

# Theory of Computation: CS-202

## Finite Automata

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## Finite Automata

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# Deterministic Finite Accepters

## Definition

A **deterministic finite accepter** or **dfa** is defined by the quintuple

$$M = (Q, \Sigma, \delta, q_0, F)$$

where  $Q$  is a finite set of **internal states**,

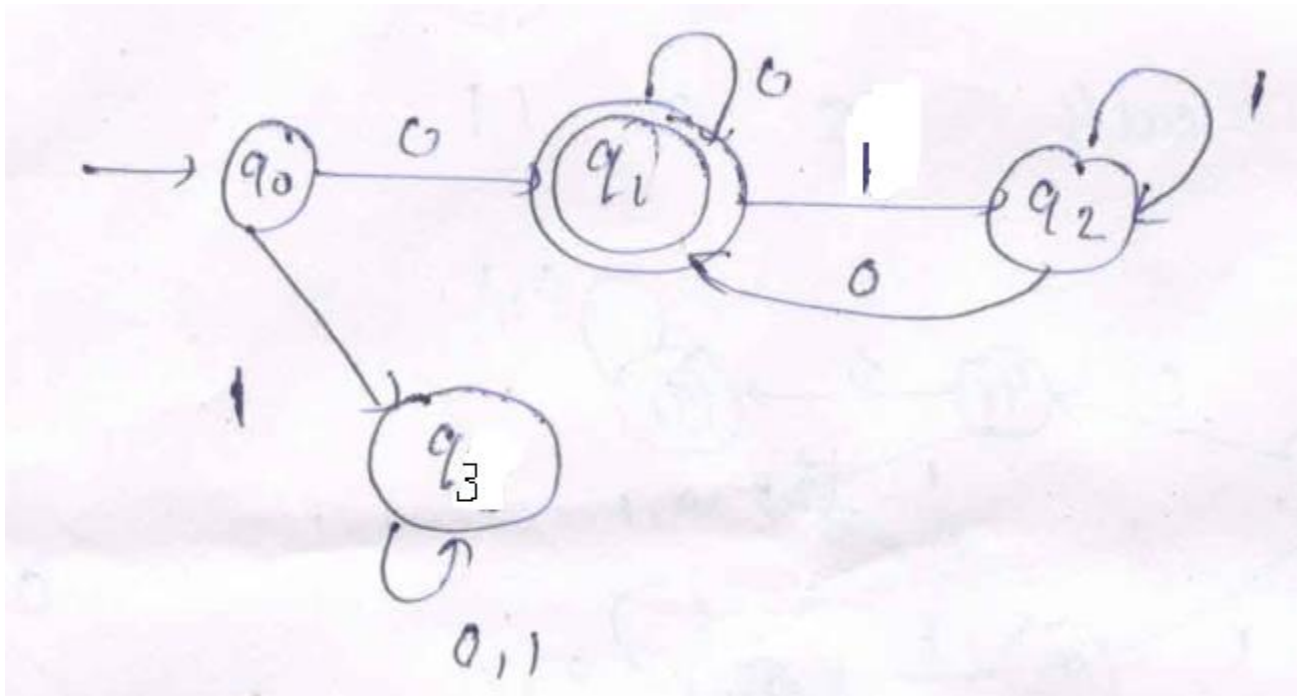
$\Sigma$  is a finite set of symbols called the **input alphabet**,

$\delta: Q \times \Sigma \rightarrow Q$  is a **total** function called the **transition function**,

