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Q-1). A random variable X has the following pmf :-

$X = x_i$	-2	-1	0	1	2	3
$P(X=x_i)$:	0.1	k	0.2	$2k$	0.3	$3k$.

(i). Find k

Ans :- $0.1 + k + 0.2 + 2k + 0.3 + 3k = 1$

$\Rightarrow 0.6 + 6k = 1$

$\Rightarrow 6k = 0.4$

$\Rightarrow k = \frac{0.4}{6} = \frac{4}{60} = \frac{1}{15}$

(ii). Obtain distribution function.

$$F(x) = P(-\infty < X \leq x), \quad x \in (-\infty, \infty)$$

when $-\infty < x < -2$, $F(x) = 0$.

when $-2 \leq x < -1$, $F(x) = 0.1$.

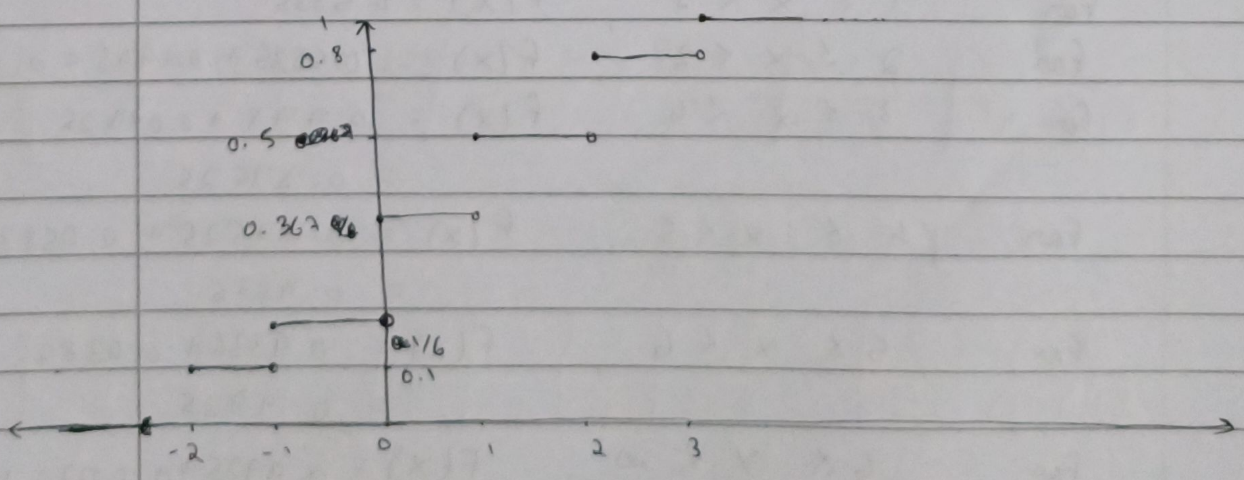
when $-1 \leq x < 0$, $F(x) = \frac{1}{15} + 0.1 = \frac{1}{6}$

when $0 \leq x < 1$, $F(x) = 0.2 + \frac{1}{6} = 0.367$

when $1 \leq x < 2$, $F(x) = \frac{2}{15} + 0.367 = 0.5$

when $2 \leq x < 3$, $F(x) = 0.3 + 0.5 = 0.8$

when $3 \leq x < \infty$, $F(x) = \frac{3}{15} + 0.8 = 1$



Q-27. Spectrum of the random variable X consist of the points :- 1, 2, 3, 4, 5, 6 and probability $P(X=i)$ is proportional to $\frac{1}{i(i+1)}$ determine the distribution function

$X=i$:	1	2	3	4	5	6
$P(X=i)$:	$\frac{k}{2}$	$\frac{k}{6}$	$\frac{k}{12}$	$\frac{k}{20}$	$\frac{k}{30}$	$\frac{k}{42}$
	$= 0.5835$	$= 0.1945$	$= 0.09725$	$= 0.05835$	0.0389	0.0277

~~For $-\infty < x < 1$, $F(x) = 0$~~

~~for $1 \le x < 2$, $F(x) = \frac{1}{2}$~~

~~for $2 \le x < 3$, $F(x) = \frac{1}{2} + \frac{1}{6} =$~~

~~for $3 \le x <$~~

Now, $k\left(\frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42}\right) = 1$.

$\Rightarrow k \times 0.85714 = 1$

$\Rightarrow k = 1.167$

$\therefore$ for $-\infty < x < 1$, $F(x) = 0$.

for $1 \le x < 2$, $F(x) = 0.5835$

for $2 \le x < 3$, $F(x) = 0.5835 + 0.1945 = 0.778$

for $3 \le x < 4$, $F(x) = 0.778 + 0.09725$

$= 0.87525$

for $4 \le x < 5$, $F(x) = 0.87525 + 0.05835$

$= 0.9336$

for $5 \le x < 6$, $F(x) = 0.9336 + 0.0389$

$= 0.9725$

for $6 \le x < \infty$, $F(x) = 0.9725 + 0.0277 = 1$

16/7/24

For Mod - I, II and IV → Introduction to Probability by Ismail Hoque [Excluding Markov Chain and statistics - I].

Joint Probability distribution

- Discrete → Sample space → Enumerable (Countable - maybe finite or infinite)
  - finite → Binomial
  - infinite → Poisson
- Continuous → Sample space → Non - enumerable (not even countable)
  - Normal
  - Expo.
  - Uniform.

* Marginal for $X = P(X = i) = f_{x_i} = \sum_{j=-\infty}^{\infty} f_{ij}$, for any $i \in S$

[∵ S → sample space].

Marginal for $Y = P(\cancel{X}^{Y} = j) = f_{y_j} = \sum_{i=-\infty}^{\infty} f_{ij}$, for any $j \in S$

* If $P(X = i, Y = j) \neq P(X = i) . P(Y = j)$ for some i, j then X and Y are not independent.

* $$P(X = i / Y = j) = \frac{P(X = i, Y = j)}{P(Y = j)}$$

* $$P(X \leq n) = \sum_{j=-\infty}^{\infty} \sum_{i=1}^{n} f(i, j)$$

(2, 2)

(1, 1).
(2, 1)
(1, 2).

__/__/____

Q-1). Let us consider the experiment of tossing two unbiased dice. Let X and Y be the random variables representing maximum of the face values and sum of the face values respectively. Find (i) the joint pmf of (X, Y), (ii) Marginal pmf of (X, Y). (iii). Are the two variables X and Y independent?, (iv). Find the conditional pmf of X given Y = 6.

Sol^n.

(i).

X \ Y	2	3	4	5	6	7	8	9	10	11	12	
1	1/36	0	0	0	0	0	0	0	0	0	0	1/36
2	0	2/36	1/36	0	0	0	0	0	0	0	0	3/36
3	0	0	2/36	2/36	1/36	0	0	0	0	0	0	5/36
4	0	0	0	2/36	2/36	2/36	1/36	0	0	0	0	7/36
5	0	0	0	0	2/36	2/36	2/36	2/36	1/36	0	0	9/36
6	0	0	0	0	0	2/36	2/36	2/36	2/36	2/36	1/36	11/36
	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36	1

X → Max Value
Y → Sum of the values.

pmf → probability mass function.

