Understanding the spectrum of a blackbody

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Overview

Part 1

The blackbody problem

Rayleigh's intensity function

The ultraviolet catastrophy

Planck's empirical formula

Part 2

Intensity of a blackbody spectrum and the problem

Einstein's work on blackbodies

Overview

Part 3

Amplitude-based description of identical particles

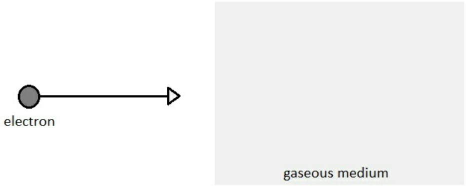
n-Boson systems

Probability-based emission and absorption of blackbodies

Part 1

An idealised object that can **absorb**, and subsequently **emit**, all the radiation incident on it is called a blackbody.

Rayleigh’s law



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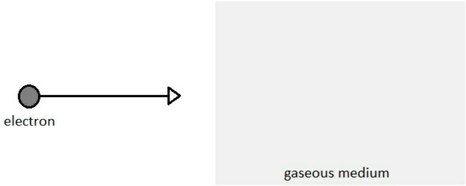

- Consider a gaseous medium into which a charged particle,say an electron, enters.

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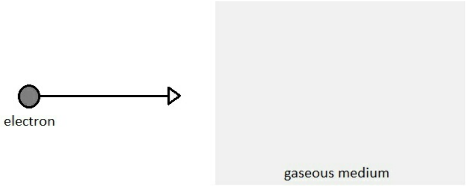

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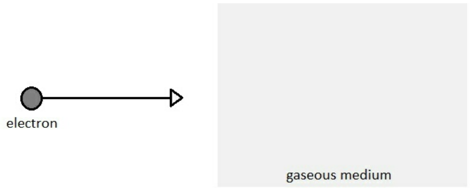

- Consider a gaseous medium into which a charged particle,say an electron, enters.
- Due to the atoms in the gas, the electron collides with the atoms several times
- As a result the gaseous atoms impart some of their kinetic energy to the electron.

Kinetic energy of the electron is,

$$\frac{1}{2}\,kT$$

And the total kinetic energy will be,

$$kT$$

Rayleigh's law

- Since the electron is radiating energy
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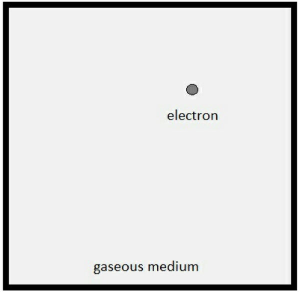

Rayleigh’s law

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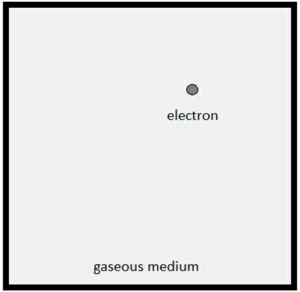
- The inner walls of the box is totally reflective.



Rayleigh's law

The electron behaves like an oscillator whose energy radiated away per second is be given by,

$$\frac{1}{Q} = \frac{\left(\frac{dW}{dt}\right)}{\omega_0 W}$$

Q is the radiation reaction

ω_0 is the natural frequency of vibration

W is the total energy content of the oscillator

Also, we have

$$\frac{1}{Q} = \frac{\gamma}{\omega_0}$$

Where γ is the damping constant.

Rayleigh's law

Average energy loss due to radiation is given by,

$$\left\langle \frac{dW}{dt} \right\rangle = \gamma kT$$

Since the degree of freedom is 3

$$\left\langle \frac{dW}{dt} \right\rangle = 3\gamma kT$$

Rayleigh's law

- $\mathbf{I}(\omega)d\omega$ **is the incident light energy** within a frequeny range $d\omega$.
- **Asumption:** The radiation which falls on a *cross-section* σ_S is absorbed completely.
- The total energy scattered is the product of $\mathbf{I}(\omega)d\omega$ and σ_S.

