Homework 2

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Answer by filling the pdf, save it as pdf, and upload to UGLA latest on Monday 4.9.2017.

## **Problems:**

1. Let  $\mathcal{P}$  denote the set of polynomials with integer coefficients, i.e., functions  $\mathbb{R} \to \mathbb{R}$  of form  $a_0 + a_1 x + a_2 x^2 + \ldots + a_n x^n$ , where the  $a_i$  are integers. Hence, e.g.,  $f(x) = 1 - x^2$  belongs to  $\mathcal{P}$ , while x/2 and  $\sin(x)$  do not. Define mappings (functions)

$$I(f) = \int_0^x f(s) ds, \qquad D(f) = \frac{d}{dx} f(x) = f'(x),$$

i.e., a definite integral and derivative of a given function f, yielding another function as a result.

- a) Is  $\mathcal{P}$  closed under integral I(f)?
- Yes/No:
- b) Is  $\mathcal{P}$  closed under derivation D(f)?
- Yes/No:

2. Let A be a regular language,  $B = A^*$ , and  $C = A \circ B$ .

- a) Is A = B? \_\_\_\_\_ (yes/no)
- b) Is B = C? \_\_\_\_\_ (yes/no)
- c) Is C = A? \_\_\_\_\_ (yes/no)
- 3. Consider the two NFAs of Problem 1.16 from the book. Which one of the following strings the machines accept:

	Machine (a)	Machine (b)
aaaaa	Yes/No:	Yes/No:
bbbbb	Yes/No:	Yes/No:
abba	Yes/No:	Yes/No:
baba	Yes/No:	Yes/No:

- 4. Does a self-transition with symbol  $\epsilon$  make any sense with NFAs? Yes/No:
- 5. Suppose  $\Sigma = \{0, 1\}$  and let  $M_i$  denote an NFA that recognizes the strings with the ith last symbol 1, and  $\bar{M}_i$  its complement, i.e., an NFA that recognizes string where the ith last bit is zero.
  - a) Does  $M_1 \cap M_2 \cap \bar{M}_3 \cap M_4$  recognize string 11101101011?  $\underline{Y}_3$

Yes/No:

b) Does  $M_1 \cup M_2 \cup \bar{M}_3 \cup M_4$  recognize string 111111111111?

Yes/No:

c) Does  $(M_1)^*$  recognize string 1011?

Yes/No:

- 6. Consider the Exercise 1.24 from the book and FST  $T_1$ .
  - a) What is the output on input 011:
  - b) What is the output on input 211:
  - c) What is the output on input 0202: