

**CS221: Digital Design**

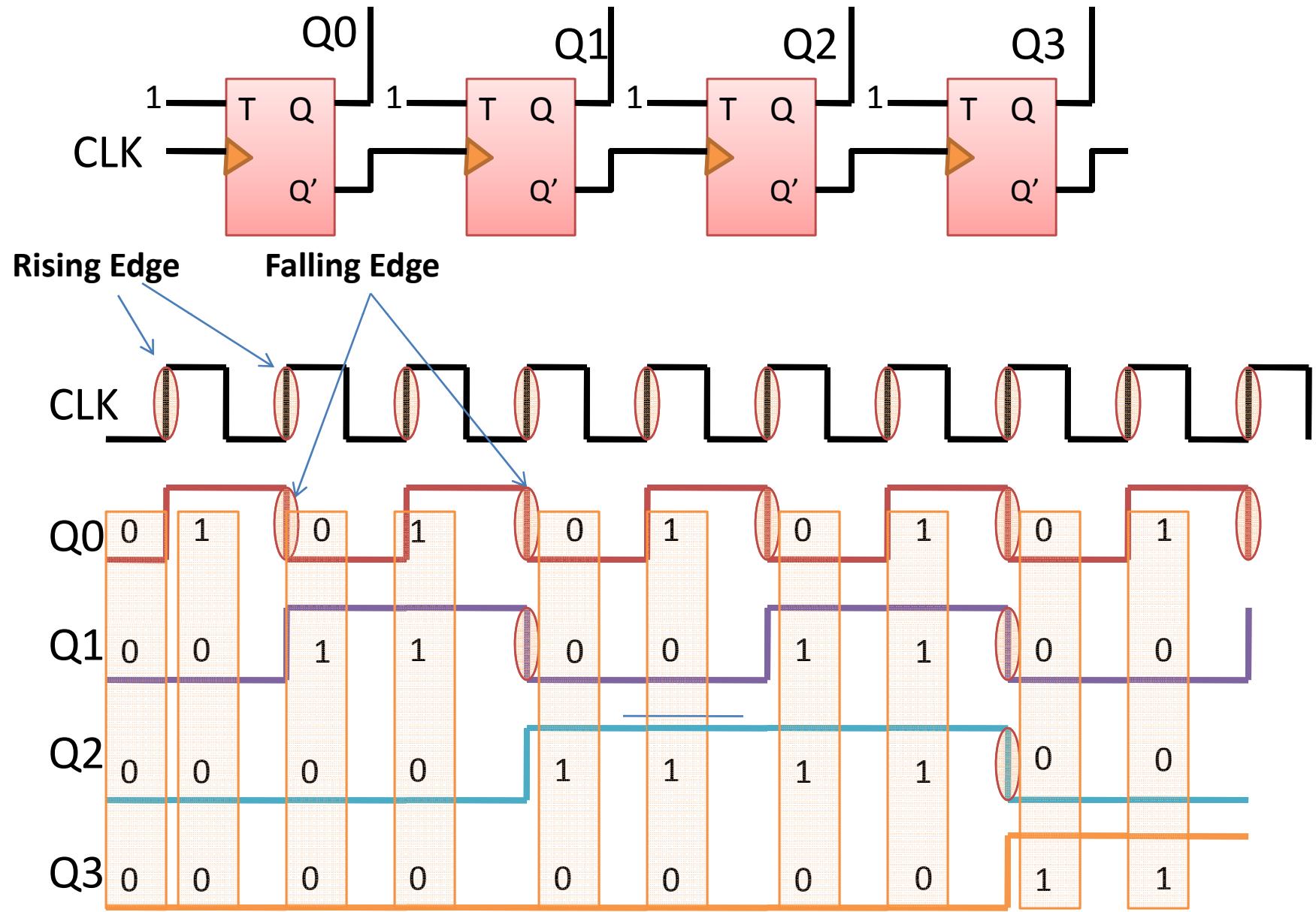
# **Counter & Registers**

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# Outline

- Counter : Synchronous Vs Asynchronous
- Counter: Finite State Machine
  - A register and Combinational logic
- Counter & FSM Controller:
  - Using JK FFs, RS FF and T FFs
- Up/Down Counter, Clear, Load Counter
- Registers
  - Preset/Clear, Load, Left/Right Shift
- Counter Based on Shift Registers

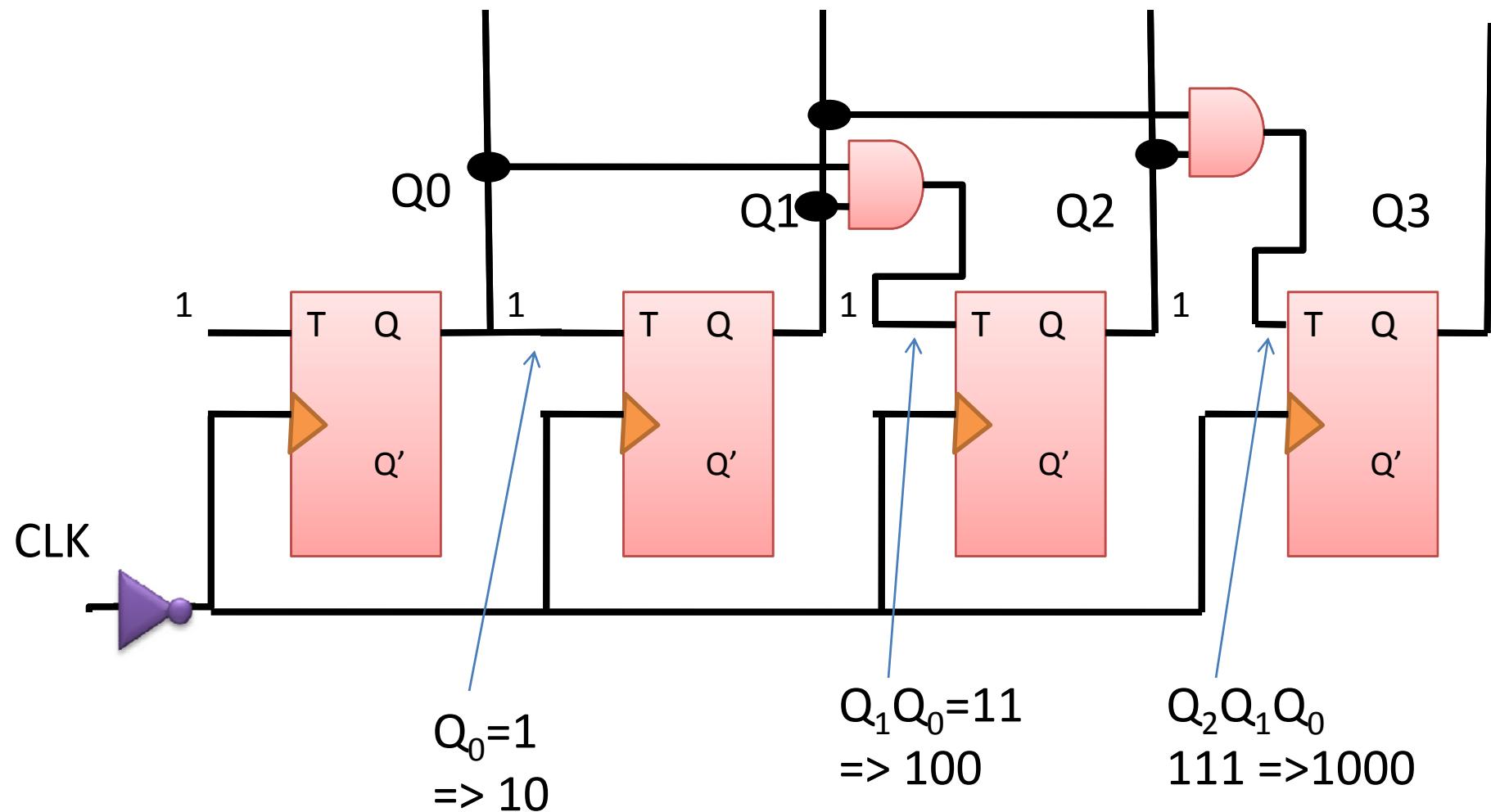
# Binary Counter: Asynchronous



# Counter : Sync Vs Async

- **Asynchronous Counter : Ripple Counter**
  - Change in State of  $Q_{i-1}$  is used to Toggle  $Q_i$
  - Clock is applied at FF0, it propagate through to FF $n$
  - Input Clock to FF1= Skewed version of Clk of FF0
    - Clock + Propagation delay of FF
  - Rippling : Overall time delay of occurrence of count pulse and when stabilized count appear at O/P
  - When counter : 1111..11 to 0000..00, toggle signal must propagate through all FFs
  - **Wrost case Settling time:  $n \times t_{pd}$**   
where  $t_{pd}$  =Propagation delay of a FF
- **Synchronous Counter:** One single clock to all FF