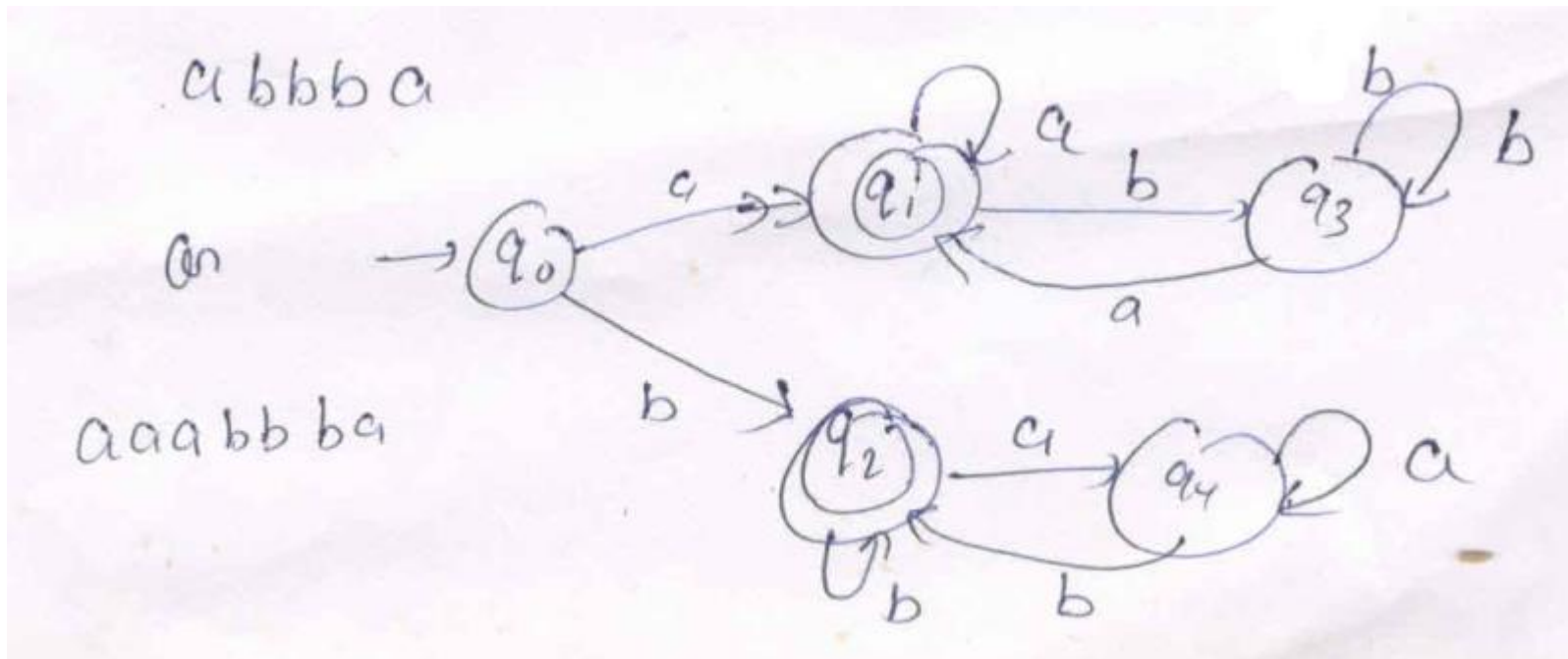
$q_0 \in Q$  is the **initial state**,

$F \subseteq Q$  is a set of **final states**.

