MGT 422 Operations Engine

Lecture 8: Inventory Models II

2026 // Spring 1 // Core

Yale SCHOOL OF MANAGEMENT

Announcements

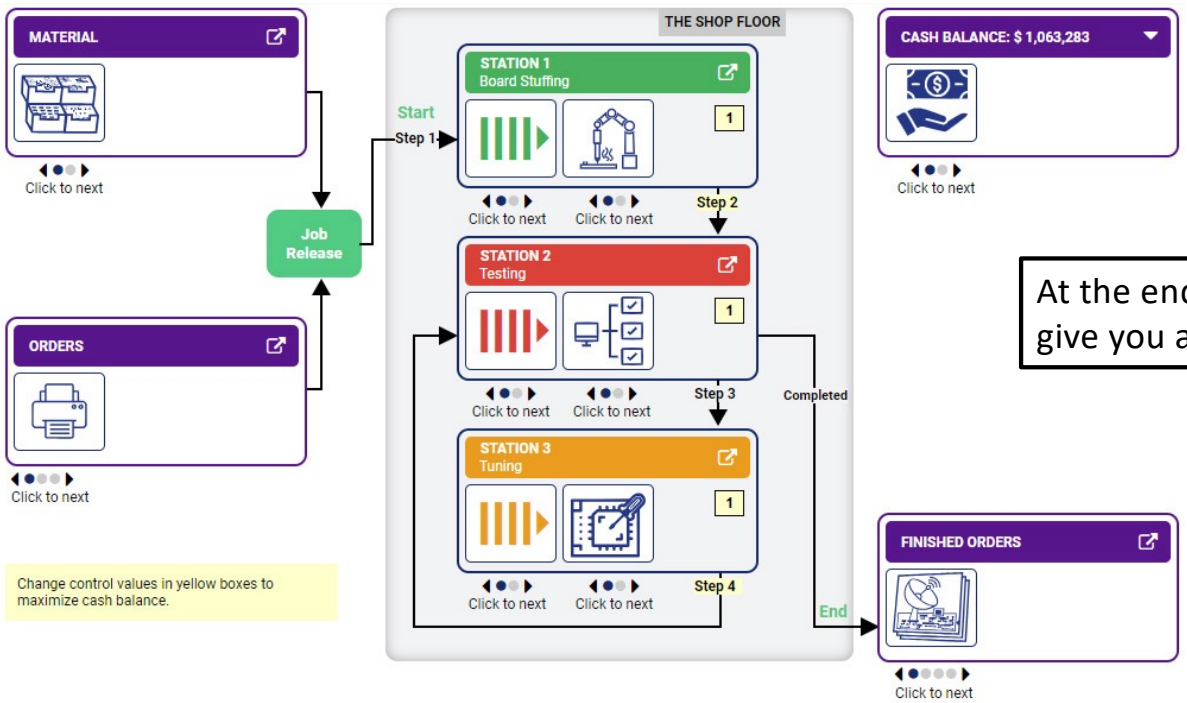
- Littlefield Simulation access is now open
  - One-page **Action Plan** is due on **Sunday, Feb. 15 at 5pm**

- The game starts running on **Sunday, Feb 15 at 5pm**
  - The website is open now.
  - You should see historical demand for the first 50 days.
  - You cannot make decision yet, but please familiarize yourself with the interface.

- After today, we are done with most of the quantitative models in this course

- No homework next week! ☺
  - But please do read the cases…

Littlefield Simulation Overview



At the end of this lecture, I will give you a hint about this…

- You run a manufacturing facility where electronic equipment is assembled
- Decide on capacity at each manufacturing step
- Decide on materials sourcing rule – assembly will be stopped if there is not enough materials!
- Generate revenue by selling equipment; you can raise debt to continue operations

Practice Problem

- During the flu season, patients arrive at a vaccination clinic every 8 minutes, on average, as a Poisson process. The clinic has 2 workers vaccinating patients, taking an average of 13 minutes per patient, with a standard deviation of 6 minutes (you may assume that vaccinated patients can leave immediately after the vaccine and do not need to wait at the clinic).

1. On average, how long does a patient have to wait in line prior to their vaccination?

2. The clinic is trying to determine how many chairs to provide for waiting patients. What is the number of patients waiting at any given time, on average?

3. The parking authority at the clinic is trying to determine the number of parking spots to allocate for vaccination visitors. If every patient drives a car to the clinic for their vaccination, what is the expected number of parking spaces required, on average?

Solutions are shown on the next slide, so you can try this practice problem before looking at the answers!

Solution

- During the flu season, patients arrive at a vaccination clinic every 8 minutes, on average, as a Poisson process. The clinic has 2 workers vaccinating patients, taking an average of 13 minutes per patient, with a standard deviation of 6 minutes (you may assume that vaccinated patients can leave immediately after the vaccine and do not need to wait at the clinic).

1. On average, how long does a patient have to wait in line prior to their vaccination?

   Using the Tq formula = **15.56 minutes**

1. The clinic is trying to determine how many chairs to provide for waiting patients. What is the number of patients waiting at any given time, on average?

   Using Little's Law for the queue, where flow time = Tq = 15.56 minutes and the flow rate is equal to the arrival rate of 0.125 arrivals per minute, we have Iq = 0.125 arrivals per minute * 15.56 minutes = **1.945 arrivals waiting on average**

2. The parking authority at the clinic is trying to determine the number of parking spots to allocate for vaccination visitors. If every patient drives a car to the clinic for their vaccination, what is the expected number of parking spaces required, on average?

   Using Little's Law for the total time in the system (Tq + p), we have a flow time of 15.56 minutes + 13 minutes = 28.56 minutes spent in the clinic in total, multiplied by a flow rate (arrival rate) of 0.125 arrivals per minute = **3.57 patients in the system on average**

Common considerations in procurement settings

- A retailer places orders with a supplier on a periodic basis. What are the key factors in devising an efficient ordering rule?
  - Don't want to hold too much inventory
  - Don't want to pay shipping costs too frequently

- *How often* should the retailer place orders, and by *what quantity*?

