UNIT II

OPTICS, FIBRE OPTICS AND LASER

2.1 INTRODUCTION TO INTERFERENCE

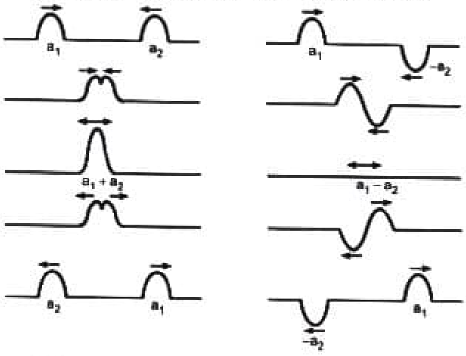
- The most common type of radiation which we come across in day to day life is electromagnetic wave or photon (the light quanta). Some of the electromagnetic waves can stimulate retina and some cannot. The part of the electromagnetic wave which can stimulate the retina is called **Light**.
- The branch of physics which deals with light is called **Optics**. Further, optics can be broadly classified as (a) **Geometrical Optics**, (b) **Physical or Wave Optics** and (c) **Quantum Optics** depending upon the basic behaviour of light assumed for explaining the optical phenomena. In the current course, our main focus will be on physical optics, where we assume the wave nature of light.
- From basic optical phenomena such as interference and diffraction, we can conclude that the light has a wave nature. In this unit, we will be studying these two basic properties of light.
- But these properties fail to explain the type of oscillations involved i.e. polarisation. The polarisation of light will be studied in later part of the text.
- The wave theory of light was proposed by Christian Huygen in 1679 but interference was demonstrated by Thomas Young only in 1802. There are several examples of interference that can be observed in everyday life. Basically, oil is colourless but a film of oil floating on water shows bright colours and also keeps on changing colour. Similarly a soap bubble, a compact disc, a thin sheet of mica or cellophane appear coloured. All this is due to interference of light.
- In engineering too, interference has wide applications such as measurement of thickness and stress, testing flatness of a surface, anti-reflecting coating etc.

2.1.1 Interference of Waves

- If two waves of same frequency travel in same direction with a constant phase difference with time, they combine so that their energy is not uniformly distributed in space, but is maximum at certain points and minimum (or zero) at other points. This phenomenon is called **Interference**.
- In interference, energy is neither created (at maxima) nor destroyed (at minima) but is redistributed so that there is more energy at certain points (maxima) and less energy at other points (minima). Even after interference the total energy of the system remains constant.
- Thus, interference is the redistribution of energy due to superposition of two or more waves.

Principle of Superposition

- The principle of superposition states that when two or more waves are superposed in space or a medium, the waves travel independently, through each other and the resultant displacement of each position is the algebraic/vector sum of the displacements due to each wave. Fig. 2.1 shows superposition of two waves.
- In Fig. 2.1 (a), two crests, with amplitude a_1 and a_2, are approaching each other and the point where they meet the resultant amplitude $(a_1 + a_2)$ is more than the individual amplitudes. After this, they pass through each other as though they have not interfered at all. Similarly, in Fig. 2.1 (b), one crest and one trough, with amplitude a_1 and $-a_2$, are approaching each other. At the point where they meet the resultant amplitude $(a_1 - a_2)$ is less than the individual amplitudes.
- The first case is called **Constructive Interference** and the second case is called **Destructive Interference**.



(a) Constructive Interference (b) Destructive interference

Fig. 2.1: Superposition

(1) Constructive Interference

- When the crest of one wave overlaps the crest of the other or the trough of one overlaps with the trough of the other, the displacement is maximum. This is called as **Constructive Interference**.

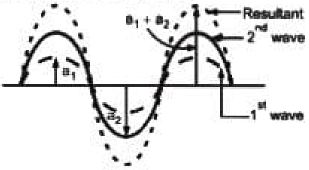


Fig. 2.2: Constructive interference

- In this case, the waves are in phase and the resultant amplitude equals the sum of the two component amplitudes.

i.e. $$A = a_1 + a_2 \quad \text{... (2.1)}$$

If $$a_1 = a_2 = a$$

then $$A = 2a \quad \text{... (2.2)}$$

The resultant intensity will be

$$I = A^2 = 4a^2 \quad \text{... (2.3)}$$

- Here the path difference between two waves is 0. Constructive interference will also take place when the path difference is λ or 2λ. In general, condition for constructive interference is,

Path difference,

$$\Delta = n\lambda \quad \text{... (2.4)}$$

where $n = 0, 1, 2, \ldots n.$

In terms of phase difference,

Phase difference,

$$\delta = k\Delta, \text{ where } k = \frac{2\pi}{\lambda}$$

$$\therefore \quad \delta = \frac{2\pi}{\lambda} \cdot n\lambda$$

$$\delta = 2n\pi \quad \text{... (2.5)}$$

(2) Destructive Interference

- In the other case, when the crest of one wave overlaps the trough of other or vice-versa, the displacement is minimum. This is called as **Destructive Interference**.

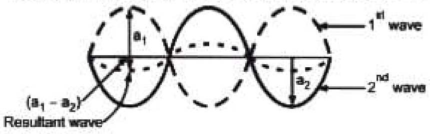


Fig. 2.3: Destructive interference

- In this case, the waves are 180° out of phase and the resultant amplitude is the difference of the two component amplitudes.

i.e. $$A = a_1 - a_2 \quad \text{... (2.6)}$$

If $$a_1 = a_2 = a$$

then $$A = 0 \quad \text{... (2.7)}$$

The resultant intensity,

$$I = A^2 = 0 \quad \text{... (2.8)}$$

- Here the path difference between two waves is λ/2. The same thing will happen when the path difference is 3λ/2 or 5λ/2. In general, destructive interference will occur when

Path difference,

$$\Delta = \left(n + \frac{1}{2}\right)\lambda \quad \text{... (2.9)}$$

where $n = 0, 1, 2, \ldots, n.$

or Phase difference,

$$\delta = k \cdot \Delta \text{ where } k = \frac{2\pi}{\lambda}$$

$$\delta = \frac{2\pi}{\lambda}\left(n + \frac{1}{2}\right)\lambda$$

$$\delta = (2n + 1)\pi \quad \text{... (2.10)}$$

2.2 INTERFERENCE OF LIGHT IN THIN FILM

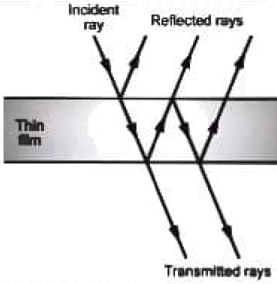


Fig. 2.4: Multiple reflections in a transparent thin film

- A film is said to be **Thin** when its thickness is of the order of wavelength of visible light (taken to be 5500 A°, which is the centre of the visible spectrum).
- A thin film may be a thin sheet of transparent material like glass, mica or an air film enclosed between two transparent sheets or a soap bubble.
- A light ray incident on a thin transparent film undergo reflection from the upper and lower surfaces of the film. They travel along different paths and may be reunited to produce **Interference**.

In a thin film, a small portion gets reflected from the upper surface while a major portion is transmitted into the film. The lower surface reflects a small portion of the transmitted component, back into the film while the rest of it emerges out of the film from other side.

Hence, a small portion of light gets partially reflected in succession several times within the film (as shown in Fig. 2.4). In a transparent thin film, the two surfaces strongly transmit and weakly reflect the incident light. In such cases, only the first few reflections at the top surface and the first few reflections at the bottom surface will be of appreciable strength. Hence, only the first two rays will be considered in the discussion.

At each reflection, the incident amplitude is divided into a reflected and transmitted component. Therefore, interference in thin films is called **Interference by Division of Amplitude**. This phenomenon was first observed by Newton and Robert Hooke but was correctly explained by Thomas Young.

In the Ongoing Discussion, Following Facts are Assumed

- When a ray of light gets **Reflected** from a **Denser Medium** into a **Rarer Medium**, it undergoes a **Phase Change of π** or a **Path Change of $\lambda/2$**.
- A **Distance 't'** traversed by light in a medium of **Refractive Index 'μ'** has an equivalent **Optical Path 'μ T'**.

2.3 INTERFERENCE DUE TO THIN FILMS OF UNIFORM THICKNESS [Dec. 17]

Consider a thin film of uniform thickness 't' and refractive index 'μ'. Let XY and X'Y' be the faces of this parallel sided film. The film is surrounded by air on both the sides.

A plane monochromatic light ray, which can be considered as a parallel beam, is incident on the upper surface of the film.

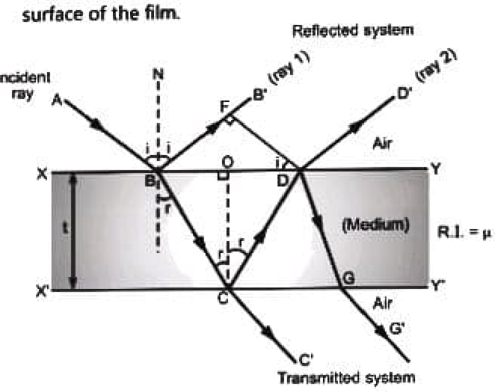


Fig. 2.5: Reflection of light from a parallel thin film

- Let AB represent one of the incident rays. The light ray travelling along AB is incident at an angle 'i' on the upper surface of the film. A part of the incident light is reflected at the upper face along BB' and a part is refracted at an angle 'r' along BC.
- At C, it is partly reflected back into the film along CD while a major portion is transmitted along CC'. The ray along CD emerges along DD' parallel to BB'.
- Both rays BB' (ray 1) and DD' (ray 2) are obtained from the same incident ray. They are, therefore, coherent and can produce interference. The condition of interference depends on the optical path difference between rays 1 and 2.
- To compute the optical path difference between the reflected ray BB' (ray 1) and the refracted ray DD' (ray 2), draw a normal DF from D on line BB'. Beyond points F and D, both rays travel equal distances. Hence will not be considered while calculating optical path difference.
- While ray BB' has covered a distance BF in air, the ray DD' has covered a distance BCD in the film of refractive index μ.
- The geometric **Path Difference** between ray 1 and ray 2 is, BC + CD − BF.

∴ Optical path difference is,

$$\Delta = \mu (BC + CD) - BF \quad \text{... (2.11)}$$

(Since, distances BC and CD are travelled in medium of R.I. = μ)

From Δ BCO and Δ DCO, $\cos r = \dfrac{CO}{BC}$ and $\cos r = \dfrac{CO}{CD}$

$$\therefore \quad \cos r = \frac{t}{BC} \text{ and } \cos r = \frac{t}{CD} \quad (\because CO = t)$$

$$\therefore \quad BC = CD = \frac{t}{\cos r} \quad \text{... (2.12)}$$

and $\quad BF = BD \sin i$ (from Δ BFD)

But $\quad BO = OD = BC \sin r$ (from Δ BCO and Δ DCO)

$$\therefore \quad BF = 2\, BC \sin r \sin i \text{ as } BD = BO + OD$$

From equation (2.12), $BF = \dfrac{2t}{\cos r} \sin r \sin i$

Dividing and multiplying by sin r,

$$BF = \frac{2t}{\cos r} \sin^2 r \, \frac{\sin i}{\sin r}$$

$$BF = \frac{2\mu t}{\cos r} \sin^2 r \quad \left(\because \frac{\sin i}{\sin r} = \mu\right) \quad \text{... (2.13)}$$

Substituting equations (2.12) and (2.13) in equation (2.11),

$$\Delta = \mu\left(\frac{t}{\cos r}+\frac{t}{\cos r}\right) - \frac{2\mu t}{\cos r}\sin^2 r$$

$$\Delta = \frac{2\mu t}{\cos r} - \frac{2\mu t}{\cos r}\sin^2 r$$

$$\Delta = (1-\sin^2 r)\frac{2\mu t}{\cos r}$$

$$\Delta = \frac{2\mu t}{\cos r}\cos^2 r \quad (\because \sin^2 r + \cos^2 r = 1)$$

$$\Delta = 2\mu t \cos r \qquad \text{... (2.14)}$$

2.3.1 In Reflected System [Dec. 17]

- The path difference given by (2.14) is not the true optical path difference between rays 1 and 2. The phase change due to reflection is to be taken into account.
- At B, the reflection is in a rarer medium. So, a path change of $\lambda/2$ occurs in the reflected ray BB'. At C, reflection is in a denser medium, therefore no path change occurs in ray 2.

  $\therefore$ Total path difference between rays 1 and 2 is given by,

Total path difference = Path difference due to thin film + Path difference due to reflections

$$\Delta = 2\mu t \cos r \pm \frac{\lambda}{2} \qquad \text{... (2.15)}$$

(i) Condition for Constructive Interference

- If the total path difference is equal to an integral multiple of λ then rays 1 and 2 meet in phase and undergo constructive interference.

i.e., $\Delta = n\lambda$

$\therefore$ $2\mu t \cos r \pm \lambda/2 = n\lambda$

$2\mu t \cos r = (2n \pm 1)\,\lambda/2$ where $n = 0, 1, 2, 3 \ldots$ (2.16)

(ii) Condition for Destructive Interference

- If the optical path difference is equal to an odd integral multiple of $\lambda/2$, then rays 1 and 2 meet in opposite phase and undergo destructive interference.

i.e. $\Delta = (2n \pm 1)\frac{\lambda}{2}$

$\therefore$ $2\mu t \cos r \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$

$2\mu t \cos r = n\lambda$ where $n = 0, 1, 2$

... and is called order of interference ...(2.17)

- The rays incident on the film at the same angle are divided into two rays which become parallel on reflection from the surfaces of the film. Parallel rays do not intersect at finite distances, hence fringes are not observed at finite distances.
- The rays are to be condensed by a lens and interference is observed in its focal plane. Else, it can be observed by the unaided eye focused at infinity. Therefore, these interference fringes are said to be **Localized at Infinity**.

(iii) Important Cases

- If the film is extremely thin i.e. $t << \lambda$ or $t \to 0$ then the path difference, $\Delta = \lambda/2$. The film will appear dark in reflected light.
- When monochromatic light is incident normal to the film then $\cos r = 1$.

$$2\mu t = (2n+1)\frac{\lambda}{2} \text{ for brightness}$$

and $2\mu t = n\lambda$ for darkness.

- This implies that the film will appear bright in reflected light if the film has thickness of

$$t = \frac{\lambda}{4\mu}, \frac{3\lambda}{4\mu}, \ldots\ldots\ldots \text{ and it will appear dark}$$

for a thickness of $t = \frac{\lambda}{2\mu}, \frac{2\lambda}{2\mu}, \frac{3\lambda}{2\mu} \ldots\ldots$

- If the incident monochromatic light is parallel, the whole film will be uniformly bright or dark as film thickness 't' and angle of refraction 'r' are constant. For a given incident wavelength (say green) the condition of constructive interference causes the incident colour to intensify (intense green).
- A change in the angle of incidence of the rays causes a change in the path difference. The optical path difference decreases with increase in angle of incidence. Hence, as inclination of the film is changed, it will appear alternately dark and bright for incident monochromatic light.
- If white light is incident on the film, the optical path difference will vary from one colour to the other as λ is different. Hence, the film will appear coloured, the colour being that of the rays which interfered constructively. Further, as the inclination of the film is changed, the film will appear coloured.
- If the incident white light is not parallel, the optical path difference will change due to change in the incident angle. Hence, the film will show different colours when viewed from different directions.

2.3.2 In Transmitted System

- When the film is observed in transmitted light, it can be shown that the path difference between rays CC' and GG' (Fig. 2.5) is equal to $2\mu t \cos r$. Reflections at C and D are in a denser medium. So, no additional path change will occur due to reflection.

Total path difference = Path difference due to thin film + Path difference due to reflections

$$\therefore \quad \Delta = 2\mu t \cos r + 0 \quad \text{... (2.18)}$$

(i) Condition for Constructive Interference

- For constructive interference the total phase difference should be an integral multiple of λ.

i.e. $\Delta = n\lambda$

$$\therefore \quad 2\mu t \cos r = n\lambda \quad \text{... (2.19)}$$

(ii) Condition for Destructive Interference

- For destructive interference the total phase difference should be an odd integral multiple of $\lambda/2$.

i.e. $\Delta = (2n \pm 1)\frac{\lambda}{2}$

$$\therefore \quad 2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2} \quad \text{... (2.20)}$$

As is evident, the condition for brightness on reflection becomes the condition for darkness on transmission and vice versa.

2.4 INTERFERENCE IN FILMS OF NON-UNIFORM THICKNESS (WEDGE SHAPED FILM)

- A **Wedge** is a plate or film of varying thickness, having zero thickness at one end and progressively increasing to a particular thickness at the other end.
- Consider two plane surfaces XY and X'Y' inclined at an angle α. The thickness of the film increases linearly from X to Y.
- When the wedge is illuminated by a parallel beam of monochromatic light, the rays reflected from its two surfaces will not be parallel. They appear to diverge from a point S near the film.
- When the film is viewed with reflected monochromatic light, **Equidistant Interference Fringes** are observed which are parallel to the line of intersection of the two surfaces. The fringes are alternately bright and dark and are localised at the surface of the film.
- On illuminating the film with monochromatic light, one system of rays is reflected from the front surface XY and the other system of rays is obtained by transmission at the back surface X'Y' (not shown in Fig. 2.6) and consequent reflections at the front surface. As both rays are obtained from a single source, they are coherent and produce interference.

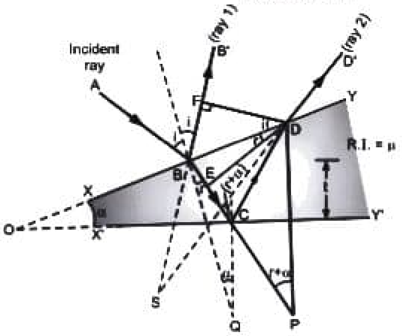


Fig. 2.6: Wedge-shaped film

The interfering rays BB' and DD' are not parallel but appear to diverge from a point S. The optical path difference between them is given by,

$$\Delta = \mu (BC + CD) - BF$$

By the definition of optical path difference,

$$BF = \mu (BE)$$

$$\therefore \quad \Delta = \mu (BE + EC + CD) - \mu (BE)$$

$$\therefore \quad \Delta = \mu (EC + CD)$$

$$\Delta = \mu (EC + CP)$$

$$(\because \text{CPD is an isosceles triangle})$$

$$\therefore \quad \Delta = \mu (EP)$$

$$\therefore \quad \Delta = 2\mu t \cos (r + \alpha) [\text{as } EP = DP \cos (r + \alpha)$$

$$= 2t \cos (r + \alpha)$$

In Reflected System:

Due to reflection, an additional path change is introduced in the reflected system at point B.

$\therefore$ Total path difference = Path difference due to thin film + Path difference due to reflections

$$\Delta = 2\mu t \cos (r + \alpha) \pm \frac{\lambda}{2} \quad \text{... (2.21)}$$

Condition for Constructive Interference:

For constructive interference the total phase difference should be an integral multiple of λ.

$$\Delta = n\lambda$$

i.e. $2\mu t \cos(r+\alpha) \pm \frac{\lambda}{2} = n\lambda$

$$\therefore \quad 2\mu t \cos(r+\alpha) = (2n \pm 1)\frac{\lambda}{2} \quad \text{... (2.22)}$$

Condition for Destructive Interference:

For destructive interference the total phase difference should be an odd integral multiple of $\lambda/2$.

$$\Delta = (2n \pm 1)\frac{\lambda}{2}$$

i.e. $2\mu t \cos(r+\alpha) \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$

$$\therefore \quad 2\mu t \cos(r+\alpha) = n\lambda \quad \text{... (2.23)}$$

Nature of Interference Pattern :

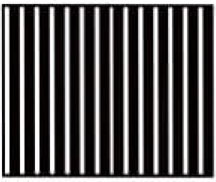


Fig. 2.7: Interference pattern in wedge-shaped film. Alternately bright and dark bands are parallel

If the film is illuminated by parallel light, then 'i' is constant everywhere and so is 'r', the angle of refraction. In addition, if monochromatic light is used, the path change will occur only due to 't'. In this case, the fringes will be of **Equal Thickness**. For a wedge shaped film, 't' remains constant only in a direction parallel to the thin edge of the wedge. So, straight fringes parallel to the edge of the wedge are obtained. The fringes are alternately bright or dark for monochromatic light. For white light, coloured fringes are obtained.

[**Note** : In transmitted system we will get exactly opposite of the reflected system.]

2.5 FRINGE WIDTH (β)

- When a wedge film is illuminated by monochromatic light of wavelength λ, it gives fringes of equal thickness. Fringe width can be calculated by knowing the position of consecutive minima or maxima. Here, for mathematical simplicity, we will consider minima.