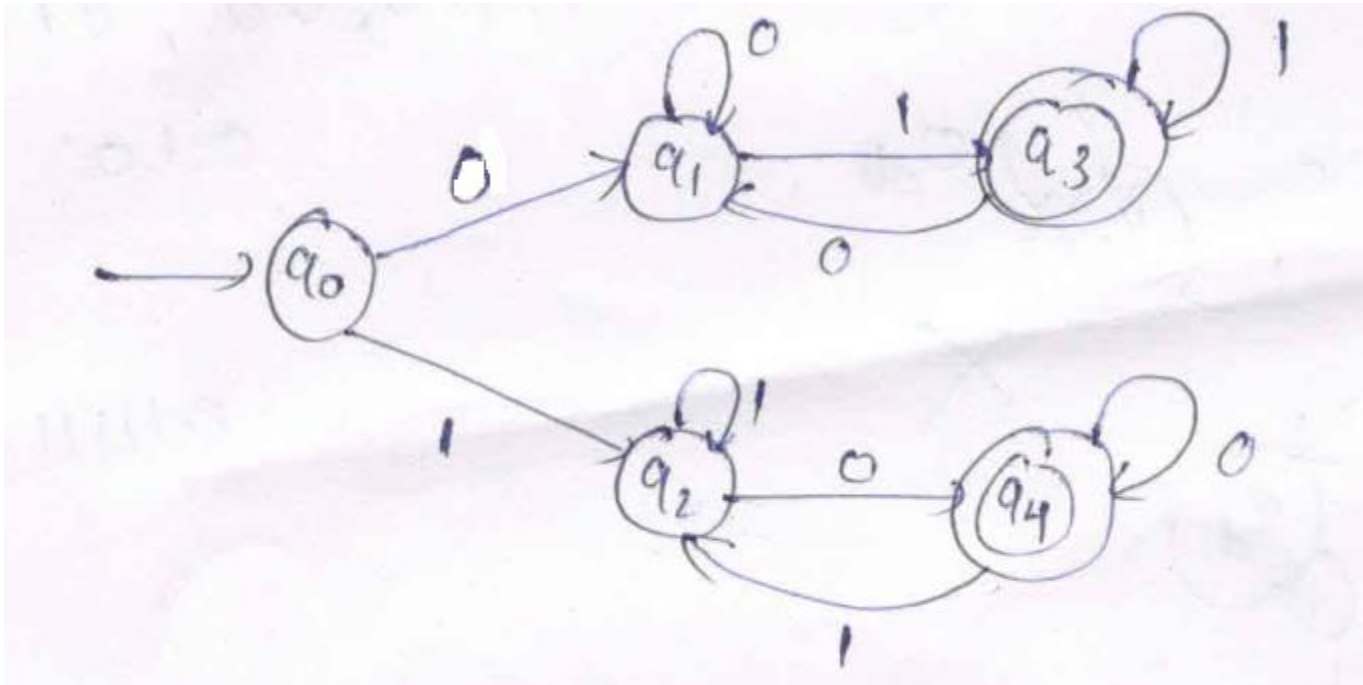
- Draw a DFA which accepts all the strings on  $\Sigma=\{0,1\}$  which must start & end with '0'.



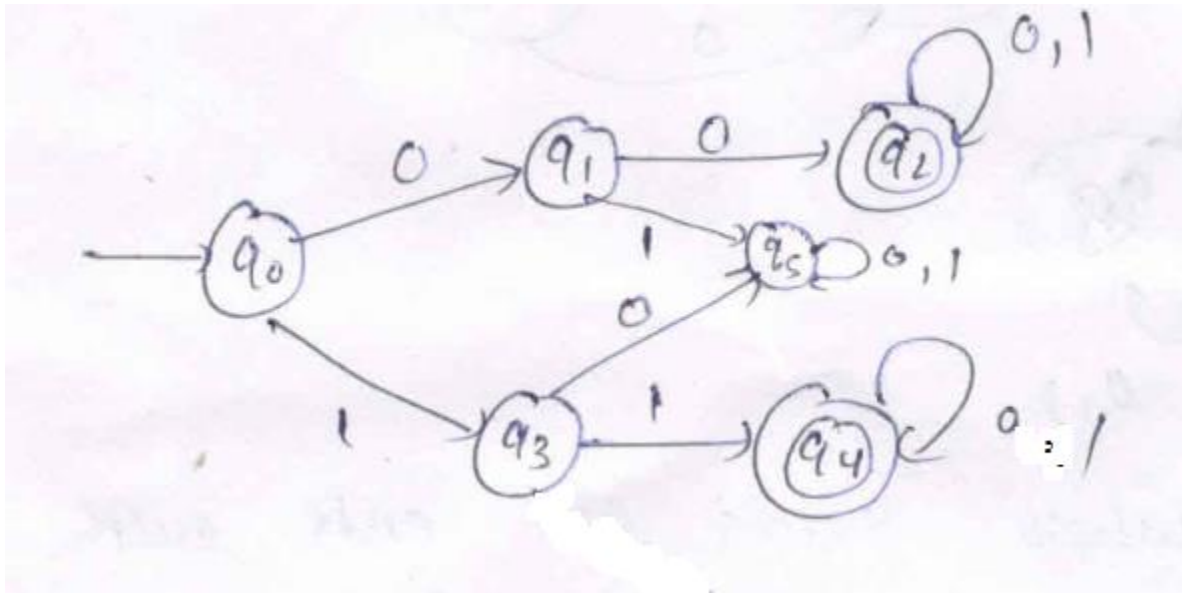
- Draw a DFA which accepts all the strings on  $\Sigma=\{a,b\}$  which starts and ends with same symbol.



- Draw a DFA which accepts all the strings on  $\Sigma=\{a,b\}$  which starts and ends with different symbol.



