Theory of Computation: CS-202

Turing Machine

Outline

- ❑ Standard Turing Machine
- ❑ Examples

Standard Turing Machine

A Turing Machine is an automaton whose temporary storage is a tape, which is divided into cells, each of which is capable of holding one symbol

Formal Definition of a Standard Turing machine

A Turing machine (TM) is defined by the seven-tuples:

$M = (Q, \Sigma, \Gamma, \delta, q_0, □, F)$

Q A finite set of internal states

Σ A finite set of input alphabet

Γ A finite set of symbols called tape alphabet

q_0 The initial/starting state, q_0 is in Q

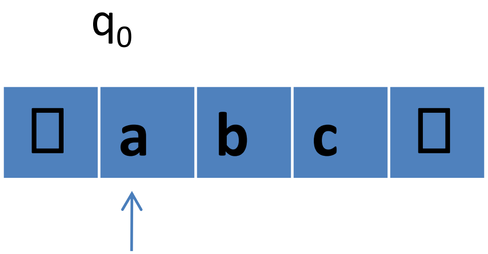
□ A special symbol called the blank symbol, is in Γ

F A set of final/accepting states, which is a subset of Q

δ A transition function, where

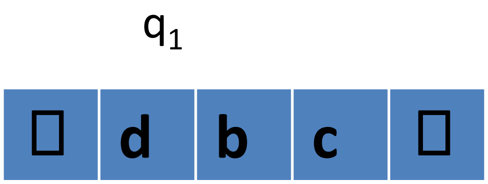
$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L/R\}$$

