# A Minimal Book Example

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## About

This is a *sample* book written in **Markdown**. You can use anything that Pandoc's Markdown supports; for example, a math equation  $a^2 + b^2 = c^2$ .

### 1.1 Usage

Each **bookdown** chapter is an .Rmd file, and each .Rmd file can contain one (and only one) chapter. A chapter *must* start with a first-level heading: # A good chapter, and can contain one (and only one) first-level heading.

Use second-level and higher headings within chapters like: ## A short section or ### An even shorter section.

The index.Rmd file is required, and is also your first book chapter. It will be the homepage when you render the book.

#### 1.2 Render book

You can render the HTML version of this example book without changing anything:

- 1. Find the Build pane in the RStudio IDE, and
- 2. Click on **Build Book**, then select your output format, or select "All formats" if you'd like to use multiple formats from the same book source files.

Or build the book from the R console:

bookdown::render\_book()

To render this example to PDF as a bookdown::pdf\_book, you'll need to install XeLaTeX. You are recommended to install TinyTeX (which includes XeLaTeX): https://yihui.org/tinytex/.

### 1.3 Preview book

As you work, you may start a local server to live preview this HTML book. This preview will update as you edit the book when you save individual .Rmd files. You can start the server in a work session by using the RStudio add-in "Preview book", or from the R console:

bookdown::serve\_book()

# Introduction

This course is an introduction to linear algebra. We will cover the following topics: vector spaces given by solutions to linear systems; linear independence; dimension; determinants; eigenvalues and eigenvectors; diagonalization; and complex numbers.

### 2.1 Textbook

The recommended textbook is Linear Algebra and its Applications (6th Edition) by D. Lay, S. Lay, and J. McDonald. The 5th Edition will equally work fine

### 2.2 Course Schedule

Week	Lecture	Topics
	Lecture 1	1.1 Systems of Linear Equations 1.2 Row Reduction and Echelon Forms
	Lecture 2	$1.3\ {\rm Vector}\ {\rm Equations}$ $1.4\ {\rm Matrix}\ {\rm Equation}\ {\rm Ax}={\rm b}$ $1.5\ {\rm Solution}\ {\rm Sets}\ {\rm of}\ {\rm Linear}\ {\rm Equations}$

# Lecture 1

### 3.1 Systems of linear equations

A system of linear equations is a set of two or more linear equations with the same variables. The goal is to find the values of the variables that satisfy all the equations in the system simultaneously. A system of linear equations can be written in matrix form as follows:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

where  $a_{ij}$  and  $b_i$  are constants and  $x_1, x_2, \dots, x_n$  are the variables. The system has m equations and n variables.

Example:

Consider the following system of linear equations:

$$2x - y = 3$$
$$3x + 2y = 1$$

This system has two equations and two variables (x and y). We can write the system in matrix form as:

$$2x - y = 3$$
$$3x + 2y = 1$$

### 3.2 Captioned figures and tables

Figures and tables with captions can also be cross-referenced from elsewhere in your book using \@ref(fig:chunk-label) and \@ref(tab:chunk-label), respectively.

See Figure 3.1.

```
par(mar = c(4, 4, .1, .1))
plot(pressure, type = 'b', pch = 19)
```

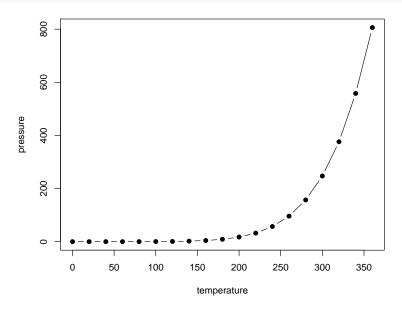


Figure 3.1: Here is a nice figure!

Don't miss Table 3.1.

```
knitr::kable(
  head(pressure, 10), caption = 'Here is a nice table!',
  booktabs = TRUE
)
```

Table 3.1: Here is a nice table!

temperature	pressure
0	0.0002
20	0.0012
40	0.0060
60	0.0300
80	0.0900
100	0.2700
120	0.7500
140	1.8500
160	4.2000
180	8.8000

# Lecture 2

You can add parts to organize one or more book chapters together. Parts can be inserted at the top of an .Rmd file, before the first-level chapter heading in that same file.