- Let's start with something simple. Assume:
  - Demand arrives over time at <u>constant</u> rate
  - No shortages allowed (retailer MUST satisfy all incoming demands)
  - Supplier can fulfill retailer orders of any size
  - No demand uncertainty ← Relaxed later
  - Delivery lead time is zero (i.e., ordered items are delivered instantaneously) ← Relaxed later

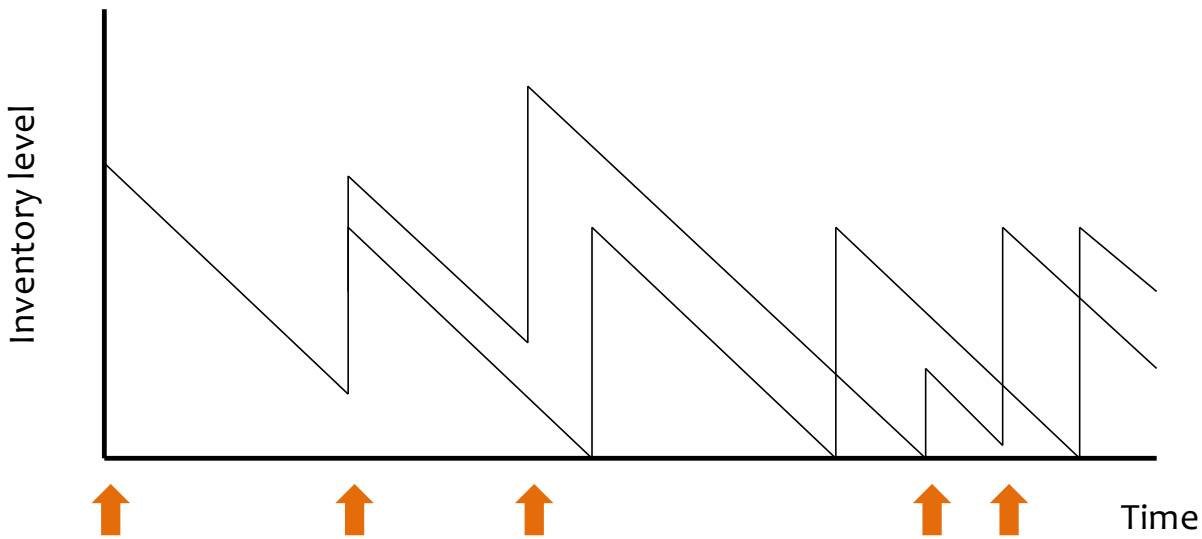
Economic Order Quantity (EOQ) Model

- Three key inputs:
  - ***A***: Annual demand rate (e.g., 10,000 units per year)
  - ***K***: **Fixed cost** of placing an order (e.g., $1,000)
  - ***H***: Annual **holding cost** per unit in inventory (e.g., $10 per unit per year)

- Note:
  - Holding cost consists of: (a) opportunity cost of capital, often expressed as **a percentage of the item cost**; and (b) direct costs such as storage, handling, etc.
  - For convenience, we will use a convention where all units are measured in yearly basis. ***Make sure that all units of measure are consistent! For example, convert weekly demand rate into annual demand rate.***

- Objective: Choose the order quantity ***Q*** that minimizes the sum of ordering costs and inventory holding costs

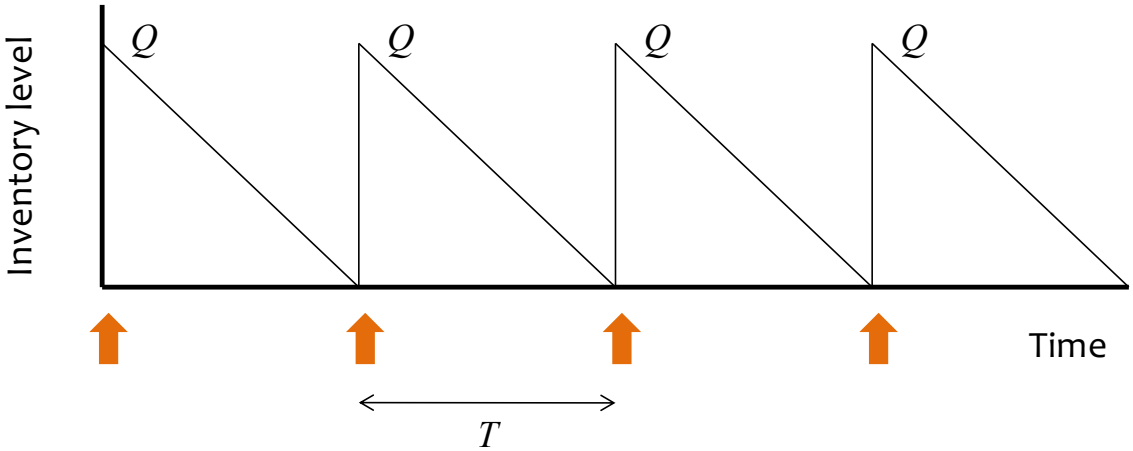
When should the retailer place orders?



What does the area below the sawtooth lines represent?

Can we do better?

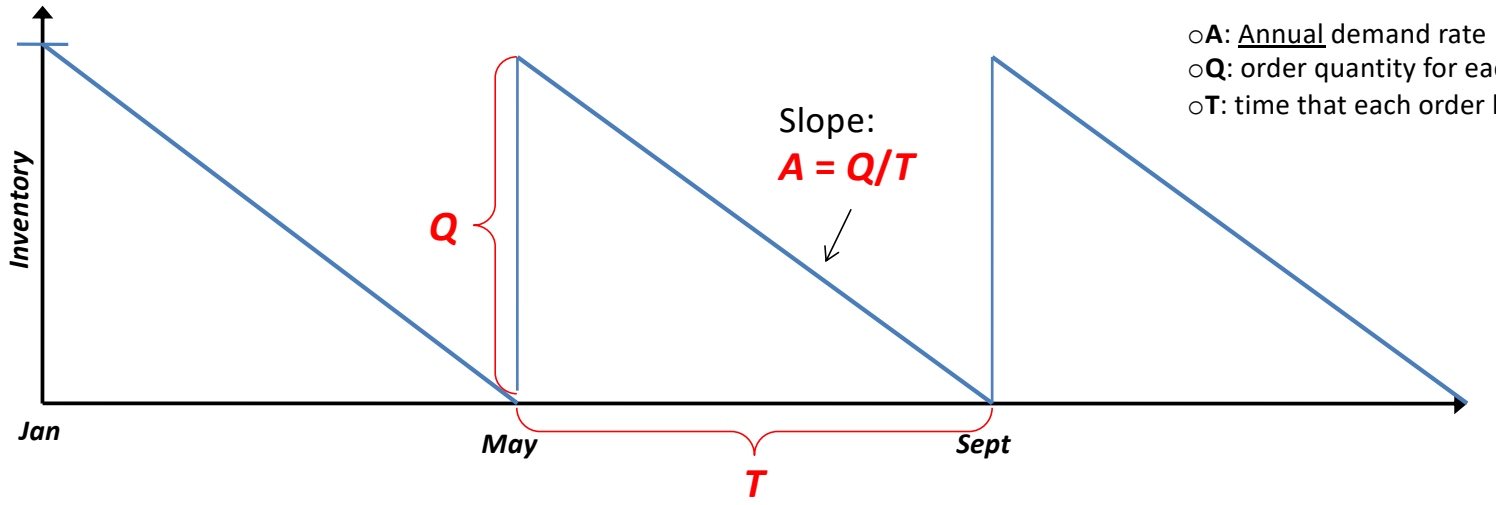
Optimal ordering policy



"Order the same amount *Q* every *T* time units"

