4.3 Stacks and Queues

In this section, we introduce two closely-related data types for manipulating arbitrarily large collections of objects: the *stack* and the *queue*. Stacks and queues are special cases of the idea of a *collection*. Each is characterized by four operations: *create* the collection, *insert* an item, *remove* an item, and test whether the collection is *empty*.

Stacks.

A *stack* is a collection that is based on the last-in-first-out (LIFO) policy. By tradition, we name the stack *insert* method `push()` and the stack *remove* operation `pop()`. We also include a method to test whether the stack is empty, as indicated in the following API:

```
public class Stack<Item> implements Iterable<Item>
```

	`Stack()`	*create an empty stack*
`boolean`	`isEmpty()`	*is the stack empty?*
`void`	`push(Item item)`	*push an item onto the stack*
`Item`	`pop()`	*return and remove the item that was inserted most recently*
`int`	`size()`	*number of items on stack*

Array implementations of stacks.

Representing stacks with arrays is a natural idea. In particular, we maintain an instance variable `n` that stores the number of items in the stack and an array `items[]` that stores the `n` items, with the most recently inserted item in `items[n-1]` and the least recently inserted item in `items[0]`. This policy allows us to add and remove items at the end without moving any of the other items in the stack.

- *Fixed-length array implementation of a stack of strings.* ArrayStackOfStrings.java implements this approach for a stack of strings whose maximum capacity is specified by the argument to the constructor. To remove an item, we decrement `n` and then return `a[n]`; to insert a new item, we set `a[n]` equal to the new item and then increment `n`.

StdIn	StdOut	n	items[]				
			0	1	2	3	4
		0					
to		1	to				
be		2	to	be			
or		3	to	be	or		
not		4	to	be	or	not	
to		5	to	be	or	not	to
-	to	4	to	be	or	not	to
be		5	to	be	or	not	be
-	be	4	to	be	or	not	be
-	not	3	to	be	or	not	be
that		4	to	be	or	that	be
-	that	3	to	be	or	that	be
-	or	2	to	be	or	that	be
-	be	1	to	be	or	that	be
is		2	to	is	or	not	to

- *Resizing array implementation of a stack of strings.* ResizingArrayStackOfStrings.java is a version of ArrayStackOfStrings.java that dynamically adjusts the length of the array `items[]` so that it is sufficiently large to hold all of the items and but not so large as to waste an excessive amount of space. First, in `push()`, we check whether there is room for the new item; if not, we create a new array of *double* the length of the old array and copy the items from the old array to the new array. Similarly, in `pop()`, we check whether the array is too large, and we *halve* its length if that is the case.

StdIn	StdOut	n	items.length	items[]							
				0	1	2	3	4	5	6	7
		0	1	null							
to		1	1	to							
be		2	2	to	be						
or		3	4	to	be	or	null				
not		4	4	to	be	or	not				
to		5	8	to	be	or	not	to	null	null	null
-	to	4	8	to	be	or	not	null	null	null	null
be		5	8	to	be	or	not	be	null	null	null
-	be	4	8	to	be	or	not	null	null	null	null
-	not	3	8	to	be	or	null	null	null	null	null
that		4	8	to	be	or	that	null	null	null	null
-	that	3	8	to	be	or	null	null	null	null	null
-	or	2	4	to	be	null	null				
-	be	1	2	to	null						
is		2	2	to	is						

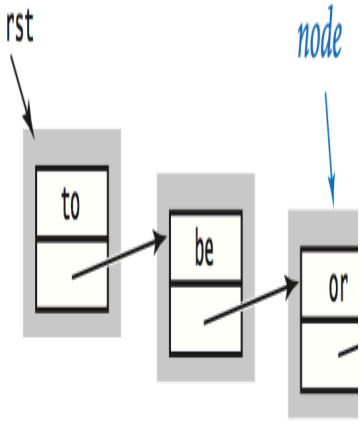
This doubling-and-halving strategy guarantees that that the stack never overflows and never becomes less than one-quarter full.

