Theory of Computation: CS-202

Regular Expression

Outline

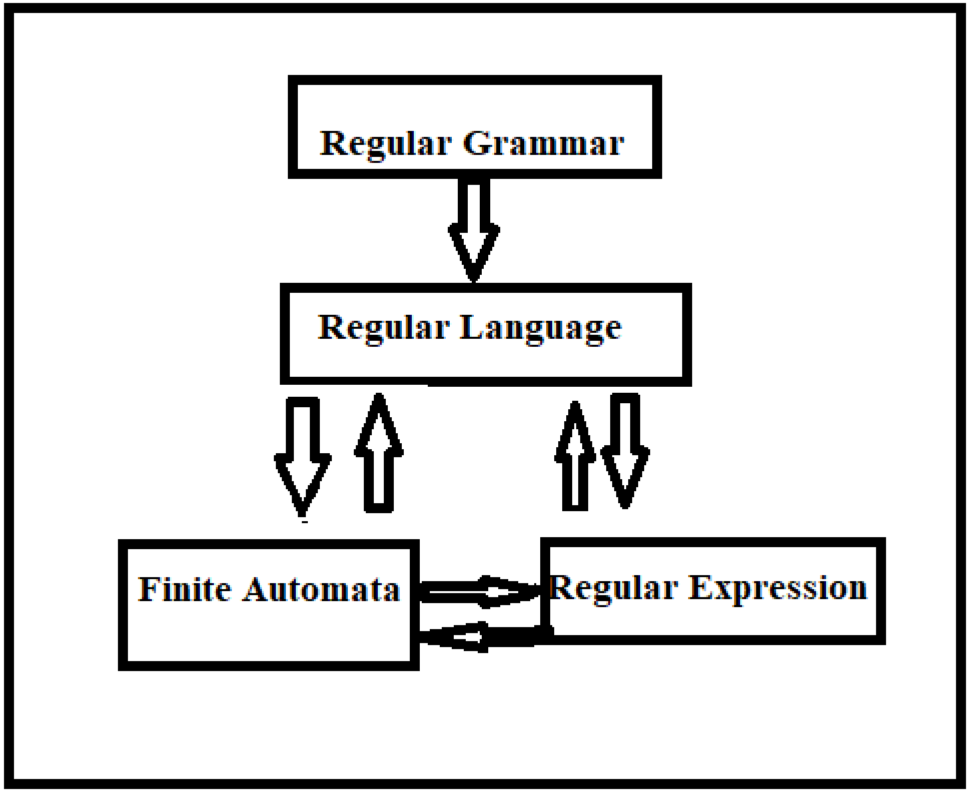
- ❑ Regular Expressions

- ❑ Regular Languages

Finite Automata

Finite Automata

- Finite Automata without output
- Finite Automata with output


Regular Grammar
Regular Language
Finite Automata
Regular Expression

Formal definition of Regular Expression

Let $\sum$ be a given input alphabet, then

1. φ, λ and $a \in \sum$ are all regular expression, called primtive Regular Expression (R.E).

2. If r_1 and r_2 are R. E. then $(r_1 + r_2)$, $(r_1 . r_2)$, r_1* and (r_1) are also R.E.

3. A string is a R.E. if we can derive it from the premitive R.E. by a finite number of application of the rule 2.
4. Order of operator, (), *, ., +

Example

Consider the expression $r=(a+b.c)^*$

Let $r_1= b$ and $r_2=c$, then r1. r2 is a regular expression.

Let $r_3= a$ and $r_4=$ r1 . r2 then $r_3 + r_4$ is a R. E.

Let $r_5=(r_3 + r_4)$

$r_6=(r_5)$

$(r_6)^*$ is also a R.E.

$\Rightarrow$ r is a R.E.

Order and precedence of operator

1. Star closure precedes concatenation.
2. Concatenation precedes union.

Language associated with Regular Expression

The language L(r) denoted by any regular expression (r) is defined by the following rules:

1. φ is a regular expression denoting empty set {}.
2. λ is a regular expression denoting $\{\lambda\}$.
3. $\forall\, a \in \sum$, 'a' is a regular expression denoting {a}.

