

This supplementary numerical analysis aims to further illustrate the sensitivity of the proposed iterative algorithm with respect to the scalar parameters β and δ . To this end, simulations were conducted for 10 logarithmically spaced values of β in the interval $[1, 100]$ and 10 logarithmically spaced values of δ in the interval $[10^{-6}, 10^{-5}]$, resulting in a total of 100 parameter combinations. For each simulation scenario, the algorithm was executed for all considered cases (Cases I–III) and for each admissible filter order $n_f \in \{4, 3, 2, 1\}$, using the same convergence tolerance $\varepsilon = 1 \times 10^{-3}$ and maximum number of iterations $\kappa_{\max} = 100$.

During this extended analysis, it was observed that for some combinations of β and δ , the execution of Algorithm 1 may terminate with a numerical issue reported by the solver (Mosek status *UNKNOWN*), despite initialization from a feasible solution. These occurrences are attributed to numerical conditioning and solver-related limitations rather than to theoretical infeasibility of the proposed conditions. Statistical indicators are therefore computed considering only successful executions of Algorithm 1; parameter combinations leading to numerical solver issues are excluded from the computation of means and standard deviations and are instead explicitly identified through robustness plots.

The numerical results corresponding to successful executions are reported in Table 1, where, for each configuration, the guaranteed cost (γ^*) and the number of iterations (n_κ) are summarized using the format *mean (standard deviation)*. Although the number of iterations is an integer-valued quantity, its mean and standard deviation are reported as real-valued statistics, rounded to one decimal place. In addition, log–log plots of the (β, δ) parameter grid are provided in Figure 1, where circles indicate successful executions of Algorithm 1 and crosses denote parameter combinations for which the solver returned a numerical failure.

Table 1: Guaranteed cost (γ^*) and number of iterations (n_κ), reported as *mean (standard deviation)* over successful executions, for different values of β and δ , considering filter order $n_f \in \{4, 3, 2, 1\}$ and Cases I–III.

Filter Order	Case I		Case II		Case III	
	γ^*	n_κ	γ^*	n_κ	γ^*	n_κ
$n_f = 4$	7.2239 (0.0008)	5.5 (0.7)	7.1975 (0.0018)	6.8 (1.8)	7.1652 (0.0000)	1.0 (0.0)
$n_f = 3$	7.2254 (0.0003)	12.3 (1.2)	7.2002 (0.0004)	6.4 (1.6)	7.1652 (0.0000)	1.0 (0.0)
$n_f = 2$	7.2658 (0.0001)	2.2 (0.4)	7.2376 (0.0000)	4.0 (0.0)	7.1652 (0.0000)	1.0 (0.0)
$n_f = 1$	7.8071 (0.0000)	4.0 (0.0)	7.7203 (0.0000)	4.0 (0.0)	7.2673 (0.0000)	7.1 (0.3)

These results clarify the role of β and δ in the initialization step and their impact on the convergence behavior of Algorithm 1. The numerical tests carried out for different values of β and δ indicate that these parameters primarily affect the convergence speed: the number of iterations displays only moderate variability, whereas the final guaranteed cost exhibits very limited dispersion across all tested cases. This observation confirms that variations in β and δ mainly influence the convergence process rather than the attained performance level.

Figure 1: Log-log map of the (β, δ) parameter grid. Circles denote successful executions of Algorithm 1, while crosses indicate parameter combinations for which the solver returned a numerical failure (Mosek status *UNKNOWN*).