For n^{th} minimum, we have

$$2\mu t \cos(r+\alpha) = n\lambda$$

For normal incidence,

$$r = 0$$

$$\therefore \quad 2t\mu \cos\alpha = n\lambda \quad \text{... (2.24)}$$

Let this n^{th} dark band be formed at a distance x_n from the thin edge.

$$\therefore \quad t = x_n \tan\alpha \quad \text{(from Fig. 2.8) ... (2.25)}$$

From equations (2.24) and (2.25),

$$2\mu x_n \tan\alpha \cos\alpha = n\lambda$$

or $$2\mu x_n \sin\alpha = n\lambda \quad \text{... (2.26)}$$

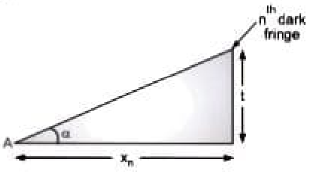


Fig. 2.8

- Similarly, if the $(n+1)^{th}$ minimum is obtained at a distance x_{n+1} from the thin edge, then

$$2\mu x_{n+1} \sin\alpha = (n+1)\lambda \quad \text{... (2.27)}$$

- Subtracting equation (2.26) from equation (2.27), we get fringe width of a bright fringe.

$$2\mu(x_{n+1} - x_n)\sin\alpha = \lambda$$

$\therefore$ Fringe width,

$$\beta = x_{n+1} - x_n = \frac{\lambda}{2\mu \sin\alpha}$$

$$\beta = \frac{\lambda}{2\mu \sin\alpha}$$

$$\cong \frac{\lambda}{2\mu\alpha} \quad \text{(for small } \alpha \text{ and in radians) ... (2.28)}$$

- For an air film ($\mu = 1$), fringe width,

$$\beta = \frac{\lambda}{2\sin\alpha} = \frac{\lambda}{2\alpha} \quad \text{(for small } \alpha) \quad \text{... (2.29)}$$

- Similarly, it can be shown that the fringe width for dark fringes is given by,

$$\beta = \frac{\lambda}{2\mu \sin\alpha} \quad \text{... (2.30)}$$

which is the same as that of a bright fringe.

- The width of a dark or bright fringe is however equal to half the fringe width.

- In equation (2.28), as all quantities on the right side are constant, β is constant for a given wedge angle α. It means that the **Interference Fringes** are **Equidistant** from each other.
- According to relation (2.29), as angle α increases, the fringes move closer. At $\alpha = 1°$, the interference pattern vanishes. If α is gradually decreased, the fringe separation increases and ultimately the fringes disappear as the faces of the film become parallel ($\alpha = 0°$).

SOLVED PROBLEMS

Problem 2.1: *In a certain region of interference we get 490th order maximum for sodium 5890 A° line. What will be the order of interference maximum at the same plane for sodium 5896 A°?*

Data: $n = 490$, $\lambda = 5890$ A°

Formula: $\Delta = n\lambda$

Solution: Path difference for nth order maximum, $\Delta = n\lambda$

∴ Path difference for 490th order maximum when λ is 5890 A°

$$\Delta = 490 \times 5890 \times 10^{-8}$$

$$\Delta = 2.89 \times 10^{-2} \text{ cm}$$

For sodium light of wavelength λ_1 = 5896 A°, the order of interference is n_1. Then the

Path difference = $n_1 \lambda_1 = 2.89 \times 10^{-2}$

$$\therefore \quad n_1 = \frac{2.89 \times 10^{-2}}{\lambda_1}$$

$$\therefore \quad n_1 = \frac{2.89 \times 10^{-2}}{5896 \times 10^{-8}} = 489.5$$

∴ The order of interference maximum = $\boxed{489}$

Problem 2.2: *Fringes are produced with monochromatic light of λ = 5450 A°. A thin glass plate of μ = 1.5 is then placed normally in the path of one of the interfering beams and the central band of the fringe system is found to move into the position previously occupied by the third bright band from the centre. Calculate the thickness of the glass plate.*

Data: λ = 5450 A°, μ = 1.5, n = 3

Formula: $t(\mu - 1) = n\lambda$

Solution: $$t = n \frac{\lambda}{(\mu - 1)}$$

Substituting, $$t = \frac{3 \times 5450 \times 10^{-8}}{1.5 - 1}$$

$$\boxed{t = 0.000327 \text{ cm}}$$

Problem 2.3: *When light falls normally on a soap film, whose thickness is 5×10^{-5} cm and whose refractive index is 1.33; which wavelength in the visible region will be reflected most strongly?*

Data: $t = 5 \times 10^{-5}$ cm, μ = 1.33

Formula: $2\mu t \cos r = (2n + 1)\lambda/2$ where n = 0, 1, 2, 3, ... etc.

Solution: For normal incidence,

$$\cos r = 1$$

$$\therefore \quad 2\mu t = (2n + 1)\frac{\lambda}{2}$$

$$\therefore \quad \lambda = \frac{2 \times 2\mu t}{2n + 1}$$

$$\lambda = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{(2n + 1)}$$

For n = 0, $$\lambda = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{1} = 2.66 \times 10^{-4} \text{ cm}$$

$$\boxed{\lambda = 26{,}600 \text{ A°}}$$

For n = 1, $$\lambda = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{3} = 8.866 \times 10^{-5} \text{ cm}$$

$$\boxed{\lambda = 8866 \text{ A°}}$$

For n = 2, $$\lambda = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{5} = 5.32 \times 10^{-5} \text{ cm}$$

$$\boxed{\lambda = 5320 \text{ A°}}$$

For n = 3, $$\lambda = \frac{4 \times 1.33 \times 5 \times 10^{-5}}{7} = 3.8 \times 10^{-5} \text{ cm}$$

$$\boxed{\lambda = 3800 \text{ A°}}$$

The wavelength 5320 A° will be most strongly reflected in the visible region.

Problem 2.4: *A parallel beam of sodium light λ = 5890 A° strikes a film of oil floating on water. When viewed at an angle of 30° from the normal, 8th dark band is seen. Determine the thickness of the film if refractive index of oil = 1.5*

Data: λ = 5890 A°, $\angle i = 30°$, μ = 1.5, n = 8

Formulae: (i) $2\mu t \cos r = n\lambda$ or $t = \dfrac{n\lambda}{2\mu \cos r}$ ... (1)

(ii) $$\mu = \frac{\sin i}{\sin r}$$

Solution: $\sin r = \dfrac{\sin i}{\mu}$

$$\cos r = \sqrt{1-\sin^2 r}$$

$$\therefore \quad \cos r = \sqrt{1-\frac{\sin^2 i}{\mu^2}}$$

$$\therefore \quad \cos r = \sqrt{1-\frac{\sin^2 30}{(1.5)^2}}$$

$$\cos r = 0.943$$

Substituting in (1),

$$\therefore \quad t = \frac{8 \times 5890 \times 10^{-8}}{2 \times 1.5 \times 0.943}$$

$$= \boxed{1.6302 \times 10^{-4}\ \text{cm}}$$

Problem 2.5: *Two glass plates enclose a wedge-shaped air film, touching at one edge and are separated by a wire of 0.03 mm diameter at a distance of 15 cm from the edge. Monochromatic light of λ = 6000 A° from a broad source falls normally on the film. Calculate the fringe width of the fringes thus formed.*

Data: $\lambda = 6000 \times 10^{-8}$ cm; For air film, $\mu = 1$

Diameter = 0.03 mm = 0.003 cm

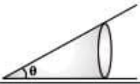
Distance of fringe from the edge = 15 cm

Formula: Fringe width,

$$\beta = \frac{\lambda}{2\mu \sin \alpha} = \frac{\lambda}{2\mu \tan \alpha}$$

Solution:

$$\beta = \frac{6000 \times 10^{-8}}{2 \times 1 \times \frac{0.003}{15}} = \boxed{0.15\ \text{cm}}$$



Problem 2.6: *Interference fringes are produced by monochromatic light falling normally on a wedge-shaped film of cellophane whose refractive index is 1.4. The angle of the wedge is 20 sec of an arc and the distance between the successive fringes is 0.25 cm. Calculate the wavelength of light.*

Data: β = 0.25 cm, μ = 1.4, θ = 20 sec

$$= \frac{20}{60 \times 60} \times \frac{\pi}{180}$$

$$= \frac{1}{180} \times \frac{\pi}{180} \text{ radians}$$

Formula: $\beta = \dfrac{\lambda}{2\mu\alpha}$

$$\therefore \quad \lambda = 2\mu\,\alpha \cdot \beta$$

Solution: $\lambda = 2 \times 1.4 \times \dfrac{\pi}{180 \times 180} \times 0.25$

$$\boxed{\lambda = 6.79 \times 10^{-5}\ \text{cm}}$$

Problem 2.7: *Two plane rectangular pieces of glass are in contact at one edge and separated by a hair at opposite edge, so that a wedge is formed. When light of wavelength 6000 A° falls normally on the wedge, nine interference fringes are observed. What is the thickness of the hair ?*

Data: $\lambda = 6000 \times 10^{-8}$ cm, n = 9, r = 0 for normal incidence

Formula: $2\mu t \cos(r + \alpha) = n\lambda$

Solution: If the fringes are seen normally and the angle of wedge is very small, then r = 0, so that

$$\cos(r + \alpha) = \cos \alpha = 1$$

For air film, $\mu = 1$

$$\therefore \quad 2\mu t = n\lambda$$

$$2 \times 1 \times t = 9 \times 6000 \times 10^{-8}$$

$$\therefore \quad t = \frac{9 \times 6000 \times 10^{-8}}{2} = \boxed{27 \times 10^{-5}\ \text{cm}}$$

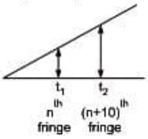
Problem 2.8: *A square piece of cellophane film with index of refraction 1.5 has a wedge-shaped section, so that its thickness at two opposite sides is t_1 and t_2. If with light of λ = 6000 A°, the number of fringes appearing on the film is 10, calculate the difference $t_2 - t_1$.*

Data: $\lambda = 6000 \times 10^{-8}$ cm, $\mu = 1.5$

Formula: $2\mu t_1 \cos(r + \alpha) = \lambda$ ... (1)

Solution: For $(n + 10)^{th}$ dark fringe,

$2\mu t_2 \cos(r + \alpha) = (n + 10)\lambda$ ... (2)



For normal incidence, r = 0 and if the angle of wedge is small,

$$\cos(r + \alpha) = \cos \alpha = 1$$

$\therefore$ Equations (1) and (2) become

$2\mu t_1 = n\lambda$ ... (3)

$2\mu t_2 = (n + 10)\lambda$ ... (4)

∴ Subtracting equation (3) from equation (4), we get

$$2\mu(t_2 - t_1) = 10\lambda$$

$$\therefore \quad t_2 - t_1 = \frac{10\lambda}{2\mu} = \frac{10 \times 6000 \times 10^{-8}}{2 \times 1.5}$$

i.e. $\boxed{t_2 - t_1 = 2 \times 10^{-4}\ \text{cm}}$

***Problem 2.9:** A parallel beam of light of wavelength 5890 A° is incident on a thin film of refractive index 1.5, such that the angle of refraction into the film is 60°. Calculate the smallest thickness of the film which will make it appear dark by reflection.*

Data: $\lambda = 5890$ A°, $r = 60°$, $\mu = 1.5$

Formula: For darkness,

$$2\mu t \cos r = n\lambda. \quad \text{Let } n = 1$$

Solution:

$$t = \frac{\lambda}{2\mu \cos r} = \frac{5890 \times 10^{-8}}{2 \times 1.5 \times \cos 60}$$

$$\boxed{t = 3.926 \times 10^{-5}\ \text{cm}}$$

***Problem 2.10:** The optical path difference between two sets of similar waves from the same source arriving at a point on the screen is 199.5 λ. Is the point dark or bright ? If the path difference is 0.012 cm, find the wavelength of the light used.*

Data: $\Delta_1 = 199.5\lambda$, $\Delta_2 = 0.012$ cm

Formula: For 199.5 λ it is odd path difference, therefore point is a dark fringe and for darkness,

$$\Delta = (2n - 1)\frac{\lambda}{2} \qquad \left(\because \left(\frac{2n-1}{2}\right) = 199.5\right)$$

Solution: $0.012 = 199.5\lambda$

$$\therefore \quad \lambda = \frac{0.012}{199.5}$$

$$\therefore \quad \lambda = 6.015 \times 10^{-8}\ \text{cm}$$

$$\boxed{\lambda = 6015\ \text{A°}}$$

***Problem 2.11:** Two pieces of plane glass are placed together with a piece of paper between the two at one edge. Find the angle in seconds of the wedge shaped air film between the plates, if on viewing the film normally with monochromatic light of wavelength 4800 A°, there are 18 bands per cm.*

Solution: 18 bands per cm

$$\therefore \text{Band width, } \beta = \frac{1}{18}$$

$$\beta = 0.0556\ \text{cm}$$

We know, $\beta = \dfrac{\lambda}{2\alpha}$

$$\alpha = \frac{\lambda}{2\beta} = \frac{4800 \times 10^{-8}}{2 \times 0.0556}$$

$$\alpha = 4.3165 \times 10^{-4}\ \text{rad}$$

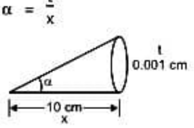
Note conversion of radians into seconds.

$$\alpha = 4.3165 \times 10^{-4} \times \frac{180}{\pi} \times 60 \times 60$$

$$\boxed{\alpha = 89.02\ \text{seconds}}$$

***Problem 2.12:** Two optically plane glass strips of length 10 cm are placed one over the other. A thin foil of thickness 0.010 mm is introduced between the plates at one end to form an air film. If the light used has wavelength 5900 A°, find the separation between consecutive bright fringes.*

Solution: $\tan\alpha = \dfrac{t}{x}$



As α is very small, $\tan\alpha = \alpha$

$$\alpha = \frac{t}{x} = \frac{0.001}{10}$$

$$\alpha = 0.0001\ \text{rad}$$

$$\beta = \frac{\lambda}{2\alpha} = \frac{5.9 \times 10^{-5}}{2 \times 0.0001}$$

$$\boxed{\beta = 0.295\ \text{cm}}$$

Note while using α in calculation it must be all the time in radians. If it is given in seconds then convert it in rad.

***Problem 2.13:** Find the thickness of a wedge-shaped film at a point where fourth bright fringe is situated. λ for sodium light is 5893 A°.*

Data: $n = 4$, $\lambda = 5893$ A°

Formula: For bright band and wedge-shaped film,

$$2\mu t \cos(r + \alpha) = (2n - 1)\frac{\lambda}{2}$$

Let normal incidence, $r = 0$ and α is very small

$$\therefore \quad \cos(r + \alpha) = 1, \mu = 1$$

$$\therefore \quad 2t = (2n - 1)\frac{\lambda}{2}$$

Solution:

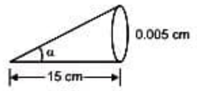
$$2t = \frac{(2 \times 4 - 1)\lambda}{2}$$

$$t = \frac{7}{4} \times 5893 \times 10^{-8}$$

$$\boxed{t = 1.031275 \times 10^{-4}\ \text{cm}}$$

Problem 2.14: *Monochromatic light emitted by a broad source of light of wavelength 6×10^{-5} cm falls normally on two glass plates which enclose a thin wedge-shaped film of air. The plates touch at one end and are separated at a point 15 cm from the end by a wire 0.5 mm in diameter. Find the width between any two consecutive bright fringes.*

Solution:



$$\alpha = \frac{t}{x} = \frac{0.005}{15} = 3.333 \times 10^{-4} \text{ rad}$$

$$\text{Bandwidth, } \beta = \frac{\lambda}{2\alpha}$$

$$\beta = \frac{6 \times 10^{-5}}{2 \times 3.333 \times 10^{-4}}$$

$$\boxed{\beta = 0.09 \text{ cm}}$$

Problem 2.15: *Fringes of equal thickness are observed in a thin glass wedge of refractive index 1.52. The fringe spacing is 1 mm and the wavelength of light is 5893 A°. Calculate the angle of wedge in seconds of an arc.*

Data:

$\mu = 1.52$

$\lambda = 5893$ A°

$\beta = 1$ mm

Formula: Fringe width is,

$$\beta = \frac{\lambda}{2\alpha}$$

Solution: $1 \times 10^{-1} = \dfrac{5893 \times 10^{-8}}{2\alpha}$

$$\therefore \quad \alpha = \frac{5893 \times 10^{-8}}{2 \times 1 \times 10^{-1}}$$

$$\alpha = 2.9 \times 10^{-4} \text{ radian}$$

or $\alpha = \dfrac{2.9 \times 10^{-4} \times 180 \times 60 \times 60}{3.14}$ of an arc

$$= \boxed{59.8 \text{ sec.}}$$

Problem 2.16: *A parallel beam of sodium light strikes a film of oil floating on water. When viewed at an angle 30° from the normal, in the reflected light, eighth dark band is seen. Determine the thickness of the film. Refractive index of oil is 1.46 and λ = 5890 A°.*

Data:

$i = 30°$

$\mu = 1.46$

$\lambda = 5890$ A°

Formulae: (i) By Snell's law,

$$\frac{\sin i}{\sin r} = \mu$$

(ii) $2\mu t \cos r = n\lambda$

Solution: (i) $\dfrac{\sin 30}{\sin r} = 1.46$

$$\sin r = \frac{0.5}{1.46} = 0.34247$$

$$\therefore \quad \boxed{r = 20°}$$

(iii) The thickness is given by relation (condition for minima)

$$2\mu t \cos r = n\lambda$$

$$t = \frac{8 \times 5890 \times 10^{-8}}{2 \times 1.46 \times \cos 20°}$$

$$\boxed{t = 1.7 \times 10^{-4} \text{ cm}}$$

Problem 2.17: *A soap film of refractive index 4/3 and thickness 1.5×10^{-4} cm is illuminated by white light incident at an angle of 45°. The light reflected by it is examined by a spectroscope in which is found a dark and corresponding to wavelength of 5×10^{-5} cm. Calculate the order of interference band.*

Data:

$\mu = 4/3 = 1.33$

$t = 1.5 \times 10^{-4}$ cm

$i = 45°$

$\lambda = 5 \times 10^{-5}$ cm

Formulae: (i) By Snell's law,

$$\frac{\sin i}{\sin r} = \mu$$

(ii) $2\mu t \cos r = n\lambda$

Solution: (i) $\dfrac{\sin 45°}{\sin r} = 1.33$

$$\sin r = \frac{0.707}{1.33}$$

$$\sin r = 0.53038$$

or $\boxed{r = 32°}$

(ii) The order of interference will be given by (condition for dark band)

$$2\mu t \cos r = n\lambda$$

$$n = \frac{2 \times 1.33 \times 1.5 \times 10^{-4} \times \cos 32°}{5 \times 10^{-5}}$$

$$\boxed{n = 6.78}$$

The order of interference,

$$n = 6$$

***Problem 2.18:** A wedge shaped air film having an angle of 40 seconds is illuminated by monochromatic light and fringes in reflected system are observed through a microscope. The distance between the consecutive bright fringes was measured as 0.12 cm. Calculate the wavelength of light used.*

Data:

$$\alpha = 40 \text{ sec.}$$
$$\beta = 0.12 \text{ cm}$$
$$\alpha = 40 \text{ sec}$$
$$\alpha = \frac{40 \times \pi}{60 \times 60 \times 180} \text{ radian}$$

Formula: $\alpha = 1.9 \times 10^{-4}$ rad.

The fringe width is given by,

$$\beta = \frac{\lambda}{2\alpha}$$
$$\lambda = 2\beta\alpha$$

Solution:

$$\lambda = 2 \times 0.12 \times 1.9 \times 10^{-4}$$
$$\lambda = 5 \times 10^{-5} \text{ cm}$$
$$\boxed{\lambda = 5000 \text{ A}^\circ}$$

***Problem 2.19:** A parallel beam of monochromatic light of wavelength λ = 5890 A° is incident on a thin film of μ = 1.5 such that the angle of refraction is 60°. Find the maximum thickness of the film so that it appears dark for normal incidence, what is the thickness required ?*

Data:

$$\lambda = 5890 \text{ A}^\circ$$
$$r = 60^\circ$$
$$\mu = 1.5$$

Formula: For dark band,

$$2\mu t \cos r = n\lambda$$
$$t = \frac{n\lambda}{2\mu t \cos r}$$

Solution: For maximum thickness, n = 1

$$t = \frac{1 \times 5890 \times 10^{-8}}{2 \times 1.5 \times \cos 60}$$
$$t = 4 \times 10^{-5} \text{ cm}$$

For normal incidence, r = 0 and hence cos r = 1.

$$\therefore \quad t = \frac{1 \times 5890 \times 10^{-8}}{2 \times 1.5 \times \cos 0}$$
$$\boxed{t = 2 \times 10^{-5} \text{ cm}}$$

***Problem 2.20:** An oil drop of volume 0.2 cc is dropped on the surface of a water tank of area 1 sq. m. The thin film spreads uniformly over the whole surface and white light reflected normally is observed through a spectrometer. The spectrum is seen to contain a first dark band whose centre has a wavelength of 5.5×10^{-5} cm. Find the refractive index of oil.*

Data:

$$V = 0.2 \text{ cc}$$
$$A = 1 \text{ sq. m.}$$
$$n = 1$$
$$\lambda = 5.5 \times 10^{-5} \text{ cm}$$

Formulae: (i) Volume = Area × thickness (ii) $2\mu t \cos r = n\lambda$

Solution: (i) $0.2 = 1 \times 10^4 \times t$

$$t = 2 \times 10^{-5} \text{ cm}$$

(ii) For minima,

$$2\mu t \cos r = n\lambda$$

Let, $r = 0$

$$\mu = \frac{n\lambda}{2t}$$
$$\mu = \frac{1 \times 5.5 \times 10^{-5}}{2 \times 2 \times 10^{-5}}$$
$$\boxed{\mu = 1.375}$$

***Problem 2.21:** A beam of monochromatic light of wavelength 5.82×10^{-7} m falls normally on a glass wedge of wedge angle of 20 seconds of an arc. If the refractive index of glass is 1.5, find the number of dark interference fringes per cm of the wedge length.*

Data:

$$\lambda = 5.82 \times 10^{-7} \text{ m}$$
$$\theta = 20 \text{ seconds}$$
$$\mu = 1.5$$

The angle in degrees,

$$\theta = \frac{20}{60 \times 60} \times \frac{\pi}{180}$$
$$\theta = 9.69 \times 10^{-5}$$

Formula:

The fringe width, $\beta = \dfrac{\lambda}{2\mu\theta}$

Solution:

$$\beta = \frac{5.82 \times 10^{-7}}{2 \times 1.5 \times 9.69 \times 10^{-5}}$$
$$\therefore \quad \beta = 0.2 \times 10^{-2} \text{ m} = 0.2 \text{ cm}$$

$\therefore$ Number of dark fringes/cm

$$= \frac{1}{\beta} = \frac{1}{0.2} = \boxed{5}$$

***Problem 2.22:** A parallel beam of sodium light of wavelength 5890×10^{-8} cm is incident on a thin glass plate of refractive index 1.5, such that the angle of refration into the plate is 60°. Calculate the smallest thickness of the plate which will make it appear dark by reflection.*

Data:

$$\lambda = 5890 \times 10^{-8} \text{ cm}$$
$$\mu = 1.5$$
$$r = 60^\circ$$

Formula: The condition for dark fringe in reflected system is

$$2\mu t \cos r = n\lambda$$

Solution: Taking $n = 1$

$$2 \times 1.5 \times t \times \cos 60 = 5890 \times 10^{-8}$$

$$\therefore \quad t = 3.926 \times 10^{-3} \text{ cm}$$

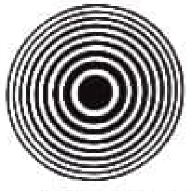
$$= \boxed{3.926 \times 10^{-5} \text{cm}}$$

2.6 NEWTON'S RINGS [May 18, 19]

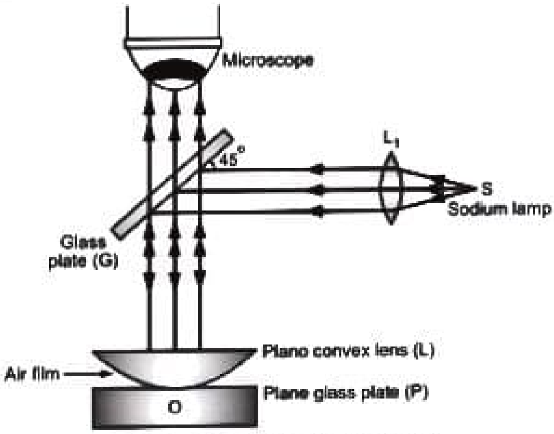
- When a **Plano-Convex Lens** of large focal length with its convex surface is placed in contact with a **Plane Glass Plate**, an air film of gradually increasing thickness is formed between them. The thickness of the film at the point of contact is zero and increases gradually outwards.
- If monochromatic light is allowed to fall normally, and the film is viewed in reflected light, alternate **Bright and Dark Rings** are observed. These rings are concentric around the point of contact between the lens and the glass plate. These fringes are called as **Newton's Rings** as they were discovered by Newton.