(ii). Marginal pmf for X :-

$X = i$	1	2	3	4	5	6
$P(X = i)$	1/36	3/36	5/36	7/36	9/36	11/36

Marginal pmf for Y :-

$Y = j$	2	3	4	5	6	7	8	9	10	11	12
$P(Y = j)$	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

__/__/____

(iii). if $P(X=i, Y=j) \neq P(X=i) \cdot P(Y=j)$ for some i, j,

then the random variables are not independent.

Here, for $P(X=4, Y=6) = \frac{2}{36}$

Now $P(X=4) = \frac{7}{36}$

and $P(Y=6) = \frac{5}{36}$

Clearly $P(X=4, Y=6) \neq P(X=4) \cdot P(Y=6)$

$\therefore$ X and Y are not independent.

(iv) $$P(X/Y=6) = \frac{P(X=i, Y=6)}{P(Y=6)}$$

$$= \begin{cases} 0, & i=1,2,6 \\ \frac{1/36}{5/36}, & i=3 \\ \frac{2/36}{5/36}, & i=4,5 \end{cases} = \begin{cases} 0, & i=1,2,6 \\ \frac{1}{5}, & i=3 \\ \frac{2}{5}, & i=4,5 \end{cases}$$

(Ans).

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d). Two dice are thrown. Let X = No. shown on the first die and Y = Larger of the two no.s find (i) the distribution of the 2-dimensional random variable (X, Y), (iii) $P(Y=4 / X=2)$, (ii) find the marginal distribution of X and Y, (iv). Are the random variables X and Y independent?

Soln.

$X \to$ No. on 1st die
$Y \to$ Larger of the 2 no.s

(i).

X \ Y	1	2	3	4	5	6	
1	1/36	1/36	1/36	1/36	1/36	1/36	6/36
2	0	2/36	1/36	1/36	1/36	1/36	6/36
3	0	0	3/36	1/36	1/36	1/36	6/36
4	0	0	0	4/36	1/36	1/36	6/36
5	0	0	0	0	5/36	1/36	6/36
6	0	0	0	0	0	6/36	6/36
	1/36	3/36	5/36	7/36	9/36	11/36	1

(iii). $P(Y=4 / X=2) = \cancel{\dfrac{P(X=2, Y=4)}{P(Y=}}$

$= \dfrac{P(Y=4, X=2)}{P(X=2)}$

$= \dfrac{1/36}{6/36} = \dfrac{1}{6}$

(iv). $P(X=2, Y=4) = \dfrac{1}{36}$

$P(X=2) \times P(Y=4) = \dfrac{6}{36} \times \dfrac{7}{36}$

$= \dfrac{42}{36}$

__/__/__

$\therefore P(X=2, Y=4) \neq P(X=2).P(Y=4)$, so X and Y are not independent.

Q). A 2-dimensional random variable (X, Y) has the spectrum $\{(i,j), i,j = 1,2,3\}$ and the pmf is given by $f_{ij} = K.i.j$. Find :-

(i) k (ii) $P(1 \leq X \leq 3, Y \leq 2)$

(iii) $P(X \geq 2)$ (iv) $P(Y < 2)$

(v) $P(X = 2)$ (vi) Are X & Y are independent.

Soln:-

(i).

X\Y	1	2	3	
1	k	2k	3k	6k
2	2k	4k	6k	12k
3	3k	6k	9k	18k
	6k	12k	18k	

$\therefore 6k + 12k + 18k = 1$.

$\Rightarrow k = \frac{1}{36}$.

OR,

$$\sum_{j=1}^{3}\sum_{i=1}^{3} f_{i,j} = 1$$

$$\Rightarrow \sum_{j=1}^{3}\sum_{i=1}^{3} kij = 1$$

$$\Rightarrow k\sum_{j=1}^{3} j(1+2+3) = 1$$

$$\Rightarrow 6k\sum_{j=1}^{3} j = 1$$

$$\Rightarrow 6k(1+2+3) = 1$$

$$\Rightarrow 36k = 1$$

$$\Rightarrow k = \frac{1}{36}.$$

(ii). $P(1 \leq x \leq 3,\ y \leq 2)$

$$= \sum_{j=1}^{2} \sum_{i=1}^{3} f_{ij}$$

$$= \sum_{j=1}^{2} \sum_{i=1}^{3} kij$$

$$= \frac{1}{36} \sum_{j=1}^{2} j(1+2+3).$$

$$= \frac{6}{36}(1+2) = \frac{12}{36} = \frac{1}{3}.$$

(iii). $P(X \geq 2)$

$$= \sum_{j=1}^{3} \sum_{i=2}^{3} kij$$

$$= \frac{1}{36} \sum_{j=1}^{3} j(2+3)$$

$$= \frac{5}{36}(1+2+3) = \frac{5}{6}.$$

(iv). $P(y \leq 2)$

$$= \sum_{j=1}^{2} \sum_{i=1}^{3} kij$$

$$= \frac{1}{36} \sum_{j=1}^{2} 6j$$

$$= \frac{1}{6}(1+2) = \frac{1}{2}.$$

(v). $P(X \leq 2)$.

$$= \sum_{j=1}^{3} \sum_{i=1}^{2} kij$$

$$= \frac{1}{36} \sum_{j=1}^{3} (1+2)j$$

$$= \frac{1}{12}(1+2+3)$$

$$= \frac{1}{2}$$

(vi). $P(X=2) = \frac{1}{36} \sum_{j=1}^{3} \sum_{i=2} ij = \frac{1}{36} \cdot 2(6)$

$$= \frac{1}{3}.$$

(viii). $P(X = i, Y = j) = \frac{1}{36} ij$

Now, $P(X = i) = \sum_{j=1}^{3} f_{ij} = \frac{i}{36} \sum_{j=1}^{3} j$

$= \frac{i}{36} \cdot 6$

$= \frac{i}{6}$

and $P(Y = j) = \frac{j}{36} \sum_{i=1}^{3} i$

$= \frac{j}{36} \cdot 6 = \frac{j}{6}$

$\therefore P(X = i, Y = j) = P(X = i) . P(Y = j) = \frac{ij}{36}$

$\therefore$ They are independent.

Q). A 2-dimensional random variable X and Y has the spectrum $\{(i, j) : i = 0, 1, 2, 3$
$j = 1, 2, 3, 4\}$ and the probability masses are given by $f_{ij} = k(3i + 4j)$.

Find (i) the value of k

(ii). marginal of X and Y

(iii). $P(Y = 2 / X = 3)$

(iv). Are they independent.