# Binary Counter: Synchronous

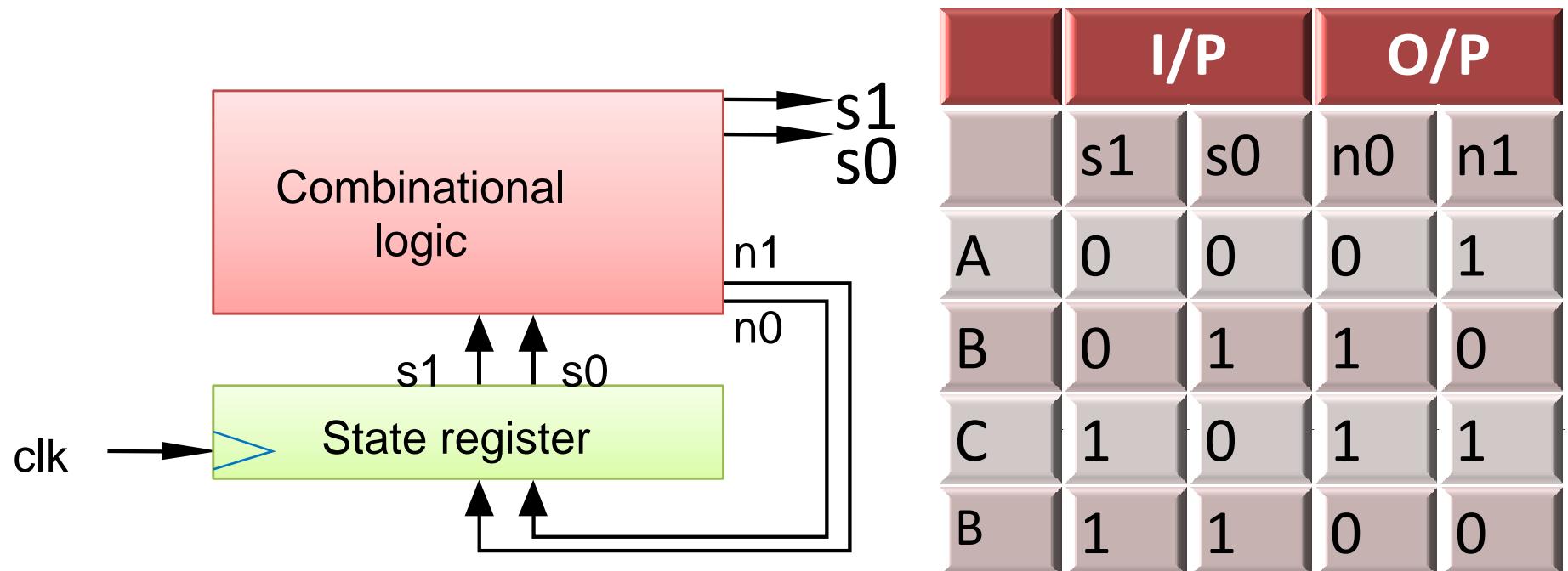
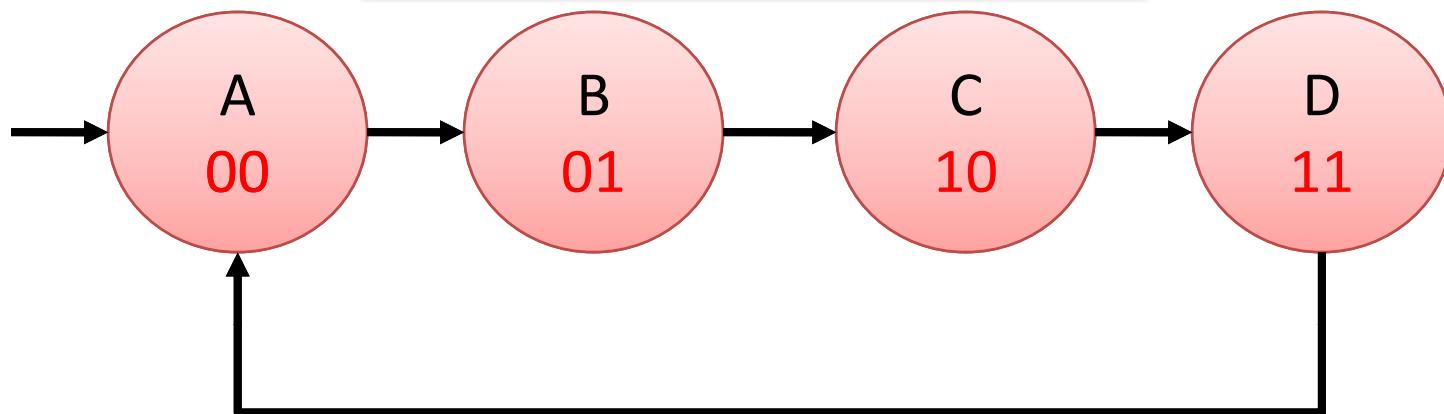


# Counter

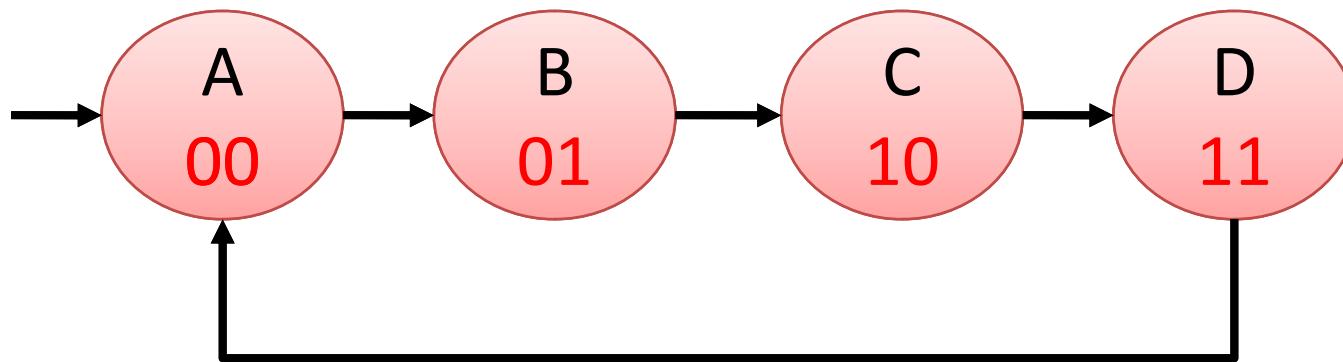
- Example of 4 bit counter
- Count from 0000 to 1111 and repeat
- Up counter : 0000 to 1111
- Down counter : 1111 to 0000
- Mod N counter:
  - Mod 10 counter : 0000 to 1001 (0 to 9) and repeat
  - Mod 6 counter : 000 to 101 (0 to 5) and repeat

