

## Tutorial 1

### Probability MB5042

1. Consider the integers between 0 and 999 inclusive.
  - (a) In how many of these numbers does the digit 9 occur at least once?
  - (b) In how many of these numbers does the digit 9 occur exactly once?
  - (c) How many of these numbers contain both the digits 8 and 9 at least once?
  
2.
  - (a) How many different four-digit whole numbers can be formed from the digits 1, 2, 3 and 4 if repeated digits are allowed?
  - (b) What is the answer if no repeated digits are allowed?
  - (c) What is the answer if the digit 4 may be repeated but the digits 1, 2 and 3 are allowed to occur at most once?
  - (d) How many integers between 100 and 999 contain three different digits? (Remember, 0 can be digit, but it cannot be the first digit.)
  
3. In how many ways can you choose four different *even* numbers from the numbers 1, 2, 3, ..., 20?
  
4. In a football league, there are twenty teams and each team plays each other team twice, once at home and once away. What is the total number of games played in the course of this league's season?

5. A committee of four people is to be chosen from six men and six women. In how many ways can this be done? In how many of the committees so formed are there more men members than women.

6. (a) Show that

$$\frac{n!}{(n-2)!} + \frac{(n-1)!}{n!} = \frac{n^3 - n^2 + 1}{n}.$$

- (b) Let  $n \geq 3$  be a whole number. If

$$3\binom{n}{3} = 5\binom{n}{2},$$

find  $n$ .

- (c) Let  $r \geq 1$  be a whole number. If

$$\binom{15}{r} = \binom{15}{2r},$$

find  $r$ .

7. (a) Expand

$$\left(x^2 - \frac{1}{x}\right)^6$$

using the binomial theorem

- (b) Find the coefficient of  $x^3$  in the expansion of  $(x+1)(x-2)^5$ .

8. (a) In how many ways may 12 white draughts pieces be placed on the 32 black squares of a draughts board if no more than one can be placed on each black square?
- (b) In how many ways may 12 white draughts pieces be placed on the 32 black squares of a draughts board if any number of pieces from 0 to 12 may be placed on a black square? (Draughts pieces of the same colour are taken to be indistinguishable. The squares are all distinct.)