- Draw a DFA which accepts all the strings on  $\Sigma=\{0,1\}$  which starts with '00' or '11'.



# Practice Problems

1. Draw a DFA which accepts all the strings on  $\Sigma=\{0,1\}$  which never ends with '100'.
2. Draw a DFA which accepts all the strings on  $\Sigma=\{0,1\}$  which contains sub-string '00'.



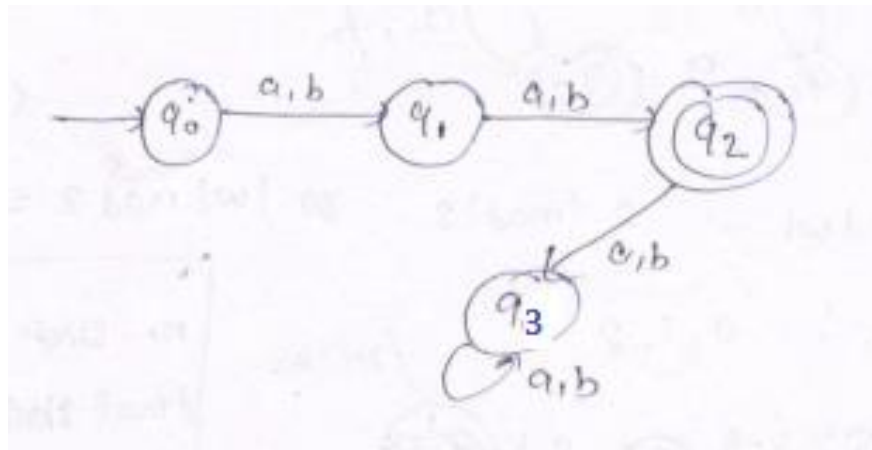
1. Draw a DFA over  $\Sigma=\{a,b\}$  which accepts all the strings of length 2.

Input strings

b	a	a
---	---	---

not accepted by given DFA.

a	b
---	---

 Accepted

2. Draw a DFA over  $\Sigma = \{a, b\}$  which accepts all the strings of length less or equal to '2'.

Input string

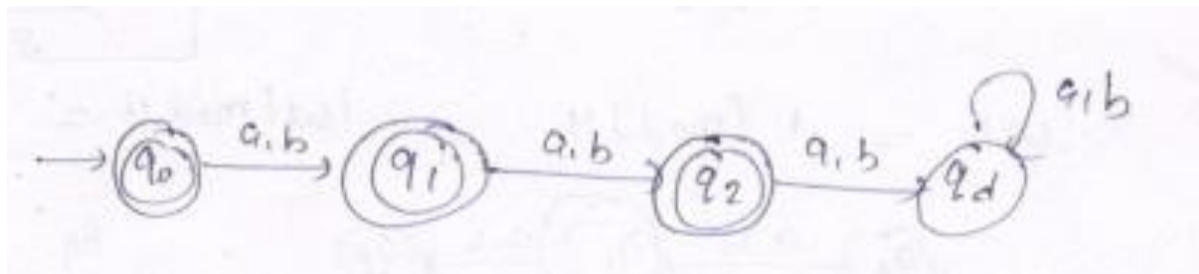
**b**

Accepted by given DFA.