Sol^n. (i). $\sum_{j=1}^{4} \sum_{i=0}^{3} k(3i + 4j) = 1$

$\Rightarrow \sum_{j=1}^{4} 4k(3(0+1+2+3) + 4j) \Rightarrow 1$

$\Rightarrow 16k \sum_{j=1}^{4} (18 + 4j) = 1$

$\Rightarrow 16k(18 + 4(1+2+3+4)) = 1$

$\Rightarrow 16k(18 + 40) = 1 \quad \Rightarrow k = 1/928$

$\Rightarrow 3k \sum_{j=1}^{4} \sum_{i=0}^{3} i + 4k \sum_{i=0}^{3} \sum_{j=1}^{4} j = 1$

$\Rightarrow 3k \sum_{j=1}^{4} (6) + 4k \sum_{i=0}^{3} (10) = 1.$

$\Rightarrow 3k \times 4 \times 6 + 4k \times 4 \times 10 = 1.$

$\Rightarrow 72k + 160k = 1$

$\Rightarrow 232k = 1.$

$\Rightarrow k = 1/232.$

(ii). Marginal of X :-

$X=i$	0	1	2	3
$P(X=i)$	$40/232$	$52/232$	$64/232$	$76/232$

$P(X=0) = \sum_{j=1}^{4} k(0+4j).$

$= 4k(1+2+3+4) = \frac{40}{232}$

$P(X=1) = \sum_{j=1}^{4} k(3+4j)$

$= k\sum_{j=1}^{4} 3 + 4k \sum_{j=1}^{4} j = 12k + 40k$

$= \frac{52}{232}$

$P(X=2) = \sum_{j=1}^{4} k(6+4j)$

$= k \times 24 + 40k = 64/232.$

$P(X=3) = \sum_{j=1}^{4} k(9+4j)$

$= k \times 36 + 40k = \frac{76}{232}.$

Marginal of y:-

$y=j$	1	2	3	4
$P(y=j)$	$34/232$	$50/232$	$66/232$	$82/232$

$$P(y=1) = \frac{1}{232}\sum_{i=0}^{3}(3i+4)$$

$$= \frac{18}{232} + \frac{16}{232} = \frac{34}{232}$$

$$P(y=2) = k\sum_{i=0}^{3}(3i+8)$$

$$= 18k + 32k = \frac{50}{232}$$

$$P(y=3) = k\sum_{i=0}^{3}(3i+12)$$

$$= 18k + 48k = \frac{66}{232}$$

$$P(y=4) = k\sum_{i=0}^{3}(3i+16)$$

$$= 18k + 64k = \frac{82}{232}$$

(iii). $$P(y=2/x=3) = \frac{P(X=3,\ y=2)}{P(X=3)} = \frac{\frac{1}{232}(9+8)}{76/232}$$

$$= \frac{17}{76}$$

(iv). $$P(X=3,\ y=2) = \frac{17}{232}$$

$$P(X=3) = \frac{76}{232}\ ; \quad P(y=2) = \frac{50}{232}$$

$\therefore$ Not independent

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Joint Continuous Probability Distribution

In discrete → f_{ij} → dependent on i & j

↳ $\sum_i \sum_j f_{ij} = 1$. Marginals → $f_X(x) = \sum_j f_{ij}$

↳ $f_Y(y) = \sum_i f_{ij}$

In continuous → $f(x,y)$ → depend on $a \leq x, y \leq b$

↳ $\int_x \int_y f(x,y)\, dx\, dy = 1$.

Marginals → $f_X(x) = \int_y f(x,y)\, dy$

$f_Y(y) = \int_x f(x,y)\, dx$.

Checking independence → $f_X(x)\, f_Y(y) = f(x,y)$

Q). Let $f(x,y) = \begin{cases} \sin x \sin y, & 0 < x < \pi/2, \quad 0 < y < \pi/2 \\ 0, & \text{elsewhere.} \end{cases}$

Show that $f(x,y)$ is a possible two dimensional probability ~~distribution~~ density function. Find the marginal density functions and prove that the random variables are independent.

Soln

$$\int_{x=0}^{\pi/2} \int_{y=0}^{\pi/2} \sin x \sin y \, dy\, dx$$

$$= \int_0^{\pi/2} \sin x \int_0^{\pi/2} \sin y \, dy\, dx$$

$$= \int_0^{\pi/2} \sin x \left[-\cos y\right]_0^{\pi/2} dx$$

$$= \int_0^{\pi/2} \sin x \left[0 - (-1)\right] dx$$

$$= \left[-\cos x\right]_0^{\pi/2} = 1$$

$$f_X(x) = \int_{y=0}^{\pi/2} \sin x \sin y \, dy.$$

$$= \sin x \left[-\cos y\right]_0^{\pi/2}$$

$$= \sin x, \quad 0 < x < \frac{\pi}{2}$$

$$f_Y(y) = \int_{x=0}^{\pi/2} \sin x \sin y \, dx$$

$$= \sin y \left[-\cos x\right]_0^{\pi/2}$$

$$= \sin y, \quad 0 < y < \frac{\pi}{2}.$$

Now,

$f_X(x) \cdot f_Y(y) = f(x, y) = \sin x \sin y$, so the random variables are independent.

d). The pdf of (X, Y) is defined as follows :-

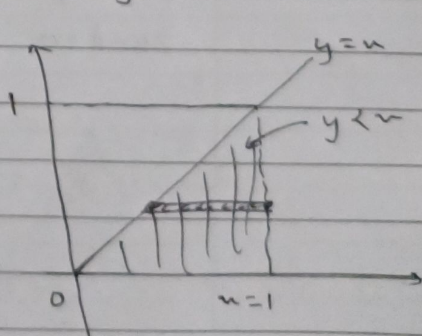
$$f(x,y) = \begin{cases} cx, & 0 < y < x < 1 \\ 0, & \text{elsewhere} \end{cases}$$

Find (i) the value of c

(ii) Marginal density functions.

(iii) Are they independent?

$y = x$



Sol^n.

$$\int_{y=0}^{1} \int_{x=y}^{1} cx \, dx \, dy = 1$$

$$\Rightarrow c \int_{y=0}^{1} \left[\frac{x^2}{2}\right]_y^1 dy = 1 \quad \Rightarrow c \int_{y=0}^{1} \left(-\frac{y^2}{2} + \frac{1}{2}\right) dy = 1$$

$$\Rightarrow \frac{c}{2} \left[-\frac{y^3}{3} + y\right]_0^1 = 1$$

_/_/_

$$\Rightarrow \frac{c}{2}\left[-\frac{1}{3} + 1\right] = 1$$

$$\Rightarrow \frac{c}{2}\left(\frac{2}{3}\right) = 1$$

$$\Rightarrow c = 3$$

$$f_X(x) = \int_{y=0}^{x} f(x,y)\,dy$$

$$= \int_0^x cx\,dy$$

$$= 3x\,[y]_0^x$$

$$= 3x^2, \quad 0 < x < 1$$

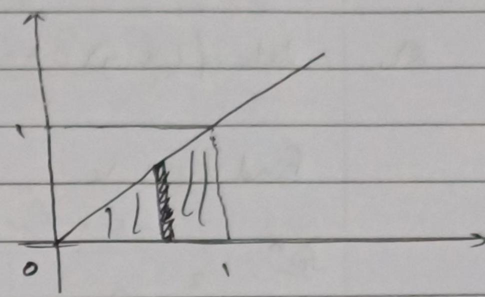
$$f_Y(y) = \int_{x=y}^{1} f(x,y)\,dx$$

$$= \int_y^1 3x\,dx$$

$$= \frac{3}{2}\left[x^2\right]_y^1 = \frac{3}{2}(1-y^2), \quad 0 < y < 1$$

Clearly, $f_X(x) \cdot f_Y(y) \neq f(x,y)$; so the random variables are not independent.

