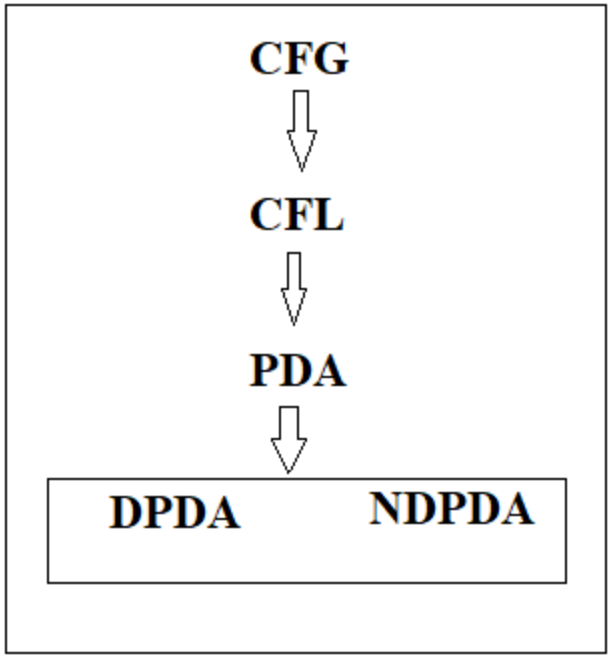
Theory of Computation
CS-202

Outline

- Push Down Automata
  - Deterministic Push Down Automata
  - Non Deterministic Push Down Automata

Context free Grammar, Language and PDA



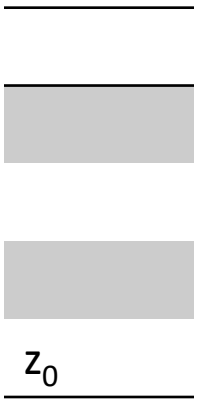
Formal Definition of a deterministic PDA

A pushdown automaton (PDA) is defined by the seven-tuples:

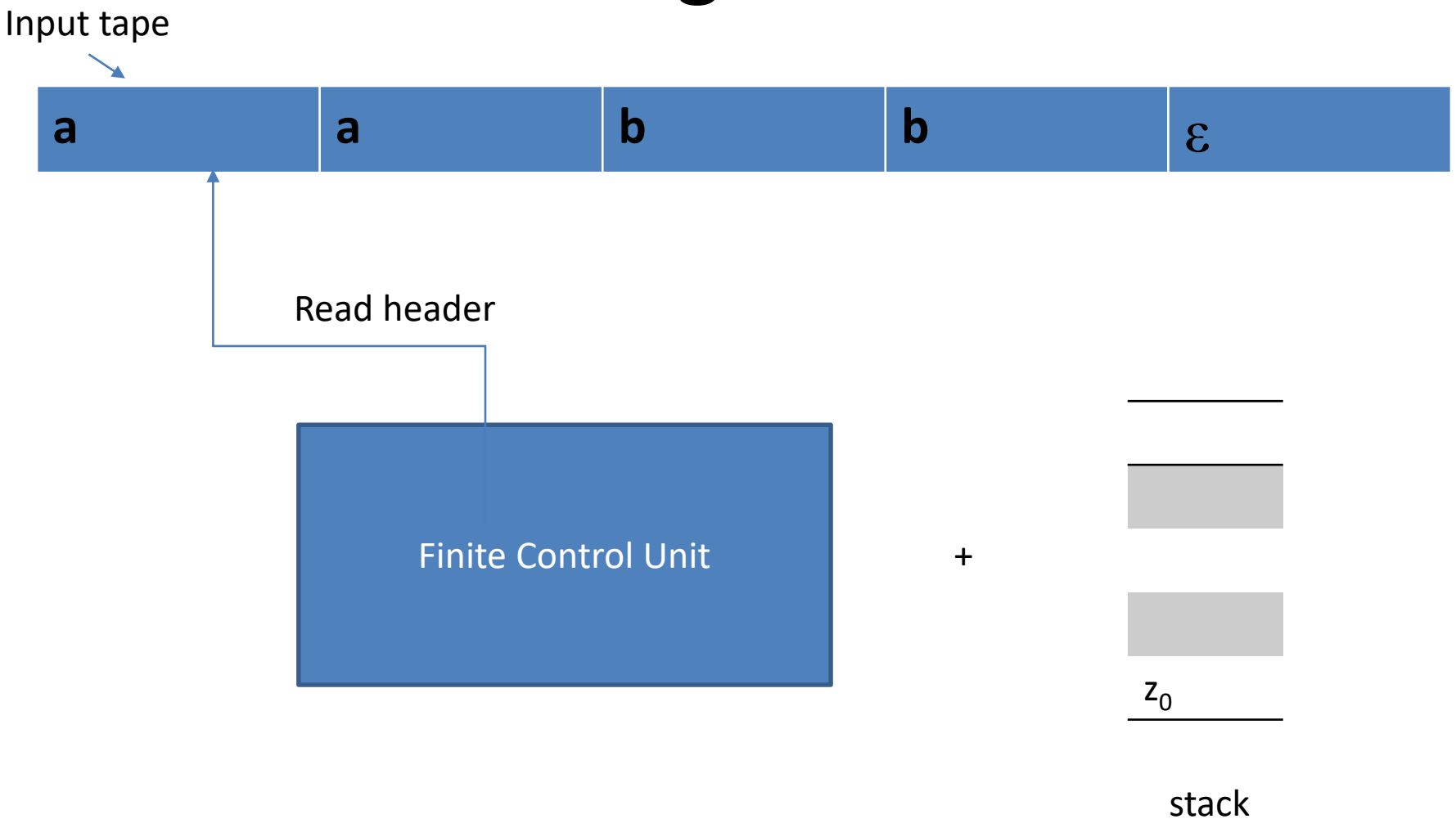
$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

Q	A finite set of states
Σ	A finite set of input alphabet
Γ	A finite set of stack alphabet
q_0	The initial/starting state, q_0 is in Q
z_0	A starting stack symbol, is in Γ
F	A set of final/accepting states, which is a subset of Q
δ	A transition function, where

$$\delta: Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \rightarrow Q \times \Gamma^*$$



Block diagram of PDA



Formal Definition of a NPDA

A non-determininstic pushdown automaton (NPDA) is defined by the seven-tuples:

$M = (Q, \Sigma, \Gamma, \delta, q_0, z_0, F)$

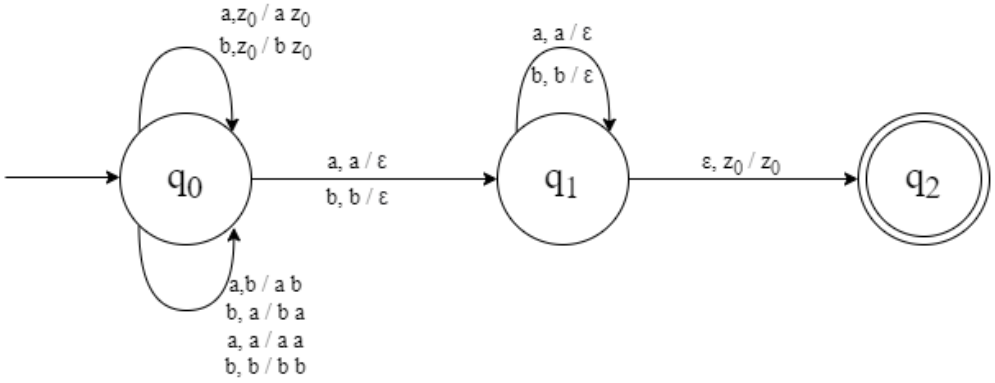
Q	A finite set of states
Σ	A finite set of input alphabet
Γ	A finite set of stack alphabet
q_0	The initial/starting state, q_0 is in Q
z_0	A starting stack symbol, is in Γ
F	A set of final/accepting states, which is a subset of Q
δ	A transition function, where

