SYLLABUS

Module 1 : Basic Concepts **(7 Lectures)**

Objectives of Engineering Analysis and Design, Idealization of Engineering Problems, Simplification of real 3D problems to 2-D and 1-D domain, Basis of Assumptions, types of supports, types of load, free body diagram, Laws of Motion, Fundamental principles, Resolution and composition of a forces, Resultant, couple, moment, Varignon's theorem, force systems, Centroid of composite shapes, moment of inertia of planer sections and radius of gyration.

Module 2 : Equilibrium **(7 Lectures)**

Static equilibrium, analytical and graphical conditions of equilibrium, Lami's theorem, equilibrium of coplanar concurrent forces, coplanar non concurrent forces, parallel forces, beams reactions, Simple trusses (plane and space), method of joints for plane trusses, method of sections for plane trusses.

Friction: Coulomb law, friction angles, wedge friction, sliding friction and rolling resistance.

Module 3 : Kinematics **(7 Lectures)**

Types of motions, kinematics of particles, rectilinear motion, constant and variable acceleration, relative motion, motion under gravity, study of motion diagrams, angular motion, tangential and radial acceleration, projectile motion, kinematics of rigid bodies, concept of instantaneous center of rotation, concept of relative velocity,

Module 4 : Kinetics **(6 Lectures)**

Mass moment of inertia, kinetics of particle, D'Alemberts principle : applications in linear motion, kinetics of rigid bodies, applications in translation, applications in fixed axis rotation.

Module 5 : Work, Power, Energy **(6 Lectures)**

Principle of virtual work, virtual displacements for particle and rigid bodies, work done by a force, spring, potential energy, kinetic energy of linear motion and rotation, work energy equation, conservation of energy, power, impulse momentum principle, collision of elastic bodies.

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• **Solved University Question Papers (Oct. 2017 to May 2019)**

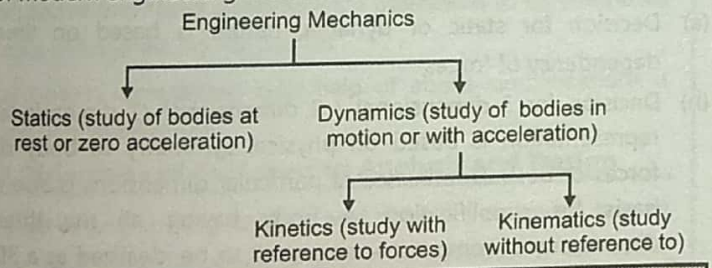
UNIT - I

COPLANAR FORCE SYSTEM

A – Basic Concept and Concurrent Forces System

1.1 INTRODUCTION

Origin of word "mechanics" can be traced to Greek words "mekhane", "mekhanikos" meaning manual labor for repetitive tasks. With development of tools, idea of mechanisms and subsequently that of machines evolved. Need of explanation of movement of connected bodies resulted in birth of "Mechanics". With development to cover wide range of problems "Engineering Mechanics" took position as a starter course based on simplifying assumptions such as effect of forces without reference to deformations. This is a subject dealing with application of knowledge of Physics and Mathematics to real life problems requiring solution through engineering analysis and design. It provides basic platform for developing further understanding of subjects such as solid mechanics, fluid mechanics, soil mechanics, structural mechanics, theory of machines and mechanisms, design of structures, design of machines etc. Understanding of "action" as a cause and investigation in to "effect" as a consequence is central theme of mechanics. Apart from such a direct relevance, this subject help to cultivate a systematic approach to simplification of real complex phenomena in to idealized small modules. Solution of such modularized small set of problem is logically interconnected to generate solution for entire system. Today's engineering mechanics is based on Isaac Newton's laws of motion. Modern practice of their applications can be traced back to Stephen Timoshenko, who is often said to be "the father of modern engineering mechanics" which is classified as:



1.2 BASIC CONCEPTS

1.2.1 Idealization of Engineering Problems

Let's imagine a task of designing a simple stool (utility furniture). This problem will be divided in to three distinct phases after selection of material of choice: (1) We need to know the geometrical detail of commodity, (2) depending on use, we need to know source of load and how that is to be transferred, (3) calculations for load propagation from source to intermediate members to destination. Clearly, we need to start with drawing a picture of stool as how it is expected to look like. Then we go for proportioning of various features height, length and width of platform, etc. Then we decide about possible load to be carried by this commodity as load of a standing person may be about 80 kg. We need to think about how this is transferred through seat to legs of stool and then to ground. We need to ensure that various components will carry and transfer the load to fulfill the objective of use. Lot of engineering is hidden in described process. If we imagine to replace making of stool with any other commodity, we find that product will change but thought process, logic and approach to arrive at final product will remain same and that is the engineering thread involved in to it. Steps narrated as above can be listed in formal engineering language and terminology as under.

1. At stage of sketching size and shape of object we are trying to prepare a graphical model. It starts with drawing a picture containing coarser details. Picture as such need to be converted in to technical drawing. (Fig. 1.1) Representation as a picture and a technical drawing will differ in characteristics. Technical drawing will aim at showing only required details such as person will be replaced with an arrow indicating the load magnitude, stool shown with respect to main members as a line diagram, etc. Picture thus transformed in to technical drawing represent graphical model of the phenomena. It contains dimensions, load magnitudes, support features, etc. Decision to solve the problem as a 3 dimensional entity or 2 dimensional entity is also made at this stage. (Fig. 1.2) It is possible to make such assumption on understanding of symmetry if involved.

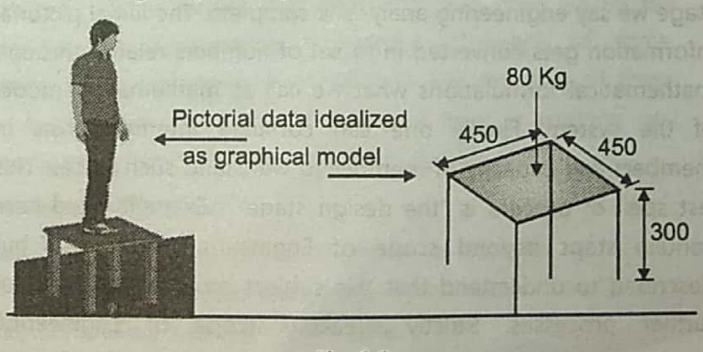


Fig. 1.1

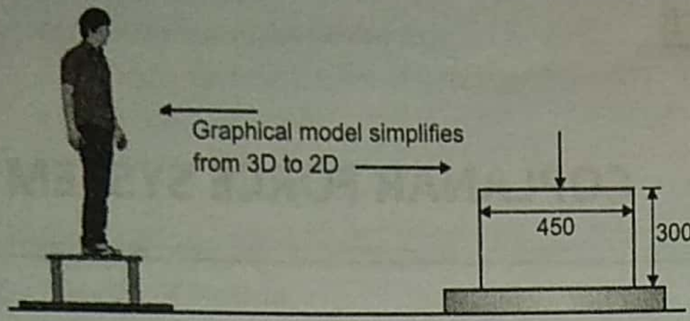


Fig. 1.2

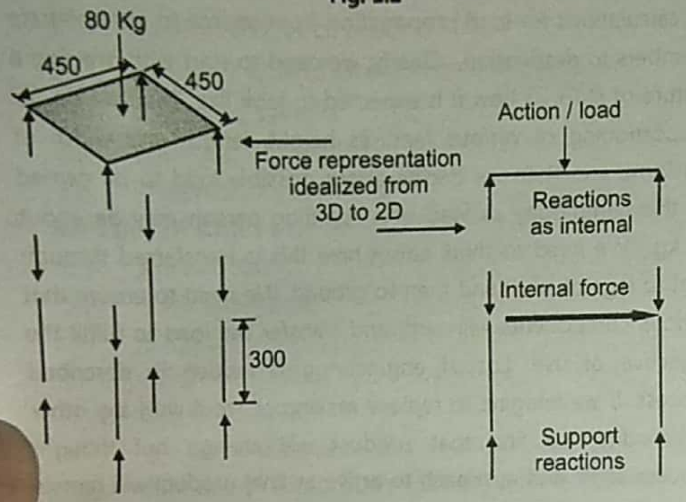


Fig. 1.3

2. Being ready with graphical model we move on to understand load transfer from platform to vertical legs and from vertical legs to supporting ground. Nature of forces in members will depend on structural action of interconnected members. At this stage all the forces will be known (or assumed) in direction and position. External loads will be known in magnitude also. Objective will be to determine magnitudes of unknown forces such as compression in legs, support reactions at base, etc. At the end of this stage we find that loads and support sections are the external forces while forces in members are internal to the system. An idea arise for balance among external forces. (Fig.1.3)

3. In order to solve for unknowns we try to relate loads and reactions and then knowing reactions proceed to calculate internal forces in members. Mathematical statements are formulated to relate (a) loads and reactions; (b) loads, reactions and internal forces. Solution of such mathematical equations leads to knowledge of total force distribution in system. At this stage we say engineering analysis is complete. The initial pictorial information gets converted in to set of numbers related through mathematical formulations what we call as mathematical model of the system. Finally one can compare internal forces in members and capacity of member to withstand such forces. This last spell of process is "the design stage". Example cited here contain steps beyond scope of Engineering Mechanics but described to understand that this subject provides platform for further processes. Strictly speaking scope of Engineering Mechanics will be to restrict the learners in dealing with effect of external forces on bodies and no reference will be made to internal forces and effects such as member deformations. This is because at the stage when we consider interaction of external loads and reactions, it is not required to complicate the consideration by involving deformations. As a phased approach, treatment will be restricted to study of forces without reference to deformations.

Conventionally, it has been said that Engineering Mechanics deals with "Rigid Bodies" or assumption is made as "bodies are rigid". Better approach is to understand that bodies are deformable under forces but knowledge of deformations is not required for solution of type of problems that are handled in scope of engineering mechanics.

For "engineering analysis and design of a utility stool" scope of Engineering Mechanics will be: (i) To develop a graphical model, (ii) To understand external loads and nature of support reactions and (iii) To develop and solve a mathematical model describing relationship of loads and reactions. Part of solution comprising of finding internal forces and checking member capacities will belong to solid mechanics, structural mechanics and design related subjects dealing with elastic deformability of bodies under given forces.

This approach of engineering analysis is useful for all branches. Concept of graphical and analytical modeling provide means of simulation of actual behavior of a physical system or prototype. A physical system can be considered in broader sense to include civil engineering structures, mechanical engineering machines, mechanisms, and motors in electrical engineering or electronic devices in other disciplines. A complex system involving numerous elements is often simplified by taking advantage of the behaviour of the components and behavior of the system under particular loading. To summarize, assumptions are made to simplify or idealize the actual problem to facilitate formulation of manageable mathematical relations. Salient steps applicable to most cases are:

(a) Decision for static or dynamic nature is based on time dependency of forces

(b) Decision for 3 dimensional / 2 dimensional /1 dimensional representation is based on physical symmetry of body or forces or both. Dominance of particular dimensions is also a basis for simplification. A body having all the three dimensions of comparable size need to be idealized as a 3D problem, e.g., machine components, automobile chassis, framed constructions, etc. A body having comparable length and width but comparatively smaller thickness is idealized as 2D Problem, e.g, wheels, gears, walls, roofs, display boards, etc. A body having significant length but comparatively smaller cross-sectional dimensions is idealized as 1D Problem, e.g, chain, belt, ropes, members of frames like columns, beams, etc.

(c) Forces are idealized as a point load, distributed load, varying load depending on area in contact and total area of body. A point load or a concentrated load is symbolic representation for case where load acts on very small area, e.g., person standing on a floor is assumed to transmit a point load. Load transferred by a wall of uniform height will be a uniformly distributed load. Load transmitted by a wall of varying height or pressure exerted on side walls are examples of varying loads

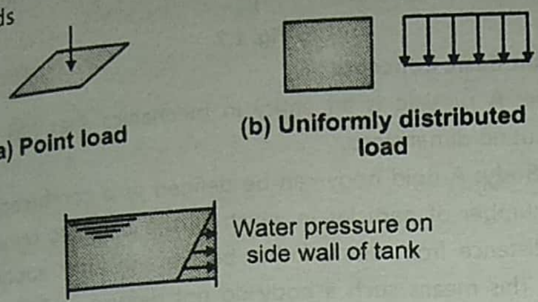


Fig. 1.4

(d) Supports are the locations in space where a body can rest against. Depending on manner of connection and restraints to motion offered supports can be idealized for 3D, 2D and 1D problems. For illustration, supports in 2D problems are as under:

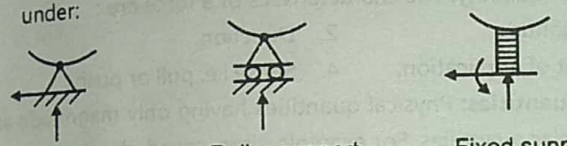


Fig. 1.5: Types of 2D supports

Concept of idealization of loads and supports will be clearer when these are used in subsequent sections. Time being, it is to be noted that a real point of contact will have nature of forces as shown above and these constitute the unknowns to be computed in Statics of Engineering Mechanics

A real system represented with help of above simplifications is nothing but "Idealization"

1.2.2 Objectives of Engineering Analysis and Design

Objective of engineering analysis and design is to enrich human life by facilitating development and improvement of tools, machines, equipment, structures and many such utility facilities. Many tasks those were attempted manually are being got done by tools and machines. This saves resources such as time, money and energy. Situations posing life threat or health hazard are best attended by machines like robots.

Development of technology or technological tools starts with engineering analysis of the phenomena. Graphical modeling, mathematical modeling and numerical solution of governing equations constitute Analysis of given system. Approach for numerical solution depend on complexity of problem and availability of computational tools. For simple systems it may be the case of manual hand computations. In every case analysis results in to knowledge of force distribution in the body or that for system of interconnected members. Engineering Design a process to ensure that forces and effects computed in analysis are well within capacities of associated bodies. Very often output of design process is set of technical drawings containing all information required for transforming details on paper a real prototype. It prescribe requirements for construction of structures or production or manufacturing of machines, engines or devices.

1.2.3 Force, Couple, Free Body Diagram

These terms will be frequently referenced to in "Mechanics". Formal definition of force will follow in next section. Here it is of interest to point out that forces are originating from two sources, i) within or internal of body and ii) outside or external to body. Internal sources are presence of body itself under influence of gravity what we call as a self-weight and resistance developed by body as consequence of external effects. External forces are explicit application of some physical action such as pushing or pulling a cart, making a blow with a hammer. Generally a direct force is expected to induce a translational displacement. Couple is a specific combination of two direct forces that are directionally opposite and while acting on given body there is separation by perpendicular distance between lines of action. It can be easily anticipated that such a pair of forces will cause the body to rotate. Free body diagram is a representation of an individual body along with all external forces acting on it in a given system. Fig. 1.3 shows platform and legs of stool in a separated form. Forces offered by legs on platform are the means of support to platform. Legs are subjected to load transferred from platform and support reactions offered by ground. An important lesson in drawing free body diagrams is understanding of external and internal forces. Only the internal forces will appear as a oppositely directed pair of collinear forces.

1.3 FUNDAMENTAL PRINCIPALS

1.3.1 Newton's Laws

It has been stated earlier that Newton's laws have paved the foundation for Engineering Mechanics. Concept of force, resultant, static and dynamic equilibrium are the gifts of these laws. Application of these concepts and laws is nothing but Newtonian Mechanics or Engineering Mechanics.

1. Newton's First Law:

Statement: A body continues to be in its state of rest or of uniform motion unless it is acted upon by an external unbalanced agency compelling it to change it's state.

Explanation: This law is a qualitative explanation of concept of force. Effect of "external unbalanced agency" in reference is said to be the reason for displacement of body.

2. Newton's Second Law:

Statement: The rate of change of momentum of a body is directly proportional to the force acting on it and takes place in the direction of force.

$$\frac{d(m\vec{v} - m\vec{u})}{dt} \;\alpha\; \vec{F}$$

$$\therefore \quad m\frac{d\vec{v} - \vec{u}}{dt} = k\vec{F}$$

$$\therefore \quad \vec{F} = m \times \vec{a}$$

$$k = 1$$

Explanation: This law provides a quantitative measure of force. If the resultant force acting on a particle is not zero, a particle will have an acceleration proportional to the magnitude of the resultant force and in the direction of this resultant force along a straight line. Alternatively it also defines concept of equilibrium as state of zero unbalanced force in a system.

3. Newton's Third Law:

Statement: For every action, there is equal and opposite reaction.

Explanation: This law describes effect of action of a force. The forces of action and reaction between bodies in contact have the same magnitude, same line of action, but opposite in direction.

In mechanics we consider bodies resting on supports to establish equilibrium. Supports are the means of availing of reactions for countering the effects of actions.

4. Newton's Law of Gravitation :

It states that two bodies of mass 'm' and 'M' are mutually attracted with equal and opposite forces of same magnitude. Refer Fig. 1.6

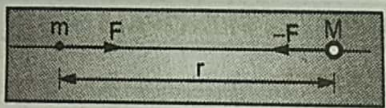


Fig. 1.6

$$\boxed{F = G\frac{Mm}{r^2}}$$ where, G = Constant of gravitation

Principle of Transmissibility of Forces:

The state of rest or motion of a rigid body will remain unchanged if a force acting at a given point of the rigid body is replaced by a force of same magnitude and same direction acting at any point on the same line of action. Refer Fig. 1.7

Sense of force has changed from push at A to pull at B.

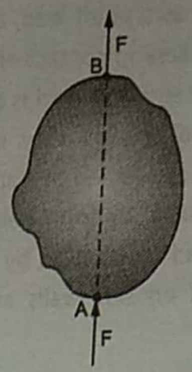


Fig. 1.7

More on Basic Concepts

Particle: A particle is an entity in mechanics that has definite mass but no dimensions.

Rigid Body: A rigid body can be defined as a combination of a large number of particles in which all the particles remain at a fixed distance from one another before and after applying the forces. This means such a body do not deform under action of forces. This is imaginary. Earlier it was pointed out that such an assumption is not required if it is understood that deformations are neither part of required solution nor the part of data required for solution.

Force: Force can be defined as any action that tends to change the state of rest or motion of the body upon which it acts. A force is a vector quantity. The characteristics of a force are :

1. Magnitude.
2. Direction.
3. Point of application.
4. Sense i.e. pull or push.

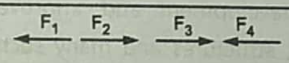
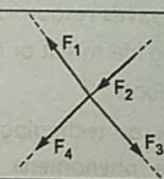
Scalar Quantities: Physical quantities having only magnitude are called scalar quantities. For example, time, speed, density etc.

Vector Quantities: Physical quantities having both magnitude as well as direction are called vector quantities. For example, force, velocity displacement etc.

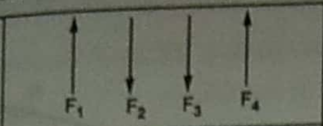
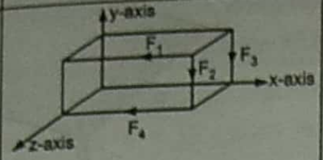
System of Forces -

A force system can be classified into 2-D force system or coplanar force system, and 3-D force system or space forces. The characteristics of force system are as follows:

Table 1.1

Sr. No.	Force System	Diagram	Description
1.	Coplanar forces		Lines of action of all forces lie in the same plane.
2.	Collinear forces		Lines of action of all forces lie in the same line.
3.	Concurrent forces		Lines of action of all forces intersect at a single point.

...Contd.

4.	Parallel forces		Lines of action of all forces are parallel to each other.
5.	Non-concurrent and non-parallel forces		Lines of action of all forces that do not intersect at a single point and are not parallel to each other.

PRINCIPLES OF STATICS

1.3.2 Force: Resolution

Resolution of a force is the process of replacing a force by several component forces having the same effect as that of a single force.

A force is resolved into two component forces acting at a point along any direction by using law of parallelogram (Analytically).

Force $\bar{R}$ can be resolved along directions OA and OB into component forces $\bar{P}$ and $\bar{Q}$ by law of parallelogram. Refer Fig. 1.8.

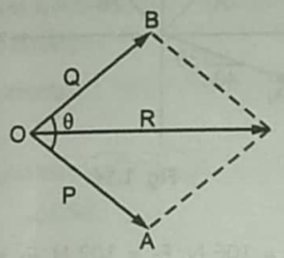


Fig. 1.8

A force is resolved into two component forces making an angle of 90° with each other. Component forces are called rectangular components of a force.

Magnitudes of rectangular components of force $\bar{R}$ along x-axis and y-axis are $F_x = R \cos \theta$ and $F_y = R \sin \theta$ respectively. Refer Fig. 1.9.

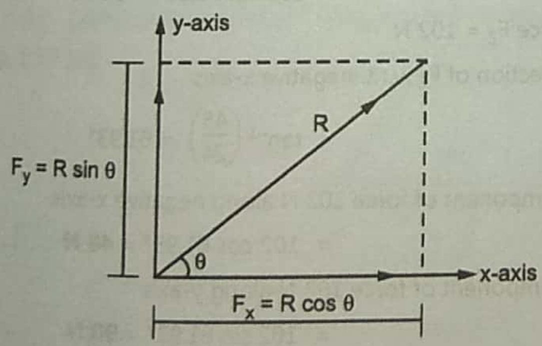


Fig. 1.9

A force is resolved into two component forces along any direction by using law of triangle. (Graphically).

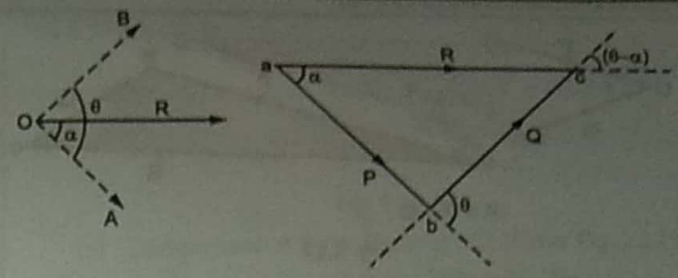


Fig. 1.10

Force $\bar{R}$ can be resolved along directions OA and OB as shown in Fig. 1.10. Side 'ab' represents component force $\bar{P}$ along direction OA. Side 'bc' represents component force $\bar{Q}$ along direction OB. Side 'ac' represents a force $\bar{R}$. Refer Fig. 1.10.

1.3.3 Resultant of Forces

Composition of forces is the process of adding two or more forces to obtain a single force having same effect as that of number of its component forces. This single force is called resultant of number of forces. Resultant of two forces acting at a point is obtained by law of parallelogram. Resultant of two forces can be obtained by law of triangle which is also known as a Graphical method.

Law of Parallelogram: Two coplanar concurrent forces can be replaced by a single resultant force by law of parallelogram. If two concurrent forces are represented by the adjacent sides of the parallelogram, then the diagonal of the parallelogram passing from a common point of two forces represents resultant in magnitude and direction.

Forces $\bar{P}$ and $\bar{Q}$ are acting at point A making an angle θ with each other. Refer Fig. 1.11.

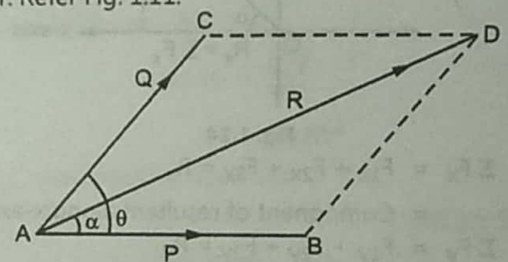


Fig. 1.11

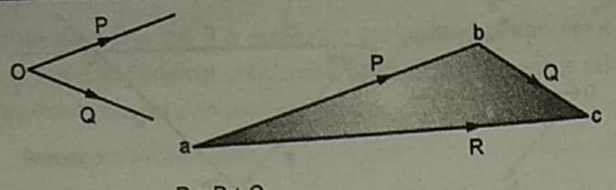
Magnitude of resultant,

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

Direction of resultant w.r.t. force $\bar{P}$, $\tan \alpha = \dfrac{Q \sin \theta}{P + Q \cos \theta}$

Triangle Law of Forces: If two forces are represented in clockwise or anticlockwise order as the adjacent sides of a triangle, the resultant force is represented by the third closing side of the triangle directed from starting point of first force to end point of second force.

Side 'ab' represents force $\bar{P}$. Side 'bc' represents force $\bar{Q}$. Side 'ac' represents force $\bar{R}$. Refer Fig. 1.12



R = P + Q

Fig. 1.12

Resultant of Concurrent Force System by Analytical Method

- Resolve each force along x and y axes. Refer Fig. 1.13
- Sum up components of forces along x-axis. (ΣF_X) i.e. R_X.
- Sum up components of forces along y-axis. (ΣF_Y) i.e. R_Y.
- Magnitude of resultant, $R = \sqrt{(\Sigma F_X)^2 + (\Sigma F_Y)^2}$.
- Direction of resultant w.r.t. x-axis = $\alpha = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$. Refer Fig. 1.14

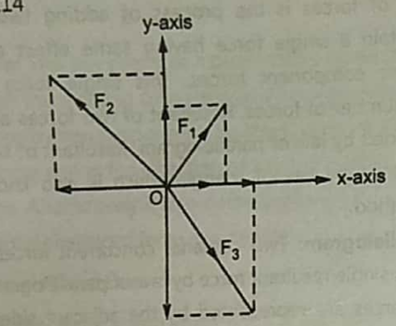


Fig. 1.13

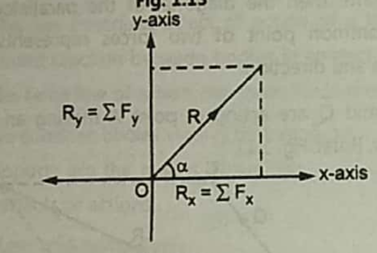


Fig. 1.14

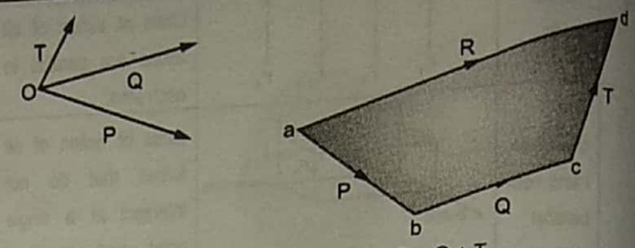
$$\Sigma F_X = F_{1X} + F_{2X} + F_{3X} = R_X$$
$$= \text{Component of resultant along x-axis}$$
$$\Sigma F_Y = F_{1Y} + F_{2Y} + F_{3Y} = R_Y$$
$$= \text{Component of resultant along y-axis}$$

Let R be magnitude of resultant.

$$\boxed{R = \sqrt{R_x^2 + R_y^2} = \sqrt{(\Sigma F_X)^2 + (\Sigma F_Y)^2}, \quad \theta = \tan^{-1}\left(\frac{R_Y}{R_X}\right) = \tan^{-1}\left(\frac{\Sigma F_Y}{\Sigma F_X}\right)}$$

Graphical Method : Resultant of several concurrent forces can be obtained by law of polygon.

Law of Polygon : *If number of forces are represented sequentially by the sides of polygon in clockwise or anticlockwise order, the resultant force is represented by the closing side of the polygon directed from starting point of first force to end point of last force.*



R = P + Q + T

Fig. 1.15

Side 'ab' represents force $\bar{P}$. Side 'bc' represents force $\bar{Q}$. Side 'cd' represents force $\bar{T}$. Side 'ad' represents resultant force $\bar{R}$. Refer Fig. 1.15

NUMERICAL EXAMPLES ON COMPOSITION AND RESOLUTION OF CONCURRENT FORCES

Example 1.1 : *Determine the x and y components of each of the forces shown.*

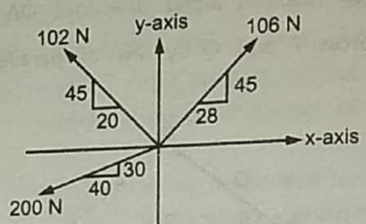


Fig. 1.16

Solution :

Give data : F_1 = 106 N, F_2 = 102 N, F_3 = 200 N. Dimensions as per Fig. 1.16

To find : x and y components of each force.

(a) Force F_1 = 106 N

Direction of F_1 w.r.t. x-axis $= \tan^{-1}\left(\frac{45}{28}\right) = 58.11°$

Component of force 106 N along x-axis

$= 106 \cos 58.11° = \mathbf{56\ N}$ **... Ans.**

Component of force 106 N along y-axis

$= 106 \sin 58.11° = \mathbf{90\ N}$ **... Ans.**

(b) Force F_2 = 102 N

Direction of F_2 w.r.t. negative x-axis

$= \tan^{-1}\left(\frac{45}{24}\right) = 61.93°$

Component of force 102 N along negative x-axis

$= 102 \cos 61.93° = \mathbf{48\ N}$ **... Ans.**

Component of force 102 N along y-axis

$= 102 \sin 61.93° = \mathbf{90\ N}$ **... Ans.**

(c) Force F_3 = 200 N

Direction of F_3 w.r.t. negative x-axis

$= \tan^{-1}\left(\frac{30}{40}\right) = 36.87°$

Component of force 200 N along negative x-axis

$$= 200 \cos 36.87° = \mathbf{160\ N} \quad \text{... Ans.}$$

Component of force 200 N along negative y-axis

$$= 200 \sin 36.87° = \mathbf{120\ N} \quad \text{... Ans.}$$

Example 1.2 : *For a particular position, connecting rod BA of engine exerts a force $P = 25$ kN on the crank-pin at A. Resolve force into two rectangular components :*

(a) P_h and P_v acting horizontally and vertically.

(b) P_r and P_t along radius AO and perpendicular to AO respectively.

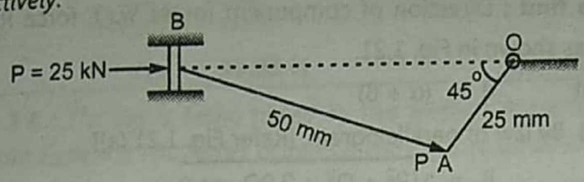


Fig. 1.17

Solution :

Given data : P = 25 kN acting at point A.

l (AB) = 50 mm, l (AO) = 25 mm, ∠ BOA = 45°

To find : (a) Components of force P along horizontal (P_h) and vertical (P_v) direction.

(b) Components of force P along AO and perpendicular to AO (P_r and P_t).

(a) From geometry of triangle BAO, [Refer Fig. 1.17 (a)]

Let ∠OBA = α

By sine rule,

$$\frac{25}{\sin \alpha} = \frac{50}{\sin 45°}$$

$$\therefore \quad \sin \alpha = \frac{25 \times \sin 45°}{50}$$

$$\therefore \quad \alpha = 20.7°$$

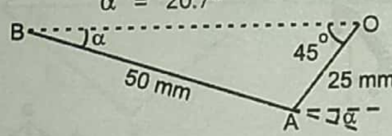


Fig. 1.17 (a)

(b) Component of force P along horizontal direction, [Refer Fig. 1.17 (b)]

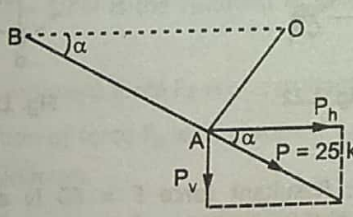


Fig. 1.17 (b)

$$P_h = P \cos \alpha = 25 \cos 20.7°$$

$$= \mathbf{23.39\ kN} \quad \text{... Ans.}$$

Component of force P along vertical direction,

$$P_v = P \sin \alpha = 25 \sin 20.7°$$

$$= \mathbf{8.84\ kN} \quad \text{... Ans.}$$

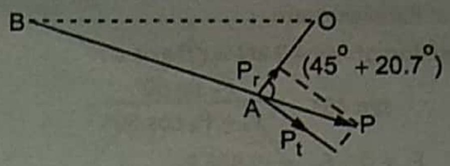


Fig. 1.17 (c)

(c) Component of force P along AO, [Refer Fig. 1.17 (c)]

$$P_r = P \cos (45° + 20.7°)$$

$$= 25 \cos 65.7° = \mathbf{10.3\ kN} \quad \text{...Ans.}$$

Component of force normal to AO,

$$P_t = P \sin (65.7°) = 25 \sin 65.7°$$

$$= \mathbf{22.8\ kN} \quad \text{... Ans.}$$

Example 1.3 : *Find value of α if resultant of given three forces is parallel to the inclined plane. Also find corresponding magnitude of the resultant.*

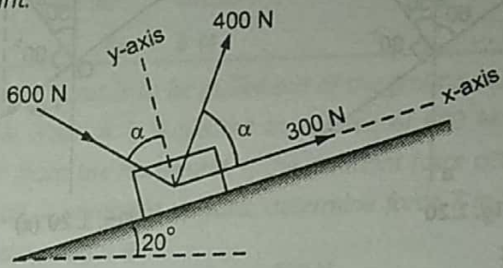


Fig. 1.18

Solution :

Let axis parallel to inclined plane be X-axis.

(a) Since resultant of three forces along X-axis is

$$\Sigma F_y = 0$$

$$\therefore \quad 400 \sin \alpha - 600 \cos \alpha = 0$$

$$\therefore \quad 400 \frac{\sin \alpha}{\cos \alpha} - 600 = 0$$

$$\therefore \quad 400 \tan \alpha - 600 = 0$$

$$\therefore \quad \tan \alpha = \frac{600}{400}$$

$$\therefore \quad \alpha = \mathbf{56.31°} \quad \text{... Ans.}$$

(b) Magnitude of resultant $= \Sigma F_x$

$$\therefore \quad \Sigma F_x = 300 + 400 \cos 56.31° + 600 \sin 56.31°$$

$$= \mathbf{1021.11\ N} \quad \text{...Ans.}$$

Example 1.4 : *Determine components of 2 kN force along oblique axes a and b. Determine projections of F on a and b axes.*

Solution :

(a) By triangle law : Refer Fig. 1.19 (a)

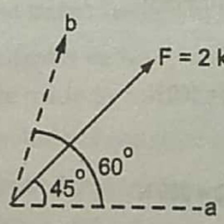


Fig. 1.19

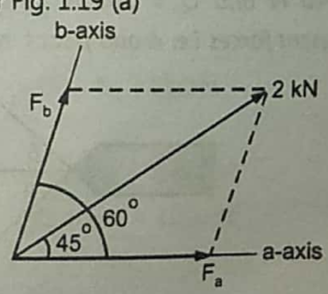


Fig. 1.19 (a)

By Law of Parallelogram :

(a) Direction of force 2 kN w.r.t. axis a :

$$\tan 45° = \frac{F_b \sin 60°}{F_a + F_b \cos 60°}$$

$$\therefore \quad F_a + 0.5\, F_b = 0.866\, F_b$$

$$\therefore \quad F_a = 0.366\, F_b \qquad \ldots (1)$$

(b) $$(2)^2 = F_a^2 + F_b^2 + 2F_a \cdot F_b \cos 60° \qquad \ldots (2)$$

Using equations (1) and (2), we get

$$F_a = \mathbf{1.633\ kN} \qquad \ldots \textbf{Ans.}$$

$$F_b = \mathbf{0.597\ kN} \qquad \ldots \textbf{Ans.}$$

Example 1.5 : *Determine components of the 1000 N force shown along aa' and bb' axes shown in Fig. 1.20*

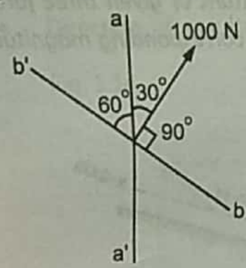


Fig. 1.20

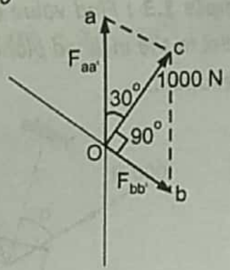


Fig. 1.20 (a)

Solution :

By law of parallelogram (Refer Fig. 1.20 (a).

Side of parallelogram opposite to force $F_{bb'}$ is equal to $F_{bb'}$ in magnitude and direction.

Redrawing half portion of parallelogram as shown in Fig. 1.20 (b)

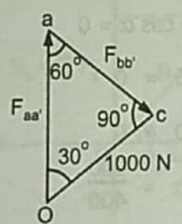


Fig. 1.20 (b)

Applying sine rule to triangle oac,

$$\frac{1000}{\sin 60°} = \frac{F_{aa'}}{\sin 90°} = \frac{F_{bb'}}{\sin 30°}$$

$$\therefore \quad F_{aa'} = \mathbf{1154.7\ N} \qquad \ldots \textbf{Ans.}$$

$$F_{bb'} = \mathbf{577.35\ N} \qquad \ldots \textbf{Ans.}$$

Example 1.6: *Resultant force R = 400 N has two component forces P = 240 N and Q = 200 N as shown. Determine direction of component forces i.e. α and β w.r.t. resultant force.*

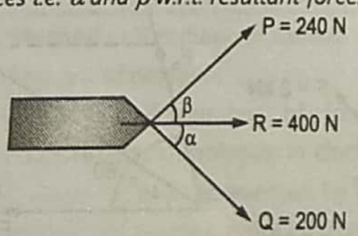


Fig. 1.21

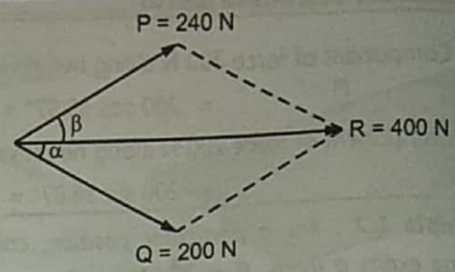


Fig. 1.21 (a)

Solution :

Given data : Resultant force R = 400 N

Components of force R are P = 240 N and Q = 200 N.

To find : Direction of component forces w.r.t. force R i.e. β and α as shown in Fig. 1.21.

Let $\theta = (\alpha + \beta)$

(a) By law of parallelogram, [Refer Fig. 1.21 (a)]

$$R = \sqrt{P^2 + Q^2 + 2\,PQ \cos\theta}$$

$$\therefore \quad (400)^2 = (240)^2 + (200)^2 + 2 \times 240 \times 200 \times \cos\theta$$

$$\therefore \quad \cos\theta = 0.65$$

$$\therefore \quad \theta = 49.46°$$

$$\therefore \quad \alpha + \beta = 49.46°$$

(b) Direction of resultant force 400 N w.r.t. force 240 N :

$$\tan\beta = \frac{200 \sin(\alpha + \beta)}{240 + 200 \cos(\alpha + \beta)}$$

But $(\alpha + \beta) = 49.46°$

$$\therefore \quad \beta = \mathbf{22.33°} \text{ and } \alpha = \mathbf{27.12°} \qquad \ldots \textbf{Ans.}$$

Example 1.7 : *The vertical force F = 60 N acts downward at A as shown in Fig. 1.22. Determine the angle θ (0° ≤ θ ≤ 90°) of member AB so that the component of F acting along the axis of AB is 80 N. What is the magnitude of the force component acting along the axis of member AC ?*

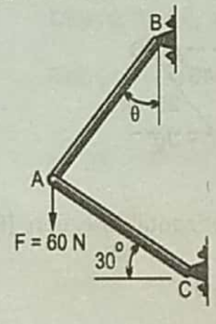


Fig. 1.22

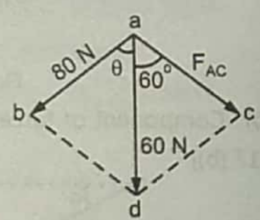


Fig. 1.22 (a)

Solution :

Given data : Resultant force F = 60 N acting vertically downward. F_{AB} = 80 N and F_{AC} are components of force F.

To find : Direction of F_{AB} w.r.t. vertical = θ = ?

Component force, F_{AC} = ?

Triangle abd as shown in Fig. 1.22 (b) represents half of parallelogram shown in Fig. 1.22 (a).

Angle θ can be determined by law of sine :

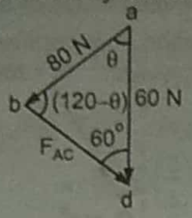


Fig. 1.22 (b)

$$\frac{80}{\sin 60^\circ} = \frac{60}{\sin (120 - \theta)^\circ}$$

$\therefore$ $\theta = \mathbf{79.49^\circ}$ ... **Ans.**

$$\frac{80}{\sin 60^\circ} = \frac{F_{AC}}{\sin \theta}$$

$\therefore$ $F_{AC} = \mathbf{90.83\ N}$ ... **Ans.**

Example 1.8 : *The log is being towed by two tractors A and B. If the resultant F_R of the two forces acting on the log is to be directed along positive x-axis and have a magnitude of 10 kN, determine the angle θ of the cable attached to B such that the force F_B in this cable is a minimum. What is the magnitude of force in each cable for this situation?*

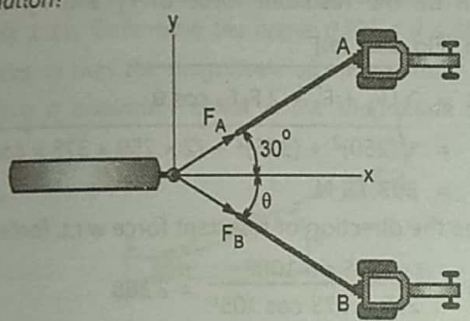


Fig. 1.23

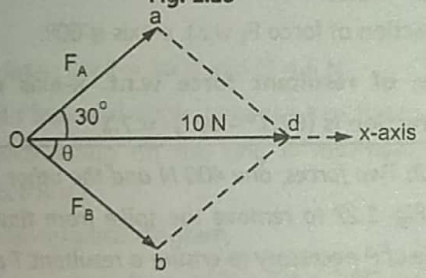


Fig. 1.23 (a)

Solution:

Given data: F_R = 10 N is the resultant of component forces F_A and F_B.

Direction of component force F_A w.r.t. resultant F_R = 30°.

To find: Direction of force F_B w.r.t. resultant F_R = θ = ? when F_B in this cable is minimum.

Redrawing half portion of the parallelogram as triangle oad.

(a) Side of parallelogram opposite to F_B is equal to F_B in magnitude and direction. (Refer Fig. 1.23 (a).

Hence, Refer Fig. 1.23 (b)

Force F_B is minimum, if forces F_A and F_B are perpendicular to each other.

i.e. from Fig. 1.23 (b).

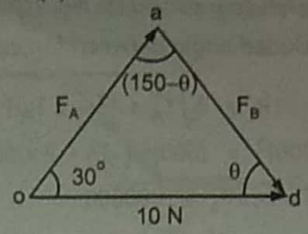


Fig. 1.23 (b)

$150 - \theta = 90^\circ$

$\therefore$ $\theta = \mathbf{60^\circ}$ ... **Ans.**

(b) By sine rule to triangle oad,

$$\frac{10}{\sin 90^\circ} = \frac{F_A}{\sin 60^\circ}$$

$\therefore$ $F_A = \mathbf{8.66\ N}$ ... **Ans.**

$$\frac{10}{\sin 90^\circ} = \frac{F_B}{\sin 30^\circ}$$

$\therefore$ $F_B = \mathbf{5\ N}$ ... **Ans.**

Example 1.9 : *The post is to be pulled out of the ground using two ropes A and B. Rope A is subjected to a force of 600 N and is directed at 60° from the horizontal. If the resultant force acting on the post is 1200 N vertically upward, determine force T in rope B and corresponding angle θ.*

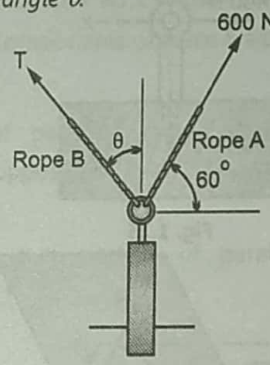


Fig. 1.24

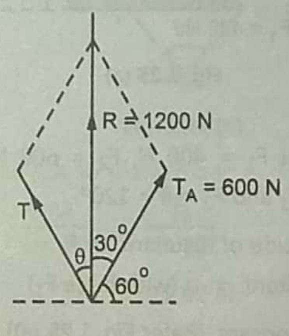


Fig. 1.24 (a)

Solution :

Given data : Tension in the rope A = T_A = 600 N

Resultant is vertically upward.

Angle made by force T_A with resultant R = α = 30°.

Magnitude of resultant of force T_A and T = 1200 N.

To find : Tension in rope = T = ?

Direction of T = θ w.r.t. resultant R.

(a) By law of parallelogram. [Refer Fig. 1.24 (a)]

Let θ_1 be the included angle between forces T_A and T.

$$R = \sqrt{T_A^2 + T^2 + 2\,T_A \cdot T\cos\theta_1}$$

$$\therefore \quad (1200)^2 = (600)^2 + T^2 + 2\times 600\times T\cos\theta_1$$

$$\therefore \quad T^2 + 1200\,T\times\cos\theta_1 = 1080000 \qquad \text{... (1)}$$

$$\tan\alpha = \frac{T\sin\theta_1}{T_A + T\cos\theta_1}$$

$$\therefore \quad \tan 30^\circ = \frac{T\sin\theta_1}{600 + T_B\cos\theta_1}$$

$$\therefore \quad 0.577\,(600 + T_B\cos\theta_1) = T\sin\theta_1$$

$$\therefore \quad 346.2 + 0.577\,T\cos\theta_1 = T\sin\theta_1 \qquad \text{... (2)}$$

By solving equations (1) and (2) simultaneously,

$$T = 743.6\text{ N and } \theta_1 = 54.79^\circ$$

$$\theta = \theta_1 - 30 = 53.79 - 30 = 23.79^\circ$$

$$\therefore \quad T = \mathbf{743.7\ N} \text{ and } \theta = \mathbf{23.79^\circ} \text{... Ans.}$$

Example 1.10 : *Determine resultant R of two forces 400 N and 600 N.*

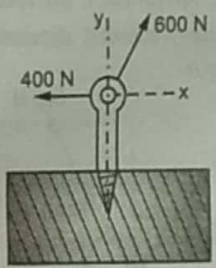


Fig. 1.25

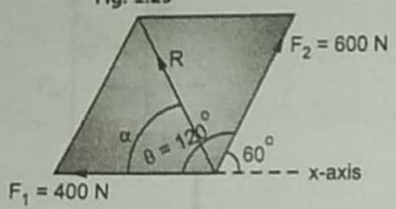


Fig. 1.25 (a)

Solution :

Given data : Let F_1 = 400 N, F_2 = 600 N, Included angle between two forces F_1 and $F_2 = \theta = 120^\circ$.

To find : Magnitude of resultant = R

Direction of resultant = α (with force F_1)

By law of parallelogram, [Refer Fig. 1.25 (a)]

(a) $R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$

$\therefore \quad R = \sqrt{(400)^2 + (600)^2 + (2\times 400\times 600\times\cos 120^\circ)}$

$\therefore \quad R = \mathbf{529.15\ N}$ **... Ans.**

(b) $\tan\alpha = \dfrac{F_2\sin\theta}{F_1 + F_2\cos\theta} = \dfrac{600\sin 120^\circ}{400 + 600\cos 120^\circ}$

$= 5.196$

$\therefore \quad \alpha = \tan^{-1}(5.196) = \mathbf{79.10^\circ}$ **... Ans.**

Example 1.11: *Determine magnitude of the resultant force of F_1 and F_2. Determine its direction from positive x-axis.*

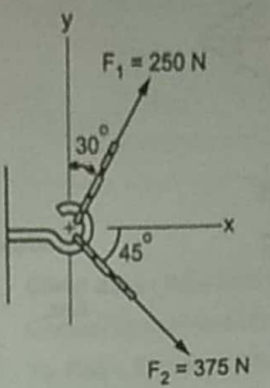


Fig. 1.26

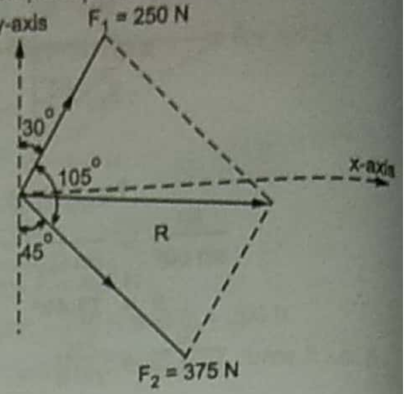


Fig. 1.26 (a)

Solution :

Given data : F_1 = 250 N, F_2 = 375 N, Included angle θ between F_1 and F_2 = 105°.

To find : Magnitude of resultant force R.

Direction of resultant force w.r.t. x-axis.

(a) Let R be the resultant force of F_1 and F_2. By law of parallelogram, [Fig. 1.26 (a)]

$$R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\theta}$$

$$= \sqrt{(250)^2 + (375)^2 + (2\times 250\times 375\times\cos 105^\circ)}$$

$$= \mathbf{393.18\ N} \qquad \textbf{... Ans.}$$

(b) Let θ be the direction of resultant force w.r.t. force F_1.

$$\therefore \quad \tan\theta = \frac{375\sin 105^\circ}{250 + 375\cos 105^\circ} = 2.368$$

$$\therefore \quad \theta = 67.1^\circ$$

Since direction of force F_1 w.r.t. x-axis is 60°.

Direction of resultant force w.r.t. x-axis measured in clockwise direction is (67.1° - 60°) = 7.1° ...Ans.

Example 1.12: *Two forces, one 400 N and the other P, are applied as shown in Fig. 1.27 to remove the spike from timber. Compute the magnitude of P necessary to ensure a resultant T directed along the spike. Also find T.*

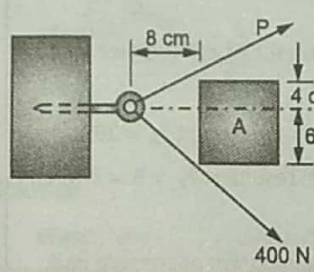


Fig. 1.27

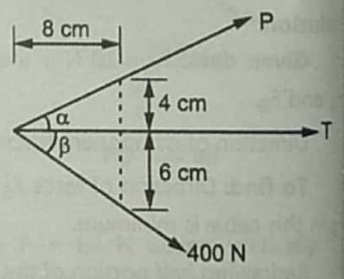


Fig. 1.27 (a)

Solution :

Given data : Resultant of force P and 400 N is force T acting along x-axis as shown.

From geometry,

Direction of force P w.r.t. resultant

$$= \tan^{-1}\left(\frac{4}{8}\right) = 26.56^\circ$$

Direction of force 400 N w.r.t. resultant

$$= \tan^{-1}\left(\frac{6}{8}\right) = 36.87^\circ$$

Included angle between component forces = $\alpha + \beta = 63.43^\circ$

To find : Magnitude of component force P.

Magnitude of resultant force T.

(a) By law of parallelogram, [Refer Fig. 1.27 (a)]

$$T = \sqrt{P^2 + (400)^2 + 2 \times P \times 400 \times \cos 63.43^\circ}$$

$$\therefore \quad T^2 = P^2 + 160000 + 357.83\ P \quad \text{... (1)}$$

$$\tan 36.87^\circ = \frac{P \sin 63.43^\circ}{400 + P \cos 63.43^\circ} = \frac{0.894\ P}{400 + 0.447\ P}$$

$$\therefore \quad P = \mathbf{536.67\ N} \quad \text{... Ans.}$$

Substituting P = 536.67 N in equation (1),

$$T = \mathbf{800\ N} \quad \text{... Ans.}$$

Example 1.13: *Determine the angle θ ($0^\circ \le \theta \le 90^\circ$) between the two forces so that the magnitude of the resultant force acting on the ring is a minimum. What is the magnitude of the resultant force?*

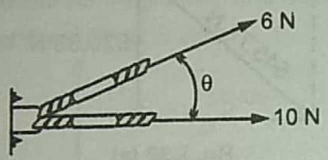


Fig. 1.28

Solution :

Given data : Let P = 10 N and Q = 6 N

To find : Included angle between two forces = θ = ? When resultant force acting on the ring is minimum, magnitude of resultant force R = ?

(a) By law of parallelogram,

$$R = \sqrt{P^2 + Q^2 + 2\ PQ \cos \theta}$$

$$\therefore \quad R^2 = P^2 + Q^2 + 2\ PQ \cos \theta \quad \text{... (1)}$$

R is minimum when last term of equation (1) equals to zero.

i.e. $\quad 2\ PQ \cos \theta = 0$

$\Rightarrow \quad \cos \theta = 0$

$\Rightarrow \quad \theta = 90^\circ\ (0^\circ \le \theta \le 90^\circ)$

$\therefore$ **R is minimum when θ = 90°** ... **Ans.**

(b) Substituting $\theta = 90^\circ$ in equation (i), we get,

$$R^2 = P^2 + Q^2$$

$$\Rightarrow \quad R = \sqrt{P^2 + Q^2}$$

$$\Rightarrow \quad R = \sqrt{(10)^2 + (6)^2}$$

$$\therefore \quad R = \mathbf{11.67\ N} \quad \text{... Ans.}$$

Example 1.14 : *Resolve 60 N force into components acting along u and v axes.*

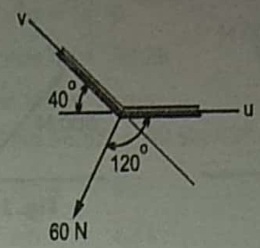


Fig. 1.29

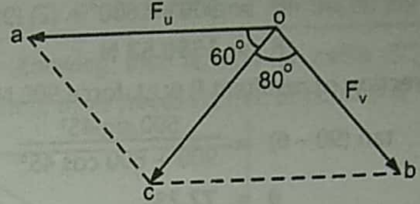


Fig. 1.29 (a)

Solution :

Given data : Force F = 60 N

Direction of force F w.r.t. negative u-axis = 60°

Direction of force F w.r.t. negative v-axis = 80°

To find : Components of force F along u and v directions : F_u and F_v.

By law of parallelogram (Refer Fig. 1.29 (a), side of parallelogram opposite to force F_v is equal to F_v in magnitude and direction.

Redrawing half portion of parallelogram, as shown in Fig. 1.29 (b).

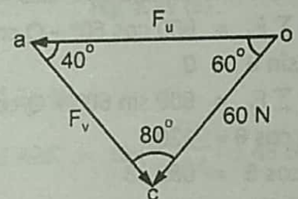


Fig. 1.29 (b)

Applying sine rule to triangle oac

$$\frac{60}{\sin 40^\circ} = \frac{F_u}{\sin 80^\circ}$$

$$\therefore \quad F_u = \mathbf{91.92\ N} \quad \text{... Ans.}$$

$$\frac{60}{\sin 40^\circ} = \frac{F_v}{\sin 60^\circ}$$

$$\therefore \quad F_v = \mathbf{80.83\ N} \quad \text{... Ans.}$$

Example 1.15: *Two forces are shown in Fig. 1.30. Knowing that magnitude of P is 600 N. Determine :*

(a) The required angle θ if the resultant R of the two forces is to be vertical.

(b) The corresponding value of R.

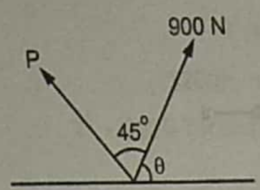


Fig. 1.30

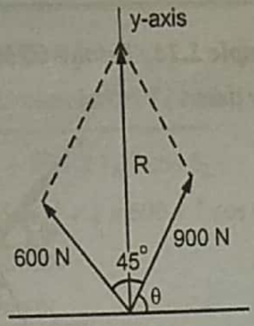


Fig. 1.30 (a)

Solution :

By law of parallelogram (Refer Fig. 1.30 (a).

(a) $R^2 = 900^2 + 600^2 + (2)(900)(600)\cos 45°$

$\therefore$ $R = \mathbf{1390.57\ N}$ ... **Ans.**

(b) Direction of resultant R w.r.t. force 900 N is $(90 - \theta)$.

$$\tan(90 - \theta) = \frac{600 \sin 45°}{900 + 600 \cos 45°}$$

$\therefore$ $\theta = \mathbf{72.23°}$ ... **Ans.**

Example 1.16: *The resultant of two forces P and Q is 1200 N vertical. Determine force Q and the corresponding angle θ for the system of forces as shown in Fig. 1.31*

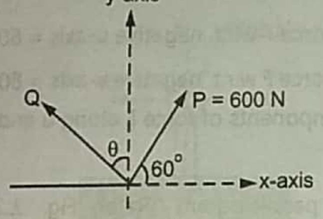


Fig. 1.31

Solution :

Resultant is vertical.

So, Component of resultant along X-axis = 0

and Component of resultant along Y-axis = 1200 N.

$\therefore$ $\Sigma F_x = 600 \cos 60° - Q \cos(90 - \theta) = 0$

$\therefore$ $300 - Q \sin\theta = 0$ ... (1)

$\Sigma F_y = 600 \sin 60° + Q \cos\theta = 1200$

$\therefore$ $519.61 + Q\cos\theta = 1200$

$\therefore$ $Q \cos\theta = 680.38$ ... (2)

Solving equation (1) and (2),

$\theta = \mathbf{23.75°}$ ... **Ans.**

$Q = \mathbf{743.33\ N}$ ... **Ans.**

Example 1.17 : *Determine the magnitude and direction of the resultant force.*

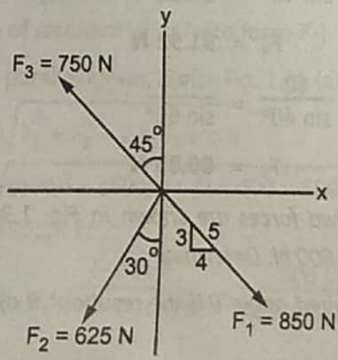


Fig. 1.32

Solution :

Given data : $F_1 = 850$ N, $F_2 = 625$ N and $F_3 = 750$ N are acting at a point.

Directions of forces are as shown in Fig. 1.32

$\theta_3 = 45°$, $\theta_2 = 30°$ and $\theta_1 = 53.13°$ w.r.t. y-axis in respective quadrants.

To find : Magnitude and direction of resultant.

(a) Resolving forces along x-axis,

$\Sigma F_x = 850 \sin 53.13° - 625 \sin 30° - 750 \sin 45°$

$= -162.83$ N

Resolving forces along y-axis,

$\Sigma F_y = -850 \cos 53.13° - 625 \cos 30° + 750 \cos 45°$

$= -520.93$ N

Magnitude of resultant $= \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$= \sqrt{(-162.83)^2 + (-520.93)^2}$

$= \mathbf{545.78\ N}$ ... **Ans.**

Direction of resultant, [Refer Fig. 1.32 (a)]

$$\tan\alpha = \frac{\Sigma F_y}{\Sigma F_x} = \frac{520.93}{162.83}$$

$\therefore$ $\alpha = 72.64°$ w.r.t. **negative x-axis... Ans.**

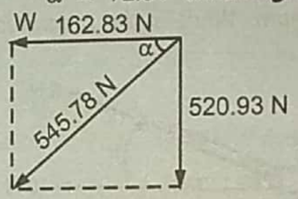


Fig. 1.32 (a)

Example 1.18 : *Determine the range of values for the magnitude of P so that the magnitude of the resultant force does not exceed 2500 N. Force P is always directed to the right.*

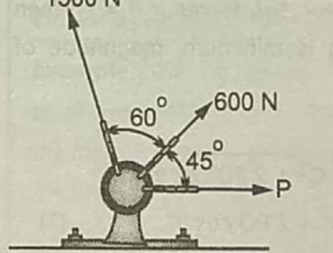


Fig. 1.33

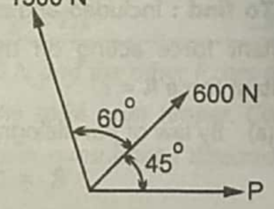


Fig. 1.33 (a)

Solution :

Given data : Magnitudes of two forces are 1500 N and 600 N.

Direction of three forces are as shown in Fig. 1.33

Resultant $R \leq 2500$ N.

To find : Force P.

(a) By law of parallelogram, [Refer Fig. 1.33 (a)]

Magnitude of resultant of forces 600 N and 1500 N

$= \sqrt{(600)^2 + (1500)^2 + (2 \times 600 \times 1500 \cos 60°)}$

$= 1873.5$ N

Direction of resultant w.r.t. force 600 N

$$= \tan^{-1}\left(\frac{1500 \sin 60^\circ}{600 + 1500 \cos 60^\circ}\right)$$

$$= 43.9^\circ$$

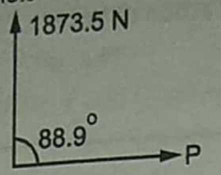


Fig. 1.33 (b)

$\therefore$ Resultant of forces 1500 N and 600 N is 1873.5 N.