   If r_1 and r_2 are regular expression , then

4. $L(r_1 + r_2)= L(r_1) \cup L(r_2)$
5. $L(r_1 . r_2)= L(r_1) . L(r_2)$
6. $L((r_1))=L(r_1)$
7. $L(r_1)^*= (L(r_1))^*$

Find Language associated with Regular Expression r=(a+b)* (a+bb)

We have, r=(a+b)*(a+bb)

$$
\begin{aligned}
L(r) &= L((a+b)^*(a+bb)) \\
&= L((a+b)^*)\, L(a+bb) \\
&= (L(a+b))^*\, L(a) \cup L(bb) \\
&= (L(a) \cup L(b))^*\, L(a) \cup L(bb) \\
&= \{a, b\}^*\, \{a, bb\} \\
&= \{\lambda, a, b, aa, bb, ab, ba, aaa\ldots\}\ \{a, bb\} \\
&= \{a, b, aa, bb, ba, aaa\ldots\ldots\ bb, abb, aabb, \ldots\ldots\ldots\ldots\}
\end{aligned}
$$

So, L(r) is the set of all the strings on {a, b} terminated by either a or bb.

$L(r_1 + r_2)= L(r_1) \cup L(r_2)$

$L(r_1 . r_2)= L(r_1) . L(r_2)$

$L((r_1))=L(r_1)$

$L(r_1)^*= (L(r_1))^*$

Find Language associated with Regular Expression $r=(a)^* (a+b)$

We have, $r=(a)^*(a+b)$

$L(r) = L((a)^*(a+b))$

$= L((a)^*)\ L(a+b)$

$= (L(a))^*\ L(a) \cup L(b)$

$= (L(a))^*\ L(a) \cup L(b)$

$= \{a\}^*\ \{a, b\}$

$= \{\lambda, a, aa, aaa\ldots\}\ \{a, b\}$

$= \{a, aa, aaa\ldots\ldots\ b, ab, aab,\ldots\ldots\ldots\ldots\}$

$L(r_1 + r_2)= L(r_1) \cup L(r_2)$

$L(r_1 . r_2)= L(r_1) . L(r_2)$

$L((r_1))=L(r_1)$

$L(r_1)^*= (L(r_1))^*$

Find Language associated with Regular Expression r=(aa)* (bb)*b

We have, r=(aa)*(bb)*b

$L(r) = \{ a^{2m} b^{2n+1} : m, n \geq 0$

1. Write Regular Expression for the set of strings over {0, 1} which starts with ’01’

R. E. $r = 01(0+1)^*$

2. Write Regular Expression for the set of strings over {0, 1} which starts and ends with ’0’

R. E. $r = 0(0+1)^*0$

3. Write Regular Expression for the set of strings over {0, 1} which starts and ends with different symbol

R. E. r= $0\,(0+1)^*1 + 1\,(0+1)^*\,0$

4. Write Regular Expression for the set of strings over {0, 1} which starts and ends with ’0’

R. E. r= $0\,(0+1)^*\,0$

5. Write Regular Expression for the set of strings of length 2 over $\{0, 1\}$

R. E. $r = (0+1)(0+1)$

6. Write Regular Expression for the set of strings of length ≥ 2 over $\{0, 1\}$

R. E. $r = (0+1)(0+1)(0+1)^*$

7. Write Regular Expression for the set of strings (w) over{a, b}, where, $n_a(w)=2$

R. E. r= $b^* \, a \, b^* \, a \, b^*$

8. Write Regular Expression for the set of strings (w) over{a, b}, where, $n_b(w) \bmod 3=0$

R. E. r= $a^*(a^* \, b \, a^* \, b \, a^* \, b \, a^*)^*$

0, 3, 6, …..

9. Write Regular Expression for the set of strings (w) over{a, b}, where, |w| mod 3=0

R. E. $r = ((a+b)(a+b)(a+b))^*$

0, 3, 6,..........

10. Write Regular Expression for the set of strings (w) over{a, b}, where, |w| mod 3=2

R. E. $r = (a+b)(a+b)((a+b)(a+b)(a+b))^*$

Practice Problems

1. Write Regular Expression for the set of strings of length ≤ 2 over $\{0, 1\}$.

2. Write Regular Expression for the set of strings (w) over{a, b}, where, $n_a(w) \bmod 3=1$.

3. Write Regular Expression for the set of strings (w) over{a, b}, where, $|w| \bmod 4=3$.

Suggested readings

1. An introduction to FORMAL LANGUAGES and AUTOMATA by PETER LINZ.
2. Introduction to Automata Theory, Languages, And Computation by JOHN E. HOPCROFT, RAJEEV MOTWANI, JEFFREY D. ULLMAN
3. Theory of computer science: automata, languages and computation **by** K.L.P MISHRA

Thank you