(i) Experimental Arrangement

- A plano-convex lens L of large radius of curvature is placed on a plane glass plate P. The point of contact between them is O. The light from an extended monochromatic source (sodium lamp) falls on a glass plate G held at an angle of 45° with the vertical.
- The glass plate G reflects normally a part of the incident light towards the air film between the lens L and the glass plate P. A part of the incident light is reflected by the curved surface of the lens L and a part is transmitted which is reflected back from the plane surface of plate P (i.e. rays are reflected from the top and bottom surfaces of the air film). These two reflected rays interfere and produce an **Interference Pattern** in the form of **Circular Rings**.
- These rings are **Localised** in the air film and can be seen with a microscope focused on the film.



(a) Typical Newton's rings pattern observed in reflected light



(b) Experimental arrangement for observing Newton's rings

Fig. 2.9

(ii) Explanation of the Formation of Newton's Rings

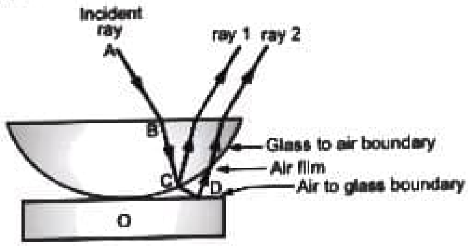


Fig. 2.10: Formation of Newton's rings

- When a monochromatic ray of light AB, is incident on the system, it gets partially reflected at C, the bottom of curved surface of the lens (glass-air boundary). This goes out in the form of ray 1 without any phase reversal. The other part is refracted along CD.
- At D, the top surface of the plane glass plate, it gets partially reflected to form ray 2. This ray has a phase reversal as it is reflected from air to glass boundary.
- As the rays 1 and 2 are derived from the same source and are coherent, so they interfere to form fringes. Interference does not take place between rays reflected from the surfaces of lens and glass plate due to their thickness which is much larger than wavelength of light.

(iii) Derivation

- The radius of curvature of plano-convex lens is very large and the small section of the air film trapped between lens and the glass plate will be similar to a wedged air film. Therefore, the optical path difference will be same as that of wedged air film.

The optical path difference for wedge film is,

$$\Delta = 2\mu t \cos(r + \alpha) \quad \text{... (2.31)}$$

For air film, $\mu = 1$, for normal incidence

$$\cos r = 1 \text{ and } \alpha \approx 0$$

$$\therefore \quad \Delta = 2t \quad \text{... (2.32)}$$

In Reflected System

Total optical path difference = Path difference due to thin film + Path difference due to reflections

$$\therefore \quad \Delta = 2t \pm \frac{\lambda}{2} \quad \text{... (2.33)}$$

Condition for Constructive Interference

For constructive interference the total phase difference should be an integral multiple of λ.

$$\Delta = n\lambda$$

$$\therefore \quad 2t \pm \frac{\lambda}{2} = n\lambda$$

$$2t = \left(n \pm \frac{1}{2}\right)\lambda \quad \text{... (2.34)}$$

Condition for Destructive Interference

For destructive interference the total phase difference should be an odd integral multiple of $\lambda/2$.

$$\Delta = (2n \pm 1)\frac{\lambda}{2}$$

$$\therefore \quad 2t \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

$$2t = n\lambda \quad \text{... (2.35)}$$

Radii of Bright Rings

- The plano-convex lens LOL' is placed on a glass plate AB. The point C is the centre of the sphere of which LOL' is a part. Let R be the radius of curvature of the lens and r_n be the radius of the n^{th} Newton's rings corresponding to the constant film thickness 't'.

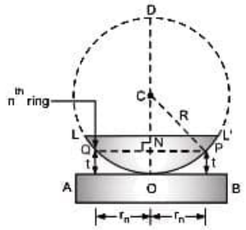


Fig. 2.11

- By the property of circle (theorem of intersecting chords),

$$NP \times NQ = NO \times ND$$

i.e. $$r_n \times r_n = t(2R - t) = 2Rt - t^2 \approx 2Rt$$

(as t^2 is very small)

$$\therefore \quad r_n^2 = 2Rt \quad \text{... (2.36)}$$

or $$t = \frac{r_n^2}{2R} = \frac{D_n^2}{8R}$$

(D_n being diameter of n^{th} bright ring) ... (2.37)

From equations (2.37) and (2.34), $\frac{2r_n^2}{2R} = (2n \pm 1)\frac{\lambda}{2}$

$$2 \cdot \frac{D_n^2}{8R} = (2n \pm 1)\frac{\lambda}{2}$$

$$D_n^2 = (2n \pm 1) \cdot 2\lambda R$$

$$D_n = \sqrt{2\lambda R} \cdot \sqrt{2n \pm 1} \text{ i.e. } D_n \propto \sqrt{2n \pm 1} \quad \text{... (2.38)}$$

Equation (2.38) shows that diameter of a bright ring is proportional to the square root of odd natural numbers.

Radii of Dark Rings [Dec. 18]

The condition for formation of dark Newton's ring is,

$$2t = n\lambda \quad \text{... (2.39)}$$

Substituting for t from (8),

$$2 \cdot \frac{D_n^2}{8R} = n\lambda$$

$$D_n^2 = 4n\lambda R \quad \text{... (2.40)}$$

$$D_n = 2\sqrt{n\lambda R} \text{ i.e. } D_n \propto \sqrt{n} \quad \text{... (2.41)}$$

Thus, the diameter of a dark ring is proportional to the square root of a natural number.

2.6.1 Properties of Newton's Rings

Rings get Closer Away from the Centre: Consider equation (2.41) giving the diameter of a dark ring. We have

$$D_n \propto \sqrt{n} \text{ and } D_{n+1} \propto \sqrt{n+1}$$

$$\therefore \quad D_{n+1} - D_n \propto (\sqrt{n+1} - \sqrt{n})$$

If constant of proportionality is taken as 1, then

$$D_{n+1} - D_n = \sqrt{n+1} - \sqrt{n}$$

$$\therefore \quad D_2 - D_1 = \sqrt{2} - \sqrt{1} = 0.414$$

$$D_3 - D_2 = \sqrt{3} - \sqrt{2} = 0.317$$

Therefore, the **Fringe Width Decreases** with the order of the fringe and the fringes get closer as the order increases. This can be shown for bright rings too.

This can also be explained in another way. The angle of the wedge increases as one moves away from the centre. From the equation for fringe spacing $\beta = \frac{\lambda}{2\mu\alpha'}$ the fringe separation decreases as the wedge angle α increases. Hence, the rings come closer with increase in their radii.

- **Dark Central Spot:** At the point of contact of the lens with the glass plate, the thickness of the air film t = 0. From equation (3), it can be seen that the path difference between rays reflected from the top and bottom surfaces of the film is $\lambda/2$. Hence, the interfering waves at the centre are opposite in phase and interfere destructively. Thus, a **Dark Spot** is produced at the centre.
- **Fringes of Equal Thickness:** It can be seen from equations (4) and (5) that **Maxima and Minima Occur Alternately** due to variation in the thickness t' of the film. Each maxima or minima is, therefore, a locus of constant film thickness. Hence, the fringes are called fringes of equal thickness.
- **Circular Fringes:** The circular wedge of air film may be regarded as having an axis passing through the point of contact O. This film bulges from the point of contact to outward with gradually increasing thickness of air film. The locus of points having the same thickness falls on a circle having its centre at the point of contact. Thus the thickness of the air film is the same at all points on any circle having O as the centre. The fringes are therefore circular. If the thickness satisfies the condition for constructive interference, the **Circular Fringe** is bright; otherwise it is dark.
- **Localised Fringes:** When the system is illuminated with a parallel light beam, the reflected rays are not parallel. They interfere near to the top surface of the film. When viewed from the top, the rays appear to diverge. As the fringes are seen at the upper surface of the film, they are said to be localised in the film.
- **White Light:** With white light, few **Coloured Fringes** are seen at centre. Away from centre they overlap.

2.7 APPLICATIONS OF NEWTON'S RINGS

2.7.1 Determination of Wavelength of Incident Light or Radius of Curvature of Plano-Convex Lens

- The experimental arrangement is shown in Fig. 2.9 (b). Let R be the radius of curvature of the lens and λ the wavelength of the light used. If D_n is the diameter of the n^{th} dark rings, then

$$D_n^2 = 4\, n\, \lambda\, R \qquad \text{... (2.42)}$$

Similarly, for $(n + p)^{th}$ dark rings,

$$D_{n+p}^2 = 4\,(n + p)\, \lambda\, R \qquad \text{... (2.43)}$$

Subtracting (2.42) from (2.43), we get,

$$D_{n+p}^2 - D_n^2 = 4\, p\, \lambda R$$

$$\therefore \quad \lambda = \frac{D_{n+p}^2 - D_n^2}{4\, p\, R} \qquad \text{... (2.44)}$$

- The microscope is adjusted to obtain Newton's rings. The centre of the cross wire is made to coincide with the central dark fringe. Counting the central fringe as n = 0, the cross wire is moved to n^{th} and $(n + p)^{th}$ dark fringe to the left and position of microscope is noted on micrometer screw gauge.
- In the same way position of n^{th} and $(n + p)^{th}$ fringe is noted on right. Subtracting position on left and right for n^{th} and $(n + p)^{th}$ fringe gives diameter of n^{th} and $(n + p)^{th}$ fringe respectively.
- Radius of curvature '**R**' is found using a spherometer.
- The wavelength λ of monochromatic source of light is found using relation (2.44). If λ is known, then same relation may be used to find R.

2.7.2 Determination of Refractive Index of a Liquid

- Firstly, perform the experiment when there is an air film between the glass plate and plano-convex lens. The system is placed in a metal container. The diameter of n^{th} and $(n + p)^{th}$ dark rings are determined using a travelling microscope.

For air,

$$D_{n+p}^2 - D_n^2 = 4\, p\, \lambda\, R \qquad \text{... (2.45)}$$

- Pour the liquid, whose refractive index is to be determined, in the container without disturbing the arrangement. The air film between the lower surface of the lens and the upper surface of the plate is replaced by the liquid. Now, measure the diameter of the n^{th} and $(n + p)^{th}$ dark rings.

- For liquid, we have $2\mu t \cos (r + \alpha) = n\lambda$ as the condition for darkness.

For normal incidence, $r = 0$ and $\alpha = 0$.

$$\therefore \quad 2\mu t = n\lambda \qquad \text{... (2.46)}$$

But
$$t = \frac{r_n'^2}{2R} = \frac{D_n'^2}{8R} \qquad \text{... (2.47)}$$

From (2.46) and (2.47),

$$2\mu \cdot \frac{D_n^2}{8R} = n\lambda$$

$$D_n^2 = \frac{4n\lambda R}{\mu} \qquad \text{... (2.48)}$$

- If $D_n'^2$ and $D_{n+p}'^2$ are the diameters of n^{th} and $(n + p)^{th}$ dark rings in liquid, then

$$D_n'^2 = \frac{4n\lambda R}{\mu} \qquad \text{... (2.49)}$$

$$D_{n+p}'^2 = \frac{4(n+p)\lambda R}{\mu} \qquad \text{... (2.50)}$$

Subtracting (2.49) from (2.50), we get

$$D_{n+p}'^2 - D_n'^2 = \frac{4p\lambda R}{\mu} \qquad \text{... (2.51)}$$

From equations (2.45) and (2.51),

$$\mu = \frac{D_{n+p}^2 - D_n^2}{D_{n+p}'^2 - D_n'^2} \qquad \text{... (2.52)}$$

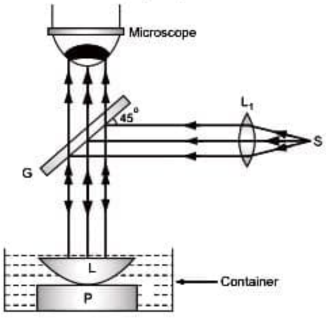


Fig. 2.12: Experimental arrangement for measurement of R.I. of liquid

Problem 2.23: *A convex lens is placed on a plane glass slab and is illuminated by a monochromatic light. The diameter of the 10^{th} dark ring is measured and is found to be 0.433 cm. The radius of curvature of the lower surface of the lens is 70 cm. Find the wavelength of the light used.*

Data: $R = 70$ cm, $n = 10$, $D_n = 0.433$ cm

Formula: $D_n^2 = 4nR\lambda$

Solution: $(0.433)^2 = 4 \times 10 \times 70 \times \lambda$

$$\therefore \quad \lambda = \frac{(0.433)^2}{4 \times 10 \times 70}$$

$$\lambda = 6.696 \times 10^{-5} \text{ cm}$$

$$= \boxed{6696 \text{ A}^\circ}$$

Problem 2.24: *In a Newton's rings experiment, the diameter of the 15^{th} dark ring was found to be 0.590 cm and that of the 5^{th} dark ring was 0.336 cm. If the radius of the plano-convex lens is 100 cm, calculate the wavelength of the light used.*

Data: $D_{15} = 0.590$ cm, $D_5 = 0.336$ cm, $R = 100$ cm, $m = 10$

Formula: $\lambda = \frac{(D_{n+m})^2 - (D_n)^2}{4mR}$

Solution: $\lambda = \frac{D_{15}^2 - D_5^2}{4 \times 10 \times R} = \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 100}$

$$\lambda = 5.880 \times 10^{-5} \text{ cm}$$

$$\boxed{\lambda = 5880 \text{ A}^\circ}$$

Problem 2.25: *The diameter of a dark ring in Newton's rings experiment decreases from 1.4 cm to 1.2 cm when air is replaced by a liquid as medium between lens and flat surface. Calculate the refractive index of the liquid.*

Data: $D_{air} = 1.4$ cm, $D_{liquid} = 1.2$ cm

Formula: $\mu = \frac{D_{air}^2}{D_{liquid}^2}$

Solution: $\mu = \frac{(1.4)^2}{(1.2)^2}$

$$\boxed{\mu = 1.36}$$

Problem 2.26: *The diameter of the tenth dark ring in Newton's rings experiment is 0.5 cm. Calculate the radius of curvature of the lens and the air thickness at the position of the ring. The wavelength of light used is 5000 A°.*

Data: $D_{10} = 0.5$ cm, $n = 10$, $\lambda = 5000 \times 10^{-8}$ cm

Formulae: (i) $D_n^2 = 4nR\lambda$, (ii) $t = \frac{D_n^2}{8R}$

Solution: (i) $R = \frac{D_n^2}{4n\lambda} = \frac{0.5^2}{4 \times 10 \times 5000 \times 10^{-8}}$

$$\boxed{R = 125 \text{ cm}}$$

(ii) Thickness is given by

$$t = \frac{D_n^2}{8R} = \frac{0.5^2}{8 \times 125} = \boxed{2.5 \times 10^{-4}\ \text{cm}}$$

***Problem 2.27:** In a Newton's ring experiment, find the radius of curvature of the lens surface in contact with the glass plate when with a light of wavelength 5890 A°, the diameter of the third dark ring is 0.32 cm. The light is incident normally.*

Data: λ = 5890 A°, D_3 = 0.32 cm, n = 3

Formula: $D_n^2 = 4Rn\lambda$

Solution:

$$R = \frac{D_n^2}{4n\lambda}$$

$$R = \frac{(0.32)^2}{4 \times 3 \times 5890 \times 10^{-8}}$$

$$\boxed{R = 144.87\ \text{cm}}$$

***Problem 2.28:** In Newton's rings, the diameter of a certain bright ring is 0.65 and that of tenth ring beyond it is 0.95 cm. If λ = 6000 A°, calculate the radius of curvature of a convex lens surface in contact with the glass plate.* **[May 18]**

Data: D_n = 0.65 cm, D_{n+p} = 0.95 cm,

$\lambda = 6 \times 10^{-5}$ cm

Formula:

$$\frac{(D_{n+p})^2 - D_n^2}{4m\lambda}$$

Solution:

$$R = \frac{(0.95)^2 - (0.65)^2}{4 \times 10 \times 6 \times 10^{-5}}$$

$$\boxed{R = 200\ \text{cm}}$$

***Problem 2.29:** In a Newton's ring experiment, a drop of water $\left(\mu = \frac{4}{3}\right)$ is placed between the lens and the plate. In this case, the diameter of the 10th ring was found to be 0.6 cm. Calculate the radius of curvature of the face of the lens in contact with the plate. Given: λ = 6000 A°.*

Data: μ = 1.3333, D_{10} = 0.6 cm, $\lambda = 6 \times 10^{-5}$ cm, n = 10

Formula: $D_n^2 = \frac{4n\lambda R}{\mu}$

Solution:

$$R = \frac{D_n^2 \times \mu}{4n\lambda} = \frac{(0.6)^2 \times 1.3333}{4 \times 10 \times 6 \times 10^{-5}}$$

$$\boxed{R = 200\ \text{cm}}$$

***Problem 2.30:** Newton's rings are observed in reflected length of λ = 5900 A°. The diameter of the 5th dark ring is 0.4 cm. Find the radius of curvature of the lens and thickness of the air film.*

Data: $\lambda = 5.9 \times 10^{-5}$ cm, n = 5, D_5 = 0.4 cm, $\therefore$ r = 0.2 cm

Formula: $D_n^2 = 4nR\lambda$

Solution:

$$R = \frac{(0.4)^2}{4 \times 5 \times 5.9 \times 10^{-5}}$$

$$R = 135.59\ \text{cm}$$

$$t = \frac{r^2}{2R} = \frac{(0.2)^2}{2 \times 135.59}$$

$$\boxed{t = 1.475 \times 10^{-4}\ \text{cm}}$$

***Problem 2.31:** In a Newton's ring experiment, the diameters of 4th and 12th dark rings are 0.4 cm and 0.7 cm respectively. Calculate the diameter of 20th dark ring.*

Data: m = 12, n = 4, D_m = 0.7 cm, D_n = 0.4 cm.