Direction of this resultant is 88.9° w.r.t. force P.

(b) Resultant of 1873.5 N and force P [Refer Fig. 1.33 (b)] does not exceed 2500 N.

$$P = 0 \text{ OR}$$

By law of parallelogram,

$$2500 = \sqrt{(1873.5)^2 + P^2 + (2 \times P \times 1873.5) \cos 88.9^\circ}$$

$$\therefore (2500)^2 = (1873.5)^2 + P^2 + 72\,P$$

$$\therefore P^2 + 72\,P - 2739997.75 = 0$$

$$\therefore P = 1619.72 \text{ N}$$

Range of P is between 0 N and 1619.72 N ...**Ans.**

Example 1.19: *Determine magnitude and direction of force F so that the resultant of three forces is zero.*

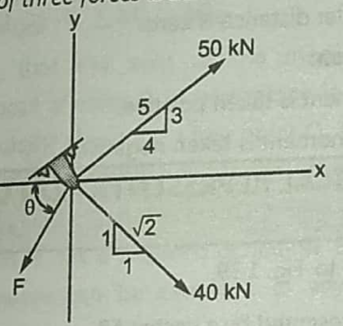


Fig. 1.34

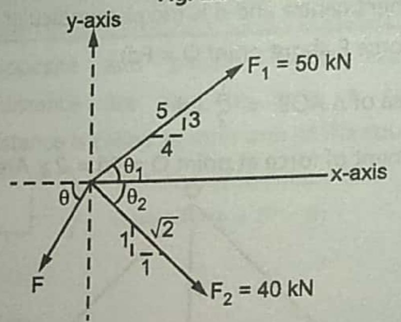


Fig. 1.34 (a)

Solution :

Given data : Magnitude of forces $F_1 = 50$ kN, $F_2 = 40$ kN

Direction of forces : $\theta_1 = 36.87^\circ$, $\theta_2 = 45^\circ$ w.r.t. positive x-axis.

Resultant R = 0

To find : Magnitude and direction of force.

(a) Since resultant is zero, $\Sigma F_x = 0$ and $\Sigma F_y = 0$

Resolving forces along x-axis,

$$\Sigma F_x = 50 \cos 36.87^\circ + 40 \cos 45^\circ - F \cos \theta = 0$$

$$\Rightarrow \quad 68.28 - F \cos \theta = 0$$

$$\Rightarrow \quad F \cos \theta = 68.28 \quad \text{... (1)}$$

Resolving forces along y-axis,

$$\Sigma F_y = 50 \sin 36.87^\circ - 40 \sin 45^\circ - F \sin \theta = 0$$

$$\Rightarrow \quad 1.71 - F \sin \theta = 0$$

$$\Rightarrow \quad F \sin \theta = 1.71 \quad \text{... (2)}$$

From equations (1) and (2),

θ = **1.43°** and F = **68.30 kN** ... **Ans.**

Example 1.20 : *Knowing that tension in cable BC is 145 N, determine resultant of three forces exerted at point B of beam AB.*

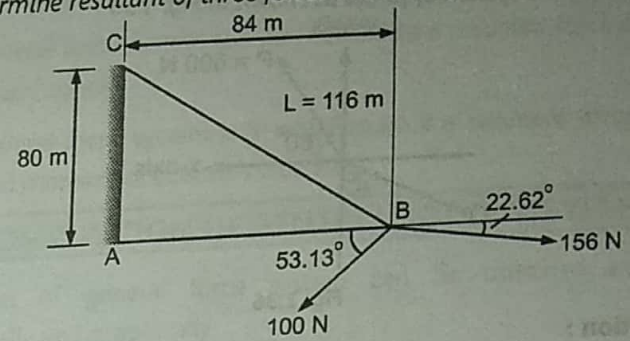


Fig. 1.35

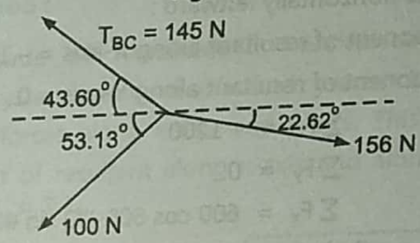


Fig. 1.35 (a)

Solution :

(a) From Fig. 1.35 (a),

$$\angle ABC = \tan^{-1}\left(\frac{80}{84}\right) = 43.60^\circ$$

(b) Resultant of forces at B,

$$\Sigma F_x = -145 \cos 43.60^\circ - 100 \cos 53.13^\circ + 156 \cos 22.62^\circ$$

$$= -21 \text{ N}$$

$$= 21 \text{ N} (\leftarrow)$$

$$\Sigma F_y = 145 \sin 43.60^\circ - 100 \sin 53.13^\circ - 156 \sin 22.62^\circ$$

$$\Sigma F_y = -40 \text{ (N)} = 40 \text{ N} (\downarrow)$$

Magnitude of resultant :

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(21)^2 + (40)^2}$$

$$= \mathbf{45.18\ N} \quad \text{...Ans.}$$

Direction of resultant

$$\alpha = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

$$= \tan^{-1}\left(\frac{40}{21}\right) = \mathbf{62.30^\circ} \quad \text{... Ans.}$$

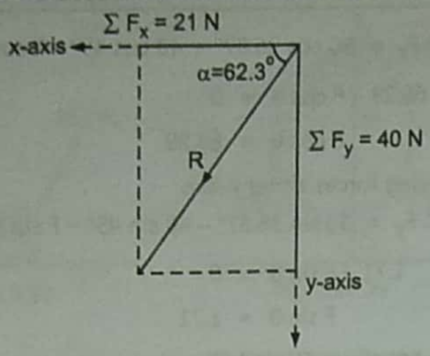


Fig. 1.35 (b)

***Example 1.21 :** The resultant of two forces P and Q is 1200 N horizontally leftward. Determine the force Q and the corresponding angle θ for the system of forces as shown in Fig. 1.36*

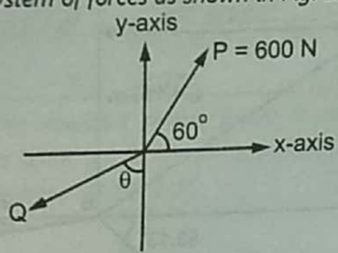


Fig. 1.36

Solution :

Resultant is horizontally leftward :

Component of resultant along X-axis = – 1200 N

So, Component of resultant along Y-axis = 0.

$$\Sigma F_x = -1200$$

$$\Sigma F_y = 0$$

$$\therefore \quad \Sigma F_x = 600 \cos 60° - Q \sin \theta = -1200 \quad \dots (1)$$

$$Q \sin \theta = 1500$$

$$\Sigma F_y = 600 \sin 60° - Q \cos \theta = 0 \quad \dots (2)$$

$$Q \cos \theta = 519.61$$

Solving equations (1) and (2),

$Q = $ **1587.45 N** ... **Ans.**

$\theta = $ **70.89°** ... **Ans.**

B - PARALLEL AND GENERAL FORCE SYSTEM

1.4 MOMENT OF A FORCE

The moment of a force is the cause of rotation. The magnitude of rotation or turning depends on the extent of moment.

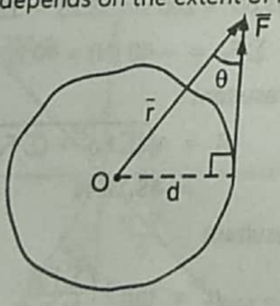


Fig. 1.37 : Moment of a force about a point O

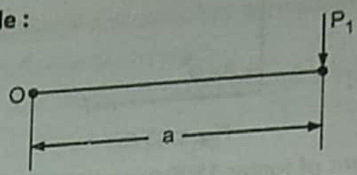
$\boxed{\bar{M}_O = \bar{r} \times \bar{F}}$ $\bar{r}$ is the position vector.

$$\therefore \quad \boxed{M_O = F r \sin \theta}$$

$$\boxed{M_o = Fd}$$

M_O = Force × Perpendicular distance between moment centre and line of action of force.

For example :

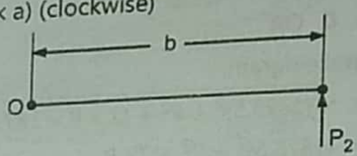


(a) Clockwise moment

Fig. 1.38

Here the moment of force P_1 about point O is given by

$M_O = (P_1 \times a)$ (clockwise)



(b) Anticlockwise moment

Fig. 1.38

Here the moment of force P_2 about point O is given by

$M_O = (P_2 \times b)$ (anticlockwise)

The moment is zero when either the force is zero or when the perpendicular distance is zero.

Sign Convention:

Clockwise moment is taken positive.

Anticlockwise moment is taken negative.

1.5 GRAPHICAL REPRESENTATION OF MOMENT OF FORCE

With reference to Fig. 1.39.

Force $\bar{F}$ is represented by a vector AB.

'O' is the moment centre and d is the perpendicular distance.

(Moment of force F about point O = Fd)

$$\text{Area of } \Delta \text{ AOB} = \frac{1}{2} \text{ Fd}$$

$\therefore$ Moment of force at point O = Fd = 2 × Area of Δ AOB.

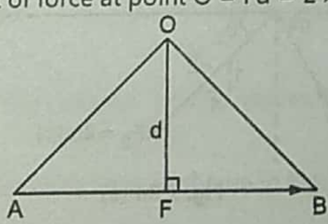


Fig. 1.39

1.6 VARIGNON'S THEOREM OF MOMENTS

Varignon's Theorem states that algebraic sum of moments of all forces about any point is equal to moment of their resultant about the same point. Moment of forces about point 'O' is

$$\boxed{-(P_1 \times d_1) + (P_2 \times d_2) - (P_3 \times d_3) = -(R \times d)}$$

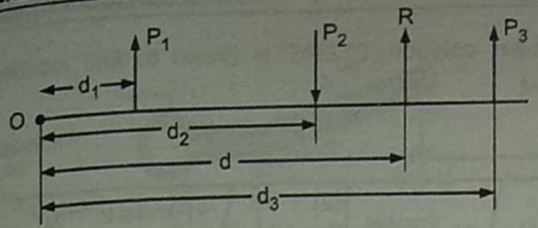


Fig. 1.40

Proof : Two concurrent forces P and Q acting at A are shown in Fig. 1.41. The resultant of P and Q is R, given by the diagonal AD of the parallelogram ABDC. Extend DC to O, which is considered as the moment centre for all forces.

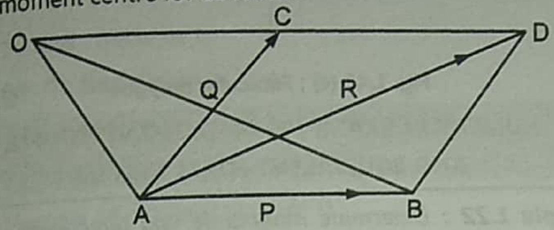


Fig. 1.41

Moment of P about O= $M_{PO} = 2\,\Delta\,ABO$

Moment of Q about O= $M_{QO} = 2\,\Delta\,ACO$

Adding, $M_{PO} + M_{QO} = \Sigma M_O = 2\,[\Delta\,ABO + \Delta\,ACO]$

But, $\Delta\,ABO = \Delta\,ABD = \Delta\,ACD$

$\therefore$ $\Sigma M_O = 2\,[\Delta\,ACD + \Delta\,ACO] = 2\,\Delta\,ADO$

$= M_{RO} =$ moment of R about O.

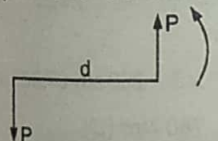
Thus, $(M_{PO} + M_{QO}) = \Sigma M_O = M_{RO}$

Now we conclude, that the sum of the moments of two concurrent forces about a point O in their plane is equal to the moment of their resultant about the same point O. This is known as Varignon's theorem of moments.

Only for simplicity we had taken the moment centre on the line DC extended, but this is not a required condition. By successive application, the theorem can be extended to any number of concurrent forces.

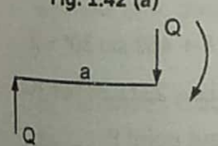
1.7 COUPLE

Two equal, opposite and parallel forces separated by perpendicular distance are said to form a couple. The perpendicular distance is called as lever arm of the couple



Anticlockwise couple of magnitude = $(P \times d)$

Fig. 1.42 (a)



Clockwise couple of magnitude $= (Q \times a)$

Fig. 1.42 (b)

Characteristics of a Couple :

- The sum of two forces along any direction is zero.
- The couple does not translate a body but tends to rotate it.
- Moment of couple about any point is equal to the product of the force and perpendicular distance between the two forces.

  $M = (P \times d)$ where, 'd' is the distance between two forces and is known as lever arm.
- Moment of couple is independent of the distance of the point "O" about which moment is taken i.e. couple is always constant.

1.8 COPLANAR GENERAL FORCE SYSTEMS

Coplanar general force system consists of several non-concurrent and non-parallel forces acting at different points in a plane.

The general force system can be reduced to a resultant force and a resultant couple.

The general force system is in equilibrium if a resultant force is zero and moment of couple is also zero.

1.9 COMPOSITION OF GENERAL FORCE SYSTEM

Resultant of general force system can be obtained both analytically and graphically.

Analytical Method :

(i) Magnitude of Resultant : Resolve each force in general force system along x-axis and y-axis. Sum up the components of forces along x-axis and y-axis. This summation gives component of resultant along x-axis and along y-axis i.e. $R_x = \Sigma F_x$ and $R_y = \Sigma F_y$.

Magnitude of Resultant = $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

(ii) Direction of Resultant : Let 'α' be the inclination of the resultant with x-axis.

$$\tan\alpha = \left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

Direction of Resultant = α

$$= \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right).$$

(iii) Location of Resultant w.r.t. any Point : Let 'd' be the perpendicular distance of resultant from point 'O'.

By Varignon's theorem,

Moment of resultant about point 'O' = Σ Moment of component forces about point 'O'.

i.e. $$R \times d = F_1 d_1 + F_2 d_2 + F_3 d_3 + \ldots\ldots$$

where F_1, F_2, F_3 ... are component forces and d_1, d_2, d_3, ... are perpendicular distances of component forces from point 'O'.

Graphical Method :

Magnitude, direction and location of the resultant of general force system can be found graphically as follows :

(i) Space Diagram : Spaces on both sides of force are named by alphabets A, B, C, D,. .... This is called space diagram. **Position of resultant is shown on space diagram.** Refer Fig. 1.43 (a).

(ii) Vector Polygon : Forces are drawn to the scale to form a vector polygon. Line joining starting of the first force and end of last force represents **magnitude and direction of resultant**. Refer Fig. 1.43 (b).

(iii) Polar Diagram : Any point 'O' is selected arbitrarily near vector polygon as a pole. Vertices of vector polygon are joined to a pole. The connecting lines are rays. This diagram is called 'polar diagram'. Refer Fig. 1.43 (b).

(iv) Funicular Polygon : Starting from any arbitrary point in space, a line parallel to ray 'Oa' is drawn in space A to meet first force. From this point, second line is drawn in space B parallel to ray 'Ob' and so on. These lines are called 'links'. First and last lines are open links. These two open links are produced to intersect each other. This diagram is called a Funicular Polygon. Resultant passes through the point of intersection. **This funicular polygon gives position of resultant in the Space Diagram.** Refer Fig. 1.43 (c).

Find magnitude, direction of resultant as well as location of resultant w.r.t. point M. Four forces P, Q, S and T are acting at different points as shown. Spaces are designated by Bow's notation as shown.

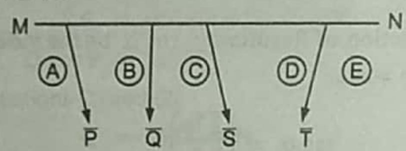


Fig. 1.43 (a)

A vector polygon abcde is drawn to find magnitude and direction of the resultant $\bar{R}$ (i.e. closing side 'ae' of polygon).

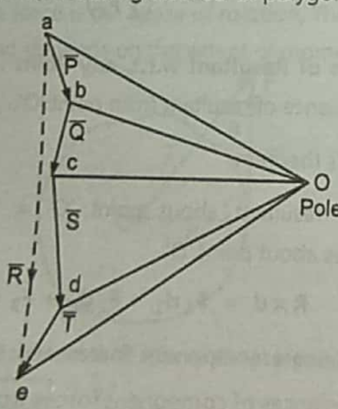


Fig. 1.43 (b) : Vector polygon and polar diagram

A funicular polygon '012345' is drawn to find location of the resultant.

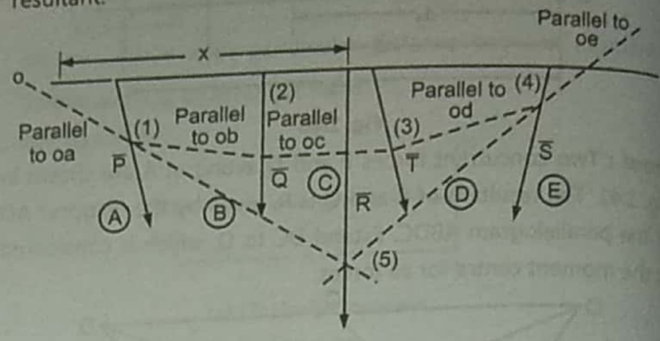


Fig. 1.43 (c) : Funicular polygon

NUMERICAL EXAMPLES ON MOMENT, COUPLE AND EQUIVALENT FORCE SYSTEM

Example 1.22 : *Determine magnitude and directional sense of resultant moment of the forces about point P.*

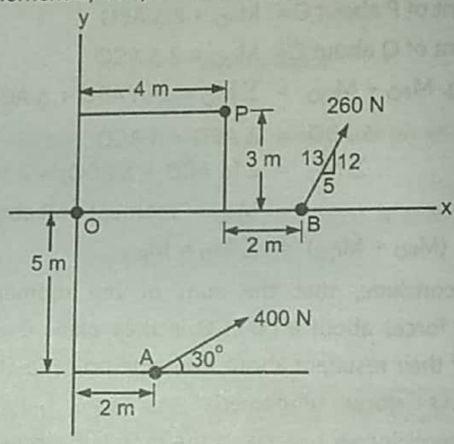


Fig. 1.44

Solution :

Given data : $F_1 = 260$ N, $\theta_1 = \tan^{-1}\left(\frac{12}{5}\right) = 67.38°$

$F_2 = 400$ N, $\theta_2 = 30°$

To find : Magnitude and direction of resultant moment at point P.

(a) Moment of 260 N about point P,

$= -260 \cos 67.38° \times 3 - 260 \sin 67.38° \times 2$

$= -780$ Nm i.e. 780 Nm (↺)

(b) Moment of 400 N about point P

$= -400 \cos 30° \times 8 + 400 \sin 30° \times 2$

$= -2371.28$ Nm i.e. 2371.28 Nm (↺)

∴ Resultant moment of forces about point P

$=$ **3151.28 Nm (↺)** ... **Ans.**

Example 1.23 : *Replace the couple and force shown by a single force F applied at a point D. Locate D by determining the distance b.*

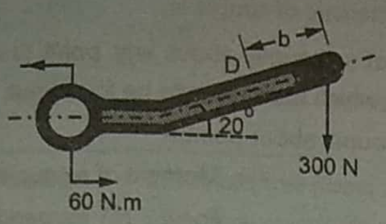


Fig. 1.45

Solution :

Given data : Force = 300 N at angle 20° with horizontal.

Moment of couple = 60 Nm.

To find : Distance b.

(a) To find distance 'b' :

Resultant force = 300 N acts at D

By Varignon's theorem,

Moment of resultant at point D

$= \sum$ Moment of forces at point D.

$\therefore \quad 300 \times 0 = 300 \times b \cos 20° - 60$

$281.9\, b - 60 = 0$

$\therefore \quad b = \mathbf{0.213\ m}$ ... **Ans.**

Example 1.24 : *A force F = 60 N is applied to the gear. Determine moment of F about point O, r = 100 mm.*

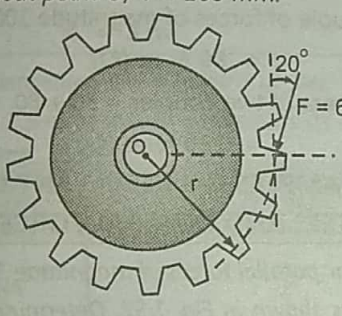


Fig. 1.46

Solution :

Force F = 60 N is resolved along x-axis and y-axis.

Component of force 60 N along x-axis

$F_x = 60 \sin 20°$

Component of force 60 N along y-axis

$F_y = 60 \cos 20°$

$= 56.38$ N

Moment of F_x about centre O = 0 N-m

Moment of F_y about centre O = 56.38×0.1

= 5.638 N-m (↻)

By Varignon's theorem,

Moment of force F about point O = 0 + 5.638

= **5.638 N-m** (↻) ...**Ans.**

Example 1.25 : *Calculate the moment of 250 N force on the handle about centre of the bolt.*

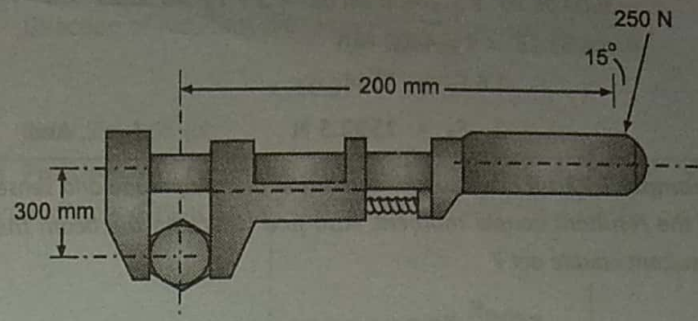


Fig. 1.47

Solution :

Force 250 N is resolved along x-axis and y-axis.

Component of force 250 N along x-axis.

$F_x = 250 \sin 15° = 64.7$ N

Component of force 250 N along y-axis

$F_y = 250 \cos 15° = 241.48$ N

Moment of F_x about centre of bolt = $F_x \times 0.03$

= 1.94 N-m (↺)

Moment of F_y about centre of bolt = $F_y \times 0.2 = 48.3$ N-m (↻)

By Varignon's theorem,

Total moment of force 250 N about centre of bolt

= – 1.94 + 48.3=– 46.36 N-m = **46.36 N-m (↺)...Ans.**

Example 1.26 : *If the resultant moment about point A is 4800 Nm clockwise, determine the magnitude of F_3 if F_1 = 300 N and F_2 = 400 N.*

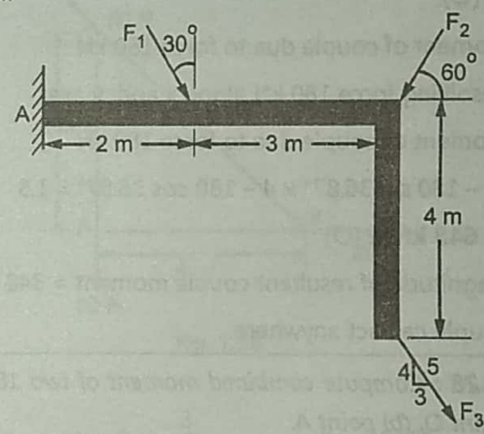


Fig. 1.48

Solution :

Given data : Moment of forces about point

A = 4800 Nm (↻)

F_1 = 300 N, F_2 = 400 N.

Direction of force $F_3 = \tan^{-1}\left(\dfrac{4}{3}\right) = 53.13°$ with horizontal.

Dimensions are as shown in the Fig. 1.48.

To find : Magnitude of force F_3.

(a) Resultant moment about point A = 4800 Nm.

i.e. $\sum$ Moment of forces about point A = 4800 Nm.

$\therefore$ $300 \cos 30° \times 2 + 400 \sin 60° \times 5 + F_3 \sin 53.13° \times 5 - F_3 \cos 53.13° \times 4 = 4800$ Nm

$\therefore$ $1.6 F_3 = 2548$

$\therefore$ $F_3 = $ **1592.5 N** ... **Ans.**

Example 1.27 : *If F = 180 kN, determine the magnitude and sense of the resultant couple moment. Also find where on the beam the resultant couple act ?*

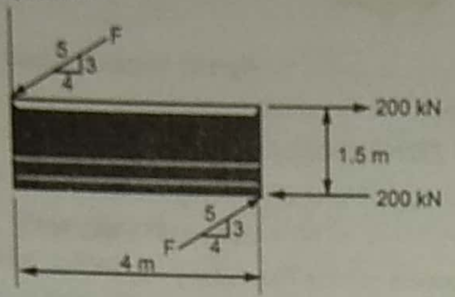


Fig. 1.49

Solution :

Given data : Forces of magnitude 200 kN and 180 kN are acting as shown in Fig. 1.49.

To find : Magnitude and sense of the resultant couple moment.

(a) To find magnitude and sense of resultant couple moment.

(i) Moment of couple due to force 200 kN = 200 × 1.5 = 300 kNm (↻).

(ii) Moment of couple due to force 180 kN :

Resolving force 180 kN along x and y axes

$\therefore$ Moment of couple due to force 180 kN

$= -180 \sin 36.87° \times 4 - 180 \cos 36.87° \times 1.5$

= **648 kNm** (↺) ...**Ans.**

(iii) Magnitude of resultant couple moment = 348 kNm (↺).

(iv) Couple can act anywhere.

Example 1.28 : *Compute combined moment of two 180 N forces about (a) point O, (b) point A.*

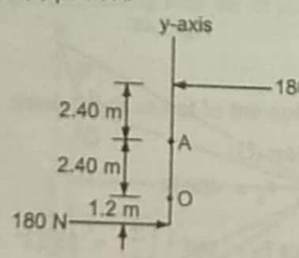


Fig. 1.50

Solution :

Since given two forces of magnitude 180 N each are parallel and opposite in direction, so they form a couple.

The characteristic of couple is :

Moment of a couple about any point is independent of distance about which moment is to be found out.

Moment of a couple about point A

= Moment of a couple about point O

= Force × Perpendicular distance between the two forces

= 180 × 6

= **1080 N-m (↺)** ...**Ans.**

Example 1.29 : *Find resultant moment of two couples for the loading as shown in Fig. 1.51.*

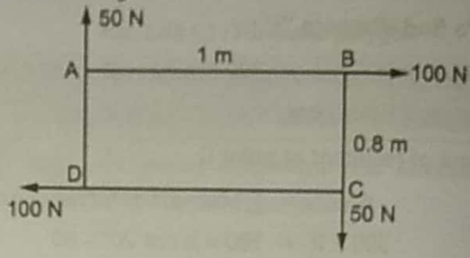


Fig. 1.51

Solution :

Moment of couple of forces of magnitude 50 N

= 50 × 1 = **50 N-m (↻)** ...**Ans.**

Moment of couple of forces of magnitude 100 N = 100 × 0.8

= **80 N-m (↻)** ...**Ans.**

$\therefore$ Resultant moment of two couples = 50 + 80

= **130 N-m (↻)** ... **Ans.**

NUMERICAL EXAMPLES ON RESULTANT OF PARALLEL FORCE SYSTEM AND GENERAL FORCE SYSTEM

Example 1.30 : *Four parallel forces of magnitude 100 N, 200 N, 50 N and 400 N are as shown in Fig. 1.52. Determine the magnitude of the resultant and also the distance of the resultant from point A.*

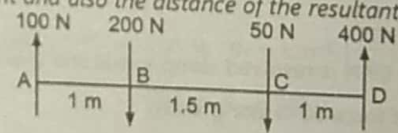


Fig. 1.52

Solution:

Magnitude of resultant of four parallel forces

= – 100 – 200 – 50 + 400 = **250 N (↑)** ... **Ans.**

Position of resultant from point A :

Assume resultant R = 250 N is at a perpendicular distance x from point A.

Fig. 1.53

By Varignon's theorem

Taking moments about point A,

$$-R \times x = 100 \times 0 + 200 \times 1 + 50 \times 2.5 - 400 \times 3.5$$

$\therefore -250 \times x = -1075$

$\therefore \quad x = \mathbf{4.3\ m}$

Distance of resultant is 4.3 m from point A to the right. **...Ans.**

Example 1.31 : *If resultant R = 600 N of three forces 100 N, F and 300 N is acting as shown in Fig. 1.54, find magnitude of force F and its distance 'x' from point A.*

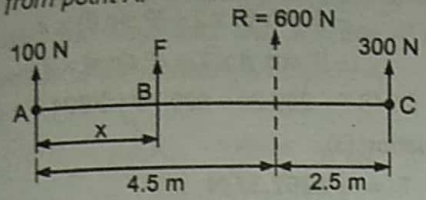


Fig. 1.54

Solution:

Magnitude of resultant of three parallel forces = 100 + F + 300 = 600 N

$\therefore$ Magnitude of force F = **200 N** **...Ans.**

Position of force F i.e. distance x.

By Varignon's theorem,

Taking moments about point A,

$$-600 \times 4.5 = 100 \times 0 - F \times x - 300 \times 7$$

$$\therefore \quad -2700 = -200x - 2100$$

$$\therefore \quad x = \mathbf{3\ m}$$ **...Ans.**

Example 1.32 : *Determine the resultant of four forces tangent to the circle of radius 1.5 m as shown. Determine its location w.r.t. 'O'.*

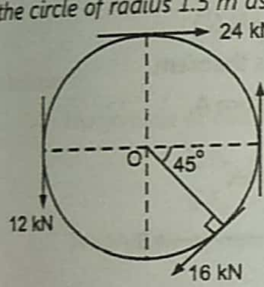


Fig. 1.55

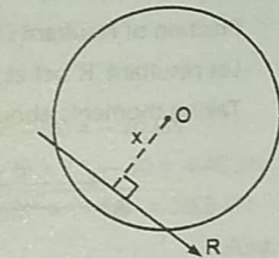


Fig. 1.55 (a)

Solution :

Given data : Forces F_1 = 24 kN, F_2 = 12 kN, F_3 = 8 kN, F_4 = 16 kN are acting as shown in Fig. 1.55.

To find : Magnitude and direction of resultant. Location of resultant w.r.t. point O.

(a) Magnitude and direction of resultant :

$$\Sigma F_x = 24 - 16 \cos 45° = 12.68 \text{ kN}$$

$$\Sigma F_y = -12 - 16 \sin 45° + 8 = -15.31 \text{ kN}$$

Magnitude of resultant $= \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(12.68)^2 + (-15.31)^2}$

$$R = \mathbf{19.88\ kN}$$ **... Ans.**

Direction of resultant $= \theta = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right) = \tan^{-1}\left(\frac{15.31}{12.68}\right) = 50.37°$

Refer Fig. 1.55 (b).

(b) Position of resultant R : Let resultant R act at a perpendicular distance of x from centre 'O. Refer Fig. 1.55 (a).

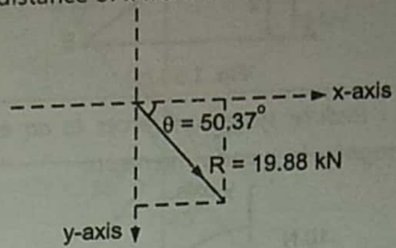


Fig. 1.55 (b) : Resultant

By Varignon's theorem,

Moment of resultant at point O = Σ Moment of forces at point O

$$\therefore \quad -R \times x = 16 \times 1.5 - 8 \times 1.5 + 24 \times 1.5 - 12 \times 1.5$$

$$\therefore \quad -19.88 \times x = 30$$

$$\therefore \quad x = \mathbf{-1.51\ m}$$ **...Ans.**

Since, value of x is –ve, assumed position of resultant is wrong, resultant acts on the right hand side of centre at perpendicular distance 1.51 m.

Example 1.33 : *Find the resultant and its point of application on y-axis for the force system acting on triangular plate as shown in Fig. 1.56.*

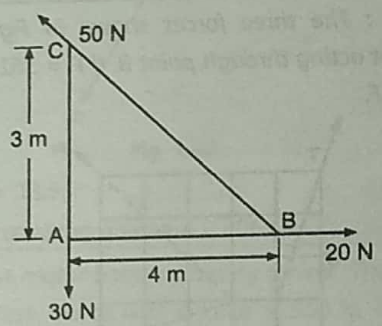


Fig. 1.56

Solution :

$\tan \theta = \frac{3}{4} \qquad \therefore \ \theta = 36.87°$

Magnitude of resultant of 20 N, 30 N and 50 N forces.

$$\therefore \quad \Sigma F_x = 20 - 50 \cos 36.87° = -20 \text{ N}$$

$$\Sigma F_y = -30 + 50 \sin 36.87° = 0 \text{ N}$$

$\therefore$ Magnitude and direction of resultant

$$= \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \mathbf{20\ N\ (\leftarrow)}$$ **... Ans.**

Position of Resultant :

By Varignon's theorem,

Let position of resultant be as shown in Fig. 1.56 (a). Taking moments at point B,

$$-R \times x = 50 \times 0 - 30 \times 4$$

$$\therefore \quad x = \frac{30 \times 4}{20} = \mathbf{6\ m} \qquad \text{... Ans.}$$

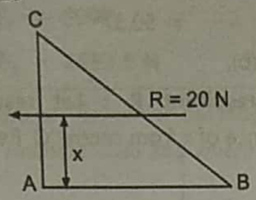


Fig. 1.56 (a)

Example 1.34 : *Reduce system of forces to an equivalent force. Determine its magnitude, x and y intercepts.*

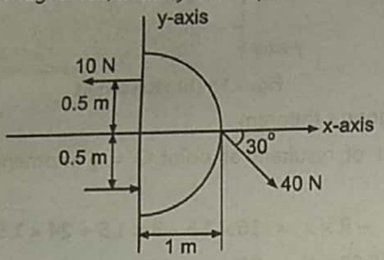


Fig. 1.57

Solution :

$$\Sigma F_x = -10 + 10 + 40 \cos 30° = 34.64 \text{ N } (\rightarrow)$$

$$\Sigma F_y = -40 \sin 30° = 20 \text{ N } (\downarrow)$$

Magnitude of resultant $= \sqrt{(34.64)^2 + (20)^2} = \mathbf{40\ N}$ **... Ans.**

Direction of resultant $= \alpha = \tan^{-1}\left(\frac{20}{34.64}\right) = \mathbf{30°}$ **... Ans.**

Example 1.35 : *The three forces shown in Fig. 1.58 create a vertical resultant acting through point B. If P = 361 N, compute the values of T and F.*

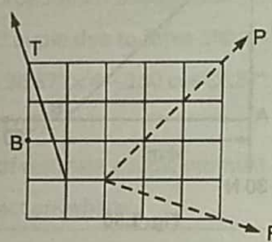


Fig. 1.58

Solution :

Inclination of forces :

Inclination of force T $= \alpha = \tan^{-1}\left(\frac{3}{1}\right) = 71.56°$

Inclination of force P $= \beta = \tan^{-1}\left(\frac{3}{2}\right) = 56.31°$

Inclination of force F $= \gamma = \tan^{-1}\left(\frac{1}{2}\right) = 25.56°$

Magnitude of resultant of forces T, P and F be i.e. $\Sigma F_x = 0$ *and let* $\Sigma F_y = R$.

$$\therefore \quad \Sigma F_x = -T \cos 71.56° + P \cos 56.31° + F \cos 26.56°$$

$$= -0.316\,T + 200.24 + 0.894\,F = 0 \qquad \text{... (1)}$$

$$\Sigma F_y = T \sin 71.56° + P \sin 56.31° - F \sin 26.56°$$

$$= 0.948\,T + 300.37 - 0.447\,F \qquad \text{... (2)}$$

Position of resultant : Resultant passes through point F.

By Varignon's theorem,

Taking moments of forces about point B,

$$R \times 0 = (T \cos \alpha) \times 1 - (T \sin \alpha) \times 1$$
$$- (P \cos \beta) \times 1 - (P \sin \beta) \times 2$$
$$- (F \cos \gamma) \times 1 + (F \sin \gamma) \times 2 \qquad \text{... (3)}$$

$$0.316\,T - 0.948\,T - 200.24 - 600.74 - 0.894\,F + 0.894\,F = 0$$

From equation (3),

$$T = \mathbf{-1267.37\ N} \qquad \text{... Ans.}$$

From equation (1),

$$F = \mathbf{-671.95\ N} \qquad \text{... Ans.}$$

Example 1.36 : *A beam AB of 6 m span is subjected to a concentrated load and a distributed load as shown. Replace system of forces by an equivalent force-couple system at supports A and B.*

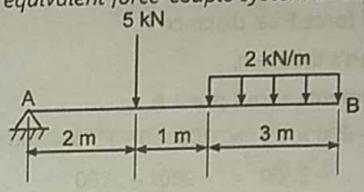


Fig. 1.59

Solution :

Magnitude of resultant :

$\Sigma F_y = -5 - (2 \times 3) = -11$ kN $= 11$ kN $(\downarrow)$

Position of resultant : By Varignon's theorem,

Let resultant 'R' act at distance 'd' from A.

Taking moments about point A,

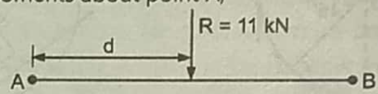


Fig. 1.59 (a)

$$R \times d = 5 \times 2 + (2 \times 3) \times (3 + 1.5)$$

$$\therefore \quad 11 \times d = 10 + 27$$

$$\therefore \quad d = 3.36 \text{ m}$$

(a) Equivalent force-couple system at 'A' :

(i) Adding two forces of 11 kN with opposite direction at point A, so that effect of these two forces is zero.

(ii) It forms a force = 11 kN (↓) acting at point A and a couple of moment = 11 × 3.36 = **36.96 kN-m (↻)** **...Ans.**

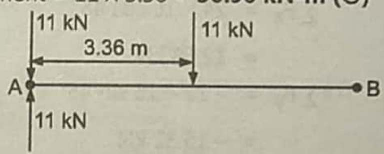


Fig. 1.59 (b)

(b) Equivalent force-couple system at 'B' :

(i) Adding two forces of 11 kN with opposite direction at point B, so that effect of these two forces is zero.

(ii) It forms a force = 11 kN (↓) at point B and a couple of moment = 11×2.64 = **29 kN-m (↺)** **... Ans.**

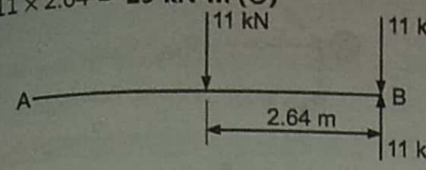


Fig. 1.59 (c)

Example 1.37 : *System of forces acting on a frame is as shown in Fig. 1.60. Calculate the magnitude and direction of the resultant. Also find the position of the resultant w.r.t. point A.*

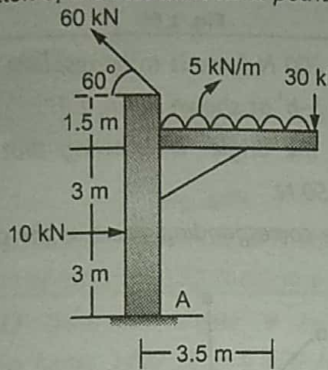


Fig. 1.60

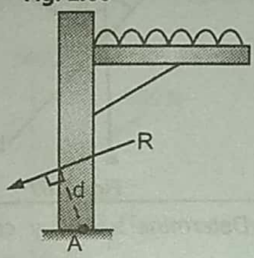


Fig. 1.60 (a)

Solution :

(a) Magnitude of resultant :

$$\Sigma F_x = -60 \cos 60° + 10 = -20 \text{ kN}$$

$$\Sigma F_y = 60 \sin 60° - (5 \times 3.5) - 30 = 4.46 \text{ kN}$$

$\therefore$ Magnitude of resultant $= \sqrt{(-20)^2 + (4.46)^2} = 20.5$ kN **... Ans.**

(b) Direction of resultant $= \alpha = \tan^{-1}\left(\frac{4.46}{20}\right) = 12.57°$ **...Ans.**

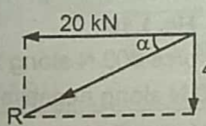


Fig. 1.60(b)

(c) Position of resultant w.r.t. point A : Refer Fig. 1.60 (a).

Let resultant acts at a perpendicular distance 'd' from point A.

By Varignon's theorem, (↻ + and ↺ –).

$$-R \times d = -60 \cos 60° \times 7.5 + 30 \times 3.5 + \left(5 \times 3.5 \times \frac{3.5}{2}\right) + 10 \times 3$$

$\therefore$ d = **2.89 m from A** **... Ans.**

PROBLEMS FOR PRACTICE

Problem No. 1 : *R = 18 kN is the resultant of four concurrent forces out of which only three are known. Find the fourth force in magnitude and direction.*

Answer : (1) P = 31.44 kN

(2) θ = 24.5° (fourth quadrant)

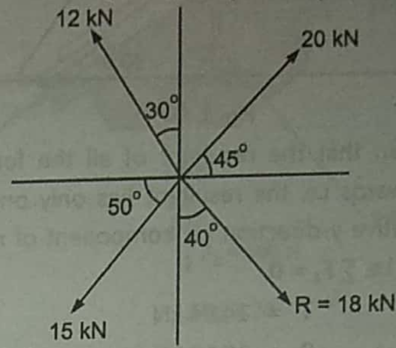


Fig. 1.61

Problem No. 2 : *A force P of magnitude 800 N is to be resolved into two components along the lines aa and bb. If the component of force P along the line bb is 300 N, determine the angle α and component of the force along line aa.*

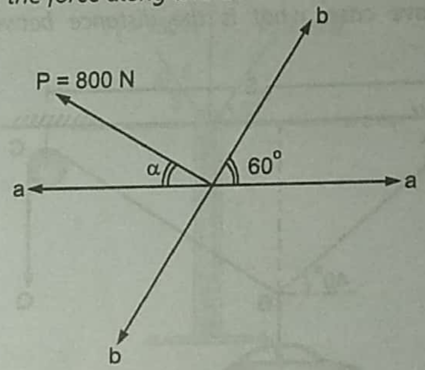


Fig. 1.62

Answer : (1) α = 18.95°

(2) Force along line aa = – 906.64

Problem No. 3 : *A motor boat is crossing a river. The flow of water pushes the boat East- wards with a force of 500 N. The wind tends to push the boat with a force of 200 N towards North-west, while the engine of the boat drives it towards the North with a force of 1000 N. Find the resultant force on the boat and the direction in which it will finally move.*

Answer : (1) R = 1196.11 N

(2) θ = 72.56° North of East.

Problem No. 4 : *A vertical pole 60 m high has a socket connection at the base and is pulled by two wire forces of 4 kN and 8 kN making an angle of 5° and 25° with the horizontal. Another wire which is attached to the top of the pole is anchored at the ground at a point 80 m from the base of the pole. If the resultant of all the wire forces at the top of the pole is known to act vertically downwards, towards the centre of the base, calculate the tension (force) in the wire and the magnitude of the resultant.*

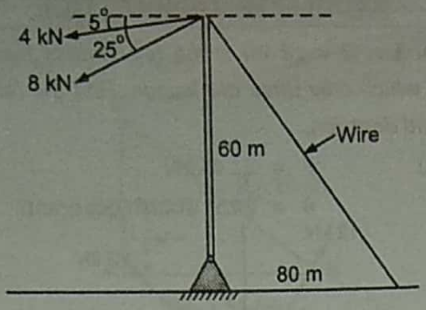


Fig. 1.63

Hint : It is given that the resultant of all the forces is acting vertically downwards i.e. the resultant has only one component which is in negative y-direction i.e. component of resultant in x-direction is zero i.e. $\sum F_x = 0$.

Answer : (1) T = 14.04 kN

(2) R = 12.15 kN (downwards)

Problem No. 5 : *A car is to be lifted in the vertical direction with a force of 20 kN. This is done by applying two forces P and Q as shown in Fig. 1.64.*

(a) Determine the value of force P if force Q is to be minimum.

(b) In the above case, what is the distance between the two pulleys ?

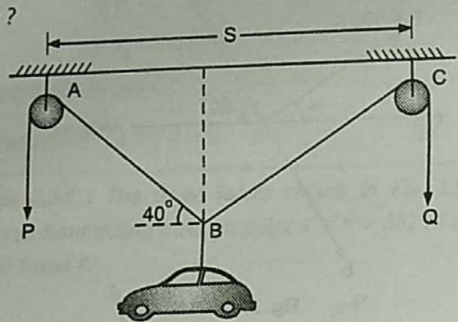


Fig. 1.64

Answer : (a) For the force Q to be minimum, the angle ABC should be 90°.

Q_{min} = 15.33 kN

P = 12.86 kN

(b) S = (5.959 + 4.195) = (10.154) m

Problem No. 6 : *Resolve the force of 500 kN into its components along the directions OA and OB as shown in Fig. 1.65.*

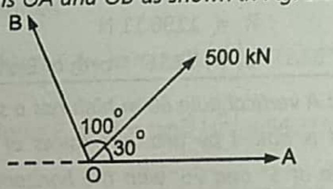


Fig. 1.65

Problem No. 7 : *Fig. 1.66 shows free body diagram of joints 'A' and 'B' of truss which is in equilibrium. Draw representative force polygon for both the joints.*

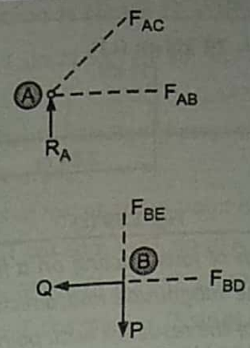


Fig. 1.66

Problem No. 8 : *A 200 N force is to be resolved into components along lines a-a' and b-b' as shown in Fig. 1.67.*

(i) Determine the angle 'α' knowing that the component along a-a' is to be 150 N.

(ii) What is the corresponding value of the component along b-b' ?

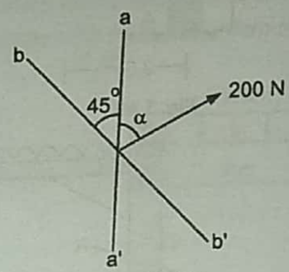


Fig. 1.67

Problem No. 9 : *Determine x and y components of the 800 N force.*

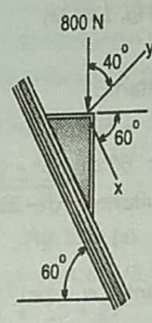


Fig. 1.68

Answer : (a) Component of force 800 N along x-axis = 514.23 N

(b) Component of force 800 N along negative y-axis = 612.83 N

Problem No. 10 : *Resolve a force of 20 N into components acting (a) along the n and t axes, (b) along the x and y axes.*

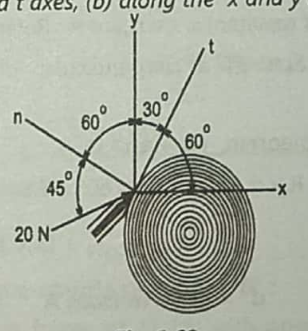


Fig. 1.69

Answer : (a) Component of force 20 N along negative n-axis = 14.14 N

Component of force 20 N along t-axis = 14.14 N

(b) Component of force 20 N along x-axis = 19.32 N

Component of force 20 N along y-axis = 51.76 N

Problem No. 11 : *Resolve resultant force of force L = 1500 N and force D = 200 N along x and y axes. $\alpha = 5^\circ$.*

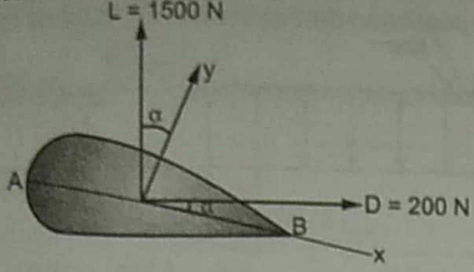


Fig. 1.70

Answer : Component of force 1513.27 N along x-axis = 68.65 N

Component of force 1513.27 N along y-axis = 1511.71 N

Problem No. 12 : *Determine angle θ so that magnitude of resultant of two forces 1400 N and 800 N is 2000 N. For this condition, determine angle α between resultant and vertical force of magnitude 1400 N.*

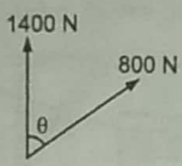


Fig. 1.71

Answer : $\theta = 51.32^\circ$, $\alpha = 18.2^\circ$

Problem No. 13 : *Express resultant force R exerted on pulley by two tensions in vector notation. Determine magnitude of R. Pulley is of negligible radius.*

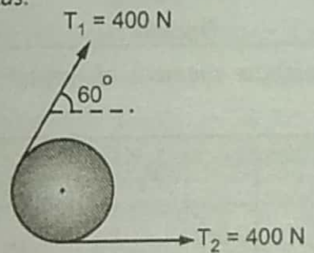


Fig. 1.72

Answer : R = 692.66 N.

Problem 14 : *Replace two forces 800 N and 900 N by two equivalent forces along x-axis and a-axis.*

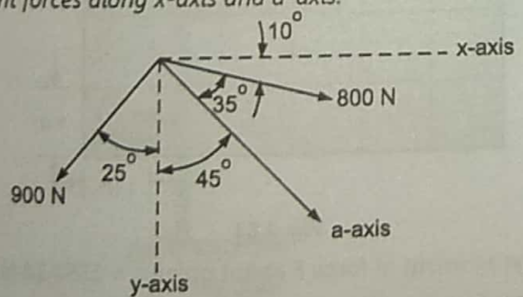


Fig. 1.73

Answer : $F_x = 547.53$ N, $F_a = 1350.2$ N

Problem No. 15 : *Show that the resultant force is zero for the given force system.*

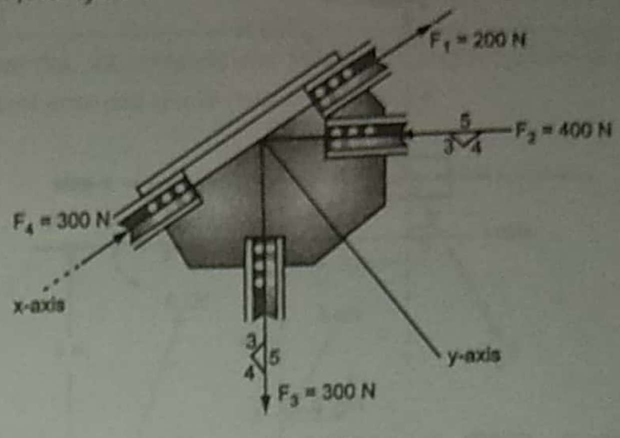


Fig. 1.74

Answer : Resultant force is zero for the given force system.

Problem No. 16 : *Determine resultant of given force system.*

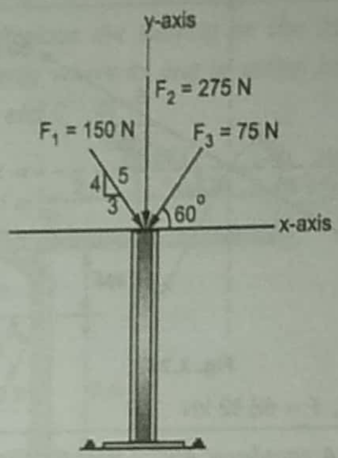


Fig. 1.75

Answer : Magnitude of resultant = 463 N, Direction of resultant = 83.49° w.r.t. positive x-axis in clockwise direction.

Problem No. 17 : *Determine the magnitude of force F so that magnitude of the resultant of the three forces is as small as possible. What is the minimum magnitude of the resultant ?*

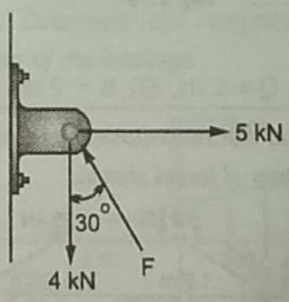


Fig. 1.76

Answer : R = 2.36 kN, F = 5.96 kN

Problem No. 18 : *If resultant of three concurrent forces is zero, determine θ and required magnitude of F_3.*

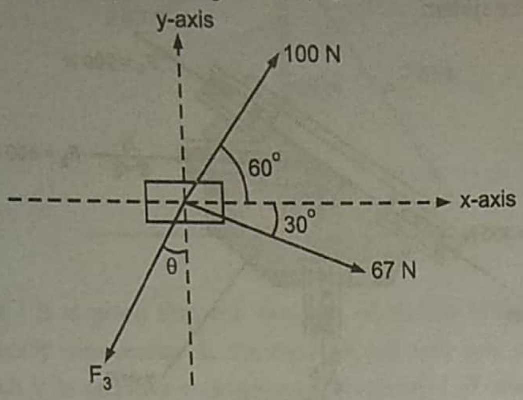


Fig. 1.77

Answer : θ = 63.82°, F_3 = 120.36 N

Problem 19 : *Determine magnitude F and direction θ of force F, so that resultant of three forces acting on the hook is zero.*

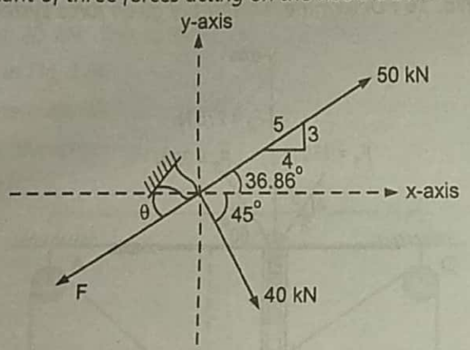


Fig. 1.78

Answer : θ = 1.43°, F = 68.32 kN

Problem No. 20 : *A cantilever ABC 1.8 m long is fixed at end A and carries loads P, Q, R as shown. Due to these loads, there is a pull of 4 kN at end A and an anticlockwise moment of 3.5 kN-m at end A. Determine the values of forces P, Q, R.*

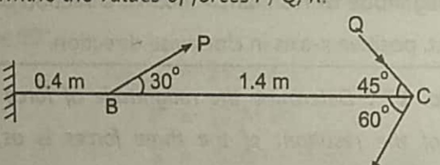


Fig. 1.79

Answer :

(1) P = 5 kN, (2) Q = 1 kN, (3) R = 2 kN

Problem No. 21 : *Find the magnitude, direction and position of the resultant of the system of forces shown.*

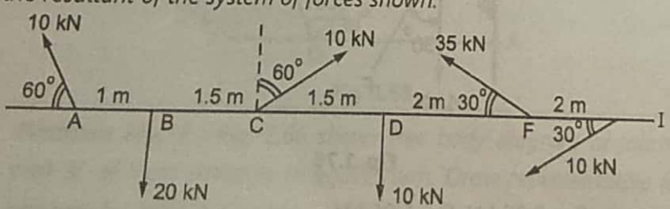


Fig. 1.80

Answer : (1) R = 35.5 kN

(2) θ = 6.2° (third quadrant)

(3) x = – 4.55 m (on LHS of point A)

Problem No. 22 : *(a) Compute the simplest resultant force for the loads shown acting on the cantilever beam.*

(b) What moment is transmitted by this force on the supporting wall at A ?

(c) Find the position on the beam where the resultant force acts.

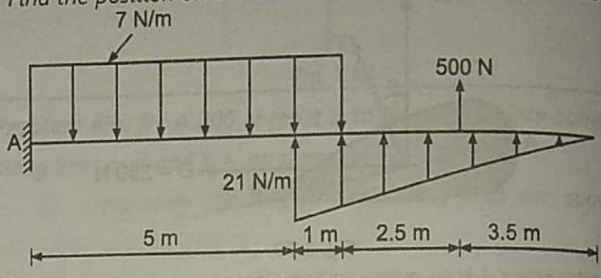


Fig. 1.81

Answer : (a) R = 531.5 N (↑)

(b) M_A = 4662.75 Nm

(c) x = 8.77 m (from A)

Problem No. 23 : *A Z-shaped lamina of uniform width of 20 mm is subjected to four forces as shown in Fig. 1.82. Find equilibrant in magnitude and direction.*

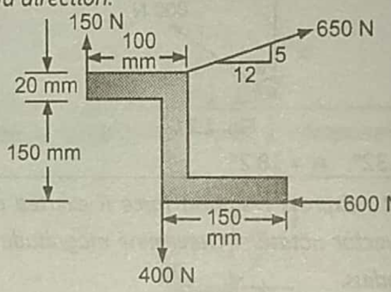


Fig. 1.82

Problem No. 24 : Compute moment of force F = 450 N about points A and B.

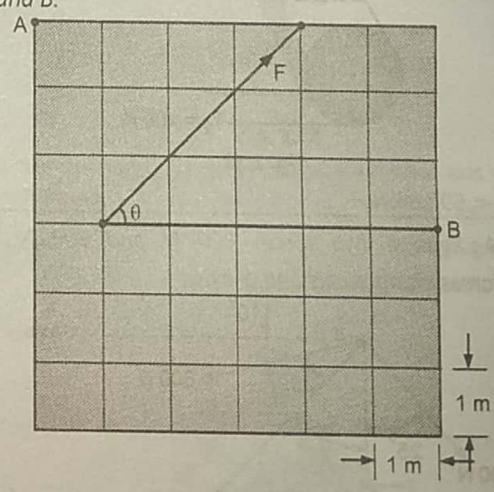


Fig. 1.83

Answer : (a) Moment of force F about point A = 1350.14 N-m (↺)

(b) Moment of force F about point B = 1350 N-m (↺)

Problem No. 25 : *The towline exerts a force of P = 4 kN at the end of 20 m long crane boom. If θ = 30°, determine the placement x of the hook at A so that this force creates a maximum moment about point O. What is this moment ?*

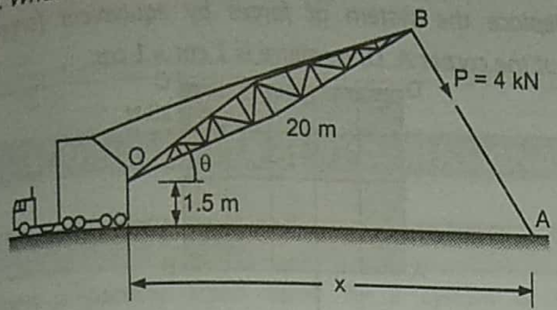


Fig. 1.84

Answer : x = 23.954 m, Maximum M_o = 80 Nm

Problem No. 26 : *Two couples act on the frame. If d = 4 m, determine the resultant couple moment. Compute the resultant by resolving each force into x and y components (a) finding the moment of each couple, (b) summing the moments of all the force components about point B.*

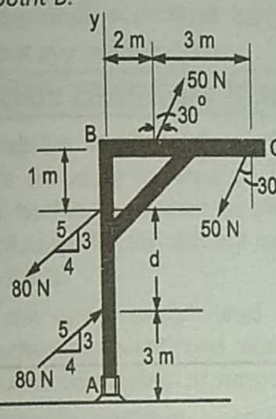


Fig. 1.85

Answer : (a) Moment of couple of 50 N force = 130 Nm (↻)

Moment of couple of 80 N force = 256 Nm (↺)

Resultant moment of couple = 126 Nm (↺)

(b) Resultant moment of forces about point B = 126 Nm (↺)

Problem No. 27 : *A 400 N force is applied at an angle θ = 20°. Determine the equivalent force-couple system acting at (a) point A and (b) point O.*

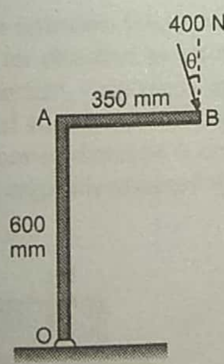


Fig. 1.86

Answer : Moment of couple of 400 N force = 131.55 Nm (↻)

Resultant force acting at A = 400 N at angle 20° with vertical

Moment of couple = 213.64 Nm

Resultant force acting at O = 400 N at an angle 20° with vertical

Problem No. 28 : *Replace the forces and couple system by an equivalent force and couple moment at point P.*

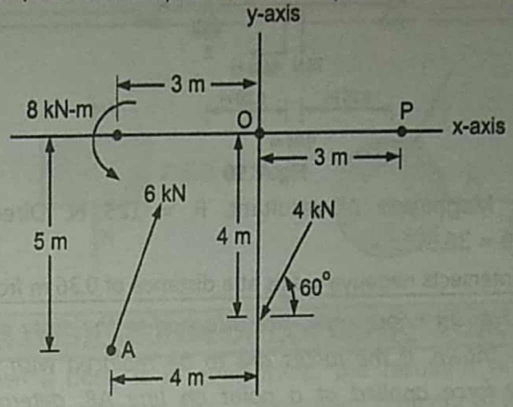


Fig. 1.87

Answer : Force acting at point P = 2.09 kN

Problem No. 29 : *Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member CD measured from end C.*

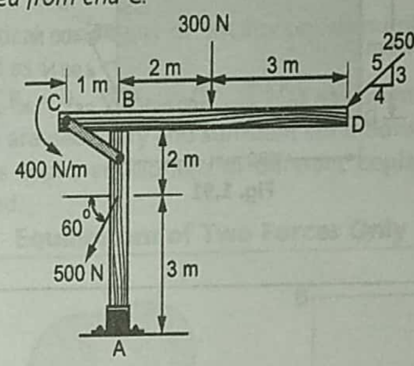


Fig. 1.88

Answer : R = 990.16 N, θ = 63°, x = 2.64 m from point C on the member CD.