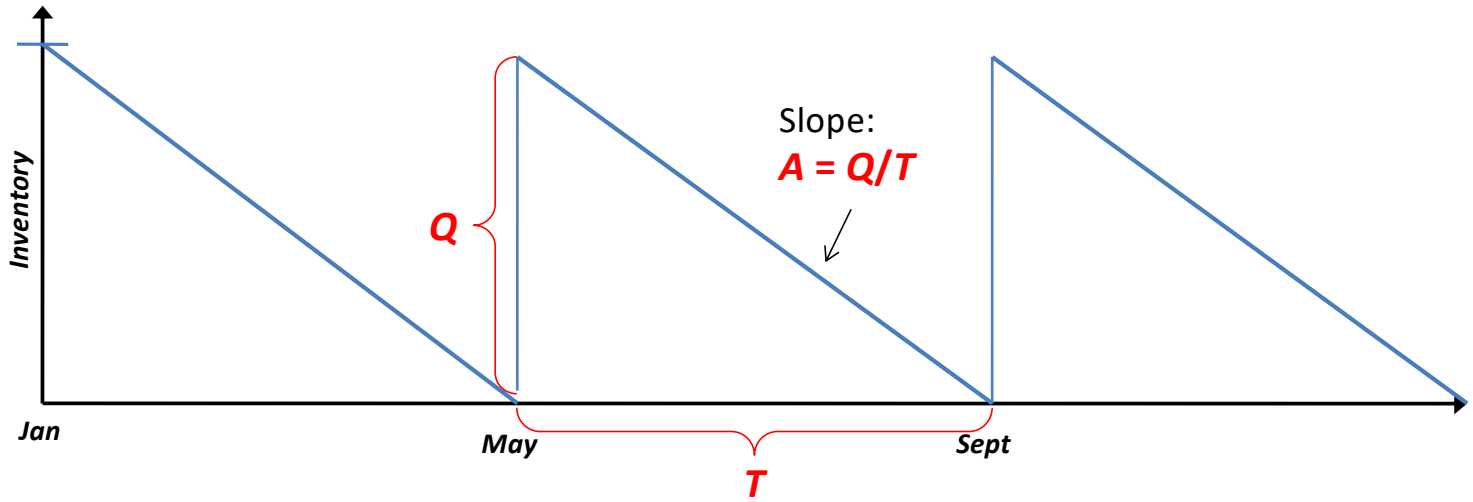
What are the optimal values for *Q* and *T*?

Calculating ordering cost and holding cost



- **A**: <u>Annual</u> demand rate
- **Q**: order quantity for each order
- **T**: time that each order lasts for

Calculating ordering cost and holding cost



Example:
A = 26 bags of coffee beans (units) per year
Q = 1 unit (bag of beans)
T = 1/26 years (how long an order lasts) = 0.03846 years * 365 days = 14 days

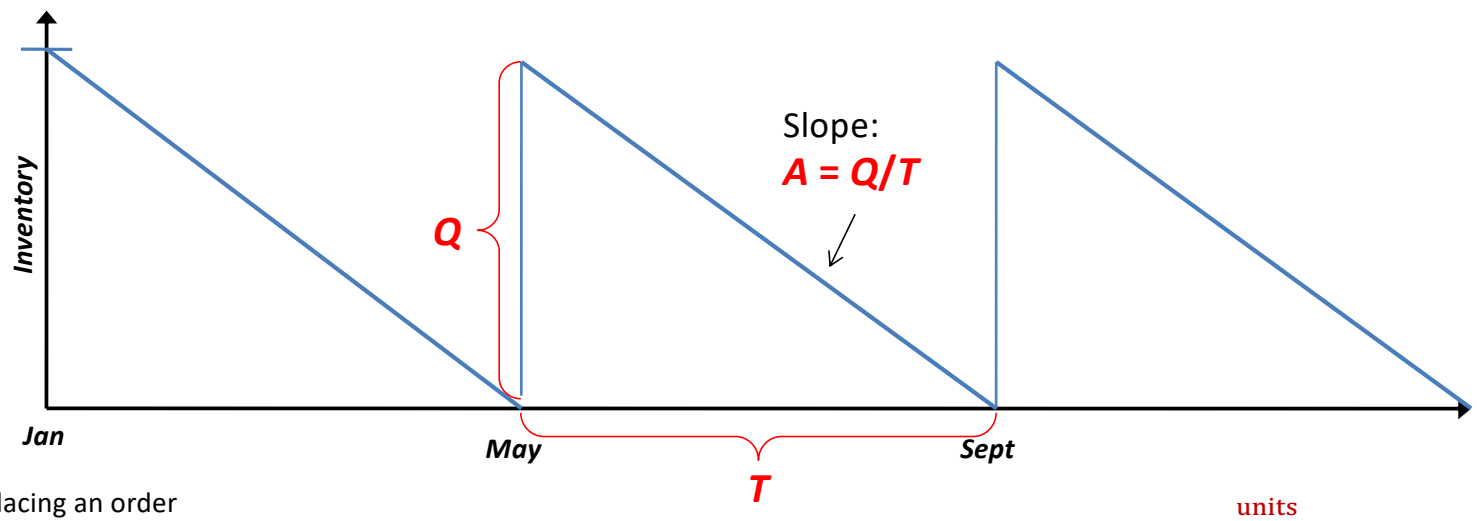
Orders per year = 1/T = 26 orders / year = 1 per two weeks

Since A = Q/T

Then T = Q/A

So Orders per Year (1/T) = A/Q

Calculating ordering cost and holding cost



- **K**: **Fixed cost** of placing an order
- **H**: Annual **holding cost** per unit per time

Units for Annual ordering cost: $\$ * \frac{\frac{\text{units}}{\text{year}}}{\text{units}} = \$ * \frac{\cancel{\text{units}}}{\text{year}} * \frac{1}{\cancel{\text{units}}} = \frac{\$}{\text{year}}$