- *Resizing array implementation of a generic stack.* ResizingArrayStack.java implements a generic stack using a resizing array. For technical reasons, a cast is needed when allocating the array of generics.

Linked lists.

A *singly linked list* comprises a sequence of *nodes*, with each node containing a reference (or *link*) to its successor. By convention, the link in the last node is *null*, to indicate that it terminates the list. With object-oriented programming, implementing linked lists is not difficult. We define a class for the node abstraction that is *recursive* in nature:

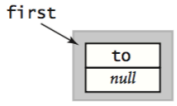
```
class Node {
   String item;
   Node next;
}
```



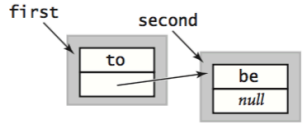
A `Node` object has two instance variables: a `String` and a `Node`. The `String` is a placeholder in this example for any data that we might want to structure with a linked list (we can use any set of instance variables); the instance variable of type `Node` characterizes the linked nature of the data structure.

- *Linking together a linked list.* For example, to build a linked list that contains the items `"to"`, `"be"`, and `"or"`, we create a `Node` for each item:

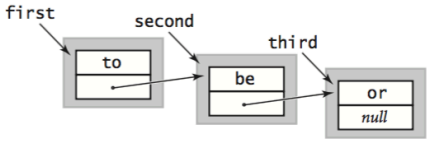
```
Node first  = new Node();
first.item  = "to";
```




```
Node second = new Node();
second.item = "be";
first.next  = second;
```



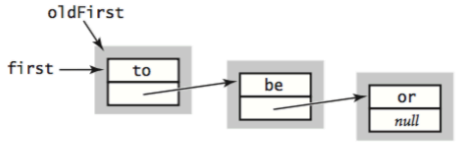

```
Node third  = new Node();
third.item  = "or";
second.next = third;
```



- *Insert.* Suppose that you want to insert a new node into a linked list. The easiest place to do so is at the beginning of the list. For example, to insert the string `not` at the beginning of a given linked list whose first node is `first`, we save `first` in a temporary variable `oldFirst`, assign to `first` a new `Node`, and assign its `item` field to `not` and its `next` field to `oldFirst`.

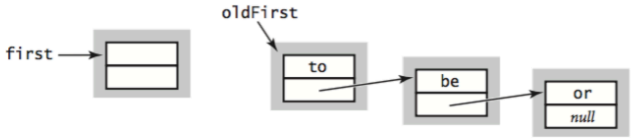
save a link to the first node in the linked list

```
Node oldFirst = first;
```



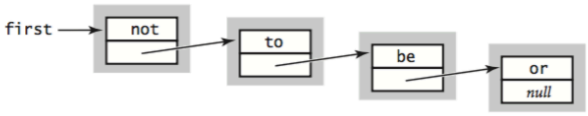
create a new node for the beginning

```
first = new Node();
```



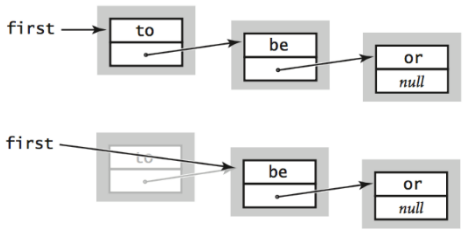
set the instance variables in the new node

```
first.item = "not";
first.next = oldFirst;
```