# FSM of Counter : 2 bit

**State bits = Output bits**

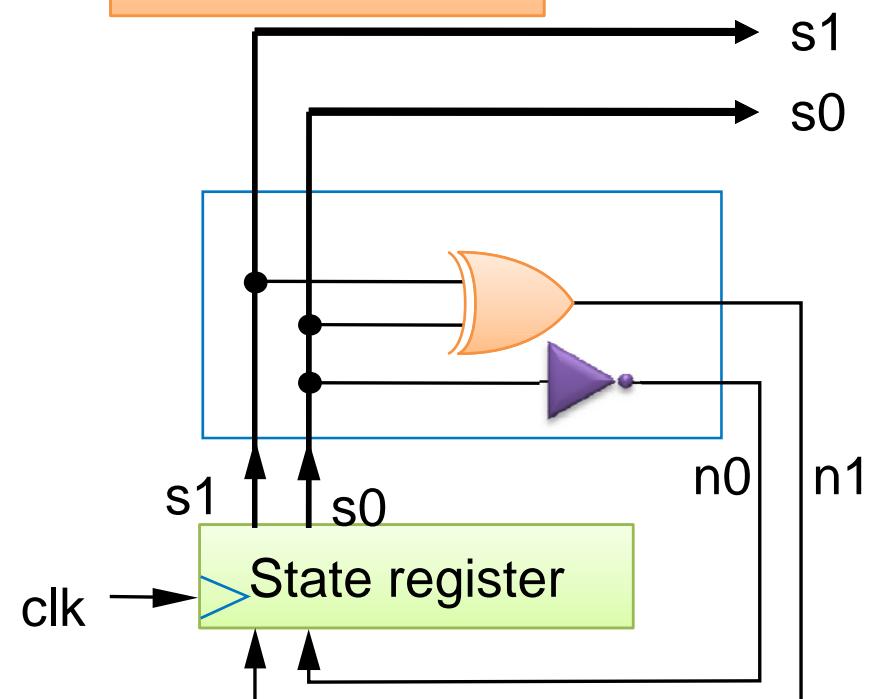


# FSM Controller: Binary Counter



$$\begin{aligned} n1 &= s1 \text{ xor } s0 \\ n0 &= s0' \end{aligned}$$

	I/P	O/P		
	s1	s0	n0	n1
A	0	0	0	1
B	0	1	1	0
C	1	0	1	1
D	1	1	0	0

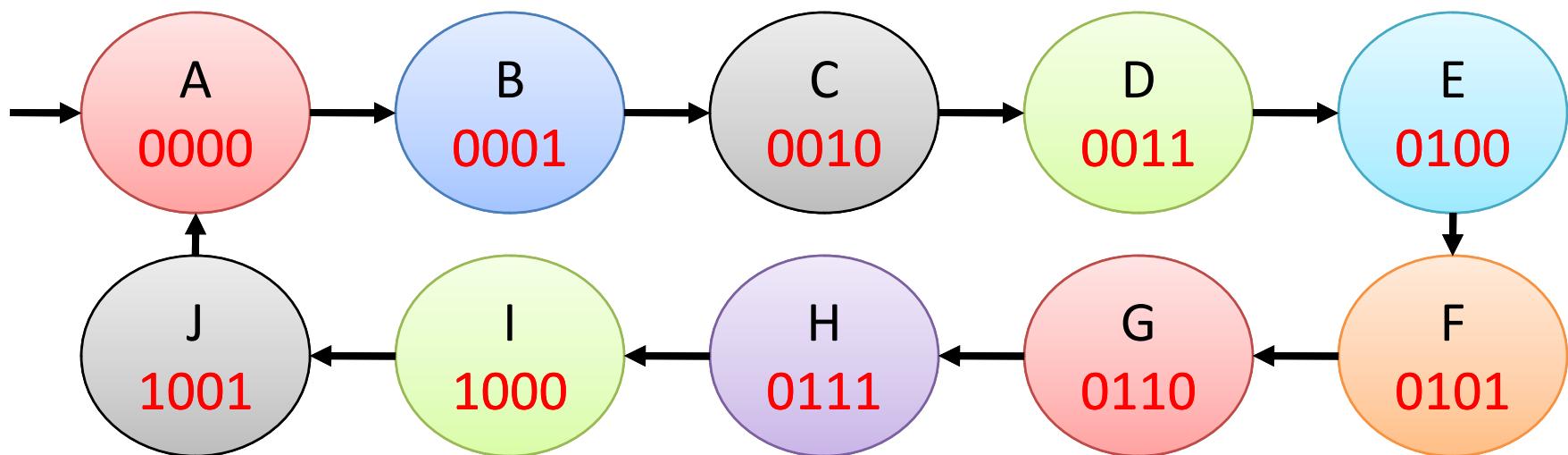


# Synchronous Counter : Design

- Together: Create FSM, Encode Bit
- State Table
- Design Combination Circuit

# Mod 10 Counter: BCD Counter

- Count from 0000 to 1001 (0-9 the Reset)
- FSM with Encoding **done**



# State Table Creation

Present State				Next State			
S3	S2	S1	S0	N3	N2	N1	N0
0	0	0	0	0	0	0	1
0	0	0	1	0	0	1	0
0	0	1	0	0	0	1	1
0	0	1	1	0	1	0	0
0	1	0	0	0	1	0	1
0	1	0	1	0	1	1	0
0	1	1	0	0	1	1	1
0	1	1	1	1	0	0	0
1	0	0	0	1	0	0	1
1	0	0	1	0	0	0	0

