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Sid Assawaworrarit and Shanhui Fan ✉

Department of Electrical Engineering, Ginzton Laboratory, Stanford University, Stanford, CA, USA. ✉e-mail: shanhui@stanford.edu

Robust and efficient wireless power transfer using a switch-mode implementation of a nonlinear parity-time symmetric circuit: Supplementary Information

Sid Assawaworrarit and Shanhui Fan*

Department of Electrical Engineering, Ginzton Laboratory, Stanford University, Stanford, CA 94305.

** To whom correspondence should be addressed. Email: shanhui@stanford.edu.*

S1: PT-symmetric wireless power transfer model and its transfer efficiency

In this section, we consider the PT-symmetric wireless power transfer model and derive the transfer efficiency. We begin with the model of Eq. (1) in the manuscript where the amplitudes of the source and receiver resonators, $\boldsymbol{a} = [a_s,\ a_r]^T$, are governed by $i\frac{d}{dt}\boldsymbol{a} = H(|a_s|)\boldsymbol{a}$ with the following nonlinear Hamiltonian:

$$H(|a_s|) = \omega_0 \begin{bmatrix} 1 + i\dfrac{g(|a_s|)}{2} & \dfrac{k}{2} \\ \dfrac{k}{2} & 1 - i\dfrac{\gamma}{2} \end{bmatrix} \qquad (S1.1)$$

To find the steady-state oscillation frequency, ω_{ss}, we look for solutions to Eq. (S1.1) in the form $\boldsymbol{a} \sim e^{-i\omega_{ss}t}$ and obtain the following characteristic equation:

$$\left(1 - \frac{\omega_{ss}}{\omega_0} + i\frac{g}{2}\right)\left(1 - \frac{\omega_{ss}}{\omega_0} - i\frac{\gamma}{2}\right) - \left(\frac{k}{2}\right)^2 = 0. \qquad (S1.2)$$

Unlike the common approach used for the study of linear PT symmetric system, where one imposes an externally fixed value of gain g_1 and computes the eigenfreqeuncy which in general is complex, here we allow g to vary freely and constrain ω_{ss} to be real (i.e. zero imaginary part) to obtain steady-state solutions. The strength of g corresponding to such a steady-state solution then becomes the saturated gain level g_{ss}. We obtain:

$$\omega_{ss} = \begin{cases} \omega_0, & k < \gamma \\ \omega_0\left[1 \pm \frac{1}{2}\sqrt{k^2 - \gamma^2}\right], & k \geq \gamma \end{cases} \qquad (S1.3)$$

and

$$g_{ss} = \begin{cases} k^2/\gamma, & k < \gamma \\ \gamma, & k \geq \gamma \end{cases}. \tag{S1.4}$$

Stability analysis (Ref. 3 in the manuscript) shows that these steady-state solutions are stable for any gain saturation model $g(|a_s|)$. This gain saturation (Eq. (S1.4)) then determines the source amplitude in steady state, $|a_{s,ss}|$. These steady-state solutions can be plugged back into Eq. (S1.1) to obtain the resonator amplitudes' distribution:

$$\left|\frac{a_{r,ss}}{a_{s,ss}}\right| = \begin{cases} k/\gamma, & k < \gamma \\ 1, & k \geq \gamma \end{cases}. \tag{S1.5}$$

Here we note the steady-state amplitudes' ratio which remains robust with respect to the coupling factor in the strong coupling region. This result, together with the saturated gain of Eq. (S1.4) which keeps source amplitude constant, makes the receiver amplitude robust against variations in the coupling factor in the strong coupling region, too. This fact allows for robust power transfer from the source to the receiver where delivered power is invariant to variations in the transfer distance in the strong coupling region. The power delivered to the load on the receiver can be written as:

$$P_{load} = \omega_0 \gamma |a_{r,ss}|^2 = \begin{cases} \frac{\omega_0 k^2}{\gamma} |a_{s,ss}|^2, & k < \gamma \\ \omega_0 \gamma |a_{s,ss}|^2, & k \geq \gamma \end{cases}. \tag{S1.6}$$

where $|a_{s,ss}|$ can be calculated from the gain saturation in Eq. (S1.4).

