

Following are the complexity classes:

1. **P Class**
2. **NP Class**
3. **CoNP Class**
4. **NP-hard**
5. **NP-complete**

P Class

The P in the P class stands for **Polynomial Time**. It is the collection of decision problems (problems with a “yes” or “no” answer) that can be solved by a deterministic machine in polynomial time.

Features:

1. The solution to P problems is easy to find.
2. P is often a class of computational problems that are solvable and tractable.
3. Tractable means that the problems can be solved in theory as well as in practice.
4. But the problems that can be solved in theory but not in practice are known as intractable.

This class contains many natural problems:

1. **Calculating the greatest common divisor.**
2. **Finding a maximum matching.**
3. **Decision versions of linear programming.**

NP Class

The NP in NP class stands for **Non-deterministic Polynomial Time**.

It is the collection of decision problems that can be solved by a non-deterministic machine in polynomial time.

Features:

1. The solutions of the NP class are hard to find since they are being solved by a non-deterministic machine but the solutions are easy to verify.
2. Problems of NP can be verified by a Turing machine in polynomial time.

Example:

- Let us consider an example to better understand the NP class.
- Suppose there is a company having a total of 1000 employees having unique employee IDs. Assume that there are 200 rooms available for them. A selection of 200 employees must be paired together, but the CEO of the company has the data of some employees who can't work in the same room due to personal reasons.
- This is an example of an NP problem.
- Since it is easy to check if the given choice of 200 employees proposed by a co-worker is satisfactory

or not i.e. no pair taken from the co-worker list appears on the list given by the CEO.

- But generating such a list from scratch seems to be so hard as to be completely impractical.

It indicates that if someone can provide us with the solution to the problem, we can find the correct and incorrect pair in polynomial time.

Thus for the NP class problem, the answer is possible, which can be calculated in polynomial time.

This class contains many problems that one would like to be able to solve effectively:

1. **Boolean Satisfiability Problem (SAT).**
2. **Hamiltonian Path Problem.**
3. **Graph coloring.**

Co-NP Class

Co-NP stands for the complement of NP Class. It means if the answer to a problem in Co-NP is No, then there is proof that can be checked in polynomial time.

Features:

1. If a problem X is in NP, then its complement X' is also in CoNP.
2. For an NP and CoNP problem, there is no need to verify all the answers at once in polynomial time,

there is a need to verify only one particular answer “yes” or “no” in polynomial time for a problem to be in NP or CoNP.

Some example problems for CO-NP are:

1. **To check prime number.**
2. **Integer Factorization.**

NP-hard class

An NP-hard problem is at least as hard as the hardest problem in NP and it is a class of problems such that every problem in NP reduces to NP-hard.

Features:

1. All NP-hard problems are not in NP.
2. It takes a long time to check them. This means if a solution for an NP-hard problem is given then it takes a long time to check whether it is right or not.
3. A problem A is in NP-hard if, for every problem L in NP, there exists a polynomial-time reduction from L to A.

Some of the examples of problems in Np-hard are:

1. **Halting problem.**
2. **Qualified Boolean formulas.**
3. **No Hamiltonian cycle.**

NP-complete class

A problem is NP-complete if it is both NP and NP-hard. NP-complete problems are the hard problems in NP.

Features:

1. NP-complete problems are special as any problem in NP class can be transformed or reduced into NP-complete problems in polynomial time.
2. If one could solve an NP-complete problem in polynomial time, then one could also solve any NP problem in polynomial time.

Some example problems include:

1. **Decision version of 0/1 Knapsack.**
2. **Hamiltonian Cycle.**
3. **Satisfiability.**
4. **Vertex cover.**

Complexity Class

Characteristic feature

P

Easily solvable in polynomial time.

NP

Yes, answers can be checked in polynomial time.

Co-NP

No, answers can be checked in polynomial time.

NP-hard

All NP-hard problems are not in NP and it takes a long time to check them.

NP-complete

A problem that is NP and NP-hard is NP-complete.