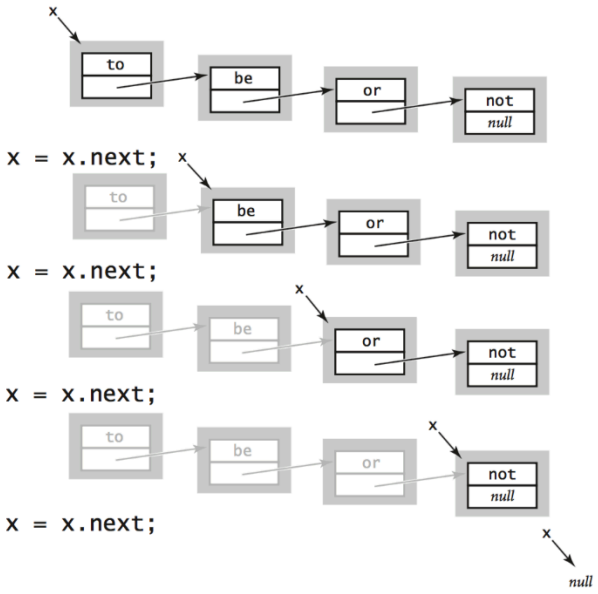
- *Remove.* Suppose that you want to remove the first node from a list. This operation is even easier: simply assign to `first` the value `first.next`.

```
first = first.next;
```



- *Traversal.* To examine every item in a linked list, we initialize a loop index variable `x` that references the the first `Node` of the linked list. Then, we find the value of the item associated with `x` by accessing `x.item`, and then update `x` to refer to the next `Node` in the linked list, assigning to it the value of `x.next` and repeating this process until `x` is `null` (which indicates that we have reached the end of the linked list). This process is known as *traversing* the list, and is succinctly expressed in this code fragment:

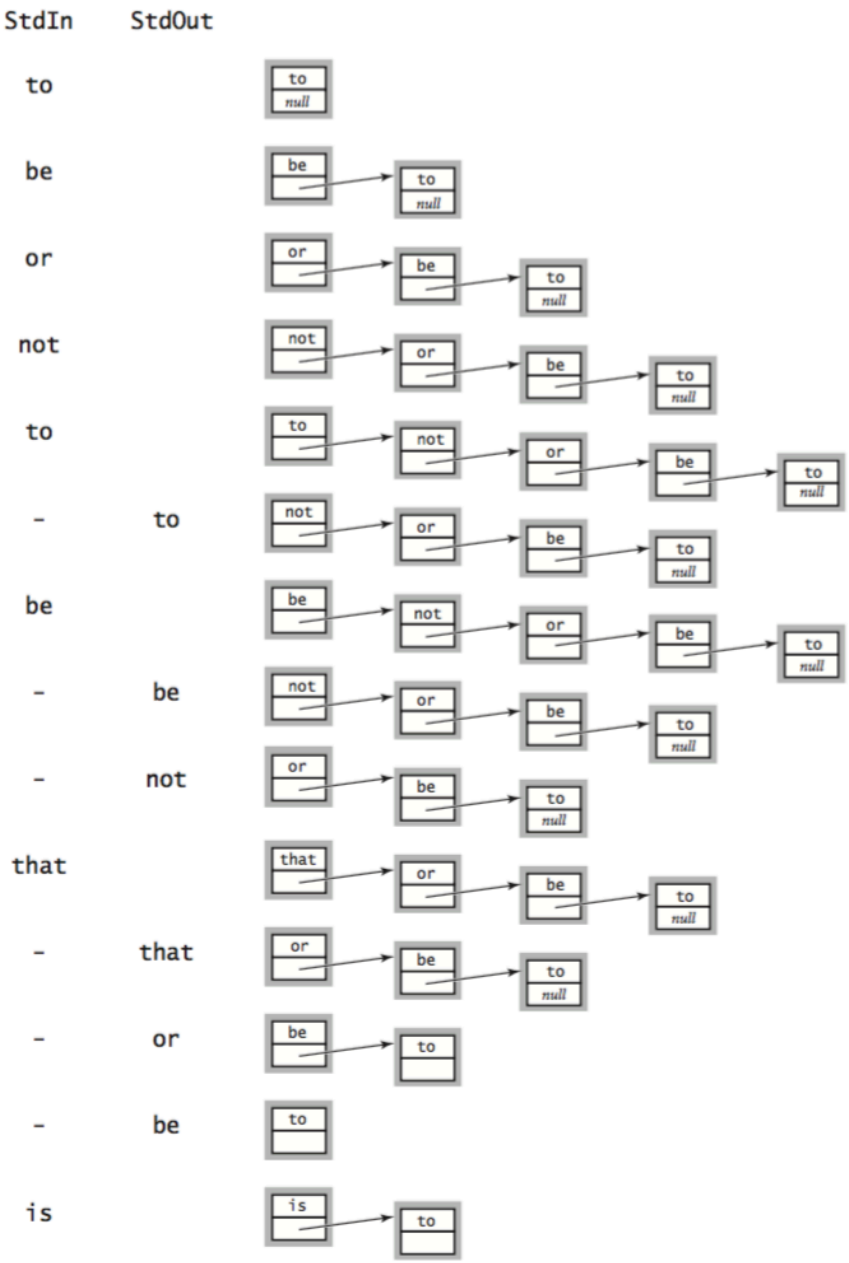
```
for (Node x = first; x != null; x = x.next)
    StdOut.println(x.item);
```



Implementing stacks with linked lists.

Representing stacks with linked lists is a natural idea. In particular, we maintain an instance variable `first` that stores a reference to the most recently inserted item. This policy allows us to add and remove items at the beginning of the linked list without accessing the links of any other items in the linked list.

- *Linked-list implementation of a stack of strings.* LinkedStackOfStrings.java uses a linked list to implement a stack of strings. The implementation is based on a *nested class* `Node` like the one we have been using. Java allows us to define and use other classes within class implementations in this natural way. We designate the nested class as `private` because clients do not need to know any of the details of the linked lists.



- *Linked-list implementation of a generic stack.* Stack.java implements a generic stack using a singly linked list.

Queue.

A *queue* supports the insert and remove operations using a first-in first-out (FIFO) discipline. By convention, we name the queue insert operation *enqueue* and the remove operation *dequeue*, as indicated in the following API:

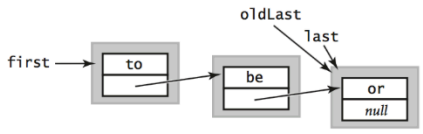
```
public class Queue<Item> implements Iterable<Item>
```

	`Queue()`	*create an empty queue*
`boolean`	`isEmpty()`	*is the queue empty?*
`void`	`enqueue(Item item)`	*insert an item onto queue*
`Item`	`dequeue()`	*return and remove the item that was inserted least recently*
`int`	`size()`	*number of items on queue*

- *Linked-list implementation of a queue.* Queue.java implements a FIFO queue of strings using a linked list. Like `Stack`, we maintain a reference `first` to the least-recently added `Node` on the queue. For efficiency, we also maintain a reference `last` to the most-recently added `Node` on the queue.

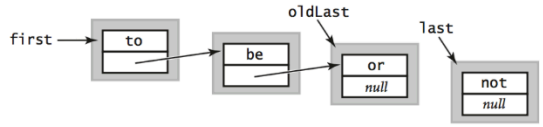
save a link to the last node

```
Node oldLast = last;
```



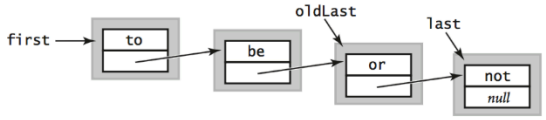
create a new node for the end

```
Node last = new Node();
last.item = "not";
```



link the new node to the end of the list

```
oldLast.next = last;
```



- *Resizing array implementation of a queue.* ResizingArrayQueue.java implements a queue using a resizing array. It is similar to ResizingArrayStack.java, but trickier since we need to add and remove items from opposite ends of the array.

StdIn	StdOut	n	lo	hi	items[]							
					0	1	2	3	4	5	6	7
		0	0	0	null							
to		1	0	1	to	null						
be		2	0	2	to	be						
or		3	0	3	to	be	or	null				
not		4	0	4	to	be	or	not				
to		5	0	5	to	be	or	not	to	null	null	null
-	to	4	1	4	null	be	or	not	to	null	null	null
be		5	1	6	null	be	or	not	to	be	null	null
-	be	4	2	6	null	null	or	not	to	be	null	null
-	or	3	3	6	null	null	null	not	to	not	null	null
that		4	3	7	null	null	null	not	to	not	that	null

Generics.

