Department of Electrical and Computer Engineering

EEE 488 - Numerical Optimization Techniques and Computer Applications

Optimal Resource Allocation with Node and Link Capacity Constraints in Complex Networks

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1 Introduction

The project is intended to investigate Optimal Resource Allocation optimization problem using the approach described in [1]. In order to evaluate performance of proposed algorithm two different network models were used, *Barabsi-Albert* (BA) scale-free network model [2] and *Erdos-Renyi* (ER) random network [3]. Considered network consists of N nodes, L links and$S$ traffic flows. V = {1,2,...,N} denotes the set of nodes, E= {1,2,...,L} denotes the set of links and F = {1,2,...,S} denotes the set of flows. For flow s , V (s) represents the set of nodes flow s goes through, E(s) represents the set of links flow s passes. D_n is capacity of node n, C_l is capacity of link l and traffic flow s has flow rate x_s. Total of the whole network is fixed and it is represented by M, $\sum_n D_n = M$.

F(l) = $\{s \in F | l \in E(s)\}$ refers to the set of traffic flows passing link l. Similarly, F(n) = $\{s \in F | n \in V(s)\}$ refers to the set of traffic flows going though node n. Using all of those variables optimization problem from the next section can be constructed.

2 Methodology

2.1 Problem Formulation

The main optimization problem is defined by the following objective function and constraints:

$$\max_{x_s \in I_s, C_l \geq 0, D_n \geq 0} \quad \sum_s U_s(x_s) \tag{1a}$$

$$\text{subject to} \quad \sum_{s \in F(n)} x_s \leq D_n, n = 1, 2, ..., N \tag{1b}$$

$$\sum_{s \in F(l)} x_s \leq C_l, l = 1, 2, ..., L, \tag{1c}$$

$$\sum_n D_n == M. \tag{1d}$$

Capacity of link l_{mn}:, where m and n represent source and destination nodes, can be represented by the following expression:

$$C_{l_{mn}} = T_{mn}\alpha(D_m + D_n) \tag{2}$$

where T represents adjacency matrix of the network, in which $T_{mn} = T_{nm} = 1$ if nodes m and n are connected and $T_{mn} = T_{nm} = 0$ otherwise. α is proportional constant. Therefore constraint (1c) can be rewritten using equation (2) and our final optimization problem can be rewritten as:

$$\max_{x_s \in I_s, D_n \geq 0} \quad \sum_s U_s(x_s) \tag{3a}$$

$$\text{subject to} \quad \sum_{s \in F(n)} x_s \leq D_n, n = 1, 2, ..., N \tag{3b}$$

$$\sum_{s \in F(l_{mn})} x_s \leq T_{mn}\alpha(D_m + D_n), m, n = 1, 2, ..., N, \tag{3c}$$

$$\sum_n D_n == M. \tag{3d}$$

2.2 Algorithm

Since given optimization problem is convex, it can be solved using *CVX* package of *MATLAB* [4]. Firstly, we have defined our network models by using BA and ER models using *scalefree* and *randomGraph* functions respectively [5-6]. Functions *isSymmetric*, *selfLoops* and *numEdges* are all parts of ER network model (each network model creates matrix that corresponds to our T adjacency matrix, which is square matrix NxN). Shortest paths between nodes that have links were calculated and named as $shortest_{path}$. This values were used to calculate shortest path of each flow, where each edge has weight of 1. *V(s)*, *F(l)* and *F(n)* sets were created. Finally, *CVX* package were used, with objective function of (3a), decision variables x_s and D_n and constraints of (3b)-(3d). Simulation results of described algorithm are represented in the next section of the report.

3 Simulation Results

Simulation results were obtained using the parameters from [1], where N = 200, L = 600, S = 1000. Utility function of the network were taken as $U_s(x_s) = \frac{-1}{d_s^2 x_s}$, where d_s is a number of nodes flow s passes (we calculated it by finding the shortest path between two nodes).

