MVC :-

Exercise 10.1

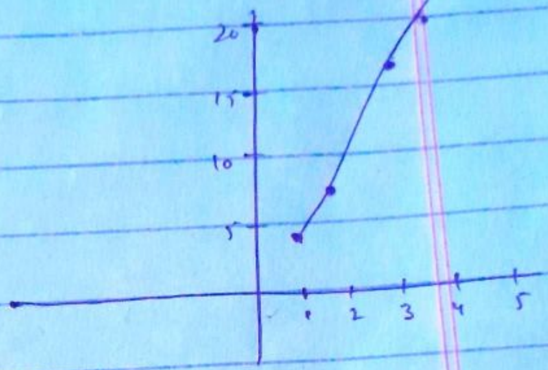
Q3 :-

$x = 3t - 4 \qquad y = 6t + 2$

$t = \frac{x+4}{3}$

$y = \cancel{6}^{2}\left(\frac{x+4}{\cancel{3}}\right) + 2$

$y = 2x + 10$



$x = 1 \qquad y = 2(1) + 10 = 12$

$x = 2 \qquad y = 2(2) + 10 = 14$

$x = 3 \qquad y = 2(3) + 10 = 16$

$x = 4 \qquad y = 2(4) + 10 = 18$

Q4 :-

$x = t - 3 \qquad y = 3t - 7 \quad (0 \leq t \leq 3)$

$t = x + 3$

As Range is b/w 0 and 3 take values of x such That t remains in the limit

$y = 3(x+3) - 7$

$y = 3x + 9 - 7$

$\boxed{y = 3x + 2}$

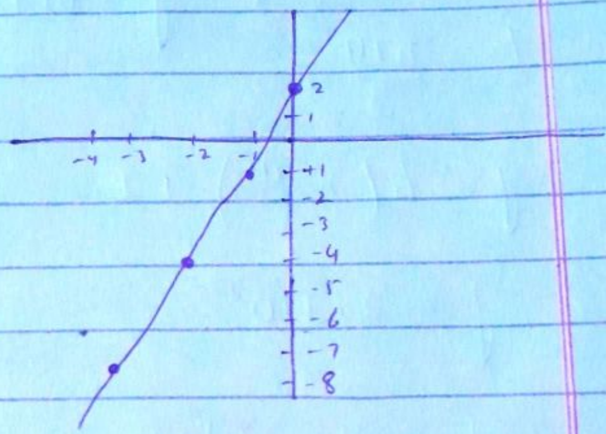
$y = 3x+2$

$x = -3 \quad y = 3(-3)+2 = -7$

$x = -2 \quad y = 3(-2)+2 = -4$

$x = -1 \quad y = 3(-1)+2 = -1$

$x = 0 \quad y = 3(0)+2 = 2$



Q5

$x = 2\cos t \qquad (0 \le t \le 2\pi)$

$y = 5\sin t$

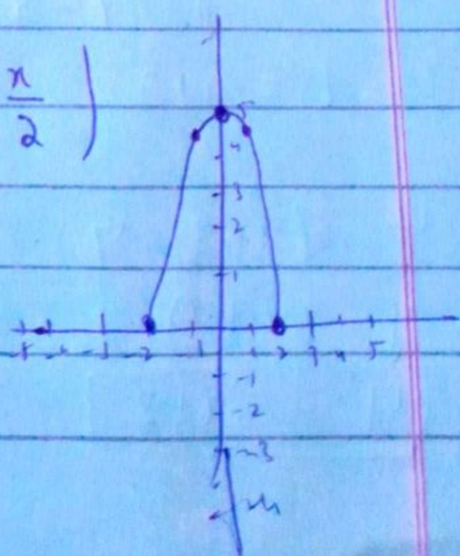
$\cos^{-1}\frac{x}{2} = t$

$y = 5\sin\left(\cos^{-1}\frac{x}{2}\right)$

$x = 0 \quad y = 5$

$x = 1 \quad y = 4.3$

$x = 2 \quad y = 0$



$x = \cancel{\ } -1$ $\quad y = 4.3$

$x = -2$ $\quad y = 0$

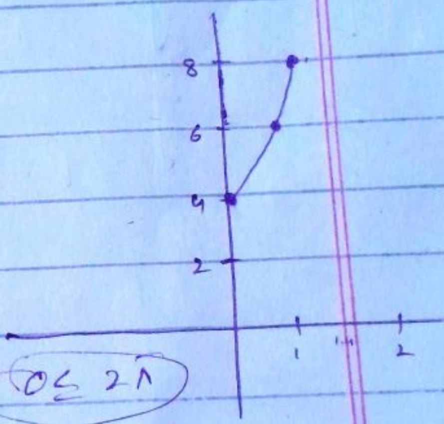
6

$$x = \sqrt{t}$$

$$y = 2t + 4$$

$$t = \frac{y-4}{2}$$

$$x = \sqrt{\frac{y-4}{2}}$$



