Markov Localization Reference

Udacity

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1 Goal

Our goal as presented in the concepts "Localization Posterior: Introduction" and "Localization Posterior: Explanation and Implementation" is to determine the belief state for pose x at time t. More formally, we wish to determine our belief state given our observations, controls, and the map: $bel(x_t) = P(x_t|z_{1:t}, \mu_{1:t}, m)$. Solving this directly requires carrying an impractical amount of data. Instead, we apply Bayes Rule, the Law of Total Probability, and the Markov assumption to arrive at a recursive state estimator in the form of $bel(x_t) = P(x_t|z_t, z_{1:t-1}, \mu_{1:t}, m) = \eta p(z_t|x_t, m)\hat{bel}(x_t)$.

2 Definitions

2.1 Localization Variables

The critical model parameters or variables are defined below.

- Observations

  $z_{1:t}$ represents the observation vector from time 0 to t (range measurements, bearing, images, etc.).

- Controls

  $\mu_{1:t}$ represents the control vector from time 0 to t (yaw/pitch/roll rates and velocities).

- Map

  m represents the map (grid maps, feature maps, landmarks).

- Pose

  x_t represents the current pose at time t (position (x,y) + orientation θ).

3   Equations

- Generalized Bayes Rule

  $P(a \mid b) = \frac{P(b|a)\, P(a)}{P(b)}$

- Bayes Rule applied to Localization

  See the concepts "Bayes Rule and Law of Total Probability" for a review.

  $bel(x_t) = \frac{(observation \quad model)\,(motion \quad model)}{normalizing \quad constant}$

- Observation Model

  $P(z_t|x_t, z_{1:t-1}, \mu_{1:t}, m)$

- Observation Model Simplified by Markov Assumption

  $P(z_t|x_t, z_{1:t-1}, \mu_{1:t}, m) = P(z_t|x_t, m)$

- Motion Model

  $P(x_t|z_{1:t-1}, \mu_{1:t}, m)$

- Bayes Filter for Localization (Markov Localization)

  $bel(x_t) = P(x_t|z_t, z_{1:t-1}, \mu_{1:t}, m) = \eta p(z_t|x_t, m)\hat{bel}(x_t)$ where $\hat{bel}(x_t)$ is our estimated belief state.

4   Concepts

Essential localization concepts

4.1   Markov Assumption

- A Markov process is one in which the conditional probability distribution of future states (ie the next state) is dependent only upon the current state and not on other preceding states. This can be expressed mathematically as:

  $P(x_t|x_{1-t}), ..., x_{t-i}, ..., x_0) = P(x_t|x_{t-1})$

  It is important to note that the current state may contain all information from preceding states. That is the case we are discussing in this lesson.

4.2   Law of Total Probability

- What is the definition?? How do we use it??

  $P(B) = \sum\limits_{i-1}^{\infty} P(B|A_i)P(A_i)$

4.3 Bayes Rule in Localization Terms

- Generalized Bayes Rule

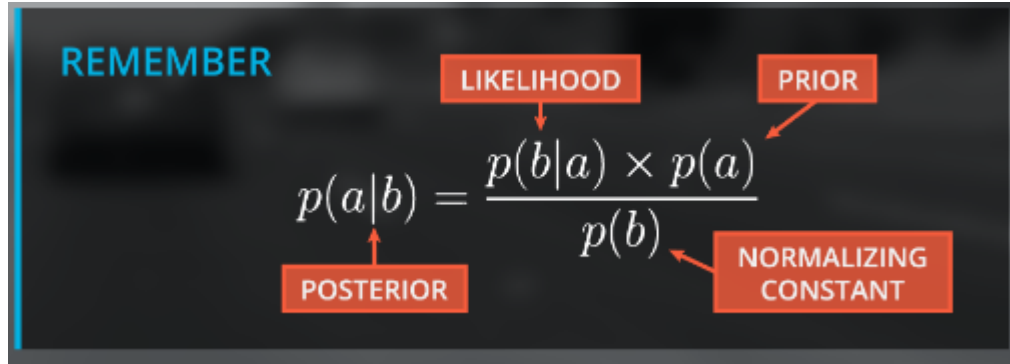$P(a \mid b) = \frac{P(b|a)\,P(a)}{P(b)}$

Figure 1: Bayes Rule

- Likelihood $P(b \mid a)$: In terms of localization, this is our observation model.