Problem No. 30 : *Find magnitude of two like parallel forces acting at a distance 2.4 m whose resultant is 200 N and its line of action is at a distance of 0.6 m from one of the forces.*

Answer : F_2 = 50 N (↓), F_1 = 150 N (↓)

Problem No. 31 : *Determine the magnitude, direction and position of the resultant of the loadings.*

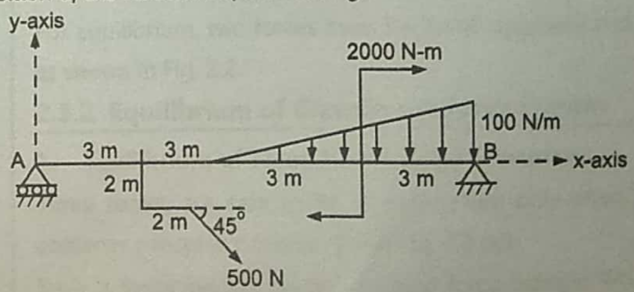


Fig. 1.89

Answer : R = 743.05 N, θ = 61.58°, x = 9.27 m

Problem No. 32 : *Compute resultant for the loads acting on the beam. Give the intercept with the axis of the beam.*

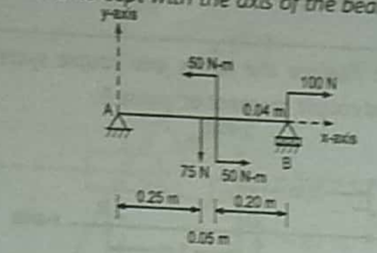


Fig. 1.90

Answer : Magnitude of resultant, R = 125 N, Direction of resultant, θ = 36.87°.

Resultant intersects negative x-axis at a distance of 0.36 m from A.

Problem No. 33 : *Four ropes are attached to the crate and exert the forces shown. If the forces are to be replaced with a single equivalent force applied at a point on line AB, determine the equivalent force and the distance from A to the point of application of the force when α = 30°.*

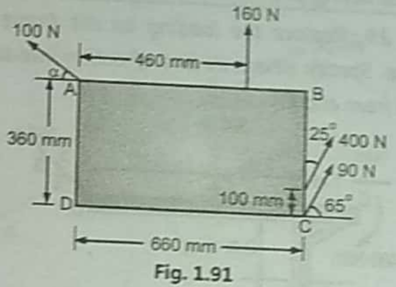


Fig. 1.91

Answer : Magnitude of resultant = 606.18 N,

Direction of resultant, θ = 78.54°, x = 714.27 mm

Problem No. 34 : *Forces are acting on a mesh as shown in Fig. 1.92. Replace the system of forces by equivalent force-couple system at the corner A. One square is 1 cm × 1 cm.*

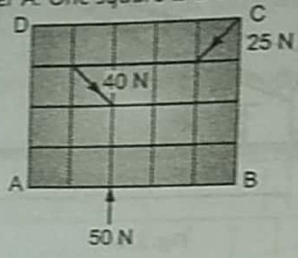


Fig. 1.92

Answer : It forms a force of x-component = 10.61 N (→) and y-component = 4.04 N (↑) and a couple of moment = 30.81 N-m (↻)

EQUILIBRIUM OF FORCES

A – EQUILIBRIUM OF TWO DIMENSIONAL FORCES

2.1 INTRODUCTION

When a particle acted upon by a system of forces is in equilibrium,

- It remains at state of rest if originally at state of rest.
- It moves with constant velocity if originally in state of motion.

This illustrates Newton's first law of motion.

In statics, when a rigid body acted upon by a system of forces is in equilibrium,

- It remains at state of rest as resultant force is zero, and
- There is no rotation of rigid body as resultant moment of forces about any point is zero.

2.2 FREE BODY DIAGRAM (F.B.D.)

Free Body Diagram is a sketch of the particle or rigid body representing it as being isolated or free from its surroundings (like supports, bodies in contact or attached cables etc.). All the known and unknown forces that act on the particle or rigid body are shown on F.B.D.

These forces are active (which tend to set the motion due to weight or attached cords etc.) and reactive (which tend to prevent the motion due to constraints or supports etc.).

F.B.D. helps to apply conditions of equilibrium correctly.

Steps to Draw F.B.D. :

- Isolate the particle or rigid body from its surroundings and sketch the shape of the same.
- Show all the active and reactive forces on the sketch. Note carefully each force by tracing around the boundary of particle/rigid body.
- Show all the known forces with proper magnitude and direction.
- Show all the unknown forces with letters for magnitude and arrow head for direction as assumed force i.e. push/pull. The correct direction becomes apparent after solving for conditions of equilibrium. If magnitude of unknown force is positive, assumed direction is correct and if negative, force is opposite to originally assumed direction.

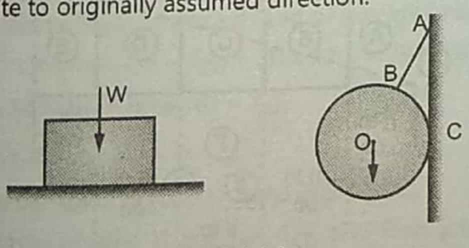


Fig. 2.1 (a)

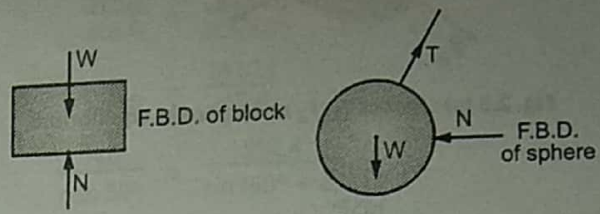


Fig. 2.1 (b)

2.3 EQUILIBRIUM OF COPLANAR FORCES

When a body is in equilibrium, the resultant of all the forces acting on it is zero. Also, the resultant moment of forces acting on it about any point is zero.

Mathematically, it may be stated as :

(i) $\bar{R} = \sum \bar{F} = 0$

(ii) $\sum$ Moment of force about any point on or off the body $= 0$.

Analytical conditions of equilibrium of number of forces may be stated as :

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M_{\text{at any point}} = 0.$$

These are necessary and sufficient conditions of equilibrium.

In this topic, equilibrium of different coplanar force systems is studied.

2.3.1 Equilibrium of Two Forces Only

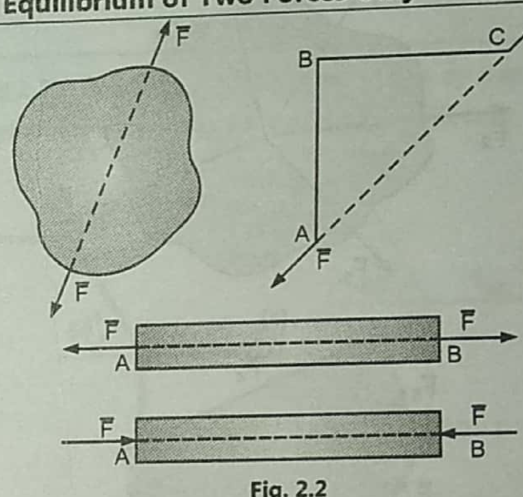


Fig. 2.2

For equilibrium, two forces must be equal, opposite and collinear as shown in Fig. 2.2.

2.3.2 Equilibrium of Concurrent Force System

1. Equilibrium of Three Forces (Lami's Theorem) :

Three forces are said to be in equilibrium only when they are coplanar concurrent forces. (Refer Fig. 2.3 (a)).

Such a force system forms a closed force triangle. Sine rule is used to find the unknown forces of force triangle.

(Refer Fig. 2.3 (b))

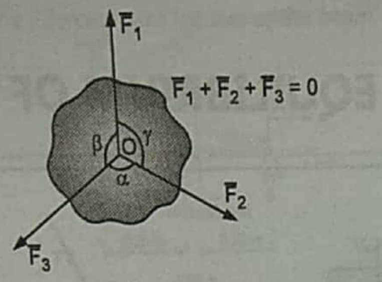


Fig. 2.3 (a) : Forces $\bar{F}_1$, $\bar{F}_2$ and $\bar{F}_3$ are in equilibrium

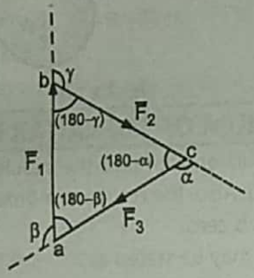


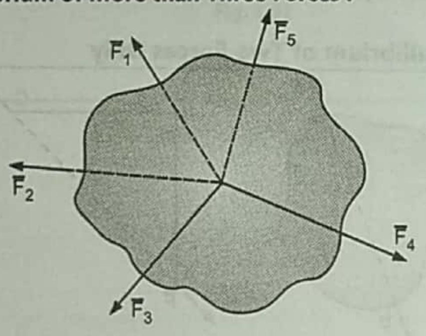
Fig. 2.3 (b) : Force triangle

Applying sine rule to the force triangle abc,

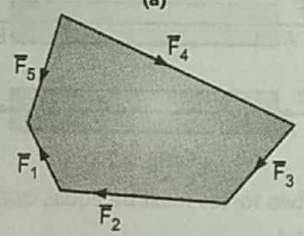
$$\frac{F_1}{\sin(180-\alpha)} = \frac{F_2}{\sin(180-\beta)} = \frac{F_3}{\sin(180-\gamma)}$$

$$\therefore \quad \boxed{\frac{F_1}{\sin\alpha} = \frac{F_2}{\sin\beta} = \frac{F_3}{\sin\gamma}} \quad \text{... Lami's theorem}$$

2. Equilibrium of More than Three Forces :



(a)



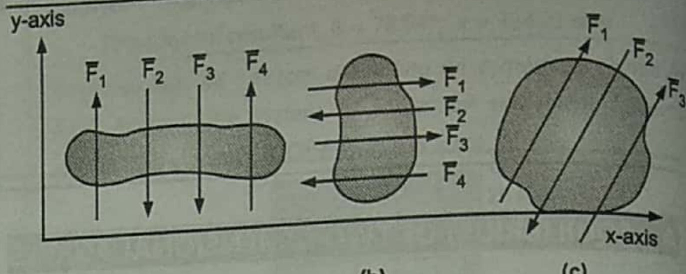
(b) Force polygon

Fig. 2.4

When a number of concurrent forces are in equilibrium, they must satisfy conditions of equilibrium as follows : $\sum F_x = 0$ and $\sum F_y = 0$

Graphically, polygon of forces must be a closed polygon. Refer Fig. 2.4 (b).

2.3.3 Equilibrium of Parallel Force System



(a) (b) (c)

Fig. 2.5

When a number of parallel forces are in equilibrium, they must satisfy conditions of equilibrium as follows :

$\sum F_y = 0$ and $\sum$ Moment of forces at y-axis = 0.

... Refer Fig. 2.5 (a)

$\sum F_x = 0$ and $\sum$ Moment of forces at x-axis = 0.

... Refer Fig. 2.5 (b)

$\sum \bar{F} = 0$ and $\sum$ Moment of forces at any point = 0.

... Refer Fig. 2.5 (c)

Graphically, polygon of forces must be a closed polygon and funicular diagram must be a closed polygon.

Find the magnitude of unknown forces R_M and R_N when rod MN is in equilibrium under several forces acting on it.

A space diagram is drawn as shown in Fig. 2.5 (d). $\bar{R}_M$ and $\bar{R}_N$ are unknown forces. Rod is in equilibrium.

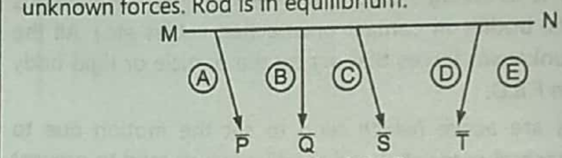


Fig. 2.5 (d)

A vector polygon 'abcde' is drawn. Polar diagram is also drawn as shown in Fig. 2.5 (e).

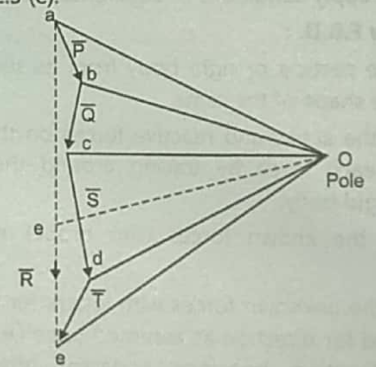


Fig. 2.5 (e) : Vector polygon and polar diagram

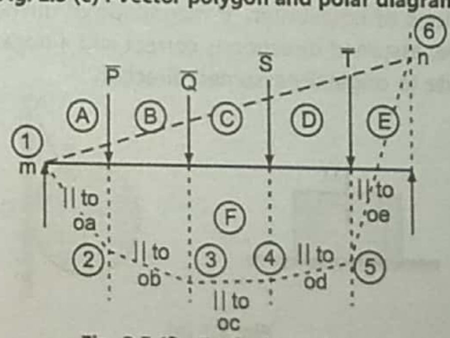


Fig. 2.5 (f) : Funicular polygon

A funicular polygon '1-2-3-4-5-6' is drawn as shown in Fig. 2.5 (f). Side '1-6' is closed as in equilibrium. Line parallel to side 1-6 is drawn on the polar diagram starting from point 'O' and intersecting vector polygon (a straight line in this case) at point f. Segment 'fa' represents force $\bar{R}_M$ and segment 'ef' represents force $\bar{R}_N$.

2.3.4 Equilibrium of General Force System

When a number of non-concurrent, non-parallel forces are in equilibrium, they must satisfy the following conditions of equilibrium :

$\sum F_x = 0$, $\sum F_y = 0$ and $\sum M = 0$.

Graphically, polygon of forces must be a closed polygon and funicular diagram must be a closed polygon.

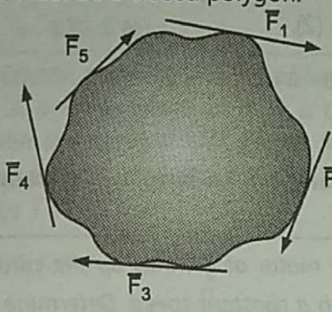


Fig. 2.6

NUMERICAL EXAMPLES ON EQUILIBRIUM OF CONCURRENT FORCE SYSTEM

Example 2.1 : *Determine the mass that must be supported at A and the angle θ of the cord in order to hold the system in equilibrium.*

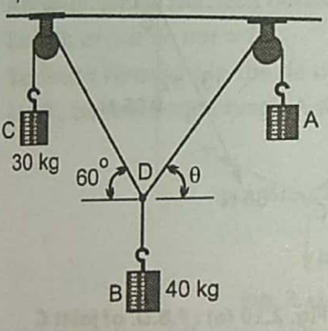


Fig. 2.7

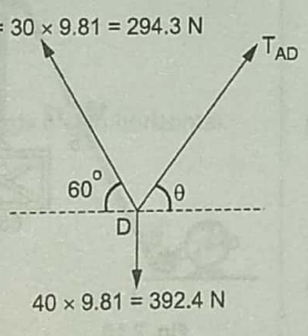


Fig. 2.7 (a) : F.B.D. of joint D

Solution :

Given data : Load = $40 \times 9.81 = 392.4$ N

Assume that pulleys are smooth and of negligible size.

Tension in the rope CD = $T_{CD} = 30 \times 9.81 = 294.3$ N

To find : Tension in rope AD = T_{AD}

Inclination of rope AD = θ

(a) Consider F.B.D. of joint D. Refer Fig. 2.7 (a).

Forces acting at joint D are T_{AD}, T_{CD} and load joint D is in equilibrium.

By Lami's theorem,

$$\frac{T_{CD}}{\sin(90^\circ + \theta)} = \frac{T_{AD}}{\sin(90^\circ + 60^\circ)} = \frac{392.4}{\sin(180^\circ - 60^\circ - \theta)}$$

$$\therefore \quad \frac{294.3}{\cos\theta} = \frac{T_{AD}}{\cos 60^\circ} = \frac{392.4}{\sin(60^\circ + \theta)}$$

$$\therefore \quad \frac{294.3}{\cos\theta} = \frac{T_{AD}}{\cos 60^\circ}$$

$$\therefore \quad T_{AD} = \frac{147.15}{\cos\theta} \quad \text{... (1)}$$

$$\frac{T_{AD}}{\cos 60^\circ} = \frac{392.4}{\sin(60^\circ + \theta)}$$

$$\therefore \quad T_{AD} = \frac{196.2}{\sin(60^\circ + \theta)} \quad \text{... (2)}$$

From equations (1) and (2),

$$\frac{147.15}{\cos\theta} = \frac{196.2}{\sin(60^\circ + \theta)}$$

$$\therefore \quad \frac{\sin(60^\circ + \theta)}{\cos\theta} = \frac{196.2}{147.15} = 1.33$$

$$\therefore \quad \frac{\sin 60^\circ \cos\theta}{\cos\theta} + \frac{\cos 60^\circ \sin\theta}{\cos\theta} = 1.33$$

$$\therefore \quad 0.866 + 0.5 \tan\theta = 1.33$$

$$\therefore \quad \theta = \mathbf{42.86^\circ} \quad \text{... Ans.}$$

$$T_{AD} = 200.74 \text{ N}$$

$$\therefore \quad \text{Mass supported at A} = \frac{200.74}{9.81}$$

$$= \mathbf{20.96 \text{ kg}} \quad \text{... Ans.}$$

Example 2.2 : *The 30 kg pipe is supported at A by a system of five cords. Determine the force in each cord for equilibrium.*

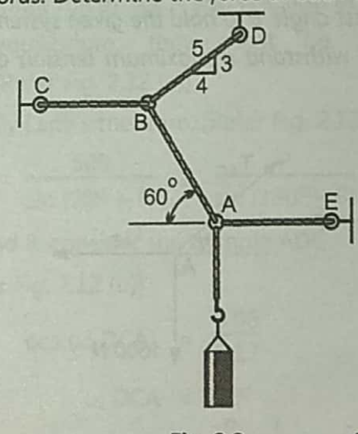


Fig. 2.8

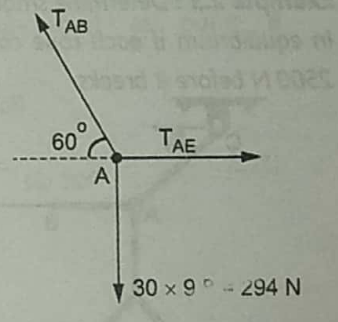


Fig. 2.8 (a) : F.B.D. of joint A

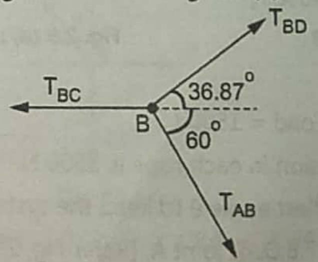


Fig. 2.8 (b) : F.B.D. of joint B

Solution :

Given data : Load attached at A = 294 N

Direction of tension in cord is as shown in Fig. 2.8.

To find : T_{AB}, T_{AE}, T_{BC}, T_{BD}.

(a) Consider F.B.D. of joint A. Refer Fig. 2.8 (a).

Forces acting at joint A are T_{AB}, T_{AE} and 294 N.

Joint A is in equilibrium.

By Lami's theorem,

$$\frac{294}{\sin 120^\circ} = \frac{T_{AB}}{\sin 90^\circ} = \frac{T_{AE}}{\sin (90^\circ + 60^\circ)}$$

$$\therefore \frac{294}{\sin 120^\circ} = \frac{T_{AB}}{\sin 90^\circ}$$

$$\therefore T_{AB} = \mathbf{339.48\ N} \quad \text{... Ans.}$$

$$\frac{294}{\sin 120^\circ} = \frac{T_{AE}}{\cos 60^\circ}$$

$$\therefore T_{AE} = \mathbf{169.74\ N} \quad \text{... Ans.}$$

(b) Consider F.B.D. of joint B. Refer Fig. 2.8 (b).

Forces acting at joint B are T_{AB}, T_{BC} and T_{BD}.

Joint B is in equilibrium.

By Lami's theorem,

$$\frac{T_{AB}}{\sin 143.13^\circ} = \frac{T_{BC}}{\sin 96.87^\circ} = \frac{T_{BD}}{\sin 120^\circ}$$

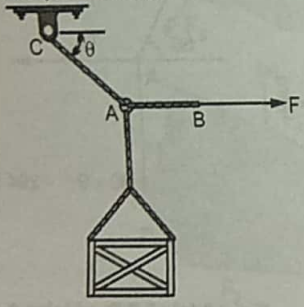
$$\therefore \frac{339.48}{\sin 143.13^\circ} = \frac{T_{BC}}{\sin 96.87^\circ}$$

$$\therefore T_{BC} = \mathbf{561.74\ N} \quad \text{... Ans.}$$

$$\frac{339.48}{\sin 143.13^\circ} = \frac{T_{BD}}{\sin 120^\circ}$$

$$\therefore T_{BD} = \mathbf{490\ N} \quad \text{... Ans.}$$

Example 2.3 : *Determine smallest angle θ to hold the given system in equilibrium if each rope can withstand a maximum tension of 2500 N before it breaks.*



Load = 1500 N

Fig. 2.9

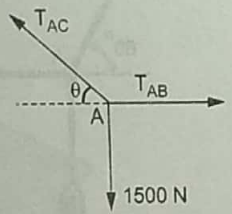


Fig. 2.9 (a) : F.B.D. of joint A

Solution :

Given data : Load = 1500 N.

Maximum tension in each rope is 2500 N.

To find : Smallest angle θ to keep the system in equilibrium.

(a) Consider F.B.D. of joint A. [Refer Fig. 2.9 (a)].

Joint A is in equilibrium.

By Lami's theorem,

$$\frac{1500}{\sin (180^\circ - \theta)} = \frac{T_{AC}}{\sin 90^\circ} = \frac{T_{AB}}{\sin (90^\circ + \theta)}$$

$$\therefore \frac{1500}{\sin \theta} = \frac{T_{AC}}{\sin 90^\circ}$$

$$\therefore T_{AC} = \frac{500}{\sin \theta} \quad \text{... (1)}$$

$$\text{and} \quad \frac{1500}{\sin \theta} = \frac{T_{AB}}{\cos \theta}$$

$$\therefore T_{AB} = 1500 \times \cot \theta \quad \text{... (2)}$$

(b) Since maximum tension in each rope is 2500 N.

From equation (1),

$$\sin \theta = 0.20$$

$$\therefore \theta = 11.54^\circ$$

From equation (2),

$$\cot \theta = \frac{2500}{500} = 5$$

$$\therefore \theta = 11.31^\circ$$

$\therefore$ **Smallest angle θ to keep the system in equilibrium is 11.31°.** **... Ans.**

Example 2.4 : *The motor at B winds up the cord attached to the 65 N crate at A with a constant speed. Determine the force in cord CD supporting pulley and the angle θ for equilibrium. Neglect size of the pulley at C.*

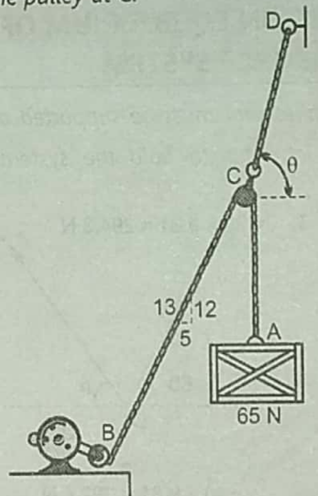


Fig. 2.10

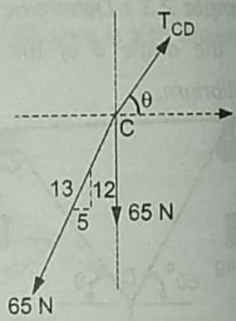


Fig. 2.10 (a) : F.B.D. of joint C

Solution :

Given data : Weight of the crate = 65 N

Tension in the cable CB = 65 N

Direction of tension with horizontal = $\tan^{-1}\left(\frac{12}{5}\right) = 67.38^\circ$

To find : Tension in the cable CD = T_{CD} = ?

Direction of T_{CD} for equilibrium = θ

(a) Forces acting at point C are : [Refer Fig. 2.10 (a)]

Weight of the crate = 65 N

Tension in the cable CB = T_{CB} = 65 N

Tension in the cable CD = T_{CD}

(b) Since forces are in equilibrium,

$$\Sigma F_X = 0 \text{ and } \Sigma F_y = 0$$

Resolving the forces along x-axis,

$$\Sigma F_X = T_{CD} \cos\theta - 65 \cos 67.38^\circ = 0$$

$$\therefore \quad T_{CD} \cos\theta = 25 \quad \text{... (1)}$$

Resolving the forces along y-axis,

$$\Sigma F_y = T_{CD} \sin\theta - 65 \sin 67.38^\circ - 65 = 0$$

$$\therefore \quad T_{CD} \sin\theta = 125 \quad \text{... (2)}$$

From equations (1) and (2),

$$\frac{T_{CD} \sin\theta}{T_{CD} \cos\theta} = \frac{125}{25}$$

$$\therefore \quad \tan\theta = 5$$

$$\therefore \quad \theta = \mathbf{78.7^\circ} \quad \text{... Ans.}$$

Substituting θ = 78.7° in equation (1),

$$T_{CD} = \mathbf{127.5\ N} \quad \text{... Ans.}$$

***Example 2.5 :** A horizontal prismatic bar AB of length l is hinged to a vertical wall at A and supported at B by a tie rod BC that makes an angle α with the horizontal. A weight P can have any position along the bar as defined by the distance x from the wall. Determine tensile force T in the bar.*

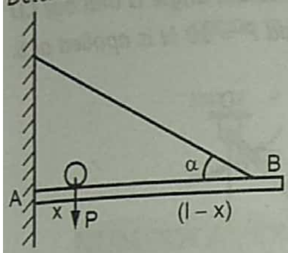


Fig. 2.11

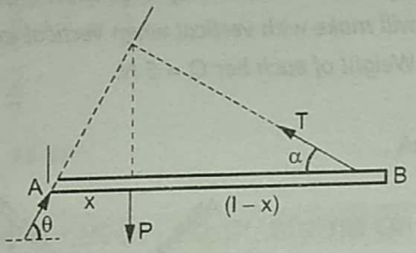


Fig. 2.11 (a)

Solution :

Given data : Weight P acting at distance x from A.

Angle of the tie bar with horizontal = α

Length of the tie bar = l

To find : Tension T in the tie bar = ?

(a) Let R_A be reaction at hinge A at an angle θ with horizontal.

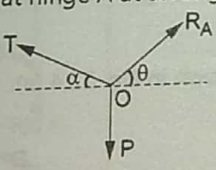


Fig. 2.11 (b)

Since forces T, P and R_A are keeping the bar in equilibrium, lines of action of three forces are intersecting at point 'O'.

Refer Fig. 2.11 (a).

By Lami's theorem, [Refer Fig. 2.11 (b)]

$$\frac{R_A}{\sin(90 + \alpha)} = \frac{T}{\sin(90 + \theta)} = \frac{P}{\sin(180 - \alpha - \theta)}$$

Since bar is in equilibrium,

Σ Moments of forces about point A = 0

$$\therefore \quad P \times x - T \sin\alpha \times l = 0$$

$$\therefore \quad \mathbf{T = \frac{Px}{l \sin\alpha}} \quad \text{... Ans.}$$

***Example 2.6 :** A roller of weight 500 N is to be pulled over a step of height h = 0.06 m by a horizontal force P applied to the end of a string wound around the circumference of the roller. Find the magnitude of P required to start the roller over the step. Radius of the roller is 0.12 m.*

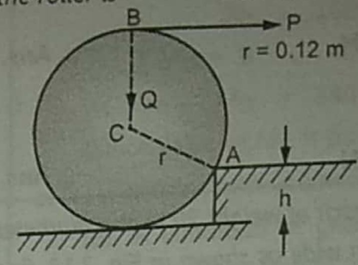


Fig. 2.12

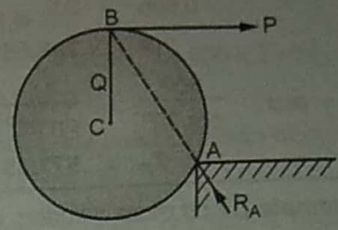


Fig. 2.12 (a)

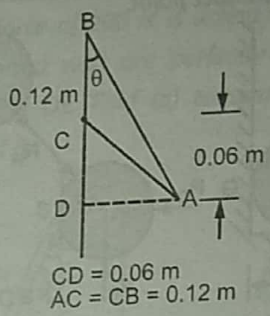


Fig. 2.12 (b)

Solution :

Given data : Radius of the roller = 0.12 m

Height of the step = 0.06 m

Weight of the roller = 500 N

To find : Horizontal force P required to start the roller.

(a) When the roller is about to roll over the point A, it loses its contact with the ground.

(b) Consider F.B.D. of the roller. Roller is in equilibrium under three forces : Q, P and reaction at A. Since, three forces are in equilibrium, they must be concurrent at point B. [Refer Fig. 2.12 (a)]

(c) By Lami's theorem, [Refer Fig. 2.12 (c)]

$$\frac{500}{\sin(90^\circ + \theta)} = \frac{P}{\sin(180^\circ - \theta)} = \frac{R_A}{\sin 90^\circ} \quad \text{... (1)}$$

To find θ, consider the triangle ADC.

[Refer Fig. 2.12 (b)]

$$\cos(\angle DCA) = \frac{0.06}{0.12}$$

$$\therefore \quad \angle DCA = 60^\circ$$

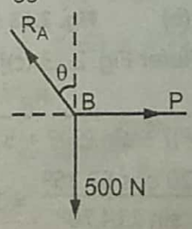


Fig. 2.12 (c)

Since AC = BC = r, and ∠ DCA = ∠ CBA + ∠ BAC = 2θ = 60°,

$$\therefore \quad \theta = 30^\circ$$

Substituting θ = 30° in equation (1),

$$\frac{500}{\cos 30^\circ} = \frac{P}{\sin 30^\circ} = \frac{R_A}{\sin 90^\circ}$$

$$\therefore \quad \frac{500}{0.866} = \frac{P}{0.5}$$

$$\therefore \quad P = \mathbf{288.68\ N} \qquad \text{... Ans.}$$

$$\text{and} \quad \frac{500}{\cos 30^\circ} = \frac{R_A}{\sin 90^\circ}$$

$$\therefore \quad R_A = \mathbf{577.35\ N} \qquad \text{... Ans.}$$

Example 2.7 : *If three cylinders, each of weight 20 N and diameter 380 mm, rest in a box of 790 mm wide as shown in Fig. 2.13, find the reactions at each contact point.*

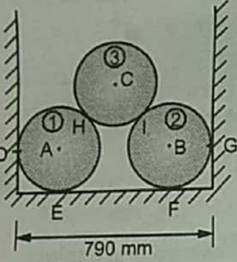


Fig. 2.13

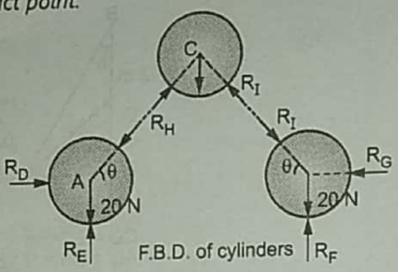


Fig. 2.13 (a) : F.B.D. of cylinders

Solution :

Given data : Weight of each cylinder = 20 N

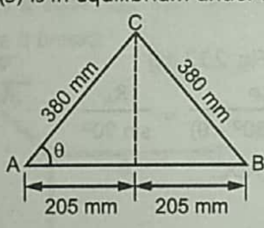
Diameter of each cylinder = 380 mm

To find : Reactions R_E, R_D, R_H, R_I, R_G, R_F of cylinders (1) and (2).

Since position is symmetrical, as well as weight and diameter are same,

$$R_D = R_G, \quad R_E = R_F \quad \text{and} \quad R_H = R_I$$

(a) Consider F.B.D. of the cylinder (3). [Refer Fig. 2.13 (a)]. Cylinder (3) is in equilibrium under reactions R_H, R_I and weight W.



$$\cos\theta = \frac{205}{380}$$

$$\therefore \theta = 57.35^\circ$$

Fig. 2.13 (b)

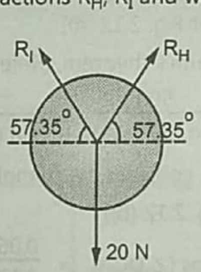


Fig. 2.13 (c) : F.B.D. of cylinder (3)

By Lami's theorem, [Refer Fig. 2.13 (c)]

$$\frac{20}{\sin[180^\circ - (2 \times 57.35^\circ)]} = \frac{R_H}{\sin(90^\circ + 57.35^\circ)}$$

$$\therefore \quad R_H = \frac{20 \cos 57.35^\circ}{\sin 114.70^\circ} = 11.88\ \text{N}$$

$$R_H = R_I = \mathbf{11.88\ N} \qquad \text{... Ans.}$$

(b) Consider F.B.D. of cylinder (1). Cylinder (1) is in equilibrium under reactions R_H, R_E, R_D and weight 20 N. [Refer Fig. 2.13 (d)]

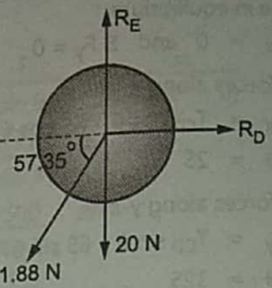


Fig. 2.13 (d) : F.B.D. of cylinder (1)

By conditions of equilibrium,

$$\Sigma F_x = 0$$

$$\therefore \quad R_D - 11.88 \cos 57.35^\circ = 0$$

$$\therefore \quad R_D = 6.41\ \text{N}$$

$$R_D = R_G = \mathbf{6.41\ N} \qquad \text{... Ans.}$$

$$\Sigma F_y = 0$$

$$\therefore \quad R_E - 20 - 11.88 \sin 57.35^\circ = 0$$

$$\therefore \quad R_E = 30\ \text{N}$$

$$\therefore \quad R_E = R_F = \mathbf{30\ N} \qquad \text{... Ans.}$$

Example 2.8 : *Two identical prismatic bars AB and CD are welded together in the form of 'T' as shown. Calculate angle α that bar CD will make with vertical when vertical load P = 10 N is applied at B. Weight of each bar Q = 5 N.*

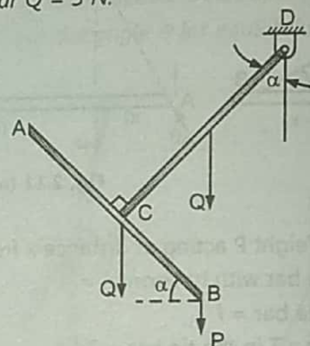


Fig. 2.14 (a)

Solution :

Given data : As shown in Fig. 2.14 (a).

To find : Angle 'α'.

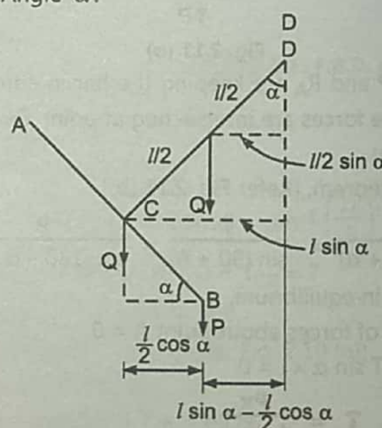


Fig. 2.14 (b)

Since bars are in equilibrium,

$$\Sigma M = 0$$

$\therefore$ Moment of force P at D

$$= -P \times \left(l \sin\alpha - \frac{l}{2}\cos\alpha\right)$$

$$\therefore \quad \Sigma M_D = 0$$

$$\therefore -Q \cdot \frac{l}{2}\sin\alpha - Q\, l \sin\alpha - P\left(l \sin\alpha - \frac{l}{2}\cos\alpha\right) = 0$$

$$\therefore -Q \sin\alpha - 2Q \sin\alpha - 2P \sin\alpha + P\cos\alpha = 0$$

$$\therefore -3Q \sin\alpha + P(\cos\alpha - 2\sin\alpha) = 0$$

$$\therefore -15 \sin\alpha + 10(\cos\alpha - 2\sin\alpha) = 0$$

$$\therefore \quad \sin\alpha - \frac{2}{3}(\cos\alpha - 2\sin\alpha) = 0$$

Dividing by sin α,

$$\therefore \quad 1 - \frac{2}{3}(\cot\alpha - 2) = 0$$

$$\frac{2}{3}(\cot\alpha - 2) = 1$$

$$\cot\alpha - 2 = \frac{3}{2}$$

$$\cot\alpha = \frac{3}{2} + 2 = \frac{7}{2}$$

$$\therefore \quad \tan\alpha = \frac{2}{7}$$

$$\therefore \quad \alpha = \mathbf{15.94°} \qquad \text{... Ans.}$$

NUMERICAL EXAMPLES ON EQUILIBRIUM OF GENERAL FORCE SYSTEM

***Example 2.9 :** Three lines of power pole exert a vertical force on pole as shown in Fig. 2.15. Determine reactions at fixed support D. Determine which line when removed creates a condition for the greatest moment reaction at D.*

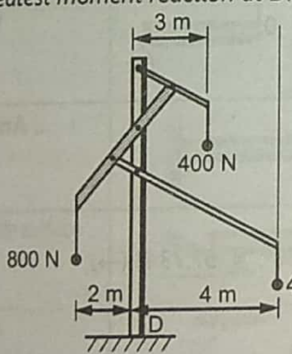


Fig. 2.15

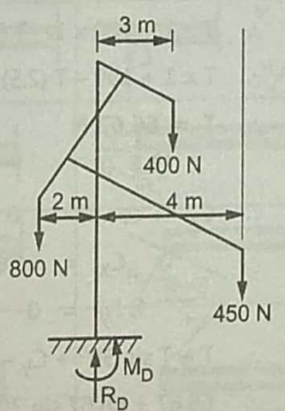


Fig. 2.15 (a)

Solution :

Given data : Forces 800 N, 400 N and 450 N are acting as shown in Fig. 2.15.

To find : Vertical reaction at D.

Moment at D.

(a) Let R_D be the vertical reaction at D and M_D be the moment at D as shown in Fig. 2.15 (a). Pole is in equilibrium.

Applying conditions of equilibrium,

$\therefore \quad \Sigma F_y = 0 \quad \therefore \quad -400 - 800 - 450 + R_D = 0$

$\therefore \quad R_D = \mathbf{1650\ N}$ ... **Ans.**

$\therefore \quad \Sigma M_D = 0 \quad \therefore \quad 400 \times 3 + 450 \times 4 - 800 \times 2 - M_D = 0$

$\therefore \quad M_D = \mathbf{1400\ Nm}$ (↺) ... **Ans.**

(Since value of M_D is positive, assumed direction of moment at D i.e. anticlockwise is correct.)

(b) A condition of greatest moment reaction is produced when a line carrying weight of 800 N is removed. ... **Ans.**

***Example 2.10 :** Force of 200 N is acting on the middle of the rod as shown. Floor and wall are perfectly smooth and slipping is prevented by string DE. Find (a) tension S in the string DE, (b) reactions at A and B.*

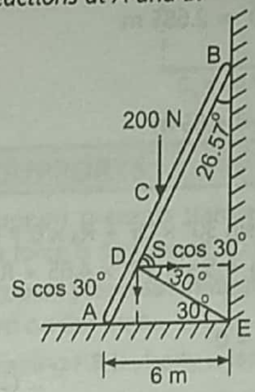


Fig. 2.16

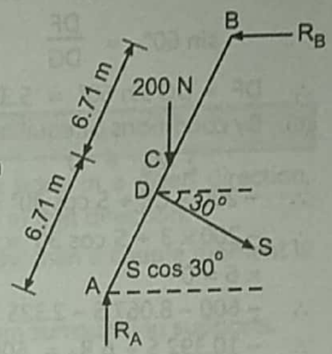


Fig. 2.16 (a) : F.B.D. of rod

Solution :

Given data : Force = 200 N is acting at the centre of the rod.

Dimensions are as shown in Fig. 2.16 (a).

To find : Tension in the string DE = S. Reactions at floor and wall i.e. R_A and R_B.

(a) Rod is in equilibrium under forces : Refer Fig. 2.16 (a).

Force = 200 N

Reaction at wall = R_A

Reaction at floor = R_B

Tension in string DE = S.

(b) From geometry, Refer Fig. 2.16 (b)

$$l(AB) = \sqrt{6^2 + 12^2} = 13.42 \text{ m}$$

$$l(BC) = l(AC) = 6.71 \text{ m}$$

Consider Δ BGA,

$$\angle BAG = \tan^{-1}\left(\frac{12}{6}\right) = 63.43°$$

Consider Δ BCE,

$$\cos 63.43° = \frac{CE}{CB} = \frac{CE}{6.71}$$

$$\therefore \quad CE = 6.71 \cos 63.43° = 3 \text{ m}$$

Consider Δ ADG,

By sine rule,

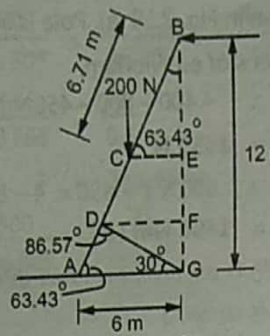


Fig. 2.16 (b)

$$\frac{6}{\sin 86.57^\circ} = \frac{DG}{\sin 63.43^\circ}$$

$$\therefore \quad DG = 5.37 \text{ m}$$

Consider Δ DFG,

$$\cos 60^\circ = \frac{FG}{DG}$$

$$\therefore \quad FG = DG \cos 60^\circ = 5.37 \cos 60^\circ = 2.685 \text{ m}$$

$$\sin 60^\circ = \frac{DF}{DG}$$

$$\therefore \quad DF = DG \sin 60^\circ = 5.37 \sin 60^\circ = 4.65 \text{ m}$$

(c) By conditions of equilibrium,

$$\Sigma M_B = 0$$

$$\therefore \quad -200 \times CE + S \cos 30^\circ \times BF + S \sin 30^\circ \times DF + R_A \times 6 = 0$$

$$\therefore \quad -200 \times 3 + S \cos 30^\circ \times (12 - FG) + S \sin 30^\circ \times 4.65 + R_A \times 6 = 0$$

$$\therefore \quad -600 - 8.067\, S - 2.325\, S + 6\, R_A = 0$$

$$\therefore \quad -10.392\, S + 6\, R_A = 600 \qquad \text{... (1)}$$

$$\Sigma F_y = 0$$

$$\therefore \quad R_A - 200 - S \sin 30^\circ = 0$$

$$\therefore \quad -0.5\, S + R_A = 200 \qquad \text{... (2)}$$

$$\Sigma F_x = 0$$

$$\therefore \quad -R_B + S \cos 30^\circ = 0$$

$$\therefore \quad R_B = 0.866\, S \qquad \text{... (3)}$$

Solving equations (1) and (2),

$$S = 81.17 \text{ N and } R_A = \mathbf{240.58\ N} \qquad \text{... Ans.}$$

Substituting $S = 81.17$ N in equation (3),

$$R_B = \mathbf{70.29\ N} \qquad \text{... Ans.}$$

Example 2.11 : *Determine tension in the cable ABD and the action at C when $\theta = 60^\circ$, $a = 1$ m, $P = 100$ N. Neglect friction in e pulley.*

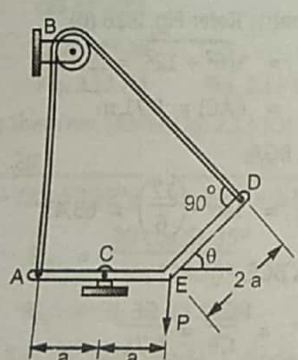
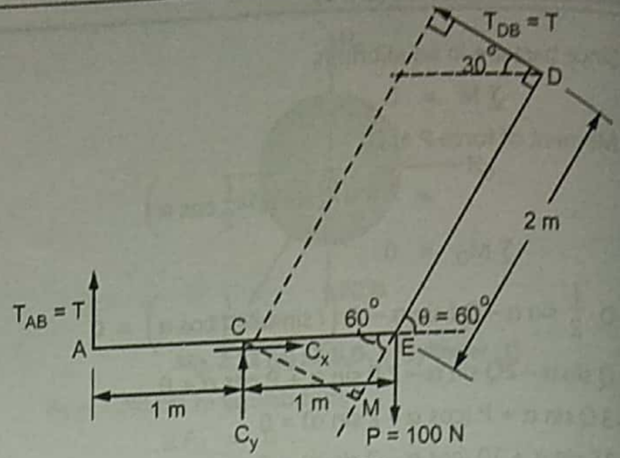


Fig. 2.17



F.B.D. of ACD

Fig. 2.17 (a) : F.B.D. of ACD

Solution :

Given data : Force P = 100 N is acting at E

Dimensions are as shown in Fig. 2.17 (a).

To find : Tension in the cable ABD i.e. $T_{AB} = T_{DB}$

Reaction at C i.e. R_C

(a) Lever ACD is in equilibrium under forces :

Reaction at C = R_C

Tension in the cable i.e. $T_{AB} = T_{DB} = T$

Force P = 100 N

(b) From geometry, Refer Fig. 2.17 (a)

Consider Δ CME,

$$\cos 60^\circ = \frac{ME}{CE} = \frac{ME}{1}$$

$$\therefore \quad ME = 0.5 \text{ m}$$

$$\therefore \quad MD = 2.5 \text{ m}$$

(c) Applying conditions of equilibrium to the lever,

$$\Sigma M_C = 0$$

$$\therefore \quad T \times 1 + 100 \times 1 - T\,(MD) = 0$$

$$\therefore \quad T \times 1 + 100 - T\,(2.5) = 0$$

$$\therefore \quad T = \mathbf{66.67\ N} \qquad \text{... Ans.}$$

$$\Sigma F_x = 0$$

$$\therefore \quad C_x - T \cos 30^\circ = 0$$

$$\therefore \quad C_x = 66.67 \cos 30^\circ = 57.73 \text{ N } (\rightarrow)$$

$$\Sigma F_y = 0$$

$$\therefore \quad T + T \sin 30^\circ + C_y - P = 0$$

$$\therefore \quad 66.67 + 66.67 \sin 30^\circ + C_y - 100 = 0$$

$$\therefore \quad C_y = 0 \text{ N}$$

$$\therefore \quad \textbf{Reaction at C} = R_C = \mathbf{57.73\ N\ (\rightarrow)} \qquad \text{... Ans.}$$

Example 2.12 : *A lever AB is hinged at C and attached to a control cable at A. If the lever is subjected to a 75 N vertical force at B, determine (a) the tension in the cable, (b) the reaction at C.*

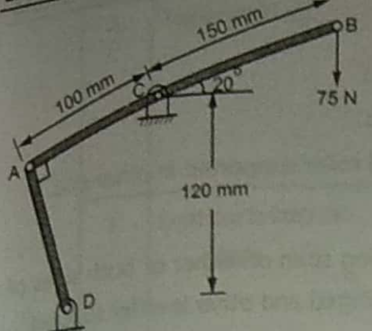


Fig. 2.18

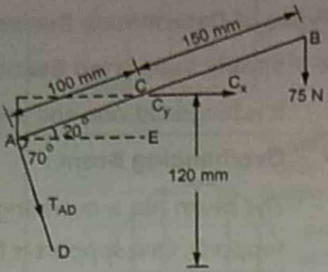


Fig. 2.18 (a)

Solution :

Given data : Force 75 N is acting at B.

Dimensions are as shown in Fig. 2.18 (a).

To find : Tension in the cable AD : T_{AD}

Reaction at C : R_C

(a) Lever AB is in equilibrium under forces : [Refer Fig. 2.18 (a)]

Force 75 N.

Tension in the cable AD i.e. T_{AD}

Reaction at hinge C i.e. R_C

(b) Applying conditions of equilibrium,

$\Sigma M_C = 0 \quad \therefore \quad -T_{AD} \times 100$

$\therefore \quad T_{AD} = \mathbf{105.71\ N}$ ... **Ans.**

$\Sigma F_X = 0$

$\therefore \quad +C_X + T_{AD}\cos 70° = 0$

$\therefore \quad C_X = -36.15\ N$

(Since value of C_X is negative, assumed direction of C_X is wrong.)

$\therefore \quad C_X = 36.15\ N\ (\leftarrow)$

$\Sigma F_y = 0$

$\therefore \quad -T_{AD}\sin 70° - 75 + C_y = 0$

$\therefore \quad C_y = \mathbf{174.33\ N\ (\uparrow)}$ ... **Ans.**

$$\text{Reaction at C} = \sqrt{C_x^2 + C_y^2} = \sqrt{(88.10)^2 + (155.44)^2}$$

$\therefore \quad R_C = \mathbf{178.04\ N}$ ... **Ans.**

$$\text{Direction of } R_C = \tan\alpha = \frac{C_y}{C_x}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{174.33}{36.15}\right) = \mathbf{78.28°}$$ ... **Ans.**

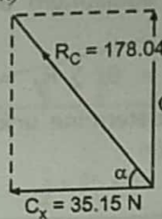


Fig. 2.18 (b)

2.4 SUPPORTS

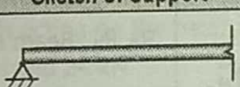
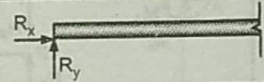
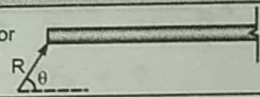
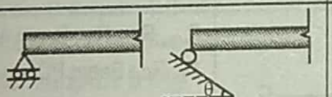
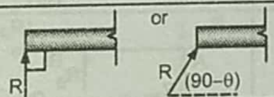
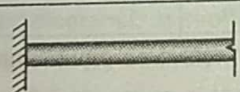
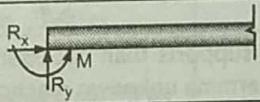
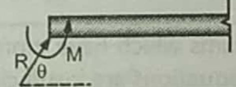
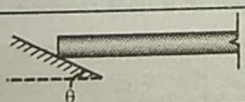
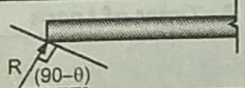
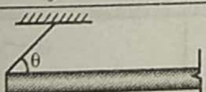
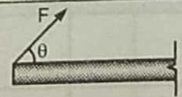
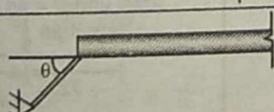
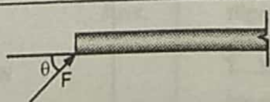
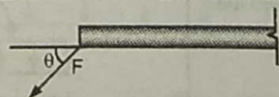
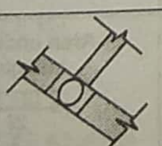
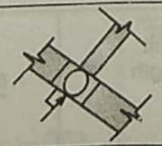
If a support prevents translation of a body in a given direction, then a force is developed on the body in that direction.

If a support prevents rotation of a body, then a couple moment is exerted on the body.

For drawing F.B.D., body is isolated from surrounding supports.

Supports are replaced by reactions as shown in Table 2.1.

Table 2.1 : Types of Supports

Type of Support	Sketch of Support	Reactions of Support		Remark
Pin or Hinge			or 	It prevents tranlation along X and Y directions.
Roller				It prevents translation in the direction perpendicular to the roller surface.
Fixed			or 	It prevents tranlastion along X and Y directions. It also prevents rotation of a body.
Smooth surface				It prevents translation in the direction perpendicular to surface.
Cable				Tension force acts along direction of cable away from the body.
Link				Reaction acts along the link away ort towards the body.
Member pin connected to collar on smooth rod				Reaction acts perpendicular to the rod.

2.5 BEAM

Beams are long and straight, bars having a constant cross-sectional area. Beams are designed to support loads applied at various points.

Beams are classified according to the way in which they are supported.

1. Determinate beams.
2. Indeterminate beams.

1. Determinate Beams :

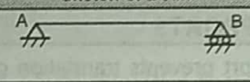
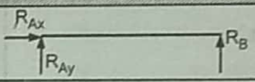
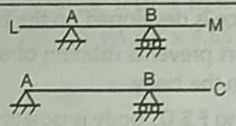
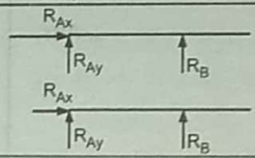
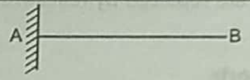
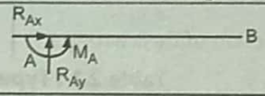
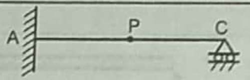
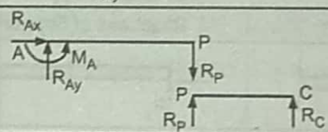
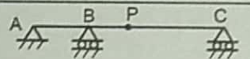
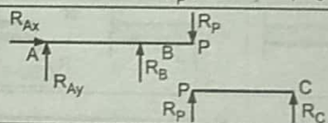
The beams which are supported by the minimum number of supports necessary to ensure equilibrium are called statically determinate beams.

Equilibrium equations i.e. $\sum F_x = 0$, $\sum F_y = 0$ and $\sum M$ at any point $= 0$ are sufficient to determine unknown reactions of beam.

Types of Determinate Beams :

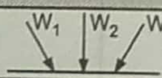
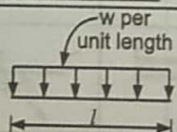
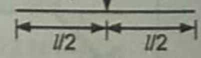
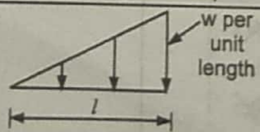
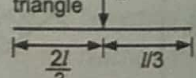
- **Simply Supported Beam :**
  It is hinged at one end and roller supported at other end.
- **Overhanging Beam :**
  The beam has a overhanging span on either or both sides of supports. One support is hinged and other is roller support.
- **Cantilever Beam :**
  It is fixed at one end and free at the other end.
- **Compound Beam :**
  Two or more beams are connected by hinges or pins to form a single continuous beam. Reactions at supports can be determined by considering F.B.D. of each beam separately as shown in Table 2.2.

Table 2.2 : Types of Determinate Beams

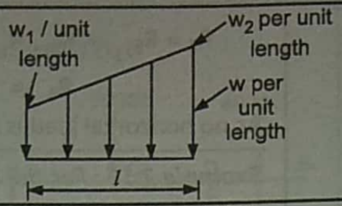
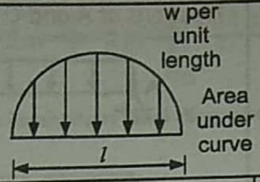
Type Beam	Sketch of Beam	F.B.D. of Beam	Unknown Reactions of Beam
Simply supported beam			R_{Ax}, R_{Ay} and R_B
Overheating beam			R_{Ax}, R_{Ay} and R_B
Cantilever beam			R_{Ax}, R_{Ay} and M_A
Compund beam			R_{Ax}, R_{Ay} and M_A : Beam AP R_P, R_B : Beam PB
			R_{Ax}, R_{Ay}, R_B : Beam AP R_P, R_C : Beam PC.

2. Indeterminate Beams : Beams which have more external supports than necessary to maintain an equilibrium are called statically indeterminate. Equilibrium equations are insufficient to determine unknown reactions.

Table 2.3 : Types of Loads

Sr. No.	Type of Load	Diagram	Load Intensity	Equivalent Load Diagram
1.	Point load or concentrated load		–	–
2.	Uniformly distributed load (u.d.l.)		wL	Area under = wl rectangle 
3.	Uniformly varying load (u.v.l)		$\frac{(0+w)l}{2} - \frac{wl}{2}$	Area under = $\frac{1}{2} \times w \times l$ triangle 

Contd...

4.	Trapezoidal load		$\frac{(w_1+w_2)}{2} \times l$	Area under trapezoid = $\left(\frac{w_1+w_2}{2}\right)l$ $\left(\frac{w_1+2w_2}{w_1+w_2}\right)\left(\frac{w_2+w_1+2}{w_1+w_2}\right)\frac{l}{3}$
5.	Load due to irregular shape		Area under curve	Area under = total load curve Centroidol area

NUMERICAL EXAMPLES ON BEAM

Example 2.13 : *Determine reactions at supports A and B.*

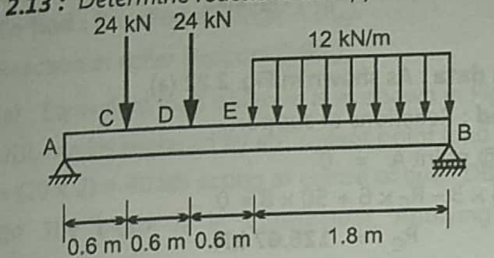


Fig. 2.19

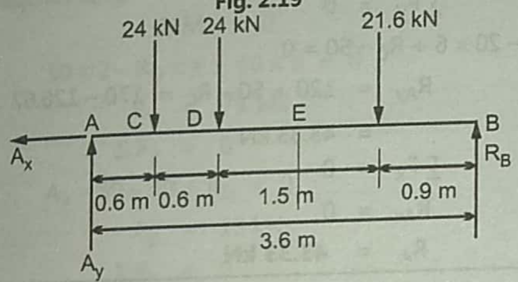


Fig. 2.19 (a) : Equivalent load diagram

Solution :

Given data : Loads acting on the beam are :

Point loads : 24 kN and 24 kN

UDL : 12 kN/m on the span 1.8 m.

To find : Reaction at hinge support A (R_A)

Reaction at roller support B (R_B)

(a) Equivalent load diagram is as shown in Fig. 2.19 (a).

UDL can be replaced by equivalent point load = $(12 \times 1.8) = 21.6$ kN acting at the centre of span EB as shown.

(b) Beam is in equilibrium.

Applying conditions of equilibrium,

$$\Sigma M_A = 0$$

$\therefore \quad 24 \times 0.6 + 24 \times 1.2 + 21.6 \times 2.7 - R_B \times 3.6 = 0$

$\therefore \quad R_B = $ **28.2 kN** ... **Ans.**

$\Sigma F_y = 0$

$\therefore \quad A_y + R_B - 24 - 24 - 21.6 = 0$

$\therefore \quad A_y = 41.4$ kN

$\Sigma F_x = 0$

$\therefore \quad A_x = 0$ kN

$\therefore$ Reaction at A is R_A = **41.4 kN** ... **Ans.**

Example 2.14 : *For the beam shown in Fig. 2.20 (a), calculate the reactions at supports.*

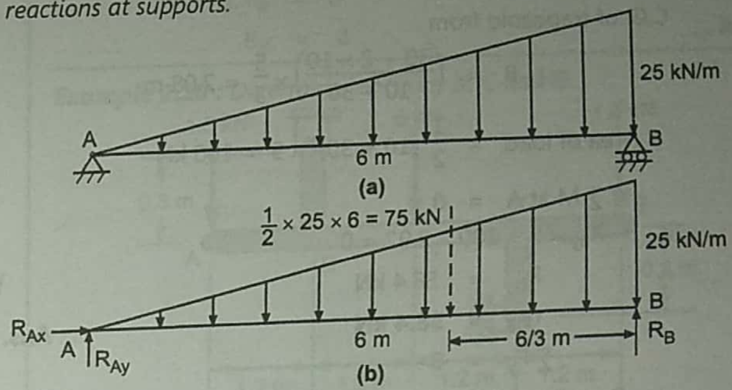


Fig. 2.20

Solution :

Given data : As shown in Fig. 2.20 (a).

