The U-plane of rank-one $4d\ \mathcal{N}=2$ KK Theories

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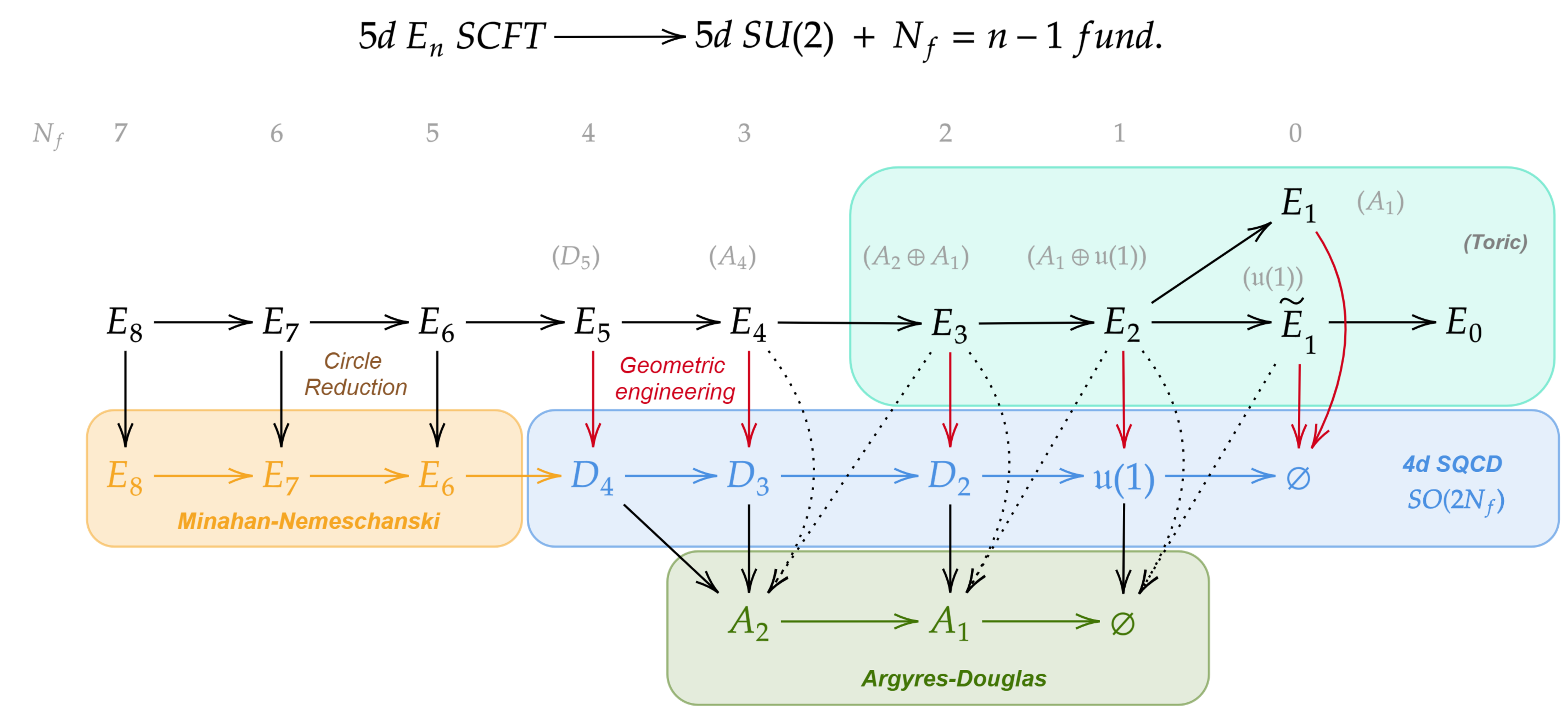
Seiberg-Witten geometry

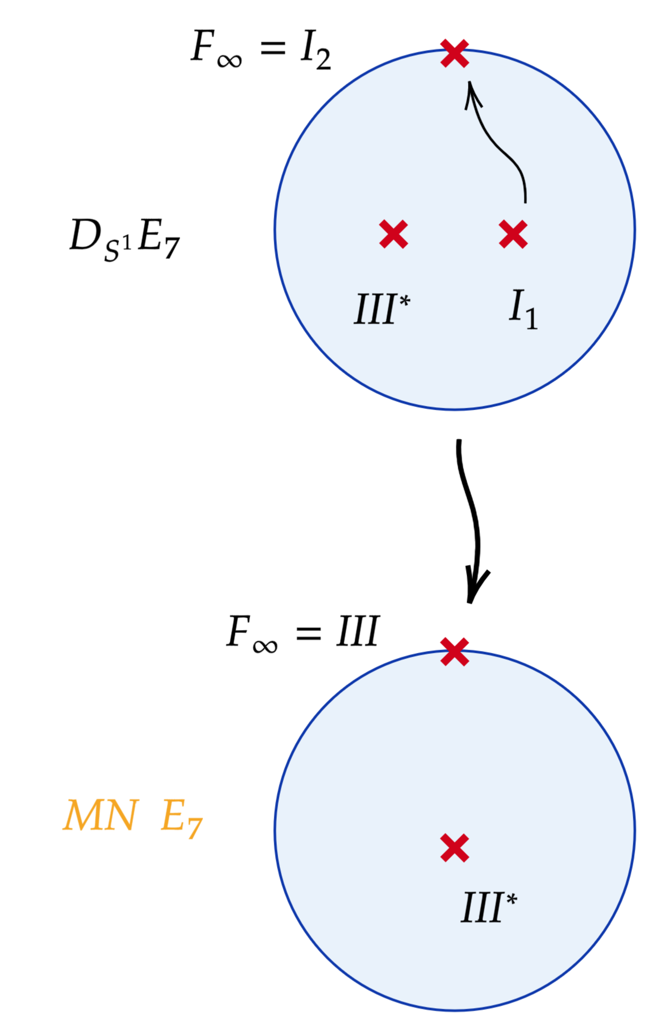
- Low energy physics on Coulomb branch (CB) of 4d $\mathcal{N}=2$ theories famously encoded in a *Rational Elliptic Surface*. CB parametrized by gauge invariant operator $u=\langle \mathrm{Tr}\,\phi^2\rangle$. For KK theories, this becomes the VEV of a Wilson line operator:
$$U = e^{2\pi i a} + e^{-2\pi i a} + \dots\ .$$
- Singular fibers classified by Kodaira. Simple constraints:
$$y^2 = 4x^3 - g_2(U,\boldsymbol{M})x - g_3(U,\boldsymbol{M})\ , \qquad \sum_v \mathrm{ord}(\Delta) = 12\ .$$
- Sections of RES form the *Mordell-Weil* group:
$$\Phi = \mathbb{Z}^r \oplus \Phi_{tor}\ .$$
- Configurations of singular fibers of RES completely classified by Persson and Miranda! A rank-one $\mathcal{N}=2$ theory is then specified by fixing the singular fiber at infinity F_∞.