**b**

**a**

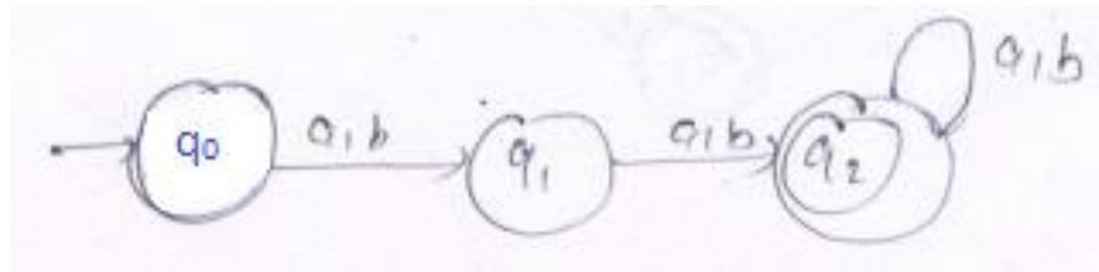
Accepted



3. Draw a DFA over  $\Sigma=\{a,b\}$  which accepts all the strings of length greater or equal to '2'.

Input string

b	a	a	b	b	b
---	---	---	---	---	---



4. Draw a DFA over  $\Sigma = \{a, b\}$  which accepts all the strings 'w' in which  $n_a(w) = 2$ .

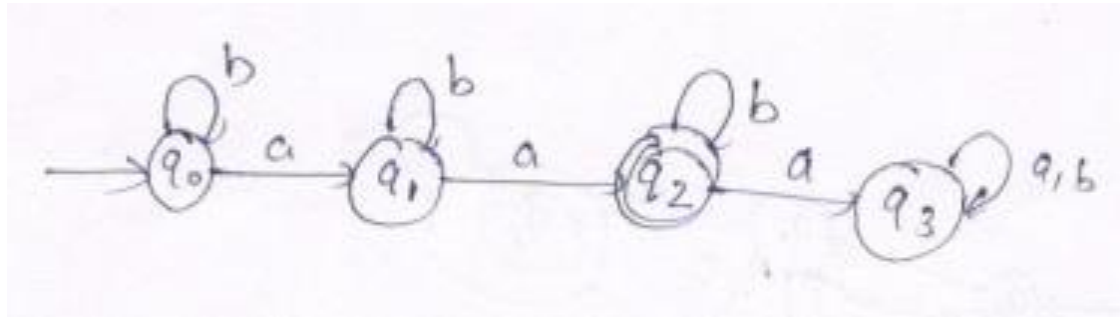
Input string 

b	a	a	b	a	b
---	---	---	---	---	---

 Not accepted

b	a	b	b	a	b
---	---	---	---	---	---

 Accepted



5. Draw a DFA over  $\Sigma = \{a, b\}$  which accepts all the strings 'w' in which  $n_a(w) \leq 2$ .

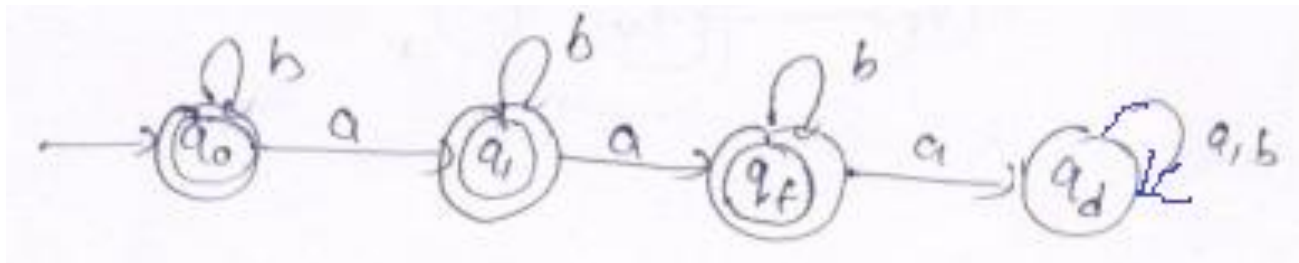
Input string 

b	a	a	b	a	b
---	---	---	---	---	---

 Not accepted

b	a	b	b	b	b
---	---	---	---	---	---

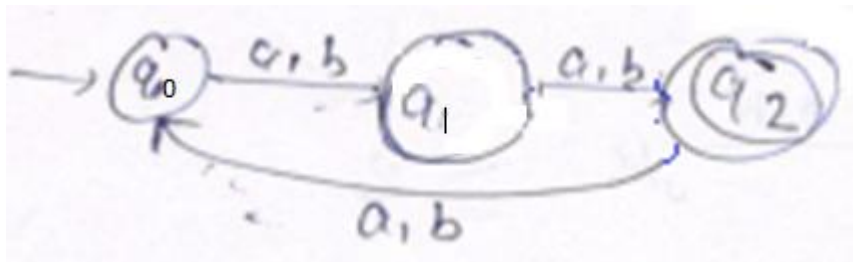
 Accepted



6. Draw a DFA over  $\Sigma=\{a,b\}$  which accepts all the strings 'w' in which  $|w| \bmod 3 = 2$ .