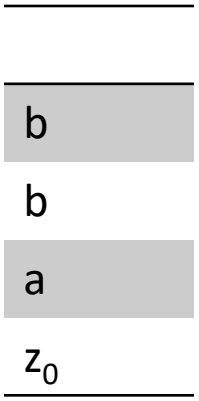
$$\delta: Q \times (\Sigma \cup \{\varepsilon\}) \times \Gamma \rightarrow 2^{Q \times \Gamma^*}$$

Design a NPDA for the language $L=\{ww^R, \quad w \in (a, b)^+\}$

Input tape

a	b	b	b	b	a	ε





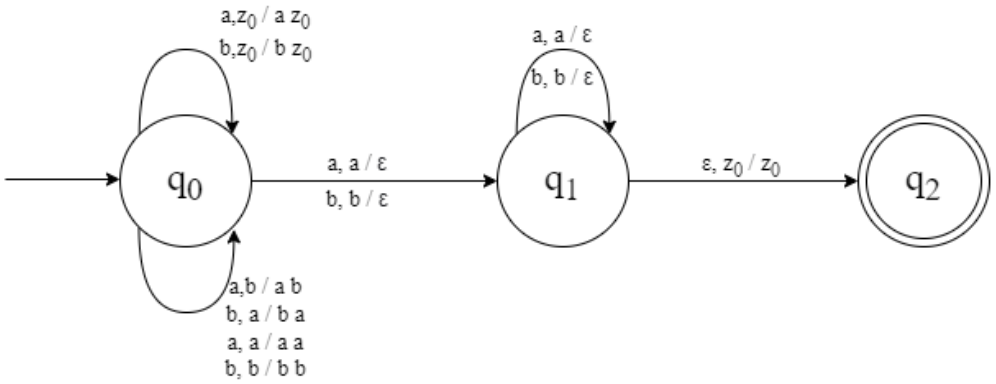
Problem: choosing middle point

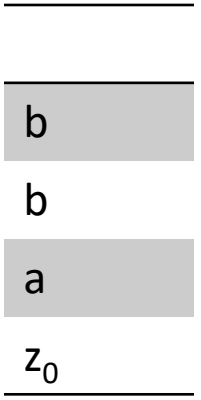
Same symbol on stack as well as input tape(can assume it centre) but not necessarily true.

Design a NPDA for the language $L=\{ww^R, \quad w \in (a, b)^+\}$

Input tape

a	b	b	b	b	a	ε





Stack

$\delta(q_0, a, a) = (q_0, aa)$ or (q_1, ε)
$\delta(q_0, b, b) = (q_0, bb)$ or (q_1, ε)

These moves make it NPDA

Power of DPDA and NPDA

Non Deterministic Pushdown Automata (NDPDA) is more powerful then Deterministic Pushdown Automata (DPDA).

Equivalence between CFG and PDA

CFG and PDA are **equivalent** in power: a **CFG** generates a context-free language and a **PDA** recognizes a context-free language.

A language is context-free iff some pushdown automaton recognizes it.

Practice Problem

1. Design a NPDA for the language
$L=\{wbw^R, \quad w \in (a, b)^+\}$

2. Design a NPDA for the language
$L=\{ww, \quad w \in (a, b)^+\}$

Suggested readings

1. An introduction to FORMAL LANGUAGES and AUTOMATA by PETER LINZ.
2. Introduction to Automata Theory, Languages, And Computation by JOHN E. HOPCROFT, RAJEEV MOTWANI, JEFFREY D. ULLMAN
3. Theory of computer science: automata, languages and computation **by** K.L.P MISHRA