Moves of the Turing Machine



q_0

□	a	b	c	□

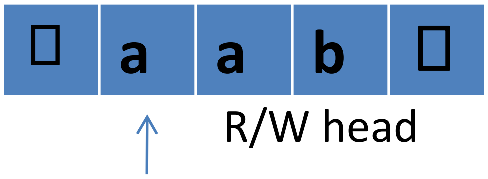


q_1

□	d	b	c	□

$\delta(q_0, a) = (q1, d, R)$

Input tape



□	a	a	b	□

R/W head

Finite control uint

Consider a Turing machine defined by

$M = (Q, \Sigma, \Gamma, \delta, q_0, \square, F)$

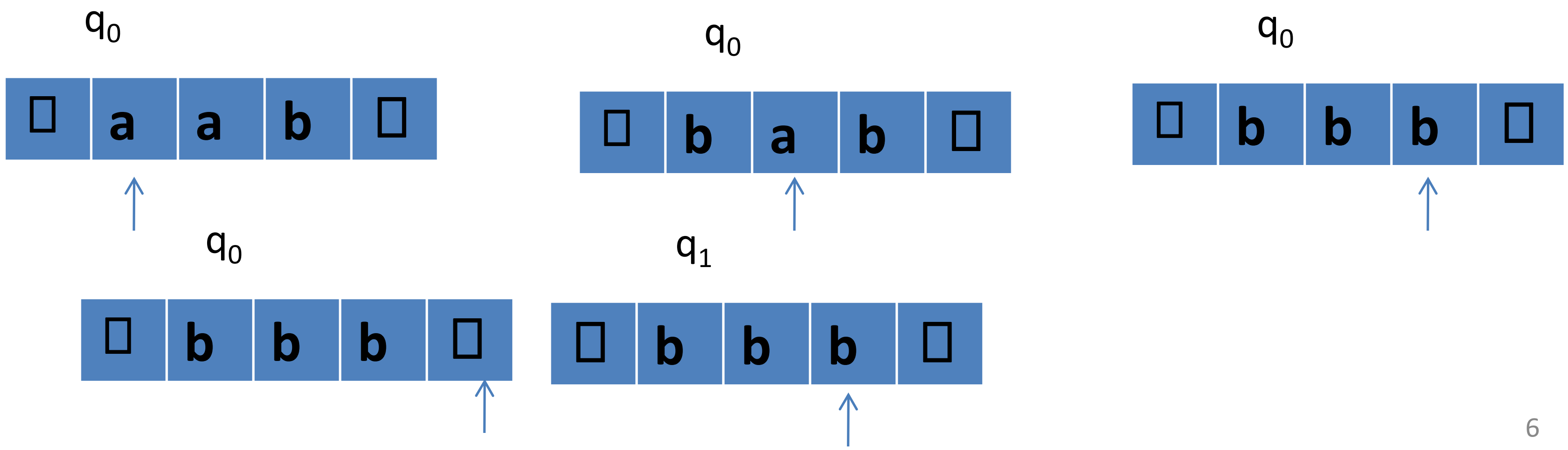
$Q \quad =\{q0, q1\}$

$\Sigma \quad =\{a, b\}$

$\Gamma \quad =\{a, b, \square\}$

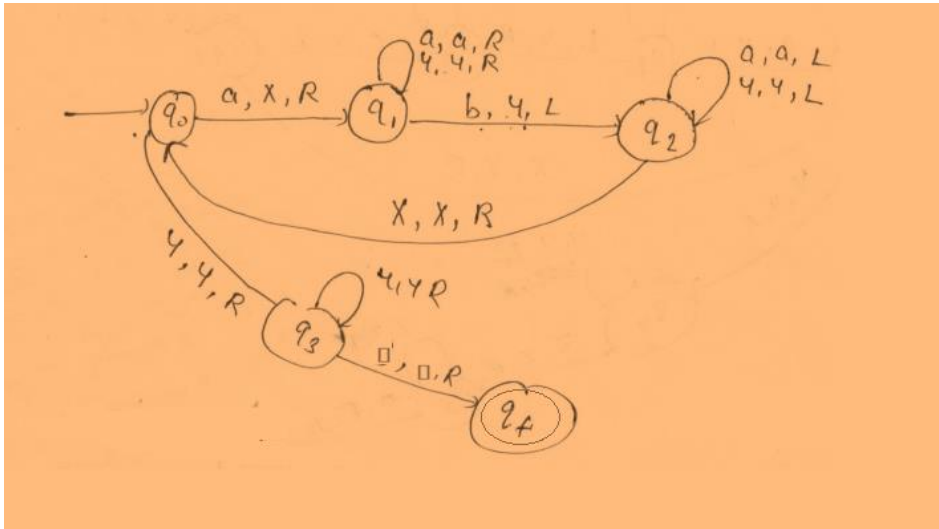
$F \quad = \{q_1\}$

& $\delta(q0, a)=(q0, b, R)$, $\delta(q0, b)=(q0, b, R)$, $\delta(q0, \square)=(q1, \square, L)$



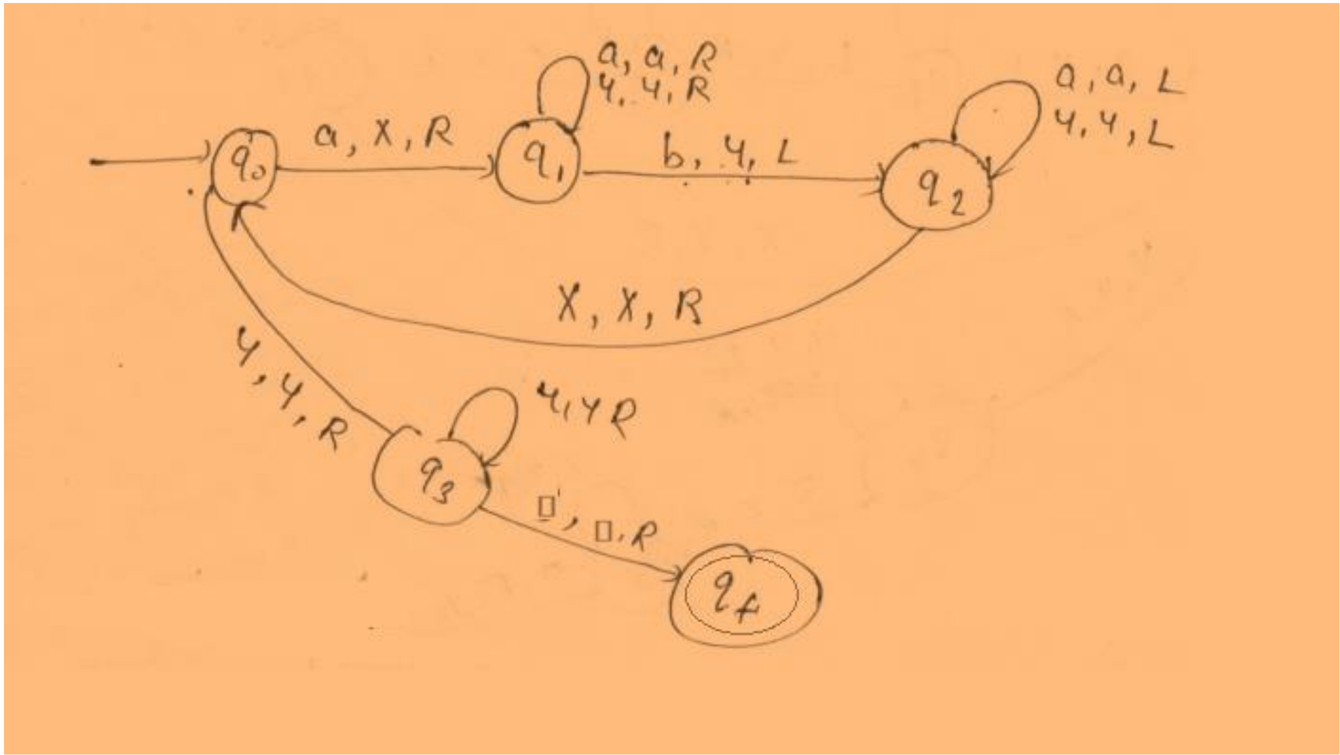
Design a Turing Machine for the language L={$a^n b^n$, n≥1}

