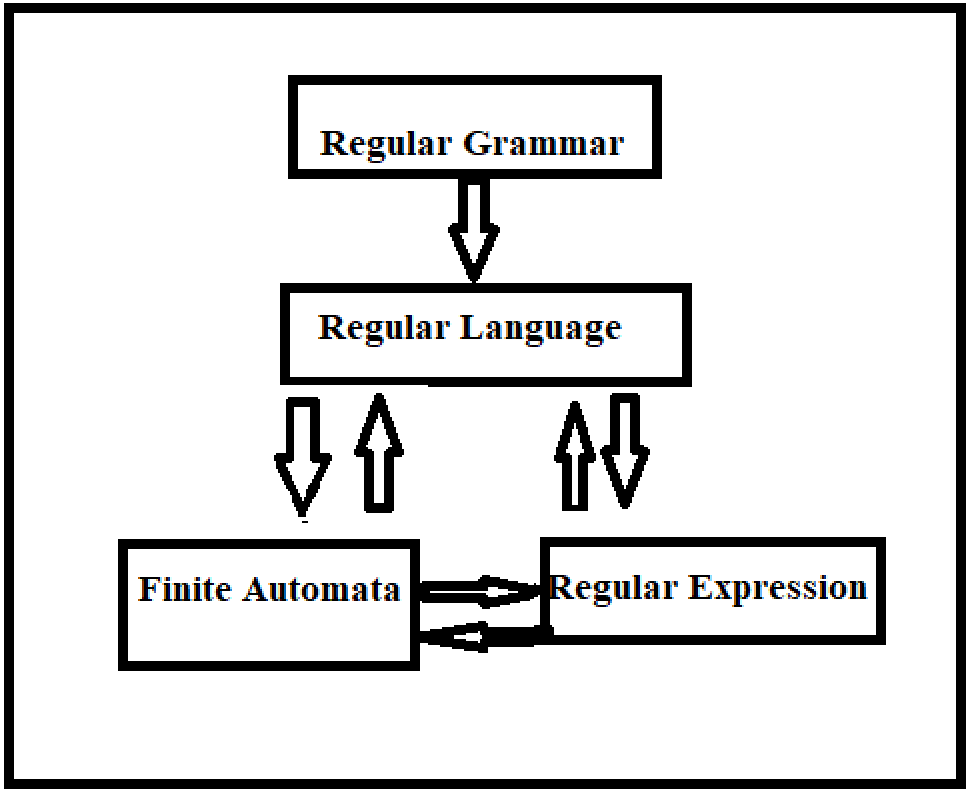
Theory of Computation: CS-202

Regular Expression

Outline

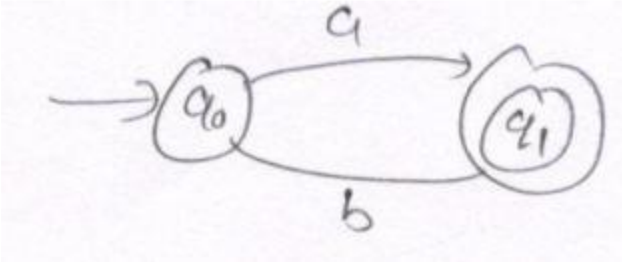
- Regular Expressions
- Regular Expression to Finite Automata
- Finite Automata to Regular Expression


Regular Grammar
Regular Language
Finite Automata
Regular Expression

Conversion from R.E. to finite Automata

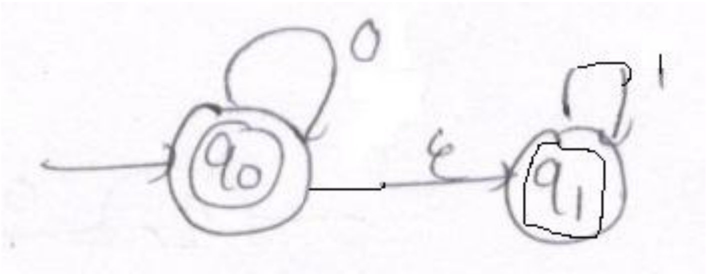
Convert the following R.E. to finite Automata

