MGT 422 Operations Engine

Lecture 7: Inventory Models I

2026 // Spring 1 // Core

Yale SCHOOL OF MANAGEMENT

Announcements

- Littlefield simulation game:
  - Simulation Website Registration – please register by Wednesday (Feb 11) at 5pm
    - Everyone needs to register!
  - Action Plan Writeup due at 5pm EST on Sunday, Feb 15
    - One page
    - Submit via Canvas, one submission for each team
  - Littlefield Simulation starts at 5pm EST on Sunday, Feb 15
- Assignment #3: Individual, due at the beginning of next class
- Syllabus update for late submissions:
  - Littlefield Action Plan (+ Assignment 1, Assignment 2): No submissions accepted after the deadline, since timely submission is essential for the class material/game to proceed.
  - Assignment 3, Assignment 4, Littlefield Summary Report: 20% late penalty applied for every 24-hour period after the deadline.

Inventory Models

Ludo Press

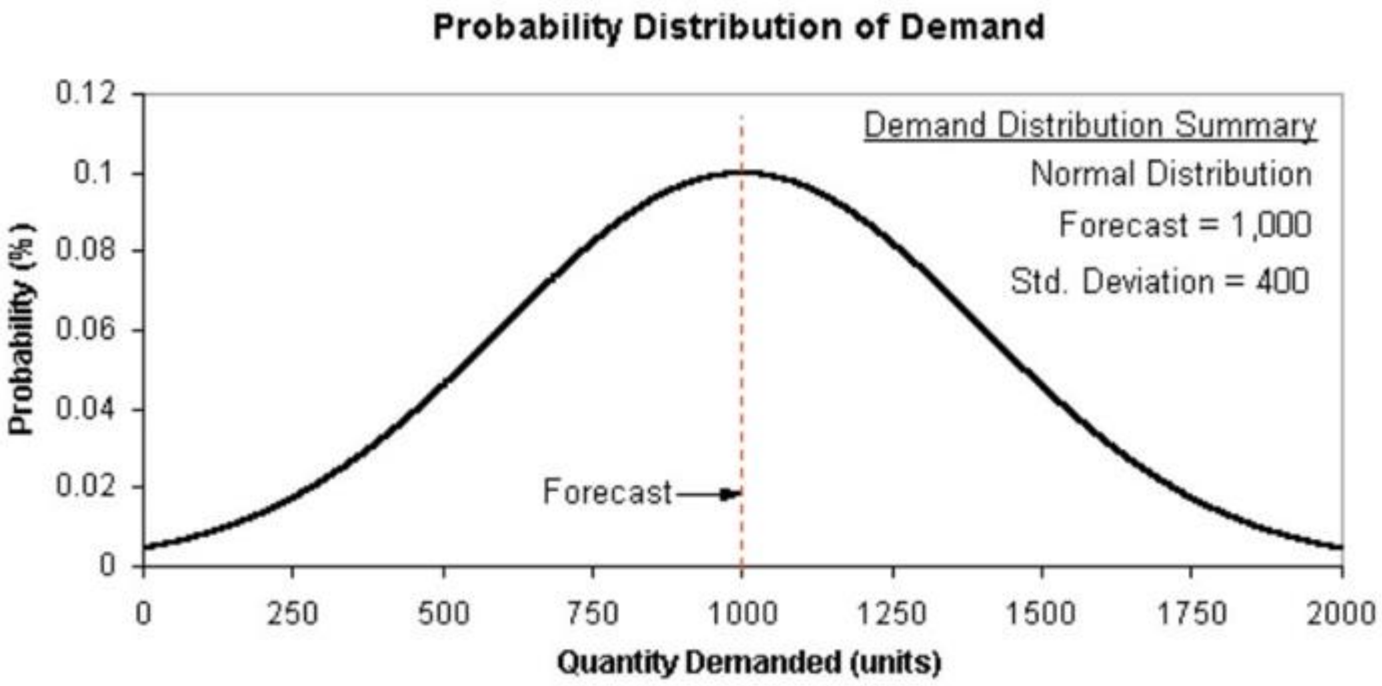
Quantity Decision for *Dado*

Historical data show that the average daily demand for *Dado* is 1000 units. How many units of *Dado* should be printed in advance?

