Classical string motion

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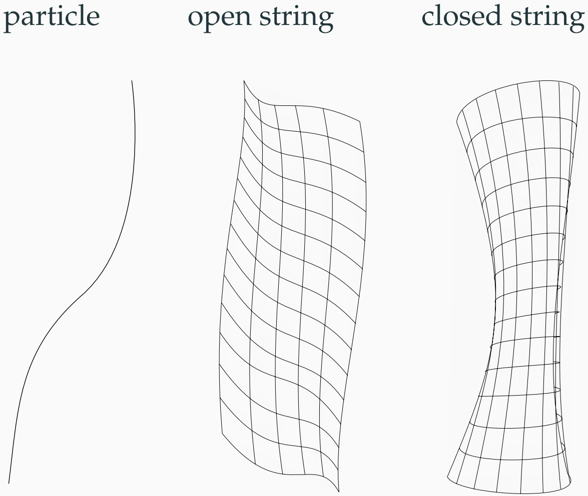
Objectives

- Find explicit solutions of classical strings and membranes in various spacetimes, for example:
  - expanding universe - de Sitter spacetime
  - gravitational wave background
  - interaction with gravitational wave pulse
  - in proximity of black hole or massive star

Strings vs particles



Nambu - Goto action for strings

- Action in terms of induced metric on string worldsheet:

$$S = -T \iint \mathrm{d}\sigma\, \mathrm{d}\tau\, \sqrt{-|\gamma|} = -T \iint \mathrm{d}\sigma\, \mathrm{d}\tau\, \sqrt{\gamma_{\sigma\tau}\gamma_{\tau\sigma} - \gamma_{\tau\tau}\gamma_{\sigma\sigma}}$$

  where τ and σ are parametric coordinates on worldsheet.

- γ in spacetime coordinates X:

$$\gamma_{\alpha\beta} = \partial_\alpha X^\mu\, \partial_\beta X^\nu\, g_{\mu\nu}$$

- Variation gives:

$$\delta S = T \iint \mathrm{d}\sigma\, \mathrm{d}\tau \Big[\partial_\alpha \Big(\sqrt{-|\gamma|}\, \gamma^{\alpha\beta} \partial_\beta X^\nu g_{\mu\nu} \Big) - \sqrt{-|\gamma|}\, \gamma^{\alpha\beta} \partial_\alpha X^\epsilon \partial_\beta X^\rho \partial_\mu g_{\epsilon\rho} \Big] \delta X^\mu$$

Gauge choice

- Choosing gauge / parametrisation to simplify equations of motion.
- Tricky because we can get trivial solutions.

Flat spacetime

- Minkowski metric - with the right gauge choice equations of motion become wave equation:

$$\frac{\mathrm{d}^2 X^\mu}{\mathrm{d}\tau^2} - \frac{\mathrm{d}^2 X^\mu}{\mathrm{d}\sigma^2} = 0$$

- Solution are of the form:

$$X^\mu = F^\mu(\sigma + \tau) + G^\mu(\sigma - \tau)$$

with parametrisation conditions:

$$\left(\frac{\mathrm{d}\vec{X}}{\mathrm{d}\sigma}\right)^2 + \left(\frac{\mathrm{d}\vec{X}}{\mathrm{d}\tau}\right)^2 = 1 \qquad \frac{\mathrm{d}\vec{X}}{\mathrm{d}\sigma} \cdot \frac{\mathrm{d}\vec{X}}{\mathrm{d}\tau} = 0$$

De Sitter spacetime

- Metric of the form:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + e^{2Ht}\left(\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right)$$

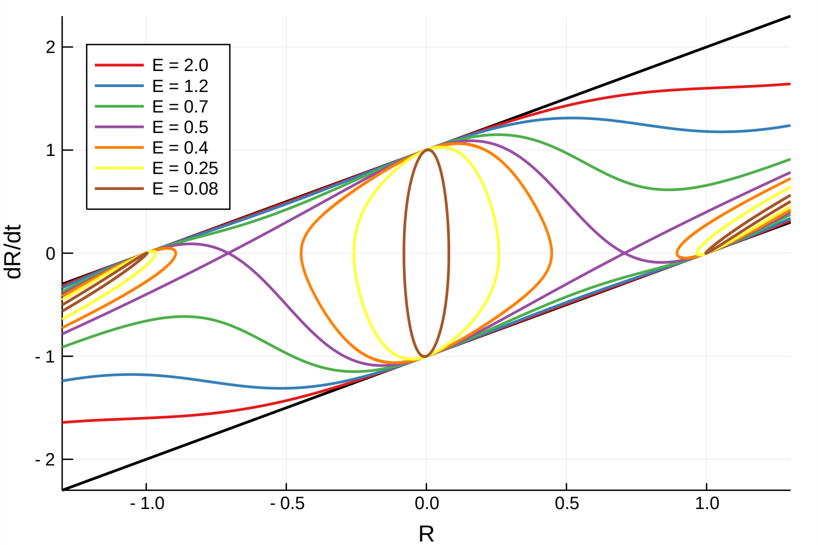
  where H is Hubble expansion rate.
- Equations of motion:

$$\frac{\mathrm{d}^2R}{\mathrm{d}t^2} = \frac{1}{R}\left[\left(\frac{\mathrm{d}R}{\mathrm{d}t}\right)^2 - 1 + 2HR\left(\frac{\mathrm{d}R}{\mathrm{d}t} - HR\right)^3 - 3HR\left(\frac{\mathrm{d}R}{\mathrm{d}t} - HR\right)\right]$$