Annual ordering cost = K × Number of orders per year = ***K*A/Q***

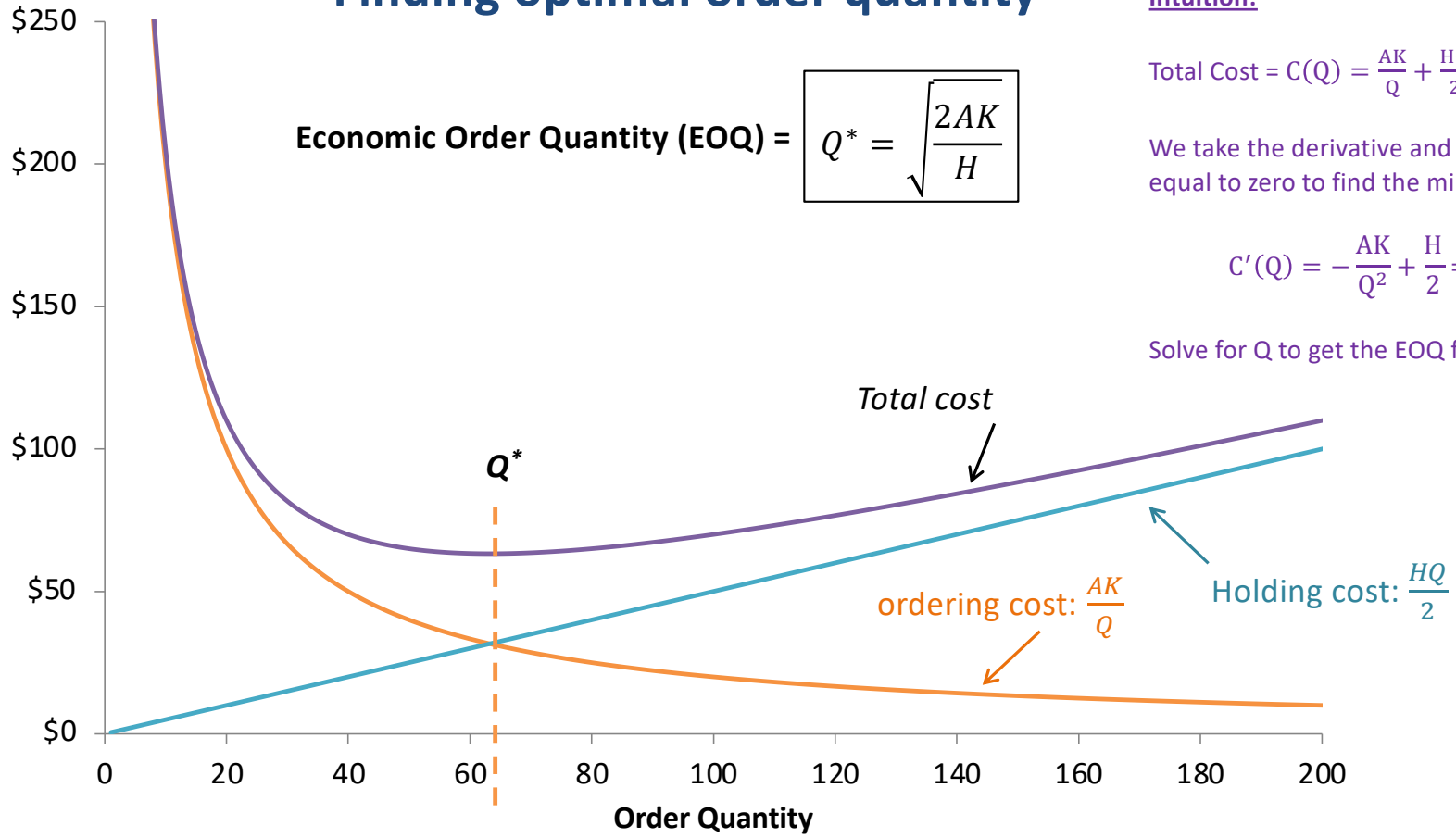
Cumulative inventory held per year = Area under the sawtooth lines
= $QT/2$ (area under triangle) × Number of orders per year = $QT/2 \times 1/T = Q/2$

Annual holding cost = H × Cumulative inventory held = ***H*Q/2***

Units for Annual Holding Cost: $\frac{\$}{\text{units}*\text{year}} * \text{units} = \frac{\$}{\cancel{\text{units}}*\text{year}} * \cancel{\text{units}} = \frac{\$}{\text{year}}$

Finding optimal order quantity

Economic Order Quantity (EOQ) = $\boxed{Q^* = \sqrt{\frac{2AK}{H}}}$



Intuition:

Total Cost = $C(Q) = \frac{AK}{Q} + \frac{HQ}{2}$

We take the derivative and set that equal to zero to find the minimum.

$$C'(Q) = -\frac{AK}{Q^2} + \frac{H}{2} = 0$$

Solve for Q to get the EOQ formula!

What about the cost of the goods?

- Under the assumptions of EOQ model, we satisfy <u>all</u> yearly demands with ordered quantities (no mismatch between demand and supply)
  - ✓ Annual procurement quantity = annual demanded quantity

- As a result, annual procurement cost is just equal to the unit cost multiplied by A, the quantity demanded. This is independent of *Q*, our decision variable.

Example: Amazon FBA Inventory Replenishment Decision

A startup sells a popular reusable water bottle through Amazon using Fulfillment by Amazon (FBA). The company regularly sends inventory to Amazon's fulfillment centers to maintain stock availability.
Each shipment to Amazon incurs a fixed logistics and administrative cost of **$24** (coordination, packaging, inbound shipping, and handling).

- The wholesale cost of each unit is **$50**.
- Amazon storage fees and the cost of capital result in an annual holding cost equal to **20% of unit cost** ($10 per unit per year).
- Demand for the product is stable, with annual sales of approximately **9,000 units**.

1) What is the optimal order quantity (Q*)?
2) What is the total cost?

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1) What is the optimal order quantity (Q*)?

A = 9000
K = 24
H = 10

$$\boxed{Q^* = \sqrt{\frac{2AK}{H}} = \sqrt{\frac{2 * 9000 * 24}{10}} = 207.8 \text{ units}}$$

$$(\text{units}) = \sqrt{\frac{\frac{\text{units}}{\text{year}} * \$}{\frac{\$}{\text{units} * \text{year}}}} = \sqrt{\frac{\text{units}}{\cancel{\text{year}}} * \cancel{\$} * \frac{\text{unit} * \cancel{\text{year}}}{\cancel{\$}}}$$

$$= \sqrt{\text{units}^2} = \text{units}$$

2) What is the total cost?

Annual ordering fixed cost = K*A/Q = 24*9000/208 = $1,038 per year

Annual holding cost = H*Q/2 = 10*208/2 = $1,040 per year

Sum of Ordering and Holding Costs ("Total Cost") = $1,038 + $1,040 = **$2,078**

Annual inventory procurement cost = c*A = $50 * 9000 = $450,000 per year

For the exam, it will be made clear which costs we are asking for.

