CS-208:Artificial Intelligence
Topic-15: Resolution

Resolution Proof

The resolution theorem proving strategy is to show that the negation of a theorem can not be true.

Step1: *Assume that the negation of the theorem is true.*

Step2: *Show that axioms and assumed negation of the theorem together determines something to be true that can not be true*.

Step3: Conclude that the assumed theorem can not be true, since it leads to contradiction.

Step4: *Conclude that the theorem is true since the assumed negation of the theorem can not be true*.

Proving a theorem by showing its negation can not be true is called

<u>Proof by Refutation</u>

Resolution Procedure

- The resolution operates by taking two clauses that contain the same literal which must occur +ve form in one clause and-ve form in the other.

- The resolvent is obtained by combing all the literals from both the two parent clauses except the ones that cancels.

- If the clause that is produced is an empty clause then contradiction has been found.

- If no contradiction exist then it is possible that the resolution procedure will never terminates.

Conversion of WFF into Clause Form or Canonical Form or Conjunctive Normal Form

Step1: Eliminate $\rightarrow$: By using $a \rightarrow b$ is equivalent to $\neg a \vee b$

Step2: Reduce the scope of ¬ by using

1. $\neg(\neg a) = a$
2. De Morgan's Laws
   - i. $\neg(a \vee b) = \neg a \wedge \neg b$
   - ii. $\neg(a \wedge b) = \neg a \vee \neg b$
3. Standard correspondence between quantifiers
   - i. $\neg\forall x: P(x) = \exists x: \neg P(x)$
   - ii. $\neg\exists x: P(x) = \forall x: \neg P(x)$

Step3: Standardize Variables by binding unique variable to each quantifier

$\forall x: P(x) \wedge \forall x: Q(x)$ has to be changed in to $\forall x: P(x) \wedge \forall y: Q(y)$

Step4: Move all the quantifiers to the left of the formula

Prefix of quantifiers followed by a matrix

<u>At this point the formula is in Prenex Normal Form</u>

Step5: Eliminate the existential quantifiers by using Skolem Constant and Skolem Function.

∃y:**Presidnt**(y) ⟹ **President**(***S1***)

(Here S1 is a function with no arguments and produce a value that satisfies President.)

If the existential quantifier occurs within the scope of universal quantifier, then the value that satisfy the predicate will depend on the value of the universally quantified variable

∀x: ∃y: **fatherof**(y,x) ⟹ ∀x: **fatherof**(**s2**(x), x)

(Here the function S2 that takes the argument *x* and produce the value which satisfies the predicate.)

These generated functions S1 and S2 are called Skolem functions. Sometime ones with no argument are called Skolem constants.

Step6: Drop the prefix of quantifiers: At this point all the variables are universally quantified so prefix can be dropped

Step7: Convert the matrix into conjunction of disjuncts: $(\_ \vee \_ \vee \_ \vee \_) \wedge (\_ \vee \_ \vee \_) \wedge (\_ \vee \_ \vee \_)$ ... This conversion can be done by using the associative and distributive laws. $(a \wedge b) \vee c = (a \vee c) \wedge (b \vee c)$

Step8: Call each conjuncts as a separate clause: In order for WFF to be true, all the clauses that are generated from it must be true.

Step9: Standardize apart the variables in the set of clause generated: Here we rename the variables so that no two clauses make reference to the same variable.