Taub-NUT Spacetime

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- The *Taub-NUT metric* is given by:

$$ds^2 = -f(r)\left(d\bar{t} - 2lcos\theta d\phi\right)^2 + \frac{dr^2}{f(r)} + (r^2 + l^2)(d\theta^2 + sin^2\theta d\phi^2)$$

$$f(r) = \frac{r^2 - 2mr - l^2}{r^2 + l^2}$$

- Often the change of variables $\bar{t} = t + 2l\phi$ is performed to obtain:

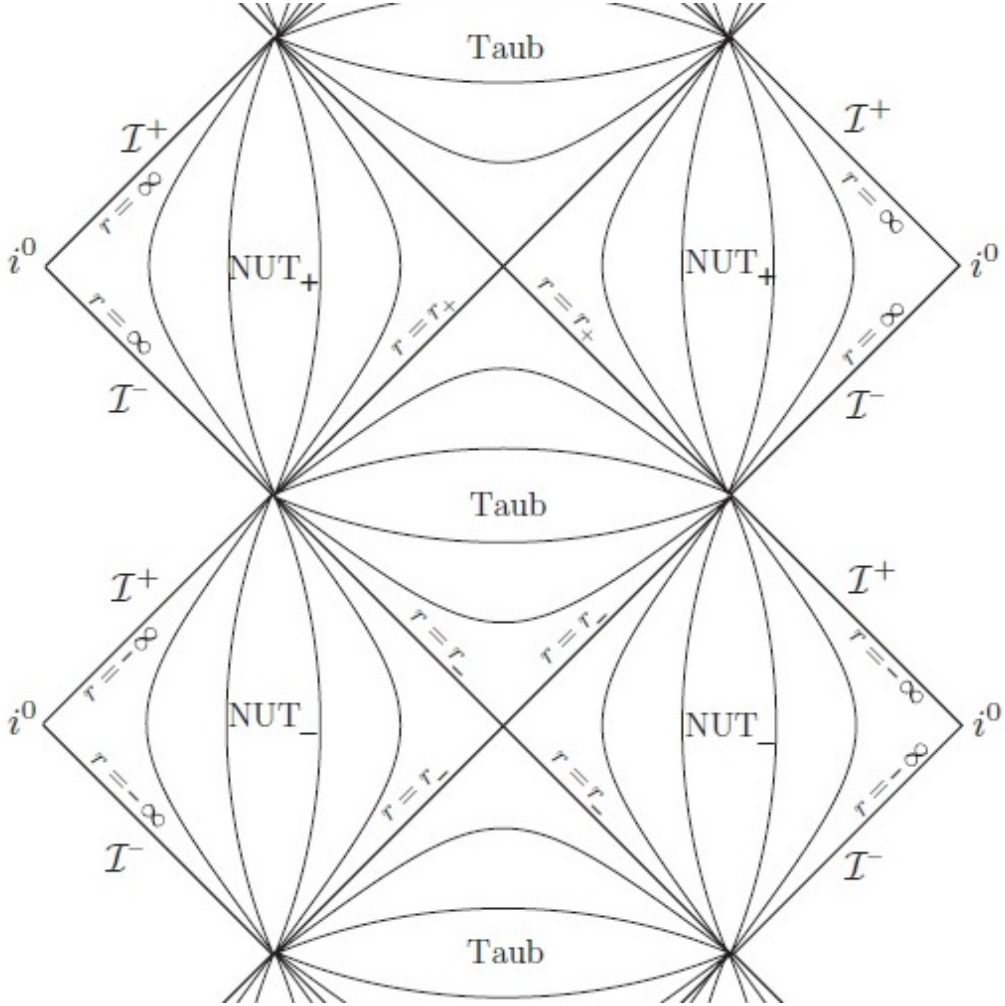
$$ds^2 = -f(r)\left(dt + 4lsin^2\frac{1}{2}\theta d\phi\right)^2 + \frac{dr^2}{f(r)} + (r^2 + l^2)(d\theta^2 + sin^2\theta d\phi^2)$$

- For constant t, r and θ, the line element is:

$$ds^2 = -\Big[2l^2 f(r)(1 - cos\theta)^2 - (r^2 + l^2)sin^2\theta\Big]d\phi^2$$

- Note that for $f(r) > 0$, the interval becomes timelike when:

$$cos\theta < -\frac{r^2 + l^2 - 4l^2 f(r)}{r^2 + l^2 + 4l^2 f(r)}$$



Penrose Diagram of the Taub-NUT spacetime in the Bonnor interpretation (the lines drawn have constant r). Note that $f(r) > 0$ in NUT regions.

- We can alternatively introduce one 'coordinate patche' for each hemisphere, i.e. $0 < \theta < \pi/2$ and $\pi/2 < \theta < \pi$.

$$ds_u^2 = -f(r)\Big(dt + 4lsin^2(\theta/2)d\phi\Big)^2 + ...$$

$$ds_d^2 = -f(r)\Big(dt - 4lcos^2(\theta/2)d\phi\Big)^2 + ...$$

- The second metric is obtained from the coordinate transformation: $t \to t - 4l\phi$.