Q. $f(x,y) = \begin{cases} kxy, & 0 < x < 1, \quad 0 < y < x \\ 0, & \text{elsewhere.} \end{cases}$



$$\int_{x=0}^{1} \int_{y=0}^{x} kxy \, dy \, dx = 1$$

$$\Rightarrow k \int_{x=0}^{1} x \int_{y=0}^{x} y \, dy \, dx = 1$$

$$\Rightarrow k \int_{x=0}^{1} x \left(\frac{y^2}{2} \right]_0^x dx = 1$$

$$\Rightarrow \frac{k}{2} \int_{x=0}^{1} x \, [x^2] \, dx = 1$$

$$\Rightarrow \frac{k}{2} \int_0^1 [x^3] \, dx = 1$$

$$\Rightarrow \frac{k}{2} \left[\frac{x^4}{4} \right]_0^1 = 1.$$

$$\Rightarrow \frac{k}{2} \left[\frac{1}{4} \right] = 1 \qquad \Rightarrow k = 8$$

Now, $f_X(x) = \int_{y=0}^{x} kxy \, dy = \frac{8x}{2} \left(y^2 \right]_0^x$

$= 4x^3, \quad 0 < x < 1.$

And $f_Y(y) = 8y \int_{x=y}^{1} x \, dx = \frac{8y}{2} \left[x^2 \right]_y^1 = 4y, \quad 0 < y < x.$

$= 4y(1 - y^2) = 4y - 4y^3, \quad 0 < y < x$

Clearly, $f_X(x) \cdot f_Y(y) \neq f(x,y)$; so the random variables are not independent.

Q. Let $f(x,y) = \begin{cases} k(3x+y), & 1 < x < 3,\ 0 < y < 2 \\ 0, & \text{elsewhere} \end{cases}$

Find (i) k, (ii) $P(X+Y < 2)$

Sol^n.

$$k \int_{x=1}^{2} \int_{y=0}^{2} (3x+y)\, dy\, dx = 1$$

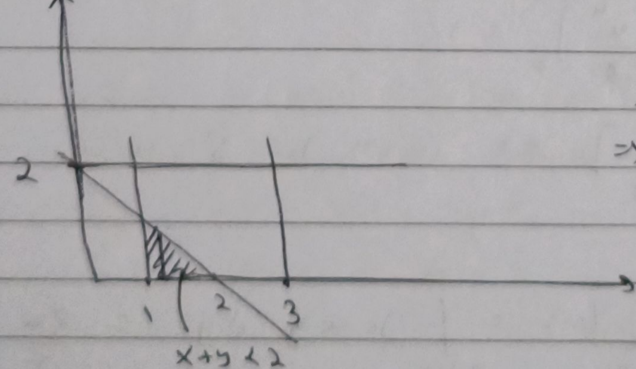
$$\Rightarrow k \int_{x=1}^{3} \left(3xy + \frac{y^2}{2}\right]_0^2 dx = 1.$$

$$\Rightarrow k \int_{x=1}^{3} [6x + 2]\, dx = 1.$$

$$\Rightarrow k \left[\frac{6x^2}{2} + 2x\right]_1^3 = 1.$$

$$\Rightarrow k[27 + 6 - 3 - 2] = 1$$

$$\Rightarrow k = \frac{1}{28}.$$



$X + Y = 2$

$\Rightarrow \frac{X}{2} + \frac{Y}{2} = 1$

$$\frac{1}{28} \int_{x=1}^{2} \int_{y=0}^{2-x} (3x+y)\, dy\, dx$$

$$= \frac{1}{28} \int_{x=1}^{2} \left[3xy + \frac{y^2}{2} \right]_0^{2-x} dx$$

$$= \frac{1}{28} \int_{x=1}^{2} \left[3x(2-x) + \frac{(2-x)^2}{2} \right] dx.$$

$$= \frac{1}{28} \int_{x=1}^{2} \left[6x - 3x^2 + \frac{4 + x^2 - 4x}{2} \right] dx.$$

$$= \frac{1}{28} \left[\frac{6x^2}{2} - \frac{3x^3}{3} + \left(4x + \frac{x^3}{3} - \frac{4x^2}{2}\right) \frac{1}{2} \right]_1^2$$

$$= \frac{1}{28} \left[3x^2 - x^3 + 2x + \frac{x^3}{6} - x^2 \right]_1^2$$

$$= \frac{1}{28} \left[2x^2 + 2x - \frac{5x^3}{6} \right]_1^2$$

$$= \frac{1}{28} \left[\cancel{4} \; 8 + 4 - \frac{40}{6} - 2 - 2 + \frac{5}{6} \right].$$

$$= \frac{1}{28} \left[\cancel{48} \; 8 - \frac{35}{6} \right].$$

$$= \frac{1}{28} \times \cancel{\frac{48}{6}} \; \frac{13}{6} = \frac{13}{168} \text{ (Ans')}.$$

$$\begin{array}{r} 48 \\ -35 \\ \hline 13 \end{array}$$

24/7/24

Poisson Distribution (as the limiting case of Binomial Distribution)

* pmf of binomial distribution:-

$X \sim B(n, p)$.

$$P(X = r) = {}^nC_r \, p^r \, q^{n-r} \; ; \; r = 0, 1, 2, \ldots n$$

$$= 0 \quad , \text{ elsewhere.}$$

In poisson distribution, 'n' is not provided; instead, the 'average success rate' = $\boxed{np = \lambda}$ or 'observed success' or 'no. of changes per unit time period' is given.

$$\therefore \boxed{p = \frac{\lambda}{n}}$$

Now, $P(X = r) = {}^nC_r \, p^r \, (1-p)^{n-r}$

$$= \frac{n!}{r!(n-r)!} \left(\frac{\lambda}{n}\right)^r \left(1 - \frac{\lambda}{n}\right)^{n-r}$$

$$= \frac{n(n-1)(n-2) \ldots (n-(r-1)) \cancel{(n-r)!}}{r! \, \cancel{(n-r)!}} \cdot \frac{\lambda^r}{n^r} \left(1 - \frac{\lambda}{n}\right)^{n-r}$$

$$= \frac{\lambda^r}{r!} \left(\frac{n}{n} \cdot \frac{n-1}{n} \cdot \frac{n-2}{n} \cdot \frac{n-3}{n} \ldots \frac{n-(r-1)}{n}\right) \frac{\left(1 - \frac{\lambda}{n}\right)^n}{\left(1 - \frac{\lambda}{n}\right)^r}$$

$$= \frac{\lambda^r}{r!} \cdot 1 \left(1 - \frac{1}{n}\right)\left(1 - \frac{2}{n}\right)\left(1 - \frac{3}{n}\right) \ldots \left(1 - \frac{(r-1)}{n}\right).$$

_/_/_

$$\frac{\left\{\left(1-\frac{\lambda}{n}\right)^{-n/\lambda}\right\}^{-\lambda}}{\left(1-\frac{\lambda}{n}\right)^{r}}$$

Take limit as $n \to \infty$:-

$$P(X=r) = \frac{\lambda^r}{r!} \cdot 1 (1-0)(1-0)\cdots(1-0) \cdot \frac{\lim\limits_{n\to\infty}\left\{\left(1-\frac{\lambda}{n}\right)^{-n/\lambda}\right\}^{-\lambda}}{\lim\limits_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^{r}}$$

As $n \to \infty$, $\frac{\lambda}{n} \to 0$.

