Do not write in the boxes immediately below.

problem	1	2	3	4	5	6	7	8	9	10	11	12	total
points													
out of	10	8	8	8	8	8	8	8	8	8	8	10	100

Math 2321 Final Exam

December 13, 2016

Instructor's name____________________________ Your name____________________________

Please check that you have 10 different pages.

Answers from your calculator, without supporting work, are worth zero points.

1) Consider the function $f(x, y, z) = x^4 + y^4 - 4x^2y^2e^z$.

a) (3 points) Find the partial derivatives $\dfrac{\partial f}{\partial x}$, $\dfrac{\partial f}{\partial y}$ and $\dfrac{\partial f}{\partial z}$ of the function f at the point $(1, 1, 0)$.

b) (5 points) Find the linearization $L(x, y, z)$ of $f(x, y, z)$ at $(1, 1, 0)$.

c) (2 points) Use the linearization of f at $(1, 1, 0)$ to estimate the value of f at $(0.9, 1.1, 0.1)$. (The "exact" value of $f(0.9, 1.1, 0.1)$ from your calculator is worth zero points.)

2) (8 points) Suppose that the temperature at a point (x, y, z) in space is given by $T(x, y, z) = 8x^2 + 2y^2 + 3z^2$, where T is measured in degrees Celsius and x, y, z in meters. In which direction does the temperature decrease most rapidly when at the point $(1, 1, 2)$? What is the minimum rate of change of the temperature (with respect to distance) when at the point $(1, 1, 2)$?

3) Consider the surface M in $\mathbb{R}^3$ where $3x^2 - y^2 + z^2 = 3$.

a) (5 points) Find an equation for the tangent plane to the surface at the point $(0, 1, 2)$.

b) (3 points) Give a vector equation for the line which passes through the point $(0, 1, 2)$ and is normal to the surface M at $(0, 1, 2)$.

4) (8 points) Find the critical points of $f(x, y) = 6x^2 - 2x^3 + 3y^2 + 6xy$ and determine what type of critical point each of them is, i.e., classify each one as a point where f has a local maximum value, a local minimum value, or a saddle point.

5) You are designing a web site for calculating the surface area of domes (arched roofs) for buildings. You decide to model a parabolic dome using the equation $z = h - b(x^2 + y^2)$ for the dome with base in the xy-plane; here, h and b are positive constants.

a) (2 points) Give a parameterization of the dome, being careful to say what the domain of the parameterization is.

b) (6 points) Find a formula for the surface area of the dome in terms of b and h.

6) (8 points) Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y, z) = 2x + 6y + 10z$ subject to the constraint $x^2 + y^2 + z^2 = 35$, and also give the points at which the maximum and minimum values occur.

7) Consider the iterated integral $\int_0^1 \int_x^1 e^{x/y}\, dy\, dx$.

a) (3 points) Sketch the region of integration.

b) (5 points) Evaluate the integral by reversing the order of integration.

8) (8 points) A region R in $\mathbb{R}^2$ is given by $1 \leq x^2 + y^2 \leq 4$. Sketch R and evaluate the double integral $\int\int_R e^{x^2+y^2}\, dA$.

9) (8 points) Determine the mass of the solid region that lies above the xy-plane, below the half-cone where $z = 2\sqrt{x^2 + y^2}$, outside of the sphere of radius 1 and inside the sphere of radius 2, both centered at the origin, if the density is given by $\delta(x, y, z) = \frac{1}{x^2+y^2+z^2}$ kg/m^3. All lengths are measured in meters.

10) (8 points) Find the work done by the force field $\mathbf{F} = \left(\ln y, x\left(y + \frac{1}{y}\right)\right)$ Newtons, where x and y are in meters, on a particle that moves from $(1,1)$ along a horizontal line to $(4,1)$, then from $(4,1)$ to $(4,2)$ along a vertical line, and finally moves from $(4,2)$ back to $(1,1)$ along the curve $x = y^2$.

11) (8 points) Evaluate the flux integral $\int\int_S \mathbf{F} \cdot \mathbf{n}\, dS$, where $\mathbf{F} = \left(xy^2 - 1, x^2y, 2z\right)$ and S is the part of the surface given by $z = x^2 + y^2$ that is above the disk of radius 2, centered at the origin, in the xy-plane. The surface is oriented upward (i.e., oriented by the upward normal).

12) Let $\mathbf{F}(x, y, z) = (x^2 \sin z, -x + z^3, e^{xy})$ be a vector field.

a) (4 points) Compute the curl, $\vec{\nabla} \times \mathbf{F}$, of $\mathbf{F}$.

b) (6 points) Let M be the portion of the paraboloid $z = 6 - x^2 - y^2$ which sits above the plane $z = 2$, oriented upward. Compute the flux of the curl of $\mathbf{F}$ through M.