The transfer efficiency, defined as the fraction of the energy injected into the source resonator that is delivered to the load, can be calculated by including the intrinsic loss parameters in the model of Eq. (1). Let Γ_s and Γ_r be the intrinsic loss factors (inverses of quality factors) of the source and receiver resonators, and γ_l represents the loss factor attributed to the receiver load so that $\gamma = \Gamma_r + \gamma_l$. The transfer efficiency is:

$$\eta_{tf} = \frac{\gamma_l |a_r|^2}{\Gamma_s |a_s|^2 + (\Gamma_r + \gamma_l)|a_r|^2} = \begin{cases} \left(1 - \frac{\Gamma_r}{\gamma}\right) \cdot \frac{1}{1 + \frac{\Gamma_s \gamma}{k^2}}, & k < \gamma \\ \left(1 - \frac{\Gamma_r}{\gamma}\right) \cdot \frac{1}{1 + \frac{\Gamma_s}{\gamma}}, & k \geq \gamma \end{cases}. \tag{S1.7}$$

The efficiency reaches maximum and remains so in the strong coupling region, limited by the intrinsic losses of the resonators. From our estimate of the coil intrinsic losses, $\Gamma_s, \Gamma_r \approx 1/800$, the transfer efficiency of 92% is obtained in the strong coupling region. The efficiency measure above represents the maximum power transfer efficiency possible with the PT-symmetric wireless power transfer setup. For the total system efficiency to reach this level, the gain element in the source must be 100% efficient.

S2: The condition for eliminating switching loss in a switch-mode amplifier and its gain parameter

For the circuit shown in Fig. 2b, we derive the optimal condition that enables us to eliminate switching loss. A more detailed discussion can be found in Refs. 33 and 35. For this part, the input signal is applied between the gate and source terminal as a binary (low/high) voltage level. The output is the drain-to-source voltage waveform to be determined. The transistor is assumed to operate as an ideal switch such that the transistor drain-to-source path is shorted (switched on) when the input signal is high, and open with no current flow (switched off) when the input signal is low. Here we assume a square-wave input at frequency f at 50% duty cycle. A constant current source I_{DC}, realized by connecting a constant voltage source (V_{DC}) with a radio-frequency choke (RFC), supplies power to the circuit. The shunt capacitor C_0 sits across the transistor's output. The transistor's output is connected to an LC filter in series with the load of resistance R. The filter is assumed to have small reactance X at the switching frequency $\omega = 2\pi f$ and take on sufficiently high reactance value at each of the higher harmonics. In general, the load network from the transistor's output to load may be more complicated, in which case R and X are effective parameters. For the circuit in Fig. 2b, $X = \omega L_1 - \frac{1}{\omega C_1}$. The current I_{L1} flowing through L_1 and to the load can be written as

$$I_{L1}(\tau) = I_m \sin(\tau - \phi)\,. \qquad (S2.1)$$

Here $\tau = \omega t$. I_m is the amplitude and ϕ is the phase delay relative to the moment the switch is turned on. I_m and ϕ will be determined self-consistently as following: From the load current, the ω-component of the transistor's output voltage can be written as:

$$V_{DS,\omega}(\tau) = RI_m \sin(\tau - \phi) + XI_m \cos(\tau - \phi)\,. \qquad (S2.2)$$

Given the constant current supply and the waveform of the input, the current through the capacitor C_0 is,

$$I_{C0}(\tau) = \begin{cases} 0 & 0 \le \tau \le \pi \\ I_{DC} - I_m \sin(\tau - \phi) & \pi < \tau < 2\pi \end{cases}.$$

And, therefore, the voltage across C_0 is,

$$V_{DS}(\tau) = \frac{1}{\omega C_0} \int_0^{\tau} I_{C0}(\tau')\, d\tau' = \begin{cases} 0 & 0 \le \tau < \pi \\ \dfrac{1}{\omega C_0}\, [I_{DC}(\tau - \pi) + I_m(\cos(\tau - \phi) + \cos(\phi))] & \pi \le \tau < 2\pi \end{cases}. \qquad (S2.3)$$