Insights from the basic EOQ model

- Tradeoff between:
  - Ordering too much leads to high inventory holding cost
  - Ordering too little leads to high fixed costs of ordering

  ➔ EOQ formula represents the balance between the two

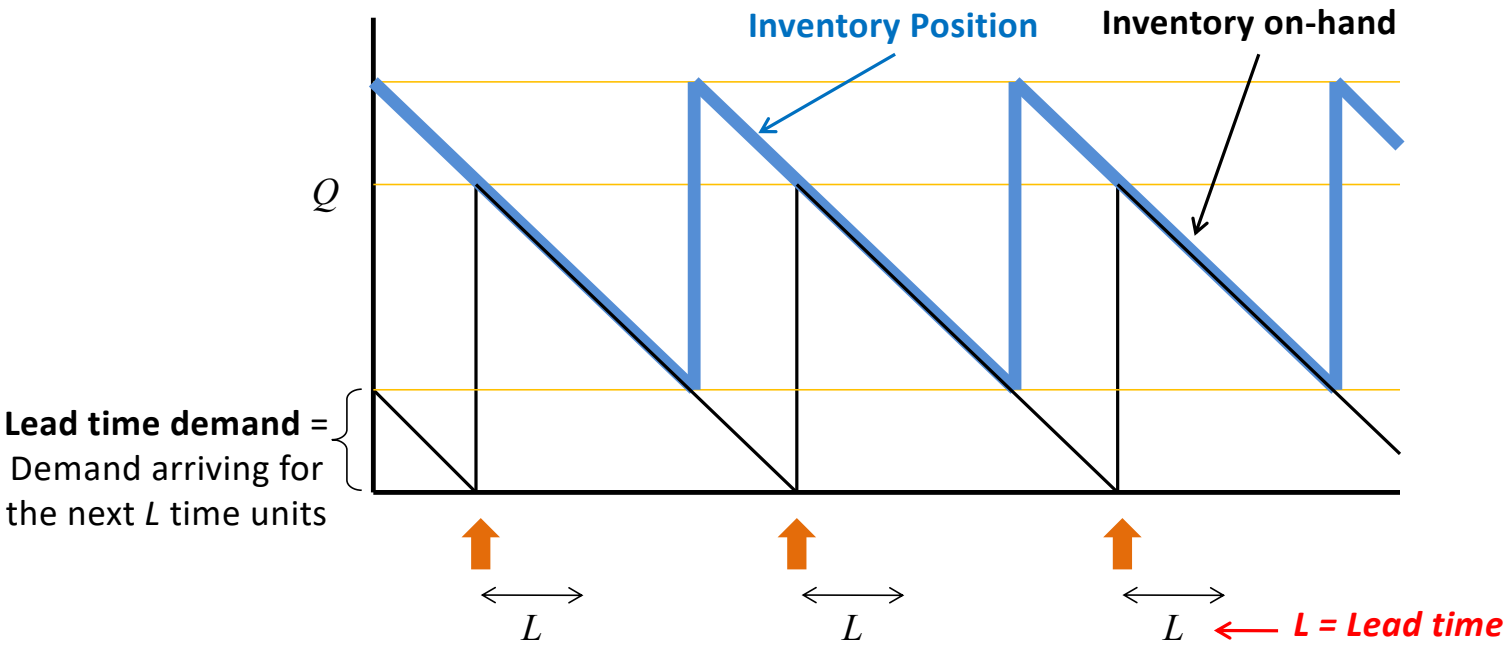
- Economies of scale
  - Doubled demand rate doesn't mean doubling order quantity!

- Establishes a relationship between optimal order quantity and optimal order frequency (since $T^* = Q^*/A$, then $1/T^* = A/Q^*$), where T* represents the time duration each optimal order quantity lasts, and (1/T*) represents the optimal order frequency.

Beyond the Basic EOQ Model:

(1) Effect of delivery lead time
(2) Effect of demand uncertainty

EOQ model with constant positive lead time *L*

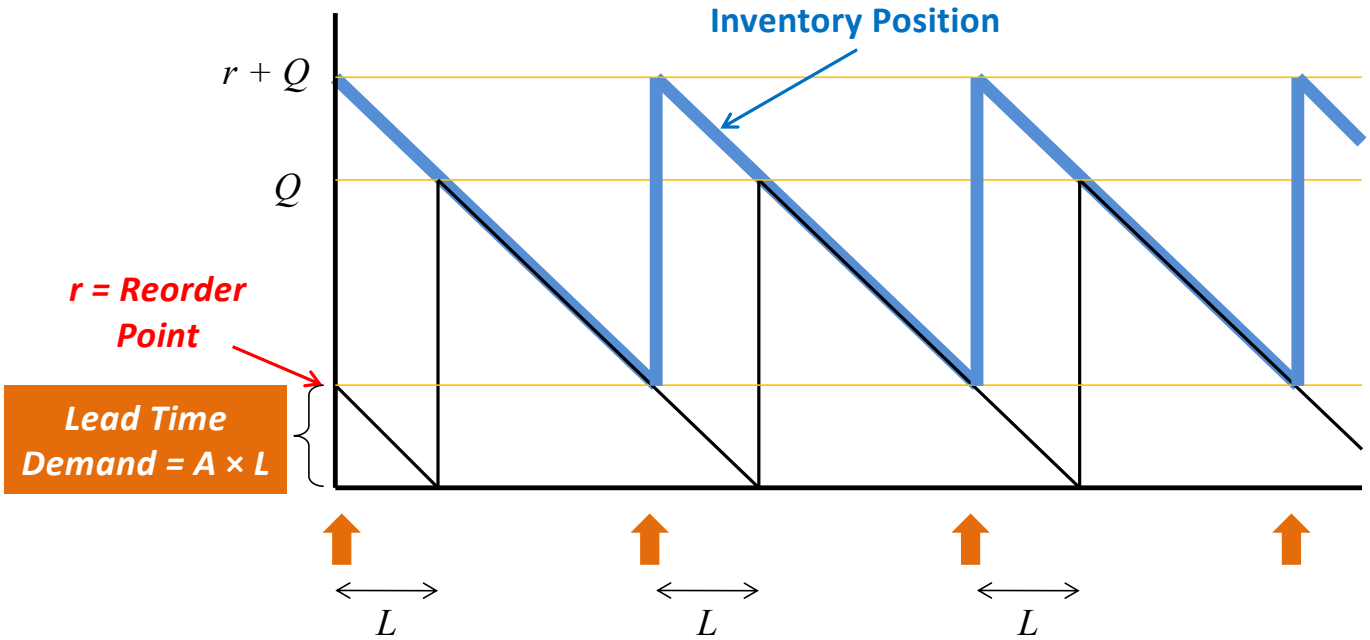


Two kinds of inventory exist when delivery lead time is positive:

- **Inventory on-hand**: Units that have been ordered and delivered to you
- Inventory on-order: Units that have been ordered but on-the-way

Inventory position = **Inventory on-hand +** Inventory on-order: Total inventory you are entitled to