We have developed stack implementations that allows us to build a stack of one particular type, such as `String`. A specific mechanism in Java known as *generic types* enables us to build collections of objects of a type to be specified by client code.

- *Implementing a generic collection.* To implement a generic collection, we specify a *type parameter*, such as `Item`, in angle brackets and use that type parameter in our implementation instead of a specific type. For example, Stack.java is generic version of LinkedStackOfStrings.java
- *Using a generic collection.* To use a generic collection, the client must specify the *type argument* when the stack is created:

```
Stack<Integer> stack = new Stack<Integer>();
```

Autoboxing.

We have designed our stacks to be *generic*, so that they objects of any type. The Java language features known as *autoboxing* and *unboxing* enable us to reuse *generic* code with primitive types as well. Java supplies built-in object types known as *wrapper types*, one for each of the primitive types: Boolean, Integer, Double, Character, and so forth. Java automatically converts between these reference types and the corresponding primitive types so that we can write code like the following:

```
Stack<Integer> stack = new Stack<Integer>();
stack.push(17);            // autoboxing   (int -> Integer)
int a = stack.pop();       // unboxing     (Integer -> int)
```

Iteration.

Sometimes the client needs to access all of the items of a collection, one at a time, without deleting them. To maintain encapsulation, we do not want to reveal the internal representation of the queue (array or linked list) to the client. To accommodate this design pattern, Java provides the *foreach* statement. You should interpret the following `for` statement in the following code fragment as *for each string s in the collection, print s*.

```
Stack collection = new Stack();
...
for (String s : stack)
    StdOut.println(s);
```

Implementing a collection that supports iteration in this way requires implementing Java's java.util.Iterator and java.util.Iterable interfaces. See the textbook for details.

Stack and queue applications.

Stacks and queues have numerous useful applications.

- *Arithmetic expression evaluation.* An important application of stacks is in *parsing*. For example, a compiler must parse arithmetic expressions written using *infix notation*. For example the following infix expression evaluates to 212.

  ```
  ( 2 + ( ( 3 + 4 ) * ( 5 * 6 ) ) )
  ```

  Evaluate.java evaluates a fully parenthesized arithmetic expression.

- *Function-call abstraction.* Most programs use stacks implicitly because they support a natural way to implement function calls, as follows: at any point during the execution of a function, define its *state* to be the values of all of its variables *and* a pointer to the next instruction to be executed. The natural way to implement the function-call abstraction is to use a stack. To call a function, push the state on a stack. To return from a function call, pop the state from the stack to restore all variables to their values before the function call and resume execution at the next instruction to be executed.
- *M/M/1 queue.* One of the most important queueing models is known as an *M*/*M*/1 queue, which has been shown to accurately model many real-world situations. It is characterized by three properties:
  - There is one server—a FIFO queue.
  - Interarrival times to the queue obey an exponential distribution with rate λ per minute.
  - Service times from a nonempty queue obey an exponential distribution with rate μ per minute.

  MM1Queue.java simulates an *M*/*M*/1 queue and plots a histogram of waiting times to standard drawing.

- *Load balancing.* LoadBalance.java simulate the process of assigning *n* items to a set of *m* servers. For each item, it chooses a sample of *s* servers and assigns the item to the server that has the fewest current items.

Exercises

1. Add a method `isFull()` to ArrayStackOfStrings.java.

4. Write a filter Reverse.java that reads strings one at a time from standard input and prints them to standard output in reverse order.

6. Write a stack client Parentheses.java that reads a string of parentheses, square brackets, and curly braces from standard input and uses a stack to determine whether they are properly balanced. For example, your program should print `true` for `[()]{}{[()()]()}` and false for `[(])`.