□	**a**	**a**	**b**	**b**	□



Transition Table

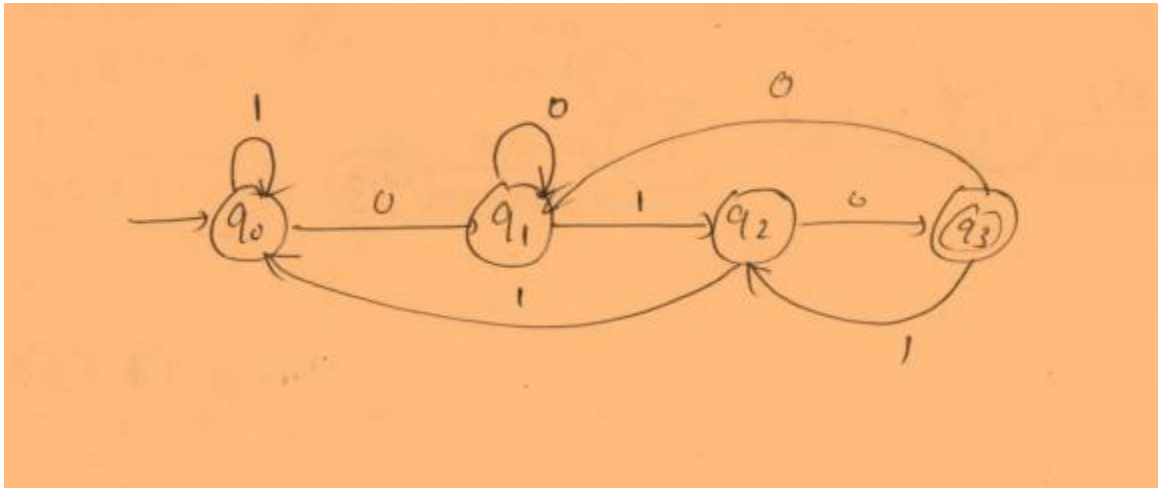
	a	b	X	Y	□
q_0	(q_1, X, R)	-	-	(q_3, Y, R)	-
q_1	(q_1, a, R)	(q_2, Y, L)	-	(q_1, Y, R)	-
q_2	(q_2, a, L)	-	(q_0, X, R)	(q_2, Y, L)	-
q_3	-	-	-	(q_3, Y, R)	(q_f, □, R)
q_f	-	-	-	-	-