$y = 4$ $\quad x = 0$ $\quad (0 \le 2\pi)$

$y = 6$ $\quad x = 1$

$y = 8$ $\quad x = \sqrt{2}$

6.2

7

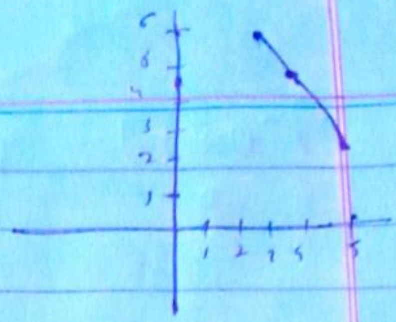
$$x = 3 + 2\cos t \qquad \frac{x-3}{2} = \cos t$$

$$y = 2 + 4\sin t \qquad t = \cos^{-1}\left(\frac{x-3}{2}\right)$$

$$y = 2 + 4\sin\left(\cos^{-1}\left(\frac{x-3}{2}\right)\right)$$

$x = 4$ $\quad y = 5.4$ $\quad \boxed{0 - 6.2}$

$x = 5$ $\quad y = 2$



$x = 3 \quad y = 6$

8 $\quad x = \sec t \quad y = \tan t \quad \left(\pi \le t \le 3\pi/2\right)$

$\pi \qquad 3\frac{\pi}{2}$

$t = \sec^{-1} x$

$y = \tan\left(\sec^{-1}(x)\right)$

$y = \tan\left(\frac{1}{\cos^{-1}(x)}\right)$

$y =$

9 $\quad x = \cos 2t$

$y = \sin t$

45 $x = \sqrt{t}$ $y = 2t + 4$ $t = 1$

$\frac{dx}{dt} = \frac{d}{dt}(t)^{1/2}$

$\frac{dx}{dt}\ \frac{1}{2}t^{-1/2}$

$\boxed{\frac{dx}{dt} = \frac{1}{2t^{1/2}}}$

$\frac{dy}{dt} = (2t + 4)$

$\boxed{\frac{dy}{dt} = 2}$

$\frac{dy}{dx} = \frac{2}{\frac{1}{2\sqrt{t}}}$

$\frac{dy}{dx} = \frac{2}{1} \times 2\sqrt{t}$

$\boxed{\frac{dy}{dx} = 4\sqrt{t}}$

$t = 1$

$\boxed{\frac{dy'}{dx} = 4}$

$\frac{d^2y}{dx^2} = \frac{dy'}{dt} \times \frac{dt}{dx}$ $\frac{d}{dt}(4\sqrt{t}) \times 2\sqrt{t}$