The Fourier component at ω of the output voltage in Eq. (S2.3) needs to be consistent with the expression in Eq. (S2.2). Thus, the parameters I_m and ϕ, which define the switch-mode amplifier's operating point, need to satisfy the following equations:

$$\begin{aligned} -\pi \cos\phi - 2\sin\phi &= \frac{I_m}{I_{DC}} \left(\pi\omega C_0 R + 2\cos^2\phi \right) \\ 2\cos\phi - \pi \sin\phi &= \frac{I_m}{I_{DC}} \left(\pi\omega C_0 X - \left(\frac{\pi}{2} - \sin 2\phi\right) \right) \end{aligned} \qquad (S2.4)$$

At the transition time corresponding to $\tau = 2\pi$, the transistor switches from the off to on states, the output voltage of the transistor drops instantly to zero, leading to a switching power loss of $\frac{fC_0V_{sw}^2}{2}$ where $V_{sw} = V_{DS}(2\pi)$ is the transistor output voltage right before the transition time. To eliminate such switching loss, the output voltage should return to zero at such transition time, i.e. $V_{sw} = 0$. In addition, we impose the condition that the output voltage should return to zero smoothly with zero derivative, $V_{DS}'(2\pi) = 0$. From Eq. (S2.2), these two conditions fix the circuit operating point: $\frac{I_m}{I_{DC}} = \frac{\sqrt{\pi^2+4}}{2} = 1.86$ and $\phi = \pi + \tan^{-1}\left(\frac{2}{\pi}\right) = 212.5°$. Using these operating parameters and Eq. (S2.4), we can find the relationships between the circuit parameters:

$$\omega C_0 R = \frac{8}{\pi(\pi^2+4)} = 0.184, \qquad (S2.5a)$$

$$\frac{X}{R} = \frac{\pi(\pi^2-4)}{16} = 1.152 \qquad (S2.5b)$$

Satisfying these relationships ensures that there is no switching loss in the amplifier.

The gain rate of the switch-mode amplifier can be calculated (Ref. 39) from $g_{\text{rate}} = \frac{P_{\text{inject}}}{2W}$ where $P_{\text{inject}} = \langle V_{DS} \cdot I_{L1} \rangle$ is the per-cycle-average power the amplifier feeds to L_1, and $W = \frac{L_1 I_m^2}{2}$ is the stored energy in L_1C_1 tank. Here we assume that, by using a phase-delay feedback, we can lock the timing of the switch signal to the I_{L1} so that ϕ remains constant. From Eqs. (S2.1) and (S2.3), the gain constant parameter that appears in Eq.1 of the manuscript can be written as,

$$\begin{aligned} g(I_m) = \frac{2g_{\text{rate}}}{\omega_0} &= \frac{2}{\omega_0^2 L C_0}\left[\frac{I_{DC}}{I_m}\left(-\frac{\cos\phi}{2} - \frac{\sin\phi}{\pi}\right) - \frac{\cos^2\phi}{\pi}\right] \\ &= \frac{2}{\omega_0^2 L C_0}\left[\frac{I_{DC}}{I_m}\left(\frac{\frac{\pi}{2}+\frac{2}{\pi}}{\sqrt{\pi^2+4}}\right) - \frac{\pi}{\pi^2+4}\right]. \qquad (S2.6) \end{aligned}$$

We may also express Eq. (S2.6) in terms of the coupled-mode amplitude, $|a_s| = \sqrt{\frac{L}{2}} I_m$. Note that in case where $V_{DS}(\tau)$ is more complicated than the expression in Eq. (S2.3) (for example, if the body diode of the FET switch turns on) then the gain constant given in Eq. (S2.6) is inaccurate and will need to be numerically evaluated.

