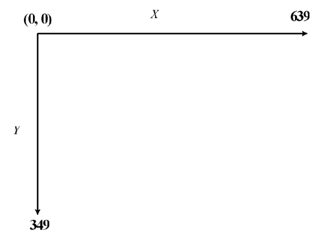
EXERCISES

1. To show graphics on a PC we must be able to identify particular points on the VDU screen of the PC. Points (pixels) on the VDU screen are specified by their coordinates. The EGA graphics adaptor can handle a set of coordinates ranging from 0 to 639 horizontally and 0 to 349 vertically with the origin of the system in the top left-hand corner



If X and Y are sets defined as: $X == \{x:\mathbb{N} \mid x<640\}$;
$Y == \{y:\mathbb{N} \mid y<350\}$
write an expression in terms of X and Y which yields all possible pixels on the screen.

2. If A = {1, 2, 3, 4}, B = {2, 4, 6, 8, 10} and C = {2, 3, 5, 7} write down the following:

(a) $\mathbb{P}C$
(b) $A \times C$
(c) $\#B$
(d) $\#A$
(e) $\#(A \times B)$

3. Given the sets A and B defined by: $A == \{p, q\}$; $B == \{1, 2, 3\}$
write down the values of:

(a) $\mathbb{P}A$
(b) $\mathbb{P}B$
(c) $\mathbb{P}(\mathbb{P}A)$
(d) $\mathbb{P}A \cap \mathbb{P}B$
(e) $A \times A$
(f) $A \times B$
(g) $A \times A \times A$
(h) $\#(\mathbb{P}A \times B)$
(i) $\#(A \times \mathbb{P}B)$
(j) $\#(\mathbb{P}A \times \mathbb{P}B)$
(k) $\{S:\mathbb{P}A \mid \#S=1\}$
(l) $\{(x, y):B\times B \mid x+y\leq 2\}$

4. Given R = {(p, 2), (q, 3), (r, 4)} and S = {(1, x), (2, y), (3, z)} write down:

(a) $\#R$
(b) **dom** R
(c) **ran** R

(d) **dom** S
(e) **ran** S
(f) R^{-1}

5. Use the definition of the inverse of a relation to argue the truth of the following statements:

(a) $(R^{-1})^{-1} = R$
(b) **dom** R = **ran** (R^{-1})
(c) **ran** R = **dom** (R^{-1})

6. Assume the following definitions for the relations *plays* and *worksby* over given sets PEOPLE, INSTRUMENT and ACTION (it should be obvious which elements go with which sets):

plays = = {(John, piano), (Williams, guitar), (Kennedy, violin), (Armstrong, trumpet), (Lennon, flute), (Lennon, piano), (Starr, piano)}
worksby = = {(piano, hammering), (guitar, plucking), (harpsichord, plucking), (trumpet, blowing), (flute, blowing), (violin, bowing), (violin, scraping)}

(a) What are the **domain** and **range** of *plays* and *worksby*?
(b) What are the **types** of *plays* and *worksby*?
(c) Write down $plays^{-1}$ and $worksby^{-1}$.
(d) What are the **domain** and **range** of $plays^{-1}$ and $worksby^{-1}$?
(e) Write out the contents of each of the following relations

(i) $\{piano, harpsichord\} \lhd worksby$?
(ii) $plays \rhd \{piano, violin\}$
(iii) $plays \unrhd \{piano\}$
(iv) $worksby \unrhd \{bowing, scraping\}$

7. Assume [COUNTRY] (the set of all countries) and [LANGUAGE] (the set of all languages) as base types with *speaks* as a relation between them (see chapter notes).

(a) Express the fact that *Latin* is not the language of any country.
(b) Express the fact that, in Switzerland, there are, officially, four languages spoken (they happen to be *French, German, Italian* and *Romansch* **but** assume that this fact is not known).

8. The set of *Users* of a computer system is the union of two other sets: *Normal_users* and *Privileged_users*. The association between the computer-users and the files that they own

is modelled by a relation *owns* over the two sets *Users* and *Files*. Write down an expression to represent the set of files owned by the members of the set *Privileged_users*.

9. Suppose *owns* and *can_read* are relations over USER × FILE. If current values are:

owns = {(Roberts, archive), (Wilson, tax), (Jones, old), (Roberts, summary)}
can_read = {(Roberts, archive), (Wilson, archive), (Wilson, tax), (Jones, tax), (Roberts, summary), (Jones, old), (Jones, archive)}

indicate which of the following predicates are *true* and which are *false*:

(a) *Roberts owns archive*
(b) $\neg$(*Roberts owns summary*)
(c) *(Jones, tax)* $\in$ *owns* $\wedge$ *(Roberts, archive)* $\in$ *can_read*
(d) *owns* $\subset$ *can_read*
(e) *#owns* = 7
(f) **dom** *owns* = **dom** *can_read*
(g) *Roberts owns archive* $\wedge$ *(Roberts, archive)* $\in$ *can_read*