$4 \cdot \frac{1}{\cancel{2\sqrt{t}}} \times \cancel{2\sqrt{t}}$

$\frac{d^2y}{dx^2} = 4$

46 $x = \frac{1}{2}t^2 + 1$

$y = \frac{1}{3}t^3 - t$ $\boxed{t = 2}$

$$\frac{dx}{dt} = \frac{d}{dt}\left(\frac{1}{2}t^2 + 1\right)$$

$$= \frac{2}{2}t + 0$$

$$\boxed{\frac{dx}{dt} = t}$$

$$\frac{dy}{dt} = \left(\frac{1}{3}t^3 - t\right)$$

$$\frac{dy}{dt} = \frac{3}{3}t^2 - 1$$

$$\boxed{\frac{dy}{dt} = t^2 - 1}$$ $t = 2$

$$\boxed{\frac{dy}{dx} = \frac{t^2 - 1}{t}}$$ $\frac{\cancel{t}\left(t - \cancel{\frac{1}{t}}\right)}{\cancel{t}}$

$$\frac{d^2y}{dx} = \frac{dy'}{dt} \times \frac{dt}{dx} = \frac{(2)^2 - 1}{2}$$

$$\frac{4-1}{2}$$

$$\boxed{\frac{dy}{dx} = \frac{3}{2}}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{t^2 - 1}{t}\right) \times \frac{1}{t^2 - 1}$$

$$\frac{t(2t) - (t^2 - 1)(1)}{t^2} \times \frac{1}{t}$$

$$= \frac{2t^2 - t^2 + 1}{t^2} \times \frac{1}{t}$$

$$\frac{t^2+1}{t^2} \times \frac{1}{t}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{t^2+1}{t^3}}$$

$t = 2$

$$\frac{(2)^2+1}{(2)^3}$$

$$= \frac{4+1}{8} = \frac{5}{8}$$

4) $x = \sec t$ $\quad y = \tan t$ $\quad t = \frac{\pi}{3}$

$$\frac{dx}{dt} = \sec t \qquad \frac{dy}{dt} = \tan t$$

$$\frac{dx}{dt} = \sec t \tan t \qquad \frac{dy}{dt} = \sec^2 t$$

$$\frac{dy}{dx} = \frac{\sec^{\cancel{2}} t}{\cancel{\sec t} \tan t}$$

$$\boxed{\frac{dy}{dx} = \frac{\sec t}{\tan t}}$$

$$\frac{dy}{dx} = \frac{\sec \frac{\pi}{3}}{\tan \frac{\pi}{3}} = \frac{2}{\sqrt{3}}$$

$$\frac{d^2y}{dx^2} \times \frac{dt}{dt} = \frac{d}{dt}\left(\frac{\sec t}{\tan t}\right) \times \frac{1}{\sec t \tan t}$$

$$\frac{1}{\sec t \tan t}$$

$$\frac{1}{\sec t \tan t}$$

$$y' = \frac{\sec t}{\tan t}$$

$$\frac{dy'}{dt} = \frac{\tan t \sec t \tan t - \sec t \sec^2 t}{\tan^2 t}$$

$$= \frac{\tan^2 t \sec t - \sec t \sec^2 t}{\tan^2 t}$$

$$= \frac{\sec t \left(\tan^2 t - \sec^2 t\right)}{\tan^2 t}$$

$$= \frac{\sec t(-1)}{\tan^2 t}$$

$$= \frac{-\sec t}{\tan^2 t} = -\frac{2}{(\sqrt{3})^2} = -\frac{2}{3}$$

$$\frac{dy'}{dt} \times \frac{dt}{dx} = \frac{-\cancel{\sec t}}{\tan^2 t} \times \frac{1}{\cancel{\sec t} \tan t}$$

$$= \frac{-1}{\tan^3 t}$$

$$= -\frac{1}{\left(\tan \frac{\pi}{3}\right)^3}$$

$$= -\frac{1}{(\sqrt{3})^3}$$

$$= \frac{-1}{3\sqrt{3}}$$

$$= \frac{-1}{3\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{-\sqrt{3}}{3(3)}$$