To find : Reactions at support.

$$\Sigma \text{ M at A} = 0$$

$\therefore \quad -R_B \times 6 + \frac{1}{2} \times 25 \times 6 \times \frac{2}{3} \times 6 = 0$

$\therefore \quad R_B = 50$ kN

$\Sigma F_y = 0$

$\therefore \quad R_{Ay} + R_B - \frac{1}{2} \times 25 \times 6 = 0$

$\therefore \quad R_{Ay} = 75 - 50 = 25$ kN

$\Sigma F_x = 0$

$\therefore \quad R_{Ax} = 0 \quad \therefore \quad R_A =$ **25 kN** ... **Ans.**

Example 2.15 : *Calculate the reactions for the trapezoidal load as shown in Fig. 2.21 (a).*

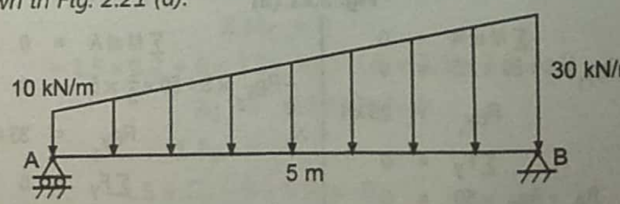


Fig. 2.21 (a)

Solution :

Given data : As shown in Fig. 2.21 (a).

To find : Reactions at support.

Method I :

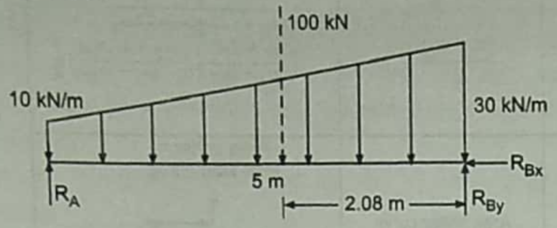


Fig. 2.21 (b)

C.G. of trapezoid from

$$A = \left(\frac{10 + 2 \times 30}{10 + 30}\right) \times \frac{5}{3} = 2.92 \text{ m}$$

C.G. of trapezoid from

$$B = \left(\frac{30 + 2 \times 10}{10 + 30}\right) \times \frac{5}{3} = 2.08 \text{ m}$$

$$\text{Area of load} = \frac{1}{2}(10 + 30) \times 5 = 100 \text{ kN}$$

$$\Sigma M \text{ at } A = 0$$

$\therefore \quad -R_{By} \times 5 + 100 \times 2.92 = 0$

$\therefore \quad R_{By} = 58.4$ kN

$R_B = \mathbf{58.4\ kN}$ **... Ans.**

$\Sigma F_y = 0$

$\therefore \quad R_A + R_{By} - 100 = 0$

$\therefore \quad R_A = 100 - 58.4 = 41.6$ kN

$\therefore \quad R_A = \mathbf{41.6\ kN}$ **... Ans.**

Method II : Divide trapezoid into rectangle (UDL) and triangle (UVL)

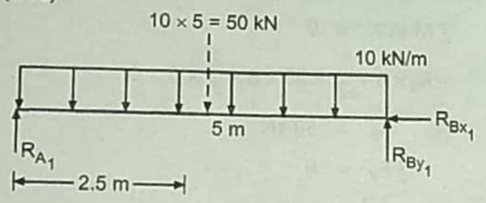


Fig. 2.21 (c)

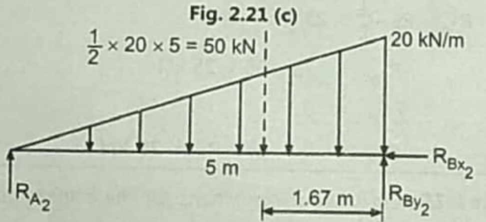


Fig. 2.21 (d)

ΣM at A = 0

$\therefore \quad -R_{By_1} \times 5 + 50 \times 2.5 = 0$

$\therefore \quad R_{By_1} = 25$ kN

$\Sigma F_y = 0$

$\therefore \quad R_{A_1} + R_{By_1} - 50 = 0$

$\therefore \quad R_{A_1} = 50 - 25 = 25$ kN

ΣM at A = 0

$-R_{By_2} \times 5 + 50 \times \frac{2}{3} \times 5 = 0$

$\therefore \quad R_{By_2} = 33.4$ kN

$\Sigma F_y = 0$

$R_{A_2} + R_{By_2} - 50 = 0$

$\therefore \quad R_{A_2} = 50 - 33.4 = 16.6$ kN

$\therefore \quad R_A = R_{A_1} + R_{A_2} = 25 + 16.6 = 41.6$ kN

$R_A = \mathbf{41.6\ kN}$ **... Ans.**

$R_B = R_{By_1} + R_{By_2} = 25 + 33.4 = 58.4$ kN

$R_B = \mathbf{58.4\ kN}$ **... Ans.**

As no horizontal load is there, therefore, R_{Bx_1} and R_{By_2} are zero.

Example 2.16 : *For the beam shown in Fig. 2.22 (a), calculate the reactions at A and C.*

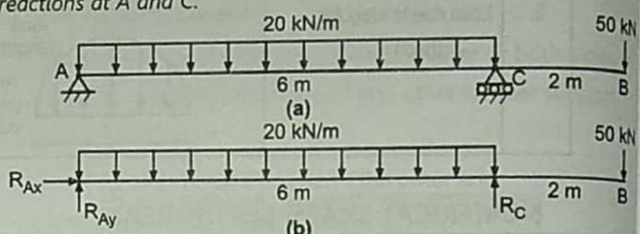


Fig. 2.22

Solution :

Given data : As shown in Fig. 2.22 (a).

To find : Reactions at support.

ΣM @ point A = 0

$20 \times 6 \times 3 - R_C \times 6 + 50 \times 8 = 0$

$\therefore \quad R_C = \mathbf{126.67\ kN}$ **... Ans.**

$\Sigma F_y = 0$

$R_{Ay} - 20 \times 6 + R_C - 50 = 0$

$\therefore \quad R_{Ay} = 120 + 50 - R_C = 170 - 126.67$

$= 43.33$ kN

$\Sigma F_x = 0$

$\therefore \quad R_{Ax} = 0$

$\therefore \quad R_A = \mathbf{43.33\ kN}$ **... Ans.**

Example 2.17 : *Determine reactions at A and B.*

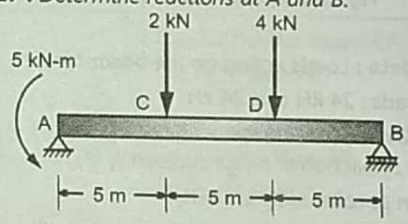


Fig. 2.23

Solution :

Given data : Loads acting on the beam are point loads 2 kN and 4 kN, couple and moment 5 kNm (↺).

To find : Reaction at hinge support A (R_A)

Reaction at roller support B (R_B).

(a) Beam is in equilibrium.

Applying conditions of equilibrium,

$\Sigma M_A = 0$

$-5 + 2 \times 5 + 4 \times 10 - R_B \times 15 = 0$

$\therefore \quad R_B = \mathbf{3\ kN}$ **... Ans.**

$\Sigma F_y = 0$

$\therefore \quad R_A + R_B - 2 - 4 = 0$

$\therefore \quad R_A = \mathbf{3\ kN}$ **... Ans.**

Example 2.18 : *Determine reactions at A and D.*

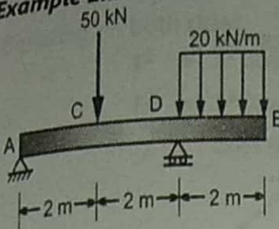


Fig. 2.24

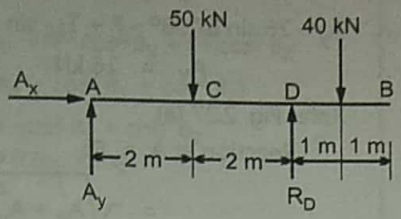


Fig. 2.24 (a) : Equivalent load diagram

Solution :

Given data : Loads acting on the beam are :

Point load : 50 kN

UDL : 20 kN/m on the span 2 m.

To find : Reaction at hinge A (R_A).

Reaction at roller support D (R_D).

(a) Equivalent load diagram is as shown in Fig. 2.24 (a).

UDL can be replaced by equivalent point load

= (20 × 2) = 40 kN acting at centre of span DB.

(b) The beam is in equilibrium. Applying conditions of equilibrium,

$$\Sigma M_A = 0$$

$$\therefore \quad 50 \times 2 - R_D \times 4 + 40 \times 5 = 0$$

$$\therefore \quad R_D = \mathbf{75\ kN} \qquad \text{... Ans.}$$

$$\Sigma F_y = 0$$

$$\therefore \quad A_y - 50 - 40 + R_D = 0$$

$$\therefore \quad A_y = 15\ \text{kN}$$

$$\Sigma F_x = 0$$

$$\therefore \quad A_x = 0$$

Reaction at A is

$$R_A = \mathbf{15\ kN} \qquad \text{... Ans.}$$

Example 2.19 : *Determine reaction and moment at B.*

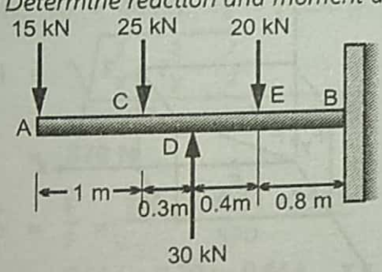


Fig. 2.25

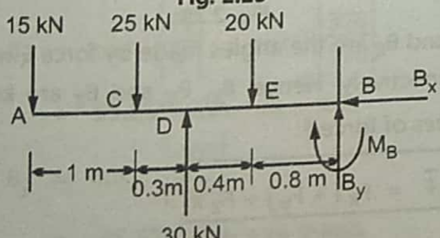


Fig. 2.25 (a) : Equivalent load diagram

Solution :

Given data : Loads acting on the beam are :

Point loads : 15 kN, 25 kN, 30 kN, 20 kN.

To find : Reactions at fixed support B i.e. B_x and B_y.

Moment at fixed support B i.e. M.

(a) Beam is in equilibrium. [Refer Fig. 2.25 (a)]

Applying conditions of equilibrium,

$$\Sigma M_B = 0$$

$$-15 \times 2.5 - 25 \times 1.5 - 20 \times 0.8 + M = 0$$

$$\therefore \quad M = \mathbf{91\ kNm}\ (\circlearrowleft) \qquad \text{... Ans.}$$

$$\Sigma F_y = 0$$

$$\therefore \quad -15 - 25 - 20 + B_y = 0$$

$$\therefore \quad B_y = \mathbf{60\ kN} \qquad \text{... Ans.}$$

$$\Sigma F_x = 0$$

$$\therefore \quad B_x = \mathbf{0} \qquad \text{... Ans.}$$

Example 2.20 : *Determine reactions at C and G.*

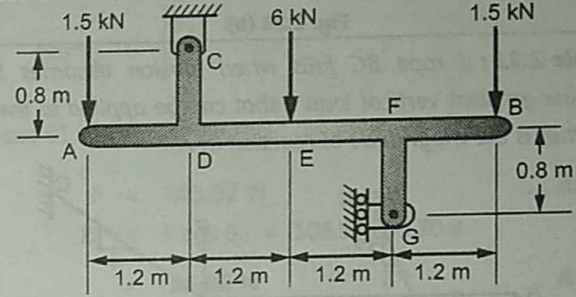


Fig. 2.26

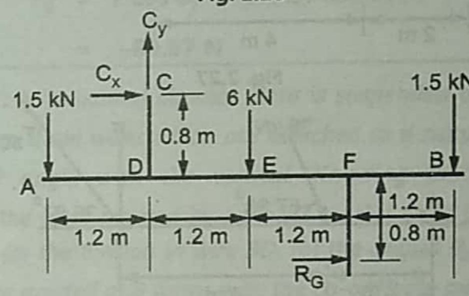


Fig. 2.26 (a)

Solution :

Given data : Loads acting on the beam AB are :

Point loads 1.5 kN, 6 kN, 1.5 kN.

To find : Reaction at hinge C (R_C)

Reaction at roller G (R_G).

(a) Beam AB is in equilibrium. [Refer Fig. 2.26 (a)]

Applying conditions of equilibrium,

$$\Sigma M_C = 0$$

$$-1.5 \times 1.2 + 6 \times 1.2 - R_G \times 1.6 + 1.5 \times 3.6 = 0$$

$$\therefore \quad R_G = \mathbf{6.75\ kN\ (\rightarrow)} \qquad \text{... Ans.}$$

$$\Sigma F_y = 0$$

$$\therefore \quad -1.5 + C_y - 6 - 1.5 = 0$$

$$\therefore \quad C_y = 9\ \text{kN}$$

$$\Sigma F_x = 0$$

$$\therefore \quad C_x + R_G = 0$$

$$\therefore \quad C_x = -6.75\ \text{kN}$$

(Since the value of C_x is negative, assumed direction is wrong)

$$C_x = 6.75 \text{ kN } (\leftarrow)$$

Refer Fig. 2.26 (b).

$$\text{Reaction at C} = R_C = \sqrt{C_x^2 + C_y^2}$$

$$= \sqrt{(6.75)^2 + (9)^2} = \mathbf{11.25\ kN} \quad \text{... Ans.}$$

$$\text{Direction of } R_C = \theta = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{9}{6.75}\right)$$

$$= \mathbf{53.13^\circ} \quad \text{... Ans.}$$

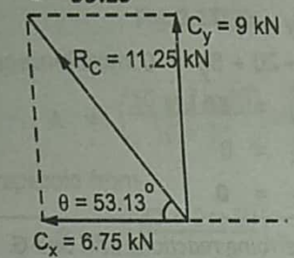


Fig. 2.26 (b)

Example 2.21 : *If rope BC fails when tension becomes 50 kN, determine greatest vertical load F that can be applied to the beam at B. What is the magnitude of reaction at A ?*

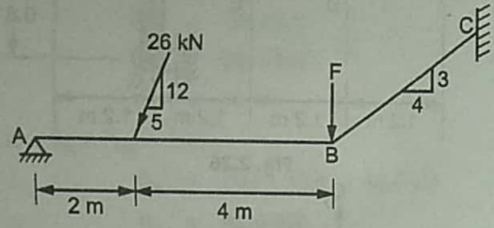


Fig. 2.27

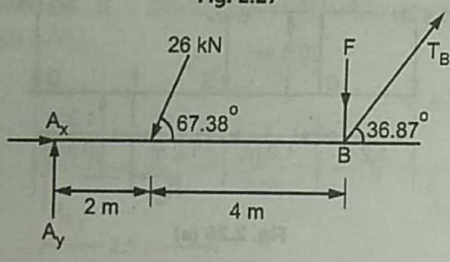


Fig. 2.27 (a)

Solution :

Given data : Loads acting on the beam AB are :

Point load 26 kN, maximum tension in rope BC, T_{BC} = 50 kN and force F.

To find : (a) Greatest vertical load F.

(b) Magnitude of reaction at A.

(a) Beam is in equilibrium. Refer Fig. 2.27 (a).

Applying conditions of equilibrium,

$$\Sigma M_A = 0$$

$$\therefore \quad 26 \times \sin 67.38^\circ \times 2 + F \times 6 - T_{BC} \sin 36.87^\circ \times 6 = 0$$

$$\therefore \quad F = \mathbf{22\ kN\ (\downarrow)} \quad \text{... Ans.}$$

$$\Sigma F_x = 0$$

$$\therefore \quad A_x - 26 \cos 67.38^\circ + T_{BC} \cos 36.87^\circ = 0$$

$$\therefore \quad A_x - 26 \cos 67.38^\circ + 50 \cos 36.87^\circ = 0$$

$$\therefore \quad A_x = -30 \text{ kN}$$

$$\therefore \quad A_x = 30 \text{ kN } (\leftarrow)$$

$$\Sigma F_y = 0$$

$$\therefore \quad A_y - 26 \sin 67.38^\circ - F + T_{BC} \sin 36.87^\circ = 0$$

$$\therefore \quad A_y = 16 \text{ kN}$$

Refer Fig. 2.27 (a).

$$\text{Reaction at A} = R_A$$

$$= \sqrt{A_x^2 + A_y^2}$$

$$= \sqrt{(30)^2 + (16)^2} = \mathbf{34\ kN} \quad \text{... Ans.}$$

$$\text{Direction of } R_A = \alpha$$

$$= \tan^{-1}\left(\frac{A_y}{A_x}\right) = \tan^{-1}\left(\frac{16}{30}\right) = \mathbf{28^\circ} \quad \text{... Ans.}$$

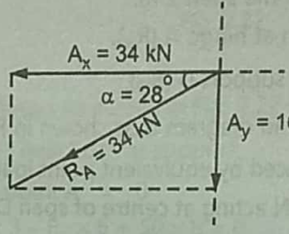


Fig. 2.27 (b)

B – SPACE FORCES

2.6 INTRODUCTION

Consider a force F passing from origin to point A in the space as shown in Fig. 2.28. The force F can be resolved into x, y, z directions.

$$\boxed{F_x = F \cos \theta_x}$$

$$\boxed{F_y = F \cos \theta_y}$$

$$\boxed{F_z = F \cos \theta_z}$$

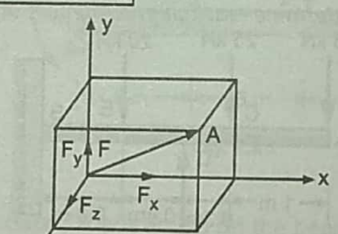


Fig. 2.28

where θ_x, θ_y and θ_z are the angles made by force F with x, y and z directions respectively. Hence, θ_x, θ_y and θ_z are known as the *direction cosines* of force F.

$$\boxed{\overline{F} = F_x i + F_y j + F_z k}$$

where i, j, k represent the unit vector along x, y, z directions respectively.

$$\therefore \quad F = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$F = \sqrt{(F \cos \theta_x)^2 + (F \cos \theta_y)^2 + (F \cos \theta_z)^2}$$

Squaring on both sides,

$$F^2 = F^2 \cos^2 \theta_x + F^2 \cos^2 \theta_y + F^2 \cos^2 \theta_z$$

$$F^2 = F^2 (\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z)$$

$$\therefore \quad 1 = \cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z$$

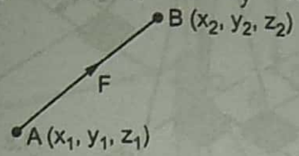


Fig. 2.29

(Provided θ_x, θ_y and θ_z are the angles with positive x, positive y and positive z axis).

Direction cosines can be calculated as,

$$\boxed{\cos \theta_x = \frac{F_x}{F}}, \quad \boxed{\cos \theta_y = \frac{F_y}{F}}, \quad \boxed{\cos \theta_z = \frac{F_z}{F}}$$

2.7 UNIT VECTOR

Consider a force F passing from point A (x_1, y_1, z_1) to B (x_2, y_2, z_2). Hence, unit vector along the direction of force is calculated as follows : [Ref. Fig. 2.29]

$$\bar{e}_{AB} = \frac{(x_2 - x_1)\, i + (y_2 - y_1)\, j + (z_2 - z_1)\, k}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}}$$

Hence, the above force can be written as,

$$\boxed{\bar{F} = F \times \bar{e}_{AB}}$$

Example 2.22 : *Determine the magnitude and direction of the force :*

$$\bar{F} = (320\ N)\, i + (400\ N)\, j - (250\ N)\, k.$$

Solution :

Given data : F = (320 N) i + (400 N) j – (250 N) k

To find : Magnitude and direction of force.

F_x = 320 N, F_y = 400 N, F_z = – 250 N

$$|F| = \sqrt{F_x^2 + F_y^2 + F_z^2}$$

$$= \sqrt{(320)^2 + (400)^2 + (-250)^2}$$

$$= \mathbf{570\ N} \qquad \text{... Ans.}$$

$$\bar{e} = \frac{\bar{F}}{F} = \frac{320\, i + 400\, j - 250\, k}{570}$$

$$= 0.56\, i + 0.70\, j - 0.44\, k$$

$$\theta_x = \cos^{-1} \left|\frac{F_x}{F}\right| = \cos^{-1} \left|\frac{320}{570}\right|$$

$$= \mathbf{55.85^\circ \text{ with +ve x-axis}} \qquad \text{... Ans.}$$

$$\theta_y = \cos^{-1} \left|\frac{400}{570}\right|$$

$$= \mathbf{45.57^\circ \text{ with +ve y-axis}} \qquad \text{... Ans.}$$

$$\theta_z = \cos^{-1} \left|\frac{250}{570}\right|$$

$$= \mathbf{63.9^\circ \text{ with –ve z-axis}} \qquad \text{... Ans.}$$

Example 2.23 : *A force acts at origin of a co-ordinate system in a direction defined by the angles θ_x = 70.9° and θ_y = 144.9°. Knowing that the z component of the force is –52 N, determine (a) the angle θ_z, (b) the other components and the magnitude of the force.*

Solution :

Given data : θ_x = 70.9°, θ_y = 144.9°, F_z = –52 N

To find : θ_z, F_x and F_y.

We have, $\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$

$$\cos^2 (70.9) + \cos^2 (144.9) + \cos^2 \theta_z = 1$$

$$\therefore \quad \theta_z = \mathbf{61.78 \text{ with –ve z-axis}} \qquad \text{... Ans.}$$

$$\text{or} \quad \theta_z = \mathbf{118.22 \text{ with +ve z-axis}} \qquad \text{... Ans.}$$

$$\cos \theta_z = \left|\frac{F_z}{F}\right|$$

$$\therefore \quad \cos 61.78 = \frac{52}{F}$$

$$\therefore \quad F = \mathbf{109.97\ N} \qquad \text{... Ans.}$$

$$F_x = F \cos \theta_x = 109.97 \cos 70.9$$

$$= \mathbf{35.98\ N} \qquad \text{... Ans.}$$

$$F_y = F \cos \theta_y = 109.97 \cos 144.9$$

$$= \mathbf{-89.97\ N} \qquad \text{... Ans.}$$

Example 2.24 : *A horizontal circular plate is suspended as shown in Fig. 2.30 from three wires which are attached to a support at D and form 30° angle with the vertical. Knowing that the z-component of the force exerted by wire BD on the plate is –32.14 N, determine : (a) the tension in wire BD, (b) the angles θ_x, θ_y and θ_z that the force exerted at B forms with the co-ordinate axes.*

Solution :

Given data : z-component of force on the plate is –32.14 N, as shown in Fig. 2.30.

To find : Tension in the wire BD, θ_x, θ_y and θ_z.

Let T be the tension in the wire BD.

Consider joint B,

$$F_h = T \sin 30 = 0.5\, T$$

$$F_z = - F_h \sin 40$$

$$-32.14 = - F_h \sin 40$$

$$\therefore \quad F_h = 50\ N$$

$$\therefore \quad F = T = 2F_h = 2 \times 50 = \mathbf{100\ N} \qquad \text{... Ans.}$$

$$F_x = -F_h \cos 40$$

$$= -50 \times \cos 40 = -38.30\ N$$

$$F_y = T \cos 30$$

$$= 100 \cos 30 = 86.60\ N$$

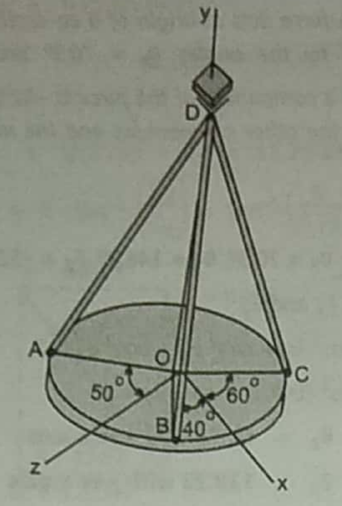


Fig. 2.30

$$\therefore \quad \theta_x = \cos^{-1}\left|\frac{F_x}{F}\right| = \cos^{-1}\left|\frac{38.30}{100}\right|$$

$$= \textbf{67.48° with –ve x-axis} \quad \textbf{... Ans.}$$

$$\theta_y = \cos^{-1}\left|\frac{F_y}{F}\right| = \cos^{-1}\left|\frac{86.60}{100}\right|$$

$$= \textbf{30° with +ve y-axis} \quad \textbf{... Ans.}$$

$$\theta_z = \cos^{-1}\left|\frac{F_z}{F}\right| = \cos^{-1}\left|\frac{32.14}{100}\right|$$

$$= \textbf{71.25° with –ve z-axis} \quad \textbf{... Ans.}$$

Example 2.25 : *A transmission tower is held by three guy wires anchored by bolts at B, C and D. If the tension in the wire AB is 2100 N, determine the components of the force exerted by the wire on the bolt at B.*

Solution :

Given data : T_{AB} = 2100 N, as shown in Fig. 2.31.

To find : Components of the force on the bolt.

Co-ordinates : A (0, 20, 0), B (–4, 0, 5)

$$\bar{e}_{BA} = \frac{\overline{BA}}{|AB|} = \frac{\bar{A}-\bar{B}}{|AB|}$$

$$= \frac{[0-(-4)]\,i + (20-0)\,j + (0-5)\,k}{\sqrt{(4)^2 + (20)^2 + (-5)^2}}$$

$$= \frac{4\,i + 20\,j - 5\,k}{21}$$

$$\bar{T}_{BA} = T_{BA} \times \bar{e}_{BA}$$

$$= 2100 \times \left(\frac{4\,i + 20\,j - 5\,k}{21}\right)$$

$$= 400\,i + 2000\,j - 500\,k$$

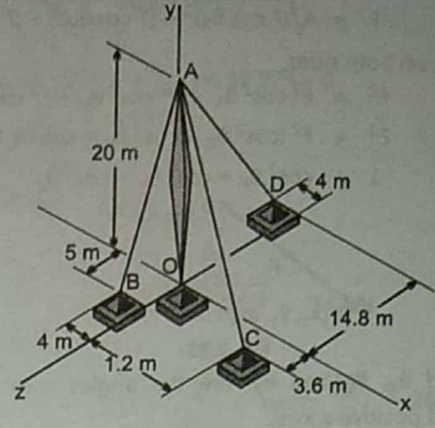


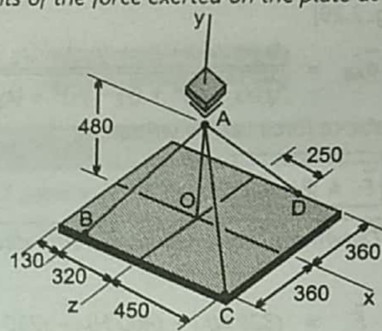
Fig. 2.31

$$\therefore \quad F_x = \textbf{400 N} \quad \textbf{... Ans.}$$

$$F_y = \textbf{2000 N} \quad \textbf{... Ans.}$$

$$F_z = \textbf{–500 N} \quad \textbf{... Ans.}$$

Example 2.26 : *A rectangular plate is supported by three cables as shown. Knowing that tension in the cable AB is 408 N, determine the components of the force exerted on the plate at B.*



Dimensions in mm

Fig. 2.32

Solution :

Given data : T_{AB} = 408 N, as shown in Fig. 2.32.

To find : Components of force.

Co-ordinates : A (0, 480, 0), B (–320, 0, 360)

Let $\bar{e}$ be the unit vector.

$$\therefore \quad \bar{e}_{BA} = \frac{\overline{BA}}{|AB|} = \frac{\bar{A}-\bar{B}}{|AB|}$$

$$\bar{e}_{BA} = \frac{[0-(-320)]\,i + (480-0)\,j + (0-360)\,k}{\sqrt{(320)^2 + (480)^2 + (-360)^2}}$$

$$\bar{e}_{BA} = \frac{320\,i + 480\,j - 360\,k}{680}$$

$$\therefore \quad \bar{T}_{BA} = T_{BA} \times \bar{e}_{BA} = 408\left(\frac{320\,i + 480\,j - 360\,k}{680}\right)$$

$$= 192\,i + 288\,j - 216\,k$$

$$\therefore \quad F_x = \textbf{192 N} \quad \textbf{... Ans.}$$

$$F_y = \textbf{288 N} \quad \textbf{... Ans.}$$

$$F_z = \textbf{–216 N} \quad \textbf{... Ans.}$$

Example 2.27 : The end of the co-axial cable AE is attached to the pole AB, which is strengthen by the guy wires AC and AD. Knowing that the tension in the wire AC is 120 N, determine (a) the components of the force exerted by the wire on the pole, (b) the angles θ_x, θ_y and θ_z that the force forms with the co-ordinate axis.

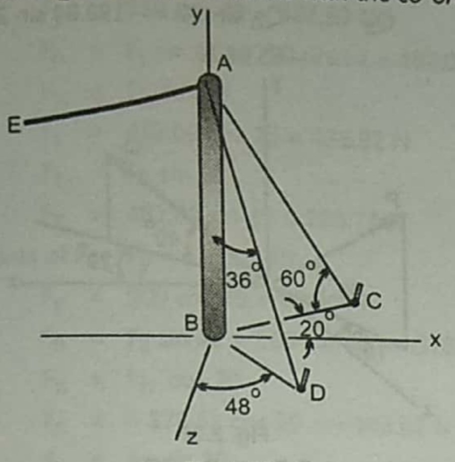


Fig. 2.33

Solution :

Given data : As shown in Fig. 2.33, T_{AC} = 120 N.

To find : F_x, F_y, F_z and θ_x, θ_y, θ_z.

$$F_h = 120 \sin 36 = 70.53 \text{ N}$$

$$F_y = -120 \cos 36 = \mathbf{-97.08 \text{ N}} \quad \text{... Ans.}$$

$$F_x = F_h \sin 48 = 70.53 \sin 48$$

$$= \mathbf{52.41 \text{ N}} \quad \text{... Ans.}$$

$$F_z = F_h \cos 48 = 70.53 \cos 48$$

$$= \mathbf{47.19 \text{ N}} \quad \text{... Ans.}$$

$$\cos \theta_x = \left|\frac{F_x}{F}\right| = \left|\frac{52.41}{120}\right|$$

$$= \mathbf{64.10° \text{ with +ve x-axis}} \quad \text{... Ans.}$$

$$\cos \theta_y = \left|\frac{F_y}{F}\right| = \left|\frac{97.08}{120}\right|$$

$$= \mathbf{36° \text{ with -ve y-axis.}} \quad \text{... Ans.}$$

$$\cos \theta_z = \left|\frac{F_z}{F}\right| = \left|\frac{47.19}{120}\right|$$

$$= \mathbf{68.84° \text{ with +ve z-axis.}} \quad \text{... Ans.}$$

Example 2.28 : Determine (a) the x, y and z components of the 450 N force, (b) the angles θ_x, θ_y and θ_z that the force forms with the co-ordinate axes.

Solution :

Given data : F = 450 N as shown in Fig. 2.34.

To find : F_x, F_y, F_z and θ_x, θ_y and θ_z.

$$F_h = F \cos 35 = 450 \cos 35 = 368.618 \text{ N}$$

$$F_h = F \cos 35 = 450 \cos 35 = 368.618 \text{ N}$$

$$F_y = F \sin 35 = 450 \sin 35$$

$$= \mathbf{258.11 \text{ N}} \quad \text{... Ans.}$$

$$F_x = -F_h \sin 40 = -368.618 \sin 40$$

$$= \mathbf{-236.61 \text{ N}} \quad \text{... Ans.}$$

$$F_z = F_h \cos 40 = 368.618 \cos 40$$

$$= \mathbf{282 \text{ N}} \quad \text{... Ans.}$$

$$\cos \theta_x = \left|\frac{F_x}{F}\right| = \left|\frac{236.61}{450}\right|$$

$$= \mathbf{58.29° \text{ with -ve x-axis}} \quad \text{... Ans.}$$

$$\cos \theta_y = \left|\frac{F_y}{F}\right| = \left|\frac{258.11}{450}\right|$$

$$= \mathbf{55° \text{ with +ve y-axis}} \quad \text{... Ans.}$$

$$\cos \theta_z = \left|\frac{F_z}{F}\right| = \left|\frac{282}{450}\right|$$

$$= \mathbf{51.2° \text{ with +ve z-axis}} \quad \text{... Ans.}$$

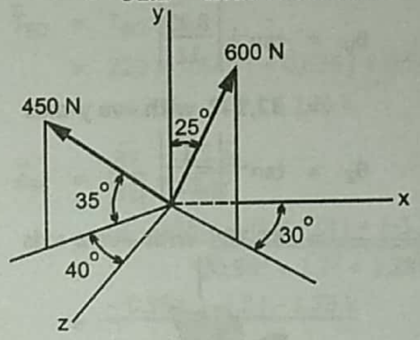


Fig. 2.34

2.8 RESULTANT IN SPACE FORCE SYSTEM

Let $\bar{F}_1$, $\bar{F}_2$, $\bar{F}_3$, ... be the forces in space.

Then,
$$\Sigma F_x = R_x$$
$$\Sigma F_y = R_y$$
$$\Sigma F_z = R_z$$
$$\bar{R} = R_x\, i + R_y\, j + R_z\, k$$
$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$
$$\cos \theta_x = \left|\frac{R_x}{R}\right|, \quad \cos \theta_y = \left|\frac{R_y}{R}\right|,$$
$$\cos \theta_z = \left|\frac{R_z}{R}\right|$$

where θ_x, θ_y and θ_z are the angles made by the resultant with x, y and z axes respectively.

Example 2.29 : To stabilize a tree partially uprooted in a storm, cables AB and AC are attached to the upper trunk of the tree and then are fastened to steel rods anchored in the ground. Knowing that tension in AB is 4.1 kN and that the resultant of forces exerted at A by cables AB and AC lies in the yz plane, determine (a) the tension in AC, (b) the magnitude and direction of the resultant of the two forces.

Solution :

Given data : Force in AB 4.1 kN and as shown in Fig. 2.35.

To find : Tension in AC and resultant.

As resultant lies in yz plane,

$$\therefore \quad \Sigma F_x = 0$$

$$-F_{AC} \cos 45 \sin 25 + 4.1 \cos 40 \cos 40 = 0$$

$$\therefore \quad F_{AC} = \mathbf{8.05\ kN} \quad \text{... Ans.}$$

$$\Sigma F_y = -F_{AC} \sin 45 - F_{AB} \sin 40$$

$$= -8.05 \sin 45 - 4.1 \sin 40 = -8.33\ \text{kN}$$

$$\Sigma F_z = F_{AC} \cos 45 \cos 25 + 4.1 \cos 40 \sin 40$$

$$= 8.05 \cos 45 \cos 25 + 4.1 \cos 40 \sin 40$$

$$= 7.17\ \text{kN}$$

$$\therefore \quad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

$$= \sqrt{(-8.33)^2 + (7.17)^2} = \mathbf{11\ kN} \quad \text{... Ans.}$$

$$\therefore \quad \theta_y = \tan^{-1} \left| \frac{8.33}{11} \right|$$

$$= \mathbf{37.14^\circ \text{ with -ve y-axis}} \quad \text{... Ans.}$$

$$\theta_z = \tan^{-1} \left| \frac{7.17}{11} \right|$$

$$= \mathbf{33.10^\circ \text{ with +ve z-axis}} \quad \text{... Ans.}$$

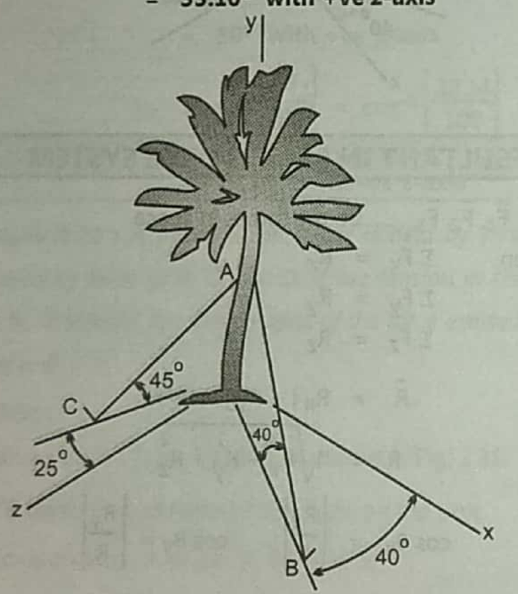


Fig. 2.35

Example 2.30 : *Find the magnitude and direction of the resultant of the two forces shown, knowing that P = 400 N and Q = 300 N.*

Solution :

Given data : P = 400 N, Q = 300 N, as shown in Fig. 2.36.

To find : Magnitude and direction of resultant.

Components of force P :

$$P_y = P \sin 30 = 400 \sin 30 = 200\ \text{N}$$

$$P_h = P \cos 30 = 400 \cos 30 = 346.41\ \text{N}$$

$$P_x = -P_h \sin 15 = -346.41 \sin 15 = -89.66\ \text{N}$$

$$P_z = P_h \cos 15 = 346.41 \cos 15 = 334.61\ \text{N}$$

Components of force Q :

$$Q_y = Q \sin 50 = 300 \sin 50 = 229.81\ \text{N}$$

$$Q_h = Q \cos 50 = 300 \cos 50 = 192.84\ \text{N}$$

$$Q_x = Q_h \cos 20 = 192.84 \cos 20$$

$$= 181.21\ \text{N}$$

$$Q_z = -Q_h \sin 20 = -192.84 \sin 20$$

$$= -65.96\ \text{N}$$

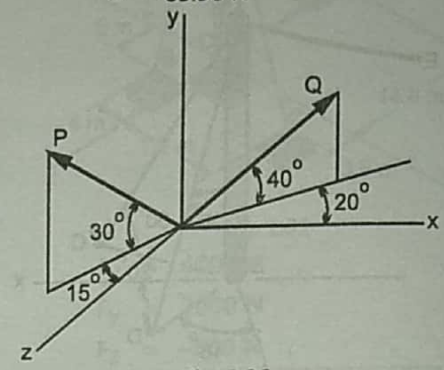


Fig. 2.36

$$R_x = \Sigma F_x = P_x + Q_x$$

$$= -89.66 + 181.21 = 91.55\ \text{N}$$

$$R_y = \Sigma F_y = P_y + Q_y$$

$$= 200 + 229.81 = 429.81\ \text{N}$$

$$R_z = \Sigma F_z = P_z + Q_z$$

$$= 334.61 - 65.96 = 268.65\ \text{N}$$

$$\therefore \quad R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

$$= \mathbf{515.06\ N} \quad \text{... Ans.}$$

$$\therefore \quad \theta_x = \cos^{-1} \left| \frac{R_x}{R} \right| = \cos^{-1} \left(\frac{91.55}{515.06} \right)$$

$$= \mathbf{79.76^\circ \text{ with +ve x-axis}} \quad \text{... Ans.}$$

$$\theta_y = \cos^{-1} \left| \frac{R_y}{R} \right| = \cos^{-1} \left(\frac{429.81}{515.06} \right)$$

$$= \mathbf{33.44^\circ \text{ with +ve y-axis}} \quad \text{... Ans.}$$

$$\theta_z = \cos^{-1} \left| \frac{R_z}{R} \right| = \cos^{-1} \left(\frac{268.65}{515.06} \right)$$

$$= \mathbf{58.56^\circ \text{ with +ve z-axis}} \quad \text{... Ans.}$$

Example 2.31 : *Find the resultant of the force system shown in Fig. 2.37.*

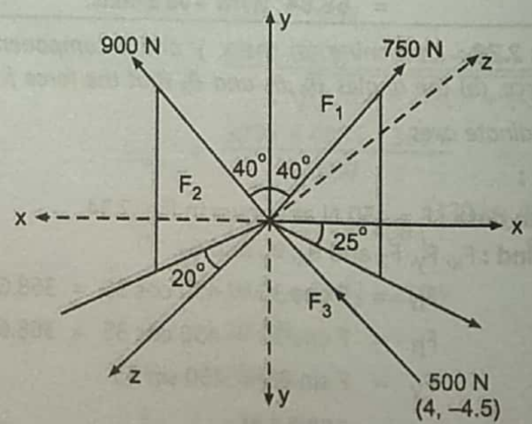


Fig. 2.37

Solution :

Given data : As shown in Fig. 2.37.

To find : Resultant.

Components of F_1 : $F_y = F_1 \cos 40$

$$F_y = 750 \cos 40 = 574.53 \text{ N}$$
$$F_h = F_1 \sin 40 = 750 \sin 40 = 482.09 \text{ N}$$
$$F_x = F_h \cos 25$$
$$F_x = 482.09 \cos 25 = 436.92 \text{ N}$$
$$F_z = F_h \sin 25$$
$$F_z = 482.09 \sin 25 = 203.74 \text{ N}$$

Components of F_2 : $F_y = F_2 \cos 40$

$$F_y = 900 \cos 40 = 689.44 \text{ N}$$
$$F_h = F_2 \sin 40 = 900 \sin 40 = 578.51$$
$$F_x = F_h \cos 20$$
$$F_x = -578.51 \cos 20 = -543.62 \text{ N}$$
$$F_z = F_h \sin 20$$
$$F_z = 578.51 \sin 20 = 197.86 \text{ N}$$

Components of F_3 : $\bar{F} = 500 \times \bar{e}$

$$\bar{e} = \frac{(0-4)\,i + (0-(-4))\,j + (0-5)\,k}{\sqrt{4^2 + 4^2 + 5^2}}$$
$$= \frac{500 \times (-4i + 4j - 5k)}{\sqrt{4^2 + 4^2 + 5^2}}$$
$$F_x = -264.91 \text{ N}$$
$$F_y = 264.91 \text{ N}$$
$$F_z = -331.13 \text{ N}$$
$$\sum F_x = 436.92 - 543.62 - 264.91 = -371.61 \text{ N}$$
$$\sum F_y = 574.53 + 689.44 + 264.91 = 1528.88 \text{ N}$$
$$\sum F_z = 203.74 + 197.86 - 331.13 = 70.47 \text{ N}$$

$\therefore$ $\bar{F} = -371.61\ i + 1528.88\ j + 70.47\ k$ **... Ans.**

Example 2.32 : *A steel rod is bent into a semicircular ring of radius 0.96 m and is supported in part by cables BD and DE which are attached to the ring at B. Knowing that the tension in the cable BD is 220 N and BE is 250 N, determine the resultant of this force exerted by the cable on the point B.*

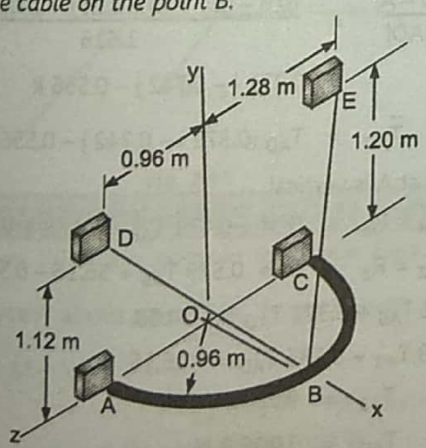


Fig. 2.38

Solution :

Given data : Tension in BD = 220 N and tension in DE = 250 N.

To find : Resultant at point B.

Co-ordinates : O(0, 0, 0), A(0, 0, 0.96), B(0.96, 0, 0), C(0, 0, –0.96), D(1, 0.12, 0.96), E(0, 0.12, –1.28).

$$\bar{e}_{BD} = \frac{\overline{BD}}{|BD|}$$
$$= \frac{\bar{D} - \bar{B}}{|BD|} = \frac{(0-0.96)\,i + (1.12-0)\,j + (0.96-0)\,k}{\sqrt{0.96^2 + 1.12^2 + 0.96^2}}$$
$$= \frac{-0.96\,i + 1.12\,j + 0.96\,k}{1.76}$$
$$= -0.545\,i + 0.636\,j + 0.545\,k$$
$$\bar{T}_{BD} = T_{BD} \times \bar{e}_{BD}$$
$$= 220\,(-0.545\,i + 0.636\,j + 0.545\,k)$$
$$= -120\,i + 140\,j + 120\,k$$
$$\bar{e}_{BE} = \frac{\overline{BE}}{|BE|} = \frac{\bar{E} - \bar{B}}{|BE|}$$
$$= \frac{(0-0.96)\,i + (1.2-0)\,j + (-1.28-0)\,k}{\sqrt{0.96^2 + 1.2^2 + 1.28^2}}$$
$$= \frac{-0.96\,i + 1.2\,j - 1.28\,k}{2}$$
$$= -0.48\,i + 0.6\,j - 0.64\,k$$
$$\bar{T}_{BE} = T_{BE} \times \bar{e}_{BE} = 250 \times (-0.48\,i + 0.6\,j - 0.64\,k)$$
$$= -120\,i + 150\,j - 160\,k$$
$$\sum F_x = R_x = -120 - 120 = -240 \text{ N}$$
$$\sum F_y = R_y = 140 + 150 = 290 \text{ N}$$
$$\sum F_z = R_z = 120 - 160 = -40 \text{ N}$$
$$\therefore \quad R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$
$$= \sqrt{(-240)^2 + (290)^2 + (-40)^2}$$
$$= \mathbf{378.55 \text{ N}}$$ **... Ans.**

$$\therefore \quad \theta_x = \cos^{-1}\left|\frac{R_x}{R}\right|$$
$$= \cos^{-1}\left|\frac{240}{378.55}\right| = 50.65 \text{ with –ve x-axis}$$
$$\theta_y = \cos^{-1}\left|\frac{R_y}{R}\right| = \cos^{-1}\left|\frac{290}{378.55}\right|$$
$$= 40° \text{ with +ve y-axis}$$
$$\theta_z = \cos^{-1}\left|\frac{R_z}{R}\right| = \cos^{-1}\left|\frac{40}{378.55}\right|$$
$$= 83.93° \text{ with –ve z-axis}$$

$\therefore$ $\bar{R} = \mathbf{-240\,i + 290\,j - 40\,k}$ **... Ans.**

Example 2.33 : *Knowing that the tension is 850 N in cable AB and 1020 N in cable AC, determine the magnitude and direction of resultant of the forces exerted at A by the two cables.*

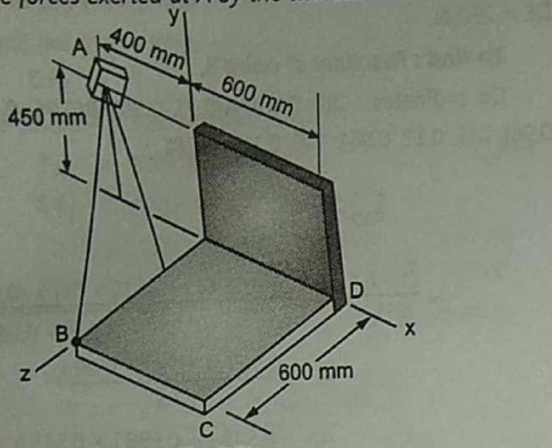


Fig. 2.39

Solution :

Given : Tension in AB = 850 N, tension in AC = 1020 N.

To find : Resultant at A.

O(0, 0, 0), A(–400, 450, 0), B(0, 0, 600), C(600, 0, 600).

$$\bar{e}_{AB} = \frac{\bar{AB}}{|AB|} = \frac{\bar{B}-\bar{A}}{|AB|} = \frac{0-(-400)\,i + (0-450)\,j + (600-0)\,k}{\sqrt{400^2 + 450^2 + 600^2}}$$

$$= \frac{400\,i - 450\,j + 600\,k}{850} = 0.471\,i - 0.529\,j + 0.706\,k$$

$$\bar{T}_{AB} = T_{AB} \times \bar{e}_{AB} = 850\,(0.471\,i - 0.529\,j + 0.706\,k)$$

$$= 400.35\,i - 449.65\,j + 600.1\,k$$

$$\bar{e}_{AC} = \frac{\bar{AC}}{|AC|} = \frac{\bar{C}-\bar{A}}{|AC|}$$

$$= \frac{600-(-400)\,i + (0-450)\,j + (600-0)\,k}{\sqrt{1000^2 + 450^2 + 600^2}}$$

$$= \frac{1000\,i - 450\,j + 600\,k}{1250}$$

$$= 0.8\,i - 0.36\,j + 0.48\,k$$

$$\bar{T}_{AC} = T_{AC} \times \bar{e}_{AC} = 1020\,(0.8\,i - 0.36\,j + 0.48\,k)$$

$$= 816\,i - 367.2\,j + 489.6\,k$$

$$\Sigma F_x = R_x = 400.35 + 816 = 1216.35\text{ N}$$

$$\Sigma F_y = R_y = -449.65 - 367.2 = -816.85\text{ N}$$

$$\Sigma F_z = R_z = 600.1 + 489.6 = 1089.7\text{ N}$$

$$\therefore \quad R = \sqrt{1216.35^2 + (-816.85)^2 + (1089.7)^2}$$

$$= 1825.98\text{ N}$$

$$\theta_x = \cos^{-1}\left|\frac{1216.35}{1825.98}\right|$$

$= \mathbf{48.23° \text{ with +ve x-axis}}$ ... **Ans.**

$$\theta_y = \cos^{-1}\left|\frac{816.85}{1825.98}\right|$$

$= \mathbf{63.43° \text{ with –ve y-axis}}$ ... **Ans.**

$$\theta_z = \cos^{-1}\left|\frac{1089.7}{1825.98}\right|$$

$= \mathbf{53.36° \text{ with +ve z-axis}}$ ... **Ans.**

Example 2.34 : *Determine the tension in cables AB and AD knowing that the tension in cable AC is 120 N and that the resultant of the forces exerted by the three cables at A must be vertical.*

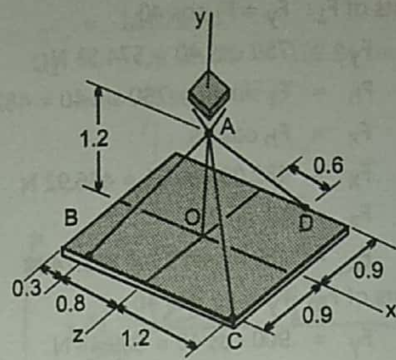


Fig. 2.40

Solution :

Given data : A(0, 1.2, 0), B(–0.8, 0, 0.9), C(1.2, 0, 0.9), D(0.6, 0, –0.9).

To find : Tension in cable AB, tension in cable AD.

$$\bar{e}_{AB} = \frac{\bar{AB}}{|AB|} = \frac{\bar{B}-\bar{A}}{|AB|}$$

$$= \frac{(-0.8-0)\,i + (0-1.2)\,j + (0.9-0)\,k}{1.7}$$

$$= -0.471\,i - 0.706\,j + 0.529\,k$$

$$\bar{T}_{AB} = T_{AB}\,(-0.471\,i - 0.706\,j + 0.529\,k)$$

$$\bar{e}_{AC} = \frac{\bar{AC}}{|AC|} = \frac{\bar{C}-\bar{A}}{|AC|}$$

$$= \frac{(1.2-0)\,i + (0-1.2)\,j + (0.9-0)\,k}{1.921}$$

$$\bar{e}_{AC} = 0.624\,i - 0.624\,j + 0.468\,k$$

$$\bar{T}_{AC} = 120\,(0.624\,i - 0.624\,j + 0.468\,k)$$

$$= 74.88\,i - 74.88\,j + 56.16\,k$$

$$\bar{e}_{AD} = \frac{\bar{AD}}{|AD|}$$

$$\frac{\bar{D}-\bar{A}}{|AD|} = \frac{(0.6-0)\,i + (0-1.2)\,j + (-0.9-0)\,k}{1.616}$$

$$= 0.371\,i - 0.742\,j - 0.556\,k$$

$$\bar{T}_{AD} = T_{AD}\,(0.371\,i - 0.742\,j - 0.556\,k)$$

Resultant at A is vertical.

$$\therefore \quad \Sigma f_x = R_x = 0 = -0.471\,T_{AB} + 74.88 + 0.371\,T_{AD}$$

$$\Sigma f_z = R_z = 0 = 0.529\,T_{AB} + 56.16 - 0.556\,T_{AD}$$

$$\therefore -0.471\,T_{AB} + 0.371\,T_{AD} = -74.88$$

$$0.529\,T_{AB} + 0.556\,T_{AD} = -56.16$$

$\therefore \quad T_{AB} = \mathbf{952\ N}$ ... **Ans.**

$T_{AD} = \mathbf{1006.8\ N}$ ... **Ans.**

2.9 DOT PRODUCT OR SCALAR PRODUCT

The dot product or scalar product of two vectors $\bar{A}$ and $\bar{B}$ is defined as -

(1) If
$$\bar{A} = A_x i + A_y j + A_z k$$
$$\bar{B} = B_x i + B_y j + B_z k$$

then
$$\bar{A}\cdot\bar{B} = (A_x B_x) + (A_y B_y) + (A_z B_z) \quad \text{... (scalar)}$$

(2)
$$\bar{A}\cdot\bar{B} = AB\cos\theta \quad \text{... (scalar)}$$

2.10 CROSS PRODUCT OR VECTOR PRODUCT

$$\bar{r}\times\bar{F} = (r\sin\theta\cdot F)\,\bar{e}$$

where, $\bar{e}$ = unit vector at right angles to the plane of $\bar{F}$ and $\bar{r}$

$$\bar{r}\times\bar{F} = \begin{vmatrix} i & j & k \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} \qquad \left\{\begin{matrix} i\times i = 0 \\ j\times j = 0 \\ k\times k = 0 \end{matrix}\right\}$$

2.11 (A) MOMENT OF FORCE ABOUT ORIGIN

Consider a force $\bar{P}$ passing through two points A (x_1, y_1, z_1) and B (x_2, y_2, z_2) in space. The moment of the force $\bar{P}$ about point O (origin) is given as :

$$M_O = \bar{r}\times\bar{P} \text{ (cross product of force and distance)}$$
$$M_O = (x_1 i + y_1 j + z_1 k)\times(P_x i + P_y j + P_z k)$$

or
$$M_O = (x_2 i + y_2 j + z_2 k)\times(P_x i + P_y j + P_z k)$$

$$M_O = \begin{vmatrix} i & j & k \\ x_1 & y_1 & z_1 \\ P_x & P_y & P_z \end{vmatrix} \text{ or } \begin{vmatrix} i & j & k \\ x_2 & y_2 & z_2 \\ P_x & P_y & P_z \end{vmatrix}$$

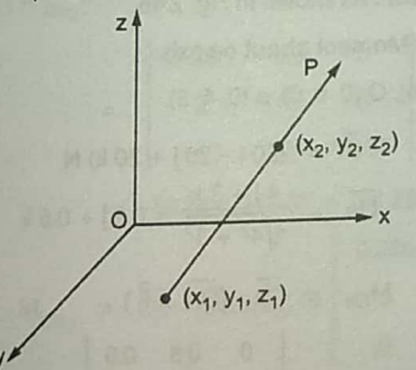


Fig. 2.41

2.11 (B) MOMENT OF FORCE ABOUT ANY POINT

Consider C (x_3, y_3, z_3) as any point in the space. Hence, moment of the above force $\bar{P}$ about point C (x_3, y_3, z_3) is given as,

$$M_C = \bar{r}\times\bar{P}$$
$$= (P_x i + P_y j + P_z k)\times[(x_1 - x_3)\,i + (y_1 - y_3)\,j + (z_1 - z_3)\,k]$$

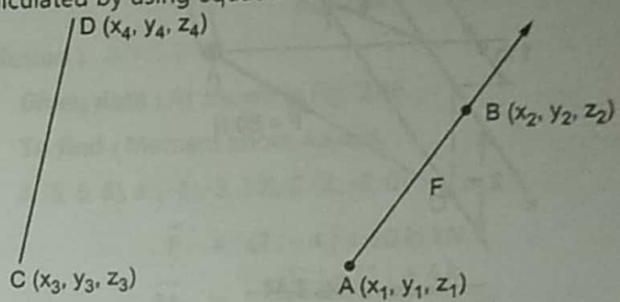
or
$$= (P_x i + P_y j + P_z k)\times[(x_2 - x_3)\,i + (y_2 - y_3)\,j + (z_2 - z_3)\,k]$$

$$M_C = \begin{vmatrix} i & j & k \\ (x_1 - x_3) & (y_1 - y_3) & (z_1 - z_3) \\ P_x & P_y & P_z \end{vmatrix}$$

or
$$\begin{vmatrix} i & j & k \\ (x_2 - x_3) & (y_2 - y_3) & (z_2 - z_3) \\ P_x & P_y & P_z \end{vmatrix}$$

2.11 (C) MOMENT OF FORCE ABOUT A LINE

Let $\bar{F}$ be the force along line AB. The moment of $\bar{F}$ along CD can be calculated by using equation of line CD.



(a) (b)

Fig. 2.42

$$\bar{F} = F\times\bar{e}_{AB}$$
$$\overline{CD} = (x_4 - x_3)\,i + (y_4 - y_3)\,j + (z_4 - z_3)\,k$$

As $\overline{CD}$ is taken, so take moment of $\bar{F}$ about point 'C'.

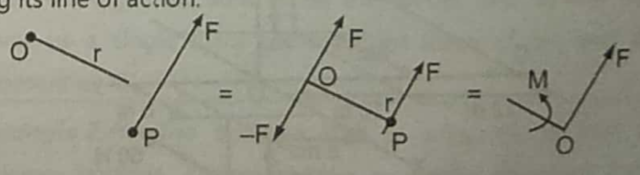
$$\therefore \quad \bar{M}_C = \overline{CA}\times\bar{F}$$

Unit vectors in matrices (i.e. i, j, k) are replaced by the co-ordinates of the line $\overline{CD}$.

$$\therefore \quad \bar{M}_{CD} = \begin{vmatrix} (x_4 - x_3) & (y_4 - y_3) & (z_4 - z_3) \\ (x_1 - x_3) & (y_1 - y_3) & (z_1 - z_3) \\ F_x & F_y & F_z \end{vmatrix}$$

2.12 EQUIVALENT FORCE SYSTEM

Resolution of a Given Force into a Force and Couple at other Point : Consider a force F acting on a rigid body at some point P. Let $\bar{r}$ be the position vector of point P from 'O'. If we have to find the force at 'O', we cannot move directly this force to 'O'. Because according to principle of transmissibility, we can move the force along its line of action.



(a) (b) (c)

Fig. 2.43

To find equivalent force system at 'O', we have to apply equal and opposite force at 'O'. Then negative force at 'O' and force at P forms a couple which is equal to cross product of position vector and force. Thus the equivalent force at 'O' due to force F at P is as shown in Fig. 2.43 (c).

Example 2.35 : *The curved rod lies in the x-y plane and has radius of curvature 3 m. If a force of F = 80 N acts at end as shown, determine the moment of this force about point B.*

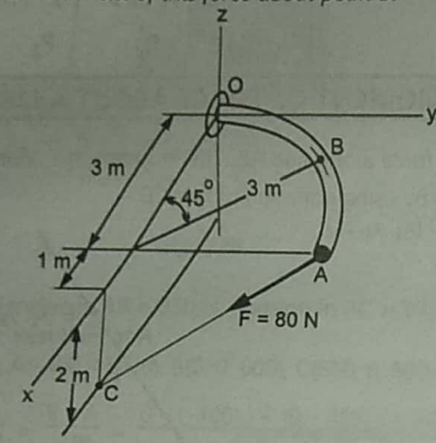


Fig. 2.44

Solution :

Given data : F = 80 N, r = 3 m as shown in Fig. 2.44.

To find : Moment about B.

A (3, 3, 0), B (0.88, 2.12, 0), C (4, 0, −2).

$$\therefore \quad \bar{F} = F\left(\frac{\bar{C}-\bar{A}}{CA}\right)$$

$$= 80\left(\frac{i-3j-2k}{\sqrt{1^2+3^2+2^2}}\right)$$

$$= 21.38\,i - 64.14\,j - 42.76\,k$$

$$\bar{M}_B = \overline{BA} \times \bar{F} = \begin{vmatrix} i & j & k \\ 2.12 & 0.88 & 0 \\ 21.38 & -64.14 & -42.76 \end{vmatrix}$$

$$= -37.63\,i + 90.65\,j - 154.79\,k$$

$\therefore$ Moment about B =

(− 37.63 i + 90.65 j − 154.79 k) N-m ... **Ans.**

Example 2.36 : *Determine the couple moment for the loading shown in Fig. 2.45.*

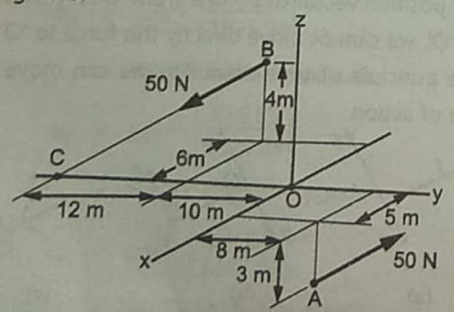


Fig. 2.45

Solution :

Given data : As shown in Fig. 2.45.