$$N0 = S0'$$

$$N1 =$$

$$N3 =$$

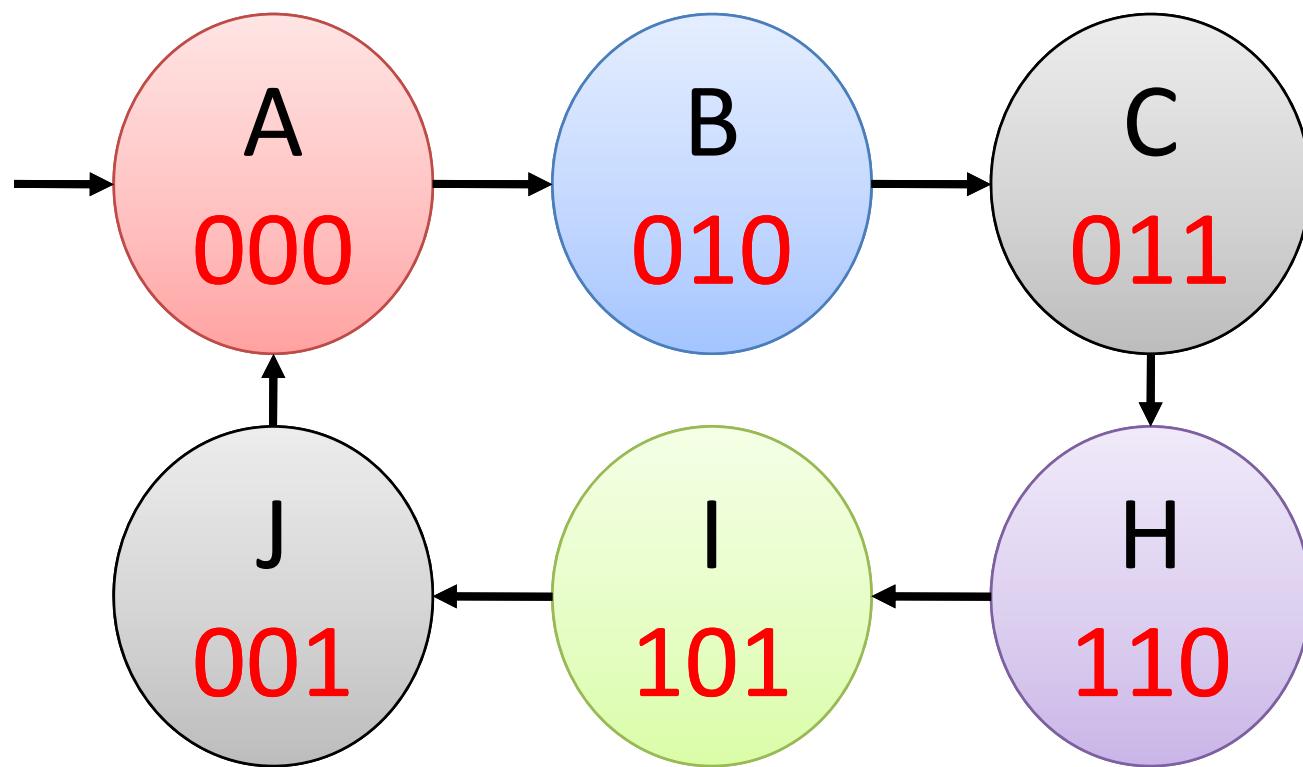
$$N4 =$$

## Using other FF in Counter

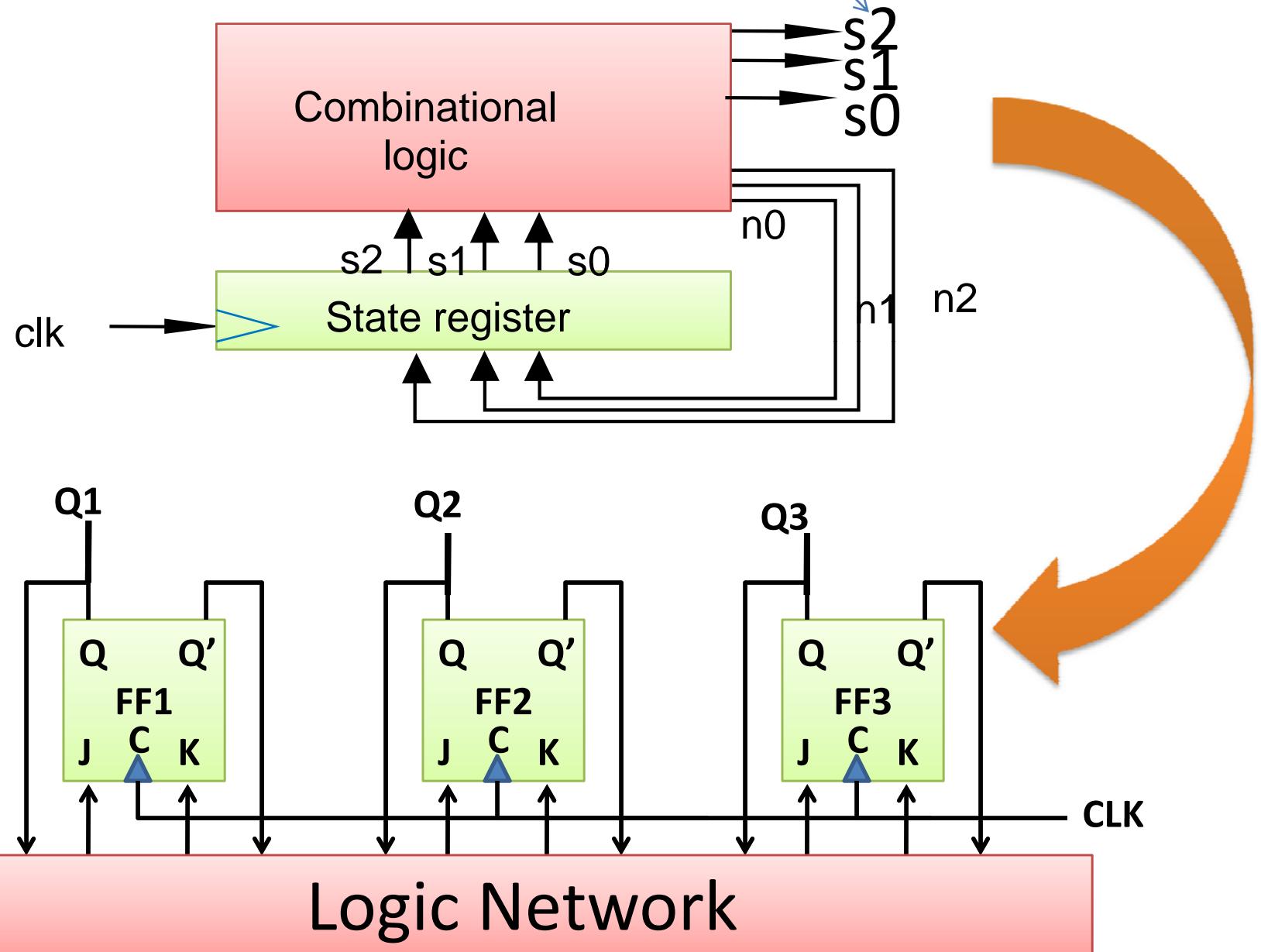
- Takes benefit of dual inputs to FF
- Counter can be implemented using Small Combinational Circuit
- More Inputs from Combinational Circuit
- Use of Excitation Table
  - How FF out changes from one to others
  - Required FF inputs to change FF output
  - 0 to 0, 0 to 1, 1 to 0 and 1 to 1

# Design of Counter: With FFs

0,2,3,4,5,1,0...



# Counter design using JK FF



# 4 Types of Flip-Flops

S	R	Q+
0	0	Qt
0	1	0
1	0	1
1	1	U



D	Q+
0	0
1	1

J	K	Q+
0	0	Qt
0	1	0
1	0	1
1	1	Qt'



T	Q+
0	Qt
1	Qt'

# Characteristic Equations

- A descriptions of the next-state table of a flip-flop
- Constructing from the Karnaugh map for  $Q_{t+1}$  in terms of the present state and input

# Characteristic tables

- The tables that we've made so far are called **characteristic tables**.
  - They show the next state  $Q(t+1)$  in terms of the current state  $Q(t)$  and the inputs.
  - For simplicity, the control input  $C$  is not usually listed.
  - Again, these tables don't indicate the positive edge-triggered behavior of the flip-flops that we'll be using.

J	K	$Q+$
0	0	$Q_t$
0	1	0
1	0	1
1	1	$Q_t'$

D	$Q+$
0	0
1	1

T	$Q+$
0	$Q_t$
1	$Q_t'$

# Characteristic equations

- We can also write characteristic equations, where the next state  $Q(t+1)$  is defined in terms of the current state  $Q(t)$  and inputs.

J	K	$Q^+$
0	0	$Q_t$
0	1	0
1	0	1
1	1	$Q_{t'}$

$$Q^+ = K'Q + JQ'$$

$$Q(t+1) = K'Q(t) + JQ'(t)$$

# Characteristic equations

- We can also write **characteristic equations**, where the next state  $Q(t+1)$  is defined in terms of the current state  $Q(t)$  and inputs.

D	$Q^+$
0	0
1	1

$$Q^+ = D$$

$$Q(t+1) = D$$

T	$Q^+$
0	$Q_t$
1	$Q_t'$

$$Q^+ = T'Q + TQ' = T \oplus Q$$

$$\begin{aligned}Q(t+1) &= T'Q(t) + TQ'(t) \\&= T \oplus Q(t)\end{aligned}$$

# Characteristic equations

		SR			
		00	01	11	10
Q		0	0	-	1
		1	1	-	1

$$Q^+ = S + R'Q \quad (SR=0)$$

Flip Flop Type	Characteristic Equation
SR	$Q^+ = S + R'Q \quad (SR=0)$
JK	$Q^+ = JQ' + K'Q$
D	$Q^+ = D$
T	$Q^+ = TQ' + T'Q = T \oplus Q$

# Excitation Table for JK Flip Flop

- It is different than Characteristic Equation
- Tabling requires **Input** to change from **Q** to **Q<sup>+</sup>**

J	K	Q <sup>+</sup>
0	0	Qt
0	1	0
1	0	1
1	1	Qt'

Characteristic Table



Q	Q <sup>+</sup>	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

Excitation Table

# Excitation Table for T Flip Flop

- It is different than Characteristic Equation
- Tabling requires **Input** to change from  $Q$  to  $Q^+$

T	$Q^+$
0	$Q_t$
1	$Q_t'$



Q	$Q^+$	T
0	0	0
0	1	1
1	0	1
1	1	0

Characteristic Table

Excitation Table

# Excitation Table for D Flip Flop

- It is different than Characteristic Equation
- Tabling requires **Input** to change from **Q** to **Q<sup>+</sup>**

D	Q <sup>+</sup>
0	0
1	1



Q	Q <sup>+</sup>	D
0	0	0
0	1	1
1	0	0
1	1	1

Characteristic Table

Excitation Table

# Excitation Table for SR Flip Flop

- It is different than Characteristic Equation
- Tabling requires **Input** to change from **Q** to **Q<sup>+</sup>**