We know, $\lim\limits_{x\to 0} (1+x)^{1/x} = e$.

$$\therefore \boxed{P(X=r) = \frac{\lambda^r}{r!} \cdot \frac{e^{-\lambda}}{1}}, \quad r = 0, 1, 2, \ldots \infty$$

$= 0$, elsewhere.

pmf under Poisson distribution.

Def^n :- A discrete random variable X is said to follow a Poisson distribution with parameter λ $(\lambda > 0)$ if the pmf of X is given by $\frac{\lambda^r e^{-\lambda}}{r!}$.

* $P(X=r) \geq 0 \quad \forall x$
* Sum of the pmf $= \sum P(X=r)$, $r = 0, 1, 2 \ldots \infty$ $= 1$

$$\sum_{r=0}^{\infty} P(X=r) = \sum_{r=0}^{\infty} \frac{\lambda^r e^{-\lambda}}{r!} = e^{-\lambda} \sum_{r=0}^{\infty} \frac{\lambda^r}{r!}$$

$$= e^{-\lambda}\left[\frac{\lambda^0}{0!} + \frac{\lambda^1}{1!} + \frac{\lambda^2}{2!} + \cdots + \infty\right]$$

$$= e^{-\lambda} \cdot e^{\lambda} = e^{0} = 1.$$

Occurrences of Poisson Distribution :-

Following are some instances where Poisson distribution maybe used :-

(i) No. of deaths from a disease or some natural calamity.

(ii) No. of accidents during a week or month or a specific time period.

(iii) No. of vehicles passing a crossing during a unit time period.

(iv) No. of defective items in a lot.

(v) No. of telephone calls received during a certain period of time.

Theorem:-

If X follows Poisson distribution with parameter λ, then :-

(i) Mean of $X = E(X)$

(ii) $Var(X)$

(iii) SD

$$E(X) = \sum_{r=0}^{\infty} r P(X=r)$$

$$= \sum_{r=0}^{\infty} r \frac{e^{-\lambda} \lambda^{r}}{r!}$$

$$= 0 + \sum_{r=1}^{\infty} \frac{r e^{-\lambda} \lambda^{r}}{r!}$$

$$= e^{-\lambda} \sum_{r=1}^{\infty} \frac{\lambda^{r}}{r!}$$

$$= e^{-\lambda} \sum_{r=1}^{\infty} \frac{\lambda^{r} \cdot \cancel{r}}{\cancel{r}(r-1)!}$$

$$= e^{-\lambda} \left[\frac{\lambda^{1}}{0!} + \frac{\lambda^{2}}{1!} + \frac{\lambda^{3}}{2!} + \dots \infty \right]$$

$= \lambda e^{-\lambda} \left(\frac{1}{0!} + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \dots \infty \right)$

$= \lambda e^{-\lambda} \cdot e^{\lambda} = \lambda$ (Ans:-)

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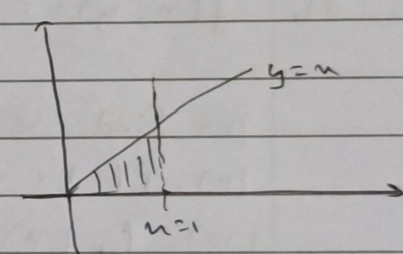
Q). The joint density function of x and y is

$$f(x, y) = \begin{cases} 2, & 0 < x < 1, \; 0 < y < x \\ 0, & \text{elsewhere.} \end{cases}$$

(i) Find the marginal density functions

(ii). $P\left(\frac{1}{4} < x < \frac{3}{4} \,\middle|\, y = \frac{1}{2}\right)$.



(i) Marginal for x:-

$f_x(x) = \int_{y=0}^{x} 2 \, dy$

$= [2x]_0^x$

$= 2x \; ; \quad 0 < x < 1$

Marginal for y:-

$f_y(y) = \int_{x=y}^{1} 2 \, dx = [2y]_y^1 = 2 - 2y ; \quad 0 < y < 1$

(ii). $P\left(\frac{1}{4} < x < \frac{3}{4} \,\middle|\, y = \frac{1}{2}\right) = \dfrac{P\left(\frac{1}{4} < x < \frac{3}{4}, \, y = \frac{1}{2}\right)}{P\left(y = \frac{1}{2}\right)}$.

Now, $P\left(y = \frac{1}{2}\right) = 2 - 2 \times \frac{1}{2} = 2 - 1 = 1$.

—/—/—

And $P(\frac{1}{4} < x < \frac{3}{4}, y = \frac{1}{2})$

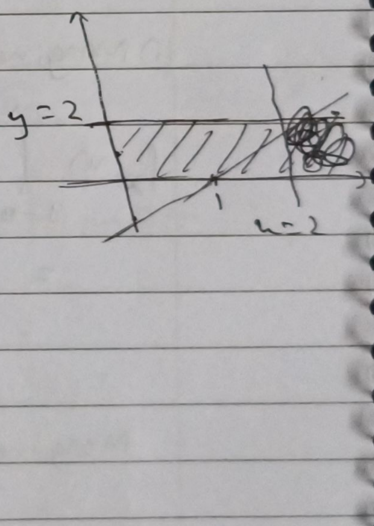
$= \int_{x=\frac{1}{4}}^{3/4} f(x, \frac{1}{2})\, dx$

$= \int_{x=1/4}^{3/4} 2\, dx = 2(x)_{\frac{1}{4}}^{3/4}$

$= 2\left[\frac{3}{4} - \frac{1}{4}\right] = 2 \times \frac{1}{2} = 1.$

$\therefore P(\frac{1}{4} < x < \frac{3}{4} / y = \frac{1}{2}) = \frac{1}{1} = 1.$

d). $f(x, y) = \begin{cases} k(x+y), & 0 \le x, y \le 2 \\ 0, & \text{elsewhere.} \end{cases}$



(i) Find k.
(ii). $P(|x - y| \le 1)$
(iii). $f_x(x)$ & $f_y(y)$
(iv). Are x and y are independent?

Soln. (i). $\int_{x=0}^{2}\int_{y=0}^{2} k(x+y) = 1$

$\Rightarrow k \int_{x=0}^{2} \left[xy + \frac{y^2}{2}\right]_0^2 = 1$

$\Rightarrow k \int_0^2 (2x + 2) = 1.$

$\Rightarrow k\left(\frac{2x^2}{2} + 2x\right]_0^2 = 1.$

$\Rightarrow k(4 + 4) = 1$

$\Rightarrow k = \frac{1}{8}.$

$x = 1 + y$

$\cancel{(iii)\ \iint \int_{x=1}^{2} \int_{y=0}}$

$|a| \leq b$

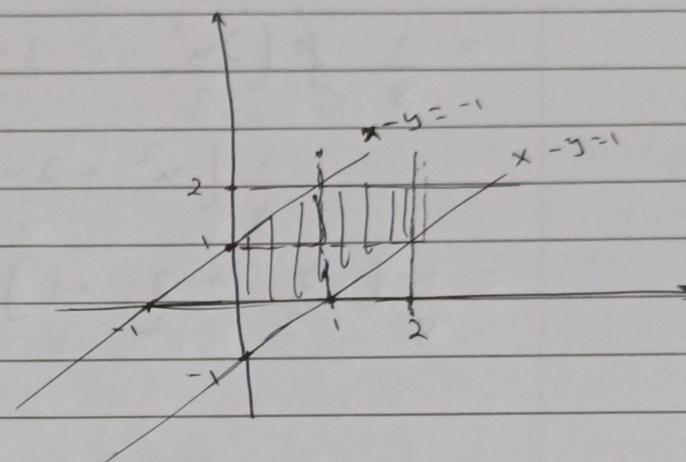
$\Rightarrow -b \leq a \leq b$

$\therefore |x-y| = 1$

$-1 \leq x - y \leq 1$

$\therefore x - y = -1$

$x - y = 1$



$\cancel{\therefore P(|x-y| \leq 1) = \int_{x=0}^{1}\int_{y=0}^{1} k(x+y)\,dy\,dx + \int_{x=0}^{1}\int_{y=1}^{x+1} k(x+y)\,dy\,dx + \iint}$