S3: Circuit design and design procedure for robust-efficient wireless power transfer circuit

Here we show that a suitably designed switch-mode wireless power transfer circuit can maintain the efficient switching condition inside the strong coupling region. The challenge is to find the circuit parameters such that the equivalent impedance being presented to the transistor output satisfy Eq. (S2.5) for a wide range of coupling constants. Consider the circuits in Fig. 1a and Fig. 1b. The receiver resonator is characterized by its resonant frequency, $\omega_0 = \frac{1}{\sqrt{L_2 C_2}}$, and loss constant, $\gamma = \frac{\sqrt{L_2}}{R_2\sqrt{C_2}}$. In the circuit of Fig. 1b, on the source side, the effect of the receiver resonator is treated by introducing impedance in series to L_1,

$$Z_{eq}(\omega, k) = \frac{\omega^2 k^2 L_1 L_2}{i\omega L_2 + R_2 || \frac{1}{i\omega C_2}}\ .$$

For sufficiently high quality-factor device coil, $\ll 1$, we can express the resistive and reactive parts of the equivalent impedance separately, $Z_{eq}(\omega, k) = R_{eq}(\omega, k) + iX_{eq}(\omega, k)$, as

$$R_{eq}(\omega, k) = \frac{A(k)\left(\frac{\gamma}{2}\right)^2}{\left(\frac{\omega-\omega_0}{\omega_0}\right)^2 + \left(\frac{\gamma}{2}\right)^2}\ , \qquad (S3.1a)$$

$$X_{eq}(\omega,k) = -\frac{A(k)\frac{\gamma}{2}\left(\frac{\omega-\omega_0}{\omega_0}\right)}{\left(\frac{\omega-\omega_0}{\omega_0}\right)^2+\left(\frac{\gamma}{2}\right)^2}, \qquad (S3.2b)$$

where $A(k)=\frac{k^2}{\gamma}\omega_0 L_1$ represents the equivalent resistive load at the device resonant frequency. From here, we can see the relationship between the eigenfrequencies of the PT-symmetric arrangement and efficient switching condition. At the eigenfrequencies of the underlying simplified nonlinear PT system, $\omega_{ss}(k)=\omega_0\left(1\pm\frac{1}{2}Re[\sqrt{k^2-\gamma^2}]\right)$ in the strong coupling region, $k\geq\gamma$, the equivalent resistance takes on a constant value $R_{eq}(\omega_{ss},k)=\gamma\omega_0 L_1$ while the equivalent reactance varies with coupling, $X_{eq}(\omega_{ss},k)=\mp\omega_0 L_1\sqrt{k^2-\gamma^2}$. The equivalent resistance represents the resistive load at the amplifier output. Thus, for moderate coupling constant values, a single value of C_0 can accommodate the optimal switching condition of Eq. (S2.5a):

$$C_{0,opt}=\frac{8}{\pi(\pi^2+4)}\frac{1}{\gamma\omega_0^2 L_1}. \qquad (S3.3)$$

The reactance present at the amplifier's output, including contributions from L_1 and C_1, is:

$$\begin{aligned} X(\omega_{ss},k) &= \omega L_1-\frac{1}{\omega C_1}+X_{eq}(\omega_{ss},k) \\ &\approx \omega_0 L_1-\frac{1}{\omega_0 C_1}+\left(L_1+\frac{1}{\omega_0^2 C_1}\right)(\omega_{ss}-\omega_0)+X_{eq}(\omega_{ss},k) \\ &\approx X_0\pm 2L_1\left(\frac{\omega_0}{2}\sqrt{k^2-\gamma^2}\right)\mp\omega_0 L_1\sqrt{k^2-\gamma^2} \\ &= X_0 \end{aligned}$$

where $X_0=\omega_0 L_1-\frac{1}{\omega_0 C_1}$ and the approximation assumes high coil quality factor ($\omega_0 L_1\gg R_{eq}$) and small resonance offset of the L_1C_1-tank from ω_0. Thus, again a single value of C_1 can maintain the optimal switching condition of Eq. (S2.5b):

$$C_{1,opt}=\frac{1}{\omega_0}\left(\frac{1}{\omega_0 L_1\left(1-\gamma\frac{\pi(\pi^2-4)}{16}\right)}\right). \qquad (S3.4)$$

S4: Modelling of the total system efficiency and transferred power

The total system efficiency can be modelled by considering the two main loss mechanisms in the system: the intrinsic losses of the resonators and the amplifier loss. The total system efficiency can be expressed in terms of the efficiencies of these two processes:

$$\eta_{total}=\eta_{tf}\cdot\eta_{amp} \qquad (S4.1)$$

Here the transfer efficiency, η_{tf}, which is given in Eq. (S1.7), accounts for the power loss through the intrinsic losses of the two resonators and represents the fraction of the power injected into the source resonator that is delivered to the load on the receiver side.

