# Paper Review: Control and Verification of High-Dimensional Surfaces with DSOS and SDSOS Programming

## Brief Summary:

Numerous problems in control theory can be posed in terms of checking the non-negativity of functions. However, even when these functions are restricted to the set of polynomials, checking the non-negativity becomes NP-hard and therefore the Sum of Squares (SOS) approach checks if a polynomial can be expressed as a sum of squares of other polynomials in order to solve the problem using Semi Definite Programming. Applications of the SOS approach limit the dimensionality of the system and therefore cannot be used in domains such as robotics where system dimensionality is higher. The authors of this paper propose replacing the SDP constraints on the Gram matrix in the SOS approach with conditions that allow the use of linear programming (LP) and Second-Order Cone Programming (SOCP) approaches which account for higher-dimensional systems.

In defining the Diagonal Dominance and Scaled Diagonal Dominance conditions, the authors initially formulate the non-negativity checking problem in terms of a semidefinite programming problem through *eq (1).* The authors then mathematically describe the Diagonal Dominance (dd) and Scaled Diagonal Dominance (sdd) features for a symmetric matrix. In the 2 following remarks, they use the fact that a set of $n*n$ dd matrices have a polytopic description to show that these matrices can be optimized over using LP and make use of Gershgorin's circle theorem to imply that Diagonal Dominance is a sufficient condition for positive semidifiniteness. Using these remarks, it can be said that *dd* is a sufficient polynomial nonnegative condition to make use of LP. Furthermore, through Theorems 3.1 and 3.2, the authors reformulate the description of the set of $n*n$ scaled-diagonally dominant matrices ($SDD_n$) as a rotated quadratic cone constraint to be imposed using an SOCP approach.

In section 4, the authors make use of the fact that the cone of nonnegative polynomials in $n$ variables and $d$ degrees ($POS_{n,d}$) is a superset of the cone of *sos* polynomials with degree $d$ in $n$ variables ($SOS_{n,d}$) to introduce cones based on the *dd* and *sdd* conditions that can be optimized over using LP and SOCP. In the following subsection *A*, the authors introduce a multiplier $(\sum x_i^2)^r$ as a regulatory term to be used as a control over the accuracy of the inner approximations to the set of nonnegative polynomials ($DSOS_{n,d}$ *and* $SDSOS_{n,d}$). Following this, the authors introduce two theorems that define conditions necessary for guaranteeing asymptotic optimization when using the $r$-multiplier that rely on the *Positivstellensatz* results from real algebraic geometry. The authors then demonstrate the advantages of their approach in terms of scalability over the SOS approach on 4 high dimensional control applications.

## Paper Contributions:

The most important contribution by the authors has been the introduction of the diagonal dominance and scaled diagonal dominance conditions that allow the use of LP and SOCP approaches and therefore their use for control applications involving high-dimensional systems. While providing detailed mathematical proofs for the introduced conditions, the authors also explore the use of the *r*-multiplier as a control parameter and provide theorems that show asymptotic guarantees on the r-multiplier based hierarchies.