$$= -\frac{\sqrt{3}}{9} \text{ Ans}$$

48 $x = \sinh t$ $y = \cosh t$ $t = 0$

$\frac{dx}{dt} = \cosh t$ $\frac{dy}{dt} = \sinh t$

$$\frac{dy}{dx} = \frac{\sinh t}{\cosh t} \quad t = 0$$

$$\frac{dy}{dx} = \frac{\sinh 0}{\cosh 0} = \tanh t$$

$$\frac{dy}{dx} = \frac{0}{1}$$

$$\boxed{\frac{dy}{dx} = 0}$$

$$\frac{dy'}{dt} \times \frac{dt}{dx} = \frac{d}{dx}(\tanh t) \times \frac{1}{\cosh t}$$

$$= \sec^2 h t \times \frac{1}{\cosh t}$$

$t = 0$

$$= \sec^2 h(0) \times \frac{1}{\cosh(0)}$$

$$= 1 \times \frac{1}{1}$$

$$\boxed{\frac{d^2y}{dx^2} = 1}$$

5. $x = \cos\phi$

$y = 3\sin\phi \qquad \phi = \frac{5\pi}{6}$

$$\frac{dx}{dt} = \frac{d}{dt}(\cos\phi) \qquad \Bigg| \qquad \frac{dy}{dt} = \frac{d}{dt}(3\sin\phi)$$

$$\frac{dx}{dt} = -\sin\phi \qquad \Bigg| \qquad \frac{dy}{dt} = 3\cos\phi$$

$$\frac{dy}{dx} = -\frac{3\cos\phi}{\sin\phi} = -\frac{3\cos 150}{\sin 150} = \frac{\frac{3\sqrt{3}}{2}}{\frac{1}{2}} = 3\sqrt{3}$$

$$\frac{d^2y}{dx^2} = \frac{d}{dt}(y') \times \frac{dt}{dx}$$

$$= \frac{d}{dt}\left(-\frac{3\cos\phi}{\sin\phi}\right)$$

$$= -3\left[\frac{\sin\phi(-\sin\phi) - \cos\phi(\cos\phi)}{(\sin\phi)^2}\right]$$

$$= -3\left[\frac{-\sin^2\phi - \cos^2\phi}{(\sin\phi)^2}\right]$$

$$= -3 \cdot -\left[\frac{\sin^2\phi + \cos^2\phi}{\sin^2\phi}\right]$$

$$= 3\,\frac{1}{\sin^2\phi} \times \frac{1}{\sin\phi}$$

$$= \frac{3}{\sin^2\phi} \quad \frac{3}{(\sin 150)^2} \times \frac{1}{\sin 150} = \frac{3}{\left(\frac{1}{2}\right)^2} \cdot \frac{1}{\frac{1}{2}} = 24$$

10.2 1 — 8
21 — 26

10.1
61, 62
65 — 70.

Q| $x = t - 3\sin t$ $\quad y = 4 - 3\cos t$

By Example 5

$$\frac{dx}{dt} = 1 - 3\cos t \qquad \frac{dy}{dt} = -3(-\sin t)$$

$$\frac{dy}{dt} = 3\sin t$$

$$\boxed{\frac{dy}{dx} = \frac{3\sin t}{1 - 3\cos t}}$$

$$= \cancel{\frac{(1-3\cos t)\frac{d}{dt}3\sin t - 3\sin t\frac{d}{dt}(1-3\cos t)}{(1-3\cos t)^2}}$$

$$\cancel{= (1-3\cos t)\,3\cos t - 3\sin t}$$

At $\boxed{t = 10}$

$$\frac{3\sin 10}{1 - 3\cos 10} = \frac{0.52}{-1.95}$$

$$= -0.2665$$

. ———— .