Design a Turing Machine that accepts a set of string ending with 010

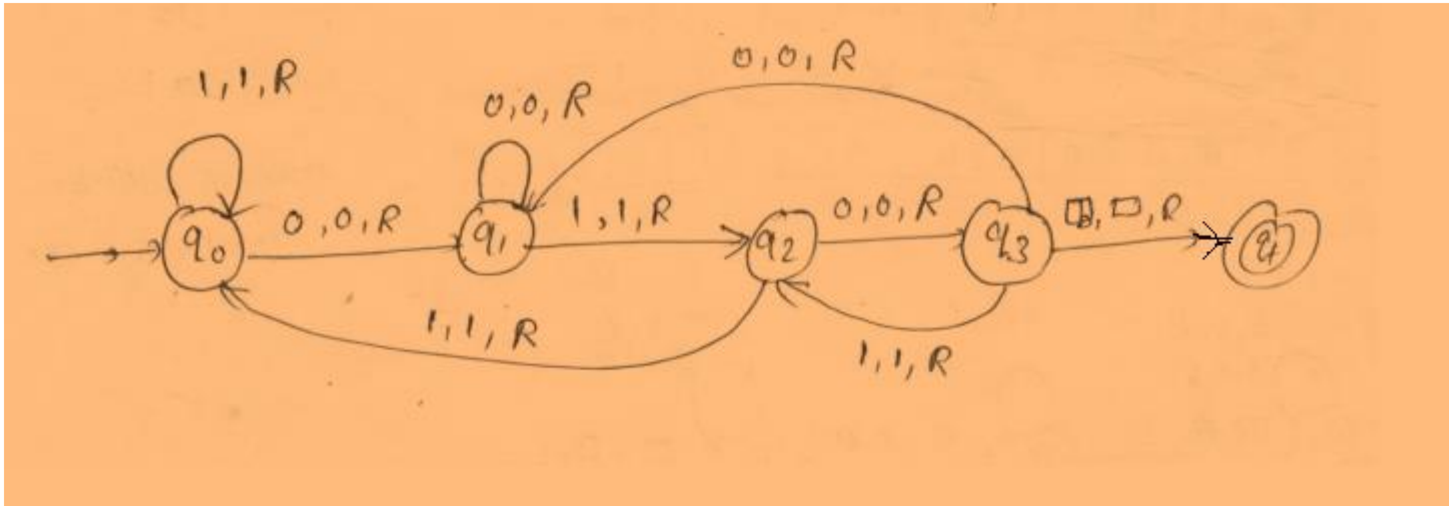
1	**0**	**1**	**0**	□

DFA



Design a Turing Machine that accepts a set of string ending with 010

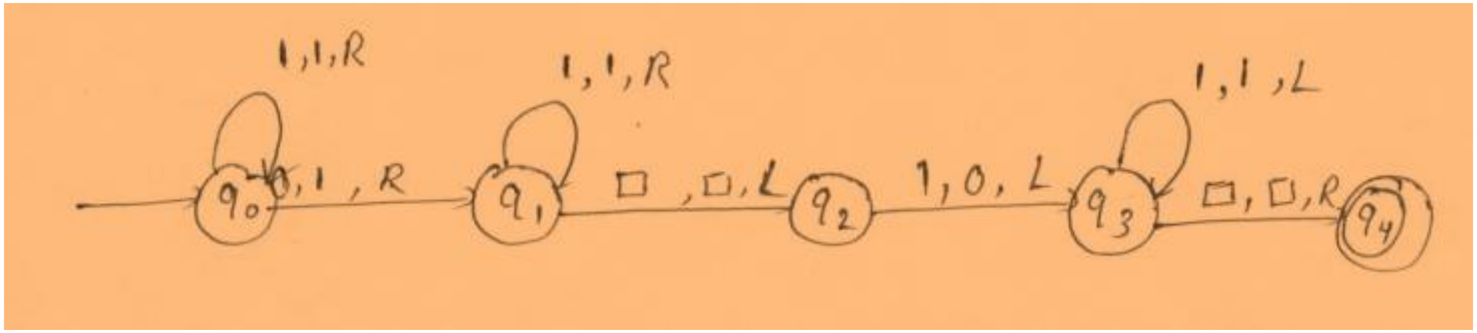
1	**0**	**1**	**0**	□



Design a Turing Machine to add two positive integers

□	1	1	1	0	1	1	□

X=3
Y=2



Design a Turing Machine to add two positive integers

1	1	1	0	1	1	□

$\delta(q_0,1) = (q_0,1,R)$

$\delta(q_0,0) = (q_1,1,R)$

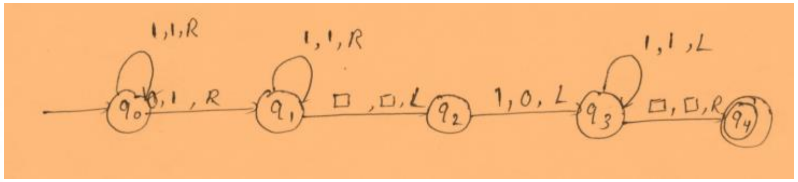
$(q_1,1) = (q_1,1,R)$

$(q_1,\square) = (q_2,\square,L)$

$(q_2,1) = (q_3,0,L)$

$(q_3,1) = (q_3,1,L)$