To find : Couple moment.

A (5, 8, −3), B (−6, −10, 4), C (0, −22, 0)

$$\therefore \quad \bar{F} = F\left(\frac{\bar{C}-\bar{B}}{|CB|}\right)$$

$$= 50\left(\frac{6i-12j-4k}{\sqrt{6^2+12^2+4^2}}\right)$$

$$= 21.42\,i - 42.84\,j - 14.28\,k$$

$$\text{Couple moment} = \overline{AC} \times \bar{F}$$

$$= \begin{vmatrix} i & j & k \\ -5 & -30 & 3 \\ 21.42 & -42.84 & -14.28 \end{vmatrix}$$

$$= 556.92\,i - 7.14\,j + 856.8\,k$$

$\therefore$ Couple moment = 556.92 i − 7.14 j + 856.8 k ... **Ans.**

Magnitude = **1021.92 Nm** ... **Ans.**

Example 2.37 : *Determine the moment of the force F about the oa axis. Express the result in cartesian co-ordinate.*

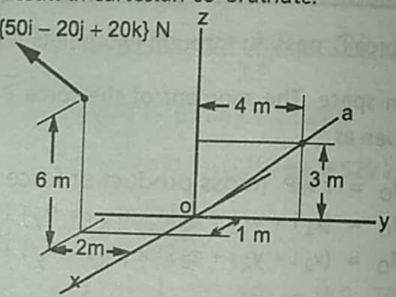


Fig. 2.46

Solution :

Given data : As shown in Fig. 2.46.

To find : Moment about oa axis.

A (1, −2, 6), O (0, 0, 0), a (0, 4, 3).

$$\bar{F} = (50\,i - 20\,j + 20\,k)\ \text{N}$$

$$\overline{Oa} = \frac{4j+3k}{\sqrt{4^2+3^2}} = 0.8\,j + 0.6\,k$$

$$M_{Oa} = \overline{Oa} \cdot (\overline{OA} \times \bar{F})$$

$$= \begin{vmatrix} 0 & 0.8 & 0.6 \\ 1 & -2 & 6 \\ 50 & -20 & 20 \end{vmatrix}$$

$$= (224 + 48) = 272\ \text{Nm}$$

In cartesian co-ordinates,

$$\bar{M}_{Oa} = 272\,(0.8\,j + 0.6\,k)$$

$$= \mathbf{(217.6\,j + 163.2\,k)\ Nm}$$ ... **Ans.**

Example 2.38 : *Determine the resultant moment of two forces about the oa axis.*

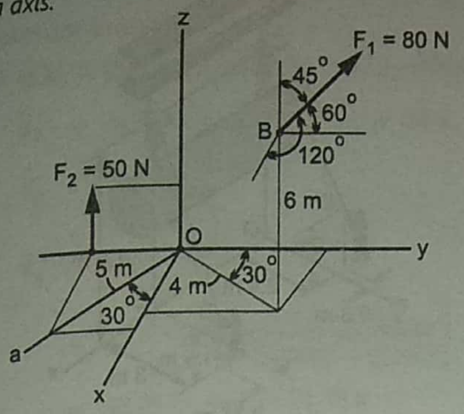


Fig. 2.47

Solution :

Given data : As shown in Fig. 2.47.

To find : Moment about oa axis.

O (0, 0, 0), a (4.33, –2.5, 0), B (2, 3.46, 6),

C (0, –5, 0).

$$\bar{F}_1 = 80 (\cos 120\, i + \cos 60\, j + \cos 45\, k)$$

$$= (-40\, i + 40\, j + 56.57\, k)\ N$$

$$\bar{F}_2 = (50\, k)\ N$$

$$\overline{Oa} = \left(\frac{\bar{a} - \bar{O}}{|Oa|}\right)$$

$$= \frac{4.33\, i - 2.5\, j}{\sqrt{(4.33)^2 + (2.5)^2}}$$

$$= 0.866\, i - 0.5\, j$$

Moment about oa axis,

$$M_{oa_1} = \overline{Oa} \cdot (\overline{OB} \times \bar{F}_1)$$

$$= \begin{vmatrix} 0.866 & -0.5 & 0 \\ 2 & 3.46 & 6 \\ -40 & 40 & 56.57 \end{vmatrix}$$

$$= -38.34 + 176.57 = 138.23\ \text{N-m}$$

$$M_{oa_2} = \overline{Oa} \cdot (\overline{OB} \times \bar{F}_2) = \begin{vmatrix} 0.866 & -0.5 & 0 \\ 0 & -2.5 & 0 \\ 0 & 0 & 50 \end{vmatrix}$$

$$= -108.25$$

$$M_{oa} = M_{oa_1} + M_{oa_2} = 138.23 - 108.25$$

$$= 29.98\ \text{N-m}$$

cartesian co-ordinates,

$$\bar{M}_{oa} = 29.98 (0.866\, i - 0.5\, j)$$

$$= \mathbf{25.96\, i - 14.49\, j\ Nm}$$ **... Ans.**

Example 2.39 : *Determine the moment of the force $\bar{F}$ about Aa axis. Express the result as a cartesian vector.*

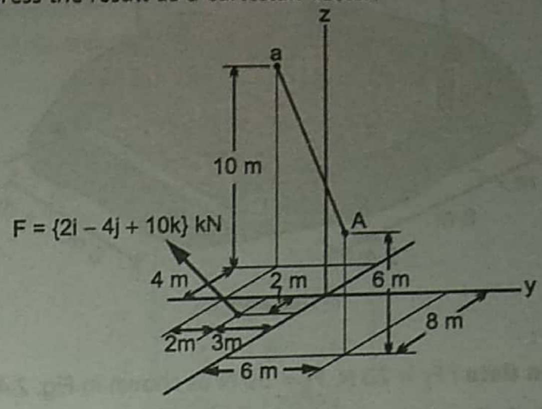


Fig. 2.48

Solution :

Given data : As shown in Fig. 2.48.

To find : Moment about Aa axis.

A (8, 6, 6), a (–4, –5, 10), C (2, –3, 0)

$$\bar{F} = (2\, i - 4\, j + 10\, k)\ kN$$

$$\overline{Aa} = \frac{-12\, i - 11\, j + 4\, k}{\sqrt{(12)^2 + (11)^2 + (4)^2}}$$

$$= -0.716\, i - 0.656\, j + 0.239\, k$$

$$\bar{M}_{Aa} = \overline{Aa} \cdot (\overline{AC} \times \bar{F})$$

$$= \begin{vmatrix} -0.716 & -0.656 & 0.239 \\ -6 & -9 & -6 \\ 2 & -4 & 10 \end{vmatrix}$$

$$= 81.62 - 31.49 + 10.04$$

$$= 60.17\ kNm$$

In cartesian co-ordinates,

$$M_{Aa} = 60.17 (-0.716\, i - 0.656\, j + 0.239\, k)$$

$$= \mathbf{(-43.08\, i - 39.47\, j + 14.38\, k)\ kNm}$$ **... Ans.**

2.13 REDUCTION OF A SYSTEM OF FORCES TO ONE FORCE AND ONE COUPLE

Consider a system of P_1, P_2, P_3, P_4 ... acting at points A, B, C, D ... having position vectors as r_1, r_2, r_3, r_4, ... and we have to reduce this system to one force and one couple at point O.

Then first we will resolve all the forces P_1, P_2, P_3, P_4 ... into forces as P_1, P_2, P_3, P_4 ...... and moments M_1, M_2, M_3, M_4 ... at point 'O'. Then the algebraic sum of all the forces and all the moments gives us a single force (as resultant force of all) and a single moment at 'O'.

Example 2.40 : *The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take F_1 = 20 N, F_2 = 50 N.*

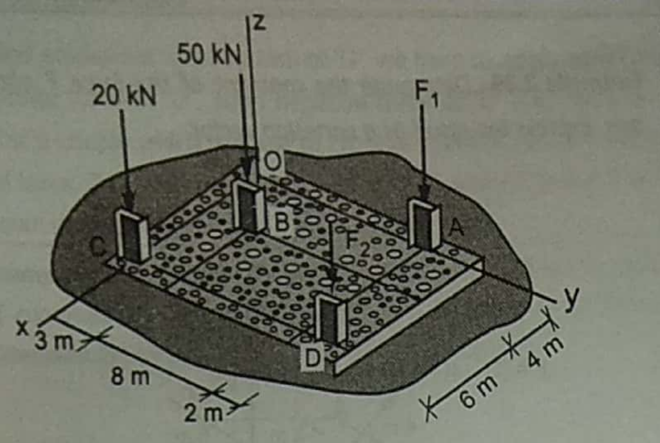


Fig. 2.49

Solution :

Given data : F_1 = 20 N, F_2 = 50 N as shown in Fig. 2.49.

To find : Resultant and its location.

O (0, 0, 0), A (0, 11, 0), B (4, 3, 0), C (10, 0, 0), D (10, 13, 0).

$\bar{F}_A = -20\,k,\ \ \bar{F}_B = -50\,k,\ \ \bar{F}_C = -20\,k,\ \ \bar{F}_D = -50\,k$

According to Varignon's theorem,

Moment of components = Moment of resultant.

$$\bar{M}_{OA} = \overline{OA} \times \bar{F}_A$$

$$\bar{M}_{OA} = \begin{vmatrix} i & j & k \\ 0 & 11 & 0 \\ 0 & 0 & -20 \end{vmatrix} = -220\,i$$

$$\bar{M}_{OB} = \overline{OB} \times \bar{F}_B = \begin{vmatrix} i & j & k \\ 4 & 3 & 0 \\ 0 & 0 & -50 \end{vmatrix}$$

$$= -150\,i + 200\,j$$

$$\bar{M}_{OC} = \overline{OC} \times \bar{F}_C = \begin{vmatrix} i & j & k \\ 10 & 0 & 0 \\ 0 & 0 & -20 \end{vmatrix} = 200\,j$$

$$\bar{M}_{OD} = \overline{OD} \times \bar{F}_D = \begin{vmatrix} i & j & k \\ 10 & 13 & 0 \\ 0 & 0 & -50 \end{vmatrix}$$

$$= -650\,i + 500\,j$$

$$\bar{R} = \bar{F}_A + \bar{F}_B + \bar{F}_C + \bar{F}_D = \mathbf{-140\,k} \quad \text{... Ans.}$$

Let resultant act at (x, y) from origin.

$\therefore\ (x\,i + y\,j)\,(-140\,k) = -1020\,i + 900\,j$

$140x\,j - 140y\,i = -1020\,i + 900\,j$

Equating i and j components, $x = 6.43$ m, $y = 7.29$ m

$\therefore$ **Resultant 140 N (↓) acts at (6.43, 7.29) m.** **... Ans.**

Example 2.41 : *Two workers use blocks and tackles attached to the bottom of an I beam to lift a large cylindrical tank. Knowing that tension in the rope AB is 324 N, replace the force exerted at A by rope AB with an equivalent force-couple system at E.*

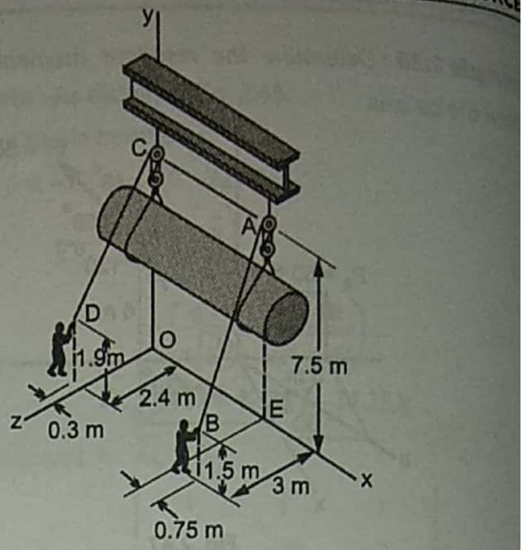


Fig. 2.50

Solution :

Given data : T_{AB} = 324 N, as shown in Fig. 2.50.

To find : Equivalent force-couple system at E.

A (x, 7.5, 0), B ((x + 0.75), 1.5, 3), E (x, 0, 0)

$$\bar{T}_{AB} = T_{AB}\left(\frac{\bar{B} - \bar{A}}{|AB|}\right) = 324\left(\frac{0.75\,i - 6\,j + 3\,k}{6.75}\right)$$

$$= 36\,i - 288\,j + 144\,k$$

$$\bar{M}_E = \overline{EB} \times \bar{T}_{AB} = \begin{vmatrix} i & j & k \\ 0.75 & 1.5 & 3 \\ 36 & -288 & 144 \end{vmatrix}$$

$$= i\,[1.5 \times 144 + 3 \times 288] - j\,[0.75 \times 144 - 36 \times 3] + k\,[0.75 \times (-288) - 1.5 \times 36]$$

$$= \mathbf{(1080\,i - 270\,k)\ Nm} \quad \text{... Ans.}$$

Example 2.42 : *A crank handle of a machine lies in the x-y plane and is subjected to the force system shown in Fig. 2.51. Replace the force system by a single resultant F acting at point O and a single couple 'M'.*

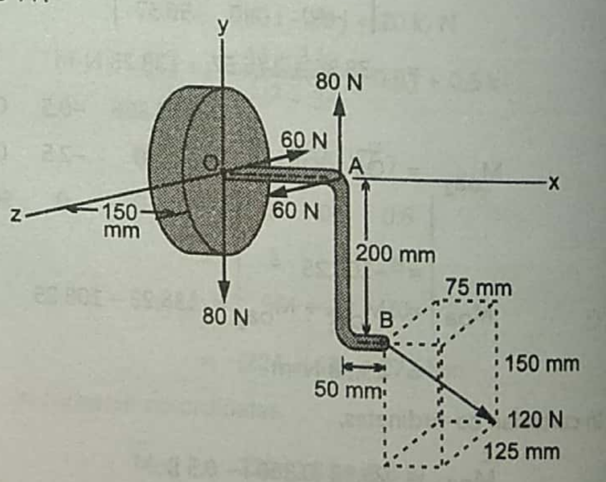


Fig. 2.51

Solution :

Given data : As shown in Fig. 2.51.

To find : Resultant force and couple.

O (0, 0, 0), A (150, 0, 0), B (200, –200, 0), C (275, –350, –125).

$$\bar{F}_1 = -80\,j,\ \ \bar{F}_2 = -60\,k,\ \ \bar{F}_3 = 60\,k,$$

$$\bar{F}_4 = 80\,j$$

$$\bar{F}_5 = 120\left(\frac{\bar{C}-\bar{B}}{|CB|}\right)$$

$$= 120\left[\frac{75\,i - 150\,j - 125\,k}{\sqrt{(75)^2 + (150)^2 + (125)^2}}\right]$$

$$= 43.05\,i - 86.1\,j - 71.75\,k$$

$$\bar{R} = -80\,j - 60\,k + 60\,k + 80\,k + 43.05\,i - 86.1\,j - 71.75\,k$$

$$= \mathbf{43.03\,i - 86.1\,j - 71.75\,k} \qquad \text{... Ans.}$$

As F_1 and F_2 pass through origin, therefore, moment is zero.

$$\therefore\ \bar{M}_1 = \bar{M}_2 = 0$$

$$\bar{M}_3 = \begin{vmatrix} i & j & k \\ 150 & 0 & 0 \\ 0 & 0 & 60 \end{vmatrix} = -9000\,j$$

$$\bar{M}_4 = \begin{vmatrix} i & j & k \\ 150 & 0 & 0 \\ 0 & 80 & 0 \end{vmatrix} = 12000\,k$$

$$\bar{M}_5 = \begin{vmatrix} i & j & k \\ 200 & -200 & 0 \\ 43.05 & -86.1 & -71.75 \end{vmatrix}$$

$$= 14350\,i + 14350\,j - 8610\,k$$

$$\bar{M} = \mathbf{(14350\,i + 5350\,j + 3390\,k)\ N\text{-}m} \qquad \text{... Ans.}$$

***Example 2.43 :** The belt passing over the pulley is subjected to two forces F_1 and F_2, each having a magnitude of 40 N. F_1 acts in –k direction. Replace these forces by an equivalent force and couple moment at point A. Express the result in cartesian vector form.*

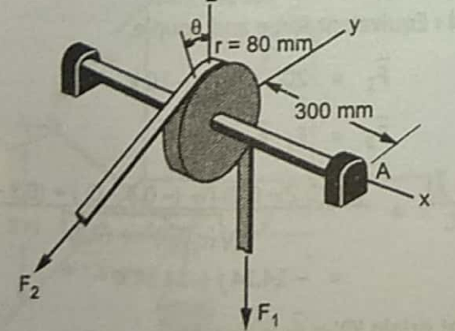


Fig. 2.52

Solution :

Given data : $F_1 = F_2 = 40$ N, as shown in Fig. 2.52.

To find : Equivalent force-couple system at A.

A (300, 0, 0), B (0, 80, 0), C (0, –56.569, 56.569)

$$\bar{F}_1 = -40\,k$$

$$\bar{F}_2 = -28.284\,j - 28.284\,k$$

$$\bar{M}_1 = \overline{AB} \times \bar{F}_1$$

$$= \begin{vmatrix} i & j & k \\ -300 & 80 & 0 \\ 0 & 0 & -40 \end{vmatrix}$$

$$= -3200\,i - 12000\,j$$

$$\bar{M}_2 = \overline{AC} \times \bar{F}_2$$

$$= \begin{vmatrix} i & j & k \\ -300 & -56.569 & -56.569 \\ 0 & -28.284 & -28.284 \end{vmatrix}$$

$$= 3200\,i - 8485.2\,j + 8485.2\,k$$

$$\therefore\ \bar{R} = \bar{F}_1 + \bar{F}_2 = \mathbf{-28.284\,j - 68.284\,k} \qquad \text{... Ans.}$$

$$\bar{M} = \bar{M}_1 + \bar{M}_2 = \mathbf{-20485.2\,j + 8485.2\,k} \qquad \text{... Ans.}$$

***Example 2.44 :** Replace the force system acting on the block in Fig. 2.53 by an equivalent system consisting of a force F acting at the origin 'O' and a couple M.*

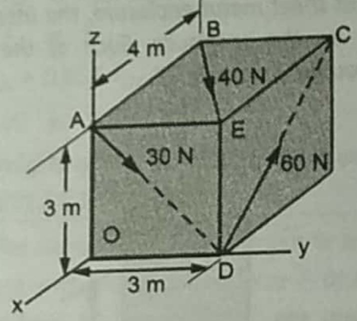


Fig. 2.53

Solution :

Given data : As shown in Fig. 2.53.

To find : Force and couple at 'O'.

O (0, 0, 0), A (0, 0, 3), B (–4, 0, 3),

C (–4, 3, 3), D (0, 3, 0), E (0, 3, 3)

$$\bar{F}_1 = F_1\left(\frac{\bar{D}-\bar{A}}{|AD|}\right) = 30\left(\frac{3\,j - 3\,k}{\sqrt{3^2 + 3^2}}\right)$$

$$= 21.21\,j - 21.21\,k$$

$$\bar{F}_2 = F_2\left(\frac{\bar{E}-\bar{B}}{|EB|}\right) = 40\left(\frac{4\,i + 3\,j}{\sqrt{4^2 + 3^2}}\right)$$

$$= 32\,i + 24\,j$$

$$\bar{F}_3 = F_3\left(\frac{\bar{C}-\bar{D}}{|CD|}\right) = 60\left(\frac{-4\,i+3\,k}{\sqrt{4^2+3^2}}\right)$$

$$= -48\,i + 36\,k$$

$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3$$

$$= -16\,i + 45.21\,j + 14.79\,k \quad \text{... Ans.}$$

$$\bar{M}_1 = \overline{OA} \times \bar{F}_1$$

$$= \begin{vmatrix} i & j & k \\ 0 & 0 & 3 \\ 0 & 21.21 & -21.21 \end{vmatrix} = -63.63\,i$$

$$\bar{M}_2 = \overline{OE} \times \bar{F}_2$$

$$= \begin{vmatrix} i & j & k \\ 0 & 3 & 3 \\ 32 & 24 & 0 \end{vmatrix} = -72\,i + 96\,j - 96\,k$$

$$\bar{M}_3 = \overline{OD} \times \bar{F}_3$$

$$= \begin{vmatrix} i & j & k \\ 0 & 3 & 0 \\ -48 & 0 & 36 \end{vmatrix} = 108\,i + 144\,k$$

$$\bar{M} = \bar{M}_1 + \bar{M}_2 + \bar{M}_3$$

$$= \mathbf{-27.63\,i + 96\,j + 48\,k} \quad \text{... Ans.}$$

***Example 2.45 :** As plastic bushings are inserted into a 60 mm diameter cylindrical sheet metal enclosure, the insertion tools exert the forces shown on the enclosure. Each of the forces with an equivalent force-couple system at 'C'.*

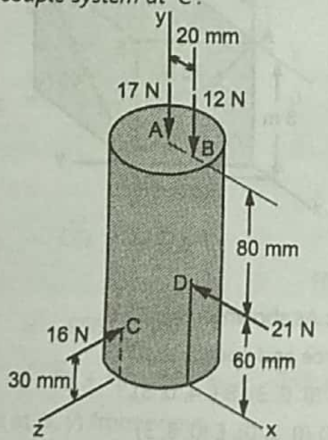


Fig. 2.54

Solution :

Given data : As shown in Fig. 2.54.

To find : Equivalent force-couple system at C.

A (0, 140, 0), B (20, 140, 0), C (0, 30, 30), D (30, 60, 0)

$$\bar{F} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3 + \bar{F}_4$$

$$= -21\,i - 17\,j - 12\,j - 16\,k$$

$$= \mathbf{(-21\,i - 29\,j - 16\,k)\ N} \quad \text{... Ans.}$$

$$\bar{M}_C = \bar{M}_1 + \bar{M}_2 + \bar{M}_3 + \bar{M}_4$$

$$\bar{M}_1 = \overline{CD} \times \bar{F}_1 = \begin{vmatrix} i & j & k \\ 30 & 30 & -30 \\ -21 & 0 & 0 \end{vmatrix}$$

$$= 630\,j + 630\,k$$

$$\bar{M}_2 = \overline{CA} \times \bar{F}_2 = \begin{vmatrix} i & j & k \\ 0 & 110 & -30 \\ 0 & -17 & 0 \end{vmatrix} = -510\,i$$

$$\bar{M}_3 = \overline{CB} \times \bar{F}_3 = \begin{vmatrix} i & j & k \\ 20 & 110 & -30 \\ 0 & -12 & 0 \end{vmatrix}$$

$$= -360\,i - 240\,k$$

$\bar{M}_4 = 0$ As F_4 passes through 'C'.

$$\therefore \quad \bar{M}_C = 630\,j + 630\,k - 510\,i - 360\,i - 240\,k$$

$$= \mathbf{(-870\,i + 630\,j + 390\,k)\ N\text{-}mm} \quad \text{... Ans.}$$

***Example 2.46 :** A pipe-bent is acted upon by three forces F_1, F_2, F_3 as shown in Fig. 2.55. Replace the force acting at the origin O and a couple.*

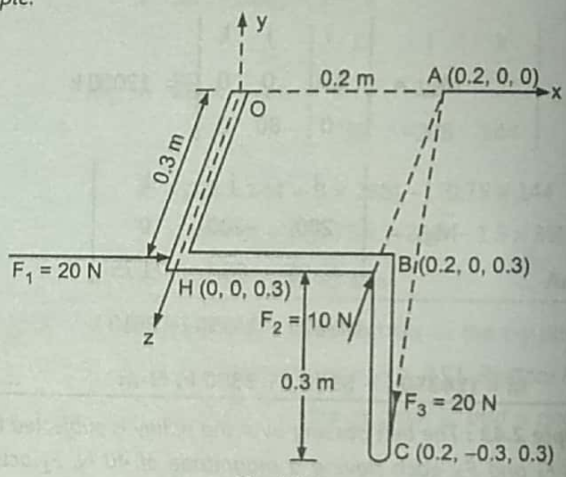


Fig. 2.55

Solution :

Given data : As shown in Fig. 2.55.

To find : Equivalent force and couple.

$$\bar{F}_1 = 20\,i, \quad \bar{F}_2 = -10\,k.$$

$$\bar{F}_3 = F_3 \cdot \bar{e}_{AC}$$

$$F_3\,\frac{(\bar{C}-\bar{A})}{AC} = \frac{20\,[(0.2-0.2)\,i + (-0.3-0)\,j + (0.3-0)\,k]}{\sqrt{(-0.3)^2 + (0.3)^2}}$$

$$= -14.14\,j + 14.14\,k$$

$$\therefore \quad \bar{F} \text{ at origin 'O'} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3$$

$$= 20\,i - 10\,k - 14.14\,j + 14.14\,k$$

$$= \mathbf{20\,i - 14.14\,j + 4.14\,k} \quad \text{... Ans.}$$

Couple : $\bar{M}_1 = \overline{OH} \times \bar{F}_1 = (0.3\,k) \times (20\,i)$

$$\therefore \quad \bar{M}_1 = \begin{vmatrix} i & j & k \\ 0 & 0 & 0.3 \\ 20 & 0 & 0 \end{vmatrix}$$

$$= j\,(20 \times 0.3) = 6\,j \text{ N-m}$$

$$\bar{M}_2 = \overline{OB} \times \bar{F}_2$$

$$\bar{M}_2 = \begin{vmatrix} i & j & k \\ 0.2 & 0 & 0.3 \\ 0 & 0 & -10 \end{vmatrix}$$

$$= -j\,[(-10 \times 0.2) - 0] = 2\,j \text{ N-m}$$

$$\bar{M}_3 = \overline{OC} \times \bar{F}_3$$

$$\bar{M}_3 = \begin{vmatrix} i & j & k \\ 0.2 & -0.3 & 0.3 \\ 0 & -14.14 & 14.14 \end{vmatrix}$$

$$= 0 - 2.83\,j - 2.83\,k$$

$$= -2.83\,j - 2.83\,k$$

$$\therefore \quad \bar{M}_O = \bar{M}_1 + \bar{M}_2 + \bar{M}_3$$

$$= 6\,j - 2\,j - 2.83\,j - 2.83\,k$$

$$= \mathbf{(5.17\,j - 2.83\,k) \text{ N-m}}$$ **... Ans.**

2.14 EQUILIBRIUM OF PARTICLE IN SPACE

A particle is in equilibrium if the resultant of all the forces acting at the point is zero.

If $\bar{P}$, $\bar{Q}$, $\bar{R}$ are three forces acting on a particle and keep the particle in equilibrium, then the resultant or equivalent force E is zero.

$$(P_x + Q_x + R_x)\,i + (P_y + Q_y + R_y)\,j + (P_z + Q_z + R_z)\,k = 0$$

Hence :

$$\Sigma F_x = (P_x + Q_x + R_x) = 0$$

$$\Sigma F_y = (P_y + Q_y + R_y) = 0$$

$$\Sigma F_z = (P_z + Q_z + R_z) = 0$$

Example 2.47 : *If each cable can sustain a maximum tension of 600 N, determine the greatest weight of the bucket and its content that can be supported.*

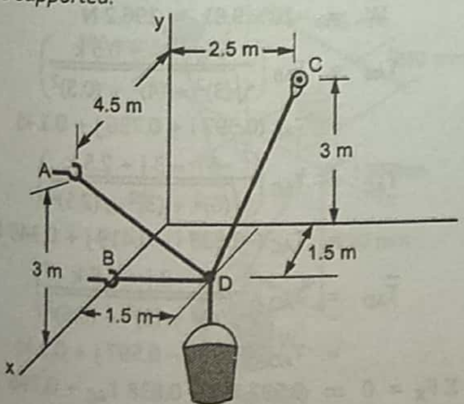


Fig. 2.56

Solution :

Given data : Maximum tension = 600 N, as shown in Fig. 2.56.

To find : Weight of bucket and its content.

A (4.5, 0, 3), B (1.5, 0, 0), C (0, 2.5, 3), D (1.5, 1.5, 0)

$$\bar{T}_{DA} = T_{DA}\left[\frac{3\,i - 1.5\,j + 3\,k}{\sqrt{(3)^2 + (1.5)^2 + (3)^2}}\right]$$

$$\bar{T}_{DA} = T_{DA}\,(0.667\,i - 0.333\,j + 0.667\,k)$$

$$\bar{T}_{DB} = \left[\frac{0 - 1.5\,j}{\sqrt{(1.5)^2}}\right] = -T_{DB}\,j$$

$$\bar{T}_{DC} = T_{DC}\left[\frac{-1.5\,i + j + 3\,k}{\sqrt{(1.5)^2 + (1)^2 + (3)^2}}\right]$$

$$= T_{DC}\,(-0.429\,i + 0.286\,j + 0.857\,k)\,T_{DC}$$

$$\Sigma F_x = 0$$

$$\Rightarrow \quad 0.667\,T_{DA} - 0.429\,T_{DC} = 0$$

$$\therefore \quad T_{DA} = 0.643\,T_{DC}$$

$$\Sigma F_y = 0$$

$$\Rightarrow \quad -1.5\,T_{DA} - T_{DB} + 0.286\,T_{DC} = 0$$

$$\therefore \quad T_{DA} = 0.76\,T_{DB}$$

$$T_{DC} = 1.56\,T_{DA}$$

$$T_{DB} = 1.36\,T_{DA}$$

$\therefore$ Maximum force is in DC,

$$T_{DC} = 600 \text{ N}$$

$$\therefore \quad T_{DA} = 385.8 \text{ N}$$

$$T_{DB} = 507.63 \text{ N}$$

$$\Sigma F_z = 0$$

$$\Rightarrow \quad 0.667\,T_{DA} + 0.857\,T_{DC} - W = 0$$

$$\therefore \quad W = 771.52 \text{ N}$$

The greatest weight of the bucket and its content that can be supported is 771.52 N. ... Ans.

Example 2.48 : *The support assembly shown is bolted in place at B, C and D and supports a downward force P at A. Knowing that the forces in members AB, AC and AD are directed along the respective members and that the force in member AB is 146 N, determine the magnitude of P.*

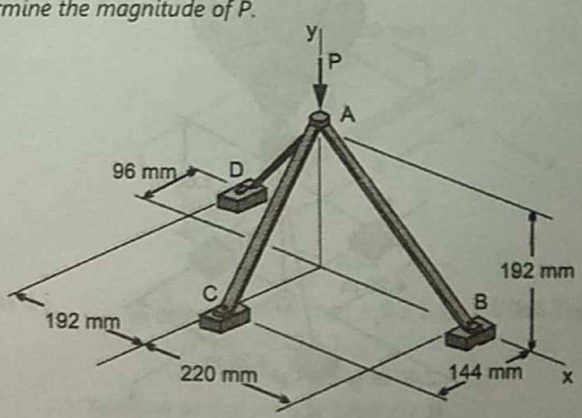


Fig. 2.57

Solution :

Given data : F_{AB} = 146 N and as shown in Fig. 2.57.

To find : Force P.

A (0, 192, 0), B (220, 0, 0), C (0, 0, 144), D (–192, 0, –96)

Assembly is in equilibrium.

$\therefore \quad \Sigma F_x = 0, \quad \Sigma F_y = 0, \quad \text{and} \quad \Sigma F_z = 0$

$$\bar{F}_{AB} = F_{AB} \times \bar{e}_{AB} = F_{AB}\left(\frac{\bar{B}-\bar{A}}{|AB|}\right)$$

$$= 146 \times \left(\frac{220\,i - 192\,j}{292}\right) = 110\,i - 96\,j$$

$$\bar{F}_{AC} = F_{AC} \times \bar{e}_{AC} = F_{AC}\left(\frac{\bar{C}-\bar{A}}{|AC|}\right)$$

$$= F_{AC}\left(\frac{-192\,j + 144\,k}{240}\right)$$

$$= -0.8\,F_{AC}\,j + 0.6\,F_{AC}\,k$$

$$\bar{F}_{AD} = F_{AD} \times \bar{e}_{AD} = F_{AD}\left(\frac{\bar{D}-\bar{A}}{|AD|}\right)$$

$$= F_{AD}\left(\frac{-192\,i - 192\,j - 96\,k}{288}\right)$$

$$= -0.67\,F_{AD}\,i - 0.67\,F_{AD}\,j - 0.33\,F_{AD}\,k$$

$$\bar{P} = -P\,j$$

$$\Sigma F_z = 0 \Rightarrow 0.6\,F_{AC} - 0.33\,F_{AD} = 0$$

$$\therefore \quad F_{AD} = 1.82\,F_{AC}$$

$$\Sigma F_x = 0 \Rightarrow 110 - 0.67\,F_{AD} = 0$$

$$\therefore \quad F_{AD} = 164.18\text{ N}, \quad F_{AC} = 90.21\text{ N}$$

$$\Sigma F_y = 0 \Rightarrow -96 - 0.8\,F_{AC} - 0.67\,F_{AD} - P = 0$$

$\therefore \quad P = \mathbf{-278.17\ N = 278.17\ N\ (\uparrow)}$ **... Ans.**

Example 2.49 : *Three cables are used together to tie a balloon as shown. Determine the vertical force P exerted by the balloon at A knowing that the tension in the cable AD is 481 N.*

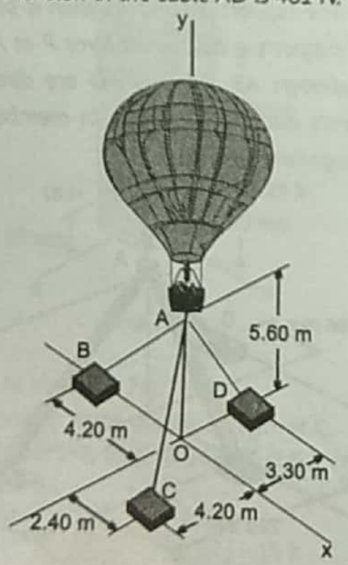


Fig. 2.58

Solution :

Given data : T_{AD} = 481 N, as shown in Fig. 2.58.

To find : Vertical force P.

Co-ordinates : O (0, 0, 0), A (0, 5.6, 0),

B (–4.2, 0, 0), C (2.4, 0, 4.2), D (0, 0, –3.3)

Concept : Body is in equilibrium.

$\therefore \quad \Sigma F_x = 0, \ \Sigma F_y = 0, \ \Sigma F_z = 0$

Consider force at joint A.

$$\bar{T}_{AB} = T_{AB} \times \bar{e}_{AB}$$

$$= T_{AB}\left(\frac{-4.2\,i - 5.6\,j}{\sqrt{(4.2)^2 + (5.6)^2}}\right)$$

$$= T_{AB}\,(-0.6\,i - 0.8\,j) \quad \text{... (}$$

$$\bar{T}_{AC} = T_{AC} \times \bar{e}_{AC}$$

$$= T_{AC}\left(\frac{2.4\,i - 5.6\,j + 4.2\,k}{\sqrt{(2.4)^2 + (5.6)^2 + (4.2)^2}}\right)$$

$$= T_{AC}\,(0.324\,i - 0.757\,j + 0.568\,k) \quad \text{... (}$$

$$\bar{T}_{AD} = T_{AD} \times \bar{e}_{AD}$$

$$= T_{AD}\left(\frac{-5.6\,j - 3.3\,k}{\sqrt{(5.6)^2 + (3.3)^2}}\right)$$

$$= T_{AD}\,(-0.861\,j - 0.508\,k)$$

$$\Sigma F_x = 0 \Rightarrow -0.6\,T_{AB} + 0.324\,T_{AC} = 0$$

$$T_{AB} = 0.54\,T_{AC}$$

$$\Sigma F_y = 0 \Rightarrow -0.8\,T_{AB} - 0.757\,T_{AC} - 0.861\,T_{AD} + P = 0$$

$$\Sigma F_z = 0 \Rightarrow 0.568\,T_{AC} - 0.508\,T_{AD} = 0$$

$$T_{AC} = 0.894\,T_{AD} = 430\text{ N}$$

$$T_{AB} = 0.54 \times 430 = 232.2\text{ N}$$

$\therefore \quad P = \mathbf{925.411\ N}$ **... An**

Example 2.50 : *Determine the tension developed in three cab required to support the traffic light, which has mass 20 kg. Ta h = 3.5 m.*

Solution :

Given data : Mass = 20 kg, h = 3.5 m, as shown in Fig. 2.59

To find : Tension in cables.

A (0, 0, 3.5), B (3, 4, 4), C (–6, –3, 6), D (4, –3, 4).

$$\bar{W} = -20 \times 9.81 = 196.2\text{ N}$$

$$\bar{T}_{AB} = T_{AB}\left(\frac{3\,i + 4\,j + 0.5\,k}{\sqrt{(3)^2 + (4)^2 + (0.5)^2}}\right)$$

$$= T_{AB}\,(0.597\,i + 0.796\,j + 0.1\,k)$$

$$\bar{T}_{AC} = T_{AC}\left(\frac{-6\,i - 3\,j + 2.5\,k}{\sqrt{(6)^2 + (3)^2 + (2.5)^2}}\right)$$

$$= T_{AC}\,(-0.838\,i - 0.419\,j + 0.349\,k)$$

$$\bar{T}_{AD} = T_{AD}\left(\frac{4\,i - 3\,j + 0.5\,k}{\sqrt{(4)^2 + (3)^2 + (0.5)^2}}\right)$$

$$= T_{AD}\,(0.796\,i - 0.597\,j + 0.1\,k)$$

$$\Sigma F_x = 0 \Rightarrow 0.597\,T_{AB} - 0.838\,T_{AC} + 0.796\,T_{AD} =$$

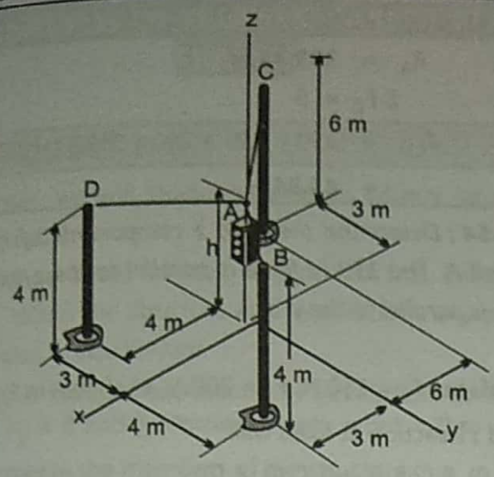


Fig. 2.59

$$\Sigma F_y = 0 \Rightarrow 0.796\, T_{AB} - 0.419\, T_{AC} - 0.597\, T_{AD} = 0$$

$$\Sigma F_z = 0 \Rightarrow 0.1\, T_{AB} + 0.349\, T_{AC} + 0.1\, T_{AD} = 196.2$$

$$T_{AB} = 347.63 \text{ N}, \quad T_{AC} = 412.76 \text{ N}, \quad T_{AD} = 173.82 \text{ N}$$

Tension in AB = **347.63 N** ... **Ans.**

Tension in AC = **412.76 N** ... **Ans.**

Tension in AD = **173.82 N** ... **Ans.**

Example 2.51 : *A container of weight W is suspended from ring A. Cable BAC passes through the ring and is attached to fixed supports at B and C. Two forces P = Pi and Q = Qk are applied to the ring to maintain the container in the position shown. Knowing that W = 1200 N, determine P and Q.*

Solution :

Given data : W = 1200 N, as shown in Fig. 2.60.

To find : P and Q.

A (0, –720, 0), B (–480, 0, –160),

C (240, 0, –130), $\bar{W} = -1200\,j$,

$\bar{P} = Pi$, $\bar{Q} = Qk$

$$T_{AB} = T_{AC} = T$$

$$\bar{T}_{AB} = T_{AB} \times \left(\frac{\bar{B}-\bar{A}}{|AB|}\right)$$

$$= T\left(\frac{-480\,i + 720\,j - 160\,k}{880}\right)$$

$$= T(-0.55\,i + 0.82\,j - 0.182\,k)$$

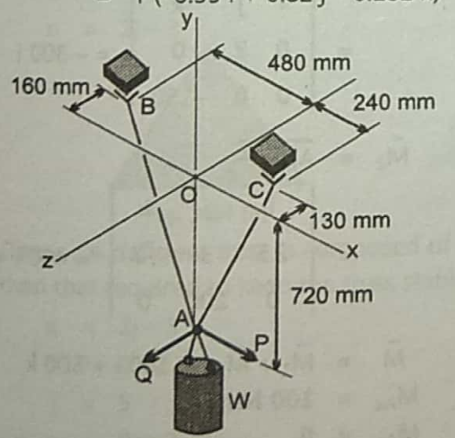


Fig. 2.60

$$\bar{T}_{AC} = T_{AC} \times \left(\frac{\bar{C}-\bar{A}}{|AC|}\right)$$

$$= T\left(\frac{240\,i + 720\,j - 130\,k}{770}\right)$$

$$= T(0.31\,i + 0.94\,j - 0.169\,k)$$

$$\Sigma F_y = 0 \Rightarrow 0.82\,T + 0.94\,T - 1200 = 0$$

$$\therefore \quad T = 681.82 \text{ N}$$

$$\Sigma F_x = 0 \Rightarrow -0.55\,T + 0.31\,T + P = 0$$

$$P = \mathbf{163.63 \text{ N}}$$ ... **Ans.**

$$\Sigma F_z = 0 \Rightarrow -0.182\,T - 0.169\,T + Q = 0$$

$$\therefore \quad Q = \mathbf{239.32 \text{ N}}$$ ... **Ans.**

Example 2.52 : *Collars A and B are connected by a 1 m long wire and can slide freely on frictionless rods. If a force P = (680 N) j is applied at A, determine (a) the tension in the wire when y = 300 mm, (b) the magnitude of the force Q required to maintain the equilibrium of the system.*

Solution :

Given data : P = (680 N) j and as shown in Fig. 2.61.

To find : Tension in the wire and force Q.

A (0, 300, 0), B (400, 0, z).

$$AB = \sqrt{(400)^2 + (300)^2 + z^2}$$

$$\therefore \quad (1000)^2 = (400)^2 + (300)^2 + z^2$$

$$\therefore \quad z = 866.03 \text{ mm}$$

$$\bar{T}_{AB} = T_{AB}\left(\frac{\bar{B}-\bar{A}}{|AB|}\right)$$

$$= T_{AB}\left(\frac{400\,i - 300\,j + 866.03\,k}{1000}\right)$$

$$= 0.4\,T_{AB}\,i - 0.3\,T_{AB}\,j + 0.866\,T_{AB}\,k$$

At A : $\Sigma F_y = 0 \Rightarrow -0.3\,T_{AB} + 680 = 0$

$$\therefore \quad T_{AB} = 2266.67 \text{ N}$$

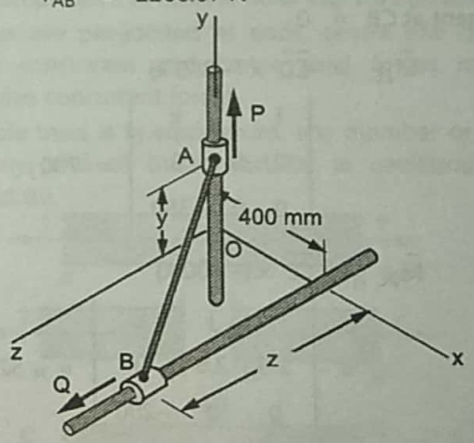


Fig. 2.61

At B : $\bar{T}_{BA} = -0.4\,T_{AB}\,i + 0.3\,T_{AB}\,j - 0.866\,T_{AB}\,k$

$$\Sigma F_z = 0 \Rightarrow -0.866\,T_{AB} + Q = 0$$

$$\therefore \quad Q = 1962.94 \text{ N}$$

$\therefore$ **Tension in the wire is 2266.67 N** ... **Ans.**

Force Q = **1962.94 N** ... **Ans.**

Example 2.53 : *Determine the force components acting on the ball and socket at A, the reaction at the roller B and the tension in the cord CD needed for equilibrium of the quarter circle plate.*

Solution :

Given data : As shown in Fig. 2.62.

To find : Reaction at B, at A and tension in CD.

Moment at AC = 0

A (3, 0, 0), B (0, 3, 0), C (0, 0, 0), D (2, 0, 0), E (1.5, 2.6, 0)

$$\bar{M}_{1C} = \overline{CB} \times (-200\,k)$$

$$= \begin{vmatrix} i & j & k \\ 0 & 3 & 0 \\ 0 & 0 & -200 \end{vmatrix} = -600\,i$$

$$\bar{M}_{2C} = \overline{CE} \times (-200\,k)$$

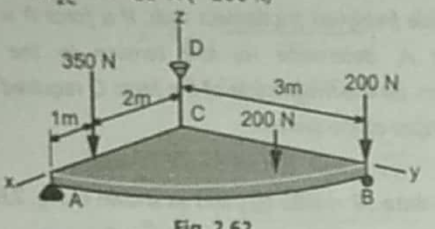


Fig. 2.62

$$= \begin{vmatrix} i & j & k \\ 1.5 & 2.6 & 0 \\ 0 & 0 & -200 \end{vmatrix} = -520\,i + 300\,j$$

$$\bar{M}_{3C} = \overline{CB} \times (R_B\,k)$$

$$= \begin{vmatrix} i & j & k \\ 0 & 3 & 0 \\ 0 & 0 & R_B \end{vmatrix} = 3R_B\,i$$

$$\bar{M}_C = 0 = -600\,i - 520\,i + 300\,j + 3R_B\,i$$

$\therefore$ R_B = **373.33 N** ... **Ans.**

Moment at CB = 0

$$\bar{M}_{1C} = \overline{CD} \times (-350\,k)$$

$$= \begin{vmatrix} i & j & k \\ 2 & 0 & 0 \\ 0 & 0 & -350 \end{vmatrix} = 700\,j$$

$$\bar{M}_{2C} = \overline{CE} \times (-200\,k)$$

$$= \begin{vmatrix} i & j & k \\ 1.5 & 2.6 & 0 \\ 0 & 0 & -200 \end{vmatrix} = -520\,i + 300\,j$$

$$\bar{M}_{3C} = \overline{CA} \times (R_A\,k)$$

$$= \begin{vmatrix} i & j & k \\ 3 & 0 & 0 \\ 0 & 0 & R_A \end{vmatrix} = -3R_A\,j$$

Σ M at CB = 0

$\therefore$ R_A = **333.33 N** ... **Ans.**

$\Sigma F_z = 0$

$\therefore$ T_{CD} = 350 + 200 + 200 − 333.33 − 373.33

= **43.34 N** ... **Ans.**

Example 2.54 : *Determine the x, y, z components of reaction at the fixed wall A. The 150 N force is parallel to the z-axis and the 200 N force is parallel to the y-axis.*

Solution :

Given data : F_1 = 150 N, F_2 = 200 N, as shown in Fig. 2.63.

To find : Reaction at fixed wall.

A (0, 0, 0), B (2.5, 3, −2), C (0, 2, 0).

$\Sigma F_x = 0$

$\Rightarrow$ $A_x = 0$

$\Sigma F_y = 0$

$\Rightarrow$ $A_y + 200 = 0$

$\therefore$ $A_y = -200$ N

$\therefore$ A_y = 200 N ($\leftarrow$)

$\Sigma F_z = 0$

$\Rightarrow$ $A_z - 150 = 0$

$\therefore$ A_z = 150 N ($\uparrow$)

Σ Moment A = 0

$$\bar{M}_1 = \overline{AC} \times \bar{F}_1$$

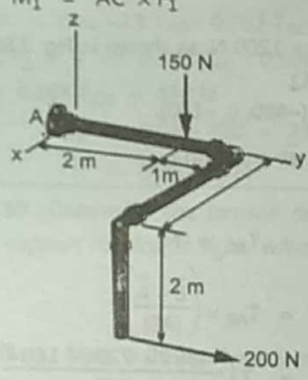


Fig. 2.63

$$= \begin{vmatrix} i & j & k \\ 0 & 2 & 0 \\ 0 & 0 & -150 \end{vmatrix} = -300\,i$$

$$\bar{M}_2 = \overline{AB} \times \bar{F}_2$$

$$= \begin{vmatrix} i & j & k \\ 2.5 & 3 & -2 \\ 0 & 200 & 0 \end{vmatrix} = 400\,i + 500\,k$$

$$\bar{M} = \bar{M}_1 + \bar{M}_2 = 100\,i + 500\,k$$

$\therefore$ M_{Ax} = **100 Nm** ... **Ans.**

M_{Ay} = **0** ... **Ans.**

M_{Az} = **500 Nm** ... **Ans.**

C – ANALYSIS OF TRUSSES, CABLES, PLANE FRAMES

2.15 INTRODUCTION

In this chapter, we will study the analysis of any structure like truss, frame, cable.

Analysis is done based on the assumption that structure is in equilibrium. When the structure is in equilibrium, any part of the structure is also in equilibrium.

Hence, analysis includes application of equilibrium conditions i.e. $\sum F_x = 0$, $\sum F_y = 0$ and $\sum$ Moment at any point = 0 to determine unknown forces in the members of the structure.

2.16 TWO FORCE MEMBER

When a member is in equilibrium under only two forces, it is called as two force member. The two forces must be collinear, equal in magnitude and opposite in direction.

If two forces tend to elongate the member, then the forces are Tensile forces (T).

If two forces tend to compress the member, then the forces are Compressive forces (C).

2.17 TRUSS

A truss is a structure made up of several bars riveted or welded together only at their ends. Plane truss lies in a single plane. It is used to support roofs and bridges.

A truss is classified depending upon number of members required for stability. The following equation is used to determine number of members in the truss.

$$\boxed{n = 2j - 3}$$

where, n = number of members

j = number of joints.

1. **Perfct Truss :** A perfect truss is composed of minimum number of members required to keep it stable.

$\therefore \quad n = 2j - 3$

$n = 3, \; j = 3$

$\therefore \quad n = 2j - 3$

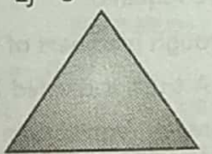


Fig. 2.64 (a)

2. **Deficient Truss :** A deficient truss is composed of number of members less than that required to keep the truss stable.

$\therefore \quad n < 2j - 3$

$n = 6$

$j = 5$

$n < 2j - 3$

$6 < 2(5) - 3$ i.e. 7

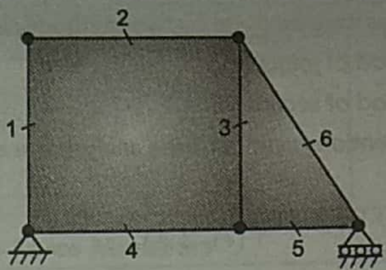


Fig. 2.64 (b)

3. **Redundant Truss :** A redundant truss is composed of number of members more than that required to keep the truss stable.

$n > 2j - 3$

$n = 6$

$j = 4$

$n > 2j - 3$

$6 > 2(4) - 3$ i.e. 5

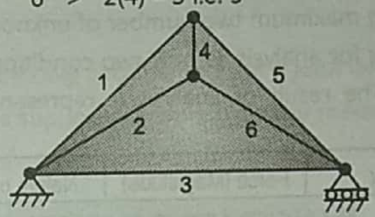


Fig. 2.64 (c)

2.17.1 Analysis of Truss

Analysis of truss includes determination of magnitude and direction of axial forces developed in members of truss due to loading. It also includes determination of magnitude and direction of support reactions of the truss.

The following assumptions are made for analysis as well as design of truss :

- A truss is a perfect truss.
- External loading is applied only at joints of the truss.
- Members of the truss are joined by pins.

Since loading is only at the joints, members of truss must be 'two force members' and forces must be axial forces acting at the ends of members, compressive or tensile. Refer Fig. 2.65 (a) and (b).

Since members are pin-jointed at ends, centre line of joining members are concurrent and hence axial forces in joining members are also concurrent forces.

When the whole truss is in equilibrium, any member or any part of truss or any joint of truss must be in equilibrium. Refer Fig. 2.65 (a) and (b).

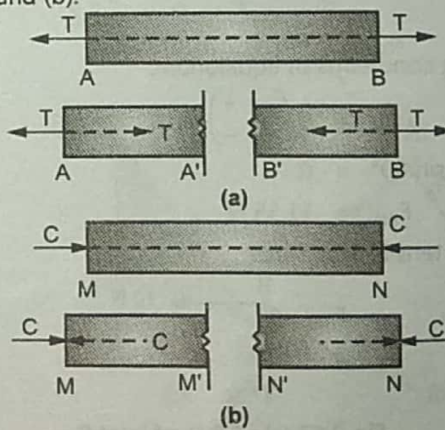


Fig. 2.65

Analysis of truss may be done by two methods :

1. Method of joints.
2. Method of sections.

Sign conventions used for truss analysis are as follows :

For forces

→ (+) ↑ (+) ↓ (–)
← (–)

For moments

↻ (+) ↺ (–)

1. Method of Joints

Since whole truss is in equilibrium, each joint of truss is also in equilibrium. Forces exerted at each joint are axial forces i.e. tension or compression. The conditions of equilibrium i.e. $\sum F_x = 0$ and $\sum F_y = 0$ are applied to each joint.

The joint having maximum two number of unknown forces can be selected initially for analysis as only two conditions of equilibrium are available. The result of analysis is represented in a tabular form.

Member	Force (Magnitude)	Nature of Force (C/T)

ILLUSTRATIVE EXAMPLE

A truss ABC is supported by a roller at A and hinged at C. It is loaded by horizontal force 10 N. The lengths of members and directions are as shown in Fig. 2.66.

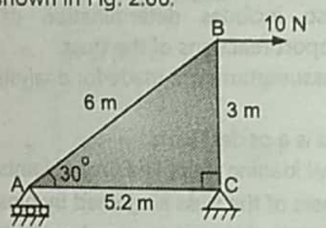


Fig. 2.66

Solution :

Let F_{AB}, F_{BC} and F_{AC} be axial forces in respective members.

Joint B has two unknown axial forces viz. F_{AB} and F_{BC}, it is selected initially for the analysis.

F.B.D. of joint B is drawn as shown in Fig. 2.67 (a). F_{AB} and F_{BC} are assumed to be tensile forces (i.e. showing arrow away from the joint).

Applying conditions of equilibrium,

$$\sum F_x = 0 \left(\begin{matrix}\rightarrow & + \\ \leftarrow & -\end{matrix}\right)$$

$$+10 - F_{AB} \sin 60° = 0$$

$$\therefore \quad F_{AB} = 11.55 \text{ N}$$

Force F_{AB} is a tensile axial force.

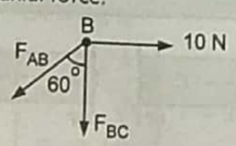


Fig. 2.67 (a) : F.B.D. of joint B

Since F_{AB} is positive, assumed direction is correct.

$$\sum F_y = 0 \qquad (\uparrow +, \downarrow -)$$

$$-F_{BC} - F_{AB} \cos 60° = 0$$

$$\therefore \quad -F_{BC} - (11.55) \cos 60° = 0$$

$$\therefore \quad F_{BC} = -5.775 \text{ N}$$

Since F_{BC} is negative, assumed direction of force is incorrect. Force F_{BC} is a compressive axial force.

Consider joint A. At A, let A_y be the support reaction due to roller.

By inspection, it can be stated, to balance the effect of horizontal component of F_{AB} (→), F_{AC} can be assumed as compressive force (←).

Applying conditions of equilibrium,

$$\sum F_x = 0$$

$$\therefore \quad + F_{AB} \cos 30° - F_{AC} = 0$$

$$\therefore \quad F_{AC} = 11.55 \cos 30° = 10 \text{ N}$$

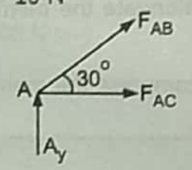


Fig. 2.67 (b)

Since F_{AC} is positive, assumed direction is correct. F_{AC} is compressive force.

$$\sum F_y = 0$$

$$+ F_{AB} \sin 30° + A_y = 0$$

$$\therefore \quad A_y = -F_{AB} \sin 30°$$

$$= -11.55 \sin 30° = -5.77 \text{ N} = 5.77 \text{ N} (\downarrow)$$

Member	Magnitude of Force	Nature of Force
AB	11.55 N	Tension
BC	5.77 N	Compression
AC	10 N	Compression

2. Method of Section

Generally, method of section is used when axial forces of a few number of members are required.

A section is passed through members of truss (whose axial forces are to be found out). A truss is divided into two portions at cut section. The conditions of equilibrium are applied to either portion of the truss. i.e. $\sum F_x = 0$, $\sum F_y = 0$, $\sum M = 0$. F.B.D. of this portion includes external loads, support reaction (if present) and internal axial forces in cut members.

A section cutting maximum three number of members, is selected as only three conditions of equilibrium are available.

In both the methods of analysis, support reactions of truss are found out initially, if required.

ILLUSTRATIVE EXAMPLE

A truss ABCDEF is supported by hinge at A and roller at F. It is loaded by 10 N at B. Length of members and their directions are as shown. Determine axial forces in members BD, CD and CE. Let F_{BD}, F_{CD} and F_{CE} be the axial forces in respective members.

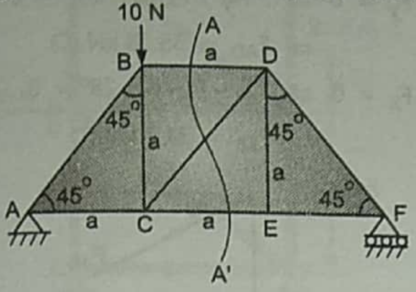


Fig. 2.68

Solution :

Support reactions at hinge and roller are determined by applying conditions of equilibrium to the whole truss.

$\sum F_x = 0 \Rightarrow A_x = 0,\ \sum F_y = 0 \Rightarrow A_y + F_y = 10,\ \sum M_F = 0 \Rightarrow$ $A_y \times 3a - 10 \times 2a = 0 \Rightarrow A_y = 6.67\text{ N } (\uparrow)$.

Let AA' be the section cutting three members, viz. BD, CD and CE.

Assume axial forces in three members be tensile.

Let us consider L.H.S. of cut section of truss.

Applying conditions of equilibrium to L.H.S. part of truss,

$\sum M \text{ at point D} = 0 \Rightarrow -F_{CE} \times a + 6.67 \times 2a - 10 \times a = 0$

$\therefore\ F_{CE} = 3.34\text{ N}$

$\therefore\ F_{CE} = 3.34\text{ N (Tensile)}$

$\sum M \text{ at point B} = 0 \Rightarrow a \times 6.67 + 23.34\ (a)$

$= 0 \Rightarrow -10 \times a + A_y \times 2a$

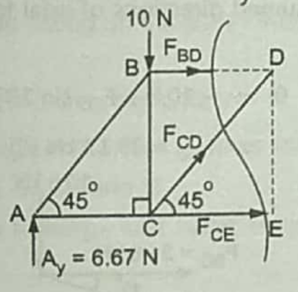


Fig. 2.68 (a)

$\sum M \text{ at point E} = 0$

$\sum M \text{ at C} = 0 \Rightarrow F_{BD} \times a + A_y \times a = 0$

$\Rightarrow F_{BD} = -A_y = -6.67$

$\therefore\ F_{BD} = 6.67\text{ N (Compression)}$

$\sum M \text{ at point B} = 0$

$\Rightarrow A_y \times a - F_{CE} \times a - F_{CD} \cos 45° \times a = 0$

$\Rightarrow 6.67 - 3.34 - F_{CD} \cos 45° = 0$

$\Rightarrow F_{CD} = 4.71\text{ N}$

$F_{CD} = 4.71\text{ N (Tension)}$

Member	Magnitude of Force	Nature of Force
BD	6.67 N	Compression
CD	4.71 N	Tension
CE	3.34 N	Tension

2.17.2 Zero Force Members

The members having no axial force are called **Zero Force Members**. These members do not support loading, but increase stability of truss during construction.

Analysis of truss is simplified, if zero force members are found by inspection initially.