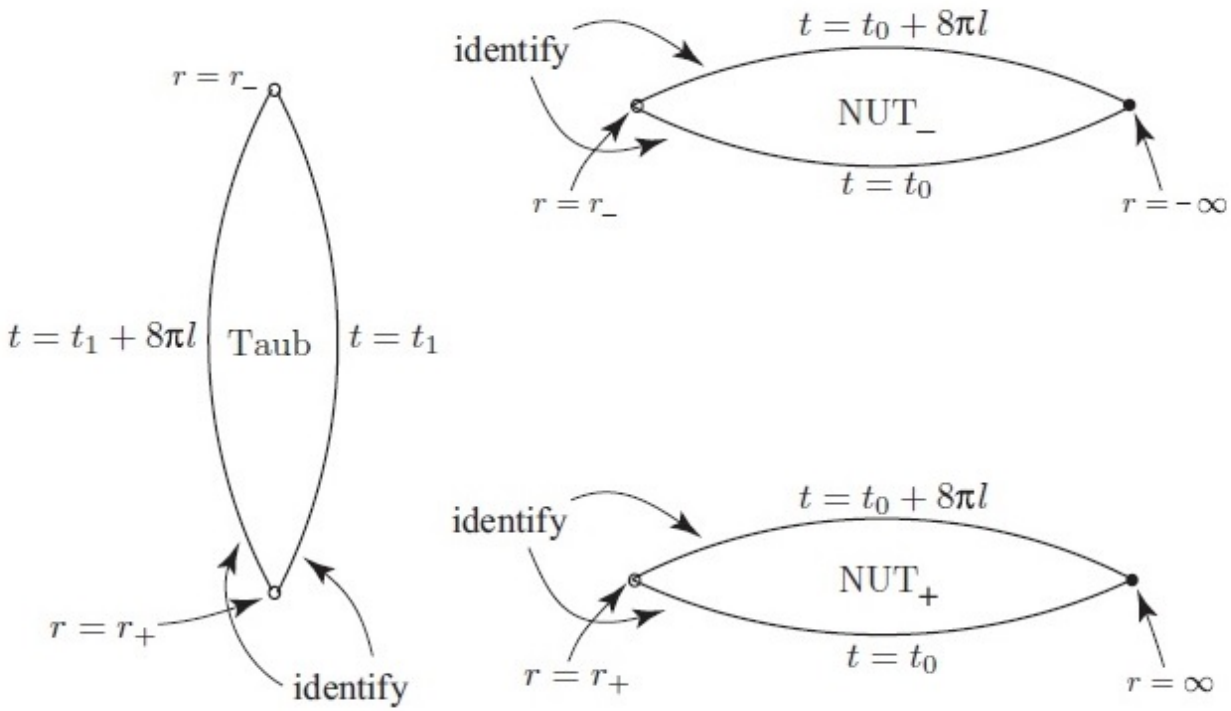
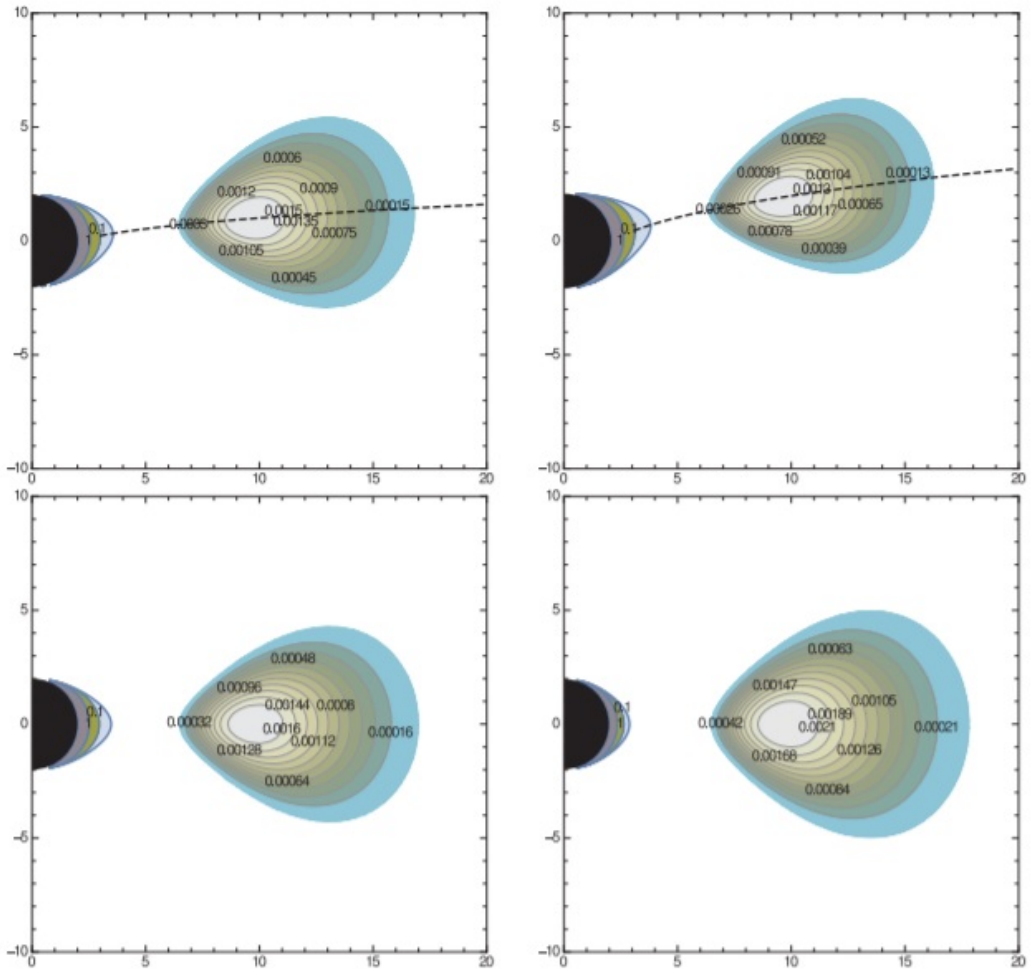


Figure: Conformal Structure in Misner Interpretation



Polish doughnuts in NUT spacetime with $l = 0.2m$ (top-left), $l = 0.4m$ (top-right), Schwarzschild (bottom-left) and Kerr spacetime (bottom-right)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