S	R	Q <sup>+</sup>
0	0	Qt
0	1	0
1	0	1
1	1	U

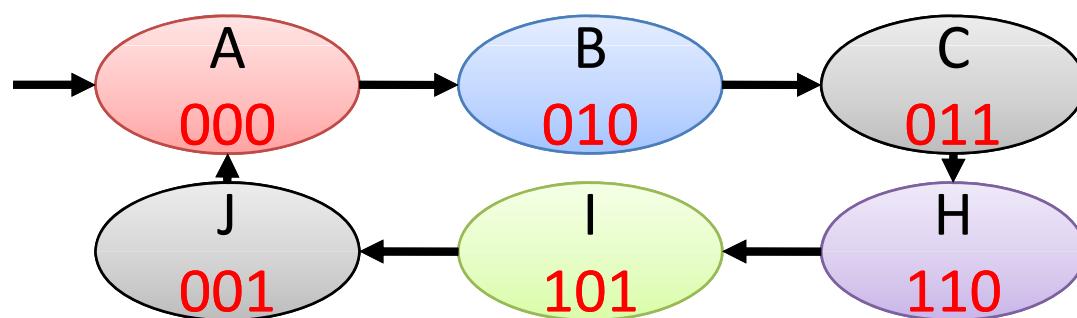
Characteristic Table



Q	Q <sup>+</sup>	S	R
0	0	0	X
0	1	1	0
1	0	0	1
1	1	X	0

Excitation Table

# Excitation Table: Sync Counter Using JK FF



Q	$Q^+$	J	K
0	0	0	x
0	1	1	x
1	0	x	1
1	1	x	0

Present State			Next State			Flip Flop Inputs					
Q1	Q2	Q3	$Q1^+$	$Q2^+$	$Q3^+$	J1	K1	J2	K2	J3	K3
0	0	0	0	1	0	0	x	1	x	0	x
0	1	0	0	1	1	0	x	x	0	1	x
0	1	1	1	1	0	1	x	x	0	x	1
1	1	0	1	0	1	x	0	x	1	1	x
1	0	1	0	0	1	x	1	0	x	x	0
0	0	1	0	0	0	0	x	0	x	x	1

# FF Input Functions

Present State			Flip Flop Inputs					
Q1	Q2	Q3	J1	K1	J2	K2	J3	K3
0	0	0	0	X	1	X	0	X
0	1	0	0	X	X	0	1	X
0	1	1	1	X	X	0	X	1
1	1	0	X	0	X	1	1	X
1	0	1	X	1	0	X	X	0
0	0	1	0	X	0	X	X	1

**J1= F (Q1,Q2,Q3)**

**J2= F (Q1,Q2,Q3)**

**J3= F (Q1, Q2,Q3)**

**K1= F (Q1,Q2,Q3)**

**K2= F (Q1, Q2,Q3)**

**K3= F (Q1, Q2,Q3)**

# Solve Each Function Using KMAP

Present State			Flip Flop Inputs					
Q1	Q2	Q3	J1	K1	J2	K2	J3	K3
0	0	0	0	X	1	X	0	X
0	1	0	0	X	X	0	1	X
0	1	1	1	X	X	0	X	1
1	1	0	X	0	X	1	1	X
1	0	1	X	1	0	X	X	0
0	0	1	0	X	0	X	X	1