Rayleigh's law

The cross-section is given by

$$\sigma_S = \frac{8\pi {r_0}^2}{3}\left[\frac{\omega^4}{(\omega^2 - {\omega_0}^2)^2 + \gamma^2\omega^2}\right]$$

At equilibrium, $\omega = \omega_0$

$$\sigma_S = \frac{2\pi {r_0}^2 3\omega_0^2}{3(\omega - \omega_0)^2 + \frac{\gamma^2}{4}}$$

Rayleigh's law

$$\frac{dW_s}{dt} = \int_0^\infty \mathbf{I}(\omega)\,\sigma_s\,d\omega = \int_0^\infty \frac{2\pi {r_0}^2 \omega_0^2\,\mathbf{I}(\omega)\,d\omega}{3\left[(\omega-\omega_0)^2 + \frac{\gamma^2}{4}\right]}$$

$\because\ \omega = \omega_0,$

$$\frac{dW_s}{dt} = \frac{2}{3}\pi {r_0}^2 \omega_0^2\,\mathbf{I}(\omega_0) \int_{-\infty}^\infty \frac{d\omega}{(\omega-\omega_0)^2 + \left(\frac{\gamma}{2}\right)^2}$$

The integral reduces to an arctan function and we get $2\pi/\gamma$.

$$\frac{dW_s}{dt} = \frac{dW}{dt} = 3\gamma kT \qquad \text{(as seen earlier)}$$

$$\therefore \mathbf{I}(\omega) = \frac{9\gamma^2 kT}{4\pi^2 r_0^2 \omega_0^2}$$

Rayleigh's law

$$\implies \gamma = \frac{2}{3}\frac{r_0\omega_0^2}{c}$$

The intensity distribution function $\mathbf{I}(\omega)$ is,

$$\mathbf{I}(\omega) = \frac{\omega^2\, kT}{\pi^2 c^2}$$

The experimental observation

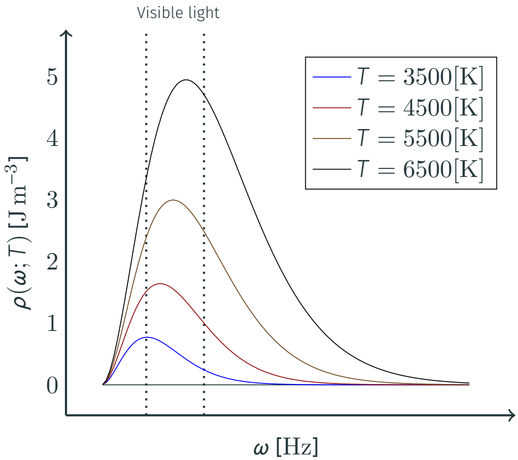
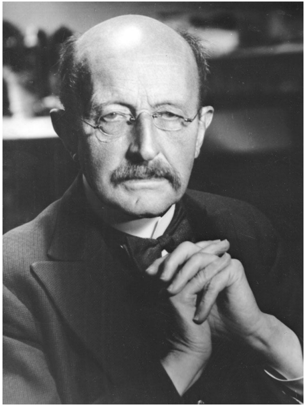


Figure 1: Spectrum of a blackbody.

A theoretical explanation for the obtained experimental results were needed.



- The harmonic oscillator can take up energies only as multiples of $\hbar\omega$.

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- The probability of occupying an energy level $\mathbf{E}$ is $\mathbf{P}(\mathbf{E}) = \alpha e^{-\hbar\omega/kT}$.

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- Then $N_n = N_0 x^n$, where $x = e^{-\hbar\omega/kT}$

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- Total energy is given by

$$E_{(total)} = N_0 \hbar\omega(0 + x + 2x^2 + \cdots)$$

- The total number of oscillators is

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- The average energy is

$$\langle E \rangle = \frac{E_{(total)}}{N_{(total)}} = \frac{N_0 \hbar\omega(0 + x + 2x^2 + \cdots)}{N_0(1 + x + x^2 + \cdots)}$$

$$Y = 1 + 2x + 3x^2 + \cdots$$

$$Yx = x + 2x^2 + 3x^3 + \cdots$$

$$Y(1 - x) = 1 + x + x^2 + \cdots$$

$$Y = \frac{1}{(1-x)^2}$$

Average energy

$$\langle E \rangle = \frac{N_0 \hbar \omega x/(1-x)^2}{N_0/(1-x)}$$

$$\langle E \rangle = \frac{\hbar\omega e^{-\hbar\omega/kT}}{1 - e^{-\hbar\omega/kT}} \qquad \because x = e^{-\hbar\omega/kT}$$

$$\boxed{\langle E \rangle = \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1}} \tag{1}$$