(1) 1000
(2) Less than 1000
(3) More than 1000

Need more information about <u>demand</u> and <u>economics</u>

Dado demand and economics



- Selling price for each unit = €1.00
- Cost of acquiring each unit = €0.25
- All unsold units are discarded (zero salvage value; €0)

“Newsvendor problem”

Newsvendor Model: Betting on Quantity

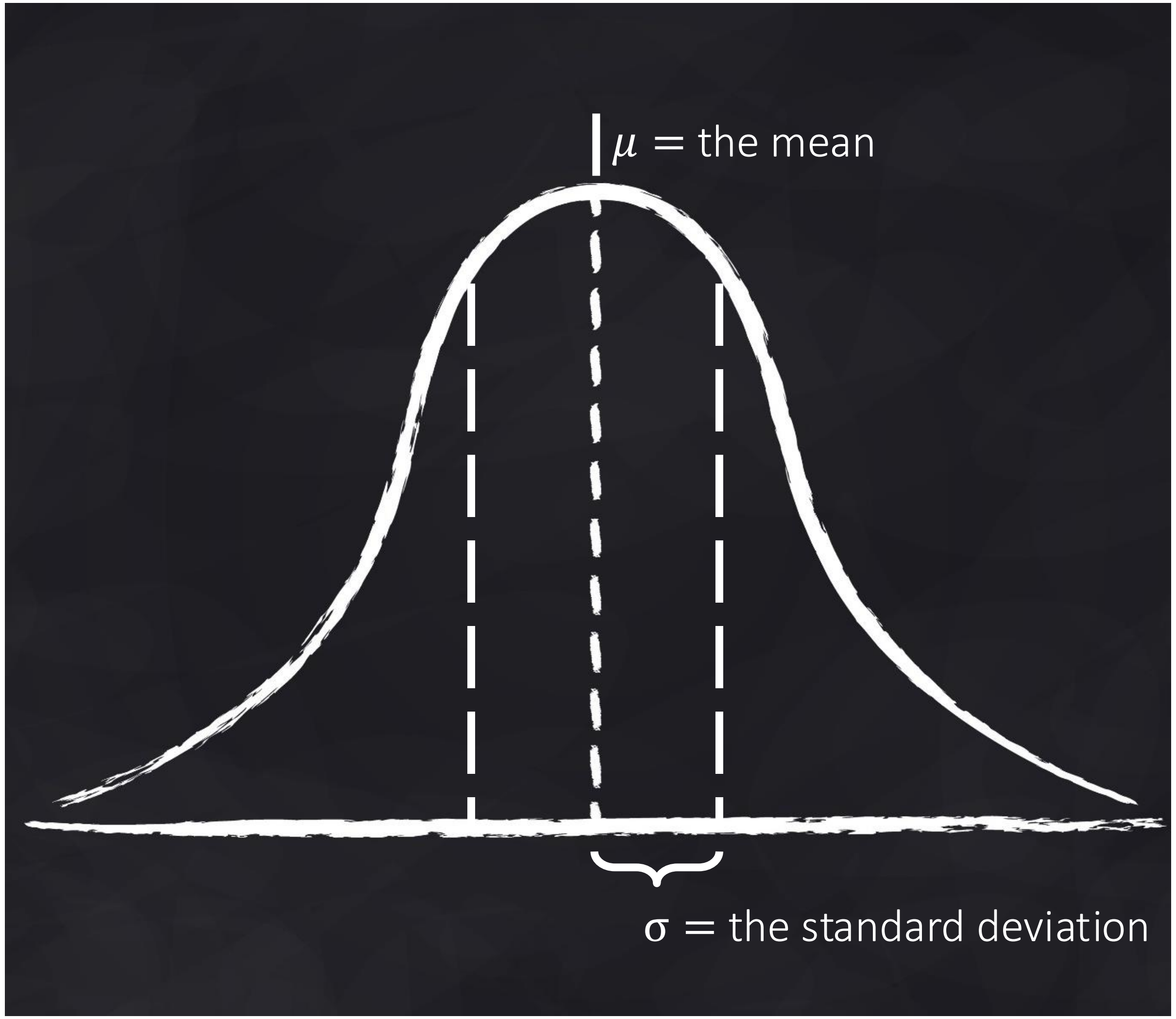
- Make a *single* bet/commitment against an uncertain future event, i.e., realization of a random demand
- Assumptions:
  - Uncertain demand
  - Single selling opportunity
  - Unit price and unit cost are fixed (no quantity discount)
  - Excess demand is lost
  - Excess supply can be sold at a "salvage" value
- Examples of random demand – commitment pair:
  - Demand for inventoried goods – Inventory
  - Number of guests at a party – Number of meals
  - Mobile data usage – phone plan data limit