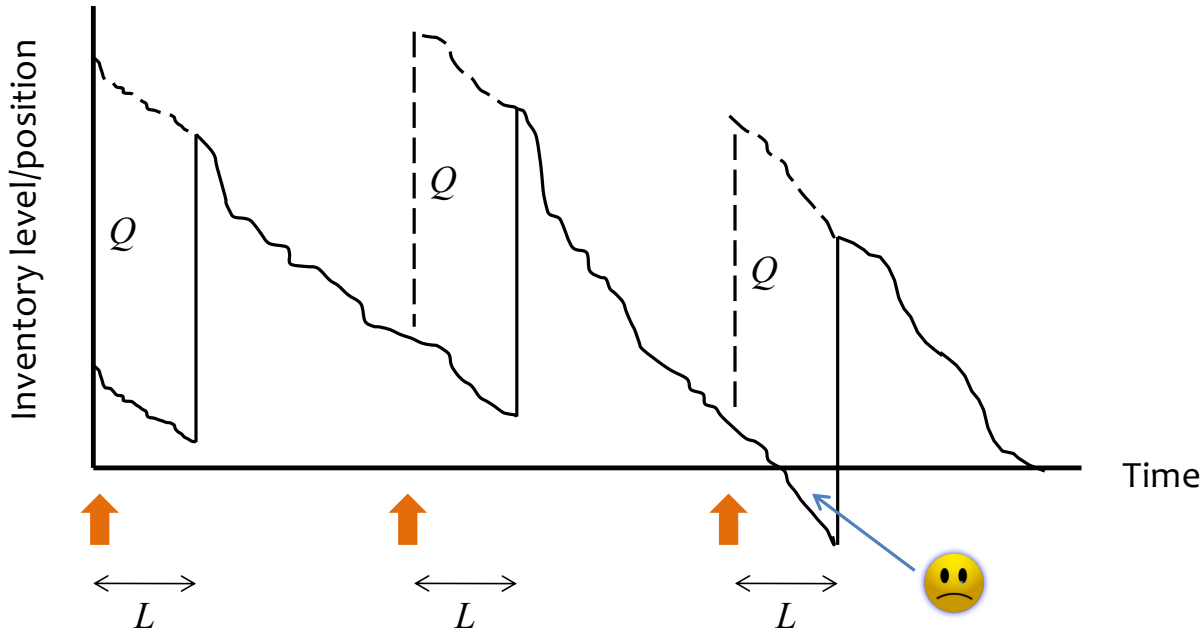
Optimal order policy with constant lead time *L*



- It is still optimal to order the EOQ quantity: $\boxed{Q^* = \sqrt{\frac{2AK}{H}}}$
- Optimal order policy can be restated as:
  "**Order *Q* units whenever <u>Inventory Position</u> reaches <u>Reorder Point</u> *r*.**"
- Optimal reorder point = $\boxed{r^* = A \times L}$ (set to match lead time demand)

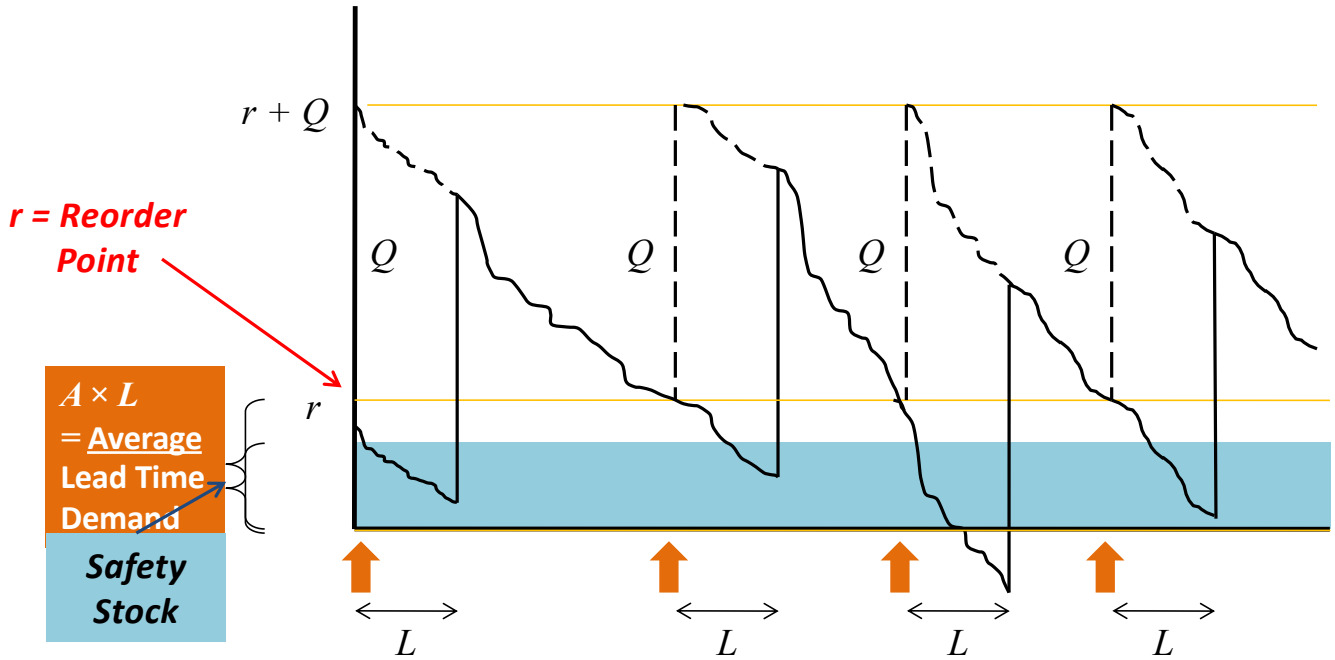
What changes if demand is uncertain?

Suppose that we order by the same amount *Q* and follow the same ordering policy as before, assuming constant positive delivery lead time of *L*.



Shortages are inevitable when demand is uncertain, however small the probability is… ***How much shortage are we willing to tolerate?***

How does ordering policy change with demand uncertainty?



- It is *near* optimal to order the EOQ quantity: $Q^* = \sqrt{\frac{2AK}{H}}$
- Optimal order policy is same as before:
  "**Order *Q* units whenever <u>Inventory Position</u> reaches <u>Reorder Point</u> *r*.**"
- Optimal reorder point = $r^* = A \times L +$ **Safety Stock**