Hence the intensity of radiation for frequency ω is given by

$$I(\omega)d\omega = \frac{\hbar\omega^3 d\omega}{\pi^2 c^2 \left(e^{\hbar\omega/kT} - 1\right)} \tag{2}$$

Graph

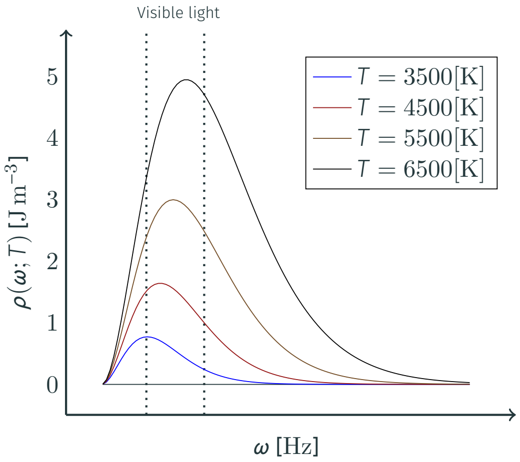

Figure 2: Spectrum of a blackbody.

Part 2

- The intensity of radiation for frequency ω is given by

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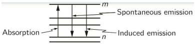
- Einstein came up with an alternate idea to solve Black body problem.

Einstein considered that even the light was quantised and considerd it to be made up of photons with energy $\hbar\omega$



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$$R_{n \to m} = N_n B_{nm} I(\omega) \tag{3}$$

- Einstein considers that there are two types of emissions that take place in an atom.

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    1. Spontaneous emission.

    2. Stimulated emission.

- The combined mathematical expression is

$$R_{m \to n} = N_m [A_{mn} + B_{mn} I(\omega)] \tag{4}$$

- At thermal equilibrium, the number of atoms going to higher energy level must be equal to the number of atoms coming to lower energy state.

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- The ratio of N_m to N_n is given by $e^{-\hbar\omega/kT}$

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$$N_n B_{nm} I(\omega) = N_m [A_{mn} + B_{mn} I(\omega)]$$

On dividing the previous equation by N_m we get,

$$B_{nm} I(\omega) e^{\hbar\omega/kT} = A_{mn} + B_{mn} I(\omega)$$

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The equation that Einstein got for the intensity of radiation for frequency is

$$I(\omega) = \frac{A_{mn}}{B_{nm} e^{\hbar\omega/kT} - B_{mn}} \tag{5}$$

Since Einstein considered that the formula given by Planck was correct, we should get

Since Einstein considered that the formula given by Planck was correct, we should get $B_{nm} = B_{mn}$

$$\frac{A_{mn}}{B_{nm}} = \frac{\hbar\omega^3 d\omega}{\pi^2 c^2}$$

This is how Einstein derived Planck’s formula but this time from a quantum mechanical viewpoint.

Part 3

P = probability

ϕ = Probability amplitude

P = $|\phi|^2$

$$\phi = \phi_1 + \phi_2$$

$$\mathbf{P} = |\phi_1 + \phi_2|^2$$

- Detector 1 is set to detect only α particles and Detector 2 is set to detect only oxygen atoms.

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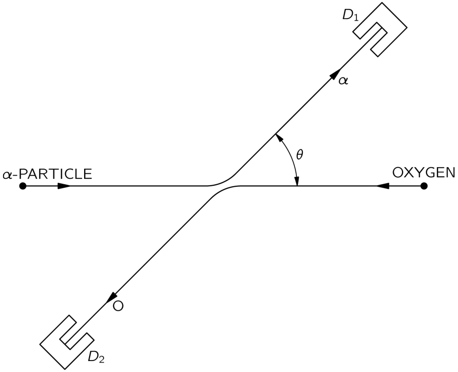
- The probability of this event = $|f(\theta)|^2$



