Paper Review: Self Scaled Regularized Robust Regression

Brief Summary:

Numerous computer vision problems such as line extraction from 2D images, image reconstruction and classification utilize robust regression at the core of their solutions. While Linear Robust Regression (LRR) is known to be an NP-Hard problem, recently proposed alternative approaches reformulate the LRR problem as a convex optimization problem with sparse regularizations so that it can be solved in polynomial time. However, these approaches fail in the presence of extreme outliers and prior conditions (ex. Same label for multiple points).

Unlike these previous approaches that focus on minimizing the set of outlier errors, the authors of this paper aim to maximize the number of inliers by sparsifying the set of model parameters (r) that explain the data (x). The authors begin by reformulating the LRR problem to a problem that aims to sparsify the model parameters (r_i) and introduce conditions under which a unique set of sparse model parameters that maximize the number of inliers can be found. However, since the reformulated problem is also NP-hard, the authors provide the tightest convex approximation to the problem by replacing the cardinality (l_0 quasinorm) of the vector sequence of model parameters with the l_∞ norm. Then, using the fact that l_1 and l_∞ norms are dual, the authors reformulate the problem and discover that in the absence of additional priors, the reformulated problem is equivalent to the l_1 regularized robust regression problem with each data point scaled by its l_1 norm ($\|x_i\|_1$).

Furthermore, the authors specifically discuss incorporating the Co-occurrence information prior as well as the prior case where matrix X is known to not have full rank. These priors are incorporated as additional constraints to the problem. Since traditional l_1 regularized robust regression focuses on minimizing the cardinality of the outlier error vector (s), it is unable to incorporate additional constraints to the data. Inspired by the work from Fazel et al [1], the authors also incorporate a reweighted heuristic in their S^2R^3 algorithm to find sparse solutions in situations where specific conditions for unique and sparse model parameters (r_i) do not hold.

Finally, the authors experimentally test their algorithm to fit a 5-dimensional hyperplane where the data is corrupted by outliers and to reconstruct 2-D face images corrupted by occlusion. The results are compared to those of regression algorithms that use randomized approaches and are combinatorial (ex. RANSAC) along with other sparsification based algorithms and the proposed S^2R^3 algorithm is shown to perform optimally both in terms of fitting errors and running times.

Paper Contributions:

The most important contribution in this paper would be the approach adopted by the authors to reformulate the Linear Robust Regression problem as an optimization problem that aims to sparsify the model parameters such that the number of inliers are maximized. Unlike the l_1 regularized robust regression algorithm that tries to sparsify the set of outlier errors (s), focusing on sparsifying the model parameters allow the authors to incorporate scaling as well as prior information conducting regression.

Additional Comments:

It is important to note that the authors have utilized convex relaxation in solving the regression problem and that the solution may not always be the most optimal solution.

References:

[1] B. S. L. M. Fazel M, "Portfolio optimization with linear and fixed transaction costs," *Annals of Operations Research* , vol. 152, pp. 341-365, 2007.