$\overrightarrow{Q_2'Q_3'}$

$Q_1'$	0	0	1	0
$Q_1$	X	X	X	X

$J_1 = Q_2 Q_3$

$J_2 = Q_3'$

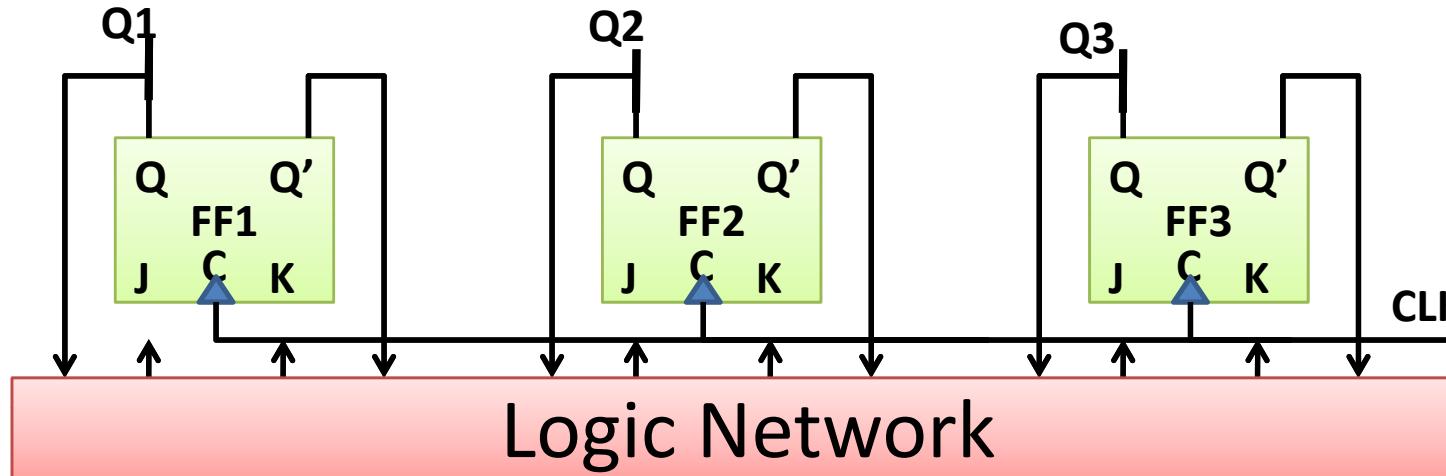
$J_3 = Q_2$

$K_1 = Q_2'$

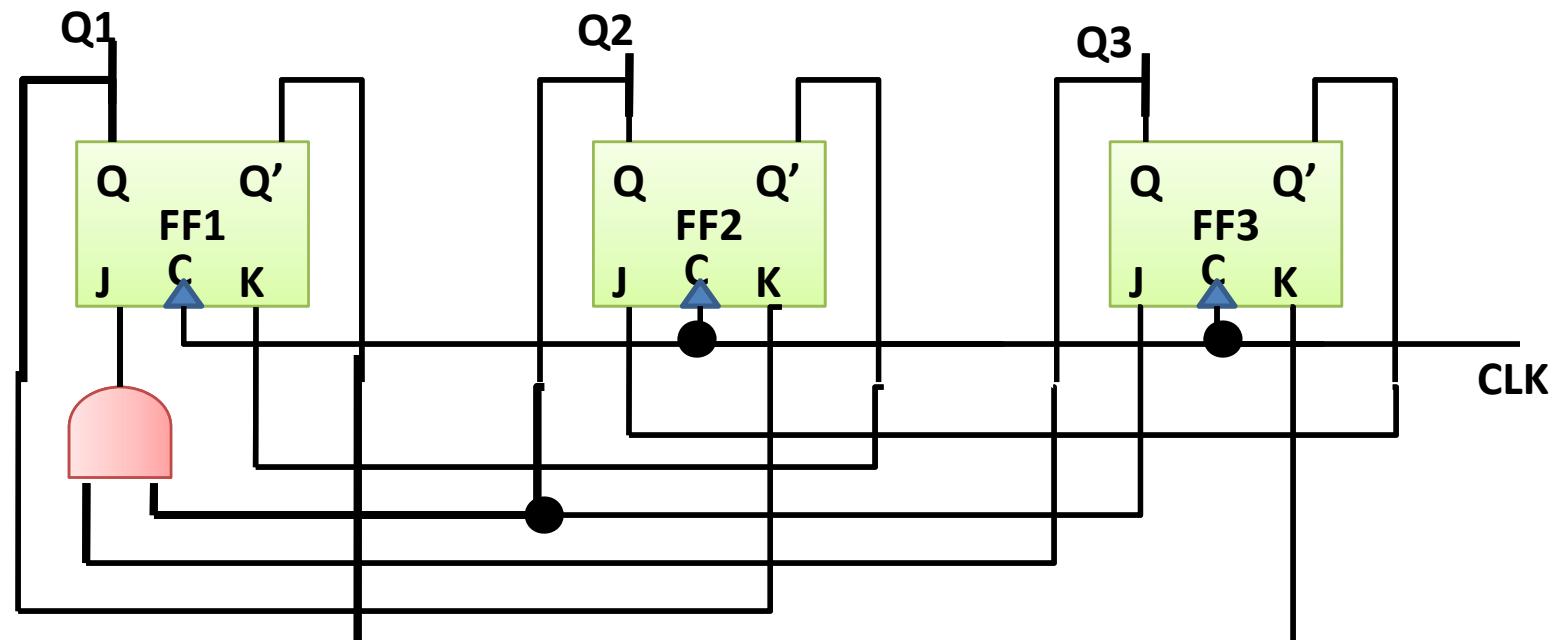
$K_2 = Q_1$

$K_3 = Q_1'$

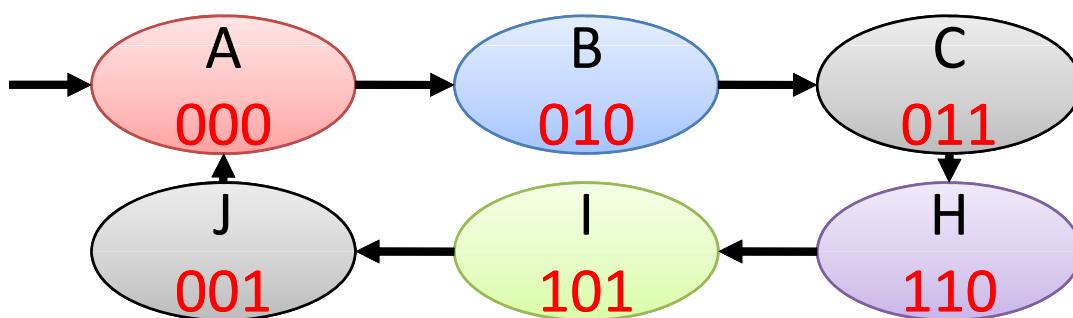
# Counter Logic Diagram



$J_1 = Q_2 Q_3$   
 $K_1 = Q_2'$   
 $J_2 = Q_3'$   
 $K_2 = Q_1$   
 $J_3 = Q_2$   
 $K_3 = Q_1'$



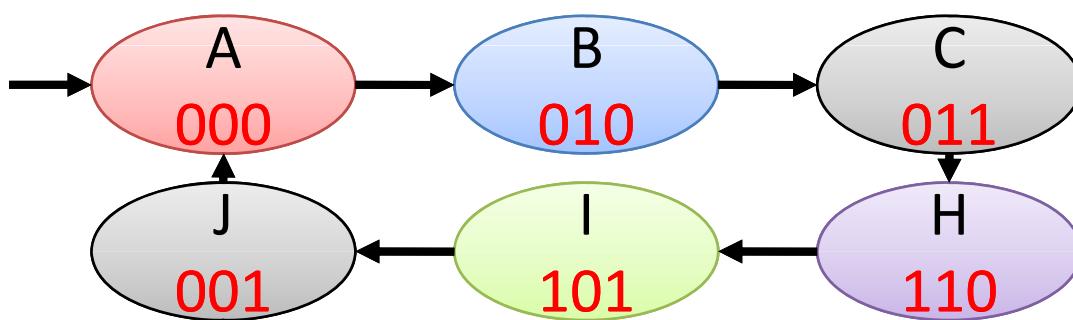
# Excitation Table: Sync Counter Using D FF



Q	$Q^+$	D
0	0	0
0	1	1
1	0	0
1	1	1

Present State			Next State			Flip Flop Inputs		
Q1	Q2	Q3	$Q1^+$	$Q2^+$	$Q3^+$	D1	D2	D3
0	0	0	0	1	0	0	1	0
0	1	0	0	1	1	0	1	1
0	1	1	1	1	0	1	1	0
1	1	0	1	0	1	1	0	1
1	0	1	0	0	1	0	0	1
0	0	1	0	0	0	0	0	0

# Excitation Table: Sync Counter Using D FF



Q	$Q^+$	D
0	0	0
0	1	1
1	0	0
1	1	1

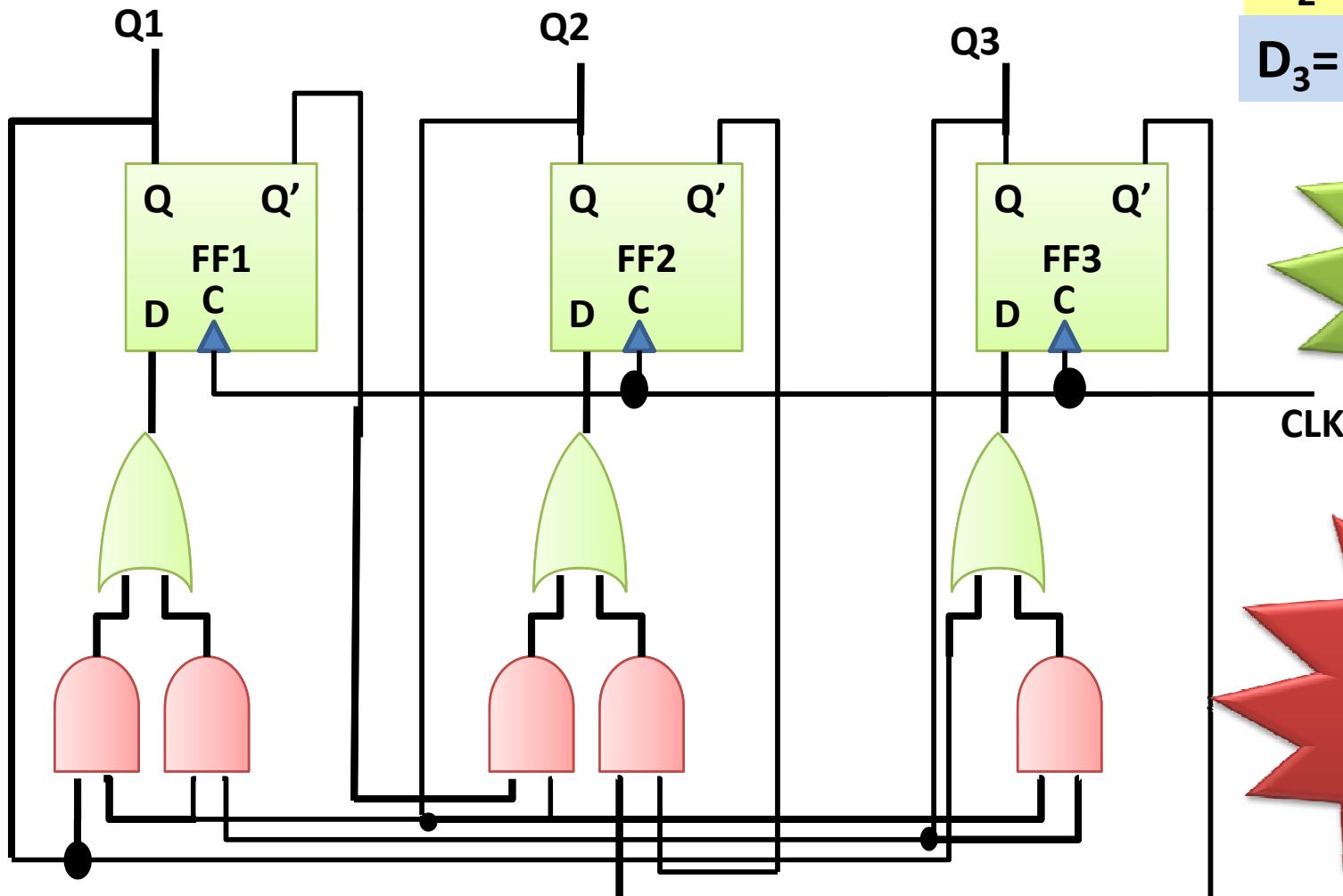
Present State			Flip Flop Inputs		
Q1	Q2	Q3	D1	D2	D3
0	0	0	0	1	0
0	1	0	0	1	1
0	1	1	1	1	0
1	1	0	1	0	1
1	0	1	0	0	1
0	0	1	0	0	0

