ECB webiste), the commodity price index as a measure for inflation (index 2015=100, averaged over quarters, not seasonally adjusted, obtained from St. Louis Fed website using the mnemonics ITACPIALLMINMEI), and its percentage change.

Our observations goes from the first quarter of 1999, until the last quarter of 2021, summing to 108 observations for each indicator. This is a period over which Italy experienced relatively stable economic growth.
In order to construct the SVAR model, three time series have been used: y taking logs of the output gap, π taking first differences and logs for the inflation, and r for the EONIA interest rate, that is computed as annualised interest rate. Moreover, our y and r time series looses the first observation, as inflation is expressed as the first difference of the consumer price index.
As we can see from the plot in Figure 1, the output remains constant over the whole span of observations, expect from the Financial Crisis of 2007-08 and the Sars COVID-19 outbreak in 2020.

I proceed to decompose the y time series into a deterministic trend (and creating a time series object for it) and a cyclical one. Then, I conducted a unit root test by applying the Augmented Dickey-Fuller test regressions to the series (selecting lags using the BIC criterion), after removing the linear time series from y. As the value of test statistic is $-6.329 < -1.95$, the results suggets that we are able to reject the null hypothesis of a unit root at a 5% significance level (Figure 2). In Figure 3 is reported the graph for the linear cycle in the output gap.

Then, I genereated a set of exogenous variables in order to manipulate these periods of sharp dips.

To estimate the model the standard form of the Cholesky decomposition is used on the matrix of contemporaneous parameters. According to the AIC the optimal lag number is $p = 3$, whereas the SC/BIC criterion indicates an optimal lag length of $p = 1$. Therefore, we should make use of a VAR(1) model (Figures 4, 5). We proceed to estimate the reduced-form VAR to obtain an object that is manipulated into the structural-form of the model. Using the $VAR()$ comand in R, we include both deterministic time trend and a constant, as well as the exogenous variable created before.

In order to set up the $A.mat$ matrix for the contemporaneous coefficients, we use a 3×3 matrix, with restrictions set as 0 and NA in all those places that would not pertain to a restriction (Figure 7). Restrictions on the $A.mat$ are imposed following an economic principle, that is, we will impose that if the EONIA interest rate r represents the policy rate, a reaction in it would be attributable to shocks in inflation π and in the output gap y in the same period. The matrix for the contemporaneous coefficients is therefore structured such that the policy rate r only affects inflation and output with a lag. This restriction suggests that a shock in output affects both the other considered variables in the same period and the lagged values in the system. Furthermore, inflation shocks affect only r contemporaneously and the lagged values in the system. We then set up the $B.mat$ matrix for the identification of individual shocks (Figure 8), which suggests that there will be no covariance terms for the residuals.

Then, I estimated the SVAR(1) model including both the matrices $A.mat$ and $B.mat$, setting the maximum number of iterations and populating values for the Hessian (Figure 9). The result highlighted in Figure 9 suggest that both the matrices have a correct functional form. Moreover, we estimated our structural parameteres to be 1.48275, -0.0049, and 0.00012

I proceed with the generation of the impulse response functions (IRFs), considering 12 quarters period ahead in the future. From Figure 10 it is noticeable that interest rate shocks are relatively persistent. Figure 11 shows the reaction on the output gap following an interest rate shock. After a slight increase, the output gap decreases, following the macroeconomic theory. Then lastly, in Figure 12 inflation increases in response to an interest rate shock and after hitting the peak response, decreases approaching zero in the medium-long run. For all of the cases considered above, the confidence intervals are relatively larger than zero, therefore the effects of the shock are not negligible.

These impulse response functions suggests that a contractionary monetary policy shock:

- increases the interest rate. This increase temporarily fades away from the system.
- has a persistent negative effect on the output gap, that is reached after a slight temporary increase.
- has a persistent positive effect on inflation, although it shows a decreasing trend in the long run.

All these effect results to be consistent with theory.

4 Theory background

Vector autoregression (VAR) is a statistical model used to capture the relationship between a set of k endogenous variables as they change over time. Moreover, VAR models generalize the univariate autoregressive model by allowing for multivariate time series .