Formulae: (i) $R = \frac{D_{n+m}^2 - D_n^2}{4(m-n)\lambda}$

(ii) $D_n^2 = 4nR\lambda$

$$\therefore \quad D_n^2 = 4n\left(\frac{D_{n+m}^2 - D_n^2}{4(m-n)\lambda}\right) \cdot \lambda$$

$$D_n^2 = \frac{4n(D_{n+m}^2 - D_n^2)}{4m}$$

Solution: $D_{20}^2 = 4 \times \frac{(0.7)^2 - (0.4)^2}{4(8)} \times 20 = \boxed{0.908\ \text{cm}}$

***Problem 2.32:** If the diameter of nth dark ring in a Newton's ring experiment changes from 0.3 cm to 0.25 cm, as liquid is placed between the lens and the plate, calculate the value of μ of the liquid.*

Data: D_{air} = 0.3 cm, D_{liquid} = 0.25 cm

Formula:

$$\mu = \frac{(D_n)^2_{air}}{(D_n)^2_{liquid}}$$

Solution:

$$\mu = \frac{(0.3)^2}{(0.25)^2} = \boxed{1.44}$$

***Problem 2.33:** In Newton's rings experiment the diameters of nth and (n + 8)th bright rings are 4.2 mm and 7.00 mm respectively. Radius of curvature of the lower surface of the lens is 2.00 m. Determine the wavelength of the light.*

Data:

$D_n = 4.2$ mm

$D_{n+8} = 7$ mm

$R = 2$ m

Formula:

$$\lambda = \frac{D_{n+m}^2 - D_n^2}{4\,m\,R}$$

Solution:

$$\lambda = \frac{D_{n+8}^2 - D_n^2}{4\,(n+8-n)\,R}$$

$$\lambda = \frac{0.7^2 - 0.42^2}{4 \times 8 \times 200}$$

$$\lambda = 5 \times 10^{-5}\ \text{cm}$$

$$\boxed{\lambda = 5000\ \text{A}^\circ}$$

Problem 2.34: *Newton's rings are formed by light reflected normally from a plano-convex lens and a plane glass plate with a liquid between them. The diameter of n^{th} ring is 2.18 mm and that of $(n+10)^{th}$ ring is 4.51 mm. Calculate the refractive index of the liquid, given that the radius of curvature of the lens is 90 cm and wavelength of light is 5893 A°.*

Data:

$D'_n = 2.18$ mm

$D'_{n+10} = 4.5$ mm

$R = 90$ cm

$\lambda = 5893$ A°

Formula:

$$R = \frac{\mu\,(D'^2_{n+m} - D'^2_n)}{4m\lambda}$$

$$\mu = \frac{4\,m\,\lambda\,R}{D'^2_{n+m} - D'^2_n}$$

Solution:

$$\mu = \frac{4 \times 10 \times 5893 \times 10^{-8} \times 90}{0.45^2 - 0.218^2}$$

$$\boxed{\mu = 1.368}$$

Problem 2.35 *: In a Newton's rings experiment, the diameter of the 5^{th} ring was 0.336 cm and that of 15^{th} ring was 0.59 cm. Find the radius of curvature of the plano-convex lens, if the wavelength of light used is 5890 A°.*

Data:

$D_{15} = 0.59$ cm

$D_5 = 0.336$ cm

$\lambda = 5890$ A°

$m = 10$

Formula:

$$R = \frac{D_{n+m}^2 - D_n^2}{4\,m\,\lambda}$$

Solution:

$$R = \frac{D_{15}^2 - D_5^2}{4 \times 10 \times \lambda}$$

$$R = \frac{(0.59)^2 - (0.336)^2}{4 \times 10 \times 5890 \times 10^{-8}}$$

$$\boxed{R = 99.83\ \text{cm}}$$

2.8 INTRODUCTION TO POLARIZATION

- The phenomenon like interference or diffraction prove the wave nature of light. But it does not tell us whether the light waves are longitudinal or transverse. Because even longitudinal waves, like sound waves, show the phenomena of interference and diffraction.
- The important difference between longitudinal and transverse wave is that the transverse waves can be polarized.
- The phenomenon of polarization can be explained only by considering the transverse nature of light. And it has been proved by electromagnetic theory that the light is transverse wave.

2.9 POLARIZATION OF WAVES

- The transverse nature of waves leads to the characteristic phenomenon called **Polarization**. The characteristic, polarization is not exhibited by longitudinal waves. Thus only transverse waves could be polarized.
- In a transverse wave, if the directions of all the vibrations at all the points are restricted to one particular plane, then the wave is called **Polarized**, more specific plane polarized. A plane polarized wave is the simplest of a transverse wave, which is also termed as **Linearly Polarized Wave**.

2.10 REPRESENTATION OF POLARIZED LIGHT

[May 18]

- According to the electromagnetic theory, light consists of electric and magnetic vectors vibrating continuously with time in a plane, transverse to the direction of propagation of light and to each other. However, **in explaining polarization only the vibrations of the electric vector are considered.**
- It does not mean that magnetic field vectors are absent, they are present. But for drawing simplicity they are not shown in the diagram.

(i) Unpolarized Light

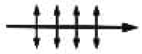
- The light having vibrations along all possible directions perpendicular to the direction of propagation of light, is called an **Unpolarized Light**. The vibrations are symmetrical about the direction of propagation of light.

Fig. 2.13: Unpolarized light

- It can be considered to consist an infinite number of waves each having its own vibration. Since unpolarized light has vibrations along all possible directions, at right angles to the directions of propagation of light, it is represented by a star.

(ii) Polarized Light

- The light having vibration only along a single plane perpendicular to the direction of propagation of light is called a **Polarized Light**. It's vibrations are one sided, therefore it is dissymmetrical about the direction of propagation of light.
- The polarized beam of light has vibrations along a single plane. If they are parallel to the plane of the paper, they are represented by arrows [See Fig. 2.14 (a)]. If they are perpendicular to the plane of the paper, they are represented by dots on a ray of light. [See Fig. 2.14 (b)].



(a) Parallel to plane of paper

(b) Perpendicular to plane of paper

Fig. 2.14: Plane polarized light

(iii) Partially Polarized Light

- A partially polarized light is a mixture of plane polarized and unpolarized light. It is represented as shown in Fig. 2.15.
- In partially polarized light the vibrations in the plane of plane polarized light dominate over the vibrations in other directions.

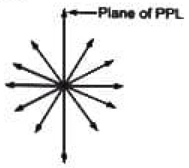


Fig. 2.15: Partially polarized light

2.11 METHODS OF PRODUCTION OF POLARIZED LIGHT

- Although polarized light has many applications in science and engineering, but the light available naturally is unpolarized. So different methods have been developed to obtain polarized light artificially.
- Every method uses one or the other optical phenomena like reflection, refraction, scattering, double refraction etc., for getting polarized light.
- Here we will be learning some of the methods for obtaining plane polarized light.

2.11.1 Production of Plane Polarized Light by Reflection [May 18]

- Polarization of light by reflection from the surface of glass was discovered by Malus in 1808. He found that polarized light is obtained when ordinary light is reflected by a plane sheet of glass.

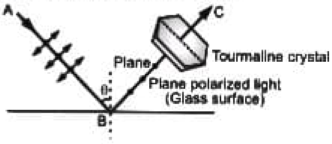


Fig. 2.16

- Consider the light incident along the path AB on the glass surface. A part of light is reflected along BC. In the path of BC, place a tourmaline crystal and rotate it slowly. It is observed that light is completely extinguished only at one particular angle of incidence.
- At any other angle of incidence there is preferential reflection of the components having vibrations perpendicular to the plane of incidence.
- This angle of incidence is equal to 57.5° for a glass surface and is known as the **Polarization Angle**.
- The vibrations of the incident light can be resolved into components-parallel to the reflecting surface (glass surface) and perpendicular to the reflecting surface. Light due to the components parallel to the reflecting surface is reflected whereas light due to the components perpendicular to the reflecting surface is transmitted i.e. the plane of the vibrations of reflecting rays are at right angles to the plane of incidence and the plane of vibrations of refracted rays are in the plane of incidence. Thus, light reflected by the surface is polarised in the plane of incidence and can be detected by tourmaline crystal.

Note

- If light is polarised perpendicular to the plane of incidence, it means that vibrations are in the plane of incidence.
- If light is polarised in the plane of incidence, it means that vibrations are perpendicular to the plane of incidence.

Polarizing Angle or Angle of Polarization

- It is defined as that angle of incidence on the reflecting surface for which reflected light is completely plane polarized.
- As the refractive index of a substance varies with the wavelength of the incident light, the polarizing angle will be different for light of different wavelength. Therefore, polarising angle will be complete only for light of a particular wavelength at a time i.e. for monochromatic light (for a given surface).

Brewster's Law

In 1811, Sir David Brewster found that ordinary light is completely polarised in the plane of incidence when it gets itself reflected from a transparent medium at a particular angle known as the **Polarizing Angle**.

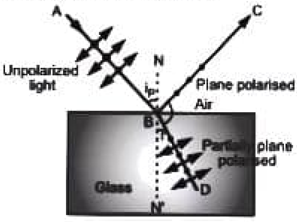


Fig. 2.17

- He was able to prove that, the tangent of the angle of polarisation is numerically equal to the refractive index of the medium. i.e. $\mu = \tan i_p$.
- Consider unpolarised light is incident on the glass surface at the polarising angle. It is reflected along BC and refracted along BD.

From Snell's law,

$$\mu = \frac{\sin i}{\sin r} \quad \text{... (2.53)}$$

From Brewster's law,

$$\mu = \tan i_p = \frac{\sin i_p}{\cos i_p} \quad \text{... (2.54)}$$

Comparing equations (2.53) and (2.54),

$$\cos i_p = \sin r = \cos\left(\frac{\pi}{2} - r\right)$$

$$\therefore \quad i_p = \frac{\pi}{2} - r$$

$$i_p + r = \frac{\pi}{2}$$

As $\quad i_p + r = \frac{\pi}{2}$,

$$\angle CBD = \frac{\pi}{2}$$

Therefore, reflected and refracted rays are at right angles to each other.

2.11.2 Production of Plane Polarized Light by Refraction: Pile of Plates

- When unpolarized light is incident at an polarizing angle on a transparent surface the reflected light is polarized completely whereas refracted light is partially polarized.

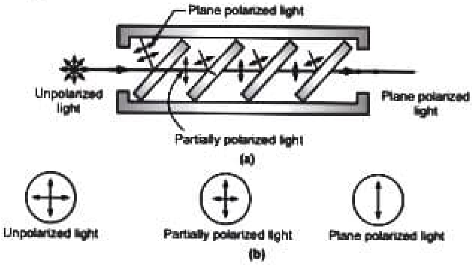


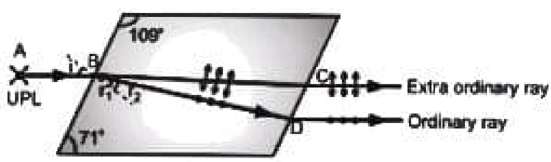
Fig. 2.18: Polarization by refraction

- If more than one refracting surface i.e. stack of glass plates are used in place of one, the process is repeated. At every surface, the unpolarized component decreases, thus the polarized component becomes prominent, giving almost plane polarized light in the direction parallel to the pile of plates.
- A pile of plates contains about 15 glass plates placed in a metal tube of suitable size. The plates are kept at 33° with the axis of tube so that the incident unpolarized light is incident at polarizing angle at the first plate.
- The unpolarized light entering in the tube will undergo successive reflection and refraction such that the emerging ray is plane polarized light.

2.11.3 Double Refraction [Dec. 17]

- The phenomenon of double refraction was discovered by Erasmus Bartholinus in 1669 during his studies on calcite. When light is incident on a calcite crystal, it is found to produce two refracted rays which are different in properties. The phenomenon of causing **Two Refracted Rays** by a crystal is called **Birefringence** or **Double Refraction**. The crystals are said to be **Birefringent**.

(a)



(b)

Fig. 2.19: Double reflection

- All anisotropic materials exhibit double refraction. The two rays formed in double refraction are linearly polarized in mutually perpendicular directions.
- One of the rays **obeys Snell's Law** of refraction and hence is called **an Ordinary Ray or O-ray**. The other ray **does not obey Snell's Law** and is called an **Extraordinary Ray or E-Ray.** Both of them are linearly/plane polarised, but plane of polarisation is perpendicular to each other. If one of the rays is eliminated, the light transmitted by the crystal will be a linearly/plane polarized light.
- When a ray of light AB is incident on the calcite crystal making an angle of incidence i, it is refracted along two paths inside the crystal: (i) along BC making an angle of refraction r_2, (ii) along BD making an angle of refraction r_1. These two rays emerge out along DO and CE which are parallel as the crystal faces are parallel.

Optic Axis

- The **Optic Axis** is the direction of symmetry of unisotropic media along which double refraction does not take place.
- A line drawn through any of the blunt corners making equal angles with each of the three edges gives the direction of the optic axis. In fact any line parallel to this line is also an optic axis. Therefore, optic axis is not a line but **It Is a Direction.**

Principal Section

- A plane containing **The Optic Axis** and **Perpendicular to the Opposite Faces** of the crystal is called the **Principal Section Of The Crystal.** The principal section cuts the surfaces of a calcite crystal in a parallelo-gram with angles 109° and 71°.

Fig. 2.20: Principal section of calcite crystal

Principal Plane

- The plane containing the optic axis and the ordinary ray is called principal plane of the ordinary ray. Similarly the plane containing the optic axis and the extraordinary ray is called the principal plane of the extraordinary ray.
- Experiments revealed that the vibrations of the ordinary rays are perpendicular to the principal section of the crystal while the vibrations of the extraordinary rays are parallel to the principal section of the crystal. Thus, the two rays are plane polarized, their vibrations being at right angles to each other.

2.11.4 Polarization by Double Refraction - Nicol Prism

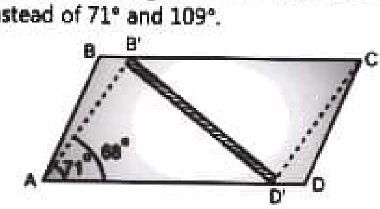
- Nicol prism is an optical device used for producing and analysing plane polarised light.

Principle

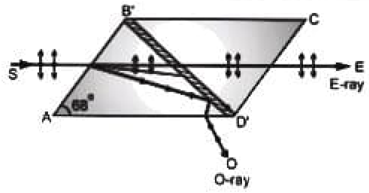
- The Nicol prism is made in such a way that it eliminates one of the refracted rays by total internal reflection i.e. O-ray is eliminated and only E-ray is transmitted through the prism.

Construction

A **calcite crystal** whose length is three times it's breadth is taken. Let ABCD be the principal section of the crystal with ∠ BAD = 71°. The end faces of the crystal are cut in such a way that they make angles 68° and 112° in the principal section instead of 71° and 109°.



(a)



(b)

Fig. 2.21

- The crystal is then cut into two pieces from one blunt corner to the other along a plane perpendicular to the principal section. The two cut faces are grounded and polished optically flat. It is then cemented together by Canada balsam whose refractive index lies between the refractive indices for the O-ray and E-ray for calcite.

  Refractive index of Calcite for O-ray

  $$\mu_o = 1.658$$

  Refractive index of Canada balsam

  $$\mu_c = 1.55 \text{ Using sodium light of } \lambda = 5893 \text{ A}^\circ,$$

  Refractive index of Calcite for E-ray

  $$\mu_e = 1.486$$

- Canada balsam layer acts as a rarer medium for O-ray and as a denser medium for E-ray. Except the end faces, the sides of the crystal are blackened.

Working

- When a ray of unpolarized light is incident on the prism surface, it splits into O-ray and E-ray. Both the rays are polarized having vibrations at right angles to each other.
- When the O-ray passes from a portion of the crystal into the layer of Canada balsam, it passes from a denser medium to rarer medium. When the angle of incidence is greater than the critical angle, the O-ray is totally internally reflected and is not transmitted.
- When the E-ray passes from calcite to the Canada balsam layer, it enters in rarer medium. Therefore, the E-ray is not affected and is transmitted through the prism.

  Refractive index for O-ray with respect to Canada balsam,

  $$\mu = \frac{1.658}{1.55}$$

  If C is the critical angle,

  $$\therefore \quad \mu = \frac{1}{\sin C}$$

  $$\sin C = \frac{1}{\mu} = \frac{1.55}{1.658}$$

  $$C = 69^\circ$$

- As the length of the crystal is large, the angle of incidence at Canada balsam surface for the O-ray is greater than the critical angle. Thus, it suffers total internal reflection while E-ray is transmitted which is plane polarized having vibrations in the principal section.

Special Cases

- If the angle of incidence is less than the critical angle for O-ray, it is not reflected and is transmitted through the prism. In this position, both the O-ray and E-ray are transmitted through the prism.
- The E-ray also has a limit beyond which it is totally internally reflected by Canada balsam surface. If E-ray travels along the optic axis, its refractive index is the same as that of O-ray i.e. 1.658. But it is 1.486 for all other directions of E-ray. Therefore depending on the direction of propagation of E-ray, μ_e lies between 1.486 and 1.658. Therefore for a particular case, μ_e may be more than 1.55 and the angle of incidence will be more than the critical angle. Then E-ray will also be totally internally reflected.

2.12 HUYGEN'S THEORY OF DOUBLE REFRACTION [Dec. 18]

Huygen explained the phenomenon of double refraction on the basis of the principle of secondary wavelets.

He assumed:

- When a beam of ordinary unpolarized light strikes a doubly refracting crystal, each point on the surface sends out **Two Wavefronts**, one for ordinary ray and the other for extraordinary ray.
- The **Ordinary-Ray** travels with the **Same Speed** v_o in all directions and the crystal has a single refractive index $\mu_o = \frac{c}{v_o}$ for this wave. Thus, the O-ray has a **Spherical Wavefront**.
- The **Speed of Extra-Ordinary Ray V_e Varies with Direction**. So, the refractive index, $\mu_e = \frac{c}{v_e}$ also varies with direction for the E-ray. Therefore, the extra-ordinary ray develops a wavefront which is **Ellipsoidal**.

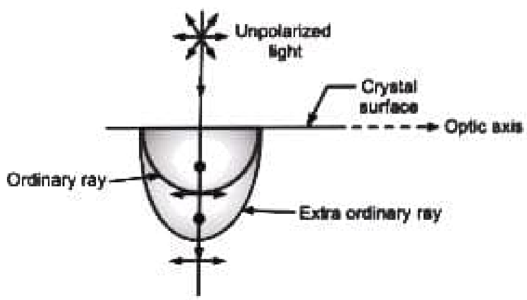
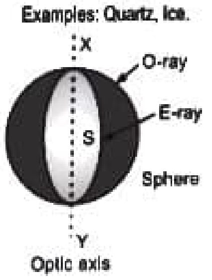
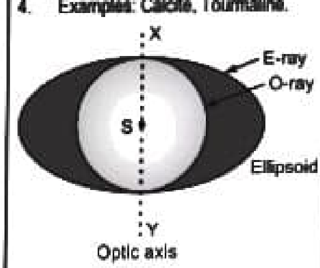


Fig. 2.22: Double refraction

- The velocity v_e measured is perpendicular to the optic axis.
- The velocities of the O-ray and E-ray are the same along the optic axis.
- When rays are incident along the optic axis, the spherical and ellipsoidal wavefronts touch each other at points of intersection with the optic axis and double refraction does not take place.
- If $v_o > v_e$ or $\mu_o < \mu_e$, the spherical wavefront lies outside the elliptical wavefront. Such crystals are called **Positive Crystals**. The examples of positive crystal are quartz, ice etc.
- If $v_e > v_o$ or $\mu_e < \mu_o$, the elliptical wavefront lies outside the spherical wavefront. Such crystals are called **Negative Crystals**. The examples of negative crystals are calcite, tourmaline, etc.

2.12.1 Positive and Negative Crystals

Positive Crystals	Negative Crystals
1. For positive crystals, $v_o > v_e$ and $\mu_o < \mu_e$.	1. For negative crystals, $v_o < v_e$ and $\mu_o > \mu_e$.
2. The velocity of O-ray is same in all directions.	2. The velocity of O-ray is same in all directions.
3. The wavefront of O-ray lies outside the wavefront of E-ray.	3. The wavefront of O-ray lies inside the wavefront of E-ray.
4. Examples: Quartz, Ice.  Fig. 2.23 (a)	4. Examples: Calcite, Tourmaline.  Fig. 2.23 (b)

2.13 CASES OF DOUBLE REFRACTION OF CRYSTAL CUT WITH OPTIC AXIS LYING IN THE PLANE OF INCIDENCE

2.13.1 Parallel to the Surface

- Fig. 2.24 shows unpolarized plane wavefront AB incident normally on the crystal surface XY. The optic axis lies along XY and is in the plane of incidence.
- At the points A and B, it develops two wavefronts, one spherical for O-ray and one elliptical for E-ray. The envelope of O-ray and E-ray gives the corresponding wavefront which is plane polarized.
- It should be noted that both O-ray and E-ray are plane polarized light. Here both O-ray and E-ray travel along the same direction with different velocities. As O-ray and E-ray travel along the same direction with different velocities, a path difference is introduced between them.
- This principle is used in the construction of quarter and half-wave plates.

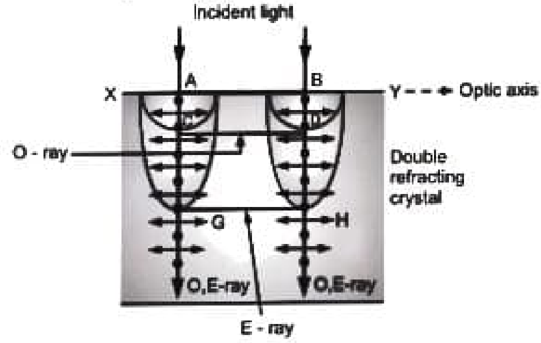


Fig. 2.24

2.13.2 Perpendicular to the Surface

- Fig. 2.25 shows unpolarized plane wavefront AB incident normally on the crystal surface XY. Optic axis lies in the plane of incidence and perpendicular to the crystal surface.
- As the light is incident in the direction of optic axis, O-ray and E-ray travel with the same speed along the optic axis. As a result O-ray and E-ray travel along the same directions with same velocity. Hence the phenomenon of **Double Refraction is Absent** in this case. Ordinary and extraordinary wavefronts CD and GH coincide at all instants.

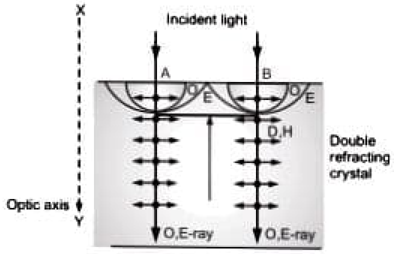


Fig. 2.25

2.13.3 Inclined to the Surface

- Fig. 2.26 shows an unpolarized plane wavefront incident normally on the crystal surface so that the optic axis makes an angle with the crystal surface.
- O-ray and E-ray travel with different velocities in different direction in the crystal. Hence double refraction is seen in this case and both O-ray and E-ray are separated by an angle depending upon the distance travelled in crystal.