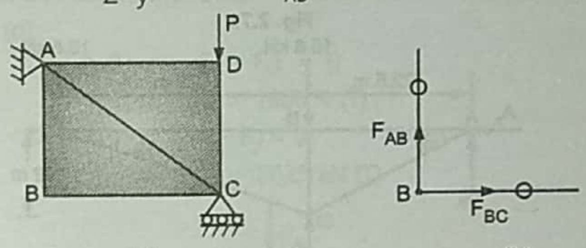
Generally, the following rules may be followed to find zero force members.

(1) If there are only two members at a joint without external load as well as support reaction, then the members are zero force members.

F.B.D. of joint B

$\sum F_x = 0 \Rightarrow F_{BC} = 0$

$\sum F_y = 0 \Rightarrow F_{AB} = 0$



(a) (b)

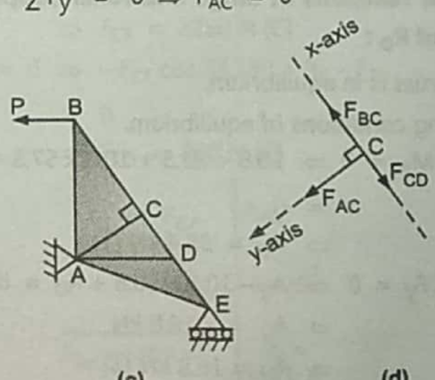
Fig. 2.69

(2) If there are only three members at a joint without external load as well as support reaction and out of three members, two are collinear then third member is a zero force member.

F.B.D. of joint C

$\sum F_x = 0 \Rightarrow F_{BC} = F_{CD}$

$\sum F_y = 0 \Rightarrow F_{AC} = 0$



(c) (d)

Fig. 2.69

NUMERICAL EXAMPLES ON METHOD OF JOINT AND METHOD OF SECTION

***Example 2.55 :** Identify zero force members in the truss.*

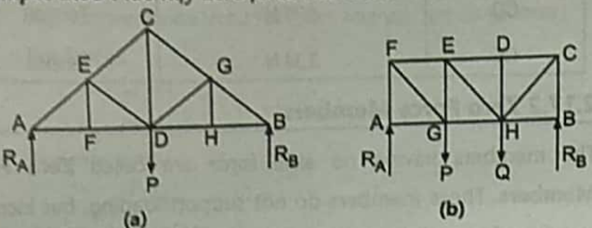


Fig. 2.70

Solution :

Zero force members : EF, ED, HG, GD.

Zero force members : BH, AG, DH.

***Example 2.56 :** Determine magnitude and nature of axial forces in the members of given truss.*

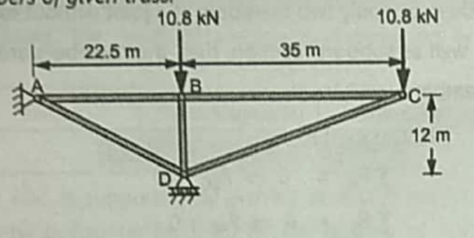


Fig. 2.71

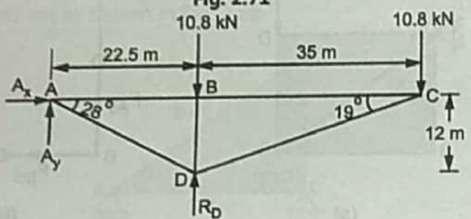


Fig. 2.71 (a)

Solution :

Given data : Forces acting on the truss are :

10.8 kN and 10.8 kN at B and C acting vertically downward.

Dimensions are as shown in Fig. 2.92.

To find : Magnitude and nature of axial force in members AB, AD, BC, BD and CD.

(a) To find reactions at hinge and roller support i.e. R_A (A_X and A_y) and R_D :

Since truss is in equilibrium,

Applying conditions of equilibrium,

$$\Sigma M_A = 0 \Rightarrow 10.8 + 22.5 + 10.8 \times 57.5 - R_D \times 22.5 = 0$$

$$\Rightarrow R_D = 38.4 \text{ kN } (\uparrow)$$

$$\Sigma F_y = 0 \Rightarrow A_y - 10.8 - 10.8 + R_D = 0$$

$$\Rightarrow A_y = -16.8 \text{ kN}$$

$$\Rightarrow A_y = 16.8 \text{ kN } (\downarrow)$$

$$\Sigma F_X = 0 \Rightarrow A_X = 0$$

Reaction at hinge support = R_A = 16.8 kN (↓)

Reaction at roller support = R_D = 38.4 kN (↑)

(b) Joint A : Assumed directions of axial forces are as shown in Fig. 2.71 (b).

$$\Sigma F_y = 0 \Rightarrow -A_y + F_{AD} \sin 28° = 0$$

$$\Rightarrow F_{AD} = 35.78 \text{ kN (C)}$$

$$\Sigma F_X = 0 \Rightarrow F_{AB} - F_{AD} \cos 28° = 0$$

$$\Rightarrow F_{AB} = 31.6 \text{ kN (T)}$$

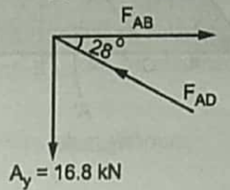


Fig. 2.71 (b) : Joint A

(c) Joint B : Assumed directions of axial forces are as shown in Fig. 2.71 (c).

$$\Sigma F_X = 0 \Rightarrow -F_{AB} + F_{BC} = 0$$

$$\Rightarrow F_{BC} = 31.6 \text{ kN (T)}$$

$$\Sigma F_y = 0 \Rightarrow -10.8 + F_{BD} = 0$$

$$\Rightarrow F_{BD} = 10.8 \text{ kN (C)}$$

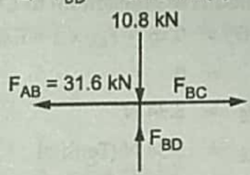


Fig. 2.71 (c) : Joint B

(d) Joint C : Assumed directions of axial forces are as shown in Fig. 2.71 (d).

$$\Sigma F_y = 0 \Rightarrow -10.8 + F_{CD} \sin 19° = 0$$

$$\Rightarrow F_{CD} = 33.17 \text{ kN (C)}$$

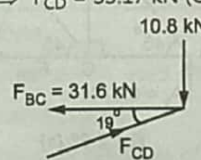


Fig. 2.71 (d) : Joint C

(e)

Sr. No.	Member	Force (kN)	Nature (C/T)
1	AB	31.6	T
2	AD	35.78	C
3	BC	31.6	T
4	BD	10.8	C
5	CD	33.17	C

Example 2.57 : Determine magnitude and nature of axial forces in the members of given truss.

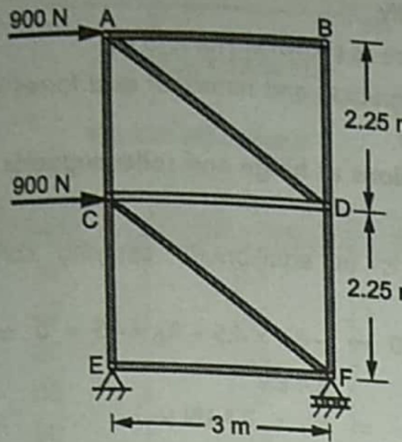


Fig. 2.72

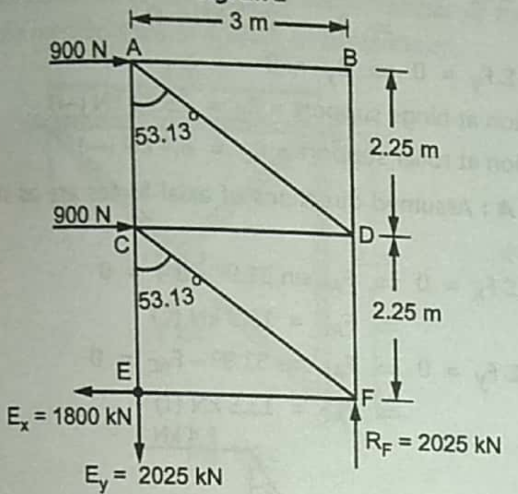


Fig. 2.72 (a)

Solution :

Given data : Forces acting on the truss : 900 N each at A and C horizontally.

Dimensions are as shown in Fig. 2.72.

To find : Magnitude and nature of axial forces in members AB, BD, AD, AC, CD, CE, CF, DF and EF.

(a) To find reactions at hinge and roller supports i.e. R_E (i.e. E_x and E_y) and R_F :

Since truss is in equilibrium, applying conditions of equilibrium,

$$\Sigma M_E = 0 \Rightarrow 900 \times 2.25 + 900 \times 4.5 - R_F \times 3 = 0$$

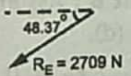
$$\Rightarrow R_F = 2025 \text{ N } (\uparrow)$$

$$\Sigma F_y = 0 \Rightarrow -E_y + R_F = 0$$

$$\Rightarrow E_y = 2025 \text{ N } (\downarrow)$$

$$\Sigma F_x = 0 \Rightarrow -E_x + 900 + 900 = 0$$

$$\Rightarrow E_x = 1800 \text{ N } (\leftarrow)$$



Reaction at hinge support = R_E = 2709 N

Reaction at roller support = R_F = 2025 kN ($\uparrow$)

Zero force members :

(b) Joint B :

$$\Sigma F_x = 0 \Rightarrow F_{AB} = 0$$

$$\Sigma F_y = 0 \Rightarrow F_{BD} = 0$$

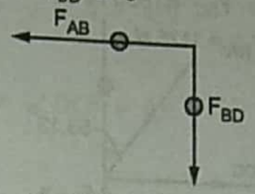


Fig. 2.72 (b) : Joint B

(c) Joint A : Assumed directions of axial forces are as shown in Fig. 2.72 (c).

$$\Sigma F_x = 0 \Rightarrow 900 - F_{AD} \sin 53.13^\circ = 0$$

$$\Rightarrow F_{AD} = 1125 \text{ N (C)}$$

$$\Sigma F_y = 0 \Rightarrow F_{AD} \cos 53.13^\circ - F_{AC} = 0$$

$$\Rightarrow F_{AC} = 675 \text{ N (T)}$$

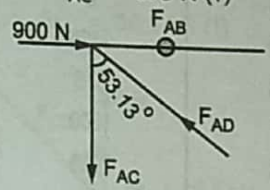


Fig. 2.72 (c) : Joint A

(d) Joint E : Assumed directions of axial forces are as shown in Fig. 2.72 (d).

$$\Sigma F_x = 0 \Rightarrow -E_x + F_{EF} = 0$$

$$\Rightarrow F_{EF} = 1800 \text{ N (T)}$$

$$\Sigma F_y = 0 \Rightarrow F_{EC} - E_y = 0$$

$$\Rightarrow F_{EC} = 2025 \text{ kN (T)}$$

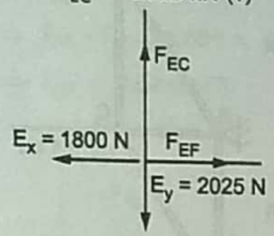


Fig. 2.72 (d) : Joint E

(e) Joint F : Assumed directions of axial forces are as shown in Fig. 2.72 (e).

$$\Sigma F_x = 0 \Rightarrow -F_{EF} + F_{CF} \sin 53.13^\circ = 0$$

$$\Rightarrow F_{CF} = 2250 \text{ N (C)}$$

$$\Sigma F_y = 0 \Rightarrow -F_{CF} \cos 53.13^\circ + R_F - F_{FD} = 0$$

$$\Rightarrow F_{FD} = 700 \text{ N (C)}$$

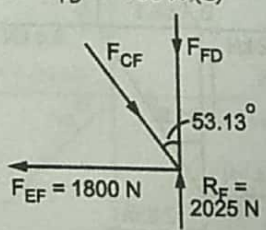


Fig. 2.72 (e) : Joint F

(f) Joint D : Assumed directions of axial forces are as shown in Fig. 2.72 (f).

$$\Sigma F_X = 0 \Rightarrow +F_{DA} \sin 53.13^\circ + F_{DC} = 0$$

$$\Rightarrow F_{DC} = 900 \text{ N (T)}$$

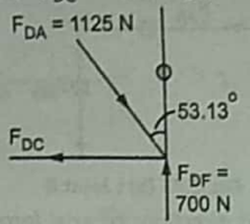


Fig. 2.72 (f) : Joint D

(g)

Sr. No.	Member	Force (N)	Nature (C/T)
1	AB	0	
2	DB	0	
3	AD	1125	C
4	AC	675	T
5	EF	1800	T
6	EC	2025	T
7	CF	2250	C
8	FD	700	C
9	DC	900	T

Example 2.58 : *Determine magnitude and nature of axial forces in the members of given truss.*

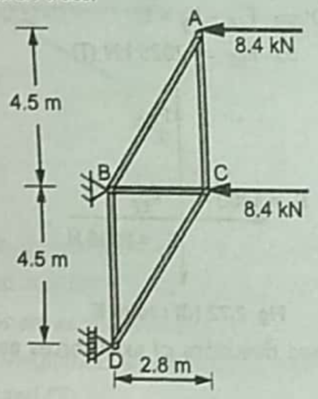


Fig. 2.73

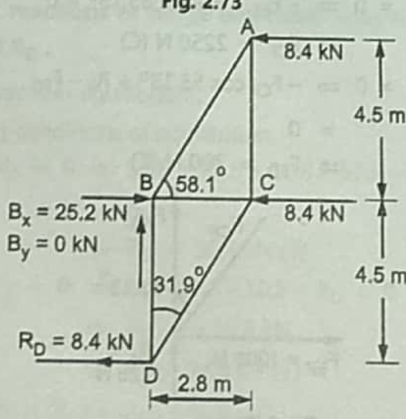


Fig. 2.73 (a)

Solution :

Given data : Forces acting on the truss : 8.4 kN and 8.4 kN at A and C horizontally.

Dimensions are as shown in Fig. 2.73 (a).

To find : Magnitude and nature of axial forces in members AC, AB, BC, CD, BD.

(a) To find reactions at hinge and roller supports i.e. R_B (B_x and B_y) and R_D :

Since truss is in equilibrium, applying conditions of equilibrium,

$$\Sigma M_B = 0 \Rightarrow -8.4 \times 4.5 + R_D \times 4.5 = 0 \Rightarrow R_D = +8.4 \text{ kN}$$

$$\Rightarrow R_D = 8.4 \text{ kN} (\leftarrow)$$

$$\Sigma F_X = 0 \Rightarrow -8.4 - 8.4 - R_D + B_X$$

$$\Rightarrow B_X = 25.2 \text{ kN} (\rightarrow)$$

$$\Sigma F_y = 0 \Rightarrow B_y = 0$$

Reaction at hinge support = R_B = 25.23 kN (→)

Reaction at roller support = R_D = 8.4 kN (←)

(b) Joint A : Assumed directions of axial forces are as shown in Fig. 2.73 (b).

$$\Sigma F_X = 0 \Rightarrow F_{AB} \sin 31.9^\circ - 8.4 = 0$$

$$\Rightarrow F_{AB} = 15.9 \text{ kN (C)}$$

$$\Sigma F_y = 0 \Rightarrow F_{AB} \cos 31.9^\circ - F_{AC} = 0$$

$$\Rightarrow F_{AC} = 13.5 \text{ kN (T)}$$

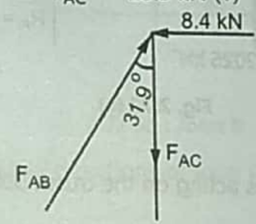


Fig. 2.73 (b) : Joint A

(c) Joint D : Assumed directions of axial forces are as shown in Fig. 2.73 (c).

$$\Sigma F_X = 0 \Rightarrow -R_D + F_{DC} \sin 31.9^\circ = 0$$

$$\Rightarrow F_{DC} = 15.9 \text{ kN (T)}$$

$$\Sigma F_y = 0 \Rightarrow -F_{DB} + F_{DC} \cos 31.9^\circ = 0$$

$$\Rightarrow F_{DB} = 13.50 \text{ kN (C)}$$

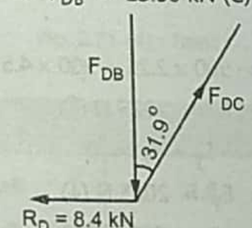


Fig. 2.73 (c) : Joint D

(d) Joint B : Assumed directions of axial forces are as shown in Fig. 2.73 (d).

$$\Sigma F_X = 0 \Rightarrow +B_X - F_{AB} \sin 31.9^\circ - F_{BC} = 0$$

$$\Rightarrow F_{BC} = 16.8 \text{ kN (C)}$$

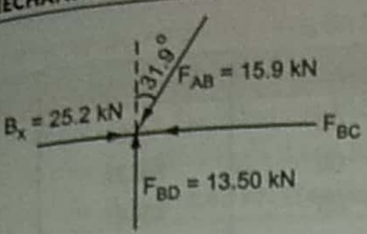


Fig. 2.73 (d) : Joint B

(e)

Sr. No.	Member	Force (kN)	Nature (C/T)
1	AB	15.9	C
2	AC	13.5	T
3	DC	15.9	T
4	DB	13.50	C
5	BC	16.8	C

Example 2.59 : *Determine the force in each member of the truss and state if the members are in tension or compression.*

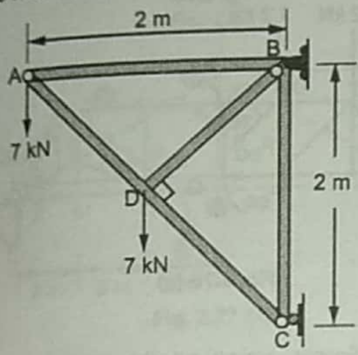


Fig. 2.74

Solution :

Given data : Forces acting on the truss : 7 kN each at A and D vertically downward.

Dimensions are shown in Fig. 2.74.

To find : Magnitude and nature of axial forces in members AB, DB, BC, AD and DC.

(a) To find reactions at hinge and roller supports i.e. R_B (B_x and B_y) and R_C :

Since truss is in equilibrium, applying conditions of equilibrium,

$$\Sigma M_B = 0 \Rightarrow -7 \times 2 - 7 \times 1 + R_C \times 2 = 0$$

$$\Rightarrow R_C = 10.5 \text{ kN } (\leftarrow)$$

$$\Sigma F_x = 0 \Rightarrow -R_C + B_x = 0$$

$$\Rightarrow B_x = 10.5 \text{ kN } (\rightarrow)$$

$$\Sigma F_y = 0 \Rightarrow -7 - 7 + B_y = 0$$

$$\Rightarrow B_y = 14 \text{ kN } (\uparrow)$$

Reaction at hinge support = R_B = 17.5 kN

Reaction at roller support = R_C = 10.5 kN (←)

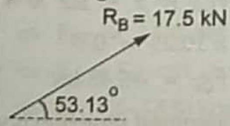


Fig. 2.74 (a)

(b) From geometry :

Consider Δ ABC.

Since AB = BC

$$\therefore \quad \angle BAC = \angle BCA = 45°$$

Consider Δ BDC.

$$\angle DBC = \angle DCB = 45°$$

Consider Δ ABD.

$$\angle DBA = \angle BAD = 45°$$

(c) Joint C : Assumed directions of axial forces are as shown in Fig. 2.74 (b).

$$\Sigma F_x = 0 \Rightarrow F_{CD} \sin 45° - R_C = 0$$

$$\Rightarrow F_{CD} = 14.85 \text{ kN (C)}$$

$$\Sigma F_y = 0 \Rightarrow F_{CB} - F_{CD} \cos 45° = 0$$

$$\Rightarrow F_{CB} = 10.5 \text{ kN (T)}$$

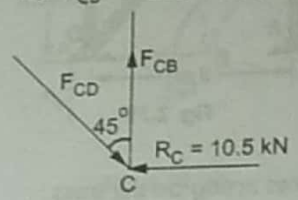


Fig. 2.74 (b) : Joint C

(d) Joint A : Assumed directions of axial forces are as shown in Fig. 2.74 (c).

$$\Sigma F_y = 0 \Rightarrow -7 + F_{AD} \sin 60° = 0$$

$$\Rightarrow F_{AD} = 8.08 \text{ kN (C)}$$

$$\Sigma F_x = 0 \Rightarrow F_{AB} - F_{AD} \cos 60° = 0$$

$$\Rightarrow F_{AB} = 4.04 \text{ kN (T)}$$

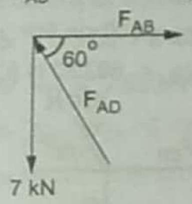


Fig. 2.74 (c) : Joint A

(e) Joint B : Assumed directions of axial forces are as shown in Fig. 2.74 (d).

$$\Sigma F_y = 0 \Rightarrow + B_y - F_{BC} - F_{BD} \sin 45° = 0$$

$$\Rightarrow F_{BD} = 4.95 \text{ kN (T)}$$

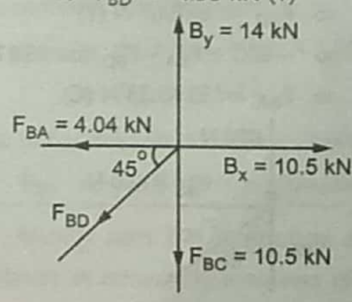


Fig. 2.74 (d) : Joint B

(f)

Sr. No.	Member	Force (kN)	Nature (C/T)
1	AB	4.04	T
2	AD	8.08	C
3	CD	14.85	C
4	CB	10.5	T
5	BD	4.95	T

Example 2.60 : *Determine the force in each member of the truss and state if the members are in tension or compression.*

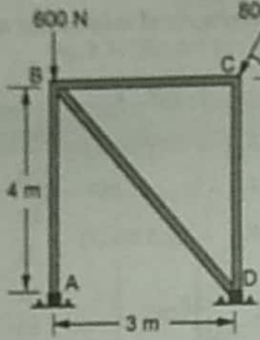


Fig. 2.75

Solution :

Given data : Forces acting on the truss :

600 N acting vertically downward.

800 N acting at 60° with horizontal.

Dimensions are as shown in Fig. 2.75.

To find : Magnitude and nature of force in members AB, BC, BD and CD.

(a) Joint C :

$$\Sigma F_x = 0 \Rightarrow -800 \cos 60° + F_{CB} = 0$$
$$\Rightarrow F_{CB} = 400 \text{ N (C)}$$
$$\Sigma F_y = 0 \Rightarrow F_{CD} - 800 \sin 60° = 0$$
$$\Rightarrow F_{CD} = 692.82 \text{ N (C)}$$

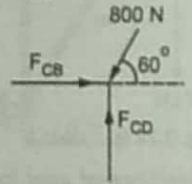


Fig. 2.75 (a) : Joint C

(b) Joint B :

$$\Sigma F_x = 0 \Rightarrow -F_{BC} + F_{BD} \sin 36.87° = 0$$
$$\Rightarrow F_{BD} = 666.67 \text{ N (T)}$$
$$\Sigma F_y = 0 \Rightarrow -600 + F_{BA} - F_{BD} \cos 36.87° = 0$$
$$\Rightarrow F_{BA} = 1133.33 \text{ N (C)}$$

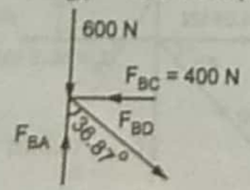


Fig. 2.75 (b) : Joint B

(c)

Sr. No.	Member	Force (N)	Nature (C/T)
1	CB	400	C
2	CD	692.82	C
3	BD	666.67	T
4	BA	1133.33	C

Example 2.61 : *Determine the force in members DF, DG and EG for truss loaded and supported as shown in Fig. 2.76.*

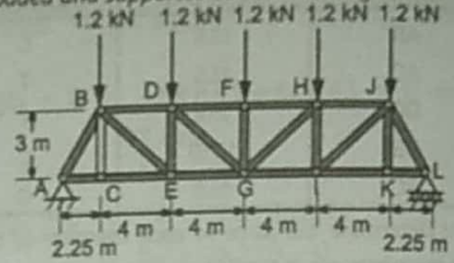


Fig. 2.76

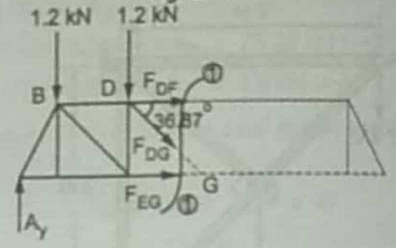


Fig. 2.76 (a)

Solution :

Given data : Forces acting on the truss are as shown in Fig. 2.76.

Dimensions are as shown in Fig. 2.76 (a).

To find : Forces in members DF, DG and EG.

(a) To find support reaction :

Applying conditions of equilibrium,

$$\Sigma M_A = 0 \Rightarrow 1.2 \times 2.25 + 1.2 \times 6.25 + 1.2 \times 10.25 + 1.2 \times 14.25 + 1.2 \times 18.25 - R_L \times 20.50 = 0$$
$$\Rightarrow R_L = 3 \text{ kN } (\uparrow)$$
$$\Sigma F_y = 0 \Rightarrow A_y + R_L - 1.2 \times 5 = 0$$
$$\Rightarrow A_y = 3 \text{ kN } (\uparrow)$$
$$\Sigma F_x = 0 \Rightarrow A_x = 0$$

(b) Section 1-1 cuts the members DF, DG and EG as shown in Fig. 2.76 (a).

Consider L.H.S. of truss.

Assume tension in members DF, DG and EG.

Applying conditions of equilibrium to L.H.S. part of truss,

$$\Sigma M_G = 0 \Rightarrow F_{DF} \times 3 - 1.2 \times 4 - 1.2 \times 8 + A_y \times 10.25 = 0$$
$$\Rightarrow F_{DF} = -5.45 \text{ kN}$$

Since value is (–) ve, assumed direction is wrong.

F_{DF} = **5.45 kN (Compression)** ... Ans.

$$\Sigma F_y = 0 \Rightarrow A_y - 1.2 - 1.2 - F_{DG}\sin 36.87^\circ = 0$$

$$\Rightarrow F_{DG} = \mathbf{1\ kN\ (Tension)} \quad \text{... Ans.}$$

$$\Sigma F_x = 0 \Rightarrow F_{DF} + F_{EG} + F_{DG}\cos 36.87^\circ = 0$$

$$\Rightarrow (-5.45) + F_{EG} + 1\cos 36.87^\circ = 0$$

$$\Rightarrow F_{EG} = \mathbf{4.65\ kN\ (Tension)} \quad \text{... Ans.}$$

Example 2.62 : *Determine the force in members BC, CK and KJ and state if these members are in tension or compression.*

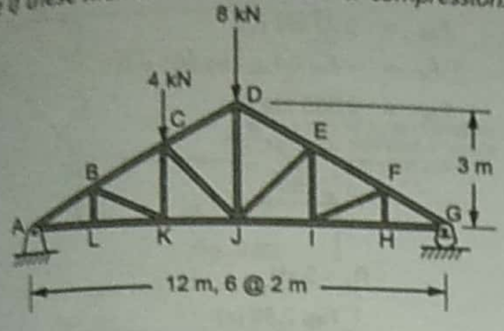


Fig. 2.77

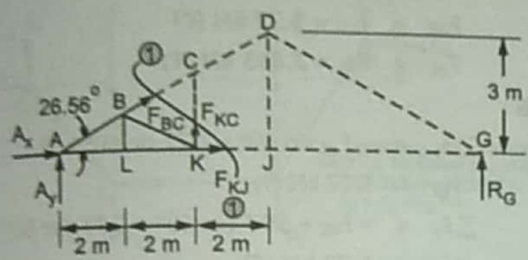


Fig. 2.77 (a)

Solution :

Given data : Forces acting on the truss are as shown.

Dimensions are as shown in Fig. 2.77 (a).

To find : Forces in members BC, CK and KJ.

(a) To find support reaction :

Applying conditions of equilibrium,

$$\Sigma M_A = 0 \Rightarrow 4 \times 4 + 8 \times 6 - R_G \times 12 = 0$$

$$\Rightarrow R_G = 5.33\ \text{kN}\ (\uparrow)$$

$$\Sigma F_y = 0 \Rightarrow A_y + R_G - 4 - 8 = 0$$

$$\Rightarrow A_y = 6.67\ \text{kN}\ (\uparrow)$$

$$\Sigma F_x = 0 \Rightarrow A_x = 0$$

(b) Section 1-1 cuts the members BC, CK and KJ as shown in Fig. 2.77 (a).

Consider L.H.S. of truss. Assume tension in members BC, CK and KJ.

Applying conditions of equilibrium to L.H.S. part of truss,

$$\Sigma M_A = 0 \Rightarrow F_{CK} \times 4 = 0$$

$$\Rightarrow \mathbf{F_{CK} = 0} \quad \text{... Ans.}$$

$$\Sigma F_y = 0 \Rightarrow F_{BC}\sin 26.56^\circ + 6.67 = 0$$

$$\Rightarrow F_{BC} = -14.9\ \text{kN}$$

$$\Rightarrow \mathbf{F_{BC} = 14.9\ kN\ (Compression)} \quad \text{... Ans.}$$

$$\Sigma F_x = 0 \Rightarrow F_{KJ} + F_{BC}\cos 26.56^\circ = 0$$

$$\Rightarrow F_{KJ} + (-14.9)\cos 26.56^\circ = 0$$

$$\Rightarrow \mathbf{F_{KJ} = 13.3\ kN\ (Tension)} \quad \text{... Ans.}$$

Example 2.63 : *Determine the force developed in members GB and GF of the bridge truss and state if these members are in tension or compression.*

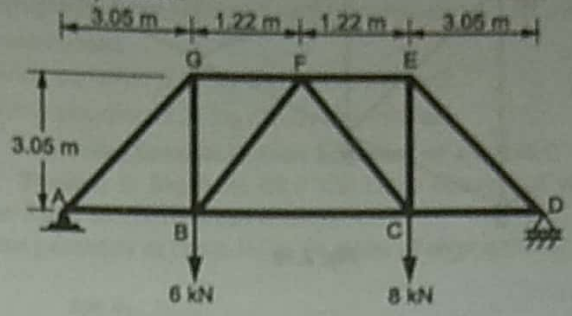


Fig. 2.78

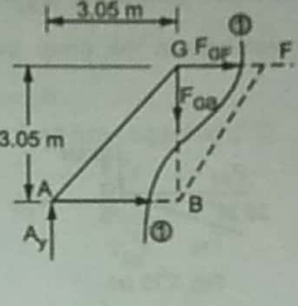


Fig. 2.78 (a)

Solution :

Given data : Forces acting on the truss are as shown in Fig. 2.78.

Dimensions are as shown in Fig. 2.78 (a).

To find : Forces in the members GB and GF.

(a) To find support reactions :

Applying conditions of equilibrium,

$$\Sigma M_A = 0 \Rightarrow 6 \times 3.05 + 8 \times 6.10 - R_D \times 8.54 = 0$$

$$\Rightarrow R_D = 7.86\ \text{kN}\ (\uparrow)$$

$$\Sigma F_y = 0 \Rightarrow A_y + R_D - 6 - 8 = 0$$

$$\Rightarrow A_y = 6.14\ \text{kN}\ (\uparrow)$$

$$\Sigma F_x = 0 \Rightarrow A_x = 0$$

(b) Section 1-1 cuts the members GF, GB and AB as shown in Fig. 2.78 (a).

Consider L.H.S. of the truss.

Applying conditions of equilibrium,

$$\Sigma M_B = 0 \Rightarrow A_y \times 3.05 + F_{GF} \times 3.05 = 0$$

$$\Rightarrow F_{GF} = -6.14\ \text{kN}$$

Since value of F_{GF} is (–) ve, assumed direction is wrong.

$$F_{GF} = \mathbf{6.14\ kN\ (Compression)} \quad \text{... Ans.}$$

Example 2.64 : *Identify zero force members and find magnitude and nature of forces in remaining members of the truss as shown in Fig. 2.79.*

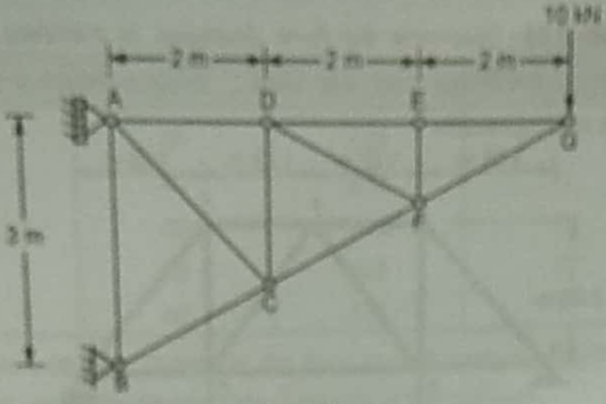


Fig. 2.79

Solution :

From geometry of truss $\angle AGB = \tan^{-1}\left(\frac{3}{6}\right) = 26.56°$

(a) Zero force members in the given truss are : EF, DE, CD, AC.

(b) Joint G.

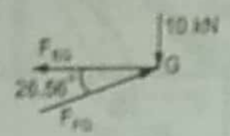


Fig. 2.79 (a)

$$\Sigma F_y = -10 + F_{FG} \sin 26.56° = 0$$
$$\Rightarrow \quad F_{FG} = 22.36 \text{ kN (C)}$$
$$\Sigma F_x = -F_{EG} + 22.36 \cos 26.36° = 0$$
$$\Rightarrow \quad F_{EG} = 20.03 \text{ kN (T)}$$

(c) Joint E : F_{ED} = F_{EG} = **20.03 kN (T)** ... **Ans.**

(d) Joint D : F_{DA} = F_{DA} = **20.03 kN (T)** ... **Ans.**

(c) Joint F : F_{FG} = F_{FC} = **22.36 kN (C)** ... **Ans.**

(d) Joint C : F_{CB} = F_{CF} = **22.36 kN (C)** ... **Ans.**

At Joint A : Reaction at roller support is horizontal with magnitude 20.03 kN.

Hence, $F_{AB} = 0$

Sr. No.	Member	Force (kN)	Nature (C/T)
1.	AB, EF, DF, CD, AC	0	-
2.	ED, EG, DA	20.03	T
3.	FG, FC, CB	22.36	C

Example 2.65 : *Determine the force in each member of the truss and state if the members are in tension or compression. Assume L = 2 m and P = 10 kN.*

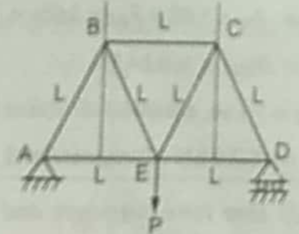


Fig. 2.80

Solution :

(a) To find support reactions at A and D :

$$R_A = 5 \text{ kN} \uparrow$$

$(\because A_x = 0)$

$$R_D = 5 \text{ kN} \uparrow$$

(b) Joint A :

$$\Sigma F_y = +5 - F_{AB} \sin 60° = 0$$
$$\Rightarrow \quad F_{AB} = 5.77 \text{ kN (C)}$$
$$\Sigma F_x = -F_{AE} - F_{AB} \cos 60° = 0$$
$$\Rightarrow \quad F_{AE} = 2.885 \text{ kN (T)}$$

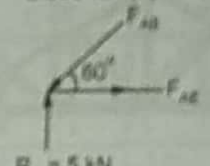


Fig. 2.80 (a)

Due to symmetry of truss.

$$F_{AB} = F_{CD} = \mathbf{5.77 \text{ kN (C)}}$$ ... **Ans.**

$$F_{AE} = F_{DE} = \mathbf{2.885 \text{ kN (T)}}$$ ... **Ans.**

(C) Joint B :

$$\Sigma F_y = -F_{BE} \sin 60° + 5.77 \sin 60° = 0$$
$$\therefore \quad F_{BE} = 5.77 \text{ kN (T)}$$
$$\Sigma F_x = -F_{BC} + 5.77 \cos 60° + 5.77 \cos 60°$$
$$\therefore \quad F_{BC} = \mathbf{5.77 \text{ kN (C)}}$$ ... **Ans.**

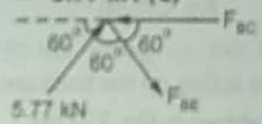


Fig. 2.80 (b)

Due to symmetry.

$$F_{BE} = F_{CE} = \mathbf{5.77 \text{ kN (T)}}$$ ... **Ans.**

2.18 CABLES

Cables are used to support the suspended loads. They are used in suspension bridges, transmission lines, etc.

Analysis of a cable subjected to concentrated loads is considered.

The following assumptions are made for analysis of a cable :

- A cable is perfectly flexible and inextensible.
- Self weight of a cable is neglected.

Since a cable is assumed to be perfectly flexible, it offers no resistance to bending, hence tensile force acting in the cable is always tangent to the cable at points along the length.

Since a cable is assumed to be perfectly inextensible, it has constant length before and after the load is applied.

Hence, once a cable is subjected to loading, whole cable or any part of it is treated as a rigid body.

When a weightless cable is subjected to several concentrated loads, it takes a form of several straight line segments, subjected to a constant tensile force.

2.18.1 Analysis of a Cable

Analysis of a cable includes determination of support reactions at both ends of a cable. It also includes determination of tension in each segment of a cable. Sometimes, it includes determination of geometry of cable viz. length of segment or sag of a point or slope of a segment, etc.

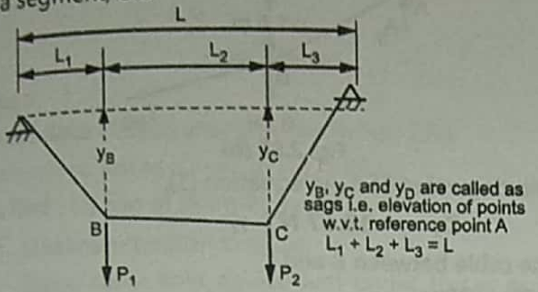


Fig. 2.81

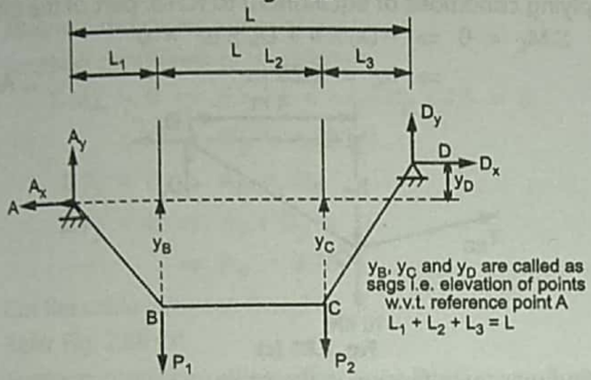


Fig. 2.81 (a)

Let us consider the cable shown in Fig. 2.81 for analysis.

1. **To Determine Support Reactions at Both Ends of a Cable :** Let A_x, A_y be components of reaction at end A. Let D_x, D_y be components of reaction at end D. Refer Fig. 2.81 (a).

To find four unknowns, four conditions of equilibrium will be required. Out of four, three conditions of equilibrium are applied to the entire cable.

$$\Sigma F_x = 0 \left(\begin{smallmatrix}\rightarrow & +\\ \leftarrow & -\end{smallmatrix}\right) \quad -A_x + D_x = 0 \qquad \text{... (2.1)}$$

$$\Sigma F_y = 0 \; (\uparrow +, \downarrow -) \quad A_y + D_y = 0 \qquad \text{... (2.2)}$$

$$\Sigma M \text{ at end } D = 0 \; (\circlearrowright +, \circlearrowleft -) + A_y \times L + A_x \times y_D - P_1 \times (L_2 + L_3) - P_2 \times (L_3) = 0 \qquad \text{... (2.3)}$$

To get fourth unknown, a cable is cut near a point where a concentrated load acts and

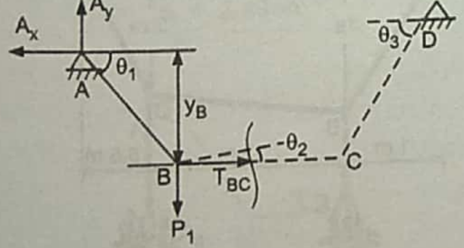


Fig. 2.82

Consider F.B.D. of L.H.S. of the cable as shown in Fig. 2.82.

Applying conditions of equilibrium,

ΣM at point B (L.H.S.) = 0 $(\circlearrowright +, \circlearrowleft -)$

$$-A_x \times y_B + A_y \times L_1 = 0 \qquad \text{... (2.4)}$$

Solving equations (2.3) and (2.4) simultaneously, A_x and A_y can be determined.

Solving equation (2.1), D_x can be determined.

Solving equation (2.2), D_y can be determined.

2. **To Determine Tension in Each Segment of a Cable :**

(i) **Tension in Segment AB :** Cut cable between A and B. Consider F.B.D. of L.H.S. part of section.

From geometry of cable, let θ_1 be slope of segment AB.

$$\tan \theta_1 = \frac{y_B}{L_1}$$

Applying conditions of equilibrium,

$$\Sigma F_x = 0 \left(\begin{smallmatrix}\rightarrow & +\\ \leftarrow & -\end{smallmatrix}\right)$$

$$-A_x + T_{AB} \cos \theta_1 = 0 \qquad \text{... (2.5)}$$

T_{AB} can be determined from equation (2.5).

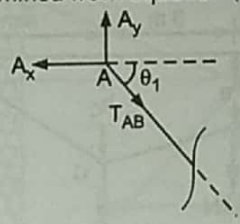


Fig. 2.83

(ii) **Tension in Segment BC :** Consider F.B.D. as shown in Fig. 2.83.

Let θ_2 be slope of segment BC.

Applying conditions of equilibrium,

$$\Sigma F_x = 0 \left(\begin{smallmatrix}\rightarrow & +\\ \leftarrow & -\end{smallmatrix}\right)$$

$$-A_x + T_{BC} \cos \theta_2 = 0 \qquad \text{... (2.6)}$$

$$\Sigma F_y = 0 \; (\uparrow +, \downarrow -)$$

$$A_y - P_1 - T_{BC} \sin \theta_2 = 0 \qquad \text{... (2.7)}$$

T_{BC} and θ_2 can be determined by solving equations (2.6) and (2.7) simultaneously.

3. **To Determine Sag at Point C :** Consider R.H.S. part of the cable when a cable is cut between points B and C. Draw F.B.D. of R.H.S. part only.

Consider equilibrium condition as follows :

$$\Sigma M \text{ at end } C = 0 \; (\uparrow +, \downarrow -) \qquad \text{... (2.8)}$$

Sag of point C w.r.t. end D can be determined from equation (2.8).

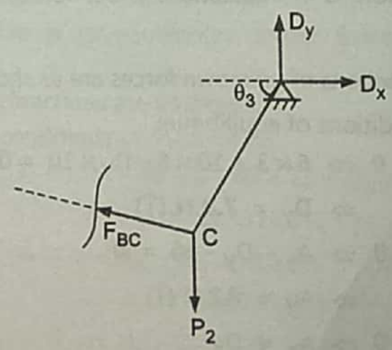


Fig. 2.84

4. To Determine Maximum Tension in Cable :

Note : If the cable is subjected to only vertical loads, horizontal component of tension in each segment is same as that of support reaction.

i.e. $A_x = D_x = T_{AB} \cos\theta_1 = T_{BC}\cos\theta_2 = T_{CD}\cos\theta_3$.

$\therefore$ Tension is maximum, when $\cos\theta$ is minimum. $\cos\theta$ is minimum, when θ is maximum. By this method, maximum tension can be determined.

OR

Find tension in each part of the cable by drawing F.B.D. to get maximum tension.

NUMERICALS ON CABLES

Example 2.66 : *Two loads are suspended as shown from the cable ABCD. Knowing that h_B = 1.8 m, determine (a) the distance h_C, (b) the components of the reaction at D, (c) the maximum tension in the cable.*

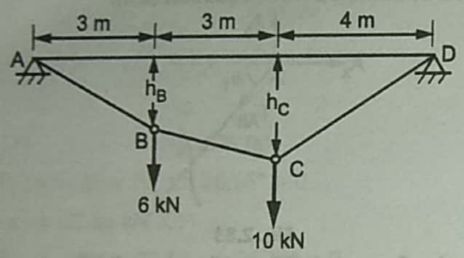


Fig. 2.85

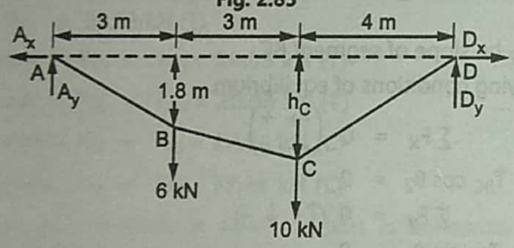


Fig. 2.85 (a)

Solution :

Given data : Loads are acting as shown in Fig. 2.85.

Dimensions are as shown in Fig. 2.85 (a).

To find : (a) Distance 'h_C'.

(b) Components of reaction at D. (c) Maximum tension in the cable.

(a) Entire cable is in equilibrium under forces as shown in Fig. 2.85 (a).

Assumed directions of unknown forces are as shown.

Applying conditions of equilibrium,

$\Sigma M_A = 0 \Rightarrow 6\times 3 + 10\times 6 - D_y \times 10 = 0$

$\Rightarrow D_y = \mathbf{7.8\ N\ (\uparrow)}$ ... **Ans.**

$\Sigma F_y = 0 \Rightarrow A_y + D_y - 16 = 0$

$\Rightarrow A_y = 8.2\ N\ (\uparrow)$

$\Sigma F_x = 0 \Rightarrow A_x = D_x$ ... (1)

(b) Cut the cable between B and C.

Refer Fig. 2.85 (b).

Applying conditions of equilibrium to L.H.S. part of the cable,

$\Sigma M_B = 0 \Rightarrow -A_x \times 1.8 + A_y \times 3 = 0$

$\Rightarrow A_x = 13.67\ N\ (\leftarrow)$

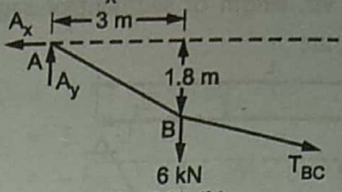


Fig. 2.85 (b)

Substituting value of A_x in equation (1),

$D_x = \mathbf{13.67\ N\ (\rightarrow)}$ ... **Ans.**

(c) Cut the cable between B and C.

Refer Fig. 2.85 (c).

Applying conditions of equilibrium to R.H.S. part of the cable,

$\Sigma M_C = 0 \Rightarrow -D_y \times 4 + D_x \times h_C = 0$

$\Rightarrow \mathbf{h_C = 2.28\ m}$ ... **Ans.**

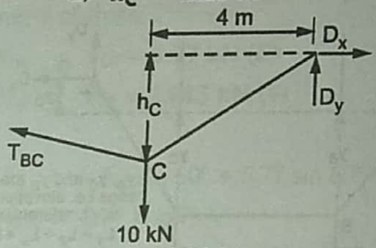


Fig. 2.85 (c)

(d) To find maximum tension in the cable :

$T_{AB}\cos\theta_1 = T_{BC}\cos\theta_2 = T_{CD}\cos\theta_3 = A_x = D_x$

i.e. Horizontal component of tension is same at all points.

Tension T is maximum when $\cos\theta$ is minimum.

$\cos\theta$ is minimum when θ is maximum.

From geometry,

$\theta_1 = 30.96^\circ,\ \theta_2 = 9.09^\circ,\ \theta_3 = 29.68^\circ$

Since θ_1 is maximum, tension in part AB of the cable is maximum.

$$\text{Maximum tension} = \frac{A_x}{\cos\theta_1} = \frac{13.67}{\cos 30.96}$$

$= \mathbf{15.94\ N}$ ... **Ans.**

$= 3.19\ kN$

Example 2.67 : *Cable ABCD supports the 4 N flowerpot E and 6 N flowerpot F. Determine the maximum tension in the cable and the sag of point B.*

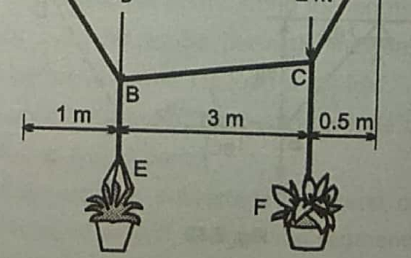


Fig. 2.86

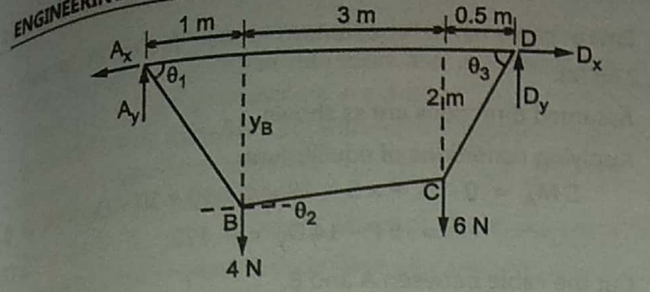


Fig. 2.86 (a)

Solution :

Given data : Loads are as shown in Fig. 2.86.

Dimensions are as shown in Fig. 2.86 (a).

To find : (a) Sag at point B : y_B

(b) Maximum tension in cable.

(a) Entire cable is in equilibrium under forces as shown in Fig. 2.86 (a).

Assumed directions are as shown.

Applying conditions of equilibrium,

$$\Sigma M_A = 0 \Rightarrow 4 \times 1 + 6 \times 4 - D_y \times 4.5 = 0$$

$$\Rightarrow D_y = 6.22 \text{ kN}$$

$$\Sigma F_x = 0 \Rightarrow A_x = D_x \quad \text{... (1)}$$

$$\Sigma F_y = 0 \Rightarrow A_y + D_y = 10$$

$$\Rightarrow A_y = 3.78 \text{ N}$$

(b) Cut the cable between B and C.

Refer Fig. 2.86 (b).

Applying conditions of equilibrium to R.H.S. part of cable,

$$\Sigma M_C = 0 \Rightarrow D_x \times 2 - D_y \times 0.5 = 0$$

$$\Rightarrow D_x = 1.555 \text{ N } (\rightarrow)$$

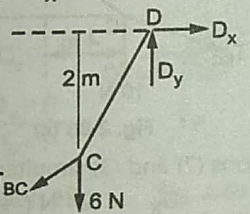


Fig. 2.86 (b) : F.B.D. of part CD

Substituting value of D_x in equation (i),

$$A_x = 1.555 \text{ N } (\leftarrow)$$

(c) Cut the cable between A and B.

Refer Fig. 2.86 (c).

Applying conditions of equilibrium to L.H.S. part of cable,

$$\Sigma M_B = 0 \Rightarrow -A_x \times y_B + A_y \times 1 = 0$$

$$\Rightarrow y_B = \mathbf{2.43 \text{ m}} \quad \text{... Ans.}$$

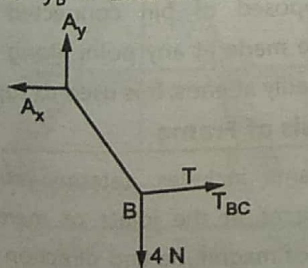


Fig. 2.86 (c) : F.B.D. of part AB

(d) To find maximum tension in the cable.

(i) Horizontal component of tension in each part of the cable is same at all points.

$$T_{AB} \cos \theta_1 = T_{BC} \cos \theta_2 = T_{CD} \cos \theta_3 = A_x \text{ or } D_x$$

Tension T is maximum when cos θ is minimum.

cos θ is minimum when θ is maximum.

From geometry, $\theta_1 = 67.63°$, $\theta_2 = 8.15°$, $\theta_3 = 75.96°$

(ii) Since θ_3 is maximum, tension in part CD of cable is maximum.

$$\text{Maximum tension} = \frac{D_x}{\cos 75.96°} = \mathbf{6.41 \text{ N}} \quad \text{... Ans.}$$

Example 2.68 : *The cable segments support the loading as shown in Fig. 2.87. Determine the distance x_B from the force at B to point A.*

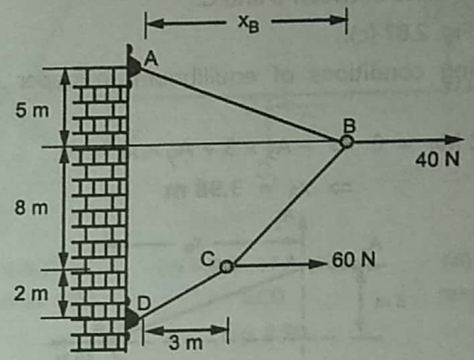


Fig. 2.87

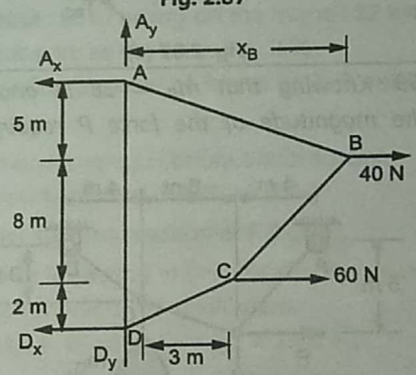


Fig. 2.87 (a)

Solution :

Given data : Loads are acting as shown in Fig. 2.87.

Dimensions are as shown in Fig. 2.87 (a).

To find : (a) Distance x_B.

(a) Entire cable is in equilibrium under forces as shown in Fig. 2.87 (a).

Assumed directions are as shown.

Applying conditions of equilibrium,

$$\Sigma M_A = 0 \Rightarrow -40 \times 5 - 60 \times 13 + D_x \times 15 = 0$$

$$\Rightarrow D_x = 65.33 \text{ N } (\leftarrow)$$

$$\Sigma F_x = 0 \Rightarrow A_x + D_x = 100 \text{ N}$$

$$\Rightarrow A_x = 34.67 \text{ N } (\leftarrow)$$

$$\Sigma F_y = 0 \Rightarrow A_y - D_y = 0 \quad \text{... (1)}$$

(b) Cut the cable between B and C.

Refer Fig. 2.87 (b).

Applying conditions of equilibrium to lower part of the cable,

$$\Sigma M_C = 0 \Rightarrow D_x \times 2 - D_y \times 3 = 0$$

$$\Rightarrow D_y = 43.55 \text{ N } (\downarrow)$$

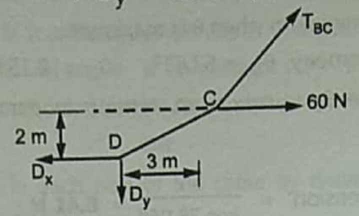


Fig. 2.87 (b) : F.B.D. of part CD

Substituting value of D_y in equation (i),

$$A_y = 43.55 \text{ N } (\uparrow)$$

(c) Cut the cable between B and C.

Refer Fig. 2.87 (c).

Applying conditions of equilibrium to upper part of the cable,

$$\Sigma M_B = 0 \Rightarrow -A_x \times 5 + A_y \times x_B = 0$$

$$\Rightarrow x_B = \mathbf{3.98 \text{ m}} \qquad \textbf{... Ans.}$$

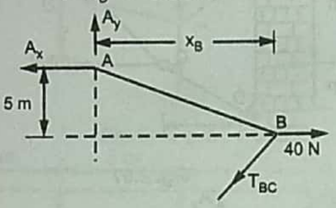


Fig. 2.87 (c)

Example 2.69 : *Knowing that m_B = 18 N and m_C = 10 N, determine the magnitude of the force P required to maintain equilibrium.*

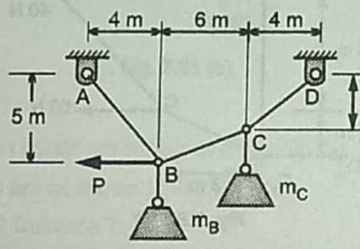


Fig. 2.88

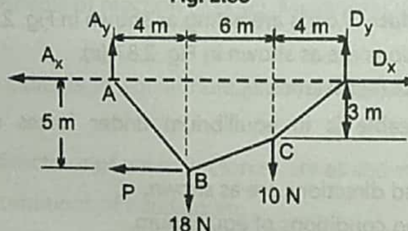


Fig. 2.88 (a)

Solution :

Given data : Loads are acting as shown in Fig. 2.88.

Dimensions are as shown in Fig. 2.88 (a).

To find : (a) Force 'P'.

(a) Entire cable is in equilibrium under forces as shown in Fig. 2.88 (a).

Assumed directions are as shown.

Applying conditions of equilibrium,

$$\Sigma M_A = 0 \Rightarrow P \times 5 + 18 \times 4 + 10 \times 10 - D_y \times 14 = 0$$

$$\Rightarrow 5P - 14 D_y = -172 \qquad ... (1)$$

(b) Cut the cable between A and B.

Refer Fig. 2.88 (b).

Applying conditions of equilibrium to R.H.S. part of the cable,

$$\Sigma M_B = 0 \Rightarrow 10 \times 6 - D_y \times 10 + D_x \times 5 = 0$$

$$\Rightarrow 5 D_x - 10 D_y = -60 \qquad ... (2)$$

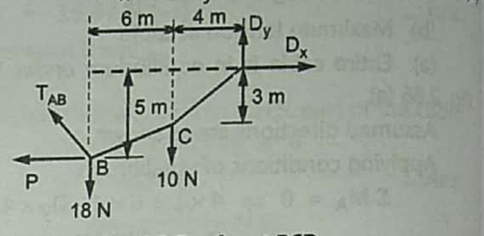


Fig. 2.88 (b) : F.B.D. of part BCD

(c) Cut the cable between B and C.

Refer Fig. 2.88 (c).

Applying conditions of equilibrium to R.H.S. part of the cable

$$\Sigma M_C = 0 \Rightarrow D_x \times 3 - D_y \times 4 = 0$$

$$\Rightarrow 3 D_x - 4 D_y = 0 \qquad ... (3)$$

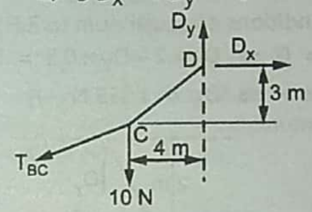


Fig. 2.88 (c)

Solving equations (2) and (3) simultaneously,

$$D_x = 24 \text{ N } (\rightarrow)$$

$$D_y = 18 \text{ N } (\uparrow)$$

Substituting D_x and D_y in equation (1),

$$P = \mathbf{16 \text{ N}} \qquad \textbf{... Ans.}$$

2.19 MULTIFORCE MEMBER

When a member is in equilibrium under more than two forces, anywhere on the member it is called multiforce member.

2.20 FRAMES

Frame is composed of pin connected multiforce members connections are made at any point along the length of member and not necessarily at ends. It is used to support loads.

2.20.1 Analysis of Frame

Analysis of frame includes determination of magnitude and direction of forces at the joints of members. It also includes determination of magnitude and direction of support reaction of frame.

For analysis, frame is disassembled in its members and isolated from supports. Two force members and multiforce members are identified. F.B.D.s of members are drawn. Forces common to any two contacting members act with equal magnitudes but opposite sense on respective members.

Conditions of equilibrium i.e.

$\sum F_x = 0$, $\sum F_y = 0$ and $\sum$ Moment about any point $= 0$ are applied to the members, determine unknown forces acting on the members.

Let us consider the frame shown in Fig. 2.89 for analysis.

Frame consists of three members AB, ADC and BDE. Pulley E and rope passing over the pulley. F.B.D.s of members are as shown.

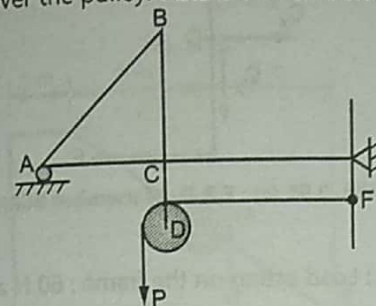


Fig. 2.89

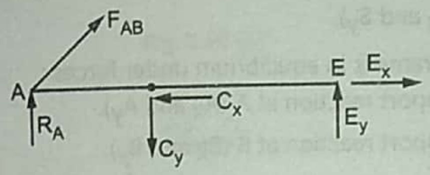


Fig. 2.89 (a) : F.B.D. of member ADC

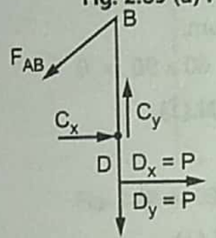


Fig. 2.89 (b) : F.B.D. of member BDE

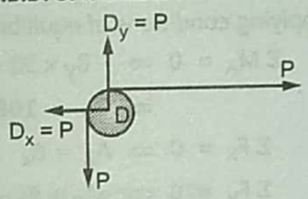


Fig. 2.89 (c) : F.B.D. of pulley pinned to member at E

Sense of forces are assumed while drawing F.B.D.s of members. Correct sense of force is determined after solving for equilibrium conditions for each member.