A SVAR has the following form:

$$A\boldsymbol{y}_t = A_1\boldsymbol{y}_{t-1} + A_2\boldsymbol{y}_{t-2} + \cdots + A_p\boldsymbol{y}_{t-p} + B\varepsilon_t, \tag{4.1}$$

where A_i is a $k \times k$ matrix of structural constants for $i = 1, \ldots, p$, and ε_t is a $k \times 1$ vector of structural shocks, assumed to be white noises.
Focusing on the structural shocks, the ε_t vectors satisfies three main properties; primarily, every error term is a zero-mean component, that is, $\mathbb{E}(\varepsilon_t) = 0$; secondly, the covariance matrix of the error terms is a positive-semidefinte matrix: $\mathbb{E}(\varepsilon_t\varepsilon_t') = \Omega$; lastly, for any non-zero k, there is no correlation across time, that is,

$\mathbb{E}(\varepsilon_t \varepsilon_\tau') = 0,$ for $t \neq \tau$. Therefore, combining the two last properties, all the elements in the off diagonal of the Ω matrix are zero.

A SVAR model can be used to identify shocks and trace these out by employing Forecast Error Variance Decomposition (FEVD) through imposing restrictions on the matrices A and/or B. Depending on the imposed restrictions on the latter two matrices, three types of SVAR models can be distinguished:

- A model: B is set to I_K (minimum number of restriction for identification is $K(K-1)/2$).
- B model: A is set ot I_K (minimum number of restrictions to be imposed for identification is the same for the A model).
- AB model: restrictions can be placed on both matrices (minimum number of restrictions for identification is $K^2 + K(K-1)/2$).

Finally, the parameters are estimated by minimizing the negative concentrated log-likelihood function:

$$\ln L_C(A, B) = -\frac{KT}{2}\ln(2\pi) + \frac{T}{2}\ln|A|^2 - \frac{T}{2}\ln|B|^2 - \frac{T}{2}\mathrm{tr}(A^\top B^{-1\top} B^{-1} A\tilde{\Sigma}_u), \tag{4.2}$$

where $\tilde{\Sigma}_u$ represents an estimate of the reduced form variance/covariance matrix for the error process[3].

5 Results and Conclusions

By structuralizing our VAR model we were able to set conditions on the behavior of the considered variables. Allowing a shock in the policy rate to start the sequence, a decrease in the interest rates allows the consumers to achieve more borrowing power, increasing the consumption. This increase in consumption is followed by an increase in the output (in this model, the potential increase in output is represented by the upper bound confidence interval) in the short run. As theory suggests, the increasing output could lead to an increase in inflation, at which the Central Bank responds by increasing the interest rate and, therefore, lowering the consumers borrowing power.

References

[1] Olivier J. Blanchard, Danny Quah (1989) *The Dynamic Effects of Aggregate Demand and Supply Disturbances*, American Economic Association.

[2] Jon Faust (1998), *The robustness of identified VAR conclusions about money*, Carnegie-Rochester Conference Series on Public Policy, *volume 49.*

[3] Bernhard Pfaff (2008), *VAR, SVAR and SVEC Models: Implementation Within R Package vars.*, Jounral os Statistical Software.