$(q_3,\square) = (q_4,\square,R)$



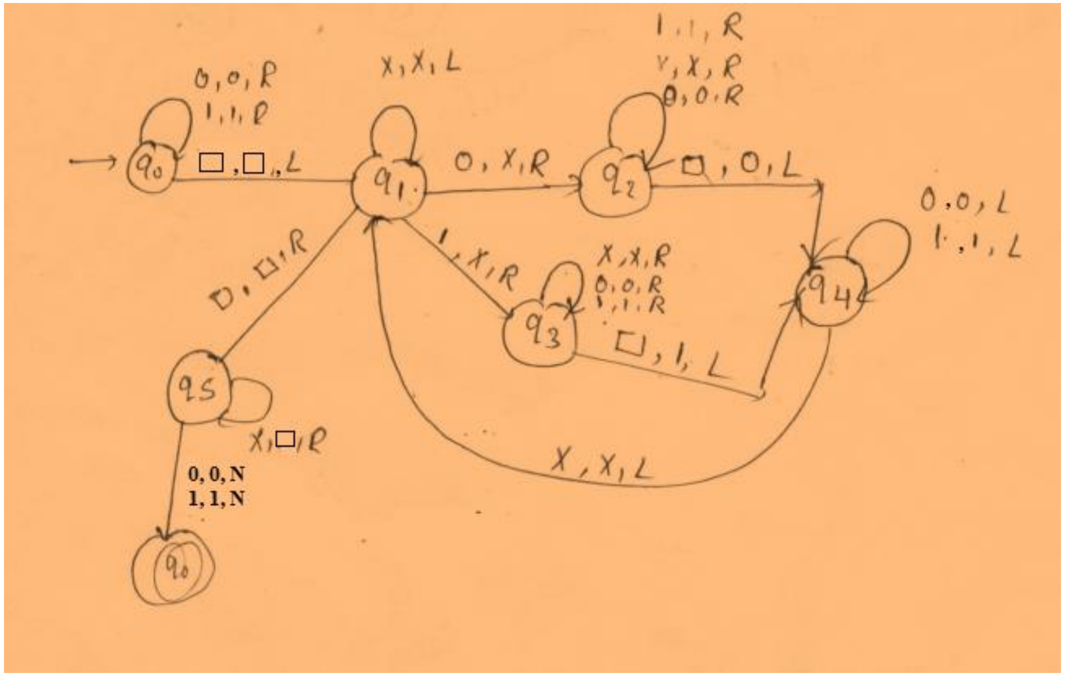
$q_0\,111011 \vdash 1q_0\,11011 \vdash 11q_0\,1011 \vdash 111q_0\,011 \vdash 1111q_1\,11$

$\vdash 11111q_1\,1 \vdash 111111q_1\,\square \vdash 11111q_2\,1 \vdash 1111q_3\,0$

$\dashv$

Design a Turing Machine that computes string reversal

□	1	0	0	□	□	□



Suggested readings

1. An introduction to FORMAL LANGUAGES and AUTOMATA by PETER LINZ.
2. Introduction to Automata Theory, Languages, And Computation by JOHN E. HOPCROFT, RAJEEV MOTWANI, JEFFREY D. ULLMAN
3. Theory of computer science: automata, languages and computation **by** K.L.P MISHRA