NUMERICAL EXAMPLES ON FRAMES

Example 2.70 : *For the frame and loading shown, determine the components of all forces acting on member ABE.*

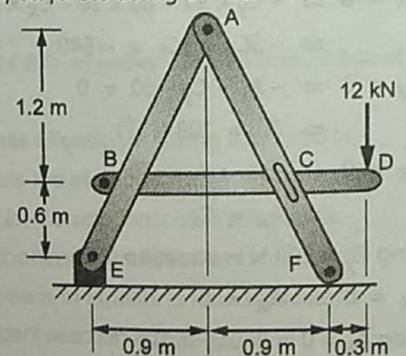


Fig. 2.90

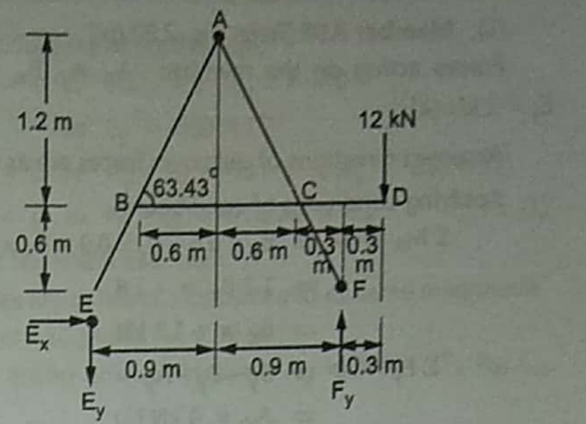
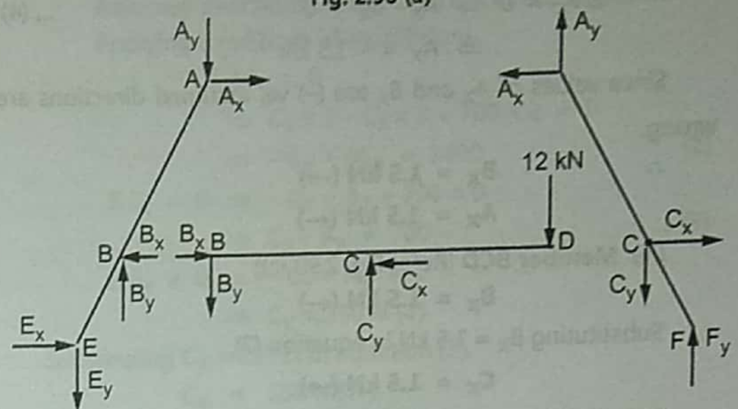


Fig. 2.90 (a)



(b) F.B.D. of member ABE (c) F.B.D. of member BCD (d) F.B.D. of member ACF

Fig. 2.90

Solution :

Given data : Load acting on the frame : 12 kN at D.

Dimensions are as shown in Fig. 2.90.

To find : Components of forces at A, B and E i.e. A_x, A_y, B_x, B_y, E_x and E_y.

(a) Whole frame is in equilibrium under forces :

(i) Support reaction at E (E_x and E_y)

(ii) Roller support reaction at F (F_y)

(iii) Load 12 kN acting at D vertically downward.

Applying conditions of equilibrium,

$$M_E = 0 \Rightarrow -F_y \times 1.8 + 12 \times 2.1$$

$$0 \Rightarrow \mathbf{F_y = 14\ kN\ (\uparrow)}$$

$$\Sigma F_y = 0 \Rightarrow -E_y + F_y - 12 = 0 \Rightarrow E_y = 2\text{ kN } (\downarrow)$$

$$\Sigma F_x = 0 \Rightarrow \mathbf{E_x = 0} \quad \text{... (1)}$$

(b) Dismember the frame as shown.

At pin, forces are collinear, opposite and of same magnitude.

(i) **Member BCD** [Refer Fig. 2.90 (c)].

Forces acting on the member : B_x, B_y, C_x, C_y and 12 kN.

Assumed directions of unknown forces are as shown.

Applying conditions of equilibrium,

$$\Sigma M_B = 0 \Rightarrow -C_y \times 1.2 + 12 \times 1.8 = 0$$

$$\Rightarrow \mathbf{C_y = 18\ kN\ (\uparrow)}$$

$$\Sigma F_y = 0 \Rightarrow -B_y + C_y - 12 = 0$$

$$\Rightarrow \mathbf{B_y = 6\ kN\ (\downarrow)}$$

$$\Sigma F_x = 0 \Rightarrow B_x = C_x \quad \text{... (2)}$$

(ii) **Member ABE** [Refer Fig. 2.90 (b)]

Forces acting on the member : A_x, A_y, B_x, B_y = 6 kN (↑), E_y = 2 kN (↓)

Assumed directions of unknown forces are as shown.

Applying conditions of equilibrium,

$$\Sigma M_A = 0 \Rightarrow B_y \times 0.6 - E_y \times 0.9 + B_x \times 1.2 = 0$$

$$\Rightarrow 1.2\, B_x = -1.8 \qquad \text{... (3)}$$

$$\Rightarrow B_x = -1.5 \text{ kN}$$

$$\Sigma F_y = 0 \Rightarrow B_y - E_y - A_y = 0$$

$$\Rightarrow A_y = 4 \text{ kN } (\downarrow)$$

$$\Sigma F_x = 0 \Rightarrow A_x - B_x = 0 \qquad \text{... (4)}$$

$$\Rightarrow A_x = -1.5 \text{ kN}$$

Since values of A_x and B_x are (–) ve, assumed directions are wrong.

∴ $\quad$ **B_x = 1.5 kN (→)**

A_x = 1.5 kN (←)

(iii) **Member BCD** (Refer Fig. 2.90 (c))

B_x = 1.5 kN (←)

Substituting B_x = 1.5 kN in equation (2),

C_x = 1.5 kN (→)

(iv) Check :

Member ACF (Refer Fig. 2.90 (d))

Forces acting on the member are :

A_x = 1.5 kN (→), A_y = 4 kN (↑), F_y = 14 kN (↑), C_x = 1.5 kN (←), C_y = 18 kN (↓)

$$\Sigma F_x = A_x - C_x = 0$$

$$\Sigma F_y = A_y - C_y + F_y = 0$$

Example 2.71 : *Determine the components of the reactions at A and B if 60 N load is applied at point D.*

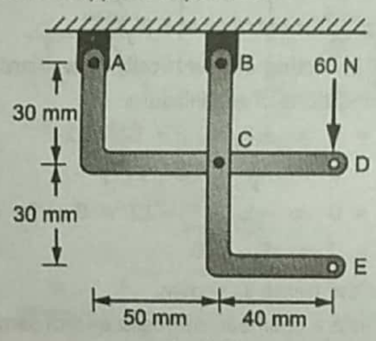


Fig. 2.91

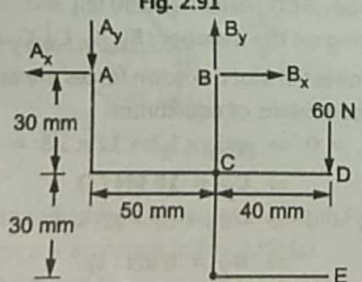


Fig. 2.91 (a)

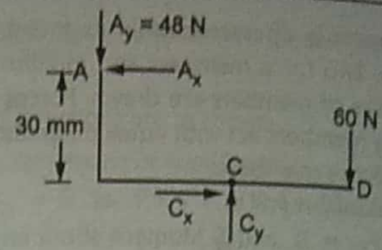


Fig. 2.91 (b) : F.B.D. of member ACD

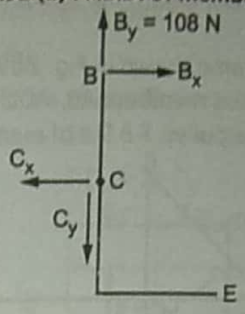


Fig. 2.91 (c) : F.B.D. of member BCE

Solution :

Given data : Load acting on the frame : 60 N at D.

Dimensions are as shown in Fig. 2.91.

To find : Components of the reactions at A (i.e. A_x and A_y) and B (i.e. B_x and B_y).

(a) Whole frame is in equilibrium under forces :

(i) Support reaction at A (A_x and A_y).

(ii) Support reaction at B (B_x and B_y).

(iii) Load 60 N acting at D vertically downward.

Applying conditions of equilibrium,

$$\Sigma M_A = 0 \Rightarrow -B_y \times 50 + 60 \times 90 = 0$$

$$\Rightarrow \mathbf{B_y = 108 \text{ N } (\uparrow)} \qquad \text{... Ans.}$$

$$\Sigma F_x = 0 \Rightarrow A_x = B_x \qquad \text{... (1)}$$

$$\Sigma F_y = 0 \Rightarrow -A_y + B_y - 60 = 0$$

$$\Rightarrow \mathbf{A_y = 48 \text{ N } (\downarrow)} \qquad \text{... Ans.}$$

(b) Dismember the frame as shown.

(i) Member ACD : [Refer Fig. 2.91 (b)]

Forces acting on the member : A_x, A_y, C_x, C_y and 60 N

Assumed directions of unknown forces are as shown.

Applying the conditions of equilibrium,

$$\Sigma M_A = 0 \Rightarrow -C_y \times 50 + 60 \times 90 - C_x \times 30 = 0$$

$$\Rightarrow -5C_y + 3C_x = -540 \qquad \text{... (2)}$$

$$\Sigma F_y = 0 \Rightarrow -A_y + C_y - 60 = 0$$

$$\Rightarrow C_y = 108 \text{ N } (\uparrow) \qquad \text{... (3)}$$

$$\Sigma F_x = 0 \Rightarrow -C_x + A_x = 0$$

$$\Rightarrow A_x = C_x \qquad \text{... (4)}$$

Substituting C_y = 108 N in equation (2),

$$\mathbf{C_x = 0 \Rightarrow A_x = 0} \qquad \text{... Ans.}$$

Substituting A_x = 0 in equation (1),

$$\mathbf{B_x = 0} \qquad \text{... Ans.}$$

Example 2.72 : *Determine components of the reactions at A and E. Radius of pulley = 0.5 m.*

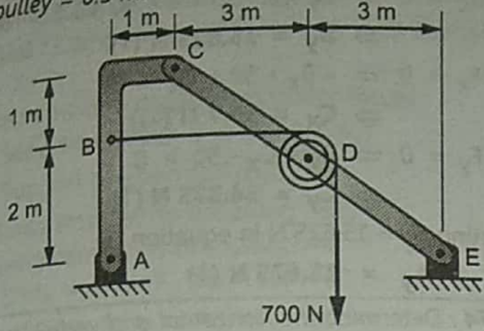


Fig. 2.92

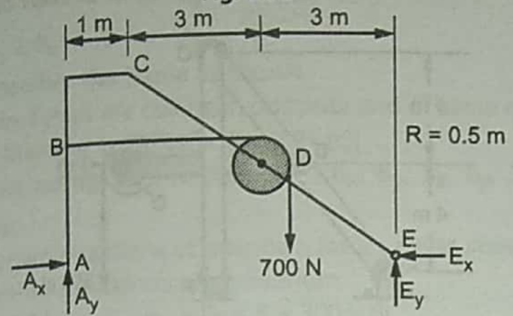


Fig. 2.92 (a)

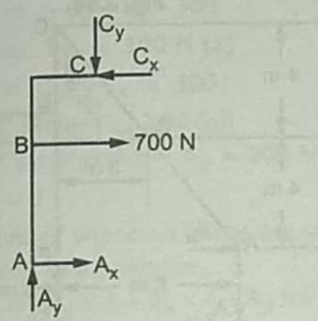
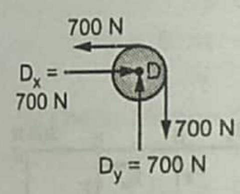


Fig. 2.92 (b) : F.B.D. of member ABC



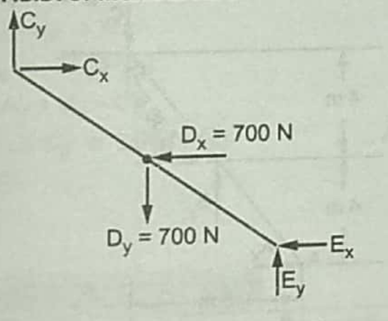


Fig. 2.92 (c) : F.B.D. of pulley **Fig. 2.92 (d) : F.B.D. of member CDE**

Solution :

Given data : Tension in string BD = 700 N

Dimensions are as shown in Fig. 2.92.

To find : (a) Components of reaction at A.

(b) Components of reaction at E.

(a) Entire frame is in equilibrium under forces :

(i) Support reaction at A.

(ii) Support reaction at E.

(iii) Load attached to string BD : 700 N

Applying conditions of equilibrium,

$$\Sigma M_A = 0 \Rightarrow 700 \times 4.5 - E_y \times 7 = 0$$

$$\Rightarrow E_y = 450 \text{ N } (\uparrow)$$

$$\Sigma F_y = 0 \Rightarrow A_y + E_y - 700 = 0 \Rightarrow A_y = 250 \text{ N } (\uparrow)$$

$$\Sigma F_x = 0 \Rightarrow A_x - E_x = 0 \Rightarrow A_x = E_x \quad \text{... (1)}$$

(b) Dismember the frame as shown.

At pin, forces are collinear, opposite and of same magnitude.

(i) Member ABC [Refer Fig. 2.92 (b)]

Forces acting on the member : A_y = 250 N (↑), A_x, C_x, C_y, 700 N (→).

Assumed directions of unknown forces are as shown.

Applying conditions of equilibrium,

$$\Sigma M_A = 0$$

$$\Rightarrow C_y \times 1 - C_x \times 3 + 700 \times 2 = 0$$

$$\Rightarrow -C_y + 3C_x = 1400 \quad \text{... (2)}$$

$$\Sigma F_x = 0 \Rightarrow -C_x + A_x + 700 = 0$$

$$\Rightarrow C_x - A_x = 700 \quad \text{... (3)}$$

$$\Sigma F_y = 0 \Rightarrow -C_y + A_y = 0$$

$$\Rightarrow C_y = 250 \text{ N } (\downarrow)$$

Substituting C_y = 250 N in equation (2),

$$C_x = 550 \text{ N } (\leftarrow)$$

Substituting C_x = 550 N in equation (3),

$$A_x = -150 \text{ N}$$

Since value of A_x is (–) ve, assumed direction of A_x is wrong.

$$\therefore \quad A_x = 150 \text{ N } (\leftarrow)$$

Reaction at support A = 291.55 N (59° R_A) **... Ans.**

(ii) Member CDE [Refer Fig. 2.92 (d)]

Forces acting on the member : C_y = 250 N (↑), C_x = 550 N (→), D_x = 700 N (←), D_y = 700 N (↓), E_x, E_y = 450 N (↑)

Applying conditions of equilibrium,

$$\Sigma F_x = 0 \Rightarrow C_x - D_x - E_x = 0$$

$$\Rightarrow E_x = -150 \text{ N}$$

Since value of E_x is (–) ve, assumed direction of E_x is wrong.

$$\therefore \quad E_x = 150 \text{ N } (\rightarrow)$$

Reaction at support E = 474.34 N (R_E 43.5°)**... Ans.**

Example 2.73 : *Determine the horizontal and vertical components of force at pins A, B and C.*

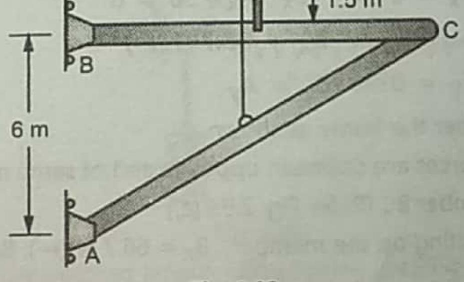


Fig. 2.93

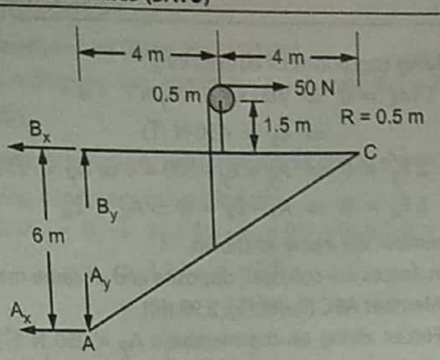
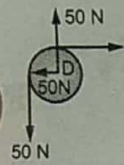
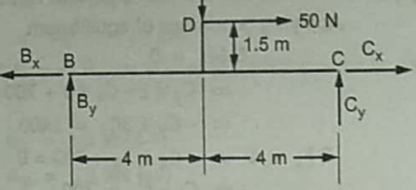


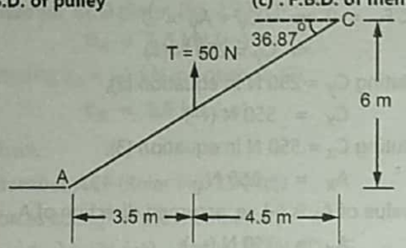
Fig. 2.93 (a)



(b) : F.B.D. of pulley



(c) : F.B.D. of member BC



(d) : F.B.D. of member AC

Fig. 2.93

Solution :

Given data : Tension in string : 50 N

Dimensions are as shown in Fig. 2.93.

To find : (a) Components of reaction at B.

(b) Components of reaction at C.

(a) Entire frame is in equilibrium under forces :

(i) Support reaction at B (B_x and B_y).

(ii) Support reaction at A (A_x and A_y).

(iii) Load attached to the string : 50 N

Applying conditions of equilibrium,

$$\Sigma M_B = 0 \Rightarrow + A_x \times 6 + 50 \times 2 = 0$$

$$\Rightarrow A_x = -16.67 \text{ N}$$

Since value of A_x is (–) ve, assumed direction of A_x is wrong.

$$\therefore \quad A_x = \mathbf{16.67\ N\ (\rightarrow)} \qquad \text{... Ans.}$$

$$\Sigma F_x = 0 \Rightarrow -B_x - A_x + 50 = 0$$

$$\Rightarrow \mathbf{B_x = 66.7\ N\ (\leftarrow)} \qquad \text{... Ans.}$$

$$\Sigma F_y = 0 \Rightarrow B_y = A_y \qquad \text{... (1)}$$

(b) Dismember the frame as shown.

At pin, forces are collinear, opposite and of same magnitude.

(i) Member BC [Refer Fig. 2.93 (c)]

Forces acting on the member : B_x = 66.7 N (←), B_y, 50 N (↓), 50 N (→), C_x, C_y.

Assumed directions of unknown forces are as shown.

Applying conditions of equilibrium,

$$\Sigma M_C = 0 \Rightarrow B_y \times 8 - 50 \times 4 + 50 \times 1.5 = 0$$

$$\Rightarrow \mathbf{B_y = 15.625\ N\ (\uparrow)} \qquad \text{... Ans.}$$

$$\Sigma F_x = 0 \Rightarrow -B_x + 50 + C_x = 0$$

$$\Rightarrow \mathbf{C_x = 16.7\ N\ (\rightarrow)} \qquad \text{... Ans.}$$

$$\Sigma F_y = 0 \Rightarrow B_y + C_y - 50 = 0$$

$$\Rightarrow \mathbf{C_y = 34.375\ N\ (\uparrow)} \qquad \text{... Ans.}$$

Substituting B_y = 15.625 N in equation (1),

$$\mathbf{A_y = 15.625\ N\ (\downarrow)} \qquad \text{... Ans.}$$

Example 2.74 : *Determine the horizontal and vertical components of force which the connecting pins at B, E and D exert on member BED.*

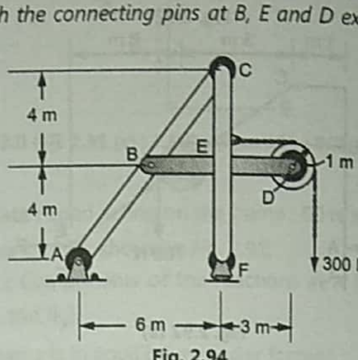


Fig. 2.94

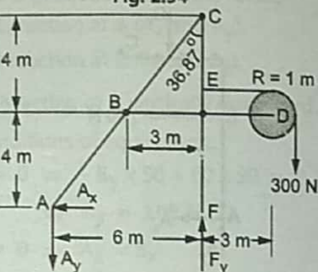
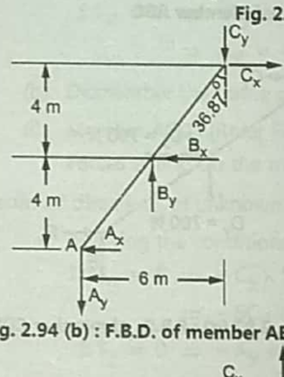


Fig. 2.94 (a)



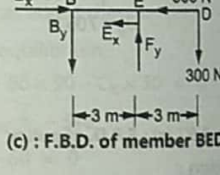


Fig. 2.94 (b) : F.B.D. of member ABC (c) : F.B.D. of member BED

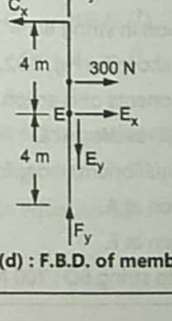


Fig. 2.94 (d) : F.B.D. of member CEF

Solution :

Given data : Load attached to the string : 300 N

Dimensions are as shown in Fig. 2.94.

To find : Components of force at B, E, D exerted on member BED.

(a) Entire frame is in equilibrium under forces :

(i) Support reaction at A (A_x and A_y).

(ii) Support reaction at F (F_y).

(iii) Load attached to the string : 300 N.

Applying conditions of equilibrium,

$$\Sigma M_A = 0 \Rightarrow -F_y \times 6 + 300 \times 10 = 0 \Rightarrow F_y = 500 \text{ N } (\uparrow)$$

$$\Sigma F_y = 0 \Rightarrow -A_y + F_y - 300 = 0$$

$$\Rightarrow A_y = +200 \text{ N } (\downarrow)$$

$$\Sigma F_x = 0 \Rightarrow A_x = 0$$

(b) Dismember the frame as shown.

At pin, forces are collinear, opposite and of same magnitude.

(i) Member BED [Refer Fig. 2.94 (c)]

Forces acting on the member : B_x, B_y, E_x, E_y, 300 N (←), 300 N (↓).

Assumed directions of unknown forces are as shown.

Applying conditions of equilibrium,

$$\Sigma M_B = 0 \Rightarrow -E_y \times 3 + 300 \times 6$$

$$= 0 \Rightarrow \mathbf{E_y = 600 \text{ N } (\uparrow)} \quad \text{... Ans.}$$

$$\Sigma F_y = 0 \Rightarrow -B_y + E_y - 300$$

$$= 0 \Rightarrow \mathbf{B_y = 300 \text{ N } (\downarrow)} \quad \text{... Ans.}$$

$$\Sigma F_x = 0 \Rightarrow B_x + E_x = 300 \quad \text{... (1)}$$

(ii) Member ABC [Refer Fig. 2.94 (b)]

Forces acting on the member : A_y = 200 N (↓), B_y = 300 N (↑), B_x, C_x and C_y, $A_x = 0$

Assumed directions of unknown forces are as shown.

Applying conditions of equilibrium,

$$\Sigma M_C = 0 \Rightarrow B_y \times 3 + B_x \times 4 - A_y \times 6 = 0$$

$$\Rightarrow \mathbf{B_x = 75 \text{ N } (\leftarrow)} \quad \text{... Ans.}$$

$$\Sigma F_x = 0 \Rightarrow C_x - B_x = 0$$

$$\Rightarrow C_x = 75 \text{ N } (\rightarrow)$$

$$\Sigma F_y = 0 \Rightarrow -A_y + B_y - C_y$$

$$= 0 \Rightarrow \mathbf{C_y = 100 \text{ N } (\downarrow)} \quad \text{... Ans.}$$

(iii) Check : Member CEF

$$\Sigma F_x = 0,$$

$$\Sigma F_y = 0$$

PROBLEMS FOR PRACTICE

Problem No. 1 : *A bar AB is hinged at 'A' and supported by rod CD. Force of 5 N is acting on the bar AB as shown. Find tensile force T in rod CD and reaction R_A at A.*

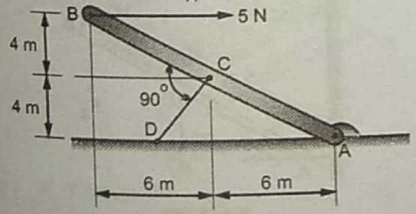


Fig. 2.95

Answer : T = 5.55 N, α = 67.43°, R_A = 5 N

Problem No. 2 : *A bar AB of weight W = 2 N is hinged to the vertical wall at A and supported at B by a cable BC. Determine the magnitude and direction of the reaction R_A at the hinge A and tensile force T in the cable BC.*

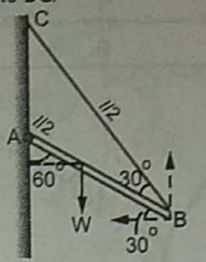


Fig. 2.96

Answer : T = 1.73 N.

Direction of reaction at hinge A = 30° with horizontal.

Magnitude of R_A = 1 N

Problem No. 3 : *A small ring A can slide without friction along the curved bar CD having radius 'a' as shown in Fig. 2.97. Determine angle α for which the system will be in equilibrium if loads W_1 and W_2 are acting as shown.*

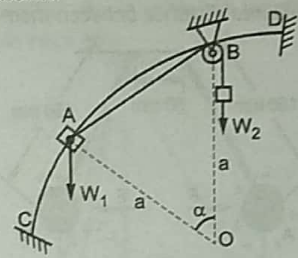


Fig. 2.97

Answer : $\alpha = \pi^c$ OR $2 \sin^{-1}\left(\frac{W_2}{2W_1}\right)$

Problem No. 4 : *The cord AB has a length of 5 m and is attached to the end B of the spring having a stiffness k = 10 N/m. The other end of the spring is attached to a roller C so that the spring remains horizontal as it stretches. If a 10 N weight is suspended from B, determine the necessary unstretched length of the spring, so that θ = 40° for equilibrium.*

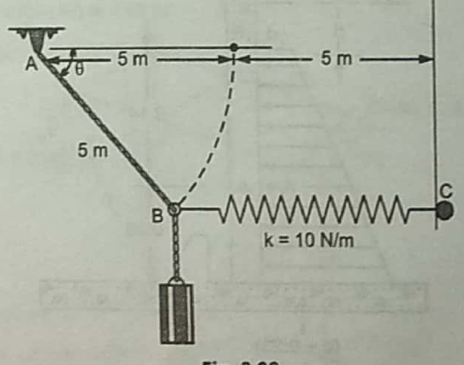


Fig. 2.98

Answer : Unstretched length of the spring = 4.9778 m

Problem No. 5 : *Determine the unstretched length of the spring AC if a force P = 80 N causes the angle θ = 60° for equilibrium. Cord AB is 2 m long.*

Spring has stiffness k = 50 N/m

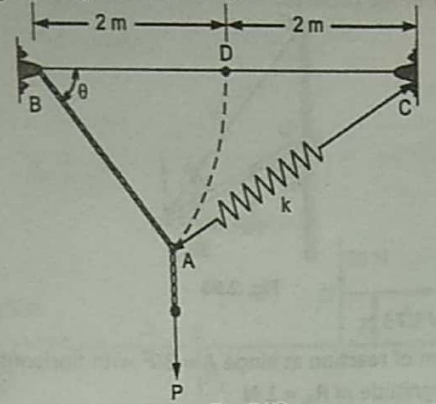


Fig. 2.99

Answer : Unstretched length of the spring = 2.664 m

Problem No. 6 : *Two electrically charged pith balls, each having a mass of 0.2 gm are suspended from light threads of equal length. Determine the resultant horizontal force of repulsion, F, acting on each ball if the measured distance between them is r = 200 mm.*

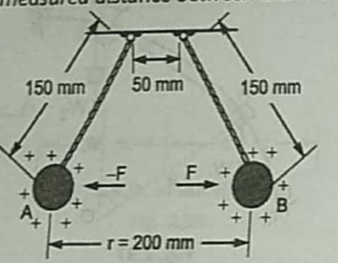


Fig. 2.100

Answer : $F = 1.133 \times 10^{-3}$ N

Problem No. 7 : *An automatic valve consists of a 230 × 230 mm square plate which is pivoted about a horizontal axis through A located at a distance 90 mm above the lower edge. The pressure exerted by water on the valve is proportional to the depth below water level as shown in Fig. 2.101. i.e. at upper edge γ × d (where γ = 10 kN/m³). Determine the minimum depth of water at the upper edge of the valve 'd' for which the valve will open.*

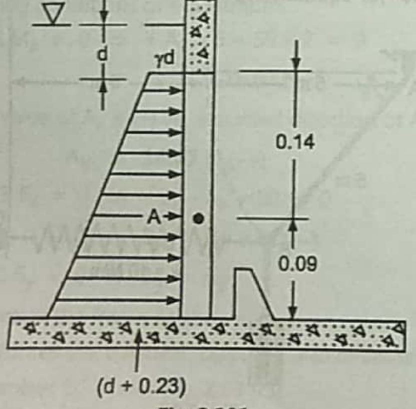


Fig. 2.101

Answer : *d = 0.0613 m*

Problem No. 8 : *A vertical gate AB is subjected to water pressure from one side as shown in Fig. 2.102. The weight per unit volume of water is 'w'. Determine P_a and P_b as reactions by supports to vertical gate.*

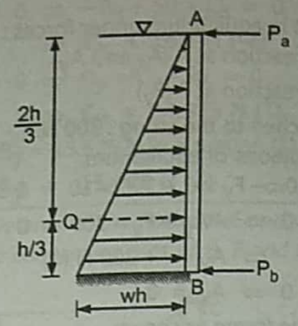


Fig. 2.102

Answer : $P_a = \dfrac{wh^2}{6}$ and $P_b = \dfrac{wh^2}{3}$

Problem No. 9 : *A concrete dam has rectangular cross-section of height h and width b, and is subjected to water pressure on one side. Determine minimum width b of this dam if the dam is not to overturn about point B when h = 4 m, ρ_{water} = 1000 kg/m³, $\rho_{concrete}$ = 2400 kg/m³.*

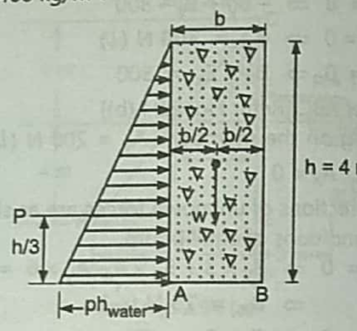


Fig. 2.103

Answer : b = 1.5 m

Problem No. 10 : *The boom supports two vertical loads F_1 and F_2. If cable CB can sustain a maximum load of 1500 N before it fails, determine the critical loads if $F_1 = 2\ F_2$. What is the magnitude of maximum reaction at pin A ?*

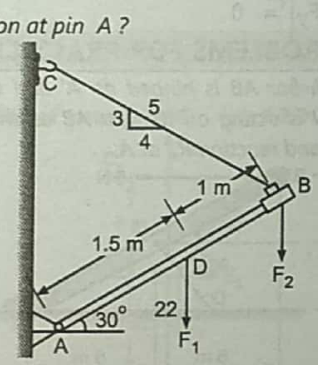


Fig. 2.104

Answer : F_2 = 723.66 N, F_1 = 1447.32 N

Problem No. 11 : *Determine reactions at the points of contact at A, B and C of the bar.*

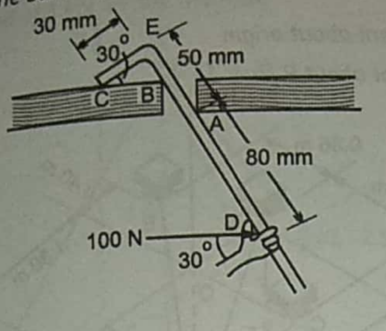


Fig. 2.105

Answer : R_A = 237.46 N, R_B = 122 N, R_C = 57.73 N

Problem No. 12 : *A tower guy wire is anchored by means of a bolt at A. The tension in the wire is 3000 N. Determine the components F_x, F_y and F_z of the force acting on the bolt.*

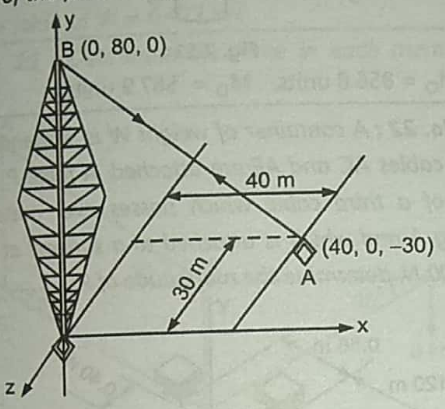


Fig. 2.106

Answers : F_x = – 1272 N, F_y = 2544 N, F_z = 954 N.

Problem No. 13 : *Knowing that the tension is 425 N in cable AB and 510 in cable AC, determine the magnitude and direction of the resultant of the forces exerted at A by the two cables.*

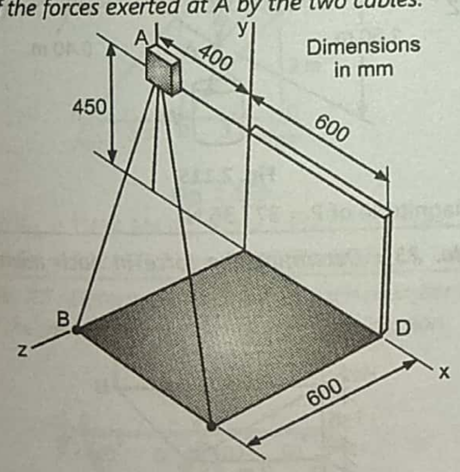


Fig. 2.107

Answer : $|\bar{R}|$ = 912.92 N, θ_x = 48.24°, θ_y = 63.41°, θ_z = 53.36°.

Problem No. 14 : *A frame ABC is supported in part by cable DBE which passes through a frictionless ring at B. Knowing that the tension in cable is 385 N, determine the magnitude and direction of the resultant of the forces exerted by the cable at B.*

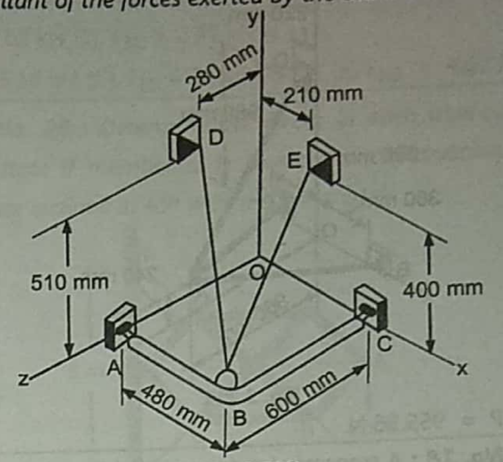


Fig. 2.108

Answer : $|R|$ = 747.83 N, θ_x = 60°, θ_y = 52.52°, θ_z = 52.04°.

Problem No. 15 : *A 12 kg circular plate of radius 175 mm is supported as shown by three wires of length 625 mm each. Determine tension in each wire.*

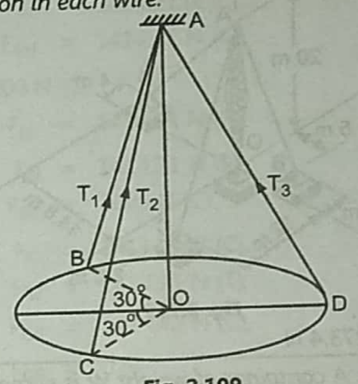


Fig. 2.109

Answer :
T_1 = 33 N
T_2 = 33 N
T_3 = 56.6 N

Problem No. 16 : *A crate is supported by three cables as shown in figure below. Determine the weight of the crate knowing that the tension in cable AB is 750 N.*

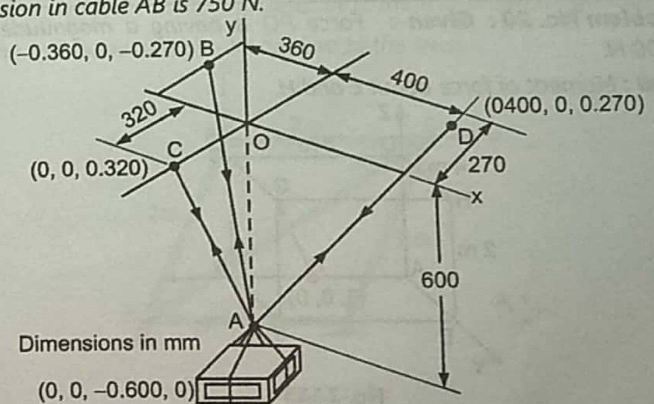


Fig. 2.110

Answer : Weight of crate is 2099 N.

Problem No. 17 : *Three cables are connected at A where force P is applied as shown in figure below. Find the value of P for which the tension in cable AD is 305 N.*

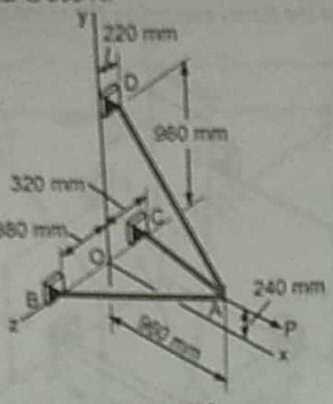


Fig. 2.111

Answer : P = 955.56 N.

Problem No. 18 : *A transmission tower is held by three guy wires attached to a pin at A and anchored by bolts at B, C and D. If the tension in the wire AB is 840 N, determine the vertical force P exerted by the tower on the pin at A.*

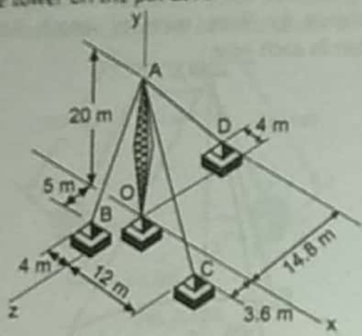


Fig. 2.112

Answer : P = 1273.4 N.

Problem No. 19 : *A container of weight W is suspended from ring A to which cables AC and AE are attached. A force P is applied to the end F of a third cable which passes over pulley at B and through ring A and which is attached to a support at D. Knowing that W = 100 N, determine the magnitude of P.*

(**Hint :** Tension is the same in all portions of the cable FBAD.)

Answer : P = 377.36 N.

Problem No. 20 : Given : *Force PQ is having a magnitude of 1600 N.*

Find : *Moment of force about E and H.*

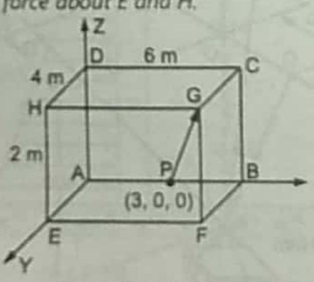


Fig. 2.113

Answer : M_E = 7724.86 N-m, M_H = 7972.30 N-m.

Problem No. 21 : $\bar{F} = 80\ i + 50\ j - 60\ k$ *passes through Q (6, 2, 5)*

Find : *(1) Moment about origin*

(2) Moment about P (3, 1, 1)

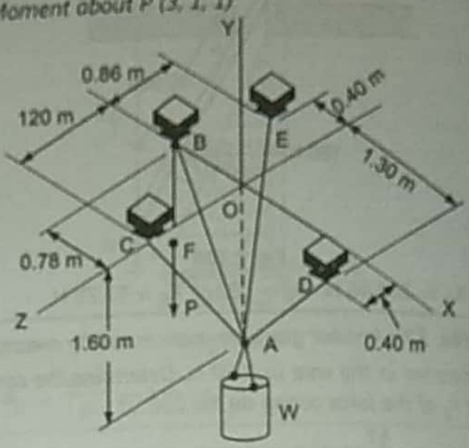


Fig. 2.114

Answer : M_O = 856.8 units, M_P = 567.9 units.

Problem No. 22 : *A container of weight W is suspended from ring A to which cables AC and AE are attached. A force P is applied to the end F of a third cable which passes over pulley at B and through ring A and which is attached to a support at D. Knowing that W = 100 N, determine the magnitude of P.*

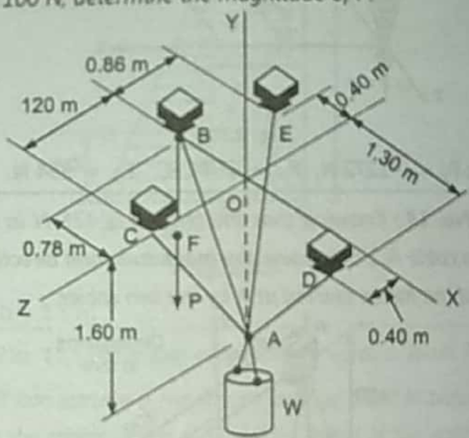


Fig. 2.115

Answer : Magnitude of P = 377.36 N

Problem No. 23 : *Determine the force in each member of the truss.*

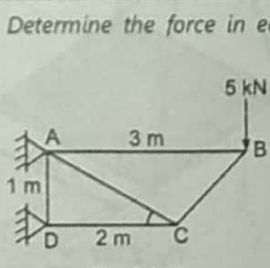


Fig. 2.116

Answer : F_{DC} = 15 kN (C), F_{AC} = 11.18 kN (T), F_{AB} = 5 kN (T), F_{BC} = 7.07 kN (C).

Problem No. 24 : *Determine the safe value of "W" that can be supported by the tripod shown in Fig. 2.117 without exceeding a compressive load of 3 kN in any member.*

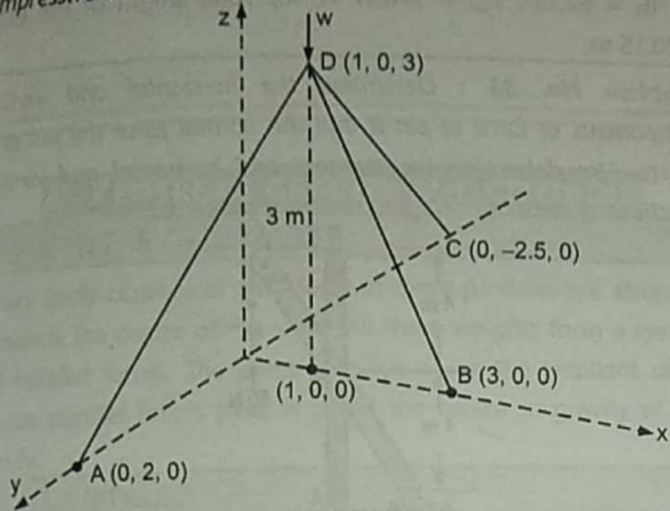


Fig. 2.117

Answer : Safe value of W = 6.477 kN

Problem No. 25 : *Determine the force in each member of the truss.*

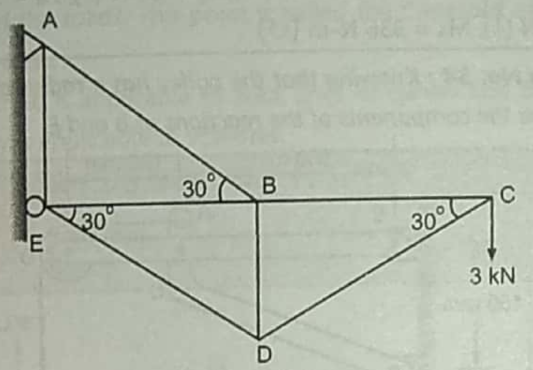


Fig. 2.118

Answer : F_{AB} = 12 kN (T), F_{AE} = 3 kN (C), F_{FD} = 6 kN (C), F_{EB} = 5.2 kN (C), F_{CD} = 6 kN (C), F_{CB} = 5.19 kN (T), F_{DB} = 6 kN (T).

Problem No. 26 : *Calculate the force in each number of truss.*

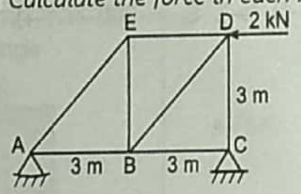


Fig. 2.119

Answer : (a) F_{AE} = 1.414 kN (C), F_{AB} = 1 kN (T); (e) F_{EB} = 1 kN (T)
(c) F_{CB} = 2 kN (T), F_{CD} = 1 kN (T); (d) F_{BD} = 1.414 kN (C)

Problem No. 27 : *Determine the force in each member of the truss and state if the members are in tension or compression.*

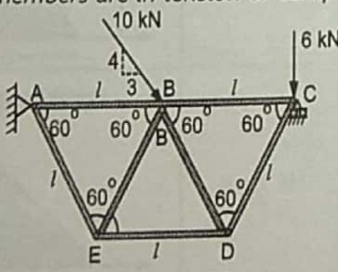


Fig. 2.120

Answer : (a) Reaction at hinge support = R_A = 15.23 kN
Reaction at roller support, R_C = 10 kN (↑), (b) F_{AE} = 16.16 kN (T),
F_{AB} = 2.08 kN (C)
(c) F_{CD} = 4.62 kN (T), F_{BC} = 2.31 kN (C),
(d) F_{EB} = 16.16 kN (C), F_{ED} = 16.16 kN (T), (e) F_{DB} = 4.62 kN (C)

Problem No. 28 : *Determine the forces in each member of the truss and state if member is in tension or compression. All the members are inclined at 45° with the horizontal.*

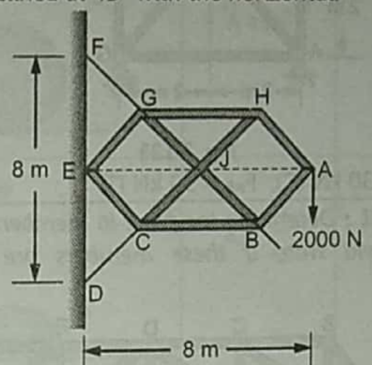
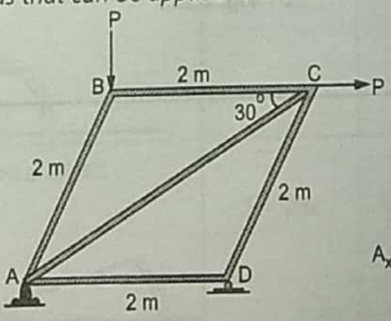


Fig. 2.121

Answer : (a) F_{AB} = 1414.21 N (C),
F_{AH} = 1414.21 N (C),
(b) F_{GH} = 2000 N (C),
F_{HJ} = 1414.21 N (C),
(c) F_{BJ} = 1414.21 N (T),
F_{BC} = 2000 N (C),
(d) F_{CJ} = 1414.21 N (C),
F_{GJ} = 1414.21 N (T),
(e) F_{GE} = 1414.42 N (C),
F_{GF} = 0 N
(f) F_{CE} = 1414.21 N (C),
F_{CD} = 2828.42 N (C)

Problem No. 29 : *The maximum allowable tensile force in the member of the truss is kN and the maximum allowable compressive force is 1.2 kN. Determine the maximum magnitude P of the two loads that can be applied to the truss.*



A_x =

Fig. 2.122

Answer : Maximum magnitude of force P = 0.76 kN.

Problem No. 30 : *Determine the forces in members AB and GF of the truss.*

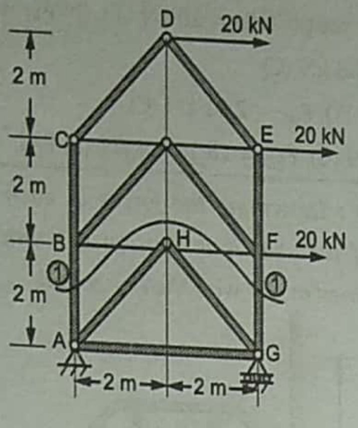


Fig. 2.123

Answer : F_{GF} = 30 kN (C), F_{AB} = 30 kN (T)

Problem No. 31 : *Determine the force in members DE, JI and DO of the truss and state if these members are in tension or compression.*

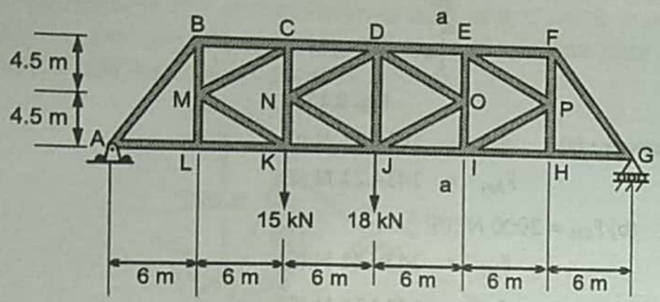


Fig. 2.124

Answer : F_{IJ} = 18.67 kN (T), F_{DE} = 18.67 kN (C), F_{OD} = 11.67 kN (C).

Problem No. 32 : *Determine the tension in each cable segment and the cable's total length.*

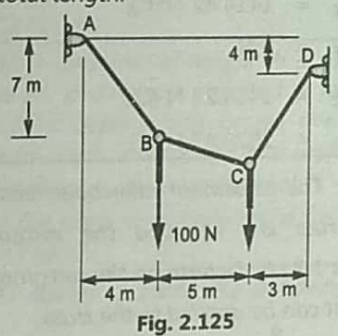


Fig. 2.125

Answer : (a) (i) θ_1 = 60.25°, T_{AB} = 166 N, (ii) θ_2 = 28.17°, T_{BC} = 93.41 N,

(iii) θ_3 = 62.15°, T_{CD} = 176.27 N, (b) Total length of the cable = 20.15 m.

Problem No. 33 : *Determine the horizontal and vertical components of force at pin B and the normal force the pin at C exerts. Also determine the moment and horizontal and vertical reactions of force at A. Neglect radius of pulley at E.*

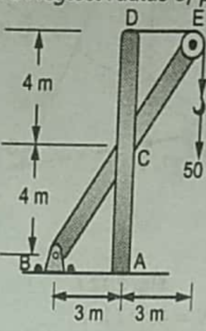


Fig. 2.126

Answer : N_c = 20 N, B_x = 34 N (→), B_y = 62 N (↑), A_x = 34 N (←), A_y = 12 N (↓), M_A = 336 N-m (↺)

Problem No. 34 : *Knowing that the pulley has a radius of 50 mm, determine the components of the reactions at B and E.*

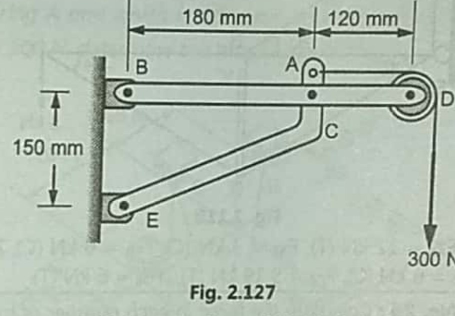


Fig. 2.127

Answer : B_x = 700 N (←), E_x = 700 N (→), B_y = 200 N (↓), E_y = 500 N (↑)

◈◈◈

CENTROID AND FRICTION

A – CENTRE OF GRAVITY AND CENTROID

3.1 CENTRE OF GRAVITY

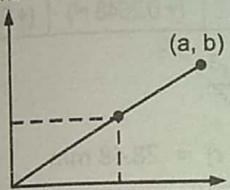
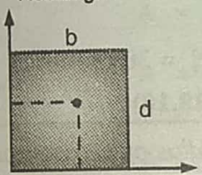
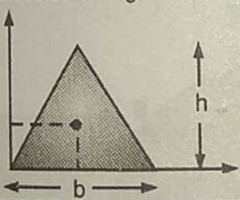
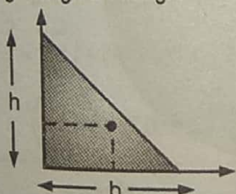
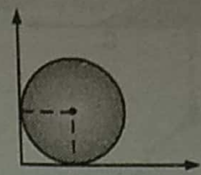
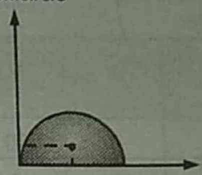
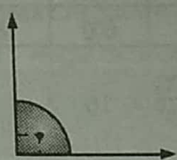
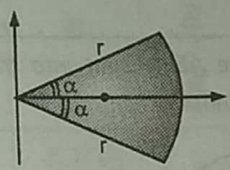
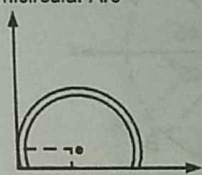
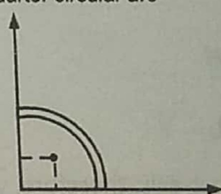
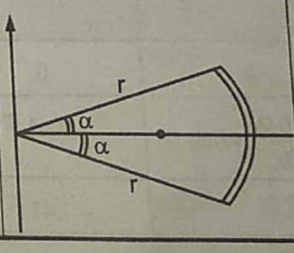
Every body consists of particles and these particles are attracted towards the centre of the earth. All these weights form a system of parallel forces. The point, through which the resultant of all these parallel forces pass, is called the centre of gravity of the body.

3.2 CENTROID

Lines, curves or geometrical figures having lengths or areas do not have any effect due to earth's attraction as they do not possess mass. These figures have a point similar to the centre of gravity of the solids. This point is called the **Centroid** of the line, curve or area.

The centroid is applicable to area, lines or curves and the centre of gravity is applicable to volumes.

3.3 CENTROIDS OF BASIC FIGURES

Sr. No.	Shape	Area	$\bar{x}$	$\bar{y}$
1.	Line 	$l = \sqrt{a^2 + b^2}$	$\frac{a}{2}$	$\frac{b}{2}$
2.	2. Rectangle 	$A = bd$	$\frac{b}{2}$	$\frac{d}{2}$
3.	Isosceles Triangle 	$A = \frac{1}{2}\,bh$	$\frac{b}{2}$	$\frac{h}{3}$
4.	Right angled Triangle 	$A = \frac{1}{2}\,bh$	$\frac{b}{3}$	$\frac{h}{3}$
5.	Circle 	$A = \pi r^2$	r	r
6.	Semicircle 	$A = \frac{\pi r^2}{2}$	r	$\frac{4r}{3\pi}$
7.	Quarter Circle 	$A = \frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
8.	Circular Sector 	$A = r^2\alpha$	$\frac{2r \sin \alpha}{3\alpha}$	0
9.	Semicircular Arc 	$l = \pi r$	r	$\frac{2r}{\pi}$
10.	Quarter circular arc 	$l = \frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$
11.	Segment of an arc 	$l = 2r\alpha$	$\frac{r \sin \alpha}{\alpha}$	0

NUMERICALS ON CENTROID

Example 3.1 : *Find the centroid of thin homogeneous wire ABCA as shown in Fig. 3.1.*

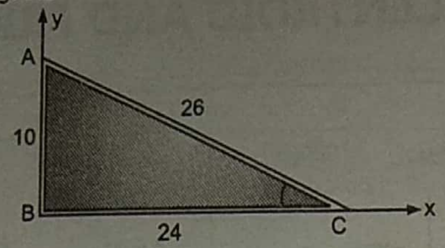


Fig. 3.1

Solution :

Given data : As shown in Fig. 3.1.

To find : Centroid of the thin wire.

Sr. No.	l	$\bar{x}$	$\bar{y}$	$l\bar{x}$	$l\bar{y}$
AB	10	0	5	0	50
BC	24	12	0	288	0
AC	26	12	5	312	130
	60			600	180

$$\bar{x} = \frac{\sum l\bar{x}}{\sum l} = \frac{600}{60} = 10$$

$$\bar{y} = \frac{\sum l\bar{y}}{\sum l} = \frac{180}{60} = 3$$

$\therefore$ **Centroid = (10, 3)** **... Ans.**

Example 3.2 : *A homogeneous wire AB is bent into the shape shown in Fig. 3.2. Determine x co-ordinate of centroid.*

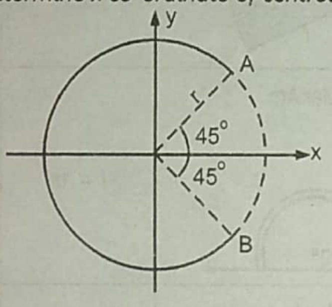


Fig. 3.2

Solution :

Given data : As shown in Fig. 3.2.

To find : x co-ordinate of centroid.

Since Fig. 3.26 is symmetrical with x-axis, therefore, $\bar{y} = 0$.

Sr. No.	l	$\bar{x}$	$l\bar{x}$
1.	$2\pi r$	0	0
2.	$-2r \sin \alpha = 1.57\ r$	$\frac{r \sin \alpha}{\alpha} = 0.9r$	$-1.41\ r^2$
	$4.71\ r$		$-1.41\ r^2$

$$\therefore \quad \bar{x} = \frac{\sum l\bar{x}}{\sum l} = \frac{-1.41\ r^2}{4.71\ r} = -0.3\ r$$

$\therefore$ **x co-ordinate of centroid is (– 0.3 r)** **... Ans.**

Example 3.3: *Locate the centroid of the shaded area with respect to the given axis.*

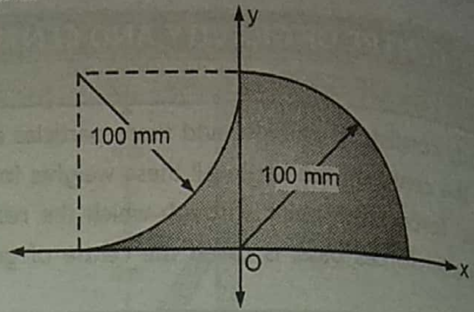


Fig. 3.3

Solution :

Given data : As shown in Fig. 3.3.

To find : Centroid of the shaded area.

Sr. No.	A	$\bar{x}$	$\bar{y}$	$A\bar{x}$	$A\bar{y}$
1.	$+\frac{\pi r^2}{4}$	$+\frac{4r}{3\pi}$	$+\frac{4r}{3\pi}$	$+0.3333\ r^3$	$+0.3333\ r^3$
2.	$+r^2$	$-\frac{r}{2}$	$+\frac{r}{2}$	$-0.5\ r^3$	$+0.5\ r^3$
3.	$-\frac{\pi r^2}{4}$	$-\left[r-\frac{4r}{3\pi}\right]$	$+\left[r-\frac{4r}{3\pi}\right]$	$+0.4515\ r^3$	$-0.4515\ r^3$
	$(+r^2)$			$(+0.2848\ r^3)$	$(+0.3818\ r^3)$

$$\bar{x} = \frac{\sum A\bar{x}}{\sum A} = \frac{0.2848\ r^3}{r^2}$$

$$= [0.2848\ r] = 28.48 \text{ mm}$$

$$\bar{y} = \frac{\sum A\bar{y}}{\sum A} = \frac{0.3818\ r^3}{r^2}$$

$$= [0.3818\ r] = 38.18 \text{ mm}$$

$\therefore$ **Centroid = (28.48, 38.18)** **... Ans.**

Example 3.4 : *A circular disc has a radius of 120 mm and portion above the line AB is removed. Locate the centroid of the remaining area.*

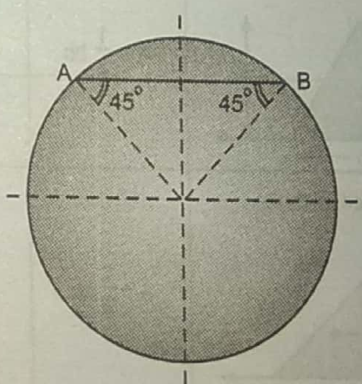


Fig. 3.4 (a)

Solution : Given data : Radius = 120 mm, as shown in Fig. 3.4 (a).

To find : Centroid.