7. What does the following code fragment print when `n` is 50? Give a high-level description of what the code fragment does when presented with a positive integer *n*.

```
Stack stack = new Stack();
while (n > 0) {
    stack.push(n % 2);
    n /= 2;
}
while (!stack.isEmpty())
     StdOut.print(stack.pop());
StdOut.println();
```

   Solution: prints the binary representation of `n` (`110010` when n is 50).

8. What does the following code fragment do to the queue `queue`?

```
Stack stack = new Stack();
while (!queue.isEmpty())
   stack.push(queue.dequeue());
while (!stack.isEmpty())
   queue.enqueue(stack.pop());
```

   Solution: reverses the order of the strings in the queue.

9. Add a method `peek()` to Stack.java that returns the most recently inserted element on the stack (without removing it).

11. Add a method `size()` to both Queue.java and Stack.java that returns the number of items in the collection.

14. Write a filter InfixToPostfix.java that converts an arithmetic expression from infix to postfix.

15. Write a program EvaluatePostfix.java that takes a postfix expression from standard input, evaluates it, and prints the value. (Piping the output of your program from the previous exercise to this program gives equivalent behavior to Evaluate.java.)

19. Develop a data type ResizingArrayQueueOfStrings.java that implements a queue wit ha fixed-length array in such a way that all operations take constant time.

21. Modify MM1Queue.java to make a program MD1Queue.java that simulates a queue for which the service times are fixed (deterministic) at rate μ. Verify Little's law for this model.

22. Develop a class StackOfInts.java that uses a linked-list representation (but no generics) to implement a stack of integers. Write a client that compares the performance of your implementation with `Stack<Integer>` to determine the performance penalty from autoboxing and unboxing on your system.

Linked-List Exercises

23. Suppose `x` is a linked-list node. What is the effect of the following code fragment?

```
x.next = x.next.next;
```

Solution: Deletes from the list the node immediately following `x`.

25. Write a method `delete()` that takes the first node in a linked list and an `int` argument `k` and deletes the `k`th node in the linked list, if it exists.

Solution:

```
// we assume that first is a reference to the first Node in the list
public void delete(int k) {
    if (k <= 0) throw new RuntimeException("Invalid value of k");

    // degenerate case - empty linked list
    if (first == null) return;

    // special case - removing the first node
    if (k == 1) {
        first = first.next;
        return;
    }
```

```
        // general case, make temp point to the (k-1)st node
        Node temp = first;
        for (int i = 2; i < k; i++) {
            temp = temp.next;
            if (temp == null) return;   // list has < k nodes
        }

        if (temp.next == null) return;  // list has < k nodes

        // change temp.next to skip kth node
        temp.next = temp.next.next;
    }
```

26. Suppose that `x` is a linked-list node. What is the effect of the following code fragment?

```
t.next = x.next;
x.next = t;
```

Solution: Inserts node `t` immediately after node `x`.

27. Why does the following code fragment not have the same effect as in the previous question?

```
x.next = t;
t.next = x.next;
```

Solution: When it comes time to update `t.next`, `x.next` is no longer the original node following `x`, but is instead `t` itself!

Creative Exercises

39. **Josephus problem.** In the Josephus problem from antiquity, *n* people are in dire straits and agree to the following strategy to reduce the population. They arrange themselves in a circle (at positions numbered from 0 to *n*−1) and proceed around the circle, eliminating every mth person until only one person is left. Legend has it that Josephus figured out where to sit to avoid being eliminated. Write a `Queue` client Josephus.java that takes two integer command-line arguments *m* and *n* and prints the order in which people are eliminated (and thus would show Josephus where to sit in the circle).
40. **Topological sort.** You have to sequence the order of n jobs that are numbered 0 to n-1 on a server. Some of the jobs must complete before others can begin. Write a program TopologicalSorter.java that takes a command-line argument n and a sequence on standard input of ordered pairs of jobs (i, j), and then prints a sequence of integers such that for each pair (i, j) in the input, job i appears before job j. First, from the input, build, for each job (1) a queue of jobs that must follow it and (2) its *indegree* (the number of jobs that must come before it). Then, build a queue of all nodes whose indegree is 0 and repeatedly

delete any job with a 0 indegree, maintaining all the data This process has many applications. For example, you can use it to model course prerequisites for your major so that you can find a sequence of courses to take so that you can graduate.