The amplifier efficiency, η_{amp}, which represents the fraction of the supplied DC power that is injected into the source resonator, can be numerically calculated. In addition to switching loss outlined in

Supplementary Information S2, here we include an analysis on the energy loss through reverse conduction which is the main loss component for our system in the weak coupling region. Reverse conduction loss can be modelled with an antiparallel diode that conducts when the voltage on the FET drain terminal is decreased beyond some voltage drop, $|V_{rc}|$, below the source terminal. In this case, the current and drain-to-source voltage (Supplementary Information S2) need to be modified as follows:

$$I_D(\tau) = \begin{cases} I_{DC} - I_m \sin(\tau - \phi) & 0 \le \tau < \pi \\ 0 & \pi \le \tau < \tau_{rc} \\ I_{DC} - I_m \sin(\tau - \phi) & \tau_{rc} \le \tau < 2\pi \end{cases}, \tag{S4.2}$$

$$V_{DS}(\tau) = \begin{cases} 0 & 0 \le \tau < \pi \\ \dfrac{1}{\omega C_0}[I_{DC}(\tau - \pi) + I_m(\cos(\tau - \phi) + \cos\phi)] & \pi \le \tau < \tau_{rc} \\ V_{rc} & \tau_{rc} \le \tau < 2\pi \end{cases}. \tag{S4.3}$$

where τ_{rc} is the (normalized) start time of reverse conduction and $V_{rc} = -2V$ for GS61008P. Here we assume that reverse conduction lasts until the end of the cycle, $I_D(2\pi^-) < 0$.

Given DC supplied current I_{DC} and phase ϕ, we can numerically solve the following equations for τ_{rc} and I_m:

$$V_{DS}(\tau_{rc}^-) = V_{rc}, \tag{S4.4a}$$

$$\int_0^{2\pi} V_{DS}(\tau) \sin(\tau - \phi)\, d\tau = I_m R_{eq} \pi, \tag{S4.4b}$$

where the top equation comes from continuity of $V_{DS}(\tau)$ and the bottom equation comes from the same consistency argument outlined in Supplementary Information S2. R_{eq} is from Eq. (S3.1a) in the Supplementary Information S3.

The supplied power, power loss through reverse conduction, and power loss through switching loss can be calculated as follows:

$$P_{DC} = \frac{I_{DC}}{2\pi} \int_0^{2\pi} V_{DS}(\tau)\, d\tau, \tag{S4.5a}$$

$$P_{rc} = \frac{V_{rc}}{2\pi} \int_{\tau_{rc}}^{2\pi} I_D(\tau)\, d\tau, \tag{S4.5b}$$