Input string

b	a	a	b	a
---	---	---	---	---



Mod 3 , Remainder: 0, 1, 2

String length	Remainder
0	0
1	1
2	2
3	0
4	1
5	2
...	...
...	...
8	2

# Nondeterministic Finite Accepters

A **nondeterministic finite accepter** or **nfa** is defined by the quintuple

$$M = (Q, \Sigma, \delta, q_0, F)$$

where  $Q$  is a finite set of **internal states**,

$\Sigma$  is a finite set of symbols called the **input alphabet**,

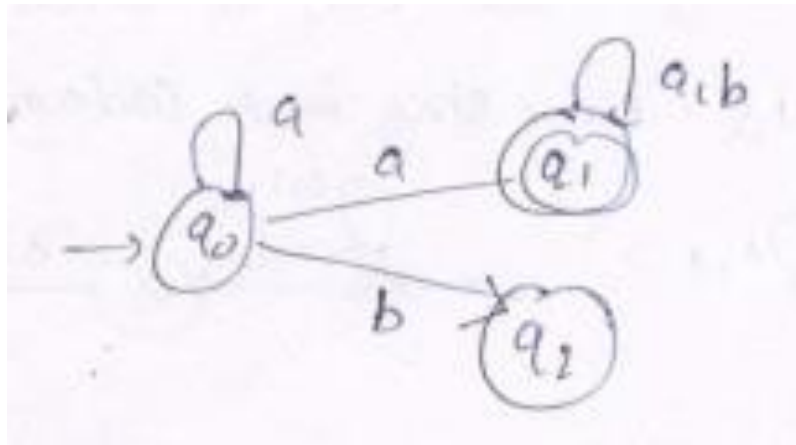
$q_0 \in Q$  is the **initial state**,

$F \subseteq Q$  is a set of **final states**.

$$\delta: Q \times (\Sigma \cup \{\lambda\}) \rightarrow 2^Q$$

**$\forall$  NFA  $\exists$  a DFA**

**$\Rightarrow$  DFA  $\subseteq$  NFA**



$$\delta(q_0, a) = \{q_0, q_1\}$$

$$\delta(q_2, b) = \{\lambda\}$$



# Transition graph

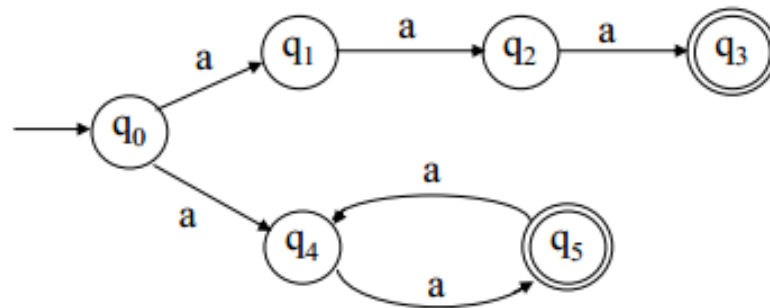
Transition Graph of an nfa  $M = (Q, \Sigma, \delta, q_0, F)$

