Paper Review: A singular value thresholding algorithm for matrix completion

Brief Summary:

The authors build on previous work by Candès and Recht which aims at solving the NP-Hard Rank Minimization problem by reformulating it to a nuclear norm minimization problem through convex relaxation. To solve this problem, Candès and Recht make use of interior point-based methods. However, these solvers become computationally very expensive when the size of the matrix to be recovered is large since large systems of linear equations need to be solved to obtain an optimal solution. To solve this issue, the authors of this paper propose an iterative algorithm that performs singular value thresholding at each step and provides a low-rank optimal solution for the recovered matrix.

The proposed algorithm makes use of a fixed singular value soft-threshold value ($\tau > 0$) and a sequence of step-sizes ($\delta_k > 0$) to iteratively update the optimal matrix until a particular stopping condition is reached. The algorithm only requires the computation of an SVD and basic matrix operations at each step which makes the algorithm quick to implement, and due to the implementation of soft-thresholding at every step, the iterative solution is low-rank and thus requires minimal storage. The authors then reformulate their proposed Single Value Thresholding (SVT) algorithm as a specific case of the Lagrange Multiplier algorithm in order to extend their method for minimizing nuclear norms with convex constraints. The authors then move on to explore the application of the SVT algorithm with linear, quadratic and general conic convex constraints.

In section 4, the authors identify particular step-size constraints to obtain an optimal solution for the matrix completion scenario and extend the analysis to propose a theorem which allows convergence to a unique solution in the presence of general convex constraints while assuming strong duality. Furthermore, the authors discuss practical implementation details in terms of selection of the step-size and obtaining a stopping criteria. The authors also implement the algorithm and provide comparison of completion times for numerical problems involving linear equality and inequality constraints before successfully applying the algorithm to real-world data with close to a billion constraints.

Paper Contributions:

The utilization of soft-thresholding of singular values at each step of the iteration, allows the algorithm to solve large scale matrix completion problems quickly while keeping the solution low-rank. Also, the algorithm is 1st order and utilizes existing linear algebra libraries and can therefore be easily implemented.

Additional Comments:

To further scrutinize their work, the authors extensively discuss a particular shortcoming (the empirical but unproven assumption that the minimum nuclear solution has low rank when the sequence of optimal solution iterates also have low rank) of the proposed algorithm as well as future work directions in the final section of the paper.