48. **Copy constructor for a stack.** Create a new constructor for the linked -list implementation of `Stack.java` so that `Stack<String> t = new Stack<String>(s)` makes `t` reference a new and independent copy of the stack `s`. You should be able to push and pop from either `s` or `t` without influencing the other.

    Recursive solution: create a copy constructor for a `Node` and use this to create the new stack.

```
public Node(Node x) {
    item = x.item;
    if (x.next != null)
        next = new Node(x.next);
}

public Stack(Stack s) {
    first = new Node(s.first);
}
```

    Non-recursive solution (untested):

```
public Node(Node x, Node next) {
    this.x = x;
    this.next = next;
}

public Stack(Stack s) {
   if (s.first != null) {
      first = new Node(s.first.value, s.first.next) {
      for (Node x = first; x.next != null; x = x.next)
         x.next = new Node(x.next.value, x.next.next);
   }
}
```

50. **Quote.** Develop a data type Quote.java that implements the following API:

public class Quote	
Quote()	*create an empty quote*
void add(String word)	*append* word *to the end of the quote*
void add(int i, String word)	*insert* word *to be at index* i
String get(int i)	*word at index* i
int count()	*number of words in the quote*
String toString()	*the words in the quote*

To do so, define a nested class `Card` that holds one word of the quotation and a link to the next word in the quotation:

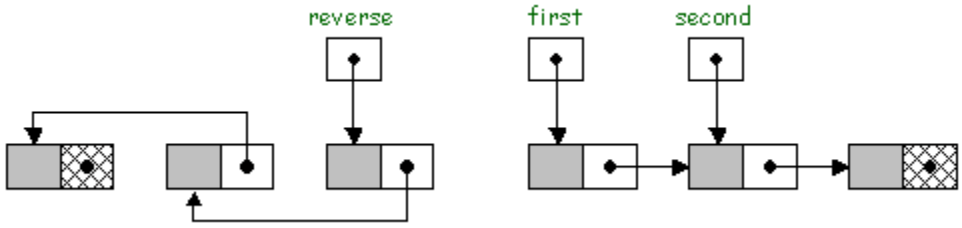
```
private class Card {
    private String word;
    private Card next;

    public Card(String word) {
        this.word = word;
        this.next = null;
    }
}
```

51. **Circular quote.** Repeated the previous exercise, but use a *circular linked list*. In a circular linked list, each node points to its successor, and the last node in the list points to the first node (instead of null, as in a standard null-terminated linked list).

    Solution: CircularQuote.java

52. **Reverse a linked list (iteratively).** Write a nonrecursive function that takes the first `Node` in a linked list as an argument, and reverses the list, returning the first `Node` in the result.

    Solution: To accomplish this, we maintain references to three consecutive nodes in the linked list, `reverse`, `first`, and `second`. At each iteration we extract the node `first` from the original linked list and insert it at the beginning of the reversed list. We maintain the invariant that `first` is the first node of what's left of the original list, `second` is the second node of what's left of the original list, and `reverse` is the first node of the resulting reversed list.




```
public static Node reverse(Node list) {
    if (first == null || first.next == null) return first;
    Node first   = list;
    Node reverse = null;
    while (first != null) {
        Node second = first.next;
        first.next  = reverse;
        reverse     = first;
        first       = second;
    }
    return reverse;
}
```

53. **Reverse a linked list (recursively).** Write a recursive function that takes the first `Node` in a linked list as an argument and reverses the list, returning the first `Node` in the result.