$$\therefore P(|X-Y| \leq 1) = \int_{x=0}^{1}\int_{y=0}^{x+1} k(x+y)\,dy\,dx + \int_{x=1}^{2}\int_{y=x-1}^{2} k(x+y)\,dy\,dx$$

$$= \frac{1}{8}\int_{x=0}^{1}\int_{y=0}^{x+1} (x+y)\,dy\,dx + \frac{1}{8}\int_{x=1}^{2}\int_{y=x-1}^{2} (x+y)\,dy\,dx$$

$$= \frac{2}{8}\int_{x=0}^{1}\left[xy + \frac{y^2}{2}\right]_0^{x+1} dx + \frac{1}{8}\int_{x=1}\left[xy + \frac{y^2}{2}\right]_{x-1}^{2}$$

$$= \frac{1}{8}\int_0^1 \left[x(x+1) + (x+1)^2\right] dx + \frac{1}{8}\int_1^2 \Big(2x + 2 - x(x-1) -$$

$1/6$

$$= \frac{1}{8}\int_0^1 (x^2+x + x^2+2x+1)\,dx +$$

$$\frac{1}{8}\int_1^2 \left(2x+2 - x^2+1 - \frac{x^2-2x+1}{2}\right)$$

$$= \frac{1}{8}\left[\frac{2x^3}{3} + \frac{3x^2}{2} + x\right]_0^1 +$$

$$\frac{1}{8}\left[x^2+2x - \frac{x^3}{3} + x - \frac{x^3}{6} + \frac{2x^2}{4} - \frac{x}{2}\right]_1^2$$

$$= \frac{1}{8}\left[\frac{2}{3} + \frac{3}{2} + 1\right] + \frac{1}{8}\left[4+4 - \frac{8}{3} + 2 - \frac{8}{6} + \frac{8}{4} - 1 - 1 - 2 + \frac{1}{3} - 1 + \frac{1}{6} - \frac{2}{4} + \frac{1}{2}\right]$$

$$= \frac{1}{8}\left[\frac{4+9+6}{6}\right] + \frac{1}{8}\left[10 - 5 - \frac{8}{3} + \frac{1}{3} - \frac{8}{6} + \frac{1}{6} + \frac{8}{4} - \frac{2}{4} + \frac{1}{2}\right]$$

$$= \frac{1}{8} \times \frac{19}{6} + \frac{1}{8}\left[5 - \frac{7}{3} - \frac{7}{6} + \frac{6}{4} + \frac{1}{2}\right]$$

$$= \frac{19}{48} + \frac{1}{8}\left[\frac{60-28-14+18+6}{12}\right]$$

$$= \frac{19}{48} + \frac{1}{8} \times \frac{\cancel{42}^{21}}{\cancel{12}_{6}} = \frac{19+21}{48}$$

$$= \frac{\cancel{40}^{\cancel{10}\,5}}{\cancel{48}_{\cancel{12}\,6}} = \frac{5}{6}$$

$P(x=2, y=2) = 1/9$

$P(x=2) = 1/3$

$P(y=2) = 1/3$

$$\frac{R_1+1}{R_1+R_2+100} = \frac{3+1}{3+6+100} = \frac{4}{109}$$

$$\therefore \frac{4}{109} + \frac{1}{4} + \frac{1}{12} + k + \frac{1}{12} = 1$$

$$\Rightarrow \frac{4}{109} + \frac{3+1+1}{12} + k = 1$$

$$\Rightarrow k = 1 - \frac{5}{12} - \frac{4}{109}$$

$$= 0.5466 \approx 0.55$$

$$P(x=2, y=3) = \frac{4}{109} = 0.036$$

$$P(x=2) = \frac{1}{3} + \frac{4}{109} + \frac{1}{4} = 0.62$$

$$P(y=3) = \frac{4}{109} + \frac{1}{12} = 0.12$$

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Th^{m}.

X follows Poisson distribution with parameter λ, then (i) $E(X) = \lambda$

(ii). $Var(X) = E(X(X-1)) - m(m-1)$

mean $= \lambda$

$$E(X(X-1)) = \sum_{r=0}^{\infty} r(r-1).\text{pmf}.$$

$$= \sum_{r=0}^{\infty} \frac{r(r-1)\, e^{-\lambda} . \lambda^{r}}{r!}$$

$$= 0 + \sum_{r=1}^{\infty} \frac{r(r-1)\, e^{-\lambda} . \lambda^{r}}{r!}$$

$$= 0 + 0 + \sum_{r=2}^{\infty} \frac{r(r-1)\, e^{-\lambda} \lambda^{r}}{r!}$$

$$= \sum_{r=2}^{\infty} \frac{\cancel{r(r-1)}\, e^{-\lambda} \lambda^{r}}{\cancel{r(r-1)}(r-2)!}$$

$$= \sum_{r=2}^{\infty} \frac{e^{-\lambda} \lambda^{r}}{(r-2)!}$$

$$= e^{-\lambda} \sum_{r=2}^{\infty} \frac{\lambda^{r}}{(r-2)!}$$

$$= e^{-\lambda} \left[\frac{\lambda^2}{0!} + \frac{\lambda^3}{1!} + \frac{\lambda^4}{2!} + \frac{\lambda^5}{3!} + \dots \infty \right]$$

$$= \lambda^2 e^{-\lambda} \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \dots \infty \right]$$

$$= \lambda^2 e^{-\lambda} . e^{\lambda}$$

$$= \lambda^2$$

$$\therefore Var(X) = \lambda^2 - \lambda(\lambda - 1)$$

$$= \lambda^2 - \lambda^2 + \lambda = \lambda \text{ (Ans:-)}$$

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(iii). Standard Deviation $= \sqrt{\lambda}$

Q-1). If X is a Poisson variate with $P(1) = P(2)$, then find mean & SD. Also find $P(X=4)$.

Sol^n.

pmf $\quad P(X=r) = \dfrac{e^{-\lambda} \cdot \lambda^r}{r!}$

Given, $P(1) = P(2)$

$\Rightarrow P(X=1) = P(X=2)$.

$\Rightarrow \dfrac{\cancel{e^{-\lambda}} \cdot \lambda^1}{1!} = \dfrac{\cancel{e^{-\lambda}} \cdot \lambda^2}{2!}$

$\Rightarrow \cancel{e^{-\lambda} \cdot \lambda \left\{ 1 - \frac{\lambda}{2} \right\} = 0}$

Either

$\Rightarrow \lambda^2 - 2\lambda = 0$.