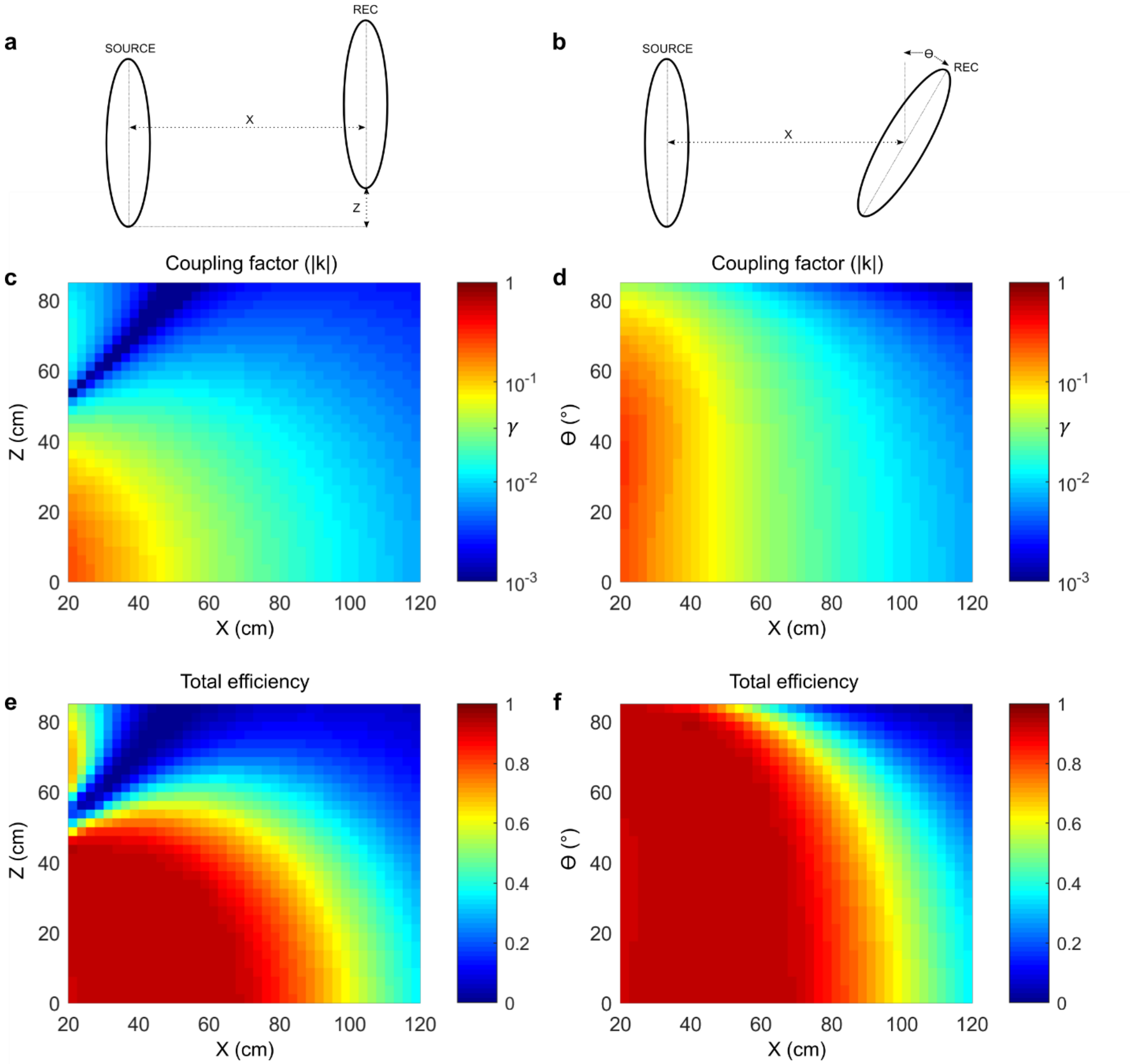
$$P_{sw} = \frac{1}{4\pi} \omega C_0 V_{DS}^2(2\pi^-). \tag{S4.5c}$$

Finally the amplifier efficiency and delivered power can be calculated:

