Understanding the spectrum of a blackbody

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Part 3

P = probability

ϕ = Probability amplitude

P = $|\phi|^2$

$$\phi = \phi_1 + \phi_2$$

$$\mathbf{P} = |\phi_1 + \phi_2|^2$$

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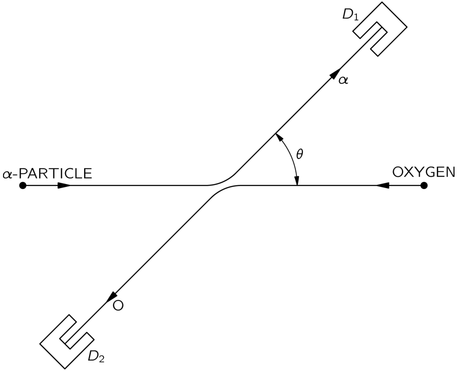
- The probability of this event = $|f(\theta)|^2$



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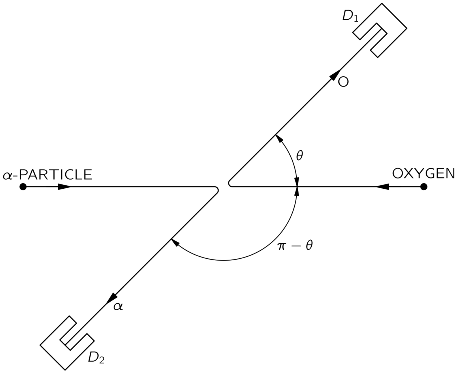
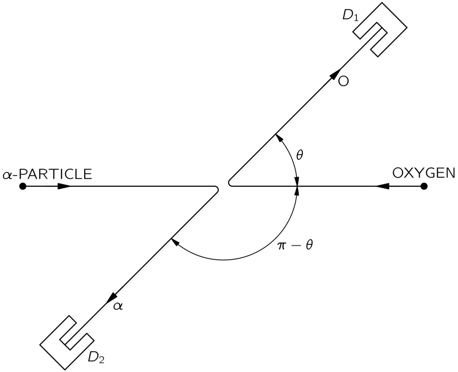
- This means that if oxygen atom in position θ , then α particle on the opposite side is at an angle $\pi - \theta$.





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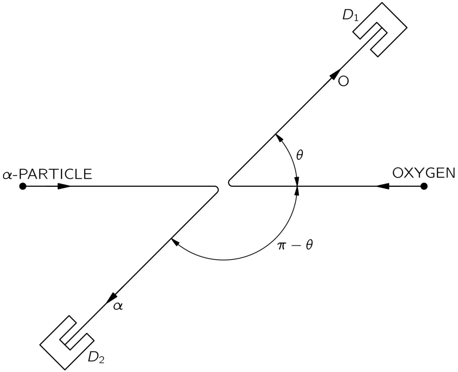

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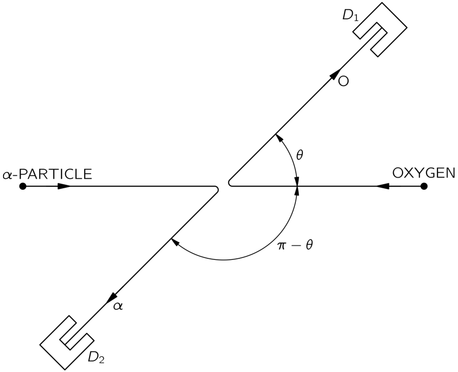
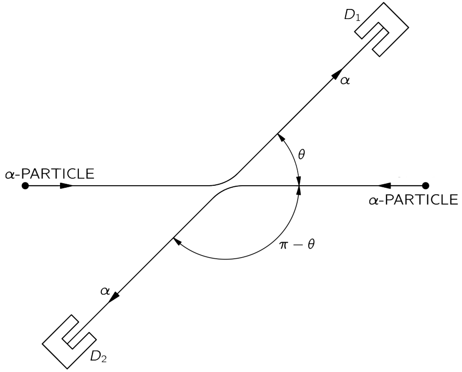


- Probability amplitude of oxygen atom = $f(\pi - \theta)$

- Probability amplitude of α particle = $f(\theta)$

- The probability of a particle being detected at detector 1 = $|f(\theta)|^2$ + $|f(\pi - \theta)|^2$



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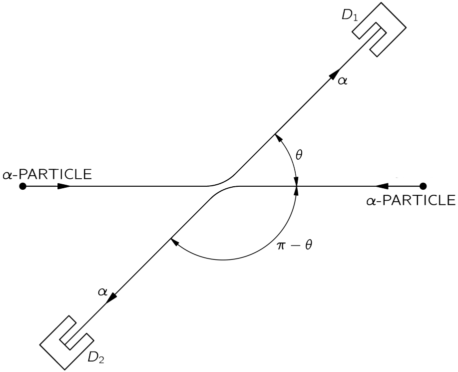