6 Code

```
#Packages
library(lubridate)
library(urca)
library(vars)
library(mFilter)
library(tseries)
library(TSstudio)
library(forecast)
library(tidyverse)

#Initializing
setwd("~/Desktop/Project_MACRO/Italy")
rm(list = ls())

#Upload the datasets for Real GDP, Inflation (measured by CPI) and
#Short term real interest rate
data = read.table(file="~/Desktop/Project_MACRO/Italy/CSV/dataset.csv",
                  header = TRUE, sep=";")
data = subset(data, r != ".")
sapply(data,class)

#Convert the observation columns to "Date" type
mydate = as.Date(data$observation_date, format = c("%d/%m/%Y"))
data$observation_date = mydate
rm(mydate)

#Subsequent conversions of the type of data (from Char to Num)
gdp <- gsub(",",".",data$gdp)
double(length=8)
my = as.double(gdp)
data$gdp = my
rm(my,gdp)

out <- gsub(",",".",data$output_gap)
my = as.double(out)
data$output_gap = my
rm(my, myout, out)

r <- gsub(",",".",data$r)
my = as.double(r)
data$r = my
rm(my, r)

cp <- gsub(",",".",data$cpi_prct)
my = as.double(cp)
data$cpi_prct = my
rm(my, cp)

cpi <- gsub(",",".",data$cpi)
my = as.double(cpi)
data$cpi = my
rm(my, cpi)
data = as.data.frame(data)
str(data)
```

```r
#Create time series object from the main variables of interest
y = ts(log(pnorm(data$output_gap[-1], log = FALSE)), start=c(1999,1,1),
       freq=4)
pi = ts(diff(log(pnorm(data$cpi_prct, log = FALSE))) * 100, start = c(1999,1,1),
        freq = 4)
r.tmp = (1 + (data$r))*0.25 - 1 # Annualized interest rate
r = ts(r.tmp[-1], start = c(1999,1,1), freq = 4)
rm(r.tmp)
r
#Construct the dataset based on the time series objects
dataset = cbind(y, pi, r)
colnames(dataset) = c("y", "pi", "r")
plot.ts(dataset, main = "Dataset")

#Decompose the output gap time series in order to highlits trend and cycles
lin.mod = lm(y ~ time(y))
lin.trend = lin.mod$fitted.values
linear = ts(lin.trend, start = c(1999,1,1), freq = 4)
lin.cycle = y - linear

#Perform the Augmented Dickey-Fuller test for Unit roots,
#without including time trends nor constants
adf.lin = ur.df(lin.cycle, type = "none", selectlags = c("BIC"))
summary(adf.lin)

#Plot of the linear cycle in the y time series
par(mfrow = c(1,1), mar = c(2.2, 2.2, 1, 1), cex = 1)
plot.ts(lin.cycle, main = "Linear cycle in the Output Gap")

#Including exogenous variables for 2008 and 2020
dum08 = rep(0, length(y))
dum08[39] = 1
dum20 = rep(0, length(y))
dum20[85] = 1

#Creating time series objects from the exogenous variables
dum20 = ts(dum20, start = c(1999,1,1), freq = 4)
dum08 = ts(dum08, start = c(1999,1,1), freq = 4)
dum = cbind(dum20, dum08)
colnames(dum) = c("dum20", "dum08")

#Model selection via Cholesky Decomposition
info.var= VARselect(dataset, lag.max = 12, type = "both")
info.var$selection
info.var$criteria

#Use VAR(1), and print out summary of the VAR(1) estimation model
var.est1 = VAR(dataset, p=1, type = "both", season = NULL, exog = dum)
summary(var.est1)

#Set up the matrix for the contemporaneous coefficients
a.mat = diag(3)
diag(a.mat) = NA
a.mat[2, 1] = NA
a.mat[3, 1] = NA
```

```r
a.mat[3, 2] = NA
print(a.mat)

#Set up the matrix for identification of the individual shocks
b.mat = diag(3)
diag(b.mat) = NA

#Estimating the SVAR(2) model.
svar.one = SVAR(var.est1, Amat = a.mat, Bmat = b.mat, max.iter = 10000,
                hessian = TRUE)
svar.one

#IRF
one.int = irf(svar.one, response = "r", impulse = "r", n.ahead = 12,
              ortho = TRUE, boot = TRUE)
par(mfrow = c(1,1), mar = c(2.2, 2.2, 1, 1), cex = 0.6)
plot(one.int)

one.gdp = irf(svar.one, response = "y", impulse = "r", n.ahead = 12,
              ortho = TRUE, boot = TRUE)
par(mfrow = c(1,1), mar = c(2.2, 2.2, 1, 1), cex = 0.6)
plot(one.gdp)

one.inf <- irf(svar.one, response = "pi", impulse = "r", n.ahead = 12,
               ortho = TRUE, boot = TRUE)
par(mfrow = c(1, 1), mar = c(2.2, 2.2, 1, 1), cex = 0.6)
plot(one.inf)
```