Add a numbered part: # (PART) Act one {-} (followed by # A chapter)

Add an unnumbered part: # (PART\\*) Act one {-} (followed by # A chapter)

Add an appendix as a special kind of un-numbered part: # (APPENDIX) Other stuff {-} (followed by # A chapter). Chapters in an appendix are prepended with letters instead of numbers.

## Lecture 2

#### 5.1 Footnotes

Footnotes are put inside the square brackets after a caret ^[]. Like this one <sup>1</sup>.

#### 5.2 Citations

Reference items in your bibliography file(s) using Okey.

For example, we are using the **bookdown** package (Xie, 2023) (check out the last code chunk in index.Rmd to see how this citation key was added) in this sample book, which was built on top of R Markdown and **knitr** (Xie, 2015) (this citation was added manually in an external file book.bib). Note that the .bib files need to be listed in the index.Rmd with the YAML bibliography key.

The bs4\_book theme makes footnotes appear inline when you click on them. In this example book, we added cs1: chicago-fullnote-bibliography.csl to the index.Rmd YAML, and include the .csl file. To download a new style, we recommend: https://www.zotero.org/styles/

The RStudio Visual Markdown Editor can also make it easier to insert citations: https://rstudio.github.io/visual-markdown-editing/#/citations

<sup>&</sup>lt;sup>1</sup>This is a footnote.

## Inverse of Matrices

The inverse of a matrix is an important concept in linear algebra. In this lecture, we will discuss what it means for a matrix to be invertible and how to find the inverse of a matrix. We will also explore some of the properties of invertible matrices and how they can be used to solve systems of linear equations.

#### 6.1 Definition

An  $n \times n$  matrix A is said to be **invertible** if there exists an  $n \times n$  matrix B such that  $AB = BA = I_n$ , where  $I_n$  is the  $n \times n$  identity matrix. In this case, B is called the **inverse** of A, denoted by  $A^{-1}$ .

#### 6.2 Theorem

Let A be an invertible  $n \times n$  matrix. Then the inverse of A is unique.

**Proof:** Suppose that A is invertible. Then  $AB = BA = I_n$  for some matrix B. Since  $AB = I_n$ , it follows that  $\operatorname{rank}(A) = n$ . Conversely, suppose that  $\operatorname{rank}(A) = n$ . Then A can be row reduced to the identity matrix  $I_n$ , which implies that there exist elementary matrices  $E_1, E_2, \dots, E_k$  such that  $E_k \cdots E_2 E_1 A = I_n$ . Hence, A is invertible, with  $A^{-1} = E_k \cdots E_2 E_1$ .

#### 6.3 Method

To find the inverse of a matrix A, we can use the following method:

- 1. Write the augmented matrix  $(A|I_n)$ , where  $I_n$  is the  $n \times n$  identity matrix.
- 2. Use row operations to transform  $(A|I_n)$  into the form  $(I_n|B)$ , where B is the inverse of A.
- 3. If  $(A|I_n)$  cannot be transformed into  $(I_n|B)$ , then A is not invertible.

#### 6.3.1 Example

Let  $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ . To find the inverse of A, we write the augmented matrix  $(A|I_2)$  and perform row operations:

$$\begin{pmatrix} 1 & 2 & 1 & 0 \\ 3 & 4 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & -2 & -3 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 3/2 & -1/2 \end{pmatrix}$$

Hence, the inverse of A is  $B = \begin{pmatrix} -2 & 1 \\ 3/2 & -1/2 \end{pmatrix}$ . You can verify that  $A \times B =$  $B \times A = I$ .