6.
$$x = t - 2\cos t$$
$$y = 2 - 2\sin t$$

a) Horizontally $\quad m = 0$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} \qquad \frac{dy}{dt} = 0$$

$$\frac{dy}{dt} = \frac{d}{dt}(2 - 2\sin t)$$
$$= 0 - 2\cos t$$
$$\frac{dy}{dt} = -2\cos t$$

$$-2\cos t = 0 \qquad \boxed{0 \le t \le 10}$$
$$\cos t = 0$$
$$t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}$$

$$\boxed{t = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}}$$ lies in Range

b)
$$\frac{dx}{dt} = t - 2\cos t$$
$$= 1 - 2(-\sin t)$$
$$\frac{dx}{dt} = 1 + 2\sin t$$

$0 = 1 + 2\sin t$

$2\sin t = -1$

$\sin t = -\frac{1}{2}$

$\boxed{t = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{19\pi}{6}}$

Q65-70

65 $x = t^2$ $\quad y = \frac{1}{3}t^3$ $\quad (0 \le t \le 1)$

$\boxed{\frac{dx}{dt} = 2t}$ $\quad \boxed{\frac{dy}{dt} = \frac{3t^2}{3}}$

$\boxed{\frac{dy}{dt} = t^2}$

$$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$= \int_0^1 \sqrt{(2t)^2 + (t^2)^2}$$

$$= \int_0^1 \sqrt{4t^2 + t^4}\, dt$$

$$= \int_0^1 (4t^2 + t^4)^{1/2}\, dt$$

$$= \int_0^1 t\,(4 + t^2)^{1/2}\, dt$$

$$= \int_0^1 \frac{1}{2}(4+t^2)^{1/2}\, 2t\, dt$$

$\therefore\ u = 4 + t^2$
$= 4 + (0)^2 = 4 \rightarrow$ lower limit
$u = 4 + t^2$
$= 4 + (1)^2 = 4 + 1 = 5$ upper limit

$$\int_4^5 \frac{1}{2}\sqrt{u}\,du$$

$$\int_4^5 \frac{1}{2}\frac{(u)^{\frac{1}{2}+1}}{\frac{1}{2}+1}\,du$$

$$\int \frac{1}{2}\frac{u^{3/2}}{\frac{3}{2}}$$

$$\int \frac{2}{2}\frac{u^{3/2}}{3}$$

$$\int_4^5 \frac{1}{3}u^{3/2}$$

$$\frac{1}{3}\left((u)^{3/2} - u^{3/2}\right)$$

$$= \frac{1}{3}(5)^{3/2} - (4)^{3/2}$$

$$= \frac{1}{3}\left(5\sqrt{5} - 2^3\right)$$

$$= \frac{1}{3}\left(5\sqrt{5} - 8\right)$$ Ans

66 $x = \sqrt{t} - 2 \qquad y = 2t^{3/4} \quad (1 \le t \le 16)$

$\frac{dx}{dt} = \frac{1}{2t^{1/2}} \qquad \frac{dy}{dt} = 2 \cdot \frac{3}{4} t^{3/4 - 1}$

$\frac{dy}{dt} = \frac{3}{2t^{1/4}}$

$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

$\int_1^{16} \sqrt{\left(\frac{1}{2\sqrt{t}}\right)^2 + \left(\frac{3}{2(t)^{1/4}}\right)^2}\, dt$

$= \int_1^{16} \sqrt{\left(\frac{1}{4t} + \frac{9}{4\sqrt{t}}\right)}\, dt$

66 $x = \sqrt{t} - 2 \qquad y = 2t^{3/4}$

finding Limit →

$u = \sqrt{t}$

$u = \sqrt{1} = 1$ Lower

$u = \sqrt{16} = 4$ Upper

let ~~$u = t$~~

$t = u^2$

$x = \sqrt{u^2} - 2 \qquad y = 2u^{2 \cdot 3/4}$

$\boxed{x = u - 2} \qquad \boxed{y = 2u^{3/2}}$

$$\boxed{\frac{dx}{du} = 1} \qquad \frac{dy}{du} = \frac{3}{2} x^{\frac{3}{2}-1} u \quad 3\sqrt{u}$$