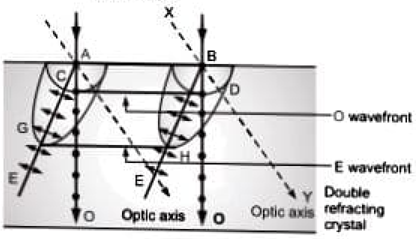


Fig. 2.26

2.14 LAURENT'S HALF SHADE POLARIMETER

2.14.1 Optical Activity

- When a beam of a plane polarized light is directed along the optic axis of quartz, the plane of polarization turns steadily about the direction of the beam and the beam emerges vibrating in some other plane than that at which it has entered.
- The amount of rotation depends upon the distance travelled in the medium and wavelength of the light. This phenomenon of rotation of the plane of polarization is called **Optical Activity**. The substances which show optical activity are sodium chlorate, turpentine, sugar crystal etc. Fig. 2.27 shows optical activity.

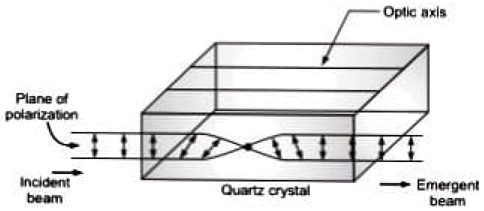


Fig. 2.27: Optical activity

- Some crystals rotate the plane of vibration to the right and some to the left. The substances which rotate to the right are called **Right Handed or Dextro-Rotatory** and those which rotate to the left are called **Left Handed** or **Laevo-Rotatory**.

2.14.2 Specific Rotation

- A striking feature of optical activity is that different colours are rotated by different amount. This rotation is nearly proportional to the inverse square of the wavelength. This gives a **Rotatory Dispersion**, violet being rotated nearly four times as much as red light. Fig. 2.28 shows rotatory dispersion.

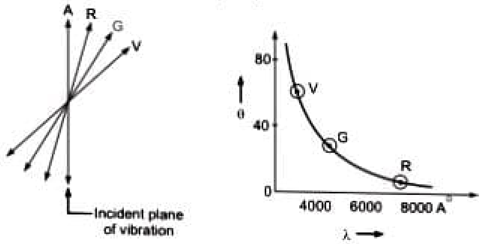


Fig. 2.28: Rotatory dispersion

- The rotation for a 1 mm thick plate is called the **Specific Rotation**.

2.14.3 Optically Active Materials

- Optical activity is exhibited by organic compounds whose molecular arrangement lacks in symmetry. Therefore, upon entering in the material, the plane polarized light changes the plane of polarization depending upon the molecular arrangement of the material.
- Most of the petroleum exhibits optical activity which are organic in nature. The optical activity is not exhibited by synthetic materials as they are mixture of left handed and right handed molecules in equal quantity, thus giving net zero rotation.

2.14.4 Laurent's Half Shade Polarimeter

- Polarimeters are instruments, used for finding the optical rotation of different solutions. When they are calibrated to read directly the percentage of cane sugar in a solution, they are named as saccharimeters.
- Polarimeters can be used to find the specific rotation of sugar solution or if the specific rotation is known, they can be used to find its concentration.

Construction :

- The essential parts of a polarimeter are as shown in Fig. 2.29. Light from a monochromatioc source S is rendered parallel by a collimating lens L. N_1 and N_2 are two Nicol prisms, N_1 acts as a polarizer while N_2 acts as a analyzer. N_2 is capable of rotation about a common axis of N_1 and N_2. The rotation of N_2 can be read on a graduated circular scale S.C. The light after passing through the polarizer N_1 becomes plane polarized with its vibrations in the principal plane of the Nicol N_1.
- The plane polarized light now passes through a half shade device HS and then through a glass tube BC containing the optically active substance. The tube is closed at the end by metal covers. The light emergent from the analyzer N_2 is viewed through a telescope T. The telescope is focused on the half shade.

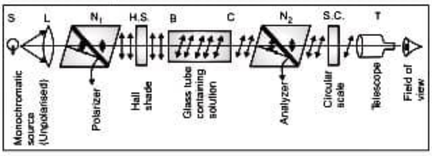


Fig. 2.29 : Laurent's half shade polarimeter

Action of Half Shade

- When an optically active substance is placed in between two crossed Nicols, the field of view is not dark. In order to make it dark the analyzer is rotated. It is observed that, when the analyzer is rotated, the field of view is not dark for a considerable region. Hence the measurement of optical is not accurate. To avoid this difficulty, a half-shade device is used. Laurent's half-shade plate consists of a semi-circular half wave plate ACB of quartz.
- The thickness of the quartz is so chosen that it introduces a phase difference of π between the ordinary and extraordinary ray passing through it. The other half ADB is made of glass and its thickness is such that is absorbs and transmits the same amount of light as done by the quartz half-plate. The two plates are cemented along the diameter AB. The optic axis of the quarts plate lies along the line AB.

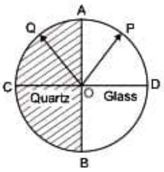


Fig. 2.30 : Laurent's half shade plate

- Let the plane polarized light coming from the polarizer be incident normally on the half shade the plate with its vibrations parallel to OP. Here OP makes an angle θ with AB. The vibrations emerge from glass plate along the plane OP. Inside the quartz plate, the incident ray would be split up into two ordinary and extraordinary components. One having vibrations along OA and the other along OD. These rays travel with unequal velocities through the quartz plate which introduces a phase difference of π between them.
- Hence on emerging from the plate, the vibrations will be along OA and PC and their resultant vibrations along OQ, where $\angle$ AOOP = $\angle$ AOQ. If the initial position of ordinary components is represented by OD then the final position is represented by OC. If the principal plane of the analyzing Nicol is parallel to OP, then the light emerging from glass portion will pass unobstructed while light from quartz will be partly obstructed.
- Due to this fact, the glass half will appear brighter then the quartz half. On the other hand, if the principal plane of the analyzer is parallel to OQ, the light from quartz portion will be unobstructed while light from glass will be partly obstructed.
- Thus, the quartz half will appear brighter than the glass half. The two halves will look equally illuminated when the analyzer is so turned that its principal plane is exactly parallel to AB. Any slight rotation in either direction produces a sharp difference in the illumination of the two halves.

Determination of Specific Rotation

- Specific rotation S is given by

$$S = \frac{\theta}{l \times c}$$

where θ is the angle of rotation in degrees, l, is the length of the solution in decimeters and c is the concentration of solution in gm/cc. Hence, to determine the specific rotation of a substance, a solution of known concentration is prepared. The length of the solution is measured directly. The value of θ is determined as follows :

- The experimental tube is filled with distilled water and placed in its position. The telescope is focused on the half-shade plane and the analyzer is rotated till equally bright position is observed in the field of view.
- The readings of two verniers on the circular scale is noted. Now, the tube is filled with the optically active solution and placed in its position. The analyzer is rotated and is brought to a position so that the whole field of view is equally bright. The new positions of the two verniers are again noted on the circular scale.
- The difference in the two readings of the same vernier gives the angle of rotation, θ produced by the solution. Thus knowing θ, l and c the specific rotation S can be calculated by the given formula.

Problem 2.36: *Two polarizing plates have polarizing directions parallel so as to transmit maximum intensity of light. Through what angle must either plate be turned if the intensity of the transmitted beam is to drop to one third ?*

Data: $I = \frac{I_0}{3}$

Formula: From Law of Malus, $I = I_0 \cos^2 \theta$

Solution:

Substituting, $\frac{I_0}{3} = I_0 \cos^2 \theta$

$$\cos^2 \theta = \frac{1}{3}$$

Or $\cos \theta = \pm \frac{1}{\sqrt{3}}$

$$\boxed{\theta = 54° \, 41' \text{ or } \pm 144° \, 40'}$$

Problem 2.37: *At a certain temperature, the critical angle of incidence of water for total internal reflection is 48° for a certain wavelength. What is the polarizing angle and the angle of refraction for light incident on the water that gives maximum polarization of the reflected light ?*

Data: Critical angle $C = 48°$

Formulae: (i) $\mu = \frac{1}{\sin C}$, (ii) $\mu = \tan i_p$

Solution:

(i) Substituting, $\mu = \frac{1}{\sin 48°}$

$$\boxed{\mu = 1.345}$$

(ii) From Brewster's law,

$$\mu = \tan i_p$$
$$1.345 = \tan i_p$$
$$i_p = \tan^{-1} (1.345)$$
$$i_p = 53° \, 22'$$

But $i_p + r = 90°$

$\therefore$ $r = 90° - i_p$

$$90° - 53° \, 22' = r$$

$\therefore$ $\boxed{r = 36° \, 38}$

Problem 2.38: *Two Nicol prisms are oriented with their principal planes making an angle of 60°. What percentage of incident unpolarized light will pass through the system ?*

Data: $\theta = 60°$

Formulae: (i) For unpolarized light,

$$I = \frac{I_0}{2}$$

(ii) For plane polarized light,

$$I_T = I \cos^2 \theta = \frac{I_0}{2} \cos^2 \theta$$

Solution: $I_T = \frac{I_0}{2} \cos^2 60°$

$$I_T = 0.125 \, I_0$$

$\therefore$ The percentage of incident unpolarized light transmitted through the system is

$$\% \, I_T = 0.125 \times 100$$

$\therefore$ $\boxed{\% \, I_T = 12.5\%}$

Problem 2.39: *A polarizer and an analyzer are oriented so that the amount of light transmitted is maximum. How can the analyzer be oriented so that the transmitted light is reduced to (1) 0.75, (2) 0.25 ?*

Data: (1) $I = 0.75 \, I_0$, (2) $I = 0.25 \, I_0$

Formula: $I = I_o \cos^2 \theta$

Solution:

Substituting $0.75\ I_0 = I_0 \cos^2\theta$

$$\frac{3}{4} = \cos^2\theta$$

$$\pm\frac{\sqrt{3}}{2} = \cos\theta$$

$$\therefore \quad \boxed{\theta = \pm 30^\circ,\ \pm 120^\circ}$$

$$0.25\ I_0 = I_0 \cos^2\theta$$

$$\frac{1}{4} = \cos^2\theta$$

$$\pm\frac{1}{2} = \cos\theta$$

$$\therefore \quad \boxed{\theta = \pm 60^\circ,\ \pm 150^\circ}$$

***Problem 2.40:** A polarizer and an anlayzer are oriented so that the maximum of light is transmitted. To what fraction of its maximum value and intensity of transmitted light reduced when the analyzer is rotated through (i) 30°, (ii) 45° and (iii) 60°?*

Solution: Law of Malus:

$$I = I_m \cos^2\theta \quad \therefore \quad \frac{I}{I_m} = \cos^2\theta$$

(i) $\theta = 30^\circ$, $\quad \frac{I}{I_m} = (\cos^2 30^\circ) = 0.75$

(ii) $\theta = 45^\circ$; $\quad \frac{I}{I_m} = (\cos^2 45^\circ) = 0.50$

(iii) $\theta = 60^\circ$, $\quad \frac{I}{I_m} = (\cos^2 60^\circ) = 0.25$

***Problem 2.41:** Find the specific rotation of cane sugar solution. If the plane of polarization is turned through 26.4°, the length of the tube containing 20% sugar solution is 20 cm.*

Data: $\theta = 26.4^\circ$, $l = 20$ cm $= 2$ dm;

$$c = 20\% = \frac{20}{100} = 0.20 \text{ gm/cc}$$

Formula: $s = \frac{\theta}{l \times c}$

Solution: $s = \frac{26.4^\circ}{2 \times 0.20}$

$$= \boxed{66^\circ \text{ (as 10 cm = 1 dm)}}$$

***Problem 2.42:** If the plane of vibration of incident beam makes an angle of 30° with the optic axis, compare the intensities of the extra-ordinary and ordinary light.*

*(**Hint:** Amplitude of E-ray = A cos θ, Amplitude of O-ray = A sin θ).*

Solution: We know,

$I \propto A^2$ and according to law of Malus, $I \propto \cos^2\theta$

For E-ray: $I_E = A^2\cos^2\theta = A^2\cos^2 30^\circ = 0.75\ A^2$

For O-ray: $I_O = A^2\sin^2\theta = A^2\sin^2 30^\circ = 0.25\ A^2$

$$\therefore \quad \frac{I_E}{I_O} = \frac{0.75}{0.25} = 3$$

$$\boxed{I_E = 3I_O}$$

***Problem 2.43:** A 20 cm long tube containing 48 c.c. of sugar solution rotates the plane of polarization by 11°. If the specific rotation of sugar is 66°, calculate the mass of sugar in the solution.*

Data: $l = 20$ cm

$s = 66^\circ$

$\theta = 11^\circ$

Formula: Specific rotation

$$s = \frac{10\theta}{l \times c}$$

$$\therefore \quad c = \frac{10\theta}{l \times s}$$

Solution: $c = \frac{10 \times 11}{20 \times 66} = \frac{1}{12}$ gm/cc

∴ 1 c.c. of sugar solution contains 1/12 gm of sugar.

∴ 48 c.c. of sugar solution will contain,

$$\frac{1}{12} \times 48 = \boxed{4 \text{ gm}}$$

***Problem 2.44:** At what angle of incidence should a beam of sodium light be directed upon the surface of diamond crystal to produce complete polarized light*

(Data Given: Critical angle for diamond = 24.5°)

Data: $i_c = 24.5^\circ$

Formula: (i) $\mu = \frac{1}{\sin i_c}$

(ii) $\mu = \tan i_p$

Solution (i) $\mu = \frac{1}{\sin 24.5}$

$\mu = 2.41$

(ii) $i_p = \tan^{-1}(2.41)$

$i_p = 67^\circ 28'$

Problem 2.45: *A 20 cm long tube containing 48 c.c. of sugar solution rotates the plane of polarization by 11°. If the specific rotation of sugar is 66°, calculate the mass of sugar in the solution.*

Data: $l = 20$ cm

$s = 66°$

$\theta = 11°$

Formula: Specific rotation

$$s = \frac{10\theta}{l \times c}$$

$$\therefore \quad c = \frac{10\theta}{l \times s}$$

Solution: $c = \frac{10 \times 11}{20 \times 66} = \frac{1}{12}$ gm/cc

$\therefore$ 1 c.c. of sugar solution contains 1/12 gm of sugar.

$\therefore$ 48 c.c. of sugar solution will contain,

$$\frac{1}{12} \times 48 = \boxed{4 \text{ gm}}$$

Problem 2.46: *At what angle of incidence should a beam of sodium light be directed upon the surface of diamond crystal to produce complete polarized light*

(Data Given: Critical angle for diamond = 24.5°)

Data: $i_c = 24.5°$

Formula: (i) $\mu = \frac{1}{\sin i_c}$

(ii) $\mu = \tan i_p$

Solution: (i) $\mu = \frac{1}{\sin 24.5}$

$\mu = 2.41$

(ii) $i_p = \tan^{-1}(2.41)$

$i_p = 67°28'$

2.15 INTRODUCTION TO LASER

- The term laser stands for **Light Amplification by Stimulated Emission of Radiation.**
- Laser is a light source which is highly coherent i.e. radiation emitted by all the emitters (atoms or molecules) in source agree in phase, direction of emission, polarisation and are essentially of one wavelength or colour (monochromatic).
- Due to coherence, a beam of laser light can travel many miles with only a negligible divergence. This makes it different from the conventional light sources which emit many wavelengths with phase and direction widely varying.
- Around 1917, Einstein first predicted the existence of two different kinds of processes by which an atom can emit radiation by (i) Spontaneous emission, (ii) Stimulated emission.
- In a laser, the process of stimulated emission is used for amplifying the light waves. The fact that stimulated emission process could be used in the construction of coherent optical sources was first put forward by Townes and Schawlow.
- The energy of an atom in any atomic system can change by
  - Absorption
  - Spontaneous emission
  - Stimulated emission.

2.16 PRINCIPLE OF LASER

2.16.1 Stimulated Emission [Dec. 17]

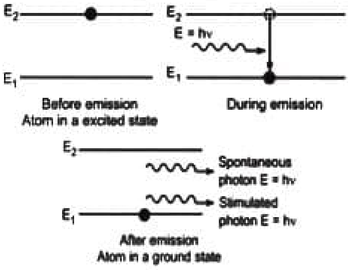


Fig. 2.31: Stimulated emission

- Consider Fig. 2.31 where the electrons are initially in the excited energy level and emission is stimulated before the spontaneous emission occurs. The excited atom is stimulated by a photon of exactly the same energy as the photon to be emitted. In such a case, two photons are emitted, one by the stimulated emission and the other stimulating photon.
- Both the photons travel in the same direction, have the same frequency and are in phase i.e. they are coherent.
- The emission of two photons with an input of only one photon implies amplification. The occurrence of spontaneous emission is directly proportional to the number of atoms in the specified energy level, whereas in stimulated emission, the rate of occurrence is proportional not only to the number of atoms in the excited state but also to the number of incident stimulating photons.

2.16.2 Population Inversion [Dec. 17]

- The process of getting a large percentage of atoms into an excited state is called as **Population Inversion**. If a large number of atoms can be excited to upper energy levels, then the probability of stimulated emission and hence light amplification becomes greater.
- The states of the system, in which the population of the higher energy state is more than the population of the lower energy state, are called as **Negative Temperature States** (negative indicates a non-equilibrium state, not the physical state of the system).
- In any atomic system, the number of particles in a higher energy state is normally less than the number of particles in a lower energy state. If N_2 denotes the number of particles in higher energy level E_2, and N_1 denotes the number of particles in lower energy level E_1, then $N_2 < N_1$ i.e. the population of higher energy level is less than the population of lower energy level. This means that under normal conditions, the ground state E_1 is heavily populated than the excited state E_2.
- If photons of energy $h\upsilon = E_2 - E_1$ are incident on the atoms, a few of the incident photons get absorbed and some of the atoms get excited to the state E_2. This process of stimulated absorption depopulates level E_1. The rate at which this process occurs is expressed as

$$R_{12} = P_a N_1 \quad \text{... (2.55)}$$

where P_a is the probability of stimulated absorption and N_1 is the population of state E_1.

- Similarly, the stimulated emission depopulates energy level E_2 resulting in the emission of photons. The rate at which this process occurs is expressed as

$$R_{21} = P_e N_2 \quad \text{... (2.56)}$$

where P_e is the probability of the process of stimulated emission and N_2 is the population of state E_2.

- At thermal equilibrium, these probabilities are equal i.e. $P_a = P_e$. Then, on comparing the two rates, it is observed that more energy is absorbed than emitted.

i.e. from (2.55) and (2.56),

$P_a N_1 > P_e N_2$ because $N_1 > N_2$.

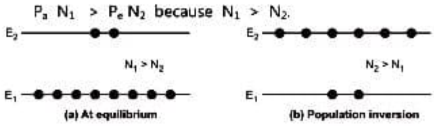


Fig. 2.32

- To produce more emission, it is essential to have $N_2 > N_1$ i.e. the number of particles in higher energy level must be made more than the number of particles in lower energy level. This is called as **Population Inversion.**
- If this inversion is achieved, there can be more emission and incoming light will be amplified coherently. A system in which population inversion is achieved is called an **Active System**.
- The method of raising atoms from lower energy levels to higher energy levels is called as **Pumping**. It can be done by subjecting the atoms to a non-uniform electric field, flooding the gas with high intensity light, etc. A more common method of pumping is **Optical Pumping**.

2.16.3 Metastable State

- The electron in an excited state has certain probability to decay or jump to a lower energy level. Generally, these probabilities are such that the jump occurs within 10^{-8} sec of excitation.
- However, there are some excited states, called **Metastable States**, which have a very low probability of decay i.e. electrons stay for longer time.
- Electrons may stay in the metastable excited states for seconds, minutes or even hours. In stimulated emission, the electrons must remain in excited level and wait for stimulating photon.
- Therefore, the active medium must have a metastable state. The population inversion can be obtained by using metastable states as the electrons rest in metastable state for long time.

2.16.4 Active Medium

- A medium in which the population inversion takes place is called the **Active Medium**. The active medium is responsible for the light amplification and hence LASER. The active medium may be a solid, liquid or gas and accordingly the lasers are classified as solid state or gas lasers.
- Out of the total active medium, only small number of atoms are responsible for lasing action and remaining atoms help only in hosting active atoms or in population inversion. The atoms which particulate in stimulated emission are called **Active Centres**.

2.16.5 Resonant Cavity

- A cavity can be constructed using mirrors such that the light rays return to their original position after travelling through the cavity for a certain number of times. Such cavities are known as **Resonant Cavities**.
- Fig. 2.33 shows cavity formed by two parallel mirrors M_1 and M_2. One of the mirrors is completely silvered (M_1) and the other is partially silvered (M_2). The laser beam emerges from the resonant cavity through the partially silvered mirror M_2.

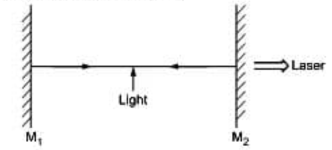


Fig. 2.33: Resonant cavity

- The active system is placed in the resonant cavity, the photon emitted will keep on reflecting back and forth within the cavity. The light which is incident parallel to the axis of optical cavity will only leak out as a laser. That is why, laser is highly directional.

2.16.6 Pumping

The method of raising atoms from lower energy levels to higher energy levels is called as **Pumping**. The pumping is used for achieving population inversion which is necessary for optical amplification to take place. There are several methods for pumping electrons. They are as follows:

1. **Optical Pumping**

- In optical pumping, an external light source (flash lamp) is used to produce a high population in some particular energy level E_2 (say) by selective absorption as shown in Fig. 2.34.