#### Properties of Invertible Matrices 6.4

Let A and B be invertible  $n \times n$  matrices. Then:

- $\begin{aligned} &1. \ \ (A^{-1})^{-1} = A \\ &2. \ \ (AB)^{-1} = B^{-1}A^{-1} \\ &3. \ \ (A^T)^{-1} = (A^{-1})^T \end{aligned}$

#### Important Theorems 6.5

Let A be a square  $n \times n$  matrix. The following statements are equivalent (meaning if one statement is true then all other statement must be true and if one is false, all others are also false)

- 1. A has an inverse
- 2. A is row equivalent to the  $n \times n$  identity matrix.
- 3. A has n pivot positions.
- 4. Ax = 0 has only the trivial solution.
- 5. The columns of A are linearly independent.
- 6. The linear transformation T(x) = Ax is one-to-one.
- 7. Ax = b has at least one solution for each  $b \in \mathbb{R}^n$ .
- 8. The columns of A span  $\mathbb{R}^n$
- 9. The linear transformation T(x) = Ax maps  $\mathbb{R}^n$  onto  $\mathbb{R}^n$ .
- 10. There exist an  $n \times n$  inverse matrix C such that CA = I and AC = I.

### Lecture 5

### 7.1 Publishing

HTML books can be published online, see: https://bookdown.org/yihui/bookdown/publishing.html

### 7.2 404 pages

By default, users will be directed to a 404 page if they try to access a webpage that cannot be found. If you'd like to customize your 404 page instead of using the default, you may add either a \_404.Rmd or \_404.md file to your project root and use code and/or Markdown syntax.

### 7.3 Metadata for sharing

Bookdown HTML books will provide HTML metadata for social sharing on platforms like Twitter, Facebook, and LinkedIn, using information you provide in the index.Rmd YAML. To setup, set the url for your book and the path to your cover-image file. Your book's title and description are also used.

This bs4\_book provides enhanced metadata for social sharing, so that each chapter shared will have a unique description, auto-generated based on the content.

Specify your book's source repository on GitHub as the repo in the \_output.yml file, which allows users to view each chapter's source file or suggest an edit. Read more about the features of this output format here:

https://pkgs.rstudio.com/bookdown/reference/bs4 book.html

Or use:

?bookdown::bs4\_book

# **Invertible Matrices**

The inverse of a matrix is an important concept in linear algebra. In this lecture, we will discuss what it means for a matrix to be invertible and how to find the inverse of a matrix. We will also explore some of the properties of invertible matrices and how they can be used to solve systems of linear equations.

XXX

## Lecture 4

### 9.1 Equations

Here is an equation.

$$f\left(k\right) = \binom{n}{k} p^{k} \left(1 - p\right)^{n - k} \tag{9.1}$$

You may refer to using \@ref(eq:binom), like see Equation (9.1).

### 9.2 Theorems and proofs

Labeled theorems can be referenced in text using \@ref(thm:tri), for example, check out this smart theorem 9.1.

**Theorem 9.1.** For a right triangle, if c denotes the length of the hypotenuse and a and b denote the lengths of the **other** two sides, we have

$$a^2 + b^2 = c^2$$

Read more here https://bookdown.org/yihui/bookdown/markdown-extensions-by-bookdown.html.

#### 9.3 Callout blocks

The bs4\_book theme also includes special callout blocks, like this .rmdnote.

You can use markdown inside a block.

```
head(beaver1, n = 5)

#> day time temp activ

#> 1 346 840 36.33 0

#> 2 346 850 36.34 0

#> 3 346 900 36.35 0

#> 4 346 910 36.42 0

#> 5 346 920 36.55 0
```

It is up to the user to define the appearance of these blocks for LaTeX output.

You may also use: .rmdcaution, .rmdimportant, .rmdtip, or .rmdwarning as the block name.

The R Markdown Cookbook provides more help on how to use custom blocks to design your own callouts: https://bookdown.org/yihui/rmarkdown-cookbook/custom-blocks.html

# **Bibliography**

Xie, Y. (2015). Dynamic Documents with R and knitr. Chapman and Hall/CRC, Boca Raton, Florida, 2nd edition. ISBN 978-1498716963.

Xie, Y. (2023). bookdown: Authoring Books and Technical Documents with R Markdown. R package version 0.34.