Do not write in the boxes immediately below.

problem	1	2	3	4	5	6	7	8	9	10	11	12	total
points													
out of	10	8	8	8	8	8	8	8	8	8	8	10	100

Math 2321 Final Exam

December 13, 2016

Instructor's name	Your name
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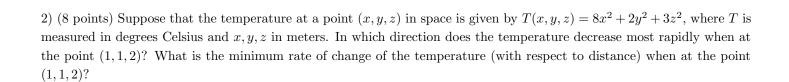
Please check that you have 10 different pages.

Answers from your calculator, without supporting work, are worth zero points.

- 1) Consider the function $f(x, y, z) = x^4 + y^4 4x^2y^2e^z$.
- a) (3 points) Find the partial derivatives $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$ of the function f at the point (1,1,0).

b) (5 points) Find the linearization L(x, y, z) of f(x, y, z) at (1, 1, 0).

c) (2 points) Use the linearization of f at (1,1,0) to estimate the value of f at (0.9,1.1,0.1). (The "exact" value of f(0.9,1.1,0.1) from your calculator is worth zero points.)



- 3) Consider the surface M in \mathbb{R}^3 where $3x^2-y^2+z^2=3$.
- a) (5 points) Find an equation for the tangent plane to the surface at the point (0,1,2).

b) (3 points) Give a vector equation for the line which passes through the point (0,1,2) and is normal to the surface M at (0,1,2).

4) (8 points) Find the critical points of $f(x,y) = 6x^2 - 2x^3 + 3y^2 + 6xy$ and determine what type of critical point each of them is, i.e., classify each one as a point where f has a local maximum value, a local minimum value, or a saddle point.

5) You are designing a web site for calculating the surface area of domes (arched roofs) for buildings. You decide to model a parabolic dome using the equation $z = h - b(x^2 + y^2)$ for the dome with base in the xy -plane; here, h and b are positive constants.
a) (2 points) Give a parameterization of the dome, being careful to say what the domain of the parameterization is.
b) (6 points) Find a formula for the surface area of the dome in terms of b and h .

6) (8 points) Use Lagrange multipliers to find the maximum and minimum values of the function f(x, y, z) = 2x + 6y + 10z subject to the constraint $x^2 + y^2 + z^2 = 35$, and also give the points at which the maximum and minimum values occur.

- 7) Consider the iterated integral $\int_0^1 \int_x^1 e^{x/y} dy dx$.
- a) (3 points) Sketch the region of integration.

b) (5 points) Evaluate the integral by reversing the order of integration.

8) (8 points) A region R in \mathbb{R}^2 is given by $1 \le x^2 + y^2 \le 4$. Sketch R and evaluate the double integral $\int \int_R e^{x^2 + y^2} dA$.

9) (8 points) Determine the mass of the solid region that lies above the xy-plane, below the half-cone where $z=2\sqrt{x^2+y^2}$, outside of the sphere of radius 1 and inside the sphere of radius 2, both centered at the origin, if the density is given by $\delta(x,y,z)=\frac{1}{x^2+y^2+z^2}$ kg/m³. All lengths are measured in meters.

10) (8 points) Find the work done by the force field $\mathbf{F} = \left(\ln y, x\left(y+\frac{1}{y}\right)\right)$ Newtons, where x and y are in meters, on a particle that moves from (1,1) along a horizontal line to (4,1), then from (4,1) to (4,2) along a vertical line, and finally moves from (4,2) back to (1,1) along the curve $x=y^2$.

11) (8 points) Evaluate the flux integral $\int \int_S \mathbf{F} \cdot \mathbf{n} \ dS$, where $\mathbf{F} = (xy^2 - 1, x^2y, 2z)$ and S is the part of the surface given by $z = x^2 + y^2$ that is above the disk of radius 2, centered at the origin, in the xy-plane. The surface is oriented upward (i.e., oriented by the upward normal).

- 12) Let $\mathbf{F}(x, y, z) = (x^2 \sin z, -x + z^3, e^{xy})$ be a vector field.
- a) (4 points) Compute the curl, $\vec{\nabla} \times \mathbf{F}$, of \mathbf{F} .

b) (6 points) Let M be the portion of the paraboloid $z = 6 - x^2 - y^2$ which sits above the plane z = 2, oriented upward. Compute the flux of the curl of ${\bf F}$ through M.