Vertex labeled with  $q_i$ : state  $q_i \in Q$ ,

Edge from  $q_i$  to  $q_j$  labeled with  $a$ :  $q_j \in \delta(q_i, a)$

Example

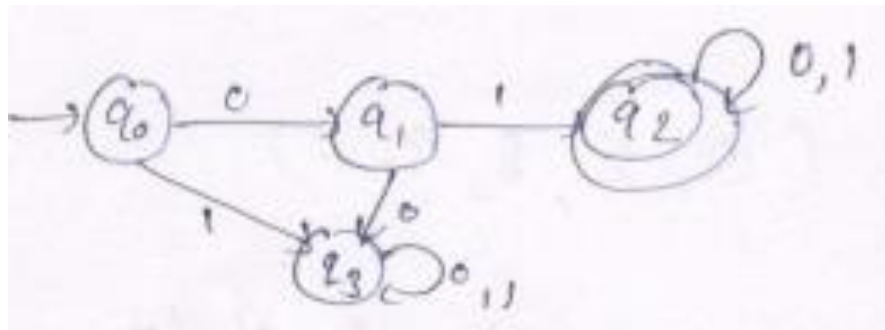
An nfa is shown as below



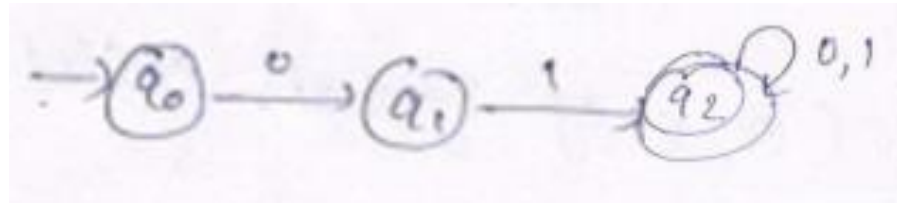
# Examples

1. Draw NFA which accepts all the strings on  $\Sigma=\{0,1\}$  which starts with '01'.

DFA



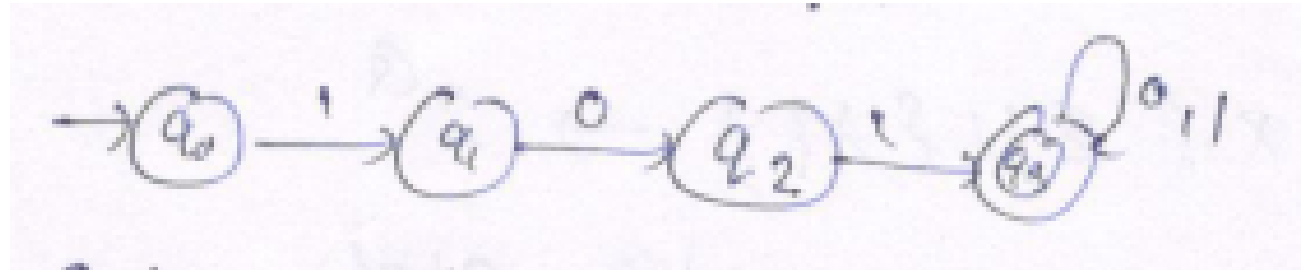
NFA



# Examples

2. Design NFA which accepts all the strings on  $\Sigma=\{0,1\}$  starting with '101'.

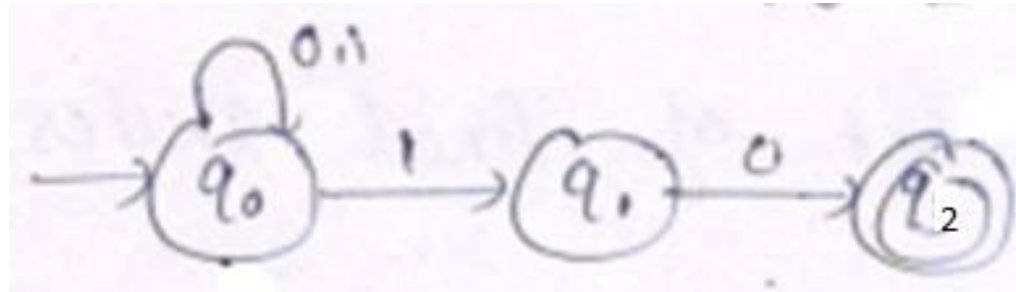
NFA



# Examples

3. Design NFA which accepts all the strings on  $\Sigma=\{0,1\}$  ending with '10'.

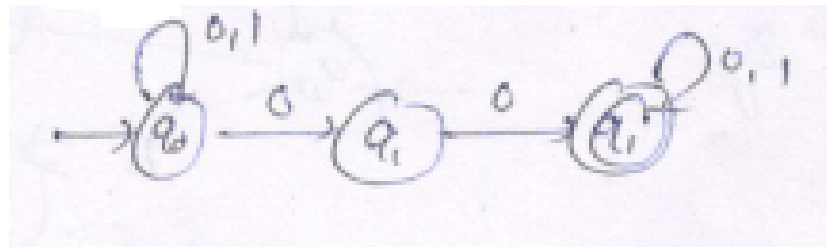
NFA



# Examples

4. Design NFA which accepts all the strings on  $\Sigma=\{0,1\}$  which contains substring '00'.

NFA



## Suggested readings

1. An introduction to FORMAL LANGUAGES and AUTOMATA by PETER LINZ.
2. Introduction to Automata Theory, Languages, And Computation by JOHN E. HOPCROFT, RAJEEV MOTWANI, JEFFREY D. ULLMAN
3. Theory of computer science: automata, languages and computation **by** K.L.P MISHRA