7 Figures

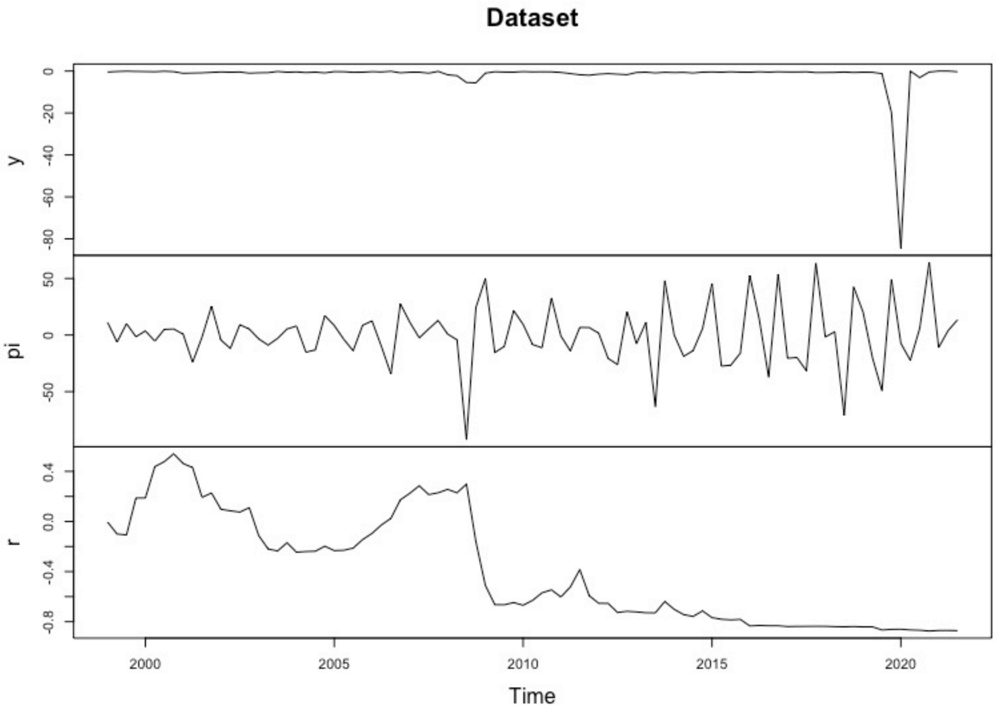

Figure 1: Plot of the constructed time series y, π, r.

```
##############################################
# Augmented Dickey-Fuller Test Unit Root Test #
##############################################

Test regression none


Call:
lm(formula = z.diff ~ z.lag.1 - 1 + z.diff.lag)

Residuals:
    Min      1Q  Median      3Q     Max
-77.212  -0.470   0.822   2.358  17.767

Coefficients:
           Estimate Std. Error t value Pr(>|t|)
z.lag.1    -0.87719    0.13860  -6.329 1.04e-08 ***
z.diff.lag  0.05331    0.10731   0.497    0.621
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.867 on 87 degrees of freedom
Multiple R-squared:  0.4173,    Adjusted R-squared:  0.4039
F-statistic: 31.15 on 2 and 87 DF,  p-value: 6.259e-11


Value of test-statistic is: -6.329

Critical values for test statistics:
     1pct  5pct 10pct
tau1 -2.6 -1.95 -1.61
```

Figure 2: Results of the Augmented Dickey-Fuller test for unit root in the Output Gap time series.

