

# Spectrum of Blackbody

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## Part 1

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An idealised object that can **absorb** and subsequently **emit**, all the radiation incident on it is called a **Blackbody**.

- Consider gaseous medium, in which we have a charged oscillator.
- Due to random motion of the gaseous atoms, it often collides with the oscillator.

## Rayleigh's law

- Consider gaseous medium, in which we have a charged oscillator.
- Due to random motion of the gaseous atoms, it often collides with the oscillator.
- As a result the gaseous atoms imparts some of it's kinetic energy to the oscillator.

- Mean kinetic energy acquired by the oscillator is,  $\frac{1}{2} kT$ .
- And the total kinetic energy will be  $kT$ .



- Since the charge is radiating energy **the system cannot attain equilibrium.**

- How do we tackle this situation?

- How do we tackle this situation?
- We confine the system.

- Inner walls of the box is totally reflective.
- After some time the system will attains **thermal equilibrium**

So, the energy radiated by the oscillator per second can be given by,

$$\frac{1}{Q} = \frac{\left(\frac{dW}{dt}\right)}{\omega_0 W}$$

$Q$  is the radiation reaction.

$\omega_0$  is the natural frequency of vibration.

$W$  is the total energy content of the oscillator.

Also, we have

$$\frac{1}{Q} = \frac{\gamma}{\omega_0}$$

Where  $\gamma$  is the damping constant.

Remember the mean energy of the oscillator is  $kT$ ? for three degrees of freedom, it is  $3kT$ .

Average energy loss due to radiation is given by,

$$\left\langle \frac{dW}{dt} \right\rangle = \gamma kT$$

- Let,  $I(\omega)d\omega$  be the incident light energy with in a frequency range  $d\omega$ .
- Assuming all the radiation which falls on a *cross section* ' $\sigma_s$ ' is absorbed completely,
- Scattered light must be the product of  $I(\omega)d\omega$  and  $\sigma_s$ .



Expression for the cross section is given by,

$$\sigma_s = \frac{8\pi r_0^2}{3} \left[ \frac{\omega^4}{(\omega^2 - \omega_0^2)^2 + \gamma^2 \omega^2} \right]$$

When  $\omega = \omega_0$ ,

$$\sigma_s = \frac{2\pi r_0^2 3\omega_0^2}{3(\omega - \omega_0)^2 + \frac{\gamma^2}{4}}$$

$$\frac{dW_s}{dt} = \int_0^\infty I(\omega) \sigma_s d\omega = \int_0^\infty \frac{2\pi r_0^2 \omega_0^2 I(\omega) d\omega}{3 \left[ (\omega - \omega_0)^2 + \frac{\gamma^2}{4} \right]}$$

since we're considering the equilibrium condition ( $\omega = \omega_0$ ), equation reduces to,

$$\frac{dW_s}{dt} = \frac{2}{3} \pi r_0^2 \omega_0^2 I(\omega_0) \int_{-\infty}^{\infty} \frac{d\omega}{(\omega - \omega_0)^2 + \left(\frac{\gamma}{2}\right)^2}$$

Since the integral reduces to arctan function we get,  $\frac{2\pi}{\gamma}$ . and as

$$\frac{dW_s}{dt} = \frac{dW}{dt}$$

,

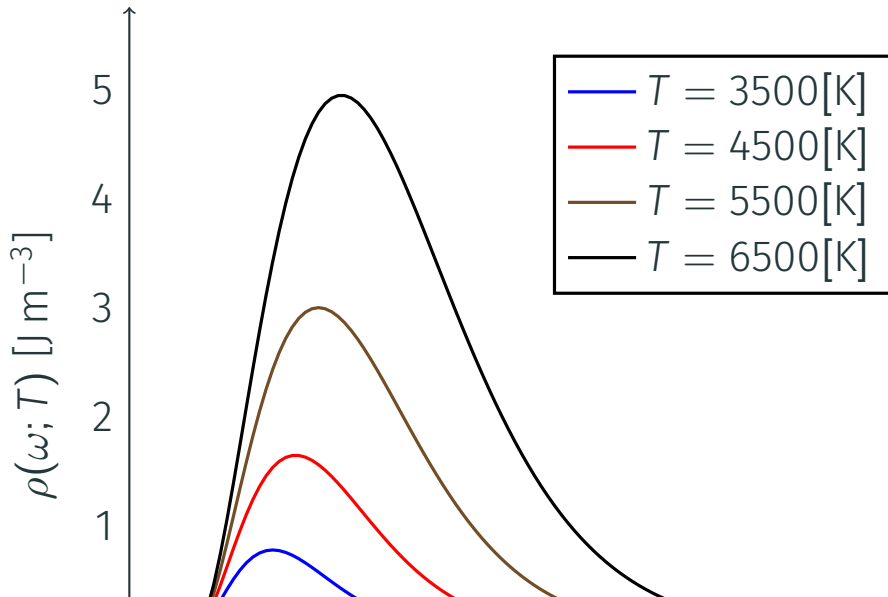
$$I(\omega) = \frac{9\gamma^2 kT}{4\pi^2 r_0^2 \omega_0^2}$$

The value of  $\gamma$  was found to be equal to,

$$\frac{23}{c} \frac{r_0 \omega_0^2}{c}$$

$$I(\omega) = \frac{\omega^2 kT}{\pi^2 c^2}$$

# Graph



- Max Planck came up with an idea.
- We had the experimental result, and what we needed was a theoretical explanation for the curve.