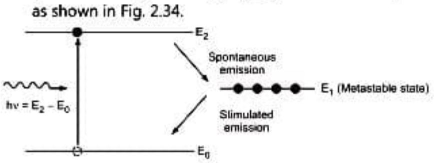


Fig. 2.34: Optical pumping

- When a flash of light falls on electrons in ground state, they absorb incident photons and get excited. After staying there for some time, some of the atoms make spontaneous transition to metastable state E_1. As the probability of spontaneous decay is less in metastable state, a large population accumulates in this level. This results in a population inversion between E_0 and E_1.
- Generally, this method is used in solid-state lasers, such as ruby laser.

2. **Inelastic Atom-Atom Collisions**

- Here suitable mixtures of gases are used. The gases are selected in such a way that their excited states are almost same. This makes the energy exchange possible between the atoms of the gases. If two gases A and B have same excited state, A^* and B^* then,

$$A^* + B \rightarrow A + B^*$$

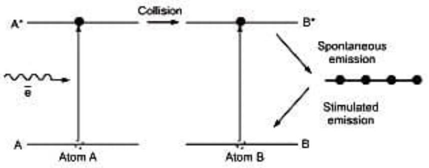


Fig. 2.35: Inelastic atom-atom collision

- The atom of gas A is excited by electric discharge. In collision with B, the energy is transferred to B. As a result, the excited level of atom B becomes more populated than lower level to which B can decay, as shown in Fig. 2.35.
- The best example is the He-Ne gas laser.

3. **Forward Biasing of a p-n Junction**

- If a p-n junction is formed with degenerate (heavily doped) semiconductors, the bands under forward bias appear as shown in Fig. 2.36.

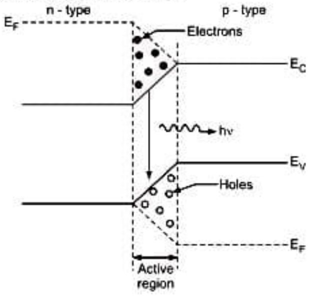


Fig. 2.36: Forward biasing of a p-n junction diode

- If the bias voltage is large enough, electrons and holes are injected into the active region. As a result, the depletion layer now contains a large number of electrons in conduction band and holes in valence band. If the population density is high enough, it gives population inversion. For a population inversion, the applied voltage should be selected in such a way that $eV > h\upsilon$ (= E_g).

- The other methods of pumping are electron excitation, chemical reactions, etc. These methods will not be discussed in detail as they are beyond the scope of the text.

2.17 EINSTEIN'S COEFFICIENTS

Consider an assembly of atoms at thermal equilibrium. The system is at temperature T with radiation of frequency υ and energy density I_υ (or photon density). Let N_1 and N_2 be the number of atoms in energy states E_1 and E_2 respectively at any time t.

The rate of transition from E_1 to E_2 will depend on the properties of energy states E_1 and E_2 and is proportional to the energy density I_υ of the radiation of frequency υ and the number of electrons N_1 in energy state E_1.

Therefore, $R_{12} \propto N_1 I_\upsilon$

or $$R_{12} = B_{12} N_1 I_\upsilon \quad \text{... (2.57)}$$

where B_{12} is the proportionality constant called **Einstein's Coefficient for Absorption**.

The electron in the excited level will make transition to a lower energy state either by spontaneous or stimulated emission.

In spontaneous emission, the rate of emission is proportional to the number of excited electron N_2.

i.e. $(R_{21})_{spont} \propto N_2$

$$\therefore \quad (R_{21})_{spont} = A_{21} N_2 \quad \text{... (2.58)}$$

where A_{21} is called the Einstein's coefficient of spontaneous emission.

But in stimulated emission the rate of emission will depend upon number of excited electron N_2 and intensity of the stimulating photons I_υ.

$\therefore (R_{21})_{stimulated} \propto N_2 I_\upsilon$

or $$(R_{21})_{stimulated} = B_{21} N_2 I_\upsilon \quad \text{... (2.59)}$$

where B_{21} is called the Einstein's coefficient of stimulated emission.

At equilibrium the rate of absorption and emission is same.

$$\therefore \quad R_{21} = (R_{21})_{spont} + (R_{21})_{stimulated} \quad \text{... (2.60)}$$

From equations (2.57), (2.58) and (2.59),

$$B_{12} N_1 I_\upsilon = A_{21} N_2 + B_{21} N_2 I_\upsilon$$

$$I_\upsilon (N_1 B_{12} - N_2 B_{21}) = A_{21} N_2$$

$$I_\upsilon = \frac{A_{21} N_2}{N_1 B_{12} - N_2 B_{21}} \quad \text{... (2.61)}$$

Dividing by $N_2 B_{21}$,

$$I_\upsilon = \frac{A_{21}/B_{21}}{\left(\frac{N_1 B_{12}}{N_2 B_{21}} - 1\right)} \quad \text{... (2.62)}$$

From Boltzmann distribution law,

$$N_1 = N_0^{-E_1/kT} \quad \text{... (2.63)}$$

and $$N_2 = N_0 e^{-E_2/kT} \quad \text{... (2.64)}$$

where N_0 = total electrons in ground state

k = Boltzmann's constant

$$\therefore \quad \frac{N_1}{N_2} = e^{-E_1 + E_2/kT}$$

But $$E_2 - E_1 = h\upsilon$$

$$\therefore \quad \frac{N_1}{N_2} = e^{h\upsilon/kT} \quad \text{... (2.65)}$$

Substituting in equation (2.62),

$$I_\upsilon = \frac{A_{21}/B_{21}}{\left(\frac{B_{12}}{B_{21}} e^{h\upsilon/kT} - 1\right)} \quad \text{... (2.66)}$$

The Planck's radiation formula is given by

$$I_\upsilon = \frac{8\pi h\upsilon^3}{c^3} \left(\frac{1}{e^{h\upsilon/kT} - 1}\right) \quad \text{... (2.67)}$$

Comparing equations (2.66) and (2.67), we have

$$B_{12}/B_{21} = 1 \quad \text{i.e.} \quad B_{12} = B_{21} \quad \text{... (2.68)}$$

and $$\frac{A_{21}}{B_{21}} = \frac{8\pi h\upsilon^3}{c^3} \quad \text{... (2.69)}$$

The equations (2.68) and (2.69) are called **Einstein's Relations**. The ratio of spontaneous to stimulated emission is proportional to υ^3.

From equations (2.58) and (2.59), the ratio of spontaneous emission to stimulated emission is

$$R = \frac{A_{21} N_2}{B_{21} N_2 I_\upsilon} = \frac{A_{21}}{B_{21} I_\upsilon} \quad \text{... (2.70)}$$

Using equations (10) and (14)

$$R = \frac{A_{21}/B_{21}}{A_{21}/B_{21}} (e^{h\upsilon/kT} - 1)$$

$$\therefore \quad R = e^{h\upsilon/kT} - 1 \quad \text{... (2.71)}$$

Therefore, at thermal equilibrium at temperature T for $\upsilon << \frac{kT}{h}$, the number of stimulated emission exceeds the spontaneous emission, while for $\upsilon >> \frac{kT}{h}$, the number of spontaneous emission exceeds the number of stimulated emission.

2.18 TYPES OF LASER

- Depending upon the energy levels involved in the pumping the lasers can be classified in following categories.
  - (a) Two level.
  - (b) Three level.
  - (c) Four level.

(a) Two Level Laser System

- A two-level laser system consists of only two energy levels, E_1 and E_2, ground state and excited state. The electrons from E_1 are pumped to E_2 as shown in Fig. 2.37.

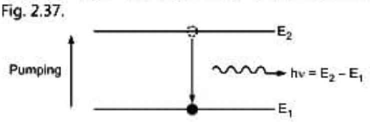


Fig. 2.37: Two-level laser system

- The electron from E_2 decays to E_1 radiating a photon of energy hυ. The best example of two-level laser system is a diode laser.

(b) Three Level Laser System

- In a three-level laser system, three energy levels E_1, E_2 and E_3 are involved as shown in Fig. 2.38. Here one of the transitions is non-radiative. The transition between E_3 to E_2 is very fast and non-radiative.
- Here the electron is pumped to E_3 directly. As decay from E_3 to E_2 is very fast, hence E_2 will be more populated and decay from E_2 to E_1 gives a photon of energy hυ. The best example of this category is ruby laser.

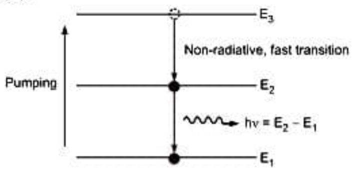


Fig. 2.38: Three-level laser system

(c) Four Level Laser System

- In a four-level laser system, the electrons are pumped to E_4 directly. The transitions will take place between E_4 to E_3, E_3 to E_2 and E_2 to E_1. Out of these three transitions, only one will be radiative and two will be non-radiative.
- Fig. 2.39 shows a four-level laser system. The best example is He-Ne laser. In a four level laser system, lasing action is always observed between E_3 and E_2.
- The life time of E_4 is very short. The transition between E_2 to E_1 is non-radiative and spontaneous. Moment electron reaches to E_1, it is pumped to level E_4, but due to short life time of energy level E_4, electron immediately jumps to E_3. Hence the levels E_1 and E_4 are free to accommodate electrons and the population of E_3 is always higher which favours for lasing action. Therefore, four level laser system works effectively.

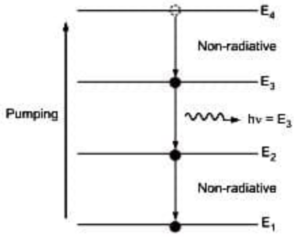


Fig. 2.39: Four-level laser system

Need of Three/Four-Level System

- If there are only two states, ground state and metastable state. When a photon is incident on it, it will be absorbed and electron will jump from ground state to metastable state. At the same time, due to stimulated emission, electron will jump to the ground state.
- During the process a situation will arise when half of the atoms are in the ground state (N_1) and half in the metastable state (N_2). i.e. $N_1 = N_2$. This will make the rate of stimulated emission and absorption equal. But to achieve population inversion, rate of absorption should be higher than stimulated emission.
- Thus if there are only two states the population inversion could not be achieved. And, therefore, laser action will not be possible.

2.19 RUBY LASER – THREE LEVEL LASER SYSTEM [Dec. 18, May 19]

A ruby laser is a solid-state laser that uses a synthetic ruby crystal. Typical ruby laser is a pulsed laser of intense red colour.

Construction

- The laser consists of a ruby rod surrounded by a flash tube. One end of the rod is highly silvered while the other end is semi-silvered. The flash tube surrounds the ruby rod in the form of a spiral.

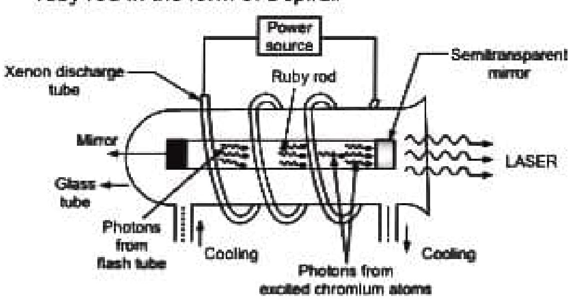
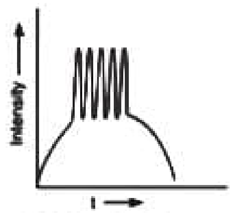


Fig. 2.40

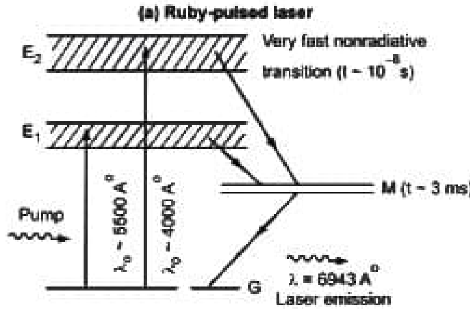
- Synthetic ruby consists of a crystal of aluminum oxide (Al_2O_3) in which a few of the aluminum atoms (Al^{3+}) are replaced by chromium atoms (Cr^{3+}). These atoms have the property of absorbing green light.
- The chromium impurity is the active atom of the laser. Doping of chromium gives ruby its characteristic red colour.

(a) Pumping and Energy Levels of Chromium

- When ruby is in a steady magnetic field, chromium acquires energy states, of which three are represented schematically as shown in Fig. 2.41 (a).
- As is clear from the figure, this is a three-level laser system. Level M actually consists of a pair of levels corresponding to wavelengths of 6943 A° and 6929 A°. However, laser action takes place only on 6943 A° line due to higher population inversion.
- The pumping of chromium atoms is performed with a Xenon or Krypton flash lamp. The chromium atoms in the ground state absorb radiation around wavelengths 5500 A° and 4000 A° and are excited to the levels marked E_1 and E_2.



(a) Ruby-pulsed laser



(b) Energy level

Fig. 2.41

(b) Assembly of Chromium Atoms to Metastable State

- The chromium atoms excited to these levels, relax rapidly through a non-radiative transition (in a time 10^{-8} to 10^{-9} sec) to the metastable state M, which has a life time of ~3 m secs. Laser emission occurs between level M and the ground state G at an output wavelength of 6943 A°.

(c) Operation

- The operational sequence starts with the ignition of the Xenon flash tube. Chromium atoms in the ruby rod are energized by absorption of the energetic photons from the flash tube.
- When the excited electrons in the chromium atoms fall back to their normal states, photons are given off by spontaneous emission emitting red light (hence ruby has a natural red colour). Some of these photons escape from the rod but many oscillate or bounce back and forth along the length of the rod with the help of the mirror at the two ends.
- When the electrons in the excited state are exposed to these radiations of the same frequency which they are about to emit, the emission process is triggered. Radiation is now emitted, which is exactly in phase with the exposed radiation.
- This cumulative process of flash tube photons exciting chromium atoms which in turn emit photons in the same direction and phase, continues until the coherent laser beam penetrates through the partially reflecting mirror on one end of the rod to give a powerful beam of red light.

(d) Pulsed Output

- A certain stage is reached when the population inversion caused by one flash of Xenon tube is used up. As soon as the flash lamp stops operating, the population of the upper level is depleted very rapidly and laser action ceases until the arrival of the next flash. Refer Fig. 2.41 (b).
- Thus, ruby is a **Pulsed Laser**. The output beam has a principal wavelength of 6943 A° equal to 4.3×10^{14} Hz frequency (lies in the visible spectrum). The duration of the output flash is about 300 μsec.
- During the operation of a ruby laser, a very high temperature is produced. To prevent any damage to the ruby rod, it is surrounded by a liquid nitrogen container and is operated to give out the beam only in pulses.
- This laser is used in many applications as its output lies in the visible region where photographic emulsions and photo detectors are more sensitive than they are in the infrared region. Ruby lasers also find application in laser holography, laser ranging, etc.

2.20 HELIUM-NEON LASER – FOUR LEVEL LASER SYSTEM [May 18]

- This is a **'Continuous Laser'** unlike the ruby laser. In this laser, the vapours of metal are used as the media.
- It is an extremely popular form of laser as it is simple, inexpensive and has an extremely broad range of emission wavelengths (0.6 to 100 μm depending on the type of gas used). The first gas laser to be operated successfully was the He - Ne laser.
- In solid-state lasers, pumping is usually done by using a flash lamp or a continuous high power lamp. Such a technique is efficient if the laser system has broad absorption bands. In gas lasers, as the atoms are characterized by sharp energy levels, an electrical discharge is generally used to pump the atoms.

Construction

- It consists of a quartz tube with a diameter of about 2-8 mm and a length of 10-100 cm. It is filled with helium and neon. The pressure of helium is approximately 10 times that of neon.
- The neon atoms provide energy states for the transitions while helium provides a mechanism for efficiently exciting neon atoms to upper metastable states i.e. helium serves merely as an energy transfer agent.
- At one end of the tube is a total reflector while at the other is a partial reflector. The gas is excited by means of a high frequency generator.

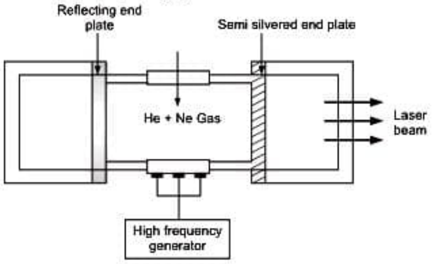


Fig. 2.42: He-ne laser

Principle

- In He-Ne laser, population inversion is produced through inelastic collisions between excited He atoms and Ne atoms in the ground state. The process can be expressed as

  $He^* + Ne \rightarrow He + Ne^*$ (* shows an excited state)

- This is possible, because the levels Ne_4 and Ne_6 of neon atoms have almost the same energy as the levels He_2 and He_3 of helium atoms as shown in the energy level diagram.

Working

(a) Electric Discharge and Excitation of Helium

- When an electrical discharge is passed through the gas, the electrons which are accelerated down the tube collide with helium and neon atoms and excite them to higher energy levels.
- The helium atoms tend to accumulate at the levels He_2 and He_3 due to their long life times of $= 10^{-4}$ secs and 10^{-6} secs respectively.

(b) Transfer of Energy from Helium to Neon and Pumping

- As the levels Ne_4 and Ne_6 of neon atoms have almost the same energy as He_2 and He_3, excited helium atoms colliding with neon atoms in the ground state can excite the neon atoms to Ne_4 and Ne_6 states.
- As the pressure of helium is ten times that of neon, the levels Ne_4 and Ne_6 of neon are selectively populated as compared to other levels of neon.

  i.e. $He^* + Ne \rightarrow He + Ne^*$ (* indicates excited state)

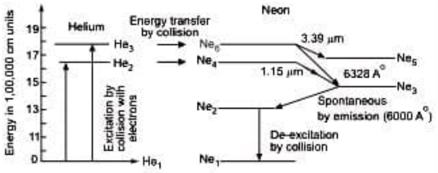


Fig. 2.43

(c) Population Inversion for Neon

- Transition between Ne_6 and Ne_3 produces the popular 6328 A° (632.8 nm) line of He - Ne laser. Neon atoms de-excite through spontaneous emission from Ne_3 to Ne_2 (life time $\sim 10^{-8}$ sec.). As this time is shorter than the life time of level Ne_6 ($\sim 10^{-7}$ sec.), steady state population inversion can be achieved between Ne_6 and Ne_3. Level Ne_2 is metastable and thus tends to collect atoms.
- The atoms from this level fall back to the ground level mainly through collisions with the walls of the tube. As Ne_2 is metastable, it is possible for the atoms in this level to absorb the spontaneously emitted radiation in $Ne_3 \rightarrow Ne_2$ transition to be re-excited to Ne_3. This tends to reduce the effect of inversion.
- It is for this reason that the gain in this laser transition is found to increase with decreasing tube diameter.

(d) Transition within Neon and Continuous Output

- The other two important wavelengths from the He - Ne laser correspond to the $Ne_4 \rightarrow Ne_3$ (1.15 µm) and $Ne_6 \rightarrow Ne_5$ (3.39 µm) transitions. The laser can be made to oscillate at 6328 A° by using optical elements (multilayer coated mirrors) in the path. These lasers are continuous, because the collision process maintains the energy states Ne_6 and Ne_4 at larger population densities than the lower states. This continued population inversion gives a continuous lasing action.
- A typical He - Ne laser operates with a current of 10 mA at a D.C. voltage of 2500 V and gives an optical output of 5 mW. Its efficiency is then $\frac{5 \times 10^{-3}}{2500 \times 10^{-2}}$ = 0.02 %.
- This is the only laser radiating in far infrared region. Hence, mostly used in laser '**Raman Spectroscopy**'.

2.21 APPLICATIONS OF LASER

2.21.1 Applications of Laser in Industry

- Laser can be focused to a very high energy density into a small image (≈ 1 micron in diameter) with the help of suitable lenses. Due to the small size of the image and the control over the energy, lasers are used extensively for cutting, welding and drilling circuits.
- **Drilling:** A laser beam is also used to drill holes of micron dimensions on printed circuit boards (PCBs). It is also used in resistance trimming in electric components industries. One can drill holes of the diameter of 10 µm through very hard substances like diamond. YAG laser is found to be very useful in such applications.
- **Welding:** Lasers are used as a heat source in welding the joints of the metals. This type of precise welding is extremely important in micro-electronics in which thin films are used. Thermocouple wires can easily be welded with the help of high power laser beam.
- **Micromachining:** Lasers are used for machining a surface in a slow and accurate manner to achieve an extraordinarily smooth finish.
- **Cutting:** Another important industrial application is metal or fabric cutting. A finely focused laser beam can cut thick and hard metal sheets with high precision and accuracy. It is also used in tailoring industries to cut thousands of layers of cloth at one instant.
- Due to its intensity and directionality, laser is used in surveying. When tunnels are to be constructed, engineers use the laser beam as a reference, to check that it is being constructed along a straight line. Similarly, it can be used to dig a ditch to a certain prescribed depth. Its most interesting use in surveying has been in measuring the distance from the earth to the moon. This distance was measured to an accuracy of 600 ft, and with the aid of reflectors to within six inches. This accuracy will allow to determine the location of the north pole to within six inches. It is further believed that a laser could be used to check whether the gravitational constant is actually a constant.
- A laser beam can determine precisely the distance, velocity and direction as well as the size and form of distant objects by means of the reflected signal as in radar. A Lidar (Laser radar), which sends out beams of laser light and detects echoes even from atmospheric layers has been developed.