1. R. E. r= (a+b)

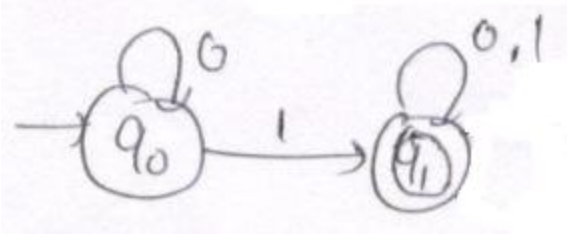


2. R. E. r= (0+1)*

3. R. E. r= 0*1*



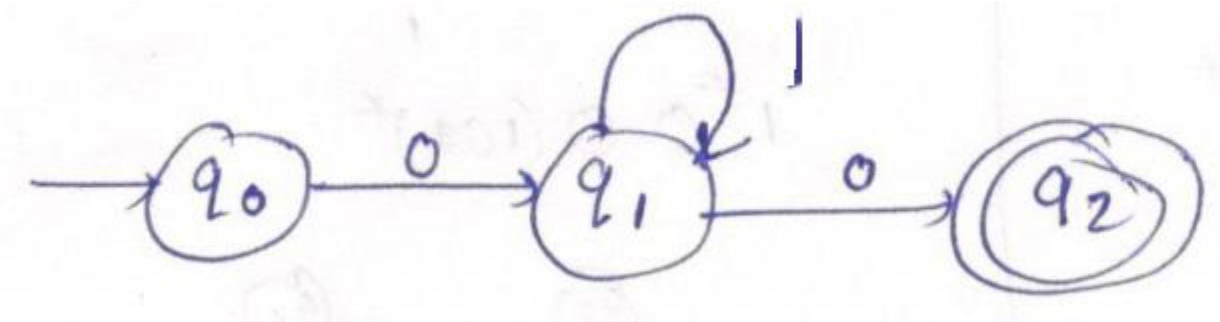
4. R. E. r= 0*1(0+1)*



Conversion from Finite Automata to R.E.

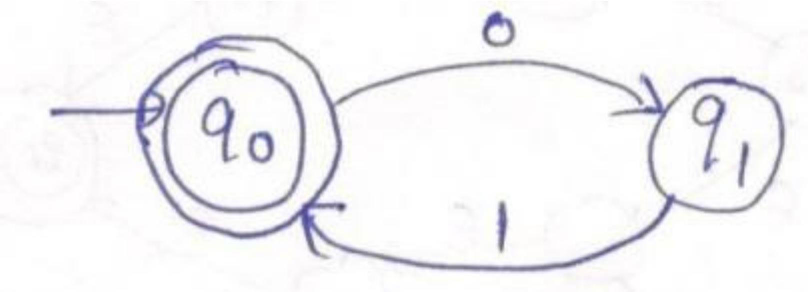
Convert the following FA to R.E.

1.



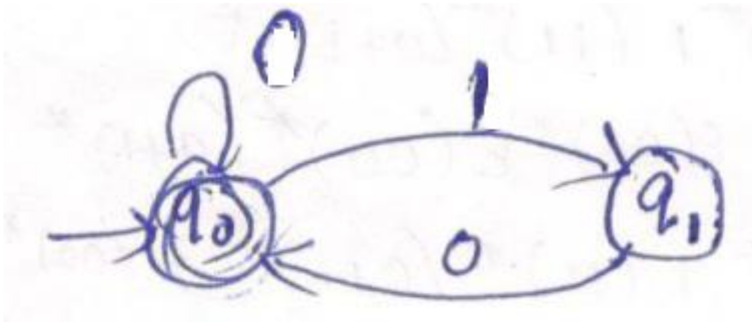
R. E. r= 01*0

2.



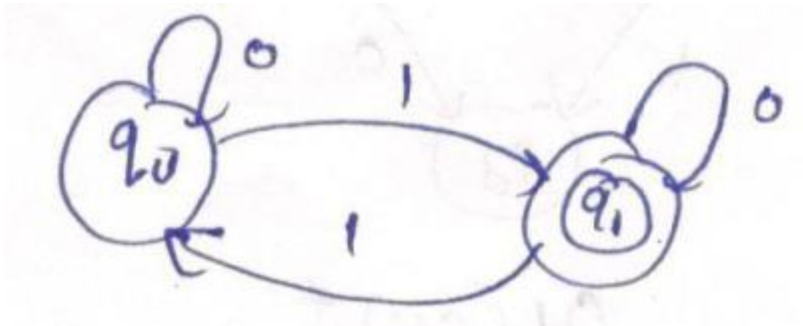
R. E. r= (01)*

3.