- He made an assumption that the energy level of the harmonic oscillator is quantized.
- The harmonic oscillator can take up energies only in the multiples of  $\hbar\omega$ .
- The probability of occupying an energy level  $E$  is  $P(E) = \alpha e^{\frac{-\hbar\omega}{kT}}$ .

## Part 2

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- The intensity of radiation for frequency  $\omega$  is given by

$$I(\omega)d\omega = \frac{\hbar\omega^3 d\omega}{\pi^2 c^2 \left( e^{\frac{\hbar\omega}{kT}} - 1 \right)}$$

- Planck considered that the matter was quantized and not light.
- Einstein came up with an alternate idea to solve Black body problem.

- Einstein considered that even the light was quantized, by taking that light is actually photons with energy  $\hbar\omega$
- a pic is needed.

- Einstein assumed that Planck's formula was right.
- Consider two energy levels , say , the  $n^{th}$  level and  $m^{th}$  level.
- A light of certain frequency is incident on atom and by absorbing a photon, transition from state n to state m occurs.

- The probability of the transition is proportional to intensity of the light.
- The proportionality constant is  $B_{nm}$  and mathematically can be written as,

$$R_{n \rightarrow m} = N_n B_{nm} I(\omega) \quad (1)$$

- Einstein considers that there are two types of emissions that take place in an atom.
- Spontaneous emission.
- Stimulated emission.

- The combined mathematical expression is,

$$R_{m \rightarrow n} = N_m [A_{mn} + B_{mn} I(\omega)] \quad (2)$$

- At thermal equilibrium, the number of atoms going to higher energy level must be equal to the number of atoms coming to lower energy state.
- The ratio of  $N_m$  to  $N_n$  is given by  $e^{\frac{-\hbar\omega}{kT}}$

- At thermal equilibrium both  $R_{n \rightarrow m}$  and  $R_{m \rightarrow n}$  are equal.
- $N_n B_{nm} I(\omega) = N_m [A_{mn} + B_{mn} I(\omega)]$
- On dividing the previous equation by  $N_m$  we get,



- $B_{nm}I(\omega)e^{\frac{\hbar\omega}{kT}} = A_{mn} + B_{mn}I(\omega)$
- So the equation that Einstein got for intensity of radiation for frequency is

$$I(\omega) = \frac{A_{mn}}{B_{nm}e^{\frac{\hbar\omega}{kT}} - B_{mn}} \quad (3)$$

- Since Einstein considered that the formula given by Planck was correct, we should get ,
- $B_{nm} = B_{mn}$  and also

$$\frac{A_{mn}}{B_{nm}} = \frac{\hbar\omega^3 d\omega}{\pi^2 c^2}$$

This is how Einstein rederived Planck's formula from a quantum mechanical viewpoint.

## Part 3

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- $P$  = probability
- $\phi$  = Probability amplitude
- $P = |\phi|^2$

- $\phi = \phi_1 + \phi_2$

- $\mathbf{P} = \left| \phi_1 + \phi_2 \right|^2$

## Case A

- Detector 1 is set to detect only  $\alpha$  particles and Detector 2 is set to detect only oxygen atoms.
- Probability amplitude of the scattering is given as  $f(\theta)$  when they are at an angle  $\theta$ .
- The probability of this event =  $\left| f(\theta) \right|^2$

- Set up the detectors such that the detectors would detect either  $\alpha$  particle or oxygen atom.
- We will not distinguish which particle is which entering the detector.
- This means that if oxygen atom in position  $\theta$  , then  $\alpha$  particle on the opposite side is at an angle  $\pi - \theta$ .



- Probability amplitude of oxygen atom =  $f(\pi - \theta)$
- Probability amplitude of  $\alpha$  particle =  $f(\theta)$
- The probability of a particle being detected at detector 1 =  $\left| f(\theta) \right|^2 + \left| f(\pi - \theta) \right|^2$

- Consider if both are  $\alpha$  particles,
- Then we would not know which particle entered the detector, so the total probability changes to,
- The probability of a  $\alpha$  particle being detected at detector 1 =  
 $\left| f(\theta) + f(\pi - \theta) \right|^2$

- If  $\theta = \frac{\pi}{2}$ , then applying this to the expression  $\left| f(\theta) + f(\pi - \theta) \right|^2$  we get,
- Probability =  $4 \left| f\left(\frac{\pi}{2}\right) \right|^2$ , if the particles are indistinguishable.

- Suppose the particles were distinguishable, then the probability for  $\theta = \frac{\pi}{2}$  when applied for  $\left| f(\theta) \right|^2 + \left| f(\pi - \theta) \right|^2$  is given by,
- Probability =  $2 \left| f\left(\frac{\pi}{2}\right) \right|^2$
- This shows that the probability gets doubled for indistinguishable particles.