$$\int_1^4 \sqrt{(1)^2 + (3\sqrt{u})^2}$$

$$\int_1^4 \sqrt{(1)^2 + 9u} \; du$$

$$\cancel{\frac{(1+9u)^{1/2+1}}{3/2}}$$

$$\cancel{\frac{2}{3}(1+9u)^{3/2}}$$

$$\int (1+9u)^{1/2} du$$

$$\int \frac{1}{9}(1+9u)^{1/2} \, 9du$$

$$= \frac{1}{9} \frac{(1+9u)^{1/2+1}}{3/2} \; du$$

$$= \frac{2}{27} \left[(1+9u)^{3/2}\right]_1^4$$

$$\cancel{\frac{2}{27}\left[\left(1+9(4)\right) - (1)\right]^{3/2}}$$

$$\cancel{= \frac{2}{27}(1+36-9)^{3/2}}$$

$$\cancel{= \frac{2}{27}(28)^{3/2}}$$

$$\frac{2}{27}\left[\left(1+9(4)\right)^{3/2}-\left(1+9(1)\right)^{3/2}\right)$$

$$\frac{2}{27}\left(37^{3/2}-10^{3/2}\right)$$

$$=\frac{2}{27}\left(37\sqrt{37}-10\sqrt{10}\right)$$

6? $x=\cos 3t \qquad y=\sin 3t$

$$\frac{dx}{dt}=-\sin 3t(3) \qquad y=\cos 3t(3)$$

$$\frac{dx}{dt}=-3\sin 3t \qquad y=3\cos 3t$$

$$\frac{dy}{dx}=\int_0^{\pi}\sqrt{(-3\sin 3t)^2+(3\cos 3t)^2}$$

$$=\int_0^{\pi}\sqrt{9\sin^2 3t+9\cos^2 3t}$$

$$=\int_0^{\pi}\sqrt{9(\sin^2 3t+\cos^2 3t)}$$

$$=\int_0^{\pi}\sqrt{9}\ (1)\ dt$$

$$=3t\Big|_0^{\pi}$$

$$=3(\pi-0)$$

$$=3\pi-0$$

$$=\boxed{3\pi}$$

Ah.

$(0 \le t \le \pi)$

68 $x = \sin t + \cos t \qquad y = \sin t - \cos t$

$\frac{dx}{dt} = \cos t - \sin t \qquad \frac{dy}{dt} = \cos t + \sin t$

$\int_0^\pi \sqrt{(\cos t - \sin t)^2 + (\cos t + \sin t)^2}\, dt$

$\int_0^\pi \sqrt{\cos^2 t + \sin^2 t - 2\cos t \sin t + \cos^2 t + \sin^2 t + 2\cos t \sin t}$

$\int_0^\pi \sqrt{1 + 1}\, dt$

$\int_0^\pi \sqrt{2}\, dt$

$\sqrt{2}(t)\Big|_0^\pi$

$= \sqrt{2}\pi$

$(-1 \le t \le 1)$

69 $x = e^{2t}(\sin t + \cos t)$

$y = e^{2t}(\sin t - \cos t)$

$\frac{dx}{dt} = e^{2t}(\sin t + \cos t)$

$= e^{2t}\sin t + e^{2t}\cos t$

$e^{2t}\cos t + 2\sin t\, e^{2t} + e^{2t}(-\sin t) + 2\cos t\, e^{2t}$

$= e^{2t}(\cos t + 2\cos t - \sin t + 2\sin t)$

$= e^{2t}(3\cos t + \sin t)$

$$\frac{dy}{dt} = e^{2t}(\sin t - \cos t)$$

$$\frac{dy}{dt} = e^{2t}\sin t - e^{2t}\cos t$$

$$= e^{2t}\sin t$$

$$= e^{2t}\cos t + \sin t\, e^{2t}(2) - \left[e^{2t}(\sin t) + \cos t\, e^{2t}(2) \right]$$