Assumption: Random Demand is Normally Distributed

All normal distributions can be converted to the standard normal that has mean = 0 and standard deviation = 1

If Q is some quantity in an actual scale, then converting Q to the standard normal scale is the same as calculating the z-score:

$$z = \frac{Q - \mu}{\sigma}$$



We can use the "Standard Normal Distribution Table" to go from z-scores to probabilities.

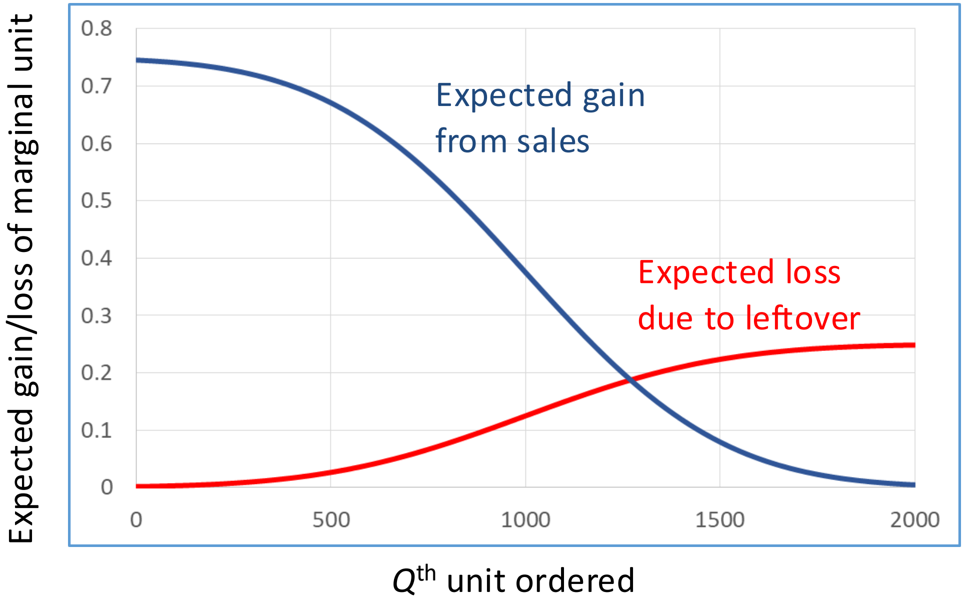
Newsvendor Model
Choosing a quantity that maximizes expected profit

The Cost of “Too Much” and “Too Little”

- C_o = **overage cost**
  - The consequence of ordering one more unit than what you would have ordered had you known the actual demand (over-commitment) → The extra unit will be left over; C_o is the cost of the unit, minus any salvage value you can get for it.
  - For *Dado*, C_o = ?
  - C_o = unit cost – unit salvage value = $c - v$ = 0.25 – 0 = **€0.25**

- C_u = **underage cost**
  - The consequence of ordering one fewer unit than what you would have ordered had you known the actual demand (under-commitment) → The extra demand will go unfulfilled (lost sale); C_u is equal to the margin you would earn on this additional sale.
  - For *Dado*, C_u = ?
  - C_u = unit price – unit cost = $p - c$ = 1.00 – 0.25 = **€0.75**