$\Rightarrow \lambda(\lambda - 2) = 0$.

Either $\lambda = 0$ or $\lambda = 2$.

But mean $= \lambda$, so $\lambda \neq 0$.

$\therefore \lambda = 2$.

$\therefore$ Mean $= 2$; SD $= \sqrt{2} = 1.414$

$P(X=4) = \dfrac{e^{-2} \cdot 2^4}{4!}$

$= \dfrac{e^{-2} \times \cancel{16}\,2}{\cancel{24}\,3} = \dfrac{2}{3e^2}$

$= 0.09$

Q-2). If X is a Poisson variate with $P(2) = 9P(4) + 90P(6)$, find mean, SD,

_/_/_

$P(2) = 9P(4) + 90P(6)$

$\Rightarrow \dfrac{e^{-\lambda}\lambda^2}{2} = \dfrac{9e^{-\lambda}\lambda^4}{24} + \dfrac{90e^{-\lambda}\lambda^6}{720}$

$\Rightarrow \dfrac{1}{2} = \dfrac{9\lambda^2}{24} + \dfrac{90\lambda^4}{720}$

$\Rightarrow 1 = \dfrac{3 \cdot 9\lambda^2}{\cancel{12}4} + \dfrac{9\lambda^4}{\cancel{36}4}$

$\Rightarrow 4 = 3\lambda^2 + \lambda^4$

$\Rightarrow \lambda^4 + 3\lambda^2 - 4 = 0.$

$\Rightarrow \lambda^4 + 4\lambda^2 - \lambda^2 - 4 = 0.$

$\Rightarrow \lambda^2(\lambda^2 + 4) - (\lambda^2 + 4) = 0.$

$\Rightarrow (\lambda^2 - 1)(\lambda^2 + 4) = 0.$

$\therefore \lambda^2 - 1 = 0$ or $\lambda^2 = -4$

$\Rightarrow \lambda = \pm 1$ $\Rightarrow \lambda = \pm 2i$

Since mean $\neq -1, \pm 2i$.

$\therefore \lambda = 1.$

$\therefore$ Mean $= 1$; SD $= 1$.

~~P(X≥2)~~

Q-3) The distribution of the no. of road accidents per day in a city is Poisson distribution with mean = 4. Find the approximate no. of days out of 100 days when there will be:-

(i). No accidents (ii) Atleast 2 accidents

(iii). At most 3 accidents (iv) Betⁿ 2 and 5 accidents (both inclusive).

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Let X denote no. of road accidents per day.

Mean $(\lambda) = 4$.

pmf

$$\frac{e^{-\lambda}\lambda^{r}}{r!} = P(X=r).$$

$$\Rightarrow P(X=r) = \frac{e^{-4}\,4^{r}}{r!}$$

(i). $P(X=0) = \frac{e^{-4}\,4^{0}}{0!}$

$= e^{-4}$

Now, no. of days (out of 100) where no accidents occur $= 100 \times e^{-4}$

$= 1.831$

≈ 2 days.

(ii). $P(X \geq 2) = 1 - (P(X=0) + P(X=1))$

$= 1 - \left(e^{-4} + \frac{e^{-4}.4^{1}}{1!}\right)$

$= 1 - e^{-4}[1 + 4]$

$= 1 - 0.0915$

$= 0.9084$

∴ No. of days when atleast 2 accidents occur $= 100 \times 0.9084$

~~$= 76.18$~~ $= 90.84$

~~≈ 76 days.~~ ≈ 91 days

(iii). $P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) + P(X=3)$

$= e^{-4} + 4e^{-4} + 8e^{-4} + \frac{32}{3}e^{-4}$

$= 0.4334$

∴ No. of days $= 100 \times 0.4334$

$= 43.34$

≈ 43 days.

(iv). $P(2 \leq x \leq 5)$

$= P(x=2) + P(x=3) + P(x=4) + P(x=5)$

$= 8 e^{-4} + \frac{64}{6} e^{-4} + \frac{4^4}{24} e^{-4} + \frac{4^5}{120} e^{-4}$

$= 0.6935$

$\therefore$ No. of days $= 100 \times 0.6935$

$= 69.35$

≈ 69 days.

Q-4). A car hire firm has 2 cars which it hires out day by day. The no. of demands of a car on each day is distributed as a Poisson distribution with average no. of demand per day 1.5. Calculate the proportion of days on which neither car is used. Also calculate the proportion of days on which some demand is refused.

Sol^n. Let x denote the no. of demands of car per day

$\lambda = 1.5$.

$$P(X = r) = \frac{e^{-1.5} \, 1.5^r}{r!}$$

$$\therefore P(X=0) = \frac{e^{-1.5} \times 1}{1}$$

$= 0.2231$

$\therefore$ Proportion of days (out of 365) on which no cars is used $= 0.2231 \times 365$

$= 81.44$

≈ 81

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$$P(X>2) = 1-(P(X=0) + P(X=1) + P(X=2))$$
$$= 1 - e^{-1.5}\left(1 + \frac{1.5}{1} + \frac{1.5^2}{2}\right)$$
$$= 1 - e^{-1.5} \times 3.625$$
$$= 1 - 0.8088$$
$$= 0.19115$$

$\therefore$ Proportion of days $= 365 \times 0.19115$
$= 69.77$
≈ 70 days.

— x —

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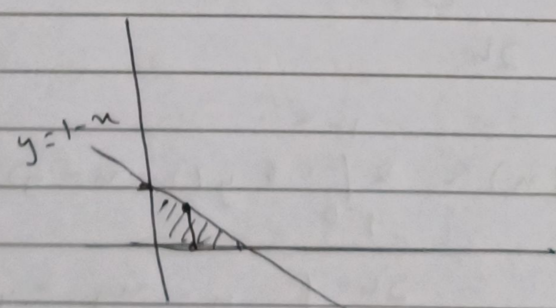
Q). $f(x,y) = \begin{cases} ky(1-x-y), & x \geq 0 \\ & y \geq 0 \\ & x+y \leq 1 \\ 0, & \text{elsewhere.} \end{cases}$

(a). Find k (b). $f_x(x)$, $f_y(y)$

(c). $P(X \leq \frac{1}{2}, Y \leq \frac{1}{2})$.