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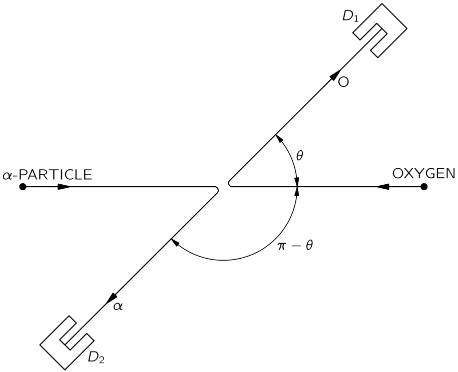
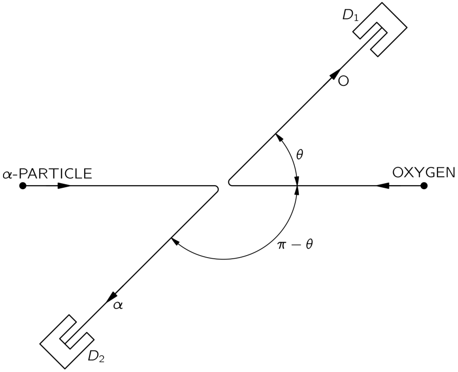
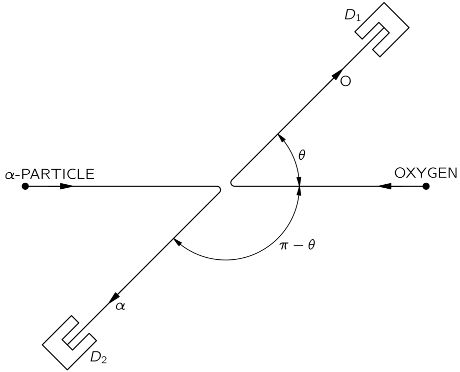
- This means that if oxygen atom in position θ , then α particle on the opposite side is at an angle $\pi - \theta$.

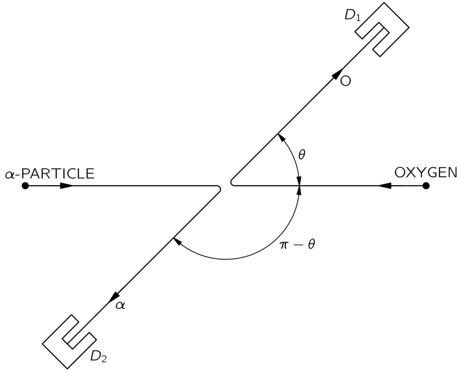




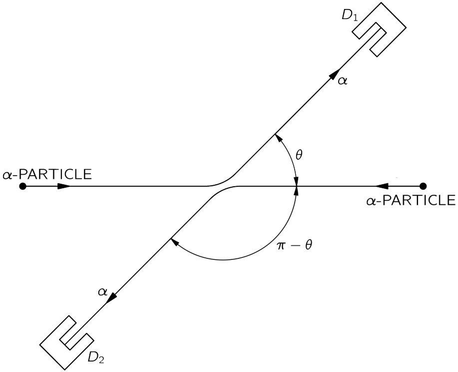
- Probability amplitude of oxygen atom = $f(\pi - \theta)$



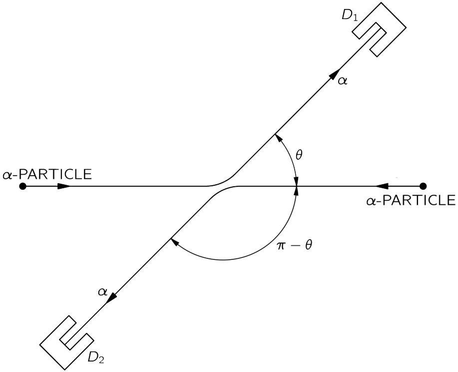
- Probability amplitude of oxygen atom = $f(\pi - \theta)$
- Probability amplitude of α particle = $f(\theta)$



- Probability amplitude of oxygen atom = $f(\pi - \theta)$
- Probability amplitude of α particle = $f(\theta)$
- The probability of a particle being detected at detector 1 = $|f(\theta)|^2$ + $|f(\pi - \theta)|^2$