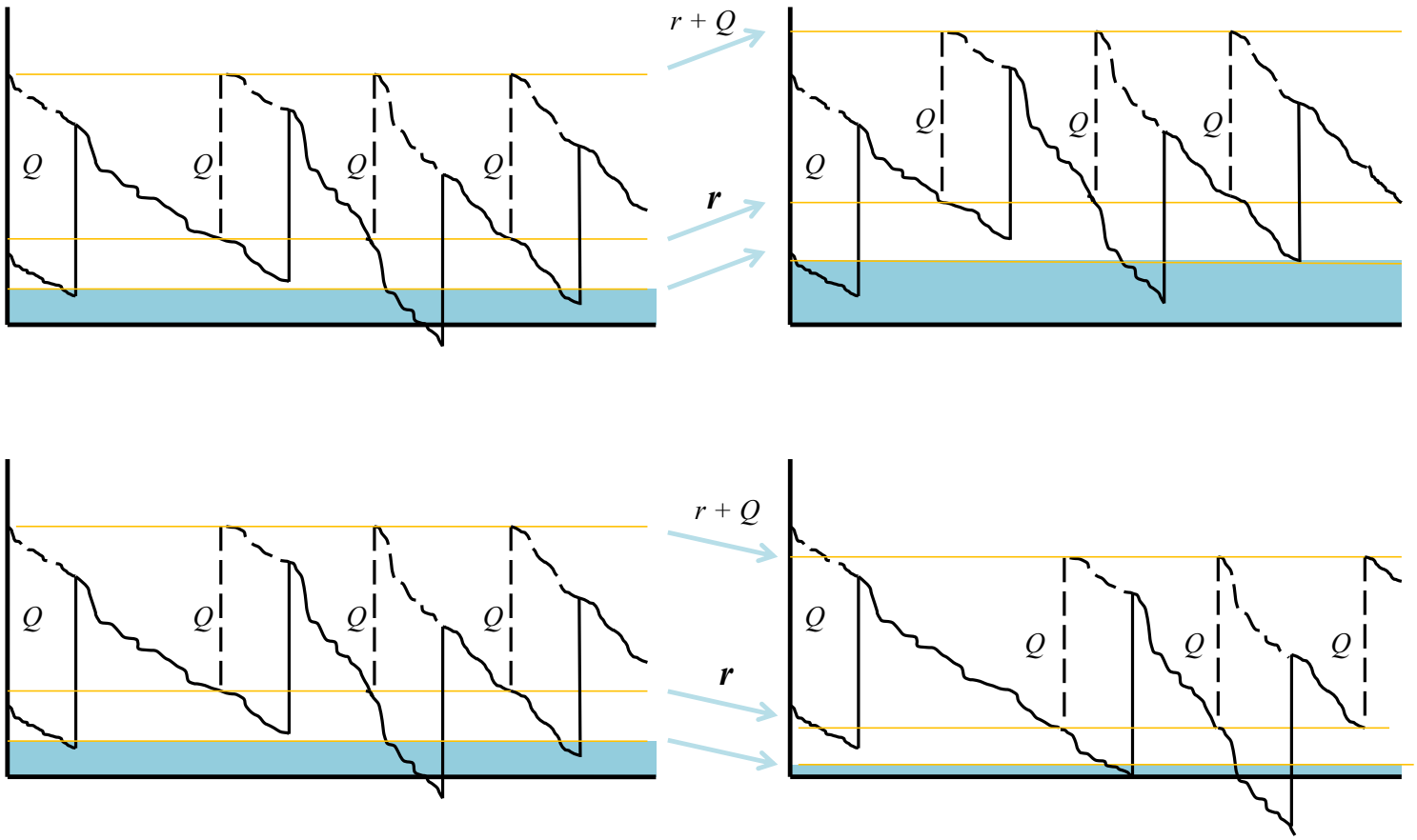
Determining safety stock size (intuitively)

- What is the relationship between safety stock size and demand uncertainty?

- It is uncertainty of demand *during lead time L* that matters

- Safety stock size also depends on *shortage penalty* (or back order cost)

“Too much/too little” problem for reorder point *r*



The Two Fundamental Inventory Models

Newsvendor Model

- Highly uncertain demand
- Short selling season
- Only one opportunity to place order/produce

Key Tradeoff

Overage	vs.	Underage
• The cost of ordering too many (i.e., leftover)		• The cost of ordering too few (i.e., stockout)
• Can be (partly) mitigated by selling after season at salvage value		• Can be (partly) mitigated by having reactive capacity

EOQ Model

- Almost stable demand (zero or low uncertainty)
- Very long selling season (i.e., infinity)
- Many opportunities to place order/produce

Key Tradeoff

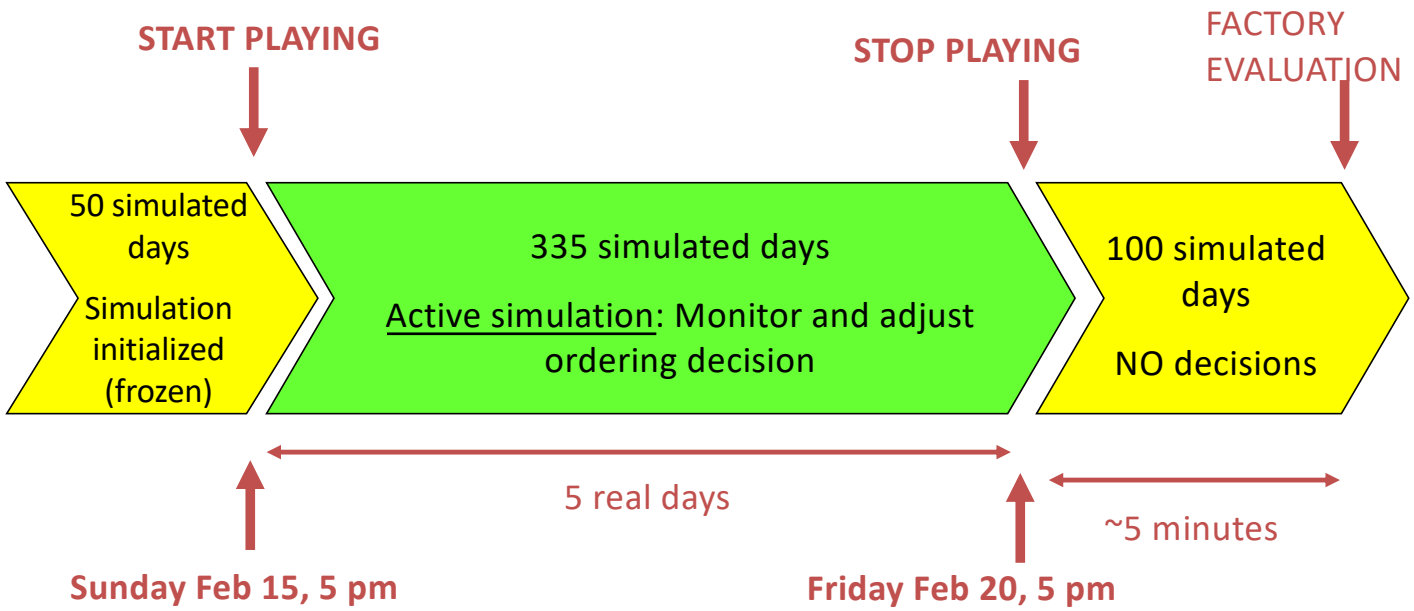
Order Frequency	vs.	Holding Inventory
• The cost of ordering is high if ordering too frequently		• The cost of holding inventory is high if ordering too infrequently
• Examples: Admin cost, transaction fee, Customs, Inconvenience, Shipping, Time, Fuel		• Examples: Storage cost, Warehouse, Electricity, Insurance, Perishability, Capital cost (lost interest)

