Gauss Elimination method:

$$x+y+z=9$$
$$2x-3y+4z=13$$
$$3x+4y+5z=40$$

$$\begin{bmatrix}1 & 1 & 1\\ 2 & -3 & 4\\ 3 & 4 & 5\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}9\\ 13\\ 40\end{bmatrix}$$

$$[A:B]=\left[\begin{array}{ccc|c}1 & 1 & 1 & 9\\ 2 & -3 & 4 & 13\\ 3 & 4 & 5 & 40\end{array}\right]$$

$$=\left[\begin{array}{ccc|c}1 & 1 & 1 & 9\\ 0 & -5 & 2 & -5\\ 0 & 1 & 2 & 13\end{array}\right]\begin{bmatrix}R_2=R_2-2R_1\\ R_3=R_3-3R_1\end{bmatrix}$$

$$=\left[\begin{array}{ccc|c}1 & 1 & 1 & 9\\ 0 & -5 & 2 & -5\\ 0 & 0 & 12 & 60\end{array}\right]\begin{bmatrix}R_3=5R_3+R_2\end{bmatrix}$$

$$x+y+z=9$$
$$-5y+2z=-5$$
$$12z=60$$

$12z=60$
$\Rightarrow z=5$

$-5y+2\cdot 5=-5$
$\Rightarrow -5y=-15$
$\Rightarrow y=3$

$x+3+5=9$
$\Rightarrow x=1$

$\therefore (x,y,z)=(1,3,5)$

Gauss Elimination method:

$$10x + y + z = 12$$

$$2x + 10y + z = 13$$

$$x + y + 5z = 7$$

$$\begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 1 & 1 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 13 \\ 7 \end{bmatrix}$$

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 2 & 10 & 1 & 13 \\ 10 & 1 & 1 & 12 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & -9 & -49 & -58 \end{array}\right]$$

~~$R_3 = R_3 - 10R_1$~~

~~$R_2 = R_3 - 5R_2$~~

$\begin{bmatrix} R_2 = R_2 - 2R_1 \\ R_3 = R_3 - 10R_1 \end{bmatrix}$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & -473 & -473 \end{array}\right]$$

$R_3 = 9R_2 + 8R_3$

$$= \left[\begin{array}{ccc|c} 1 & 1 & 5 & 7 \\ 0 & 8 & -9 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$x + y + 5z = 1$

$8y - 9z = -1$

$z = 1$

$8y - 9.1 = -1$

$\Rightarrow 8y = 8$

$\therefore y = 1$

$x + 1 + 5.1 = 7$

$\therefore x = 1$

$\therefore (x, y, z) = (1, 1, 1)$