Global aspects from the Coulomb branch

Setup: IIA on $\widetilde{\boldsymbol{X}} \longleftrightarrow$ IIB on $\hat{\boldsymbol{Y}}$, where $\widetilde{\boldsymbol{X}} = \mathrm{Tot}(\mathcal{K}\to dP_n)$. We use Hori-Vafa mirrors in F-theory uplift:
$$E\times\mathbb{C}^* \longrightarrow \hat{\boldsymbol{Y}} \longrightarrow \mathbb{C}\ .$$

1. **Non-abelian flavour symmetry algebra** is encoded in the Kodaira fibers. Fiber at infinity does not contribute to flavour symmetry!
2. **Abelian flavour symmetry** is $U(1)^r$, with r the rank of the Φ.
3. **Torsion part** of Φ restricts global form of the flavour symmetry.
$$\mathcal{Z}^{[1]} = \{P\in\Phi_{tor} : (P) \text{ intersects } \Theta_{v,0} \text{ for all } F_{v\neq\infty}\}\ ,$$
and denote by $\mathcal{F}$ the cokernel of the inclusion map $\mathcal{Z}^{[1]}\to\Phi_{tor}$. Thus, by construction, we have the short exact sequence:
$$0\to\mathcal{Z}^{[1]}\to\Phi_{tor}\to\mathcal{F}\to 0\ .$$
The flavour symmetry group of the theory $\mathcal{T}_{F_\infty}$ is given by:
$$\boldsymbol{G_F} = \widetilde{\boldsymbol{G}}_{\boldsymbol{F}}/\boldsymbol{\mathcal{F}}\ ,$$
with $\widetilde{G}_F$ the simply connected group with algebra $\mathfrak{g}_F$ from fibers.
4. We also conjecture:
$$\mathcal{Z}^{[1]} \cong \text{ 1-form symmetry}\ , \qquad \Phi_{tor}\cong \text{ 2-group symmetry}\ .$$





Examples and Results

- Simplest singular fibers describe theories:
$$F_\infty = \begin{array}{ll} I^*_{4-N_f}: & \text{4d } SU(2) + N_f \text{ flavours}\ ,\\ I_{9-n}: & \text{4d } E_n \text{ KK theories}\ . \end{array}$$
- Our analysis is in agreement with known results, but also predicts new ones. For RES with only two singular fibers:

$\{F_v\}$	$\Phi_{\rm tor}$	F_∞	4d theory	$\mathfrak{g}_F$	G_F
II^*, II	-	II^*	AD H_0	-	-
		II	E_8 MN	E_8	E_8
III^*, III	$\mathbb{Z}_2$	III^*	AD H_1	A_1	$SU(2)/\mathbb{Z}_2$
		III	E_7 MN	E_7	$E_7/\mathbb{Z}_2$
IV^*, IV	$\mathbb{Z}_3$	IV^*	AD H_2	A_2	$SU(3)/\mathbb{Z}_3$
		IV	E_6 MN	E_6	$E_6/\mathbb{Z}_3$
I_0^*, I_0^*	$\mathbb{Z}_2\times\mathbb{Z}_2$	I_0^*	$SU(2)\ N_f=4$	D_4	$\mathrm{Spin}(8)/\mathbb{Z}_2\times\mathbb{Z}_2$

- For 5d SCFT parent theories, we find that:
$$G_F(E_n) = E_n/Z(E_n)\ .$$

Modularity and BPS quivers

- When RES can be constructed as quotient $\mathbb{H}/\Gamma$, for $\Gamma\subset \mathrm{PSL}(2,\mathbb{Z})$, then U-plane can be isomorphically mapped to upper half-plane $\mathbb{H}$.
- The special points of Γ - **cusps** and **elliptic points** - get mapped to the singular fibers. The light BPS states can be simply found:
$$\tau = \frac{q}{m} \quad\longleftrightarrow\quad \pm(m,-q)\ .$$