R. E. r= (0*+(10)*)* or ((0+10)*

4.



R. E. r= 0*1 (0+10*1)*

Identities for Regular Expression

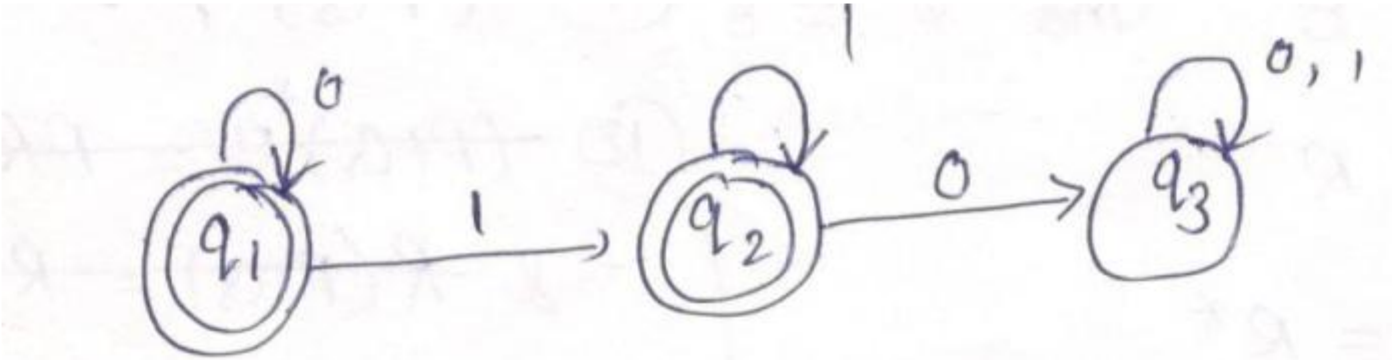
If P, Q and R are Regular Expression, then

1. $\Phi + R = R$ the identity for union
2. $\varepsilon R = R \varepsilon = R$ the identity for concatenation
3. $\Phi R + R \Phi = \Phi$ the annihilator for concatenation
4. $\varepsilon^* = \varepsilon$ and $\Phi^* = \varepsilon$
5. $R+R=R$
6. $R^*R^*=R^*$
7. $RR^*=R^*R$
8. $(R^*)^*=R^*$
9. $\varepsilon + RR^* = \varepsilon + R^*R = R^*$
10. $(PQ)^*P=P(QP)^*$
11. $(P+Q)^*=(P^*Q^*)=(P^*+Q^*)^*$
12. $(P+Q)R=PR+QR$ & $R(P+Q)=RP+RQ$

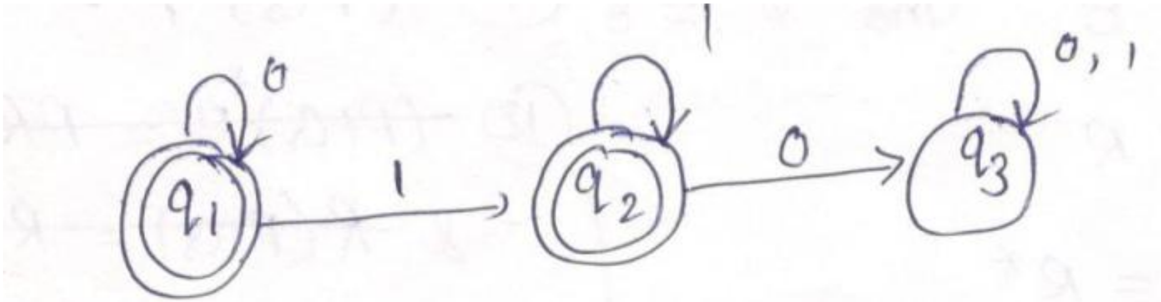
Arden's Theorem:

Let P and Q be two R.E. over Σ. If P does not contain ε, then the equation $R=Q+RP$ has a unique solution $R=QP^*$