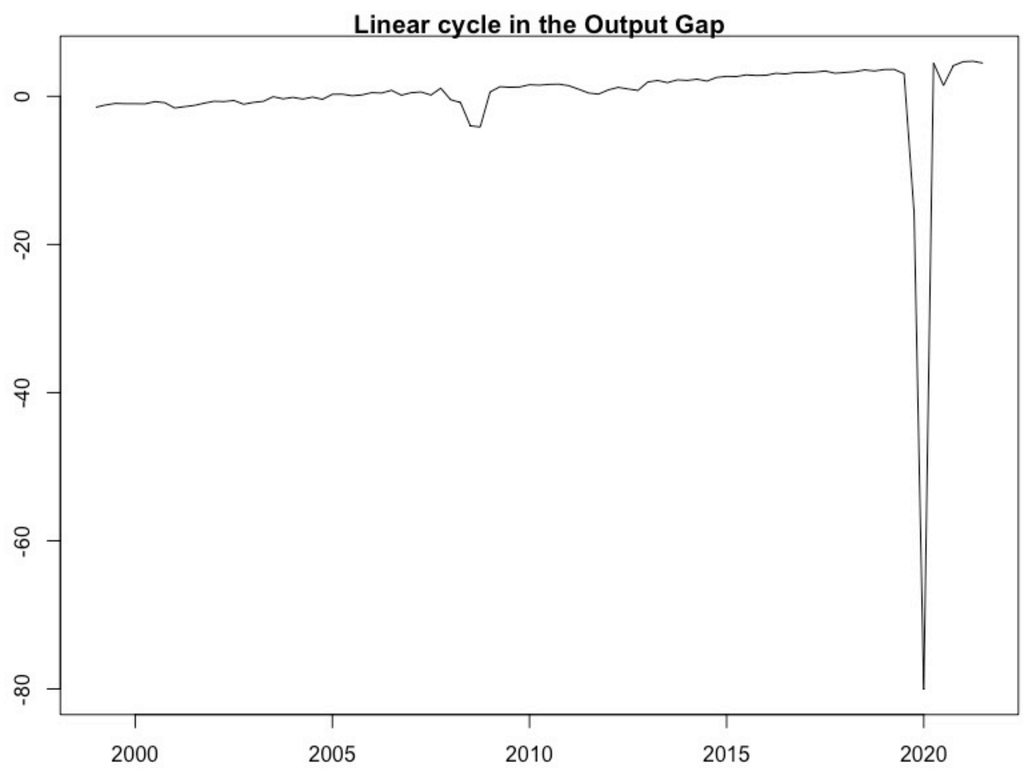

Figure 3: Linear cycle in the output gap trend. The output shocks are relative to the Financial crisis and the COVID-19 outbreak.

```
> info.var$selection
AIC(n)  HQ(n)  SC(n) FPE(n)
     3      3      1      3
```

Figure 4: Output of the optimal lag length for an unrestricted VAR with maximal length of 12.

```
> info.var$criteria
                 1          2          3          4          5          6          7          8
AIC(n)    6.434073   6.215345   6.073531   6.201733   6.315653   6.368367   6.496180   6.630710
HQ(n)     6.614315   6.503732   6.470063   6.706410   6.928475   7.089334   7.325292   7.567967
SC(n)     6.883968   6.935177   7.063300   7.461439   7.845296   8.167948   8.565698   8.970165
FPE(n) 623.021533 501.415449 436.589601 499.235623 564.621057 603.126640 697.992129 818.138332
                 9          10          11          12
AIC(n)    6.701664    6.869836    6.902025    6.709649
HQ(n)     7.747066    8.023383    8.163717    8.079487
SC(n)     9.311056    9.749165   10.051291   10.128852
FPE(n) 906.378342 1116.108742 1211.752978 1063.561838
```