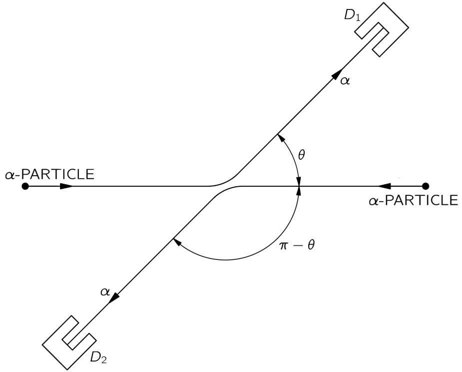


- Consider if both are α particles,

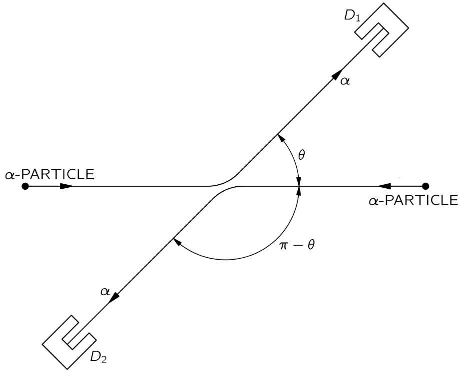


- Consider if both are α particles,

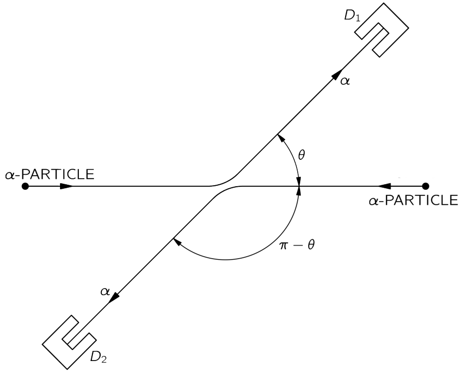
- Then we would not know which particle entered the detector, so the total probability changes to,



- Consider if both are α particles,
- Then we would not know which particle entered the detector, so the total probability changes to,
- The probability of a α particle being detected at detector 1 = $|f(\theta) + f(\pi - \theta)|^2$

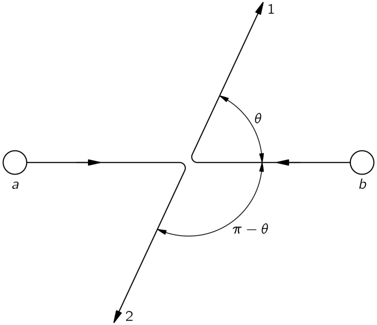


- If $\theta = \dfrac{\pi}{2}$, then applying this to the expression $|f(\theta) + f(\pi - \theta)|^2$ we get,



- If $\theta = \dfrac{\pi}{2}$, then applying this to the expression $|f(\theta) + f(\pi - \theta)|^2$ we get,

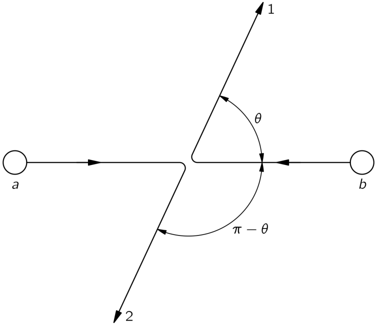
- Probability = $4|f\left(\dfrac{\pi}{2}\right)|^2$, if the particles are indistinguishable.



- Suppose the particles were distinguishable, then the probability for $\theta = \dfrac{\pi}{2}$ when applied for

$$|f(\theta)|^2 + |f(\pi - \theta)|^2$$

is given by



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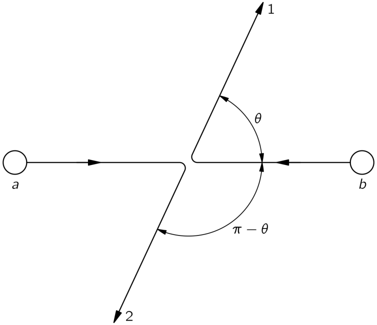

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- This shows that the probability gets doubled for indistinguishable particles.

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- *OBSERVATION*: When we have situation in which the identity of the electron which is arriving at a point is exchanged with another one, the new amplitude interfere with old one with an opposite phase.