10. If R_1, $R_2 : \mathbb{N} \leftrightarrow \mathbb{N}$ are the relations:
$R_1 = \{ 1 \mapsto 4, 2 \mapsto 3, 3 \mapsto 2, 3 \mapsto 3, 4 \mapsto 1 \}$; $R_2 = \{ 1 \mapsto 2, 2 \mapsto 3, 3 \mapsto 4\}$
write out:

(a) $R_1 \mathbin{⨾} R_2$
(b) $R_2 \mathbin{⨾} R_1$
(c) $R_1 \mathbin{⨾} R_1$
(d) $R_1 \mathbin{⨾} R_1 \mathbin{⨾} R_1$
(e) $R_1 ⦇ \{2, 3\} ⦈$
(f) $R_2 ⦇\{0\}⦈$
(g) $R_1^{-1} ⦇\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}⦈$
(h) $(R_1 \mathbin{⨾} R_2) ⦇\{3\}⦈$
(i) $((\{2\} \lhd R_1) \mathbin{⨾} (R_2^{-1})) ⦇\{2\}⦈$
(j) $(R_1 ⦇\{2\}⦈) \setminus (R_2 ⦇\{2\}⦈)$
(k) $(R_1 \setminus R_2) ⦇\{2\}⦈)$

11. The genealogy of historical personages can be investigated or, at least, recorded, with computer assistance, and the binary relation is an ideal model for a repository of such genealogical information. Suppose [PERSON] is a suitable base type, and suppose that V, A, C, L, Vy, E, Aa, D, W, G and N are distinct members of it.

Our knowledge about their relationships may be modelled by the *infix* relation:

*_is_a_parent_of_* : PERSON $\leftrightarrow$ PERSON

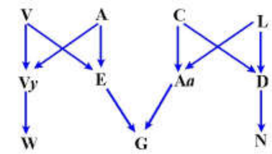
Note that the from-set and to-set are the same type.The genealogy of historical personages can be investigated or, at least, recorded, with computer assistance, and the binary relation is an ideal model for a repository of such genealogical information. Suppose [PERSON] is a suitable base type, and suppose that V, A, C, L, Vy, E, Aa, D, W, G and N are distinct members of it.

Our knowledge about their relationships may be modelled by the *infix* relation:

$$\_is\_a\_parent\_of\_ : \text{PERSON} \leftrightarrow \text{PERSON}$$

Note that the from-set and to-set are the same type.

The accompanying diagram represents a possible value of *_is_a_parent_of_*.



That V is a parent of E is illustrated by the arrow from V to E and is expressed more formally by the expression: V *is_a_parent_of* E

(a) List the *domain* and *range* of the *is_a_parent_of* relation for the value shown
(b) List the relation *is_a_parent_of* when *domain restricted* to the set {V, E, G}
(c) List the relation *is_a_parent_of* when *range subtracted* by the set {V, E, G}

12. In the preceding chapter an example was given of a *speaks* relation.

If EU : ℙ COUNTRY and *speaks_in_EU* is a restricted form of *speaks* (relating to countries of the set EU only), write down an expression involving the *speaks* and *speaks_in_EU* relations.

13. Given that [PERSON] is a given set representing the set of all people, and we have the relations: *has_father*, *has_mother, has_parent* : PERSON ↔ PERSON then the relation *has_sibling* (brother or sister) can be defined in terms of *has_parent* thus:

$$has\_sibling : \text{PERSON} \leftrightarrow \text{PERSON}$$
$$has\_sibling = (has\_parent \fatsemi has\_parent^{-1}) \setminus \mathbf{id}\ \text{PERSON}$$

(a) Use composition of relations to describe the *has_grandparent* relation in terms of the *has_parent* relation.
(b) If a person's first-cousin is defined to be a child of the person's aunt or uncle describe the *has_first_cousin* relation symbolically.
(c) Write down a definition of the relation *has_ancestor* in terms of *has_parent*.

14. Using the following relations *brother_of* and *mother_of*:

$brother\_of$ == {(John, Lynne), (Mike, Sue), (David, Mary)}
$mother\_of$ == {(Lynne, Chris), (Lynne, Matthew), (Sue, Paul)}

(a) verify that $(brother\_of \fatsemi mother\_of)^{-1} = (mother\_of^{-1} \fatsemi brother\_of^{-1})$

(b) What sort of blood relationship is defined by $(brother\_of \fatsemi mother\_of)^{-1}$?

15. The teaching rooms in a university are booked to different courses. Only Science courses can book rooms which are laboratories. Using given sets:

[ROOM]	the set of all possible rooms,
[SESSION]	the set of all possible sessions for which a room may be booked, and
[COURSE]	the set of all possible courses,

write formal expressions for the following:

(a) the relation *book* between rooms and courses

(b) the relation *book* restricted to laboratories

(c) the relation *book* restricted to Science courses

(d) a further relation *make* which links time-sessions to courses

(e) a composition of *make* with *book* (consider the need for inverses) to give the time slots when rooms are booked.