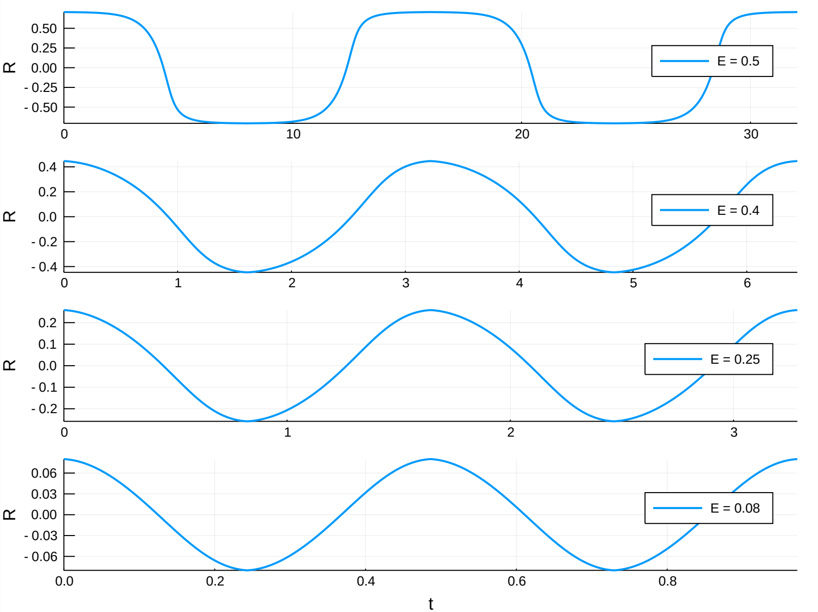
- Singularity at $R = 0$.
- In this case Hamiltonian is constant. We can get simpler solution:

$$\frac{\mathrm{d}R}{\mathrm{d}t} = \frac{R^5 - R^3 + E^2R \pm \sqrt{E^2R^4 + E^4 - E^2R^2}}{E^2 + R^4}$$

De Sitter background

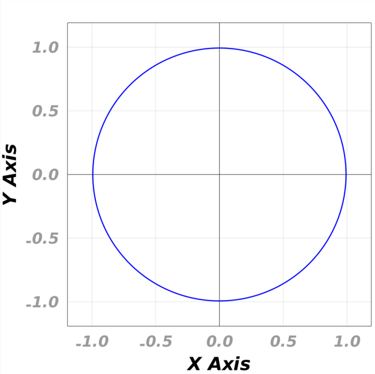


De Sitter background

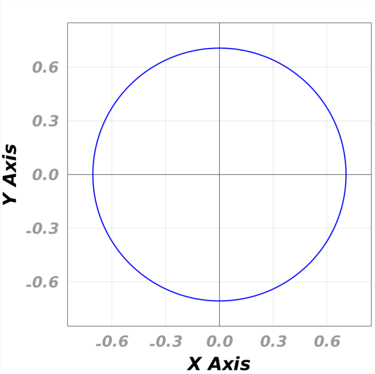


De Sitter background

flat spacetime



de Sitter spacetime



Gravitational wave background

- Metric of the form:

$$\mathrm{d}s^2 = H\,\mathrm{d}U^2 + 2\,\mathrm{d}U\,\mathrm{d}V + \mathrm{d}x^2 + \mathrm{d}y^2$$

  where $U = 1/\sqrt{2}(z-t)$ and $V = 1/\sqrt{2}(z+t)$
- $H = H(U, x, y)$ satisfies

$$\frac{\mathrm{d}^2H}{\mathrm{d}x^2} + \frac{\mathrm{d}^2H}{\mathrm{d}y^2} = 0$$

- Equations of motion:

$$\begin{aligned}\left(\partial_\tau^2 - \partial_\sigma^2\right) U &= 0\\ \left(\partial_\tau^2 - \partial_\sigma^2\right) V + \partial_U H/2\left[(\partial_\tau U)^2 - (\partial_\sigma U)^2\right]&\\ -\partial_i H\left[\partial_\tau U \partial_\tau X^i - \partial_\sigma U \partial_\sigma X^i\right] &= 0\\ \left(\partial_\tau^2 - \partial_\sigma^2\right) X_i - \partial_i H/2\left[(\partial_\tau U)^2 - (\partial_\sigma U)^2\right] &= 0\end{aligned}$$

- We found possible gauge $U = P^U\tau/T$.

Summary

- What I have so far:
    - Solution of string motion in flat spacetime.
    - Radial solution in de Sitter background.
    - Understanding of gauge choice and some tricks to simplify equations.
- In progress:
    - Motion in gravitational wave background.
    - Behavior after interacting with gravitational wave pulse.
    - Motion close to black hole or massive star.