Balancing the risk and benefit of adding a unit

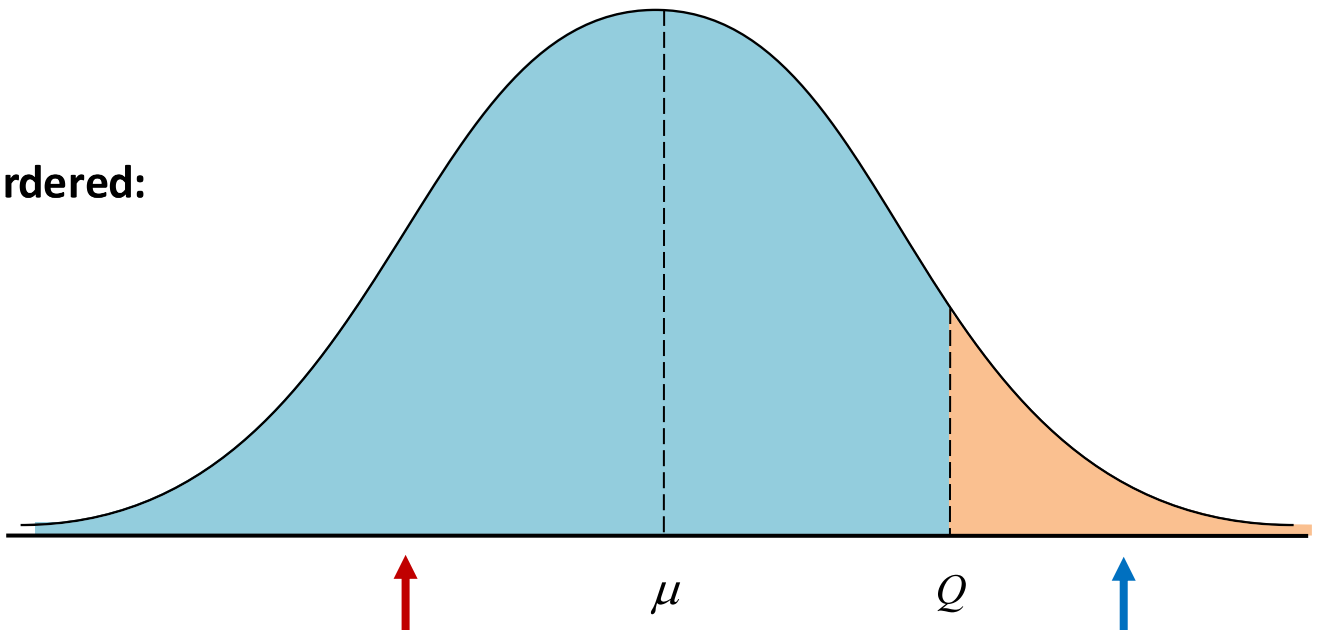


As more units are added, expected benefit from adding one unit to inventory decreases while expected loss of adding one more unit increases

- Adding one more unit to inventory increases the chance of overage:
  - **Expected loss on the Q^{th} unit** = $C_o \times F(Q)$
  - $\boldsymbol{F(Q)}$ **= Prob{*Demand* ≤ *Q*}**
- … but the benefit/gain of adding one more unit is the reduction in the chance of underage:
  - **Expected gain on the Q^{th} unit** = $C_u \times (1 - F(Q))$

Finding the Intersection Point

For the Qth unit ordered:



Expected loss on the Qth unit = $\boxed{C_O * F(Q)}$

Expected gain on the Qth unit = $\boxed{C_u * (1 - F(Q))}$

Quantity that maximizes expected profit

The optimal quantity balances the two: expected loss = expected gain

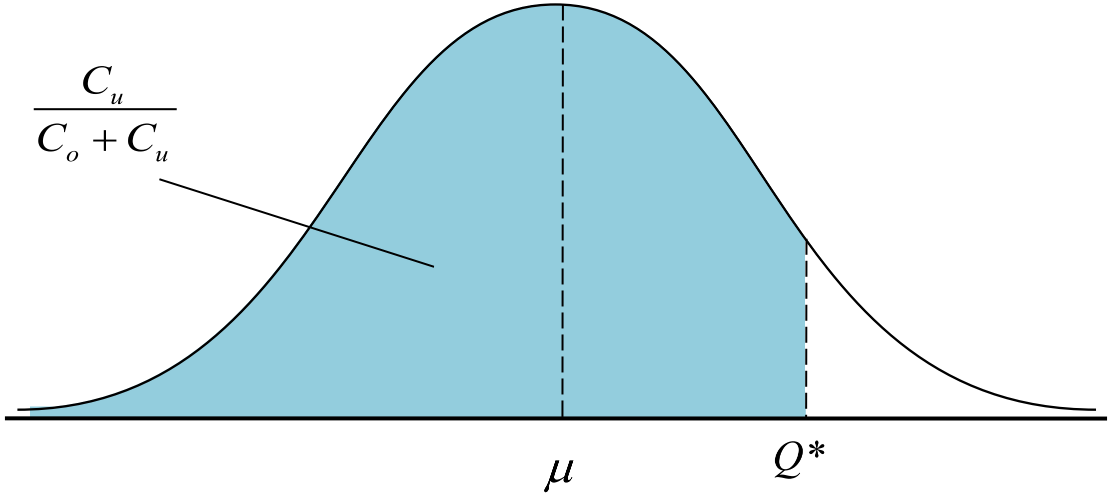
$$\boxed{C_o * F(Q)} = \boxed{C_u * (1 - F(Q))}$$

$$C_o * F(Q) = C_u - C_u * F(Q)$$

$$C_o * F(Q) + C_u * F(Q) = C_u$$

$$F(Q) * (C_o + C_u) = C_u$$

$$\boxed{\mathbf{F}(\boldsymbol{Q}^*) = \frac{\boldsymbol{C_u}}{\boldsymbol{C_o + C_u}}}$$



The right hand-side is called the **critical ratio**. And the Q corresponding to this ratio is the quantity that maximizes expected profit (denoted Q*).

Calculating the Critical Ratio for *Dado*

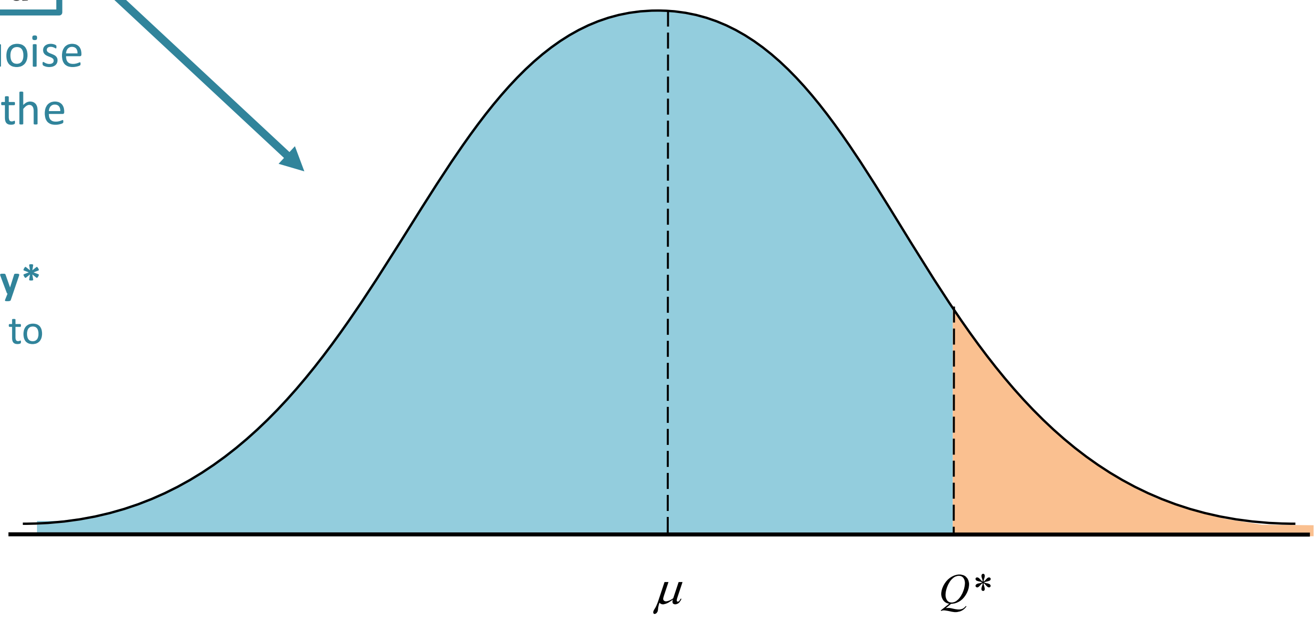
$$\mathbf{F}(\boldsymbol{Q}^{*}) = \frac{\mathrm{C_u}}{\mathrm{C_o} + \mathrm{C_u}} = \frac{0.75}{0.25 + 0.75} = 0.75$$