16. Suppose [BOOK] and [PERSON] are given sets for a Library system. Suppose, further, that *_lent_to_* is a relation such that x *lent_to* y means that the book x has been loaned to the person y (NB we shall see, later, that this relation is of a special sort).

(a) Write down an expression for the set of library books on loan to a person, p.

(b) If no borrower is allowed to have more than 8 books on loan, write down an expression restricting the number of loans for a borrower p.

17. Suppose R is a relation and suppose the domain of R is restricted to the elements of some set S using $S \lhd R$. Write down an expression connecting the *relational image* of R with the *restricted domain* of R.

18. A university operates a modular degree scheme such that students enrolled at the university are able to register for a selection of modules from a large menu. The choice is not entirely "free" and registrations for modules are subject to certain constraints. It is important that the administrators keep track of which students are doing which modules.

Basic types are: [PERSON, MODULE]

which, respectively, are the sets of people and modules that might ever be in the system.

If the ordered pair (p, m) where p is a student and m a module, represents the information that the student p is taking the module m, we might represent the entire modular degree scheme by the relation *taking*, where: $taking : \mathrm{PERSON} \leftrightarrow \mathrm{MODULE}$

The known constraints are:

- those who are registered for modules must be enrolled as students;
- the modules taken by students must be *bona fide* degree modules at the university;
- the greatest number of students allowed to take any module is *maxNum*

The following sets are to be used to model the system:

$students : \mathbb{P}\ \mathrm{PERSON}$	the set of students currently enrolled at the university, and
$degModules : \mathbb{P}\ \mathrm{MODULE}$	the set of modules currently in the modular degree scheme.

And *maxNum* will be modelled as a natural number (i.e. be of type $\mathbb{N}$).

(a) Write down a symbolic expression for the set which describes the rather unusual situation where every possible person is taking every possible module?

(b) If $firstYear : \mathbb{P}\ \mathrm{PERSON}$ represents the set of all first-year students, create a symbolic expression to define the set of all first-year students who are registered for module m.

(c) Write an expression which gives all degree modules which have **no** students registered.

(d) Derive the set of students who are registered for module m

(i) by using *range restriction* on the relation *taking*;
(ii) by using the *relational image* concept.

(e) Write symbolic expressions which reveal the connections between the sets

(i) *students* and *taking*;
(ii) *taking* and *degModules*

(f) Write an expression for the set of all students who are registered for at least one module which student s is taking.

19. Certain medicines may be given to patients diagnosed as suffering from particular illnesses. Suppose we attempt to model this system using the following given sets:

[PATIENT]	:	the set of all people who might ever be patients;
[ILLNESS]	:	the set of all illnesses from which people may suffer;

[MEDICINE] : the set of all medicines that might be used to treat illnesses.

Since

- patients may suffer from more than a single illness;
- many patients can suffer from the same illness;
- a particular illness may be treated with different medicines; and,
- a particular medicine can be used to treat many illnesses,

it seems likely that the following sets of ordered pairs could prove useful as a basis for modelling:

$$complaints : \text{PATIENT} \leftrightarrow \text{ILLNESS}$$
$$treatments : \text{ILLNESS} \leftrightarrow \text{MEDICINE}$$

(a) By referring to the simple system modelled above

(i) state what sort of mathematical structure has been used to model the two sets *complaints* and *treatments*; and

(ii) draw a suitably labelled diagram which includes representations of the sets PATIENT, ILLNESS, MEDICINE, *complaints* and *treatments*.

(b) If *medications* : PATIENT ↔ MEDICINE is a set of ordered pairs matching patients with the medicines that they are taking, write down a symbolic expression showing how *medications* can be derived from the two sets *complaints* and *treatments*.

(c) If our model needs to incorporate extra information to show that particular medicines should **not** be prescribed to treat certain illnesses, we might declare:

$$not\_permitted : \text{ILLNESS} \leftrightarrow \text{MEDICINE}$$

where $(i, m) \in not\_permitted$ only when illness i should **not** be treated with medicine m.

(i) If *banned_medications* : PATIENT ↔ MEDICINE shows those medicines patients must **not** take, write a symbolic expression to derive *banned_medications* from sets defined above.

(ii) Write down a symbolic expression for the set containing pairs of medicines which may “clash”, in the sense that the first medicine of the pair may be used to treat a particular illness while the second is forbidden for that illness.

(iii) Write down a symbolic expression for the set containing pairs of illnesses such that the first illness in the pair may be treated by some medicine while the second in the pair should not be treated by the same medicine.

(d) If at least one of the medicines m_1, m_2, m_3 is being taken by the patients in a particular set, write down a symbolic expression for that set using

(i) relational image
(ii) domain restriction
(iii) range restriction

(e) Create a symbolic expression which gives the set of medicines appropriate for illnesses i_1, i_2, i_3, or i_4 if the medicines are also suitable for patients p_1, p_2 and p_3.