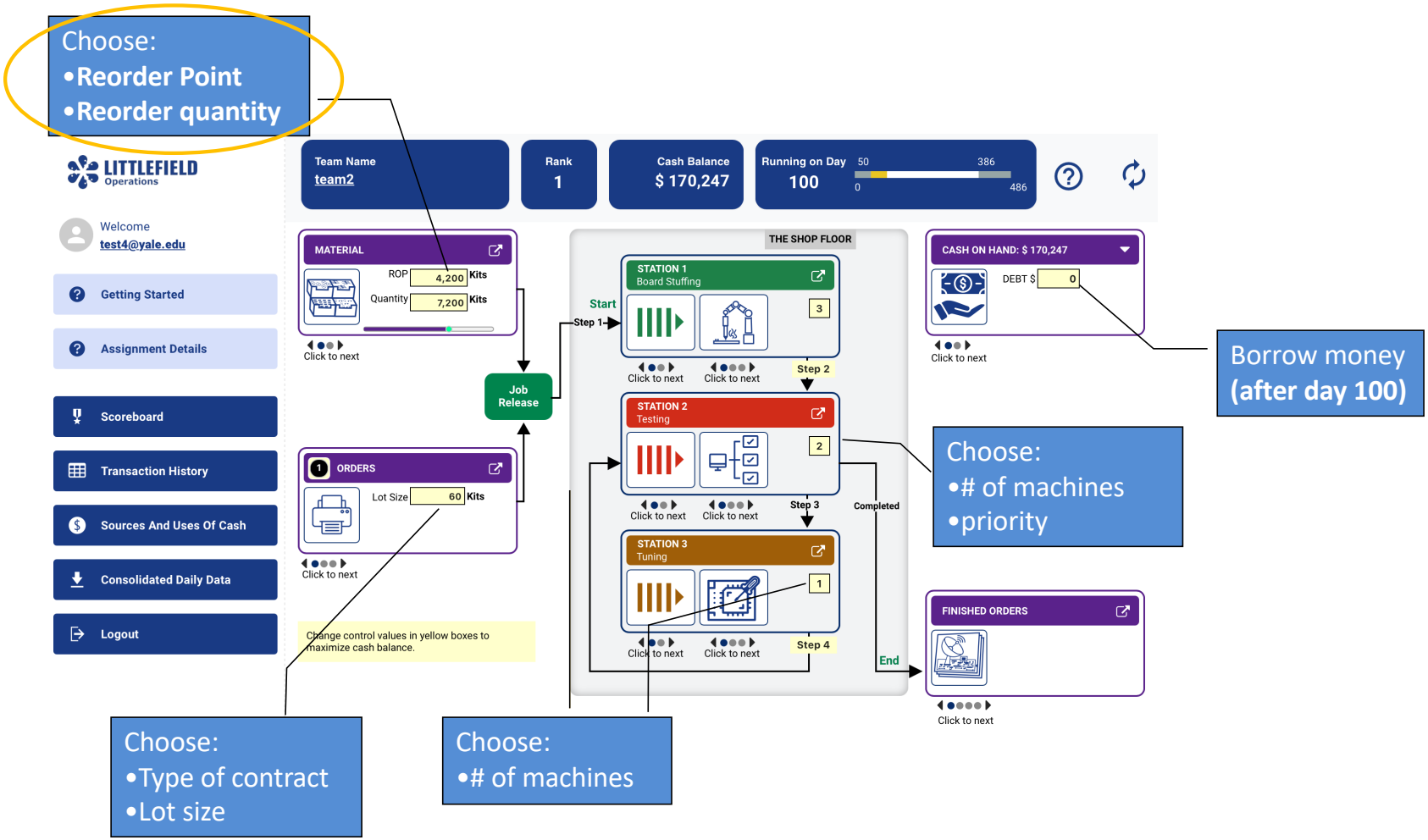
Using EOQ in Littlefield Simulation

Littlefield simulation: Rules of game

- ✓ Littlefield Technology is a manufacturing company that assembles electronic equipment
- ✓ Your team has been appointed to manage production operations
- ✓ You have 335 simulated days (= 5 real days) to manage the factory and generate cash (profit from sales – debt)
- ✓ 1 real day = 67 simulated days. No need for constant monitoring!
- ✓ You start managing the factory on Day 50; Computer runs the simulation on behalf of you in the first 50 days and last 100 days
- ✓ **The winning team is the team with the most cash at the end of the game (Day 385 + 100)**
- ✓ You can monitor ranking of your team throughout the game!

Schedule


START PLAYING
STOP PLAYING
FACTORY EVALUATION
50 simulated days
Simulation initialized (frozen)
335 simulated days
Active simulation: Monitor and adjust ordering decision
100 simulated days
NO decisions
5 real days
~5 minutes
Sunday Feb 15, 5 pm
Friday Feb 20, 5 pm


Choose:
•Reorder Point
•Reorder quantity
LITTLEFIELD Operations
Welcome
test4@yale.edu
Getting Started
Assignment Details
Scoreboard
Transaction History
Sources And Uses Of Cash
Consolidated Daily Data
Logout
Team Name
team2
Rank
1
Cash Balance
$ 170,247
Running on Day
100
50
386
0
486
MATERIAL
ROP 4,200 Kits
Quantity 7,200 Kits
Click to next
ORDERS
Lot Size 60 Kits
Click to next
Job Release
Change control values in yellow boxes to maximize cash balance.
THE SHOP FLOOR
Start
Step 1
STATION 1
Board Stuffing
3
Click to next
Click to next
Step 2
STATION 2
Testing
2
Click to next
Click to next
Step 3
Completed
STATION 3
Tuning
1
Click to next
Click to next
Step 4
End
CASH ON HAND: $ 170,247
DEBT $ 0
Click to next
Borrow money
(after day 100)
Choose:
•# of machines
•priority
FINISHED ORDERS
Click to next
Choose:
•Type of contract
•Lot size
Choose:
•# of machines

Considerations for Littlefield simulation

- Apply EOQ logic to place orders with a raw materials supplier
  - Order quantity, reorder point (think about the input parameters for EOQ)
  - Delivery lead time, random demands

- Randomness everywhere!
  - Even if you have excess capacity at processing steps, there will be jobs waiting to be processed because of *variability* in both demands and processing times → Using the initial data of first 50 simulated days, try to get an idea of (1) the rates of demand arrivals and processing at each step and (2) impact of variability

- Think carefully about simulation end-game
  - "You will lose control of the factory and the final 100 simulated days will be run over a few minutes" → Make one-time decision for the last 100 days
  - Recall that the goal is to maximize the cash position at the end of the simulation, on Day 485

Next class

- Discussion on quality management
  - Read Ritz-Carlton case
  - Read Cachon & Terwiesch 10.0 – 10.5 (posted on Canvas)