2.21.2 Applications of Laser in Medicine

- Bloodless cancer surgeries can be performed as the beam can be focused on a small area, so that only the harmful tissue can be destroyed without damaging the surrounding region.
- Laser has been successfully used in ophthalmology, in the treatment of detached retinas, in welding cornea, etc. At the command of the physician, laser produces a beam of light which is directed onto the eye under treatment, to produce a minute coagulation. A series of these lesions weld the detached retina.
- Laser is used as a tool in the study of genetics. Lasers have been built into or are devised to be attached to microscopes. As high density energy is achieved, it can be used in micro-surgery, micro-burning, etc. Such a microscopic laser can concentrate millions of watts of power per square millimeter into a selected area. For example, a focused microscope laser can be used to make tiny openings (of 25 μ in diameter) in the cell walls, of say the nervous system, heart, retina, etc. without causing irreversible damage.
- Laser microprobes can be used as dental drills giving an advantage of no heating, no anesthetic and no pain to the patient. They have also been successfully used for localized treatment of skin growths and blemishes in human beings. A large amount of energy can be transmitted through the skin to interact with deeper different biological materials or structures which are damaged.

2.21.3 Application of Laser in Communication

- In this technology, optical energy is transferred through a guided media, called the **Glass Fibre**. When a beam of light enters at one end of a transparent rod (glass rod say), the light beam is totally internally reflected and gets trapped within the rod.
- A similar behaviour is exhibited by a bundle of fine fibres. A beam enters at one end and is transmitted through the wire to the other end, even when the fibre is curved.
- One of the most important areas of application of fibre optics is in telecommunication. The communication kit consists of a transmitter, optical fibre and the receiver. The block diagram is as shown in Fig. 2.44.

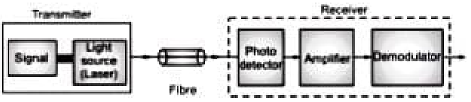


Fig. 2.44: Communication kit

- The transmitter consists of a light source, either LED or laser diode, with a signal. The light beam from the source is connected to the fibre, through optical connections.
- The carrier and signal frequency propagate through the fibre.
- At the other end, it is detected with the help of a photodetector. The received signal is demodulated and the information is stored or displayed by electronic circuits.

2.21.4 Application in Information Technology (Holography)

Holography

- This is a technique of producing an interference pattern between a direct laser beam and a laser beam reflected from an object on a photographic plate. This pattern on the developed photographic plate, when illuminated with laser in a proper manner, produces a three-dimensional image of the object called a **'Hologram'**.
- Holography deals with three-dimensional image of the object whereas photography is a two-dimensional effect. In photography, the photographic plate records only the intensity of light due to the image formed on it. In holography, both the intensity and phase distribution are recorded simultaneously using interference technique. Due to this the image produced by the technique of holography has a true three-dimensional form and is as true as the object

2.22 OPTICAL FIBRE

2.22.1 Principle of Optical Fibre

- Optical fibre is a very thin and flexible medium having a cylindrical shape consisting of three sections: (i) The core, (ii) The cladding and (iii) The outer jacket.

Principle of Light Transmission

- The principle of light transmission through optical fibre is total internal reflection. For total internal reflection to take place at the fibre wall, the following conditions should be satisfied:
  - ➢ The refractive index of the core material (μ_1) must be greater than that of the cladding (μ_2).

- At the core-cladding interface, the angle of incidence θ must be greater than the critical angle,

  where $\theta_c = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)$.

- When a light ray travels from a denser to a rarer medium, the angle of refraction is greater than the angle of incidence. As the angle of incidence increases, the angle of refraction also increases and for a particular angle of incidence, the refracted ray grazes the interface between the core and the cladding. This angle of incidence is called as the **Critical Angle θ_c**.
- If angle of incidence is greater than θ_c, the ray will be reflected back into the core, i.e. it suffers **Total Internal Reflection**. For angles equal to or greater than the critical angle the light will be totally reflected and no light will be refracted. Fig. 2.45 shows total internal reflection.
- When light is incident on core of the fibre optics, it will be refracted and will travel in the core. After some time it will strike one of the core-cladding interface say upper surface. If the angle of incidence is greater than critical angle, it will be totally reflected and remain in the core.
- Now, the reflected light will travel to the lower surface. It is then incident on the lower surface where the same process is repeated and light gets transmitted from one end to the other end as shown in Fig. 2.45.

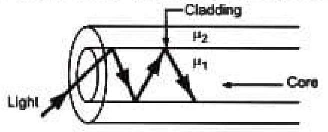


Fig. 2.45: Propagation of light in fibre optics

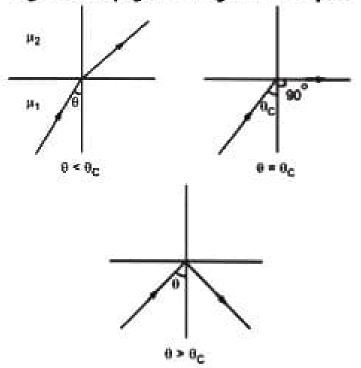


Fig. 2.46: Total internal reflection

2.22.2 Application of Fibre Optics in Communication Kit

- One of the most important areas of application of fibre optics is in telecommunication. The communication kit consists of a transmitter, optical fibre and the receiver. The block diagram is as shown in Fig. 2.47.

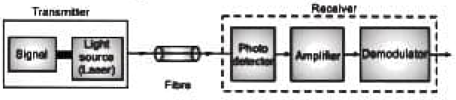


Fig. 2.47: Communication kit

- The transmitter consists of a light source, either LED or laser diode, with a signal. The light beam from the source is connected to the fibre, through optical connections.
- The carrier and signal frequency propagate through the fibre.
- At the other end, it is detected with the help of a photodetector. The received signal is demodulated and the information is stored or displayed by electronic circuits.

2.22.3 Introduction to Optical Fibre

- Efforts to device communication systems for sending from one place to another distant place have been continuing by the human being. These systems have used as optical or acoustical means like signal lamps or horns, electrical codes like Morse Code (1938), Telephones (1878) so effort. A portion of electromagnetic waves is used in telephones, amplitude modulated and frequency modulated radio, television, CB (citizen's band radio), satellite link in the recent days.
- Optical region of the electromagnetic spectrum which contains wavelengths from 50 nm (500 A°) ultraviolet) to about 1000 nm (10000 A°) (far infrared) with the visible range from 400 nm to 700 nm (nanometre, 10^{-9} m).
- Advances in technology have made it possible to use optical fibres alongwith good optical sources, photodetectors and fibre cable connectors to transmit more data at high transmission rate from one place to a distant place.

2.22.4 Structure of Optical Fibre

- Optical fibre is a dielectric waveguide and it operates at optical frequencies (5×10^4 Hz). It is generally cylindrical, the core of which has higher refractive index (n_1) than that of the surrounding material (n_2). The core and the surrounding dielectric together form an optical waveguide. Depending on the type of the waveguide, optical fibres are categorised into two steps as

1. Step Index Fibre and
2. Graded Index Fibre.

- The path through which light is propagated is called **Waveguide**. In case of optical fibre, core and cladding together work as optical waveguide.

1. Core-Cladding Fibre

Core : In this type of optical fibre, there is a single solid dielectric cylinder of radius a and refractive index n_1 as shown in Fig. 2.48. This solid cylinder is known as the **Core** of the optical fibre.

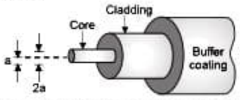


Fig. 2.48 : Construction of step index optical fibre

Cladding (as a Rarer Medium) :

- The core is surrounded by a solid dielectric (cone polymer) **Cladding** having a refractive index n_2 which is less than n_1. Cladding reduces the scattering losses due to dielectric discontinuities at the core surface.
- Also, it adds to the mechanical strength of the fibre and protects the core from absorbing the surface contamination which can come in contact with it. The cladding is made up of either glass or plastic materials.

Buffer (for Mechanical Strength) :

- Most fibres are encapsulated in an elastic abrasion resistant plastic material. This encapsulating material is called **Buffer Coating**. The buffer adds further mechanical strength to the fibre and keeps away the fibre from small geometrical irregularities, distortions or roughness of the surrounding surfaces. This also avoids random microscopic or sharp bends when the fibres are incorporated into cables or when supported on some other structures.
- The conventional optical fibre consists of a core region of refractive index, n_1, which is surrounded by a cladding of lower refractive index, n_2. The fibre or a bundle of fibres is sheathed in an outer protective covering.

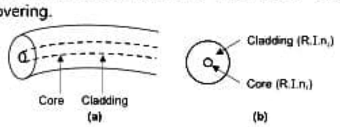


Fig. 2.49 : The fibre with core and cladding

There are two types of optical fibres, viz. (a) step index and (b) graded index.

2.22.5 Types of Optical Fibre

- **Step-Index Optical Fibre :** The core has an **Uniform Refractive Index**, n_1 and the cladding has an **Uniform Refractive Index** n_2 ($n_2 < n_1$). Let the core radius be 'a' and the outer radius of cladding be 'b'.

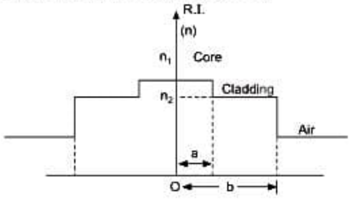


Fig. 2.50 : The refractive index profile of a step-index fibre

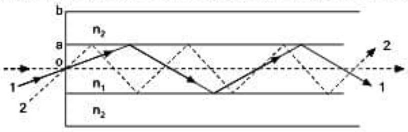


Fig. 2.51 : The paths of rays in step-index fibre

- In the step-index fibre, rays entering at different angles of incidence with the axis travel different path lengths and emerge out at different times. This results in **Pulse Dispersion**, i.e. an input pulse gets widened as it travels along the fibre.
- **Graded-Index Optical Fibre :** The **Refractive Index of the Core Varies Continuously** from n_1 at the centre to n_2 at the core-cladding interface. The cladding has the constant refractive index n_2.
- In the graded-index fibre, a ray is continuously bent and travels a periodic path along the axis. Rays entering at different angles follow different paths with the same period, both in space and time. This results into periodic **Self Focussing** of the rays as shown in Fig. 2.52. Therefore, the pulse dispersion is less as compared with the step-index fibre.

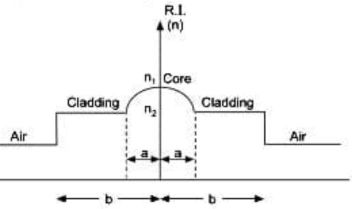


Fig. 2.52 : The R.I. profile of a graded-index fibre

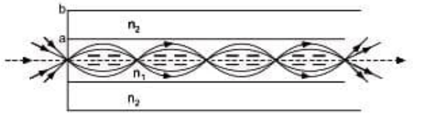


Fig. 2.53 : The paths of rays in graded-index fibre

- The core diameters (2a) in fibres in use may range between 4 μm to 100 μm. The core + cladding diameter (2b) usually ranges between 100 μm to 200 μm.
- Fibres with narrow cores (about 10 μm) allow only one wave-mode to pass and are called **Monomode Fibres**; while those with core diameters about 50 μm and above allow different wavemodes and are called **Multimode Fibres**.
- Optical fibres used commonly in telecommunication applications have 2b = 125 μm and (typical values for refractive indices are : $n_1 \equiv 1.5$ and $n_2 = n_1 (1 - \Delta)$ where Δ = 0.01 to 0.02 (1 to 2 %)).

2.22.6 Index Difference (Core-Cladding Index Difference)

- In practical step-index fibres, the core with radius has refractive index n_1. A typical value of n_1 is 1.48. The cladding surrounding this core has a refractive index slightly lower than n_1. The relation between n_1 and n_2 is given by

$$n_2 = n_1 (1 - \Delta)$$

where Δ is called core-cladding index difference or simply the index difference.

- The values of n_2 are chosen so that the index difference Δ is equal to 0.01. As the core is having higher refractive index than that of cladding, electromagnetic waves at optical frequencies propagate along the fibre due to **Total Internal Reflection** at the core-cladding interface.
- An optical fibre may be either monomode or multimode in case of both the types i.e. step index and graded-index fibre.

2.22.7 Advantages of Optical Fibres

- Optical frequencies are extremely large. Since the frequency of light used as carrier is of the order of 10^{15} Hz, the information carrying capacity of a fibre is much greater.
- The optical fibres are made of dielectric material, which offers electrical isolation between input and output parts of the circuit.
- The material used in fibres is silica glass (or SiO_2). As this is available abundantly on earth, the cost of fibre lines is much lower.
- As fibres have a high information capacity, multiple channel routes can be compressed into very small cables. This helps in reducing congestions in overcrowded cable ducts.
- As fibres are very thin, light and occupy less space, a large number of them can be used at a time. Because of this, detailed images can be obtained. This is of particular importance in medicine where an endoscope, employing fibre optics is increasingly being used to take pictures inside the human body.
- The transmission is due to internal reflection, therefore there is less loss. The cable is immune to electric, magnetic or R.F. fields in atmosphere, because it is covered with a cladding.
- Active scintillating fibres (fibre lasers) are useful in developing flexible high intensity laser probes.

2.22.8 Acceptance Angle and Numerical Aperture

[May 19]

Consider Fig. 2.54. ϕ is the angle of incidence at the core-cladding interface for a ray entering the core making an angle, i, with the fibre axis.

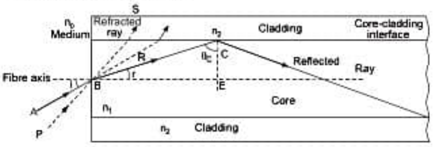


Fig. 2.54 : Ray propagation through step-index optical fibre

The condition for total internal reflection to take place is

$$\sin \phi \geq \frac{n_2}{n_1} \quad \text{... (2.72)}$$

We have, $\sin \phi = \sin (90 - r) = \cos r$ ... (2.73)

Also, $\frac{\sin i}{\sin r} = n_1, \quad \therefore \sin r = \frac{\sin i}{n_1}$ ... (2.74)

Now, $\sin r = \sqrt{1 - \cos^2 r}$

$= \sqrt{1 - \sin^2 \phi}$ ... (2.75)

The condition of total internal reflection, (2.72), therefore can be expressed as

$$\sin r \leq \sqrt{1 - \frac{n_2^2}{n_1^2}} \quad \text{... (2.76)}$$

Using (2.75), we have,

$$\frac{\sin i}{n_1} \leq \sqrt{1-\frac{n_2^2}{n_1^2}}$$

$$\therefore \quad \sin i \leq \sqrt{n_1^2-n_2^2} \quad \text{... (2.77)}$$

If i_m is the maximum angle of incidence for which total internal reflection can occur, we have

$$\sin i_m = \sqrt{n_1^2-n_2^2} \quad \text{for } n_1^2-n_2^2 < 1$$

$$= 1 \quad \text{for } n_1^2-n_2^2 \geq 1 \text{... (2.78)}$$

Light incident within *the cone of half-angle* i_m at the input end of the fibre will undergo total internal reflection and be guided along the fibre. This, therefore, is a measure of the **Light Gathering Power** of the fibre. **sin i_m** is called the **Numerical Aperture** (N.A.) of the fibre.

The numerical aperture is a function of the refractive indices of core and the claddings.

2.22.9 Acceptance Cone

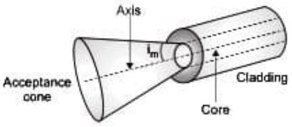


Fig. 2.55

The acceptance cone of an optical fibre decides its light gathering power and depends on acceptance angle. Larger the acceptance angle, larger is the light gathering power.

The acceptance cone is derived by rotating the acceptance angle about the fibre axis. The Fig. 2.55 shows the acceptance cone.

***Problem 2.47 :** Refractive index n_1 = 1.48 and n_2 = 1.45 in an optical fibre. Calculate numerical aperture and the maximum entrance angle $\theta_{o\ max}$ if the fibre is kept in air.*

Data : $n_1 = 1.48, \quad n_2 = 1.45$

Formula : Numerical aperture,

$$N.A. = \sqrt{n_1^2-n_2^2}$$

Solution :

$$= \sqrt{1.48^2-1.45^2}$$

$$= \sqrt{2.1904-2.125} = \sqrt{0.0879}$$

$$= 0.2964$$

Also, $N.A. = n \cdot \sin\theta_{o\ max}$ (here n = 1 for air)

$$\therefore \quad \theta_{o\ max} = \sin^{-1}(N.A.) = \sin^{-1}(0.2964)$$

$$= \boxed{17.24°} \text{ or } \boxed{17°\ 15'}$$

***Problem 2.48 :** Numerical aperture of an optical fibre is 0.5. Find the refractive index of cladding if the refractive index of the core is 1.53. Also calculate index difference.*

Data : $N.A. = 0.5, \quad n_1 = 1.53$

Formula : $N.A. = \sqrt{n_1^2-n_2^2}$

Solution : $0.5 = \sqrt{1.53^2-n_2^2}$

Squaring both sides,

$$\therefore \quad 0.25 = 1.53^2-n_2^2$$

$$\therefore \quad n_2^2 = 2.34-0.25 = 2.09$$

$$\therefore \quad n_2 = 1.446 \quad \text{(taking square roots)}$$

Now, index difference

Δ = change of refractive index per unit change of core refractive index.

$$\therefore \quad \Delta = \frac{n_1-n_2}{n_1} = \frac{1.53-1.446}{1.53} = \boxed{0.055}$$

2.22.10 Applications of Optical Fibre

The optical fibers were basically designed for the optical communication. But now they are extensively used in other fields such as medicines, electronics, military etc. Some of them are discussed below.

1. Communication Applications

In communication system optical fibre is used to transmit the information from transmitter to receiver. The details of optical fibre link are discussed in Article 2.22.

The optical fibre cable is preferred over other links as :

- They have higher information carrying capacity.
- The optical fibres are made of dielectric materials, therefore transmitter and receiver are electrically isolated.
- The material used is silica glass which is very cheap.
- As the information is transmitted by total internal reflection, the transmission losses are less.
- They are very thin, hence occupy very less space.

2. Medical Applications

The main use of optical fibre in medicine is to illuminate or burn the internal organs of human body and collect the scattered light for formation of image.

In endoscopy a bundle of optical fibre is used to illuminate the internal organs of human body where the sunlight cannot reach. Here light from artificial source of light is guided through the fibre. The light will fall on the organ and will be scattered. This scattered light is collected by another bundle of optical fibres. This scattered light is used for formation of image.

In ophthalmology a laser beam is guided by the fibre is to detach the retina or for vision correction.

A guided laser beam through an optical fibre is also used in angioplasty. A special catheter having three channels – one for guiding laser beam, second for formation of image and third a hollow tube to remove the blocking tissues. The laser beam is used to cut or burn the unwanted tissues.

3. Military Applications

The military equipments, aircrafts, ships, submarines need heavy copper wires for communication equipments. The heavy weight copper wires can be replaced by light weight optical fibres. This reduces the load increasing the overall efficiency of the instruments.

Due to high information carrying capacity they can be used to transmit the video which shows the real situations and helps the ground staff for controlling unmanned vehicles, aeroplanes or missiles.

4. Fibre Optic Sensors

The fibre optic is used to couple the sensors and detectors. The advantages of these sensors are that they are cheap and light weight. These sensors can be used to measure pressure, temperature, stress etc.

- **Temperature Sensor :** The fibre is coated with a thin silicon layer at one end. The silicon layer is backed by a reflective coating as shown in Fig. 2.56. When a light beam is passed through the coating it passes through the silicon layer and is reflected by reflective surface. This reflected light returns to the detector. The absorption of silicon varies with temperature which alters the intensity of the light received by the detector. This variation in the intensity of light is sensed as variation in temperature.

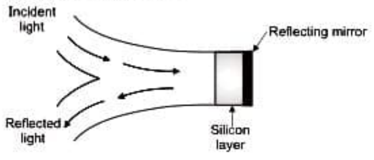


Fig. 2.56 : Temperature sensor

- **Pressure Sensor :** The concept of photoelasticity or induced double refraction is used to design pressure sensor. Fig. 2.57 shows the schematic diagram of pressure sensor.

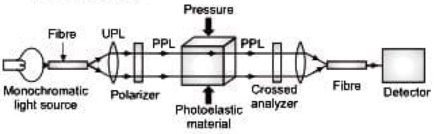


Fig. 2.57 : Pressure sensor

The photoelastic material is kept between crossed polarizer and analyzer. The monochromatic light is guided from source to polarizer using an optical fibre. Then the light is passed through the photoelastic material. The light coming out of the photoelastic material is passed through the crossed (kept at 90° w.r.t. polarizer) analyzer. When no pressure is applied, the axis of polarization remains unaltered and hence no light passes through the analyzer. When mechanical pressure is applied bire fringence is introduced. This gives O-ray and E-ray which are plane polarized in the plane perpendicular to each other. The O-ray will pass through the analyzer. Hence transmission of light occurs which is detected by the detector.

- **Smoke or Pollution Detector :** A smoke detector can be constructed by using optical fibre. A beam of light coming out off a fibre is collected by another fibre kept at some distance. If smoke or dust particles are present between fibres, light will be scattered by smoke particles. This will reduce the amount of light collected by the second fibre. The intensity of light collected will depend on the density of smoke, hence the variation in density will be detected.
- **Interference Sensor :** In this, a single mode fibre is used. A beam splitter is used to divide a laser beam into two parts. Each of these two parts are collected by two separated fibres. One of the fibres acts as sensing fibre and other as reference fibre. The sensing fibre changes the opitcal path of the light travelling through it, due to change in the length or refractive index of the fibre. Fig. 2.58 shows the Mach-Zehnder arrangement of interferroelectric sensor.

  The light entering the fibres is coherent whereas light coming out of the fibres will have phase difference due to change in the optical path due to physical parameter being measured. These two beams interfere to give interference pattern. The measurement of fringe width will give the value of physical parameter being measured.