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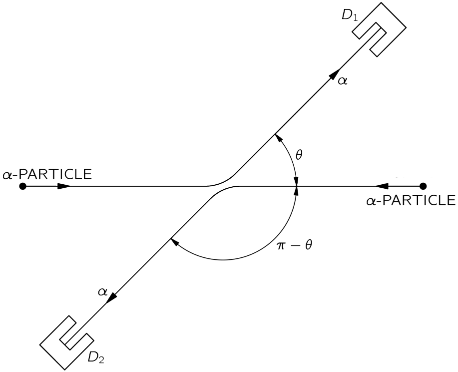

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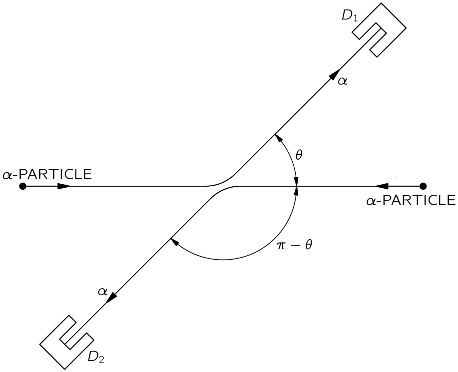
- The probability of a α particle being detected at detector 1 = $|f(\theta) + f(\pi - \theta)|^2$



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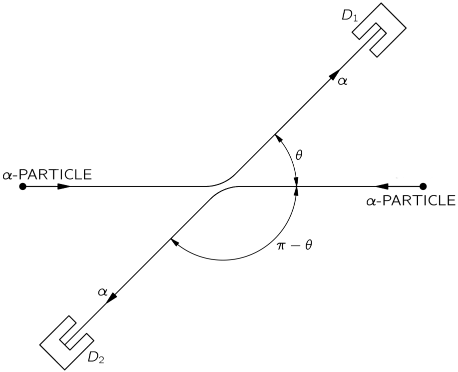
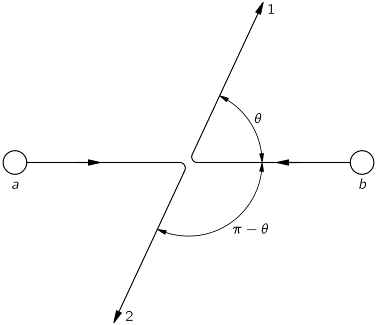


- If $\theta = \dfrac{\pi}{2}$, then applying this to the expression $|f(\theta) + f(\pi - \theta)|^2$ we get,

- Probability = $4|f\left(\dfrac{\pi}{2}\right)|^2$, if the particles are indistinguishable.



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$$|f(\theta)|^2 + |f(\pi - \theta)|^2$$

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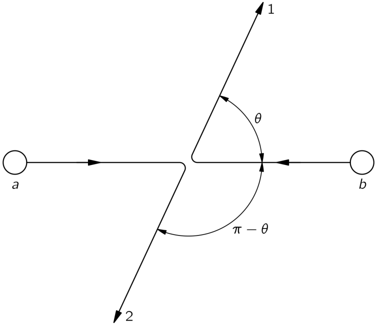

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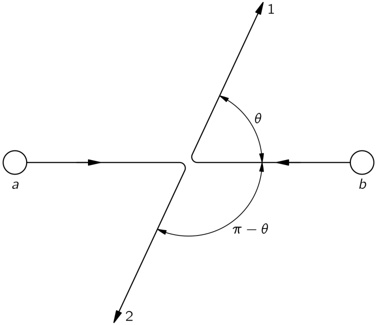

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- This shows that the probability gets doubled for indistinguishable particles.

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- In electrons case , the interfering amplitude for exchange interfere with a negative sign. Probability for electron = $|f(\theta) - f(\pi - \theta)|^2$

Fraction of cases	Particle 1	Particle 2	Spin at D1	Spin at D2	Probability
$\frac{1}{4}$	up	up	up	up	$\lvert f(\theta) - f(\pi - \theta)\rvert^2$
$\frac{1}{4}$	down	down	down	down	$\lvert f(\theta) - f(\pi - \theta)\rvert^2$
$\frac{1}{4}$	up	down	up	down	$\lvert f(\theta)\rvert^2$
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Identical particles , also called **indistinguishable particle** , are particles that cannot be distinguished from one another.

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- Consider particle ’a’ and particle ’b’.

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$$|\langle 1|a\rangle \langle 2|b\rangle + \langle 2|a\rangle \langle 1|b\rangle|^2 ds_1 ds_2$$

- Integrating over the area of the detector , if we let ds_1 and ds_2 range over the whole area (ΔS) , we could count each part of the area twice since the expression $|\langle 1|a\rangle \langle 2|b\rangle + \langle 2|a\rangle \langle 1|b\rangle|^2 ds_1 ds_2$ contains everything that can happen with any pair of surface elements ds_1 and ds_2.

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- This is just twice what we got the probability for distinguished particles.

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- Probability that each particle acting alone would go into an element of the surface ds of the detector is $|\langle \cdots \rangle|^2 ds$.

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- If the amplitude does not depend on where ds is located in the detector , then the

$$Probability = \left(|a|^2 |b|^2 \ldots\right) (ds_1 ds_2 \ldots)$$

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- $(P_n)_{BOSE} = n!(P_n)_{different}$

- What is the probability that a boson will go into particular state when there are already n particles present ?