$$D_1 = Q_1 Q_2 + Q_2 Q_3$$

$$D_2 = Q_1' Q_2 + Q_2' Q_3'$$

$$D_3 = Q_1 + Q_2 Q_3$$

# Counter Implementation using D FF



$$D_1 = Q_1 Q_2 + Q_2 Q_3$$

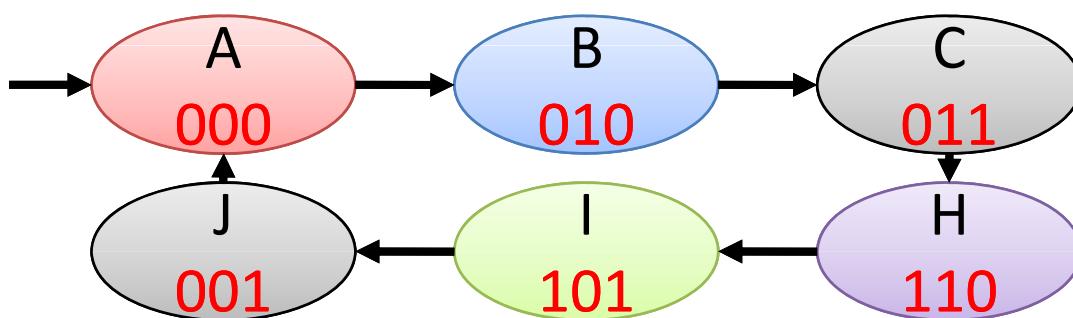
$$D_2 = Q_1' Q_2 + Q_2' Q_3'$$

$$D_3 = Q_1 + Q_2 Q_3$$

Same as  
FSM  
Controller

Require  
more Logic  
as JK FF  
Based

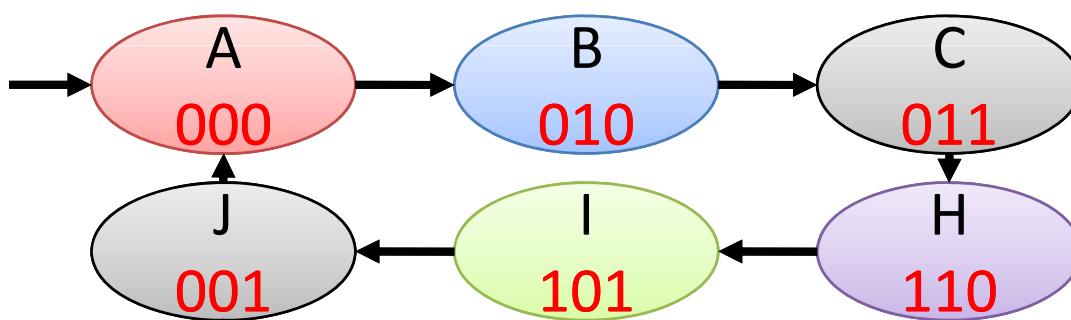
# Excitation Table: Sync Counter Using T FF



Q	$Q^+$	T
0	0	0
0	1	1
1	0	1
1	1	0

Present State			Next State			Flip Flop Inputs		
Q1	Q2	Q3	$Q1^+$	$Q2^+$	$Q3^+$	T1	T2	T3
0	0	0	0	1	0	0	1	0
0	1	0	0	1	1	0	0	1
0	1	1	1	1	0	1	0	1
1	1	0	1	0	1	0	1	1
1	0	1	0	0	1	1	0	0
0	0	1	0	0	0	0	0	1

# Excitation Table: Sync Counter Using D FF



Q	$Q^+$	T
0	0	0
0	1	1
1	0	0
1	1	1

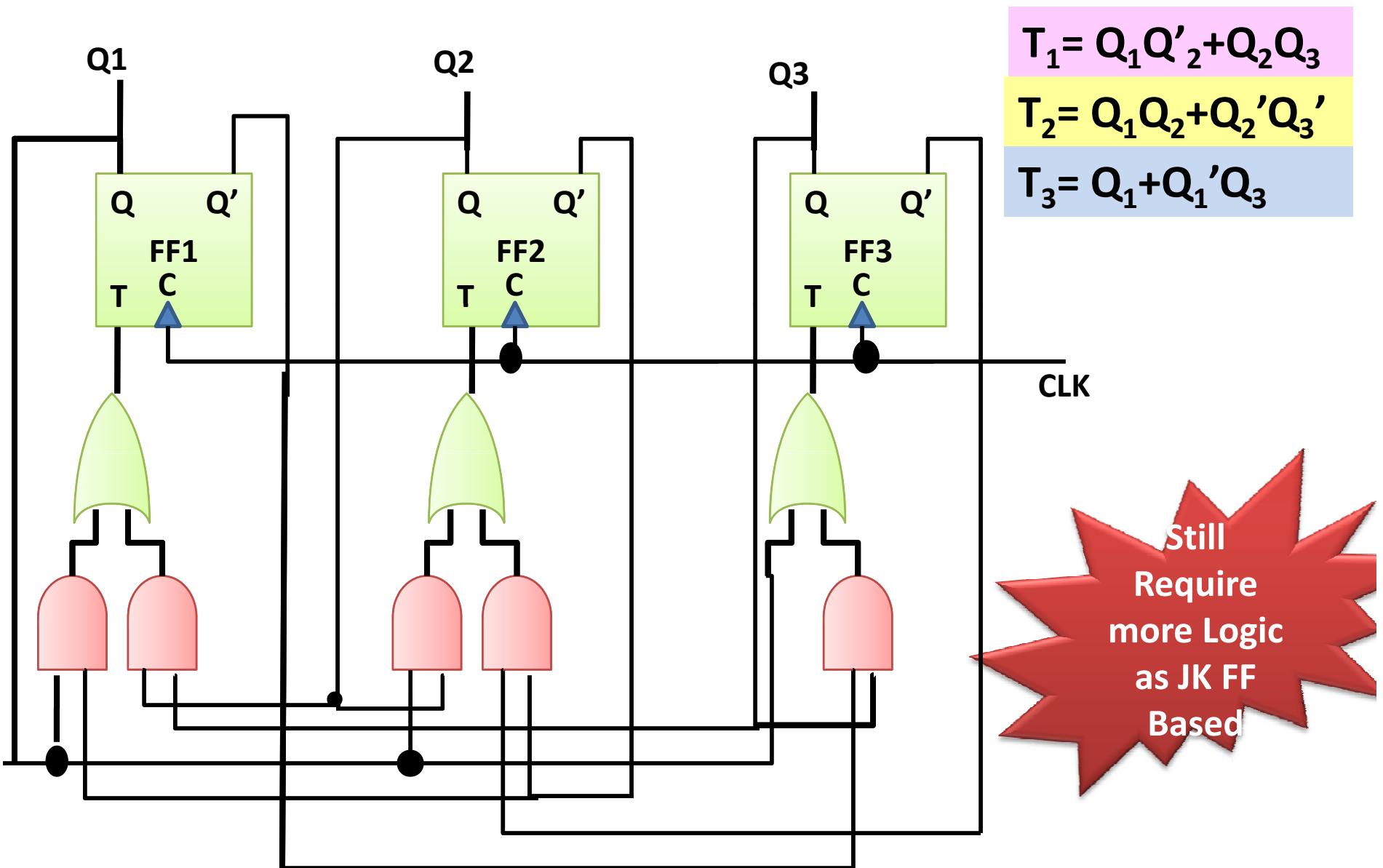
Present State			Flip Flop Inputs		
Q1	Q2	Q3	T1	T2	T3
0	0	0	0	1	0
0	1	0	0	0	1
0	1	1	1	0	1
1	1	0	0	1	1
1	0	1	1	0	0
0	0	1	0	0	1