The Critical Ratio

$$\boldsymbol{F(Q^*)} = \frac{C_u}{C_o + C_u}$$

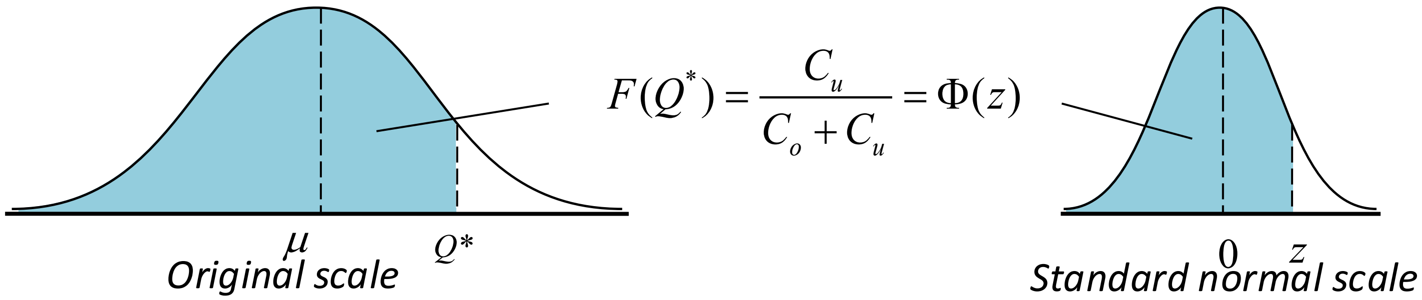
F(Q*) is the turquoise shaded region to the left of Q*

This is a ***probability***
Using this, we need to find Q*.



Evaluating profit-maximizing quantity: Step-by-step procedure

1. Determine the underage and overage costs
   - C_o = unit overage cost = unit cost – unit salvage value
   - C_u = unit underage cost = unit price – unit cost
2. Evaluate the critical ratio: $\frac{C_u}{C_o+C_u}$ now you have your F(Q*)
3. If the demand forecast follows a normal distribution with mean μ and standard deviation σ,
   - In the *Standard Normal Distribution Function Table*, find the value of z ("z-statistic") that corresponds to the value given by the critical ratio
   - Convert the z-statistic to obtain optimal quantity: $Q^* = \mu + z \times \sigma$



Production quantity for *Dado* that maximizes exp. profit

1. Determine underage and overage costs

2. Calculate critical ratio

3. Use the *Standard Normal Distribution Table*

Use the *Standard Normal Distribution Function* Table

Critical Ratio = 0.75

Standard Normal Distribution Function $\Phi(z)$
(Download from Canvas Files > Cases and Readings)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
…	…	…	…	…	…	…	…	…	…	…

0.75 lies between two entries!

