MB5042 Probability DCU PDMT

Tutorial 1

Dr. Maged Shaban

30 January 2025

Outline

Problem 1: Integers Between 0 and 999

Consider the integers between 0 and 999 inclusive.

(a) In how many of these numbers does the digit 9 occur at least once?

(b) In how many of these numbers does the digit 9 occur exactly once?

(c) How many of these numbers contain both the digits 8 and 9 at least once?

Hints:

Use complementary counting for part (a).

For part (b), consider the position of the digit 9.

For part (c), use the inclusion-exclusion principle.

Problem 2: Four-Digit Numbers

(a) How many different four-digit whole numbers can be formed from the digits 1, 2, 3, and 4 if repeated digits are allowed?

(b) What is the answer if no repeated digits are allowed?

(c) What is the answer if the digit 4 may be repeated but the digits 1, 2, and 3 are allowed to occur at most once?

(d) How many integers between 100 and 999 contain three different digits? (Remember, 0 can be a digit, but it cannot be the first digit.)

Hints:

For part (a), use the multiplication principle.

For part (b), use permutations.

For part (c), consider cases based on the number of 4s.

For part (d), ensure the first digit is not 0 and all digits are unique.

Problem 3: Choosing Even Numbers

In how many ways can you choose four different even numbers from the numbers 1, 2, 3, ..., 20?

Hints:

Identify the even numbers in the range 1 to 20.

Use combinations to select 4 numbers from the even numbers.

Problem 4: Football League Games

In a football league, there are twenty teams, and each team plays each other team twice, once at home and once away. What is the total number of games played in the course of this league's season?

Hints:

Calculate the number of unique pairs of teams.

Multiply by 2 to account for home and away games.

Problem 5: Forming a Committee

A committee of four people is to be chosen from six men and six women.

(a) In how many ways can this be done?

(b) In how many of the committees so formed are there more men members than women?

Hints:

For part (a), use combinations to select 4 people from 12.

For part (b), consider cases where the committee has 3 men and 1 woman or 4 men and 0 women.

Problem 6: Expanding Expressions

(a) Expand

$$\left(x^2 - \frac{1}{x}\right)^6$$

using the binomial theorem.

(b) Find the coefficient of x^3 in the expansion of

$$(x+1)(x-2)^5.$$

Hints:

For part (a), apply the binomial theorem to expand the expression.

For part (b), expand $(x-2)^5$ and multiply by $(x+1)$, then find the coefficient of x^3.

Problem 7: Combinatorial Identities

(a) Show that

$$\frac{n!}{(n-2)!} + \frac{(n-1)!}{n!} = \frac{n^3 - n^2 + 1}{n}.$$

(b) Let $n \geq 3$ be a whole number. If

$$3\binom{n}{3} = 5\binom{n}{2},$$

find n.

(c) Let $r \geq 1$ be a whole number. If

$$\binom{15}{r} = \binom{15}{2r},$$

find r.

Problem 8: Placing Draughts Pieces

(a) In how many ways may 12 white draughts pieces be placed on the 32 black squares of a draughts board if no more than one can be placed on each black square?

(b) In how many ways may 12 white draughts pieces be placed on the 32 black squares of a draughts board if any number of pieces from 0 to 12 may be placed on a black square? (Draughts pieces of the same color are taken to be indistinguishable. The squares are all distinct.)

Hints:

For part (a), use combinations to select 12 squares out of 32.

For part (b), use the stars and bars theorem.