(a) $\int_0^1 \int_{y=0}^{1-x} ky(1-x-y)\,dy\,dx = 1.$

$\Rightarrow k\int_0^1 \int_{y=0}^{1-x} y - xy - y^2\,dy\,dx = 1$

$\Rightarrow k\int_0^1 \left[\frac{y^2}{2} - \frac{xy^2}{2} - \frac{y^3}{3}\right]_0^{1-x} dx = 1$

$$\Rightarrow k\int_0^1 \frac{(1-x)^2}{2} - \frac{x(1-x)^2}{2} - \frac{(1-x)^3}{3}\, dx = 1$$

Let $(1-x)^2 = t^2$

$\Rightarrow 1-x = t$

$\Rightarrow -dx = dt$

$\Rightarrow dx = -dt$

$$\therefore k\int_1^0 \left(\frac{t^2}{2} - \frac{t^2(1-t)}{2} - \frac{t^3}{3}\right) -dt = 1$$

$$\Rightarrow k\int_0^1 \frac{t^2}{2} - \frac{t^2}{2} + \frac{t^3}{2} - \frac{t^3}{3}\, dt = 1$$

$$\Rightarrow k\left[\cancel{\frac{t^3}{6}} - \cancel{\frac{t^3}{6}} + \frac{t^4}{8} - \frac{t^4}{12}\right]_0^1 = 1$$

$$\Rightarrow k\left(\frac{1}{8} - \frac{1}{12}\right] = 1$$

$$\Rightarrow k\left(\frac{3-2}{24}\right] = 1$$

$$\Rightarrow k = 24$$

(b) $$f_x(x) = k\int_{y=0}^{1-x} y(1-x-y)\, dy$$

$$= 24\int_{y=0}^{1-x} y - xy - y^2\, dy$$

$$= 24\left[y^2 - \frac{xy^2}{2} - \frac{y^3}{3}\right]_0^{1-x}$$

$$= 24\left[(1-x)^2 - \frac{x(1-x)^2}{2} - \frac{(1-x)^3}{3}\right]$$

$$= 24(1-x)^2\left(1 - \frac{x}{2} - \frac{1-x}{3}\right)$$

$$= 24(1-x)^2\left(\frac{6-3x-2+2x}{6}\right)$$

_/_/_

$= 24(1-x)^2 \left(\frac{4-x}{6}\right)$

$= 4(1-x)^2(4-x).$

(b) $f_y(y) = 24\int_{x=0}^{1-y} y(1-x-y)\, dx$

$= 24y\int_0^{1-y} 1-x-y \; dx.$

$= 24y\left[x - \frac{x^2}{2} - xy\right]_0^{1-y}$

$= 24y\left[(1-y) - \frac{(1-y)^2}{2} - (1-y)y\right]$

$= 24y(1-y)\left[1 - \frac{1-y}{2} - y\right].$

$= 24y(1-y)\left[2-1+y-2y\right].$

$= 24y(1-y)(1-y).$

$= 24y(1-y)^2.$

(c). $P(X \le \frac{1}{2}, Y \le \frac{1}{2}) = 24\int_{y=0}^{1/2} y\int_{x=0}^{1/2} (1-x-y)\, dx\, dy.$

$= 24\int_{y=0}^{1/2} y\left[x - \frac{x^2}{2} - xy\right]_0^{1/2} dy$

$= 24\int_{y=0}^{1/2} y\left[\frac{1}{2} - \frac{1}{8} - \frac{y}{2}\right] dy.$

$= 24\int_0^{1/2} \frac{y}{2} - \frac{y}{8} - \frac{y^2}{2}\, dy$

$= 24\left[\frac{y^2}{4} - \frac{y^2}{16} - \frac{y^3}{6}\right]_0^{1/2}$

$= 24\left[\frac{1}{16} - \frac{1}{64} - \frac{1}{48}\right]$

$= \frac{24}{16}\left[1 - \frac{1}{4} - \frac{1}{3}\right]$

$$= \frac{6}{4} \times \frac{12-3-4}{2}$$

$$= \frac{1}{4} \times \frac{5}{2} = \frac{5}{8}.$$

——— × ———

Q.5) If the chance of being ~~dea~~ killed by a flood during a year is 1/3000. Use Poisson distribution to calculate probability that out of 3000 persons living in a village, atleast 1 will die in flood in a year.

Sol^n. Let x denote no. of deaths by flood during a year.

$$\therefore \lambda = np$$
$$= 3000 \times \frac{1}{3000}$$
$$= 1.$$

$$P(X = r) = \frac{e^{-\lambda} \lambda^r}{r!}$$

$$\Rightarrow P(X = r) = \frac{e^{-1} \cdot 1^r}{r!} = \frac{1}{e\, r!}$$

$$\Rightarrow P(X \geq 1) = 1 - P(X = 0)$$
$$= 1 - \frac{1}{e \cdot 0!}$$
$$= 1 - \frac{1}{e} = 0.632.$$

~~∴ no. of people dying out of 3000 = 0.632 × 3000~~
~~= 1896, 36~~
~~≈ 1896~~

_/_/_

Q-6) Suppose a book of 585 pages contains 43 typographical errors if these errors are randomly distributed throughout the book, then what is the probability that 10 pages selected at random, will be free from errors.

Sol^n

Let X denote the no. of errors in 10 pages.

$P(\text{getting an error}) = \frac{43}{585} = 0.0735$

$n = 10$

$\therefore \lambda = np$

$= 10 \times 0.0735$

$= 0.735$

$P(X=0) = \frac{e^{-0.735} \cdot 0.735^{0}}{0!}$

$= 0.479$

Q-7) A manufacturer who produces medicine bottles, finds that 0.1 % of the bottles are defective. The bottles are packed in boxes containing 500 bottles. Find that in 100 such boxes, how many boxes are expected to contain -
(i) No defective (ii) Atleast 2 defective boxes.

Sol^n

Let X denote the no. of defective bottles out of 500 bottles.

$n = 500$; $p = \frac{1}{1000}$; $\lambda = \frac{500}{1000} = 0.5$

(i) $P(X=0) = \frac{e^{-0.5} (-0.5)^{0}}{0!} = 0.60653$

$\therefore$ No. of defective bottles in 100 such boxes $= 100 \times 0.6065$

$= 60.65$

≈ 61.

(ii) $P(X \geq 2) = 1 - (P(X=0) + P(X=1))$

$= 1 - (0.6065 + e^{-0.5} (0.5))$

$1 - (0.6065 + 0.3032)$
$= 1 - 0.9097$
$= 0.0903$

$\therefore$ No. of defective bottles in 100 such boxes
$= 100 \times 0.0903$
$= 9.03$
≈ 9

Q-8). Some airlines find that each passenger who reserves a seat fails to turn up with probability 0.1 independently of others passengers of these airlines. Airline A always sells 10 tickets for their 9 seat airplane while Airline B sells 20 tickets for their 18 seat airplane. Using Poisson distribution, find which one of the airlines - A and B, is more often overbooked.

Solⁿ.

Let X denote the no. of passengers who fail to turn up in airline A.

$p = 0.1$; $n = 10$

$\lambda = 10 \times 0.1 = 1$.

For getting overbooked, 0 passengers failed to turn up,

$\therefore P(X = r) = \dfrac{e^{-\lambda} \cdot \lambda^{r}}{r!}$

Here, $r = 0$; $\therefore P(X = 0) = \dfrac{e^{-1} \cdot 1^{0}}{0!}$

$= \dfrac{1}{e} = 0.3678$

_/_/__

Let y denote the no. of passengers who fail to turn up in airline B.

$p = 0.1$; $n = 20$.

$\therefore \lambda = 0.1 \times 20 = 2$.

For getting overbooked, 0 passenger or 1 passenger fails to turn up,

$\therefore P(X=0) + P(X=1)$

$= \frac{e^{-2} \cdot 2^0}{0!} + \frac{e^{-2} \cdot 2^1}{1!}$

$= \frac{1}{e^2} + \frac{2}{e^2} = \frac{3}{e^2} = 0.406$

$\therefore$ Airline B is more often ~~be~~ overbooked.