$$T_1 = Q_1 Q'_2 + Q_2 Q_3$$

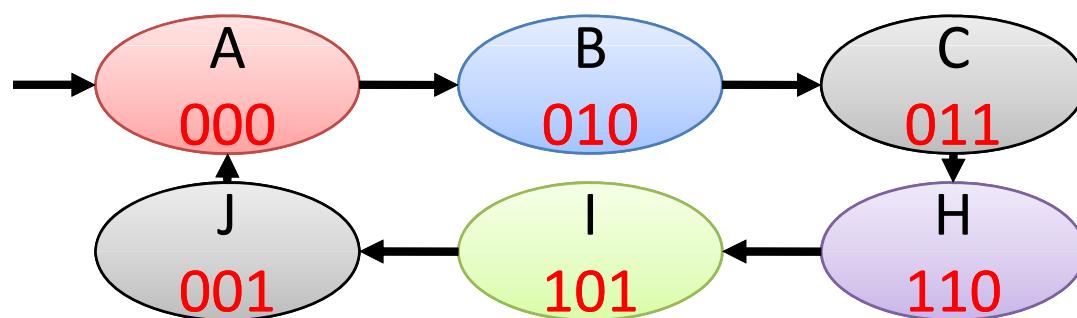
$$T_2 = Q_1 Q_2 + Q_2' Q_3'$$

$$T_3 = Q_1 + Q_1' Q_3$$

# Counter Implementation using D FF



# Excitation Table: Sync Counter Using RS FF



Q	$Q^+$	S	R
0	0	0	x
0	1	1	0
1	0	0	1
1	1	x	0

Present State			Next State			Flip Flop Inputs					
Q1	Q2	Q3	$Q1^+$	$Q2^+$	$Q3^+$	S1	R1	S2	R2	S3	R3
0	0	0	0	1	0	0	x	1	0	0	x
0	1	0	0	1	1	0	x	x	0	1	0
0	1	1	1	1	0	1	0	x	0	0	1
1	1	0	1	0	1	x	0	0	1	1	0
1	0	1	0	0	1	0	1	0	x	x	0
0	0	1	0	0	0	0	x	0	x	0	1

# FF Input Functions

Present State			Flip Flop Inputs					
Q1	Q2	Q3	S1	R1	S2	R2	S3	R3
0	0	0	0	X	1	0	0	X
0	1	0	0	X	X	0	1	0
0	1	1	1	0	X	0	0	1
1	1	0	X	0	0	1	1	0
1	0	1	0	1	0	X	X	0
0	0	1	0	X	0	X	0	1

$S1 = F(Q1, Q2, Q3)$

$S2 = F(Q1, Q2, Q3)$

$S3 = F(Q1, Q2, Q3)$

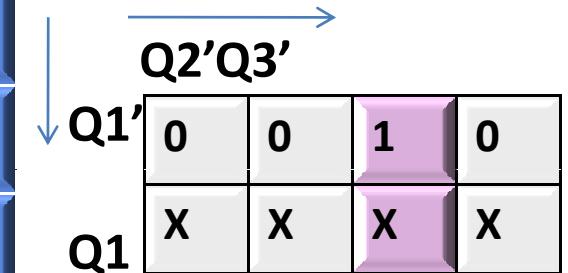
$R1 = F(Q1, Q2, Q3)$

$R2 = F(Q1, Q2, Q3)$

$R3 = F(Q1, Q2, Q3)$

# Solve Each Function Using KMAP

Present State			Flip Flop Inputs					
$Q_1$	$Q_2$	$Q_3$	$R_1$	$S_1$	$S_2$	$R_2$	$S_3$	$R_3$
0	0	0	0	X	1	X	0	X
0	1	0	0	X	X	0	1	X
0	1	1	1	X	X	0	X	1
1	1	0	X	0	X	1	1	X
1	0	1	X	1	0	X	X	0
0	0	1	0	X	0	X	X	1



$$S_1 = Q_2 Q_3$$

$$S_2 = Q_1' Q_3'$$

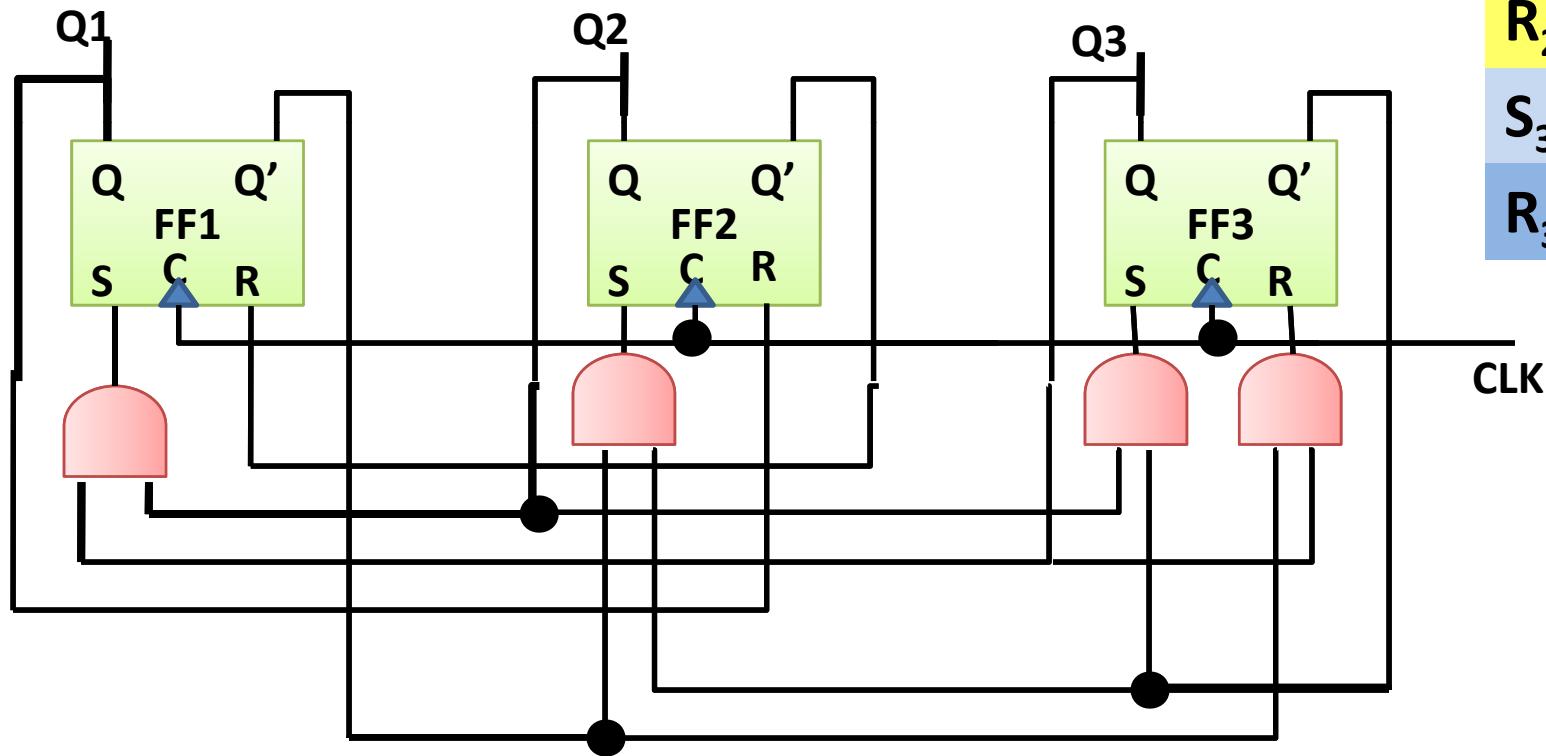
$$S_3 = Q_2 Q_3'$$

$$R_1 = Q_2'$$

$$R_2 = Q_1$$

$$R_3 = Q_1' Q_3$$

# Counter Logic Diagram



$$S_1 = Q_2 Q_3$$

$$R_1 = Q_2'$$

$$S_2 = Q_1' Q_3'$$

$$R_2 = Q_1$$

$$S_3 = Q_2 Q_3'$$

$$R_3 = Q_1' Q_3$$

**Thanks**