Figure 5: Output of the optimal lag length for an unrestricted VAR with maximal length of 12.

```
> summary(var.est1)

VAR Estimation Results:
=========================
Endogenous variables: y, pi, r
Deterministic variables: both
Sample size: 90
Log Likelihood: -510.865
Roots of the characteristic polynomial:
0.8771 0.2854 0.008364
Call:
VAR(y = dataset, p = 1, type = "both", exogen = dum)


Estimation results for equation y:
==================================
y = y.l1 + pi.l1 + r.l1 + const + trend + dum20 + dum08

       Estimate Std. Error t value Pr(>|t|)
y.l1   -0.01873    0.02473  -0.757  0.45099
pi.l1   0.02503    0.00831   3.012  0.00344 **
r.l1   -1.56560    1.04513  -1.498  0.13793
const   0.17907    0.53973   0.332  0.74089
trend  -0.03765    0.01773  -2.124  0.03662 *
dum20 -84.42347    2.14945 -39.277  < 2e-16 ***
dum08  -3.71385    2.11057  -1.760  0.08215 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 2.03 on 83 degrees of freedom
Multiple R-Squared: 0.9533,     Adjusted R-squared: 0.9499
F-statistic: 282.3 on 6 and 83 DF,  p-value: < 2.2e-16


Estimation results for equation pi:
===================================
pi = y.l1 + pi.l1 + r.l1 + const + trend + dum20 + dum08

       Estimate Std. Error t value Pr(>|t|)
y.l1    0.28469    0.29556   0.963 0.338230
pi.l1  -0.25774    0.09932  -2.595 0.011179 *
r.l1   10.29192   12.49109   0.824 0.412335
const  -3.12999    6.45065  -0.485 0.628798
trend   0.19269    0.21184   0.910 0.365674
dum20   6.63558   25.68964   0.258 0.796817
dum08 -99.63697   25.22500  -3.950 0.000163 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 24.26 on 83 degrees of freedom
Multiple R-Squared: 0.2193,     Adjusted R-squared: 0.1629
F-statistic: 3.886 on 6 and 83 DF,  p-value: 0.001812


Estimation results for equation r:
==================================
r = y.l1 + pi.l1 + r.l1 + const + trend + dum20 + dum08

        Estimate Std. Error t value Pr(>|t|)
y.l1   0.0003335  0.0011699   0.285   0.7763
pi.l1  0.0001202  0.0003931   0.306   0.7606
r.l1   0.8765957  0.0494437  17.729   <2e-16 ***
const  0.0283743  0.0255337   1.111   0.2697
trend -0.0018704  0.0008385  -2.231   0.0284 *
dum20  0.0259557  0.1016876   0.255   0.7992
dum08  0.1433318  0.0998485   1.435   0.1549
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1


Residual standard error: 0.09604 on 83 degrees of freedom
Multiple R-Squared: 0.9561,     Adjusted R-squared: 0.9529
F-statistic:   301 on 6 and 83 DF,  p-value: < 2.2e-16


Covariance matrix of residuals:
          y        pi         r
y   4.12123  -6.11077  0.019250
pi -6.11077 588.69386 -0.099344
r   0.01925  -0.09934  0.009224

Correlation matrix of residuals:
          y       pi        r
y   1.00000 -0.12406  0.09873
pi -0.12406  1.00000 -0.04263
r   0.09873 -0.04263  1.00000
```

Figure 6: Summary output of the estimation of the VAR(2).

```
     [,1] [,2] [,3]
[1,]   NA    0    0
[2,]   NA   NA    0
[3,]   NA   NA   NA
```

Figure 7: Matrix for the contemporaneous coefficients.

```
     [,1] [,2] [,3]
[1,]   NA    0    0
[2,]    0   NA    0
[3,]    0    0   NA
```

Figure 8: Matrix for the identification of individual shocks.

```
> svar.one

SVAR Estimation Results:
========================


Estimated A matrix:
           y        pi r
y    1.00000 0.0000000 0
pi   1.48275 1.0000000 0
r   -0.00449 0.0001221 1

Estimated B matrix:
      y    pi       r
y  2.03  0.00 0.00000
pi 0.00 24.08 0.00000
r  0.00  0.00 0.09553
```

Figure 9: SVAR(2) model.