- In electrons case , the interfering amplitude for exchange interfere with a negative sign. Probability for electron = $|f(\theta) - f(\pi - \theta)|^2$

Fraction of cases	Particle 1	Particle 2	Spin at D1	Spin at D2	Probability
$\frac{1}{4}$	up	up	up	up	$\lvert f(\theta) - f(\pi - \theta)\rvert^2$
$\frac{1}{4}$	down	down	down	down	$\lvert f(\theta) - f(\pi - \theta)\rvert^2$
$\frac{1}{4}$	up	down	up	down	$\lvert f(\theta)\rvert^2$
			down	up	$\lvert f(\pi - \theta)\rvert^2$
$\frac{1}{4}$	down	up	up	down	$\lvert f(\pi - \theta)\rvert^2$
			down	up	$\lvert f(\theta)\rvert^2$

Identical particles, also called **indistinguishable particle**, are particles that cannot be distinguished from one another.

Identical indistinguishable particles

- Consider particle ’a’ and particle ’b’.

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$$|\langle 1|a\rangle \langle 2|b\rangle + \langle 2|a\rangle \langle 1|b\rangle|^2 ds_1 ds_2$$

- Integrating over the area of the detector , if we let ds_1 and ds_2 range over the whole area (ΔS) , we could count each part of the area twice since the expression $|\langle 1|a\rangle \langle 2|b\rangle + \langle 2|a\rangle \langle 1|b\rangle|^2 ds_1 ds_2$ contains everything that can happen with any pair of surface elements ds_1 and ds_2.

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- $Probability_{BOSE} = \dfrac{(4\,|a\,|^2\,|b\,|^2)}{2}\,(\Delta S)$
- This is just twice what we got the probability for distinguished particles.

State with n Bosons

- Consider n particles say a, b, c...scattered in n direction say 1, 2, 3 ...

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- Consider n particles say a, b, c...scattered in n direction say 1, 2, 3 ...
- Probability that each particle acting alone would go into an element of the surface ds of the detector is $|\langle\cdots\rangle|^2 ds$.

- **Assumption**: All particles are distinguishable.

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- Probability that n particles will be counted together in n different surface elements = $|a_1 b_2 c_3 \dots|^2 ds_1 ds_2 \dots$

- If the amplitude does not depend on where ds is located in the detector , then the

$$Probability = \left(|a|^2 |b|^2 \dots\right) (ds_1 ds_2 \dots)$$

- Integrating each dS over the surface ΔS of the dectector

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- $Probability = \left(|n! abc \ldots|^2\right) (ds_1 ds_2 \ldots)$
- Integrate each ds over the area ΔS of the detector

- Probability that n particles will be counted on the n surface elements is given by
  $Probability = \left(|a_1 b_2 c_3 \ldots + a_1 b_3 c_2 \ldots|^2\right) (ds_1 ds_2 \ldots)$
- $Probability = \left(|n! abc \ldots|^2\right) (ds_1 ds_2 \ldots)$
- Integrate each ds over the area ΔS of the detector
  $(P_n)_{BOSE} = n! \left(|abc \ldots|^2\right) (\Delta S)^n$

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  $(P_n)_{different} = \left(|a|^2|b|^2\ldots\right)(\Delta S)^n$
- $(P_n)_{BOSE} = n!(P_n)_{different}$

- What is the probability that a boson will go into particular state when there are already n particles present ?

Emission and Absorption of photons

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- Consider that there are some atom emitting n photons.
- *OBSERVATION*: The probability that an atom will emit a photon into a particular final state is increased by the factor (n+1) if there are already n photons in that state.

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  where $a = \langle j | a \rangle$ is the amplitude when there are no other photons are present.

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- The above two equation shows that they are symmetric in nature.

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- The co-efficient of absorption, of induced emission and spontaneous emission are all equal and are related to the probability of spontaneous emission.

- For each light frequency ω, there are certain N number of atoms which have two energy states separated, given by the equation $E = \omega\hbar$.

The Blackbody Spectrum

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- NOTE: Each atom in the ground state can absorb a photon and go into the excited state and each atom in the excited state can emit a photon and go to the ground state.

- At equilibrium, the rate of these two process must be equal.

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- $\overline{n}$ is the average number of photons present in a given state with the frequency ω.