Round-Up Rule: Use the convention of rounding up to *higher z* → Choose 0.7517

Corresponding *z*-statistic = 0.68 (= 0.6 + 0.08)

Finally: $Q^* = \mu + z \times \sigma = 1000 + 0.68 \times 400 =$ **1272** (more precise answer = 1269.68)

Nonzero salvage value

Suppose that a local recycler is willing to pay salvage value of €0.05 for each unit of *Dado* left over at the end of each day. How would this change impact the quantity decision?

- No change in demand forecast: normal with $\mu = 1000$ and $\sigma = 400$
- No change in unit price and unit cost: p = €1.00, c = €0.25

Will the optimal quantity increase or decrease?

- C_u = ?
- C_o = ?

Nonzero salvage value

Suppose that a local recycler is willing to pay salvage value of €0.05 for each unit of *Dado* left over at the end of each day. How would this change impact the quantity decision?

- No change in demand forecast: normal with μ = 1000 and σ = 400
- No change in unit price and unit cost: p = €1.00, c = €0.25

Will the optimal quantity increase or decrease?

- C_u = unit price – unit cost = p – c = €1.00 - €0.25 = €0.75
- C_o = unit cost – salvage value = c – v = €0.25 **- €0.05** = **€0.20** (smaller than before)

Updated Critical Ratio = $\frac{0.75}{0.20+0.75}$ = ***0.7895*** (larger than before)

Use the *Standard Normal Distribution Function* Table

Critical Ratio = 0.7895

Standard Normal Distribution Function $\Phi(z)$
(Download from Canvas Files > Cases and Readings)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
...	...	...	...	...	...	...	...	...	...	...

Corresponding z-statistic = 0.81

Finally: $Q^* = \mu + z \times \sigma = 1000 + 0.81 \times 400 =$ **1324** (52 more units than before)

Newsvendor Model: Choosing quantity that satisfies a predetermined service target

A measure of service: In-stock probability

- $\boxed{\textbf{In-stock probability} = \text{Prob}(\text{Demand} \leq Q) = F(Q)}$
  - Probability that all demands are satisfied with produced quantity

- $\boxed{\textbf{Stockout probability} = 1 - \text{In-stock probability}}$

  $= \text{Prob}(\text{Demand} > Q) = 1 - F(Q)$

Quantity commitment to satisfy in-stock probability target

- Suppose that Ludo Press prints *Dado* each day to satisfy 95% in-stock probability target
  - Demand forecast: Normally distributed with $\mu = 1000$ and $\sigma = 400$
- This is not a profit maximization problem; **computing critical ratio does not apply**. We start with the target service level, then infer the quantity that satisfies that level.
- Step 1:
  - Find the *z*-statistic that yields the target in-stock probability of 0.95
  - In the *Standard Normal Distribution Function Table* we find $\Phi(1.64) = 0.9495$ and $\Phi(1.65) = 0.9505$ (see next page)
  - Choose $z = 1.65$ (round-up rule)
- Step 2:
  - Convert the *z*-statistic into an order quantity for the actual demand distribution
  - $Q = \mu + z \times \sigma = 1000 + 1.65 \times 400 =$ **1660**

Finding *z* that corresponds to in-stock probability 0.95

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
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0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
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0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	$\boxed{0.9505}$	0.9515	0.9525	0.9535	0.9545

z = 1.65

Newsvendor model summary

- The model can be applied to settings in which…
  - There is a single order/production/replenishment opportunity
  - Demand is uncertain
  - There is a "too much-too little" challenge:
    - If demand exceeds the order quantity, sales are lost
    - If demand is less than the order quantity, there is leftover inventory

- At the order quantity that maximizes expected profit, the probability that demand is less than the order quantity equals the critical ratio:
  - The expected profit maximizing order quantity balances the "too much-too little" costs

- Service target approach may be more appropriate if a short-term profitability does not capture a value proposition

Next Class

- Please submit HW#3 before the start of next class
- Please also make sure you've registered your team for Littlefield!