    Solution: Assuming the linked list has n elements, we recursively reverse the last n-1 elements, then append the first element to the end.

```
public Node reverse(Node first) {
    if (first == null || first.next == null) return first;
    Node second = first.next;
    Node rest = reverse(second);
    second.next = first;
    first.next  = null;
    return rest;
}
```

56. **Listing files.** A folder is a list of files an folders. Write a program Directory.java that takes the name of a folder as a command line argument and prints all of the files contained in that folder, with the contents of each folder recursively listed (indented) under that folder's name.

Web Exercises

1. Write a recursive function that takes as input a queue, and rearranges it so that it is in reverse order. Hint: `dequeue()` the first element, recursively reverse the queue, and the enqueue the first element.
2. Add a method `Item[] multiPop(int k)` to `Stack` that pops k elements from the stack and returns them as an array of objects.
3. Add a method `Item[] toArray()` to `Queue` that returns all N elements on the queue as an array of length N.
4. What does the following code fragment do?

```
IntQueue q = new IntQueue();
q.enqueue(0);
q.enqueue(1);
for (int i = 0; i < 10; i++) {
    int a = q.dequeue();
    int b = q.dequeue();
    q.enqueue(b);
    q.enqueue(a + b);
    System.out.println(a);
}
```

    Fibonacci

5. What data type would you choose to implement an "Undo" feature in a word processor?

6. Suppose you have a single array of size N and want to implement two stacks so that you won't get overflow until the total number of elements on both stacks is N+1. How would you proceed?
7. Suppose that you implemented `push` in the linked list implementation of `StackList` with the following code. What is the mistake?

```
public void push(Object value) {
   Node second = first;
   Node first = new Node();
   first.value = value;
   first.next = second;
}
```

   Solution: By redeclaring `first`, you are create a new local variable named `first`, which is different from the instance variable named `first`.

8. **Stack with one queue.** Show how to implement a stack using one queue. *Hint:* to delete an item, get all of the elements on the queue one at a time, and put them at the end, except for the last one which you should delete and return.
9. **Listing files with a stack.** Write a program that takes the name of a directory as a command line argument, and prints out all of the files contained in this directory and any subdirectories. Also prints out the file size (in bytes) of each file. Use a stack instead of a queue. Repeat using recursion and name your program DirectoryR.java. Modify DirectoryR.java so that it prints out each subdirectory and its total size. The size of a directory is equal to the sum of all of the files it contains or that its subdirectories contain.
10. **Stack + max.** Create a data structure that efficiently supports the stack operations (pop and push) and also return the maximum element. Assume the elements are integers or reals so that you can compare them. *Hint:* use two stacks, one to store all of the elements and a second stack to store the maximums.
11. **Tag systems.** Write a program that reads in a binary string from the command line and applies the following (00, 1101) tag-system: if the first bit is 0, delete the first three bits and append 00; if the first bit is 1, delete the first three bits and append 1101. Repeat as long as the string has at least 3 bits. Try to determine whether the following inputs will halt or go into an infinite loop: 10010, 100100100100100100. Use a queue.
12. **Set of integers.** Create a data type that represents a set of integers (no duplicates) between 0 and n-1. Support add(i), exists(i), remove(i), size(), intersect, difference, symmetricDifference, union, isSubset, isSuperSet, and isDisjointFrom.
13. **Indexing a book.** Write a program that reads in a text file from standard input and compiles an alphabetical index of which words appear on which lines, as in the following input. Ignore case and punctuation. Similar to FrequencyCount, but for each word maintain a list of location on which it appears.
14. **Copy constructor for a resizing array implementation of a stack.** Add a copy constructor to ArrayStackOfStrings.java
15. **Reorder linked list.** Given a singly linked list x1 -> x2 -> ... -> x_2n containing 2n nodes, rearrnage the nodes to be x1 -> x2n -> x2 -> x_2n-1 -> x3 -> .... *Hint*: break the

linked list in half; reverse the order of the nodes in the second linked list; merge the two lists together.

Last modified on November 05, 2020.