- The absorption rate from the state is $N_g\overline{n}|a|^2$, and the emission rate into that state is $N_e(\overline{n}+1)|a|^2$.
- At equilibrium $N_g\overline{n}|a|^2 = N_e(\overline{n}+1)|a|^2$

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- For any harmonic oscillator, the quantum mechanical energy levels are equally spaced with a seperation $\hbar\omega$.

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Considering the boson which do not interact with each other, and in that state the whole system of particles behaves (for all quantum mechanical purpose) exactly like an harmonic oscillator.

- Analysing the Electro-magnetic field in a box, it show the properties of an harmonic oscillation.
- Thus, the number of photons in a particular state in a box, can be equated to the number of energy levels associated with the particular modes of oscillation of the electromagnetic fields.

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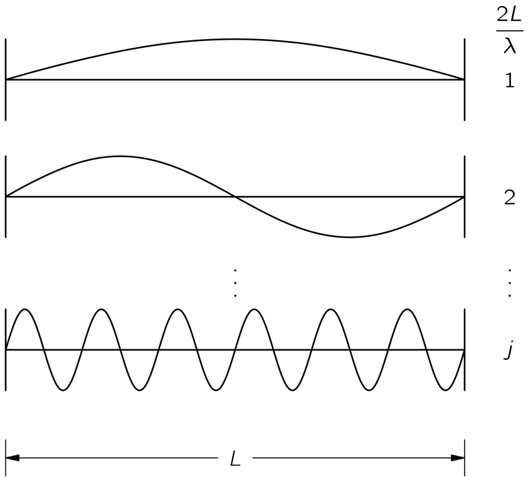
ASSUMPTION

For every mode there are some atoms in the box, which have energy levels that can radiate into that mode so that each mode can get into thermal equilibrium.

- There will be billions of modes in the box and there will be many small frequency intervals $\Delta\omega$.



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- An assumption is made that kL is large that in small interval Δk, there are many modes.

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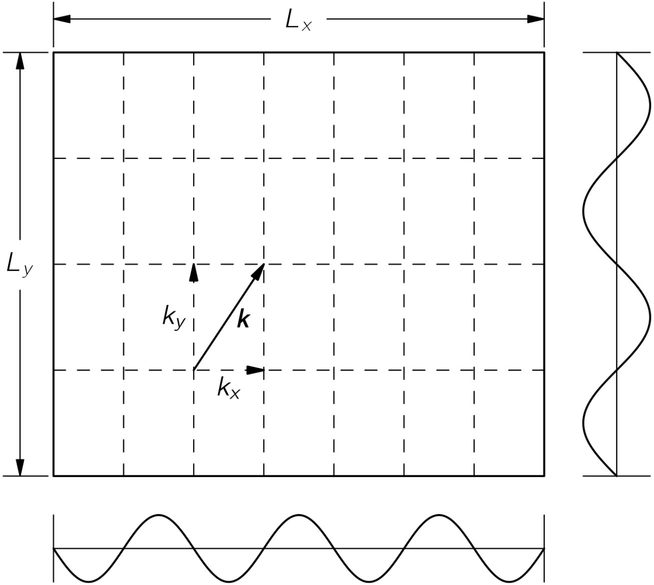
$$\boxed{\Delta W = \frac{L(\Delta k)}{2\pi}}$$


L_x
L_y
k_y
$\boldsymbol{k}$
k_x

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$$\Delta W = \frac{L_x L_y L_z}{(2\pi)^3}(\Delta k_x \Delta k_y \Delta k_z) \tag{6}$$

$$dW(K) = V \frac{d^3 k}{(2\pi)^3} \tag{7}$$

- Applying the above result to find number of photon modes for photons with frequencies in the range Δk.
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- These modes are independent, we must for double the number of modes.
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$$\Delta W(\omega) = \frac{V(\omega^3)\Delta\omega}{\pi^2 c^3} \text{ (for light).}$$

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- Multiplying This by the number of modes, we get the energy ΔE in the modes that lie in the interval $\Delta\omega$:
$$\Delta E = \left(\frac{\hbar\omega}{e^{\hbar\omega/k_BT} - 1}\right)\left(\frac{V\omega^3\Delta\omega}{\pi^2 c^3}\right)$$

The photons are the bosons, which have tendency to try to get to all into the same state.

Thank you