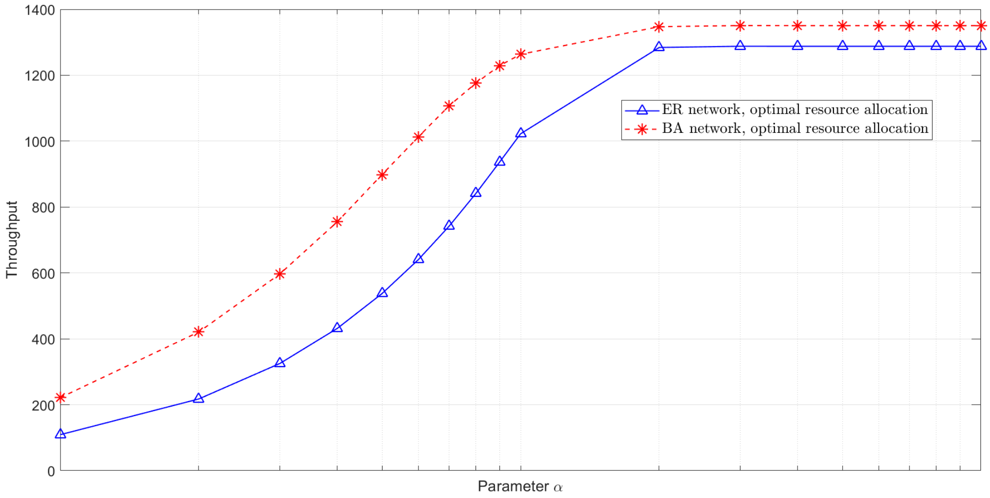

Figure 1: Relationship of throughput of the network and parameter α.

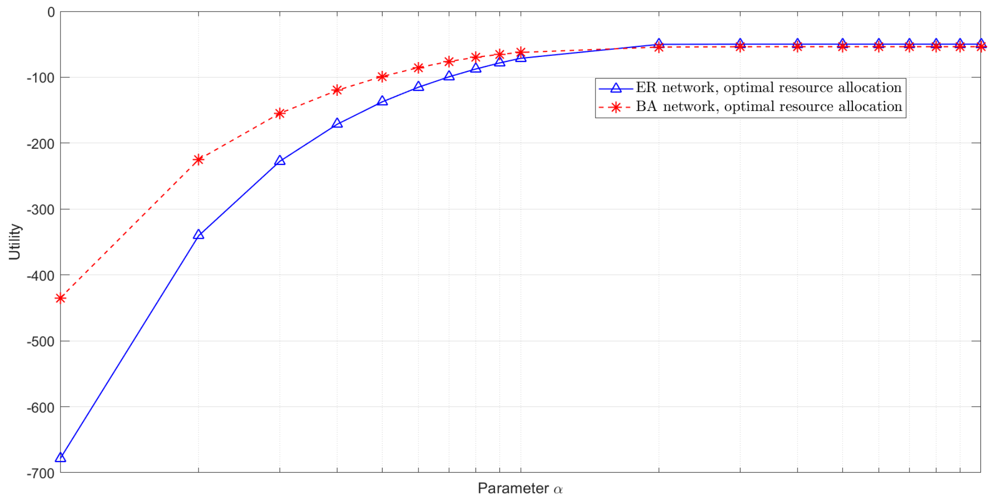

Figure 2: Relationship of Utility of the network and parameter α.

Figures 1 and 2 represent the relationship between the parametric value

α and throughput/Utility of the network. Total node capacity of the network were taken as M = 5000. As it can be observed from simulation results with increasing value of α both overall *throughput* and *utility* of the network increases and it converges when $\alpha \geq 0.2$, so it can be assumed that the optimal for α is 0.2.