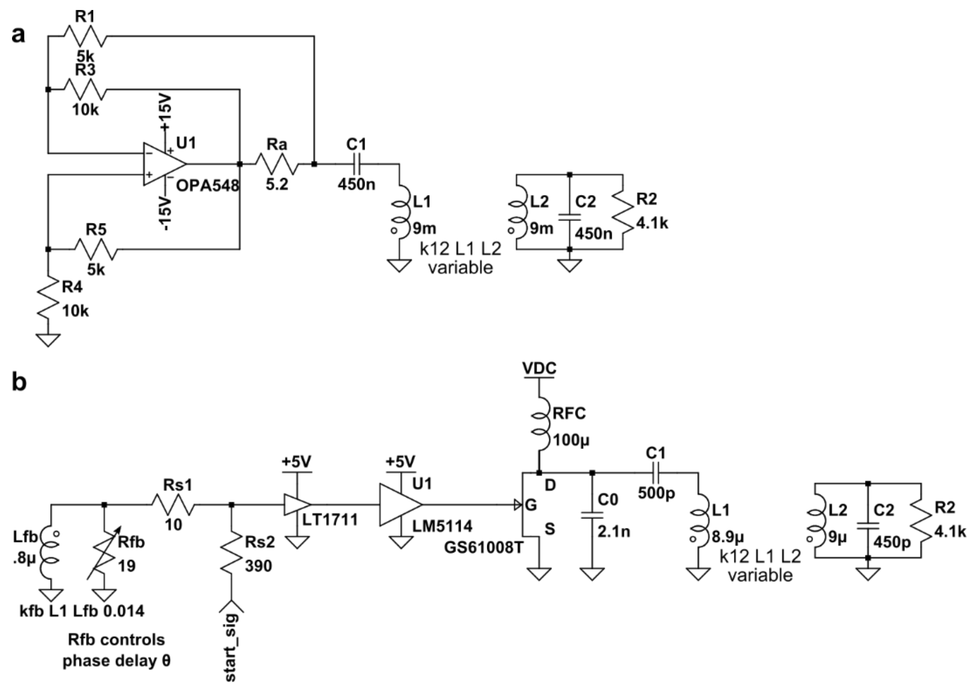
$$\eta_{amp} = 1 - \frac{P_{rc}}{P_{DC}} - \frac{P_{sw}}{P_{DC}}, \tag{S4.6}$$

$$P_{load} = \eta_{total}\, P_{DC}. \tag{S4.7}$$

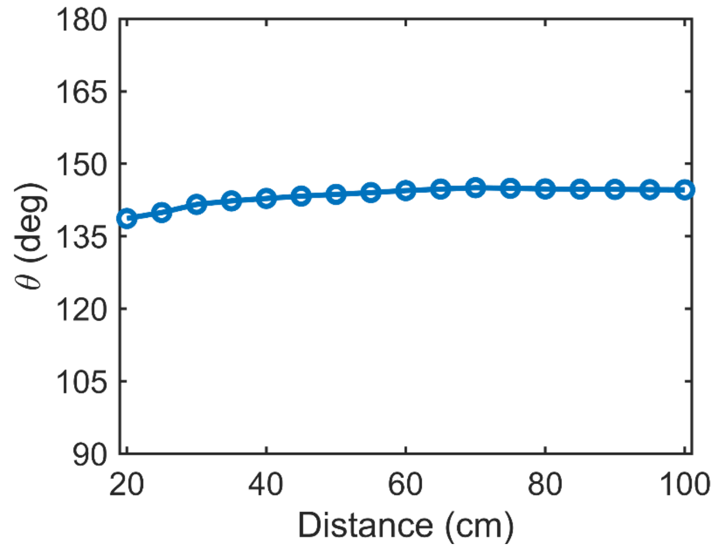
The calculated total system efficiency, delivered power, and amplifier output waveforms shown in Supplementary Fig. 4 are in good agreement with measurement results.