- *Can we apply the same logic to the electron-electron scattering ?*
- *OBSERVATION : " When we have situation in which the identity of the electron which is arriving at a point is exchanged with another one , the new amplitude interfere with old one with an opposite phase."*
- In electrons case , the interfering amplitude for exchange interfere with a negative sign.  
Probability for electron =  $\left| f(\theta) - f(\pi - \theta) \right|^2$

## SPIN PROBABILITY TABLE

**Identical particles**, also called **indistinguishable particle**, are particles that cannot be distinguished from one another.

# Identical - Indistinguishable Particles

- Consider particle 'a' and particle 'b'.
- Let the two particles collide and get scattered in two different directions say '1' and '2' over a surface element  $ds_1$  and  $ds_2$  of the detector respectively.
- If the particles are indistinguishable then the amplitudes of these processes will add up.
- Probability that the two particles arrive at  $ds_1$  and  $ds_2$  is
$$\left| \langle 1|a \rangle \langle 2|b \rangle + \langle 2|a \rangle \langle 1|b \rangle \right|^2 ds_1 ds_2$$



- Integrating over the area of the detector , if we let  $ds_1$  and  $ds_2$  rangr over the whole area ( $\Delta S$ ) , we could count each part of the area twice since the expression  $\left| \langle 1|a \rangle \langle 2|b \rangle + \langle 2|a \rangle \langle 1|b \rangle \right|^2 ds_1 ds_2$  contains everything that can happen with any pair of surface elements  $ds_1$  and  $ds_2$ .

- $Probability_{BOSE} = \frac{\left( 4 |a|^2 |b|^2 \right)}{2} (\Delta S)$

- This is just twice what we got the probability for distinguished particles.

- Consider n particles say a, b, c... scattered in n direction say 1, 2, 3 ...
- Probability that each particle acting alone would go into an element of the surface  $ds$  of the detector is  $\left| \langle \rangle \right|^2 ds$ .

- **Assumption** :All particles are distinguishable.
- Probability that n particles will be counted together in n different surface elements =  $\left| a_1 b_2 c_3 \dots \right|^2 ds_1 ds_2 \dots$
- If the amplitude does not depend on where ds is located in the detector , then the  

$$Probability = \left( \left| a \right|^2 \left| b \right|^2 \dots \right) (ds_1 ds_2 \dots)$$

- Integrating each  $dS$  over the surface  $\Delta S$  of the detector

$$(P_n)_{different} = \left( |a|^2 |b|^2 \dots \right) (\Delta S)^n$$

- Now suppose that all the particles are Bose particles.
- For  $n$  particles, there are  $n!$  different, but indistinguishable possibilities for which we must add the amplitudes.

- Probability that n particles will be counted on the n surface elements is given by

$$Probability = \left( \left| a_1 b_2 c_3 \dots + a_1 b_3 c_2 \dots \right|^2 \right) (ds_1 ds_2 \dots)$$

- $Probability = \left( \left| n! abc \dots \right|^2 \right) (ds_1 ds_2 \dots)$
- Integrate each ds over the area  $\Delta S$  of the detector

$$(P_n)_{BOSE} = n! \left( \left| abc \dots \right|^2 \right) (\Delta S)^n$$

- Comparing the probability when the particles are distinguishable and indistinguishable

$$(P_n)_{BOSE} = n! \left( |abc\dots|^2 \right) (\Delta S)^n$$

$$(P_n)_{different} = \left( |a|^2 |b|^2 \dots \right) (\Delta S)^n$$

- $(P_n)_{BOSE} = n! (P_n)_{different}$

What is the probability that a Bose particle will go into particular state when there are already  $n$  particles present ?

# Emission and Absorption of photons

- When the light is emitted, a photon is "created".
- Consider that there are some atom emitting  $n$  photons.
- *OBSERVATION* : The probability that an atom will emit a photon into a particular final state is increased by the factor  $(n+1)$  if there are already  $n$  photons in that state.



# The Blackbody Spectrum

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# The Blackbody Spectrum

- For each light frequency  $\omega$ , there are certain  $N$  number of atoms which have two energy states separated, given by the equation  $E = \omega\hbar$ .
- Let  $N_e$  and  $N_g$  be the average numbers of atoms that are in excited state and ground state.
- In thermal equilibrium at temperature  $T$ , from statistical mechanics
- NOTE: Each atom in the ground state can absorb a photon and go into the excited state and each atom in the excited state can emit a photon and go to the ground state.

- At equilibrium, the rate of these two process must be equal.
- Rate is proportional to the probability of the event and the number of atoms present.
- $\bar{n}$  is the average number of photons present in a given state with the frequency  $\omega$

- The absorption rate from the state is  $N_g \bar{n} |a|^2$ , and the emission rate into that state is  $N_e (\bar{n} + 1) |a|^2$ .
- At equilibrium  $N_g \bar{n} |a|^2 = N_e (\bar{n} + 1) |a|^2$