$$= e^{2t}(\cos t + 2\sin t + \sin t - 2\cos t)$$

$$= e^{2t}(3\sin t - \cos t)$$

$$\int_{-1}^{1} \sqrt{\left(e^{2t}(3\cos t + \sin t)\right)^2 + \left(e^{2t}(3\sin t - \cos t)\right)^2}$$

$$= \int_{-1}^{1} \sqrt{(e^{2t})^2\left((3\cos t + \sin t)^2 + (3\sin t - \cos t)^2\right)}$$

$$= \int_{-1}^{1} e^{2t}\sqrt{9\cos^2 t + \sin^2 t + \cancel{6\cos t\sin t} + 9\sin^2 t + \cos^2 t - \cancel{6\sin t\cos t}}$$

$$\int_{-1}^{1} e^{2t}\sqrt{9(\cos^2 t + \sin^2 t) + 1}$$

$$\int_{-1}^{1} e^{2t}\sqrt{9+1}\; dt$$

$$\int_{-1}^{1} \sqrt{10}\, e^{2t}\, dt$$

$$\int_{-1}^{1} \frac{1}{2}\sqrt{10}\, e^{2t}\, 2dt$$

$$= \frac{1}{2}\sqrt{10}\, e^{2t}\Big|_{-1}^{1}$$

$$= \frac{1}{2}\sqrt{10}\; e^{2(1)} - e^{2(-1)}$$

$$= \frac{1}{2}\sqrt{10}\left(e^{2} - e^{-2}\right)$$

$$\boxed{0 \le t \le \frac{1}{2}}$$

10 $x = 2\sin^{-1}t \qquad y = \ln(1-t^2)$

$$\frac{dx}{dt} = \frac{2}{\sqrt{1-t^2}} \qquad \frac{dy}{dx} = \frac{1}{(1-t^2)} - 2t$$

$$\int_0^{1/2} \sqrt{\left(\frac{2}{\sqrt{1-t^2}}\right)^2 + \left(\frac{-2t}{1-t^2}\right)^2}$$

$$\int \sqrt{\frac{4}{1-t^2} + \left(\frac{-2t}{1-t^2}\right)^2}$$

$$\int \sqrt{\frac{4}{1-t^2} + \frac{4t}{(1-t^2)^2}}$$

$$\int_0^{1/2} \sqrt{\frac{4(1-t^2) + 4t^2}{(1-t^2)^2}}$$

$$= \int_0^{1/2} \sqrt{\frac{4 - 4t^2 + 4t^2}{(1-t^2)^2}}\, dt$$

$$= \int_0^{1/2} \int \frac{4}{(1-t^2)^2}$$

$$\int_0^{1/2} \frac{2}{1-t^2} dt$$

$$= \int_0^{1/2} \cancel{\frac{2}{(1+t)(1-t)}} dt$$

$=$

$$\therefore \frac{2}{(1+t)(1-t)}$$

$$= 2 = A(1+t) + B(1-t)$$

$t=1$

$$2 = A(1+1)$$

$$\boxed{A = 1}$$

$t=-1$

$$2 = A(1-1) + B(1-(-1))$$

$$2 = 2B$$

$$\boxed{B = 1}$$

$$\int_0^{1/2} \left(\frac{1}{1+t} + \frac{1}{1-t} \right) dt$$

$$\ln|1+t|_0^{1/2} + \ln|1-t|_0^{1/2}$$

$$= \ln\left|\frac{1+t}{1-t}\right|_0^{1/2}$$

$$\ln\left|\frac{1+1/2}{1-1/2}\right| - \ln\left|\frac{1+0}{1-0}\right|$$

$$\frac{\ln 3/2}{\ln 1/2} - \ln(1)$$

$$= \ln 3 - \ln(1)$$

$$= \ln 3 \quad \text{Ans}$$