Construct R.E. corresponding to the state diagram



Construct R.E. corresponding to the state diagram



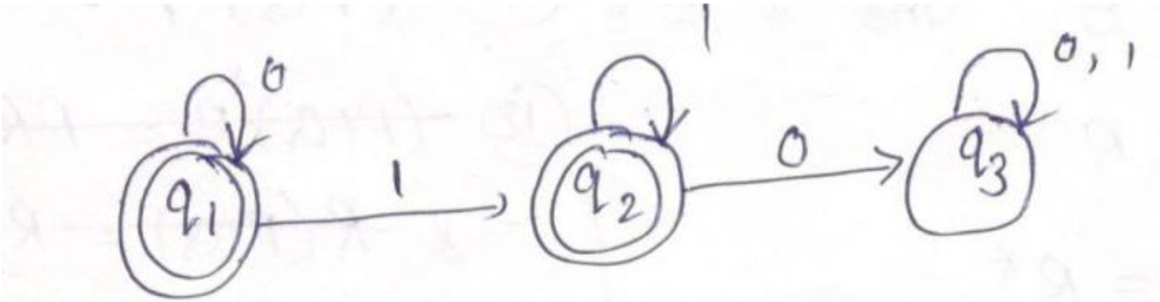
The three equation for q1, q2 and q3 can be written as:

$q_1 = q_1 0 + \varepsilon$ …………(1)

$q_2 = q_1 1 + q_2 1$ ………(2)

$q_3 = q_2 0 + q_3 0 + q_3 1$…….(3)

Construct R.E. corresponding to the state diagram



R=Q+RP
R=QP*

From eq. 1

$q_1=q_1\,0 + \varepsilon$ ………(1)

$q_1= \varepsilon 0^*$ using Arden theorem

From eq. 2

$q_2=q_1\,1+q_2\,1$ ………(2)

$q_2= \varepsilon 0^*\,1+q_2\,1= 0^*\,1+q_2\,1$

$\Rightarrow q_2 = (0^*\,1)1^*$

now from eq. 3 $q_3=q_20+q_30+q_31$…….(3)

$q_3= (0^*\,1)1^*\,0+q_30+q_31= (0^*\,1)1^*\,0+(0+1)q_3$

Construct R.E. corresponding to the state diagram

R=Q+RP
R=QP*

As q_1 and q_2 are the final states, so we need not to go for state q_3
So, the require regular expression

$R=q_1+q_2$

$R= \varepsilon 0^* + (0^*1)1^*$

$R=0^* + (0^*1)1^*$ by identity 3

$R=0^* + (0^*1)1^*$

$R=0^*(\varepsilon + 11^*)$

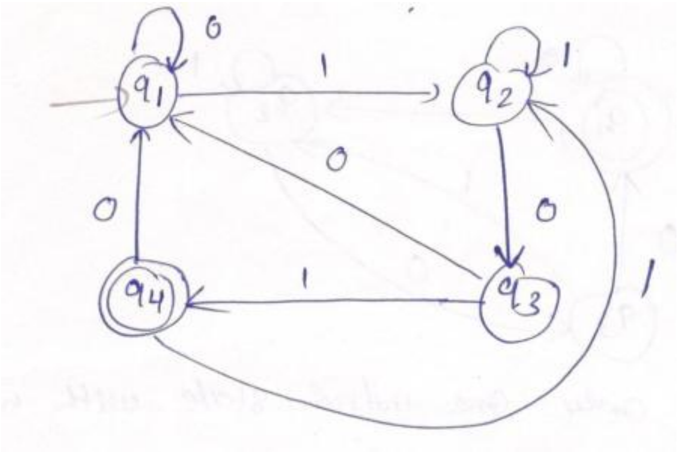
$R=0^*(1^*)$ by identity 9

$R=0^*1^*$

Practice Problems

1. Convert the R.E (a+bc*d)* to F.A.

2. Convert F.A to R.E using Arden's theorem.



Suggested readings

1. An introduction to FORMAL LANGUAGES and AUTOMATA by PETER LINZ.
2. Introduction to Automata Theory, Languages, And Computation by JOHN E. HOPCROFT, RAJEEV MOTWANI, JEFFREY D. ULLMAN
3. Theory of computer science: automata, languages and computation **by** K.L.P MISHRA

Thank you