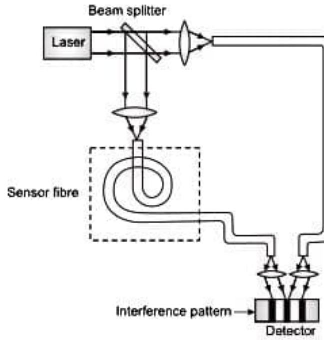


Fig. 2.58 : Interferometric sensor

SUMMARY

- **Interference:** The superposition of two waves of equal amplitude, frequency and a constant phase difference.

  (a) Constructive Interference or Bright Fringe: Whole number multiple of λ or $n\lambda$.

  (b) Destructive Interference or Dark Fringe: Odd integer multiple of $\frac{\lambda}{2}$ or $(2n + 1)\ \frac{\lambda}{2}$.

- **Thin Film:** A film is said to be thin if its thickness is of the order of a few wavelengths.

- In a thin **Film of Uniform Thickness** in reflected system, condition for

  (a) Constructive interference:

  $$2\mu t \cos r = (2n + 1)\ \frac{\lambda}{2}$$

  (b) Destructive interference:

  $$2\mu t \cos r = n\lambda$$

  (c) In transmitted system, the conditions reverse.

- In a **Wedge-Shaped Film**, the condition for

  (a) Constructive interference:

  $$2\mu t \cos (r + \alpha) = (2n + 1)\ \frac{\lambda}{2}$$

  (b) Destructive interference:

  $$2\mu t \cos (r + \alpha) = n\lambda$$

  (c) Fringe width:

  $$\beta \approx \frac{\lambda}{2\mu\alpha}$$

  (d) Fringes obtained are: equal in thickness, straight, parallel and equidistant.

- **Newton's Rings**: Fringes of circular in shape with dark fringe at the centre. The width of the fringe decreases with the order of the fringe i.e. as one moves away from the centre.

- **Diameter for**

  (a) Bright ring (reflected light)

  $$D_n^2 = (2n \pm 1) \cdot 2\lambda R$$

  (b) Dark ring (reflected light)

  $$D_n^2 = 4n\lambda R$$

- **Wavelength of Monochromatic Source of Light**:

  $$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

- **Refractive Index of Liquid:**

  $$\mu = \frac{D'^2_{n+p} - D'^2_n}{D^2_{n+p} - D^2_n}$$

- **Unpolarized Light** has the electric vector vibrating along all possible directions at right angles to the direction of propagation of light.

- **Types of Polarization** (i) Plane, (ii) Circular and (iii) Elliptical

- **Partially Polarized Light:** Mixture of plane polarized light and unpolarized light.

- If the vibrations of the electric vector in a light wave are confined to a single plane, then the light wave is **plane polarized or linearly polarized**.

- **Methods of Production of Plane Polarized Light:** (i) Reflection, (ii) Refraction, (iii) Scattering, (iv) Selective absorption and (v) Double refraction.

- **Brewster's Law:** States that, tangent of the angle of polarization is proportional to the refractive index of the medium i.e. $\mu = \tan i_p$.

- **Double Refraction or Birefringence**: When light passes through anisotropic crystals, it splits up into two rays, O-ray and E-ray.

- **Birefringence** of the crystal is given by, $\Delta\mu = \mu_e - \mu_o$.

- **Positive Crystals :** the velocity of ordinary ray is greater than that of the extraordinary ray ($\mu_e > \mu_o$).

- **Negative Crystals :** the velocity of extraordinary ray is greater than that of the ordinary ray ($\mu_o > \mu_e$).

- **Optic Axis :** O-ray and E-ray travel with the same velocity.

- **Polaroid :** Uses selective absorption for obtaining plane polarized light.
- **Nicol Prism :** Optical device used for producing and analyzing plane polarized light.
- **Optical Activity :** The phenomenon of rotation of the plane of polarization.
- **Rotatory Dispersion :** The rotation is nearly proportional to the inverse square of the wavelength. This gives a violet being rotated nearly four times as much as red light.
- **Polarimeters :** Are instruments used for finding the optical rotation of different solutions. When they are calibrated to read directly the percentage of cane sugar in a solution, they are named as **Saccharimeters.**
- **LASER:** Light **A**mplification by **S**timulated **E**mission of **R**adiation
- **Absorption:** Absorption is a process in which a photon, of energy $h\upsilon$, gets absorbed by an atom and it goes from a lower energy state E_1 to a higher energy state E_2.
- **Emission :**
- **Spontaneous Emission :** An electron which is raised to an excited state E_2 (due to absorption), spontaneously decays back to a lower energy level E_1 and radiates an energy equal to $E_2 - E_1$. Such an emission is called as spontaneous emission. This emission is random in nature and depends only on the type of atom and type of transition.
- **Stimulated Emission:** A photon of energy $h\upsilon = E_2 - E_1$ triggers an excited atom to drop to the lower energy state giving up a photon. This phenomenon of forced emission of photons is called as stimulated emission.
- **Population Inversion:** The process of getting a large percentage of atoms into an excited state is called as population inversion.
- **Active System:** A system in which population inversion is achieved is called an active system.
- **Pumping:** A method of raising atoms from lower energy levels to higher energy levels is called as pumping. It can be done by subjecting the atoms to a non-uniform electric field, flooding the gas with high intensity light (optical pumping) etc.
- **Metastable States:** Ordinary energy levels have a life time of 10^{-8} to 10^{-9} secs. Energy levels having a life time greater than ordinary energy levels ($\sim 10^{-6}$ to 10^{-3} secs) are called as metastable states.
- **Types of Lasers:** Lasers are mainly divided into the following categories: (i) Solid state laser, (ii) Gas laser, (iii) Semiconductor laser.

  They can be operated in two modes: (a) Continuous, (b) Pulsed.
- **Solid-State Laser:** Ruby laser is an example of solid-state laser. It produces an intense red beam using a three-level system with a wavelength of 6943 A°. It is a pulsed laser.
- **Gas Laser:** He-Ne laser is an example of a gas laser. It employs a four-level pumping scheme and operates in continuous mode. It produces a beam of wavelength 6328 A°.
- **Semiconductor Laser:** A semiconductor laser is a specially fabricated pn junction that emits coherent light when it is forward biased. The basic mechanism of producing laser in a semiconductor diode laser, is the electron-hole recombination at the pn junction when a current is passed through the diode.
- **Major Properties of Laser:**

  (i) Directionality, (ii) Monochromaticity,

  (iii) Coherence, (iv) Polarizability.
- **Applications:** Due to its unique properties, lasers are used in a variety of fields like welding, machining, surveying, communication, holography, cutting, drilling, information processing, surgery and related medical fields, in CD players, printers, etc.
- **Principle :** Total internal reflection.
- **Total Internal Reflection :** When light passes from denser to rarer medium and if angle of incidence is greater than critical angle, then light is totally reflected into the denser medium.
- **Optical Fibre :** It is a dielectric waveguide.
- **Waveguide :** The path through which light is propagated / guided. Optical waveguide consists of core and cladding.
- **Core :** A single dielectric cylinder of radius r and refractive index n_1.
- **Cladding :** A solid dielectric material surrounding the core with refractive index n_2 ($n_2 > n_1$).
- **Step-Index Optical Fibre :** The core has a uniform refractive index n_1 and the cladding has a uniform refractive index n_2.

- **Graded-Index Optical Fibre :** The refractive index of the core varies continuously from n_1 at the centre to n_2 at the core-cladding interface.
- **Acceptance Angle :** Light incident within the cone of half angle θ_0.
- **Numerical Aperture :** $\sin \theta_0 = \sqrt{n_1^2 - n_2^2}$.
- Modes of propagation :

  (a) Single mode, (b) Multimode.

IMPORTANT FORMULAE

- General condition for constructive interference,

  $x = n\lambda$, where $n = 0, 1, 2 ...$
- General condition for destructive interference,

  $x = (2n + 1)\frac{\lambda}{2}$, where $n = 0, 1, 2 ...$
- For uniform film,
  - Condition for constructive interference,

    $2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}, \quad n = 0, 1, 2 ...$
  - Condition for destructive interference,

    $2\mu t \cos r = n\lambda, \quad n = 0, 1, 2 ...$
- For nonuniform (wedge) film,
  - Condition for constructive interference,

    $2\mu t \cos (r + \alpha) = (2n \pm 1)\frac{\lambda}{2}, \quad n = 0, 1, 2 ...$
  - Condition for destructive interference,

    $2\mu t \cos (r + \alpha) = n\lambda, \quad n = 0, 1, 2 ...$
- Fringe width of fringes formed by wedge film,

$$\beta = \frac{\lambda}{2\mu \sin \alpha}$$

- Newton's rings,
  - Diameter of bright fringe,

    $$D_n^2 = \sqrt{2\lambda R} \cdot \sqrt{2n \pm 1} \text{ (bright)}$$
  - Diameter of dark fringe, $D_n^2 = 4n\lambda R$
- Wavelength of monochromatic source of light,

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}$$

- Refractive index of liquid, $\mu = \frac{D'^2_{n+p} - D'^2_n}{D^2_{n+p} - D^2_n}$
- Brewster's Law, $\mu = \tan i_p$
- For polarization by reflection, $i_p + r = \frac{\pi}{2}$
- The transmitted light through polarizer (Law of Malus),

$$I = I_0 \cos^2 \theta$$

- Refractive index of O-ray, $\mu_o = \frac{\sin i}{\sin r_o} = \frac{c}{v_o}$
- Refractive index of E-ray, $\mu_e = \frac{\sin i}{\sin r_e} = \frac{c}{v_e}$
- Specific rotation

$$s = \frac{10\theta}{l \times c}$$

- Rate of absorption, $R_{12} = P_a N_1$.
- Rate of stimulated emission, $R_{21} = P_e N_2$.
- At equilibrium, $P_a = P_e$.
- Population inversion, $N_2 > N_1$.
- Frequency of photons emitted, $\upsilon = \frac{E_2 - E_1}{h}$.

UNSOLVED PROBLEMS

1. A parallel beam of light of wavelength 5890 A° is incident on a thin film of refractive index 1.5, such that the angle of refraction into the film is 60°. Calculate the smallest thickness of the film which will make it appear dark by reflection.

   [Ans. 3.926×10^{-5} cm]
2. Two pin holes separated by a distance of 0.5 mm are illuminated by a monochromatic light of wavelength 6000 A°. An interference pattern is obtained on a screen placed at a distance of 100 cm from the pin holes. Find the distance on the screen between the fifth and tenth dark fringes. **[Ans.** 0.6 cm]
3. An oil drop of volume 0.2 cc is dropped on the surface of a tank of water of area 1 sq. meter. The film spreads uniformly over the whole surface and white light reflected normally is observed through a spectrometer. The spectrum is seen to contain first dark band whose centre has wavelength of 5.5×10^{-5} cm. Find the refractive index of oil. **[Ans.** 1.375]
4. A soap film of refractive index $\frac{4}{3}$ and of thickness 1.5×10^{-4} cm is illuminated by white light incident at an angle of 60°. The light reflected by it is examined by a spectroscope in which is found a dark band corresponding to a wavelength of 5×10^{-5} cm. Calculate the order of interference of the dark band.

   [Ans. n = 6]

5. The optical path difference between two sets of similar waves from the same source arriving at a point on the screen is 199.5 λ. Is the point dark or bright ? If the path difference is 0.012 cm, find the wavelength of the light used. **[Ans.** Dark, 6015 A°]

6. In a Newton's rings experiment, the diameter of the 5^{th} ring is 0.336 cm and the diameter of the 15^{th} ring is 0.590 cm. Find the radius of curvature of the plano convex lens, if the wavelength of light used is 5890 A°. **[Ans.** 99.82 cm]

7. In a Newton's rings experiment, find the radius of curvature of the lens surface in contact with the glass plate when with a light of wavelength 5890 A°, the diameter of the third dark ring is 0.32 cm. The light is incident normally. **[Ans.** 144.9 cm]

8. In Newton's rings, the diameter of a certain bright ring is 0.65 cm and that of tenth ring beyond it is 0.95 cm. If λ = 6000 A°, calculate the radius of curvature of a convex lens surface in contact with the glass plate. **[Ans.** 200 cm]

9. In a Newton's rings experiment, a drop of water $\left(\mu = \frac{4}{3}\right)$ is placed between the lens and the plate. In that case, the diameter of the 10^{th} ring was found to be 0.6 cm. Calculate the radius of curvature of the face of the lens in contact with the plate, given λ = 6000 A°. **[Ans.** 200 cm]

10. Newton's rings are observed in reflected light of λ = 5900 A°. The diameter of the 5^{th} dark ring is 0.4 cm. Find the radius of curvature of the lens and the thickness of the air film.

[Ans. 35.59 cms, 0.000295 cm]

11. In a Newton's ring experiment, the diameters of 4^{th} and 12^{th} dark rings are 0.4 cm and 0.7 cm respectively. Calculate the diameter of 20^{th} dark ring.

[Ans. 0.894 cm]

12. In a Newton's rings experiment, the source emits two wavelengths λ_1 = 6000 A° and λ_2 = 4500 A°. It is found that n^{th} dark ring due to λ_1 coincides with $(n + 1)^{th}$ dark ring due to λ_2. If the radius of curvature of the curved surface is 90 cm, find the diameter of n^{th} dark ring for λ_1. **[Ans.** 0.2538 cm]

13. If the diameter of n^{th} dark ring in a Newton's ring experiment changes from 0.3 cm to 0.25 cm, as a liquid is placed between the lens and the plate, calculate the value of μ of the liquid. **[Ans.** μ = 1.44]

14. A wedge-shaped air film, having an angle of 45 seconds, is illuminated by monochromatic light and fringes are observed vertically through a microscope. The distance measured between the consecutive fringes is 0.12 cm, calculate the wavelength of light used. **[Ans.** 5233 A°]

15. Two pieces of plane glass are placed together with a piece of paper between the two at one edge. Find the angle in seconds, of the wedge shaped air film between the plates, if on viewing the film normally with monochromatic light of wavelength 4800 A°, there are 18 bands per cm. **[Ans.** 89.1 seconds]

16. Two rectangular pieces of a plane glass are laid one upon the other and a thin wire is placed between them, so that a thin wedge shaped air film is formed between them. The plates are illuminated with sodium light of λ = 5893 A° at normal incidence. Bright and dark bands are formed, there being 10 of each per cm length of the wedge measured normal to the edge in contact. Find the angle of the wedge.

[Ans. 2.94×10^{-4} radians]

17. Two optically plane glass strips of length 10 cm are placed one over the other. A thin foil of thickness 0.010 mm is introduced between the plates at one end to form an air film. If the light used has wavelength 5900 A°, find the separation between consecutive bright fringes. **[Ans.** 0.295 cm]

18. Find the thickness of a wedge-shaped film at a point where fourth bright fringe is situated. λ for sodium light is 5893 A°. **[Ans.** 1.03×10^{-4} cm]

19. If the plane of vibrations of the incident beam makes an angle of 30° with the optic axis, compare the intensities of extraordinary and ordinary light.

$\left[\textbf{Ans. } \frac{I_e}{I_o} = 3\right]$

20. A beam of light travelling in water strikes a glass plate which is also immersed in water. When the angle of incidence is 51°, the reflected beam is found to be polarized. Calculate the refractive index of glass. **[Ans.** 1.235]

21. A glass plate is used as a polarizer. Find the angle of polarization for it. Also find the angle of refraction, given μ for glass = 1.54. **[Ans.** 57°, 33°]

22. Two polarizing sheets have their polarizing directions parallel so that the intensity of the transmitted light is maximum. Through what angle must either sheet be turned so that the intensity becomes one half the initial value ? **[Ans.** 45°, 135°**]**
23. The refractive index for plastic is 1.25. Calculate the angle of refraction for a ray of light inclined at polarizing angle. **[Ans.** 38.6°**]**
24. A beam of light is passed through two Nicol prisms in series. In a particular setting, maximum light is passed by the system and it is 500 units. If one of the Nicols is now rotated by 20°, calculate the intensity of transmitted light. **[Ans.** 441.5 units**]**
25. Two Nicol prisms are oriented with their principal planes making an angle of 30°. What percentage of incident unpolarized light will pass through the system ? **[Ans.** 37.5 %**]**
26. A polarizer and an analyzer are oriented so that the amount of light transmitted is maximum. To what fraction of its maximum value is the intensity of the transmitted light reduced when the analyzer is rotated through (i) 45°, (ii) 90° ? **[Ans.** 0.5, 0**]**

EXERCISE

1. Explain the phenomena of interference.
2. What is constructive and destructive interference ?
3. Derive the conditions for constructive and destructive interference.
4. Explain the phenomenon of interference in thin films in reflected light.
5. What are Newton's rings ? Explain how they are formed.
6. Explain the formation of colours in thin films.
7. Explain the phenomenon of interference in thin film in transmitted light.
8. How can Newton's rings be obtained in the laboratory ? How will you use them to measure the wavelength of sodium light ?
9. Explain the theory and the experimental arrangement of Newton's rings experiment.
10. What have you understood by non-reflecting films ? Explain.
11. In Newton's rings, show that the radii of dark rings are proportional to the square root of natural numbers.
12. When seen by reflected light, why does an excessively thin film appear to be perfectly black when illuminated by a white light ?
13. Explain, why colours are not observed in the case of a thick film when illuminated by a white light.
14. How can Newton's rings be used to determine the refractive index of a liquid ? Derive the formula used.
15. Prove that in reflected light Newton's rings, the diameters of bright rings are proportional to the square root of the odd natural numbers.
16. How can Newton's rings be obtained in the laboratory ? Prove that for Newton's rings in reflected light, the diameters of dark rings are proportional to the square root of natural numbers.
17. Explain the term polarization of light.
18. Define plane of polarization and plane of vibration. Explain a method to show that light waves are transverse.
19. Distinguish between polarized and unpolarized light.
20. State Brewster's law and use it to prove that when light is incident on a transparent substance at the polarizing angle, the reflected and refracted rays are at right angles to each other.
21. Explain how you would obtain plane polarized light by reflection.
22. What is pile of plates ? Explain how it can be used for producing plane polarized light.
23. What is polarizing angle ? Explain.
24. Explain the phenomenon of double refraction in calcite.
25. Describe the construction and working of a Nicol prism.
26. What is a Nicol prism ? Explain how a Nicol prism can be used as an analyzer and polarizer.
27. Explain giving diagrams the nature of refraction observed in the case of calcite crystal when:
    (a) Optic axis is parallel to the refractive surface and lying in the plane of incidence (normal incidence).
    (b) Optic axis is perpendicular to the refracting surface and lying in the plane of incidence (normal incidence).
28. Give Huygen's construction for ordinary and extraordinary wavefronts when the beam of light is refracted through a doubly refracting crystal when the optic axis is inclined to the crystal surface and lying in the plane of incidence (normal incidence).
29. What does the numerical aperture indicate ?
30. Give advantages of using optical fibre as compared to conventional cable for telecommunication.
31. What is optical activity? Explain, how Laurentz's half shade polarimeter can be used for measuring specific rotation?
32. Explain the operation of Ruby laser with a neat labelled diagram.
33. Explain the following terms:
    (i) Spontaneous emission (ii) Stimulated emission
    (iii) Population inversion.

34. Explain action of gas laser. How does stimulated emission take place with exchange of energy between Helium and Neon atoms ?
35. What is population inversion ? Explain the operation of He - Ne laser.
36. What are the different uses to which laser beams are put in industry, medicine ?
37. Define and explain the terms:
    (i) Pumping (ii) Active systems.
38. Write a note on use of lasers in fibre communication systems and information technology.
39. Explain the elements of optical fibre communication link.
40. What are various parts in the optical fibre communication system ?
41. Explain total internal reflection and its relation with the working of optical fibre.
42. Give constructional features of optical fibre.
43. Describe various parts of optical fibre ?
44. Give name of different types of optical fibre and their structure.
45. What do you mean by
    (i) Monomode fibres, (ii) Multimode fibres.
46. Explain 'Numerical Aperture' and arrive at the expression for numerical aperture.

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Better understanding of polarization with some animations:

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Animation of double refraction:

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Three dimensional diagram showing different types of polarization:

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UNIVERSITY QUESTIONS

December 2017

1. Derive an expression for the optical path difference for the reflected rays in a thin film of constant thickness and hence find the conditions for maxima and minima. **[6]**
2. What is double refraction? Explain the difference between ordinary ray (O-ray). And extra ordinary ray (e-ray) **[6]**
3. What is population inversion and stimulated emission ? Calculate the acceptance angle of an optical fibre where the refractive index of core is 1.55 and that of cladding is 1.50. **[4+2]**

May 2018

1. In case of Newton's rings in reflected light show that diameter of bright rings is proportional to the square root of odd natural numbers. In Newton's rings, the diameter of a certain bright ring is 0.65 cm and that of tenth ring is 0.95 cm. If $\lambda = 6000$ A°, calculate the radius of curvature of a convex lense.
2. Give the diagrammatic representation of polarized and unpolarized light. **[6]**
   Explain the method of producing plane polarized light by reflection.
3. Explain the construction and working of He-Ne laser with neat diagram. **[6]**

December 2018

1. In case of Newton's rings, prove $D_n \propto \sqrt{n}$, where D_n is diameter of n^{th} dark ring. **[6]**
2. Explain Double refraction using Huygen's wave theory of light. **[6]**
3. Explain the construction and working of Ruby laser with neat diagram. **[6]**

May 2019

1. Prove that for Newton's rings in reflected light, the diameter of dark ring is proportional to the square root of natural numbers. **[6]**
2. Explain construction and working of Ruby laser with neat diagram. **[6]**
3. Obtain mathematical expression for acceptance angle and numerical aperture. **[6]**

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