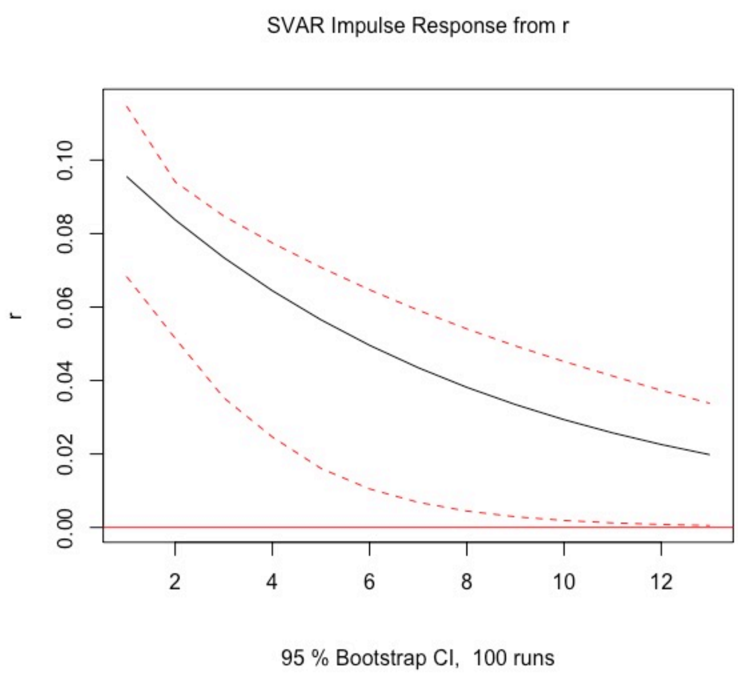

Figure 10: IRF considering the response of the key interest rate to an individual interest rate shock.

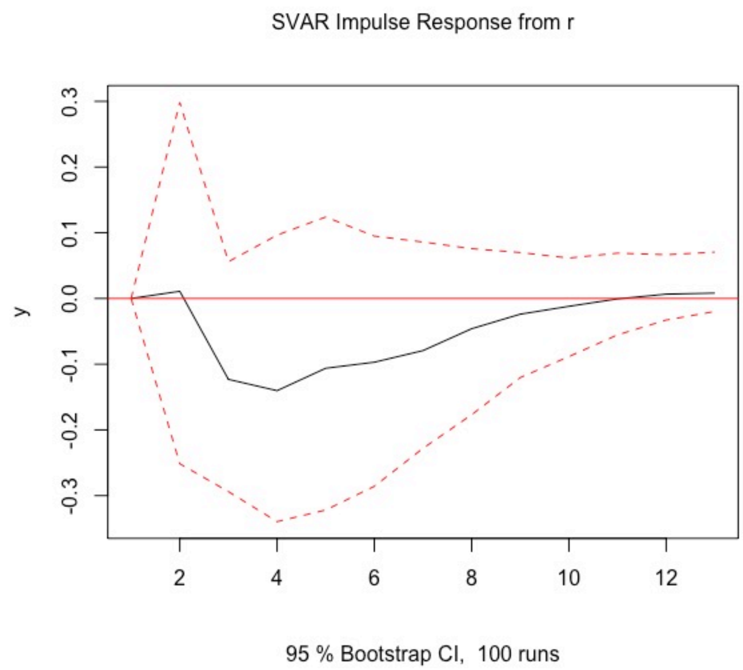

Figure 11: IRF considering the response of output interest rate shock.

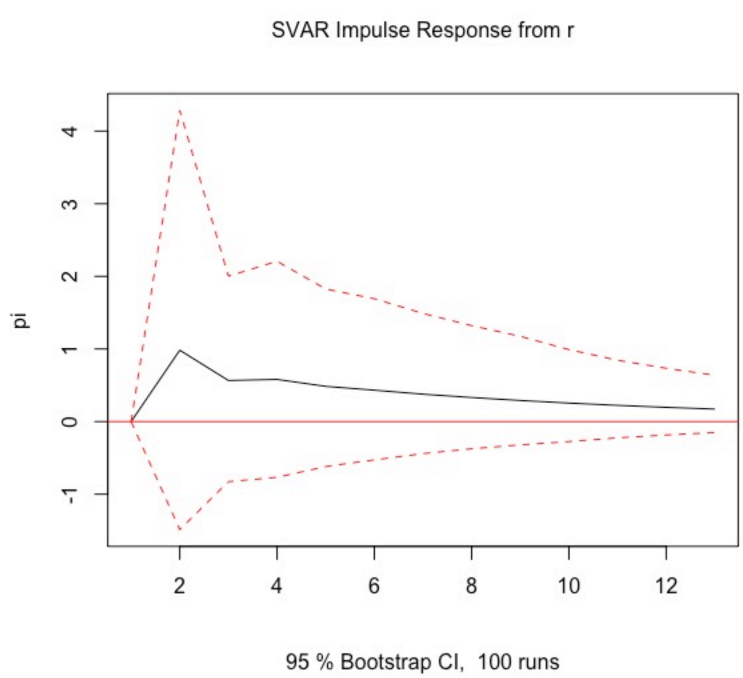

Figure 12: IRF considering the response of inflation to a key interest rate shock.