- Prior $P(a)$: In terms of localization, this is our motion model.

- Normalizing Constant (total probability of evidence b, being true) P(b): In terms of localization this is the total probability of our motion model

- Bayes Rule applied to Localization

  By substitution we arrive at Bayes Rule in localization terms

  $bel(x_t) = \frac{(observation \quad model)\,(motion \quad model)}{normalizing \quad constant}$

- Normalization Constant Eta

  To simplify we define the normalization constant as Eta, the inverse of the original normalization term. This is the sum of the products of the observation and motion models over all possible values of x_t^i

  $\sum_i p(z_t|x_t^i, z_{1:t-1}, \mu_{1:t}, m) p(x_t^i, z_{1:t-1}, \mu_{1:t}, m)$
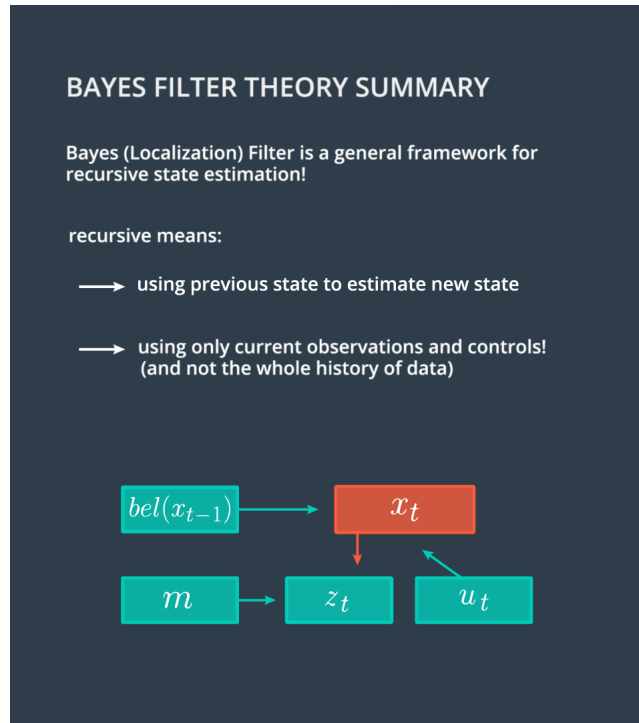
5 Bayes Filter Summary

Figure 2: Bayes Filter Summary

- The Bayes Localization Filter Markov Localization is a general framework for recursive state estimation.

  That means this framework allow us to use the previous state (state at t-1) to estimate a new state (state at t) using only current observations and controls (observations and control at t), rather than the entire data history (data from 0:t).
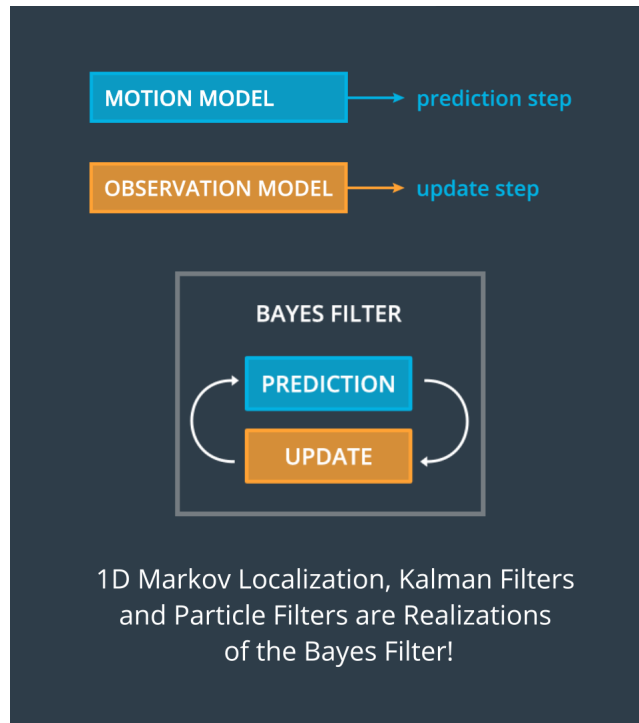
Figure 3: Bayes Filter Summary 2

- The motion model describes the prediction step of the filter while the observation model is the update step.

  The state estimation using the Bayes filter is dependent upon the interaction between prediction (motion model) and update (observation model steps) and all the localization methods discussed so far are realizations of the Bayes filter.