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- *OBSERVATION*: The probability that an atom will emit a photon into a particular final state is increased by the factor (n+1) if there are already n photons in that state.

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  where $a = \langle j|a\rangle$ is the amplitude when there are no other photons are present.

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- The above two equation shows that they are symmetric in nature.

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- The co-efficient of absorption, of induced emission and spontaneous emission are all equal and are related to the probability of spontaneous emission.

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- NOTE: Each atom in the ground state can absorb a photon and go into the excited state and each atom in the excited state can emit a photon and go to the ground state.

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- $\overline{n}$ is the average number of photons present in a given state with the frequency ω.

- The absorption rate from the state is $N_g\overline{n}|a|^2$, and the emission rate into that state is $N_e(\overline{n}+1)|a|^2$.
- At equilibrium $N_g\overline{n}|a|^2 = N_e(\overline{n}+1)|a|^2$

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- The energy of each photon is given by $\dfrac{\hbar\omega}{e^{\hbar\omega}/k_B t - 1}$
- For any harmonic oscillator, the quantum mechanical energy levels are equally spaced with a seperation $\hbar\omega$.

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Considering the boson which do not interact with each other, and in that state the whole system of particles behaves (for all quantum mechanical purpose) exactly like an harmonic oscillator.

- Analysing the Electro-magnetic field in a box, it show the properties of an harmonic oscillation.
- Thus, the number of photons in a particular state in a box, can be equated to the number of energy levels associated with the particular modes of oscillation of the electromagnetic fields.

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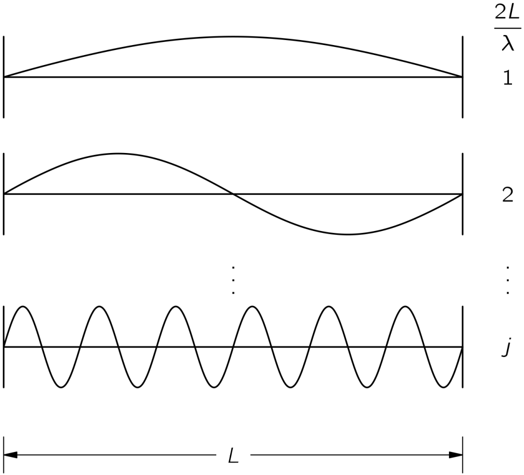
ASSUMPTION

For every mode there are some atoms in the box, which have energy levels that can radiate into that mode so that each mode can get into thermal equilibrium.

- There will be billions of modes in the box and there will be many small frequency intervals $\Delta\omega$.



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- An assumption is made that kL is large that in small interval Δk, there are many modes.

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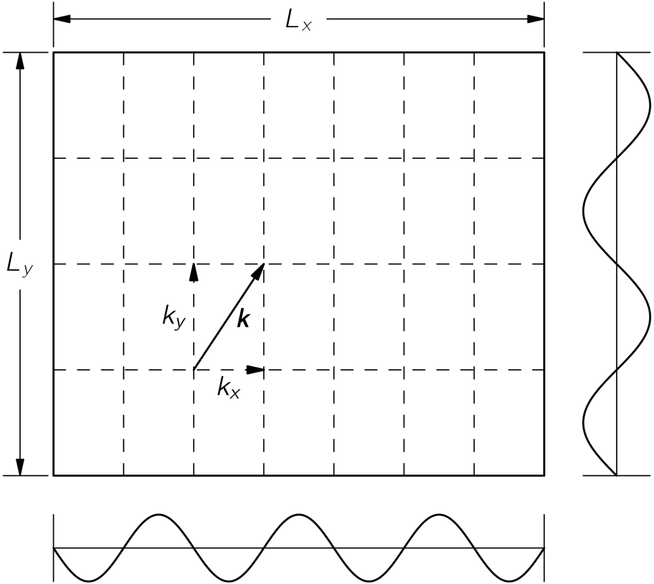
$$\boxed{\Delta W = \frac{L(\Delta k)}{2\pi}}$$


L_x
L_y
k_y
$\boldsymbol{k}$
k_x

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$$\Delta W = \frac{L_x L_y L_z}{(2\pi)^3} (\Delta k_x \Delta k_y \Delta k_z) \tag{1}$$

$$dW(K) = V \frac{d^3 k}{(2\pi)^3} \tag{2}$$

- Applying the above result to find number of photon modes for photons with frequencies in the range Δk.
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$$\Delta W(\omega) = \frac{V(\omega^3)\Delta\omega}{\pi^2 c^3} \text{ (for light).}$$

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- Multiplying This by the number of modes, we get the energy ΔE in the modes that lie in the interval $\Delta\omega$:
$$\Delta E = \left(\frac{\hbar\omega}{e^{\hbar\omega/k_BT} - 1}\right)\left(\frac{V\omega^3\Delta\omega}{\pi^2 c^3}\right)$$

The photons are the bosons, which have tendency to try to get to all into the same state.

Thank you