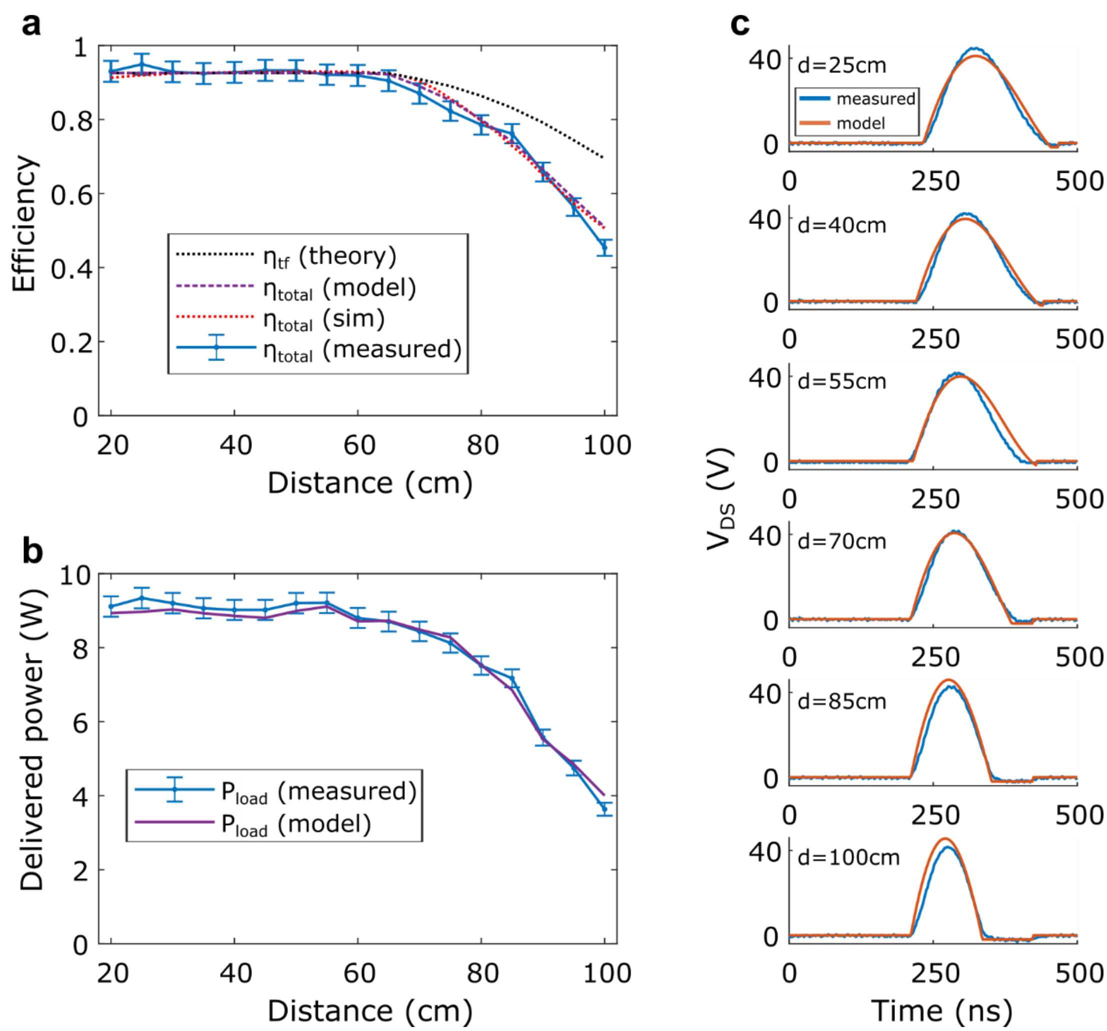
Supplementary Figure 1| Total system transfer efficiency beyond coaxial alignment. **a**,**b**, Lateral offset (a) and receiver tilt (b) misalignments. **c**,**d**, Coupling constant calculated from mutual coupling between the coils as function of the coils' separation and relative alignment for lateral offset (c) and tilt of receiver coil (d). **e**,**f**, Total power transfer efficiency (simulation) for the case of lateral offset (e) and receiver coil tilt (f).



Supplementary Figure 2| Circuit diagrams used for simulations and efficiency comparisons in Fig. 1. **a**, NIC-based circuit (Fig. 1c). **b**, Switch-mode-amplifier-based circuit (Fig. 1d).



Supplementary Figure 3| Phase delay variation with the transfer distance. Shown here is the calculated phase delay $\theta = \pi - \mathrm{atan}\left(\frac{R_{fb}}{\omega L_{fb}}\right) + \omega \Delta t_{pd}$ where $\Delta t_{pd} = 25$ ns is the total propagation delay in the circuit.



Supplementary Figure 4| Modelling and measurement of the system efficiency and delivered power. **a**,**b**, Calculation and measurement of the total system efficiency (a) and delivered power (b) as functions of the transfer distance. **c**, Measurement and model comparison of the amplifier output voltage waveforms at various transfer distances.