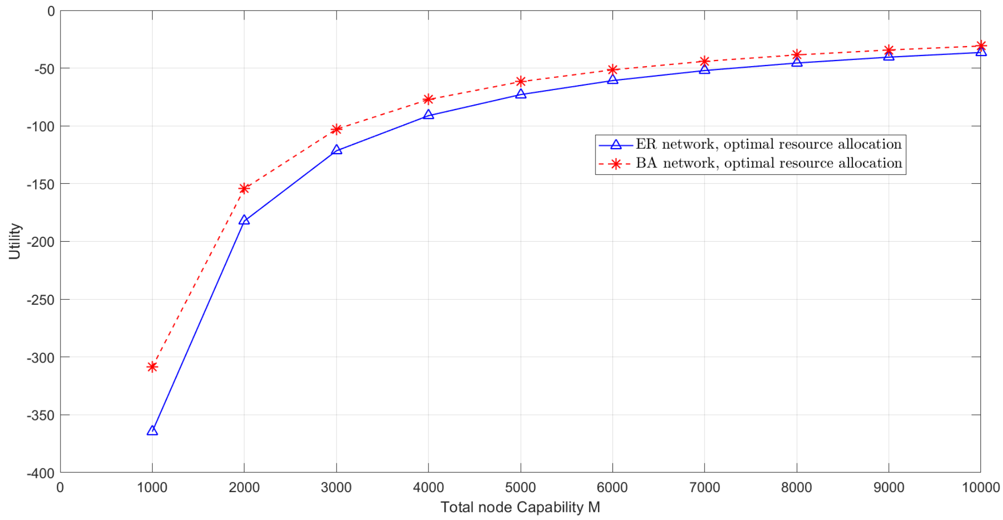

Figure 3: Relationship of Utility of the network and total node capacity M.

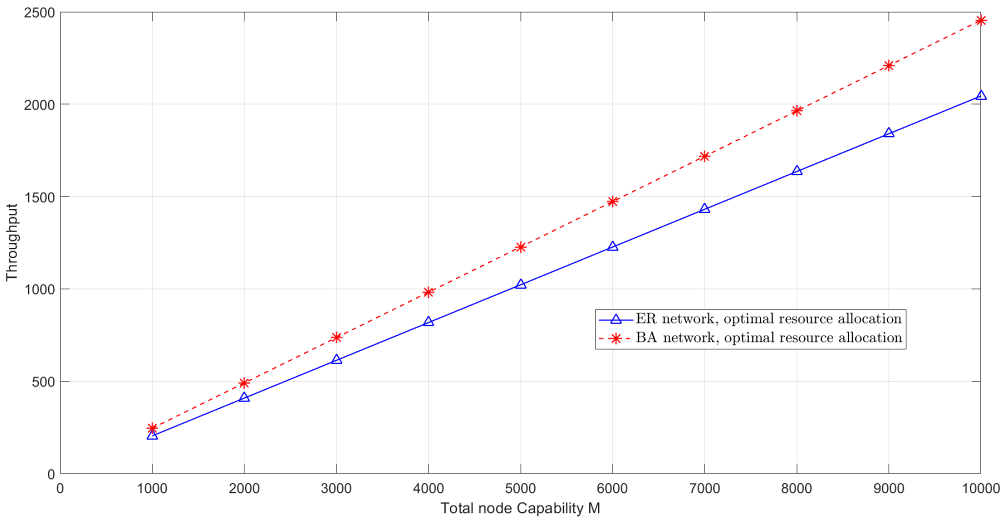

Figure 4: Relationship of throughput of the network and total node capacity M.

Figures 3 and 4 represent the relationship between the total node capacity and throughput/Utility of the network. The value of α were set to 0.2, since it is an optimal value. As it can be observed with increasing value of M both overall *throughput* and *utility* of the network increases and this values will keep increasing for larger M because according to our constraints the capacities of nodes and links will also increase. In both cases BA network model shows better performance.

4 Conclusion

In this project, we have solved optimization problem described in [1] using *CVX* tool. Since both node and link capacity constraints were considered, results mostly fit realistic conditions. According to simulation results, our implementation of the paper is successful, since our graphs match with graphs in [1]. All *MATLAB* codes are attached in .zip file, also read *README* file attached.

Bibliography

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