(a) Area of quarter circle, $A = \frac{\pi r^2}{4} = 11309.4\ \text{mm}^2$

$$\bar{x} = \bar{y} = \frac{4r}{3\pi}$$

$$(OP)^2 = (\bar{x})^2 + (\bar{y})^2$$

$$(OP)^2 = \left[\frac{4r}{3\pi}\right]^2 + \left[\frac{4r}{3\pi}\right]^2$$

$$(OP) = 72\ \text{mm}$$

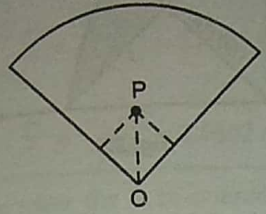


Fig. 3.4 (b)

(b)

$$\cos 45 = \frac{x_1}{r}$$

$$x_1 = 0.707\ r$$

$$AB = 2x_1 = (1.414)\ r$$

Area of triangle $ABC = \frac{1}{2} \times \text{base} \times \text{height}$

$$A = \frac{1}{2} \times (AB) \times (OQ)$$

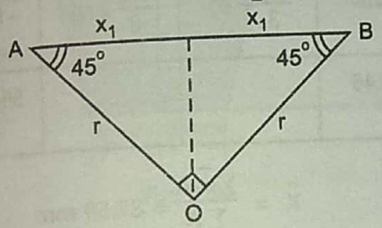


Fig. 3.4 (c)

$$A = \frac{1}{2} \times (1.414\ r)\ (0.707\ r)$$

$$A = 0.5\ r^2$$

$$A = 0.5 \times (120)^2$$

$$A = 7200$$

$$\bar{y} = \frac{2}{3}\ h \text{ from O}$$

$$= \frac{2}{3}(0.707\ r) = (0.4713\ r) = 56.56\ \text{mm}$$

Sr. No.	A	$\bar{x}$	$\bar{y}$	$A\bar{x}$	$A\bar{y}$
1.	$(\pi r^2) = 45237.6$	0	0	0	0
2.	$\left(-\frac{\pi r^2}{4}\right) = (-11309.4)$	0	72	0	– 814276.8
3.	$\left(\frac{1}{2} bh\right) = (7200)$	0	56.56	0	+ 407232
	+ 41128.2				– 407044.8

$$\therefore \quad \bar{y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{-407044.8}{+41128.2} = -9.897\ \text{mm}$$

$\therefore$ Centroid = (0, –9.897) ... **Ans.**

$\therefore$ **The centroid lies on y-axis at a distance of (9.897) mm below the original centre.** ... **Ans.**

Example 3.5 : *A slender homogeneous wire is bent into shape shown in Fig. 3.5 (a) and suspended from point A. Find the angle which the neck of the hook makes with vertical.*

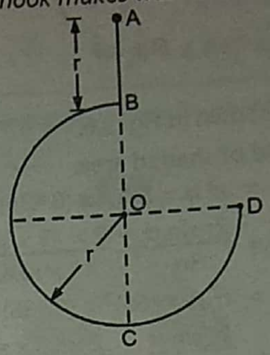


Fig. 3.5 (a)

Solution :

Given data : As shown in Fig. 3.5 (a).

To find : Angle of neck (θ).

Sr. No.	l	$\bar{x}$	$\bar{y}$	$l\bar{x}$	$l\bar{y}$
1.	r	0	$\left(r + \frac{r}{2}\right)$	0	$+ 1.5\ r^2$
2.	πr	$-\frac{2r}{\pi}$	0	$-2r^2$	0
3.	$\frac{\pi r}{2}$	$+\frac{2r}{\pi}$	$-\frac{2r}{\pi}$	$+ r^2$	$- r^2$
	(5.71 r)			$(- r^2)$	$+ (0.5\ r^2)$

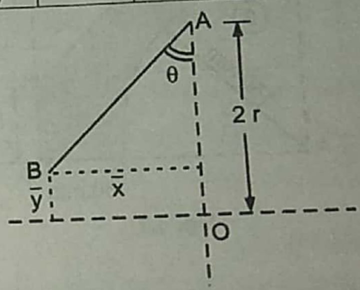


Fig. 3.5 (b)

$$\bar{x} = \frac{\Sigma l\bar{x}}{\Sigma l} = \frac{-r^2}{5.71\ r} = (-0.175\ r)$$

$$\bar{y} = \frac{\Sigma l\bar{y}}{\Sigma l} = \frac{+0.5\ r^2}{5.71\ r} = (+0.0875\ r)$$

$$\tan\theta = \frac{\bar{x}}{(2r - \bar{y})}$$

$$\tan\theta = \frac{0.175\ r}{(2r - 0.0875\ r)}$$

$$\tan\theta = 0.915$$

$$\theta = 5.23°$$ ... **Ans.**

Example 3.6 : *Locate the centroid of the shaded area.*

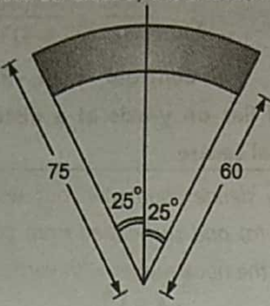


Fig. 3.6

Solution :

Given data : As shown in Fig. 3.6.

To find : Centroid of shaded area.

(a) $A_1 = r^2 \alpha = (75)^2 \times (0.436) = 2454.37$

$$y_1 = \frac{2r \sin \alpha}{3\alpha} = \frac{2 \times 75 \times \sin 25}{3 \times (0.436)} = 48.465$$

(b) $A_2 = r^2 \alpha = (60)^2 \times (0.436) = 1569.6$

$$y_2 = \frac{2r \sin \alpha}{3\alpha} = \frac{2 \times 60 \times \sin 25}{3 \times (0.436)} = 38.77$$

Now, $$\bar{y} = \frac{A_1 y_1 - A_2 y_2}{A_1 - A_2} = \frac{(2454.37)\ (48.465) - (1569.6)\ (38.77)}{(2454.37 - 1569.6)}$$

$$\bar{y} = 65.66$$

$\therefore$ **Centroid = (0, 65.66)** **... Ans.**

Example 3.7 : *Locate the centroid of the shaded lamina as shown in Fig. 3.7.*

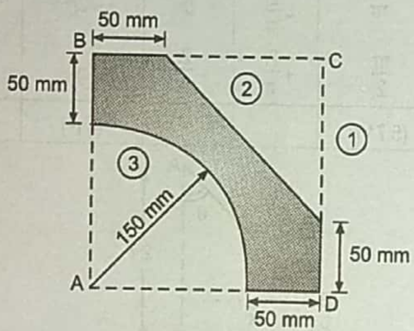


Fig. 3.7

Solution :

Given data : As shown in Fig. 3.7.

To find : Centroid of the shaded area.

Sr. No.	A	$\bar{x}$ (from A)	$\bar{y}$ (from A)	$A\bar{x}$	$A\bar{y}$
1.	200×200	100	100	4×10^6	4×10^6
2.	$-\frac{150 \times 150}{2}$	150	150	-1.69×10^6	-1.69×10^6
3.	$-\frac{1}{4}\left(\frac{\pi}{4} \times 300^2\right)$	$\frac{4 \times 150}{3\pi}$	$\frac{4 \times 150}{3\pi}$	-1.12×10^6	-1.12×10^6
	$\Sigma A = 11078.54$			$\Sigma A\bar{x} = 1.19 \times 10^6$	$\Sigma A\bar{y} = 1.19 \times 10^6$

$$\therefore \quad \bar{x} = \frac{\Sigma A \bar{x}}{\Sigma A} = \frac{1.19 \times 10^6}{11078.54} = 107.41 \text{ mm from A}$$

$$\bar{y} = \frac{\Sigma A \bar{y}}{\Sigma A} = \frac{1.19 \times 10^6}{11078.54} = 107.41 \text{ mm from A}$$

$\therefore$ **Centroid = (107.41, 107.41)** **... Ans.**

Example 3.8 : *Determine the position of centroid of the shaded area as shown in Fig. 3.8 with respect to origin O.*

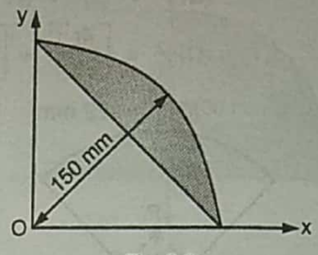


Fig. 3.8

Solution :

Composite area is divided into quarter circle and triangle. The area of triangle is indicated as negative, since it is to be subtracted from other area.

Part	A (mm²)	$\bar{x}$ (mm)	$\bar{y}$ (mm)	$\bar{x}$ A (mm³)	$\bar{y}$ A (mm³)
Quarter circle	$\frac{\pi r^2}{4} =$ 17671.46	$\frac{4r}{3\pi} = 63.66$	$\frac{4r}{3\pi} = 63.66$	1125×10^3	1125×10^3
Triangle	− 11250	$\frac{150}{3} = 50$	$\frac{150}{3} = 50$	-562.5×10^3	-562.5×10^3
Total	6421.46			562.5×10^3	562.5×10^3

$$\bar{X} = \frac{\Sigma \bar{x} A}{\Sigma A} = 87.59 \text{ mm}$$

$$\bar{Y} = \bar{X} = 87.59 \text{ mm}$$ **... Ans.**

Example 3.9 : *Determine the position of centroid of the shaded area as shown in Fig. 3.9 w.r.t. origin O.*

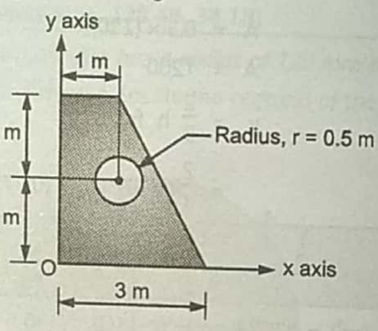


Fig. 3.9

Solution :

Composite area is divided into rectangle, triangle and circle. Area of circle is indicated as negative since it is to be subtracted from other area.

Part	A (m²)	$\bar{x}$ (m)	$\bar{y}$ (m)	$\bar{x}$ A (m³)	$\bar{y}$ A (m³)
Rectangle	$4 \times 1 = 4$	0.5	2	2	8
Triangle	$\frac{1}{2} \times 2 \times 4$ = 4	$1 + \frac{1}{3} \times 2$ = 1.667	$\frac{1}{3} \times 4$ = 1.333	6.668	5.332
Circle	$-\pi r^2$ = − 0.7854	1	2	− 0.7854	− 1.5708
Total	7.2146			7.8826	11.7612

$$\bar{X} = \frac{\Sigma \bar{x} A}{\Sigma A} = 1.0926 \text{ m} \qquad \text{... Ans.}$$

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = 1.6302 \text{ m} \qquad \text{... Ans.}$$

Example 3.10 : *Determine the Y co-ordinate of centroid of the shaded area as shown in Fig. 3.10.*

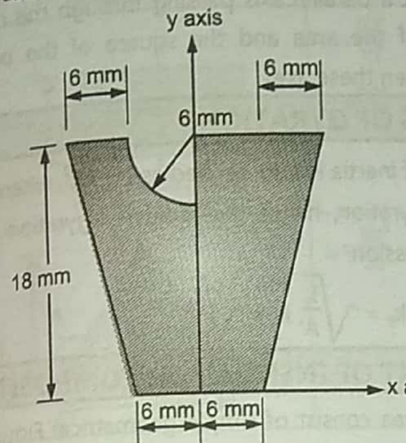


Fig. 3.10

Solution :

Composite area is symmetrical about Y-axis.

Half part of composite area is considered to find $\bar{y}$ as shown in Fig. 3.10 (a).

Area of quarter circle is indicated negative since it is to be subtracted from other area.

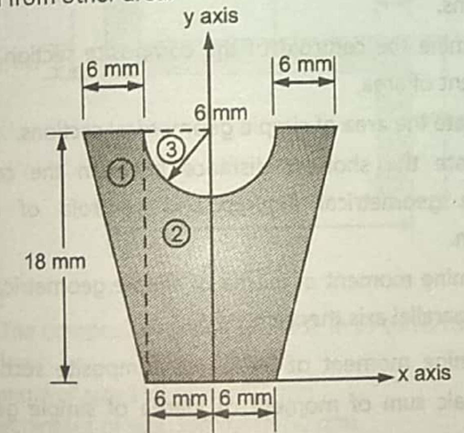


Fig. 3.10 (a)

Part	A (mm²)	$\bar{y}$ (mm)	$\bar{y}$ A (mm³)
Triangle	$\frac{1}{2} \times 6 \times 18 = 54$	$\frac{2}{3} \times 18 = 12$	648
Rectangle	$6 \times 18 = 108$	$\frac{1}{12} \times 18 = 9$	972
Quarter circle	$-\frac{\pi r^2}{4} = -28.27$	$18 - \frac{4r}{3\pi} = 15.45$	− 436.87
Total	133.73		1183.13

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \mathbf{8.847 \text{ mm}} \qquad \text{...Ans.}$$

Example 3.11 : *Determine y-co-ordinate of centroid of shaded area as shown in Fig. 3.11.*

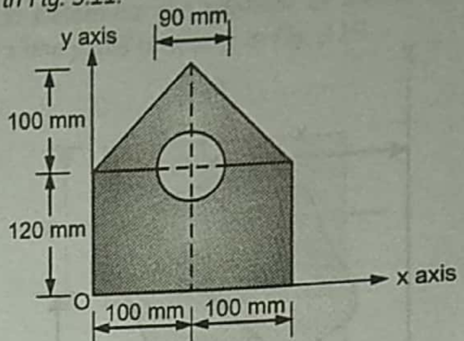


Fig. 3.11

Solution :

Composite area is divided into rectangle, triangle and circle. Area of circle is indicated negative since it is to be subtracted from other area.

Area	A (mm²)	$\bar{y}$ (mm)	$\bar{y}$ (A) (mm³)
Rectangle	120×200 = 24000	$\frac{1}{2} \times 120 = 60$	1440000
Triangle	$\frac{1}{2} \times 100 \times 200$ = 10000	$120 + \frac{1}{3} \times 100$ = 153.33	1533333.333
Circle	$\pi r^2 = -6361.725$	120	− 763407.0148
Total	27638.275		2209926.318

$$\bar{Y} = \frac{\Sigma \bar{y} A}{\Sigma A} = \mathbf{79.9589 \text{ mm}} \qquad \text{... Ans.}$$

B – MOMENT OF INERTIA

3.4 DEFINITION

In last section we determine the centroid for an area by considering the first moment of area about an axis; that is, for the composition we had to evaluate an integral of the form ∫xdA. Integrals of the second moment of an area such as $\int x^2 dA$ are referring to as moment of inertia for the area.

3.5 MOMENT OF INERTIA

Consider the area A as shown in Fig. 3.12 which lies in the x-y plane. By definition, the moment of inertia of the differential planner area dA about the x and y axis are given by, $dI_x = y^2 dA$

and $dI_y = x^2dA$ respectively. For the entire area the moment of inertia are determined by integrating equation (01) with respect area A.

$$I_x = \int_A y^2dA \text{ and } I_y = \int_A x^2dA \quad \ldots(3.1)$$

We can also formulate the second moment of differential area dA about pole O or z-axis as shown in Fig. 3.2. This is known as polar moment of inertia, dI_z or $dJo = \int r^2dA$, where r is the perpendicular distance from the pole i.e. z axis to the element dA. For the entire area the polar moment of inertia is given by equation (3.2).

$$I_z \text{ or } J_o = \int_A r^2dA = I_x + I_y \quad \ldots(3.2)$$

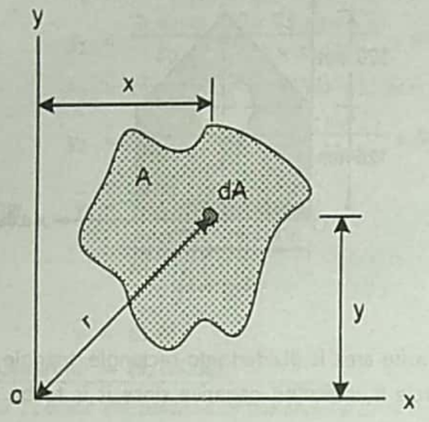


Fig. 3.12: Moment of Inertia

From the above formulation it is seen that I_x, I_y and I_z will always positive, since they involve the product of distance square and area and hence its SI unit is m^4 or mm^4.

3.6 PARALLEL AXIS THEOREM

If the moment of inertia for an area is known about centroidal axis then, it is convenient to determine the moment of inertia of the area about the parallel axis using parallel axis theorem. To derive the expression of the theorem, consider the moment of inertia of the shaded area as shown in Fig. 3.13 about x axis.

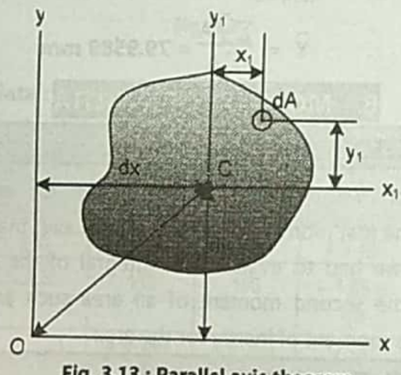


Fig. 3.13 : Parallel axis theorem

A small element dA is located at an arbitrary distance y_1 from the centroidal axis x_1, whereas the fixed distance between the parallel axis x and x_1 is defined as d_y. The moment of inertia of small element about the x axis is given by:

$$dI_x = \int(y_1 + d_y)^2dA \quad \ldots(3.3)$$

The moment of inertia for the entire area is given by integrating equation (3.3).

$$I_x = \int_A (y_1 + d_y)^2dA$$

$$= \int_A y_1^2dA + 2d_y \int_A y_1dA + d_y^2 \int_A dA \quad \ldots(3.4)$$

The first integral term represent the moment of inertia of the area about the centroidal axis, Ix_1. The second integral term is zero because x_1 passes through the centroid C. The third integral term represent the total area, therefore the final expression is,

$$I_x = I_{x1} + Ad_y^2 \quad \ldots(3.5)$$

Similarly expression for I_y is,

$$I_y = I_{y1} + Ad_x^2 \quad \ldots(3.6)$$

The expression of these equation state that the moment of inertia of an area about an axis is equal to the moment of the inertia of the area about a parallel axis passing through the centroid plus the product of the area and the square of the perpendicular distance between these axes.

3.7 RADIUS OF GYRATION

The moment of inertia is also defined as $I = Ak^2$, where k is known as radius of gyration, hence the radius of gyration is given by following expression.

$$k_x = \sqrt{\frac{I_x}{A}},\ k_y = \sqrt{\frac{I_y}{A}},\ k_z = \sqrt{\frac{I_z}{A}} \quad \ldots(3.7)$$

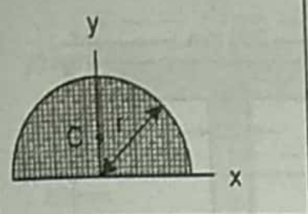
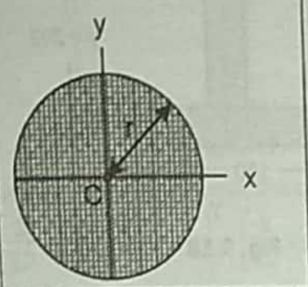
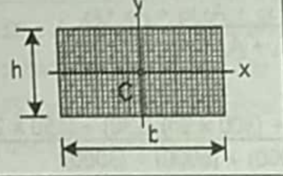
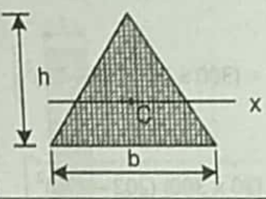
3.8 MOMENT OF INERTIA FOR COMPOSITE AREA

A composite area consist of simple geometrical Figures such as circle, semicircle, rectangle, triangle etc for which the moment of inertia about it centroid is to be known. The moment of inertia of composite section is the algebraic sum moment of inertia of simple geometrical sections.

Following procedure is adopted to determine the moment of inertia of composite section.

- Divide the composite section in to simple geometrical sections.
- Determine the centroid of the composite section using first moment of area.
- Calculate the area of simple geometrical sections.
- Calculate the shortest distance between the centroid of simple geometrical Figures and centroid of composite section.
- Determine moment of inertia of simple geometrical sections using parallel axis theorem.
- Determine moment of inertia of composite section taking algebraic sum of moment of inertia of simple geometrical sections.

3.9 MOMENT OF INERTIA OF GEOMETRICAL SECTION

Sr. No.	Geometrical Section	Moment of Inertia
1.		$I_x = \frac{\pi r^4}{8}$ $I_y = \frac{\pi r^4}{8}$
2.		$I_x = \frac{\pi r^4}{4}$ $I_y = \frac{\pi r^4}{4}$
3.		$I_x = \frac{bh^3}{12}$ $I_y = \frac{bh^3}{12}$
04		$I_x = \frac{bh^3}{36}$

***Example 3.12**: Determine the moment of inertia of the composite section about x and y axis as shown in Fig. 314.*

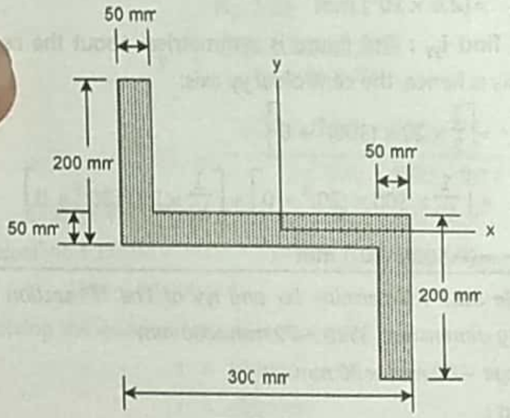


Fig. 3.14

Solution:

The composite section is divide in to three rectangles are as follows:

First rectangle of size 150 mm × 50 mm

Second rectangles of size 300 mm × 50 mm

Third rectangles of size 150 mm × 50 mm

Moment of Inertia about x axis using parallel axis theorem

$$I_x = \frac{50 \times 150^3}{12} + 150 \times 50 \times 175^2 + \frac{300 \times 50^3}{12} + \frac{50 \times 150^3}{12} + 150 \times 50 \times 175^2$$

$$= \mathbf{490.625 \times 10^6\ mm^4} \qquad \text{...(Ans.)}$$

Moment of Inertia about y axis using parallel axis theorem

$$I_y = \frac{150 \times 50^3}{12} + 150 \times 50 \times 175^2 + \frac{50 \times 300^3}{12} + \frac{150 \times 50^3}{12} + 150 \times 50 \times 175^2$$

$$= \mathbf{575.00 \times 10^6\ mm^4} \qquad \text{...(Ans.)}$$

***Example 3.13:** Determine the moment of inertia of the shaded portion about x and y axis as shown in Fig. 3.15.*

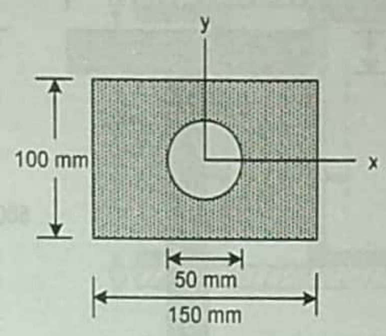


Fig. 3.15

Solution: Moment of inertia about x axis

$$I_x = \frac{150 \times 100^3}{12} - \frac{\pi \times 25^4}{4}$$

$$= \mathbf{12.193 \times 10^4\ mm^4} \qquad \text{...(Ans.)}$$

Moment of inertia about y axis

$$I_y = \frac{100 \times 150^3}{12} - \frac{\pi \times 25^4}{4}$$

$$= \mathbf{27.818 \times 10^4\ mm^4} \qquad \text{...(Ans.)}$$

***Example 3.14:** Determine the moment of inertia for the I-section about x and y axis as shown in Fig. 3.16.*

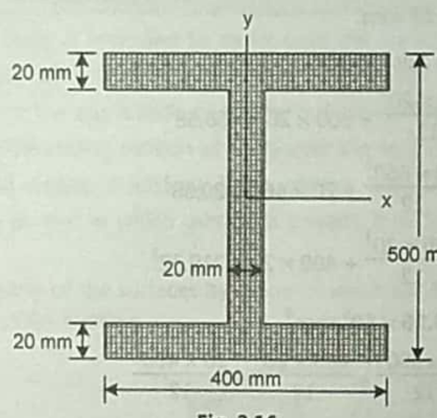


Fig. 3.16

Solution:

The section is symmetrical about both axis and the position of centroid is known.

Using standard expression of moment of inertia for rectangle, moment of inertia about x axis,

$$I_x = \frac{400 \times 500^3}{12} - \frac{380 \times 460^3}{12}$$

$$= \mathbf{1084.36 \times 10^4\ mm^4} \quad \text{...Ans.}$$

Moment of inertia about y axis,

$$I_y = \frac{500 \times 400^3}{12} - \frac{460 \times 380^3}{12}$$

$$= \mathbf{563.24 \times 10^4\ mm^4} \quad \text{...Ans.}$$

Example 3.15: *Determine the moment of inertia for the I-section about x and y axis as shown in Fig. 3.17.*

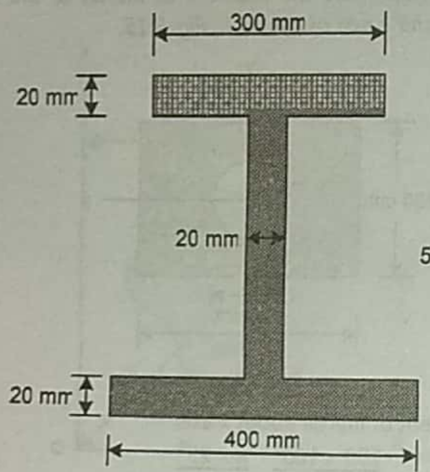


Fig. 3.17

Solution:

The given I-section is unsymmetrical about x axis. The position of centroidal axis from top of the top flange is determined using first moment of area.

$$\bar{y} = \frac{300 \times 20 \times 10 + 460 \times 20 \times 250 + 400 \times 20 \times 490}{300 \times 20 + 460 \times 20 + 400 \times 20}$$

$$= 270.68 \text{ mm}$$

Using parallel axis theorem

$$I_x = \frac{300 \times 20^3}{12} + 300 \times 20 \times 260.68^2 + \frac{20 \times 460^3}{12} + 20 \times 460 \times 20.68^2 + \frac{400 \times 20^3}{12} + 400 \times 20 \times 219.32^2$$

$$= \mathbf{959.16 \times 10^6\ mm^4} \quad \text{...Ans.}$$

$$I_y = \frac{20 \times 300^3}{12} + \frac{460 \times 20^3}{12} + \frac{20 \times 400^3}{12}$$

$$= \mathbf{151.97 \times 10^6\ mm^4} \quad \text{...Ans.}$$

Example 3.16 : *Determine the M.I. of the I section about it's central xx and yy axis. The particulars are :*

Top Flange – (300 mm × 20 mm)

Web – 20 m × 300 mm

Bottom flange – 150 mm × 20 mm

Solution :

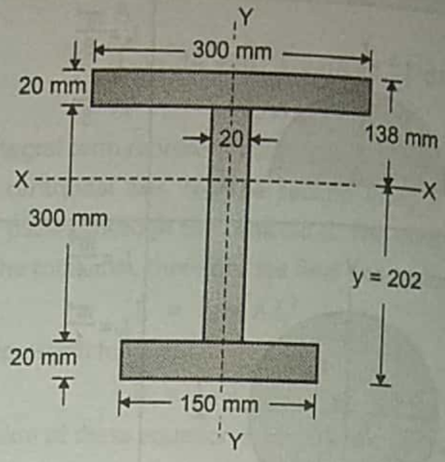


Fig. 3.18

(a) To find I_{xx} :

$$\text{Centroid} = \bar{y} = \frac{A_1 x_1 + A_2 x_2 + A_2 x_2 + A_3 + x_3}{A_1 + A_2 + A_3}$$

$$\bar{y} = \frac{(300 \times 20)(330) + (300 \times 20)(170) + (150 \times 20)(10)}{(6000) + (6000) + (3000)}$$

$$\bar{y} = 202 \text{ mm}$$

$$I_{xx} = \left[\frac{1}{12} \times (300) \times (20)^3 + (300 \times 20)(138 - 10)^2\right] + \left[\frac{1}{12} \times 20\,(300)^3 + (20 \times 300)(202 - 170)^2\right] + \left[\frac{1}{12} \times 150 \times (20)^3 + (150 \times 20)(202 - 10)^2\right]$$

$$I_{xx} = \mathbf{(2.6 \times 10^8)\ mm^4} \quad \text{...Ans}$$

(b) To find I_{yy} : The figure is symmetrical about the central yy axis. This is hence, the centroidal yy axis.

$$I_{yy} = \left[\frac{1}{2} \times 20 \times (300)^3 + 0\right] + \left[\frac{1}{12} \times 300 \times (20)^3 + 0\right] + \left[\frac{1}{12} \times 20\,(150)^3 + 0\right]$$

$$I_{yy} = \mathbf{(0.508 \times 10^8)\ mm^4} \quad \text{...Ans}$$

Example 3.17 : *Determine Ixx and Iyy of The "T" section having following dimensions. Web – 20 mm × 80 mm*

Top flange – 80 mm × 20 mm

Solution :

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$Y = \frac{(80 \times 20)(90) + (80 \times 20)(45)}{(1600 + 1600)}$$

$$\bar{y} = 67.5 \text{ mm}$$

$$I_{xx} = \left[\frac{1}{12}\times 80\,(20)^3 + (80\times 20)\,(32.5-10)^2\right]$$
$$+\left[\frac{1}{12}20\times(80)^3 + (80\times 20)\,(67.5-40)^2\right]$$
$$I_{xx} = (2.92\times 10^6)\ \text{mm}^4$$

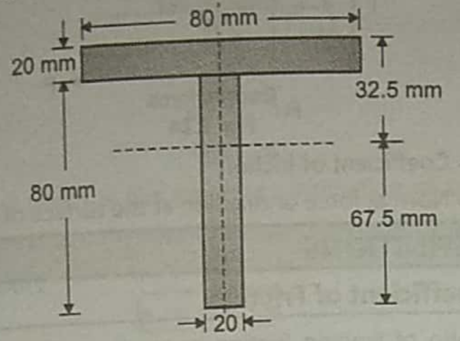


Fig. 3.19

As the figure is symmetrical about the central yy axis :

$$I_{yy} = \left[\frac{1}{12}\times 20\times(80)^3 + 0\right] + \left[\frac{1}{12}\times 80\,(20)^3 + 0\right]$$
$$I_{yy} = (0.906\times 10^8)\ \text{mm}^4 \qquad \text{...Ans}$$

***Example 3.18 :** A beam having a cross-section in the form of a channel is shown in the figure. The centroidal xx axis is at a distance of 75 mm from the base. Find the channel thickness. Also calculate I_{xx} and I_{yy}.*

Solution :

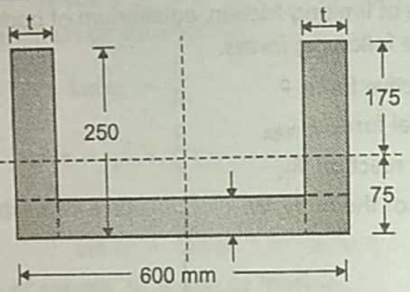


Fig. 3.20

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3}$$
$$75 = \frac{2(250\,t)\,(125) + (600-2t)\times t\times\frac{t}{2}}{2\,(250t) + (600-2t)\,t}$$
$$37500\,t + 45000t - 150t^2 = 62500\,t + 300\,t^2 - t^3$$

Calcelling t $37500 + 4500 - 1500\,t^2 = 625\,00 + 300\,t - t^2$

$\therefore\quad t^2 - 450t + 20{,}000 = 0$

Solving the quadratic equation we get,

$$t = 50\ \text{mm}$$
$$t = 400\ \text{mm}$$

(b) $$I_{xx} = 2\left[\frac{1}{12}\times 50\times(250)^3 + (50\times 250)\,(175-125)^2\right]$$
$$+\left[\frac{1}{12}\times 500\times(50)^3 + (500\times 50)\,(75-25)^2\right]$$
$$I_{xx} = (2.60\times 10^8)\ \text{mm}^4 \qquad \text{...(Ans.)}$$

$$I_{yy} = 2\left[\frac{1}{12}\times 250\times(50)^3 + (250\times 50)\,(300-25)^2\right]$$
$$+\left[\frac{1}{12}\times 50\times 500^3 + (50\times 500)\,(0)\right]$$
$$I_{yy} = (2.417\times 10^3)\ \text{mm}^4 \qquad \text{...(Ans.)}$$

***Example 3.19 :** A column section is made up of two channels and two flange plates as shown. Each plate is 210 mm × 10 mm. Compute the values of Ixx and Iyy for the compound section. The properties of each channel are.*

$$A = 1800\ \text{mm}^2$$
$$I_{xx} = (11.6\times 10^6)\ \text{mm}^4$$
$$I_{yy} = (0.84\times 10^6)\ \text{mm}^4$$

Distance of e.g. from back = 20 mm

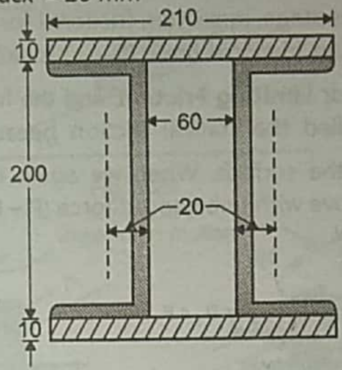


Fig. 3.21

Solution :

$$I_{xx} = 2\,[11.6\times 10^6] + 2\left[\frac{1}{12}\times 210\times 10^3 + (210\times 10)\,(105)^2\right]$$
$$I_{xx} = (69.54\times 10^6)\ \text{mm}^4$$
$$I_{yy} = 2\,[(0.84\times 10^6) + (1800)\,(30+20)^2] + 2\left[\frac{1}{12}\times 10\times 210^3\right]$$
$$I_{yy} = (26.12\times 10^6)\ \text{mm}^4 \qquad \text{...Ans}$$

C – FRICTION

3.10 INTRODUCTION

When a body is intended to move over the surface, due to the application of a tractive force P, a force which automatically appears on the application of the force P that prevent or oppose any possible sliding motion of body over the surface is known as **Frictional Force**. Frictional force always acts in a direction opposite to that in which motion is sought. It is denoted by the letter F.

The property of the surfaces by virtue of which the frictional force exists is called **friction**.

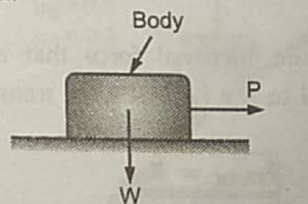


Fig. 3.22

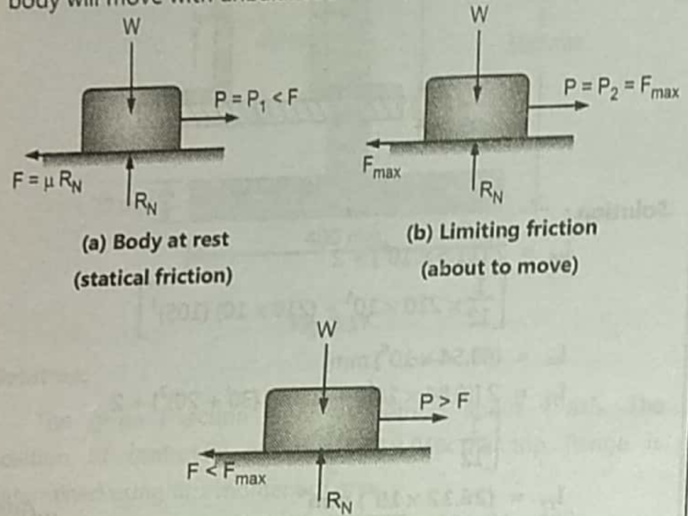
3.11 CHARACTERISTICS OF THE FRICTIONAL FORCE

The following are the basic characteristics of frictional force :

- It is a passive and self-adjusting force. This force exists only if the tractive force P is there, so it is called passive force. As we go on increasing the force P, the frictional force F also goes on increasing till the maximum frictional force.
- It always acts in a direction opposite to that in which motion occurs.

3.12 LAWS OF COULOMB FRICTION

We exert a continuously increasing force (a tractive force P), which is completely resisted by friction until the body begins to move. At this stage, maximum frictional force exists.
This maximum frictional force (F_{max}) is called the **Limiting Force of Friction** or **Limiting Friction**, and the frictional force less than F_{max}, is called the statical friction because the body remains static over the surface. When we again increase tractive force, body will move with unbalanced force ($P - F_{max}$).



(a) Body at rest (statical friction)

(b) Limiting friction (about to move)

(c) Dynamic friction

Fig. 3.23

At this instant, the friction existing between body and surface is less than the static friction and is called dynamic friction. At the condition of impending motion, it is possible to relate the frictional and normal components of force F, as given by Coulomb in 1781.

The laws are :

1. The total amount of friction that can be developed is independent of the magnitude of the area of contact but it depends upon the nature of the surface in contact with each other.
2. The maximum frictional force that can be developed is proportional to the normal force transmitted at the surface of contact.

$$\boxed{F_{max} \propto R_N}$$

or $$\boxed{F_{max} = \mu \cdot R_N}$$

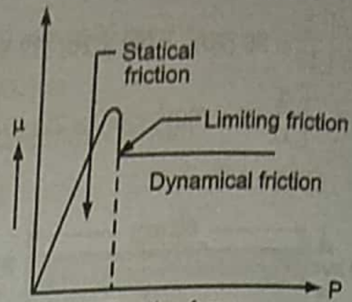


Fig. 3.24

where μ = Coefficient of friction

R_N = Normal force or reaction at the surface of contact.

3.13 DEFINITIONS

3.13.1 Coefficient of Friction

It is the ratio of limiting friction or maximum frictional force to normal reaction.

$$\mu = \frac{F_{max}}{R_N}$$

Theoretically, for ideally smooth surface, $\mu = 0$, but there exist hardly any such surface in practice. The coefficient of friction μ increases as the tractive force is gradually increased. The coefficient of limiting friction is slightly more than the coefficient of dynamic friction.

3.13.2 Total Reaction

At the stage of limiting friction, equilibrium of body, under the action of the following forces.

1. The tractive force, P
2. Frictional force, F_{max}
3. Normal reaction, R_N
4. Weight of the body, W.

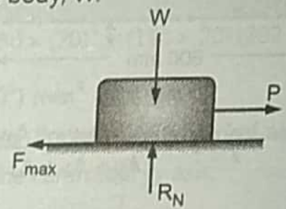


Fig. 3.25

The frictional force F_{max} and normal reaction R_N, which acts at right angles to each other, may be combined into a single force.

$$\boxed{R_T = \sqrt{F_{max}^2 + R_N^2}}$$

This single force is called the total reaction.

3.13.3 Angle of Friction (ϕ)

It is the inclination of the total reaction R, with the normal reaction (R_N).

$$\tan \phi = \frac{F_{max}}{R_N}$$

But, $$\frac{F_{max}}{R_N} = \mu, \text{ the coefficient of friction}$$

$$\tan \phi = \mu = \frac{F_{max}}{R_N} \text{ or } \frac{F}{R}$$

$\therefore$

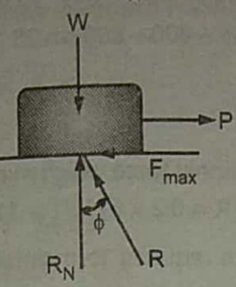


Fig. 3.26

This angle ϕ which is the maximum inclination of total reaction makes with normal reaction is called the angle of friction. From above relation,

$$\boxed{\phi = \tan^{-1} \mu}$$

3.13.4 Angle of Repose

The angle of repose is defined as the maximum inclination of a plane at which a body remains in equilibrium over the inclined plane by the assistance of friction only.

Let us consider a body of weight W rest on rough inclined plane, which makes an angle α with horizontal.

Resolving force along and perpendicular to plane at the stage of just sliding,

$$W \sin \alpha = F \quad \text{... (3.8)}$$

$$W \cos \alpha = R \quad \text{... (3.9)}$$

Dividing equation (1) by equation (2),

$$\tan \alpha = \frac{F}{R}$$

$$\tan \phi = \mu = \frac{F}{R}$$

Hence from the above equation,

$$\tan \alpha = \tan \phi \quad \therefore \quad \alpha = \phi$$

Thus, **Angle of repose = Angle of friction.**

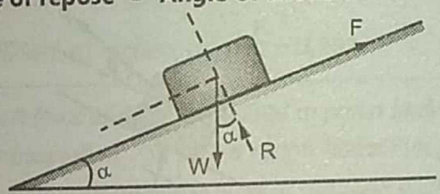


Fig. 3.27

3.13.5 Cone of Friction

It is defined as the right circular cone with vertex at the point of contact of the two bodies, axis in the direction of normal reaction (R) and semi-vertical angle equal to angle of friction (ϕ).

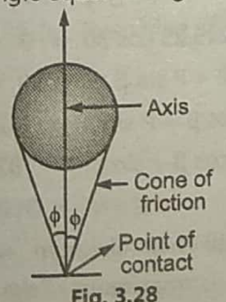


Fig. 3.28

3.14 ROUGH INCLINED PLANE

We know that an inclined plane is a plane lying at a certain angle to the horizontal. Such planes are quite common, a steep hill road or a ramp in a building. The analysis of problem related with inclined plane is similar to other problem of friction. In this case, the frictional resistance F is acting in opposite to the direction of impending motion tangentially to the plane. (If block moves up the plane, F is acting down the plane and vice-versa and reaction is acting normal to the plane).

Consider F.B.D. of block of impending motion down and up the plane and applying condition of equilibrium in any two mutually perpendicular directions. (It is convenient to resolve force along and normal to the plane.)

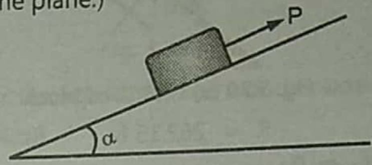


Fig. 3.29

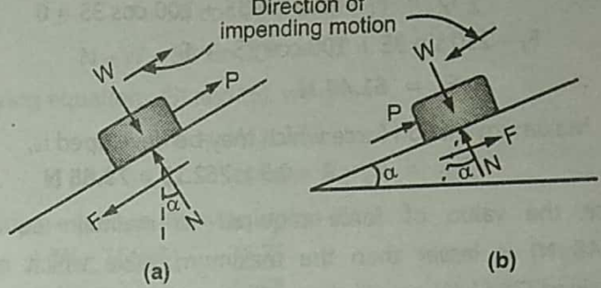


Fig. 3.29

Case - I : Impending Motion Down the Plane.

$$N - W \cos \alpha = 0 \quad \text{... (3.10)}$$

and

$$P + F - W \sin \alpha = 0 \quad \text{... (3.11)}$$

Solving these equations for required unknown force P,

$$P = W \sin \alpha - \mu W \cos \alpha$$

$$P = W (\sin \alpha - \mu \cos \alpha)$$

Case - II : Impending Motion up the Plane.

$$N = W \cos \alpha \quad \text{...(3.12)}$$

$$P = F + W \sin \alpha \quad \text{... (3.13)}$$

Solving equations (3.12) and (3.13), we get,

$$\mathbf{P = W (\sin \alpha + \mu \cos \alpha)}$$

We shall solve all problems related to inclined plane by above approach.

Example 3.20 : *Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when θ = 35° and P = 100 N.*

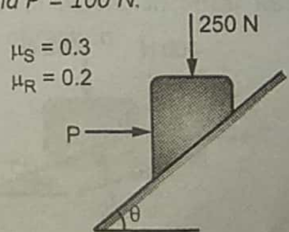


Fig. 3.30

Solution :

Given data : P = 100 N, θ = 35°, μ_s = 0.3,

To find : Friction force.

(Incline plane as x-x axis and perpendicular to incline plane as y-y axis).

Let F_r be the frictional force required to maintain equilibrium.

$$\therefore \quad \Sigma f_y = 0$$

$$-250 \cos 35 - 100 \sin 35 + R = 0$$

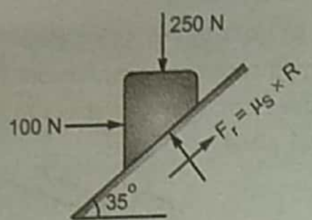


Fig. 3.30 (a) : F.B.D. of block

$$\therefore \quad R = 262.15 \text{ N}$$

Let $\Sigma f_x = 0$.

$$\therefore \quad \Sigma f_x = F_r - 250 \sin 35 + 100 \cos 35 = 0$$

$$F_r - 250 \sin 35 + 100 \cos 35 = 0$$

$$\therefore \quad F_r = \mathbf{61.48 \text{ N}} \quad \text{... Ans.}$$

Maximum friction force which may be developed is,

$$F_m = \mu_s \times R = 0.3 \times 262.15 = \mathbf{78.65 \text{ N}} \quad \text{... Ans.}$$

Since the value of force required to maintain equilibrium (61.48 N) is lesser than the maximum value which may be obtained (78.65 N), equilibrium will be maintained.

Example 3.21 : *Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when θ = 40° and P = 400 N.*

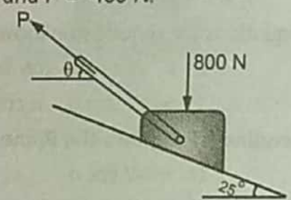


Fig. 3.31

Solution :

Given data : P = 400 N, θ = 40°.

To find : Is block in equilibrium and frictional force.

(Incline plane as x-x axis and perpendicular to plane as y-y axis).

$$\Sigma f_y = 0$$

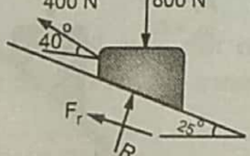


Fig. 3.31 (a) : F.B.D. of block

$$R + 400 \sin 15 - 800 \cos 25 = 0$$

For equilibrium $\Sigma f_x = 0$

$$F_r + 400 \cos 15 - 800 - 800 \sin 25 = 0$$

$$\therefore \quad F_r = -48.27 \text{ N}$$

$$\therefore \quad F_r = \mathbf{48.27 \text{ N}} \quad \text{... Ans.}$$

Maximum frictional force which may be developed.

$$F_m = \mu_s R = 0.2 \times 621.51 = \mathbf{124.30 \text{ N}} \quad \text{... Ans.}$$

Since value of force required to maintain equilibrium (48.27 N) is less than the maximum frictional force (124.3 N), so equilibrium will be maintained.

Example 3.22 : *Knowing that the coefficient of friction between the 25 kg block and the incline is μ_s = 0.25, determine (a) the smallest value P required to start the block moving up the incline, (b) the corresponding value of β*

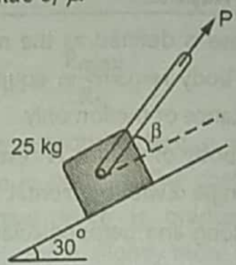


Fig. 3.32

Solution :

Given data : m = 25 kg, μ_s = 0.25, θ = 30°.

To find : P and β.

(Incline plane as x-x axis and perpendicular to incline plane as y-y axis).

$$\Sigma f_x = 0$$

$$P \cos \beta - F_r - 245.25 \sin 30 = 0$$

$$P \cos \beta - 0.25\, R - 122.63 = 0$$

$$\therefore \quad R = 4P \cos \beta - 490.52$$

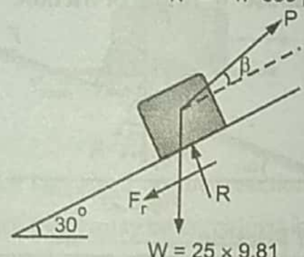


Fig. 3.32 (a) : F.B.D. of block

$$\Sigma f_y = 0$$

$$R + P \sin \beta - 245.25 \cos 30 = 0$$

$$4P \cos \beta - 490.52 + P \sin \beta - 212.39 = 0$$

$$4P \cos \beta + P \sin \beta = 702.91$$

$$\therefore \quad P (4 \cos \beta + \sin \beta) = 702.91$$

$$\therefore \quad P = 702.91/(4 \cos \beta + \sin \beta)$$

The value of P will be minimum when the denominator (4 cos β + sin β) is maximum. Take the derivative of the denominator with respect to β and set it equal to zero to determine the value of β which will make P a minimum.

$$\frac{d}{d\beta}(4\cos\beta + \sin\beta) = 0$$

$$-4\sin\beta + \cos\beta = 0$$

$$\tan\beta = 0.25$$

$$\therefore \quad \beta = \mathbf{14.03°} \quad \text{... Ans.}$$

∴ Hence, the maximum value of P is,

$$P = \frac{702.91}{(4\cos 14.03 + \sin 14.03)}$$

$$= \mathbf{170.48\ N} \quad \text{... Ans.}$$

Example 3.23 : *The force required to pull a body of weight 80 N on a rough horizontal plane is 22.5 N. Determine the coefficient of friction if the force is applied at an angle of 17° with the horizontal.*

Solution :

Consider the F.B.D. of a body at the time of motion due to force of 22.5 N acting at an angle of 17°, hence a force of friction equal to μR will be acting in the opposite direction of motion. Thus, body is in equilibrium under the action of forces shown in Fig. 3.33. Resolving force along and perpendicular to the plane,

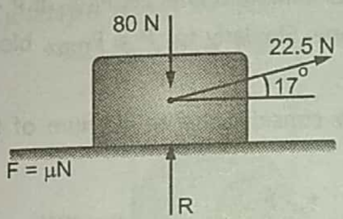


Fig. 3.33: F.B.D. of body

$$22.5\cos 17° - F = 0 \quad \text{... (1)}$$

$$R + 22.5\sin 17° - 80° = 0 \quad \text{... (2)}$$

Solving equations, we get, R = 73.422 N and F = 21.517 N

$$\text{Coefficient of friction, } \mu = \frac{F}{R} = \mathbf{0.293} \quad \text{... Ans.}$$

Example 3.24 : *A force of 40 N is required to pull a body of weight W and which is inclined at 25° with a rough horizontal plane. But the force required to push a body is 50 N. If the push is inclined 20° to the horizontal, find the weight of the body and coefficient of friction.*

Solution :

Consider the equilibrium of a body under the action of pull force as shown in Fig. 3.34 (a). Resolving the forces along and normal to the plane,

$$-F + 40\cos 25° = 0 \quad \text{... (1)}$$

$$-N + W - 40\sin 25° = 0 \quad \text{... (2)}$$

Solving equations (1) and (2), we get,

$$N = W - 16.905 \quad \text{... (3)}$$

$$\mu(W - 16.905) = 36.25 \quad \text{... (4)}$$

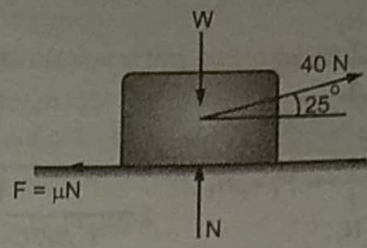


Fig. 3.34 (a)

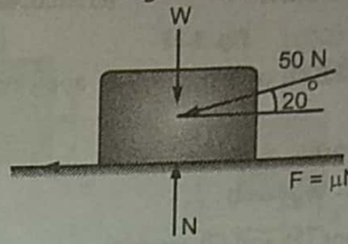


Fig. 3.34 (b)

Now, consider the equilibrium of a body under the action of pull force [Fig. 3.34(b)]

Resolving the forces along and normal to the plane,

$$F - 50\cos 20° = 0 \quad \text{... (5)}$$

$$N - W - 50\sin 20° = 0 \quad \text{... (6)}$$

Solving equations (5) and (6), we get,

$$N = W + 17.1 \quad \text{... (7)}$$

$$\mu(W + 17.1) = 46.985 \quad \text{... (8)}$$

Dividing equation (4) by equation (8),

$$\frac{\mu(W - 16.905)}{\mu(W + 17.1)} = \frac{36.25}{46.98}$$

$$\therefore \quad 46.98\,W - 794.19 = 36.25\,W + 619.875$$

$$\therefore \quad 10.73\,W = 1414.06$$

$$\therefore \quad W = \mathbf{131.79\ N} \text{ and } \mu = \mathbf{0.316} \quad \text{... Ans.}$$

Example 3.25 : *In Fig. 3.35, a block B of weight 8 kN rests on another block A of weight 16 kN. A string passing round a frictionless pulley connects both these blocks as shown. What would be the value of the horizontal force P to drag the block A towards left ? Coefficient of friction for all sliding surfaces is 0.25.*

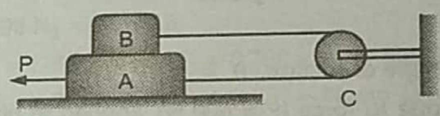


Fig. 3.35

Solution :

As block A moves towards left due to force P, block B will move towards right, due to tension in the string connect both blocks. Draw F.B.D. of both blocks, frictional forces are acting opposite to the direction of motion.

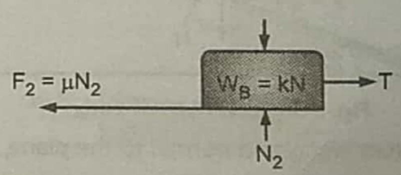
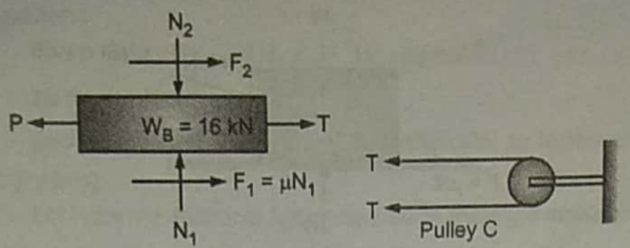


Fig. 3.35 : (a) F.B.D. of block B



(b) F.B.D. of block A (c) F.B.D. of pulley C

Fig. 3.35

Now, consider F.B.D. of block B and applying condition of equilibrium,

$$T - \mu N_2 = 0 \quad \text{... (1)}$$

$$N_2 - W_B = 0 \quad \text{... (2)}$$

Solving equations (1) and (2), we get,

$$N_2 - \text{kN} \quad \text{and} \quad T = 0.25 \times 8 = 2 \text{ kN}$$

Now, applying condition of equilibrium to block B,

$$T + F_1 + F_2 - P_{min} = 0 \quad \text{... (3)}$$

$$N_1 - W_A - N_2 = 0 \quad \text{... (4)}$$

$$N_1 = 16 + 8 = 24 \text{ kN}$$

$$F_1 = \mu N_1 = 24 \times 0.25 = 6 \text{ kN}$$

$$F_2 = \mu N_2 = 8 \times 0.25 = 2 \text{ kN}$$

Substituting all values in equation (3), we get,

$$P_{min} = T + F_1 + F_2 = 2 + 6 + 2 = 10 \text{ kN}$$

$$\mathbf{T = 2 \text{ kN}, \quad P_{min} = 10 \text{ kN}} \quad \text{... Ans.}$$

Example 3.26 : *A body of weight 1000 N is pulled upon inclined plane by a force of 600 N. The inclination of the plane is 27° to the horizontal and force is applied parallel to the plane. Determine the angle of friction.*

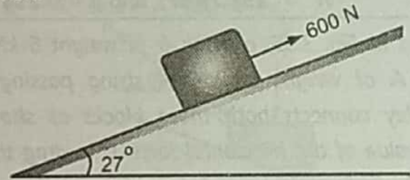


Fig. 3.36

Solution :

We know, angle of friction, $\theta = \tan^{-1}(\mu)$

Therefore, due to force (P = 600 N), applied to body parallel to the plane in upward direction, a frictional force F acting opposite to the force P. Now, consider free body diagram of block and applying condition of equilibrium.

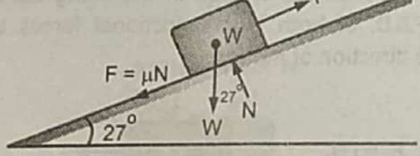


Fig. 3.36 (a) : F.B.D. of body

Resolving forces along and normal to the plane,

$$P = W \sin 27° + F \quad \text{... (1)}$$

$$N = W \cos 27°$$

$$= 1000 \cos 27° = 891 \text{ N} \quad \text{... (2)}$$

Putting P = 600, W = 1000 and F = μN in equation (1),

$$600 = 453.9 + \mu \times 891$$

$$\therefore \quad \mu = \frac{146.1}{891} = 0.164$$

$$\therefore \quad \text{Angle of friction, } \theta = \tan^{-1}(0.164) = \mathbf{9.315°} \quad \text{... Ans.}$$

Example 3.27 : *Two blocks A and B are connected by a string passing over a smooth pulley as shown in Fig. 3.37. The block A of weight 50 N, the angle of inclined plane with the horizontal is 30°. If the coefficient of friction between the plane and the block A is 0.4, find the maximum and minimum values of mass B for equilibrium of the system.*

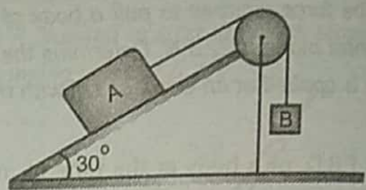


Fig. 3.37

Solution :

The possible motion of block is either downward or upward, it depends upon the magnitude of hanging mass B. At the stage when mass B has its minimum limit i.e. P_{min}. If $P < P_{min}$, block A slides down to plane. Similarly for $P > P_{max}$, block A moves up the plane.

Case 1 : Now consider the equilibrium of the system, for $(P = P_{min})$.

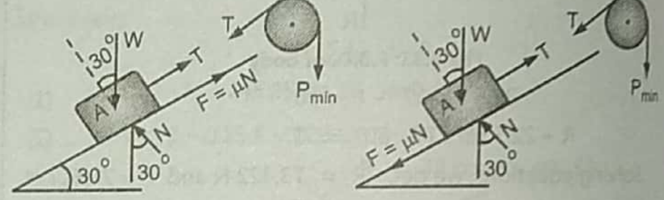


Fig. 3.37 (a) **Fig. 3.37 (b)**

Consider F.B.D. of smooth pulley.

$$P_{min} = T$$

For pulley, P_{min} = T (because pulley is smooth) for block A.

Resolving forces along and normal to the plane,

$$T + F = W \sin 30° = 25 \text{ N} \quad \text{... (1)}$$

$$N = W \cos 30° = 25\sqrt{3} \quad \text{... (2)}$$

$$N = 43.3$$

Solving we get, $T = 25 - F = 25 - 0.4 \times 25\sqrt{3}$

$$\therefore \quad T = 7.679 \text{ N}$$

$$P_{min} \approx 7.68 \text{ N}$$

Case 2 : Motion of blocks up the plane.

$$P_{max} = T \quad \text{(for pulley)}$$

For equilibrium of the system, resolving forces along and normal to the plane,

$$T = F + W \sin 30° \quad \text{... (3)}$$

$$W \cos 30° = 43.3 \text{ N} \quad \text{... (4)}$$

Solving we get, $T = 0.4 \times 43.3 + 50 \sin 30^\circ = 42.32$ N

$\therefore$ $P_{max} = 42.32$ N

Therefore range of mass B is **7.68 N to 42.32 N** ... **Ans.**

Example 3.28 : *Determine the maximum value of P which can be applied to the roller shown in Fig. 3.38 without causing it to rotate. The coefficient of friction for contact surfaces is 0.5.*

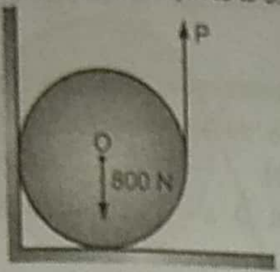


Fig. 3.38

Solution :

Consider F.B.D. of roller, as $P > P_{max}$, roller moves anticlockwise.

Thus, frictional force acting in such a direction will produce clockwise rotation.

Applying condition of equilibrium,

$$N_1 = F_2 = \mu N_2 \quad \text{... (1)}$$

$$F_1 + N_2 + P = 800$$

$$\mu N_1 + N_2 + P = 800 \quad \text{... (2)}$$

$$\mu (\mu N_2) + N_2 + P = 800$$

$$1.25 N_2 + P = 800 \quad \text{... (3)}$$

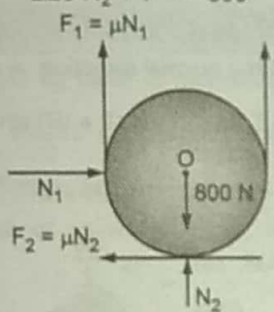


Fig. 3.38 (a)

Now taking moment of forces about O,

$$P \cdot R = \mu N_1 \cdot R + \mu N_2 \cdot R = (\mu N_1 + \mu N_2) \cdot R$$

$$P = \mu (\mu N_2) + \mu N_2 = 0.5 (0.5 N_2) + 0.5 N_2$$

$$P = 0.75 N_2$$

Substituting the value of P in equation (2),

$2N_2 = 800 \;\therefore\; N_2 = 400$ and $P = 300$ N.

Thus, P_{max} = **300 N** ... **Ans.**

Example 3.29 : *Determine force P required to start the wedge. The angle of friction for all surfaces in contact is 15°.*

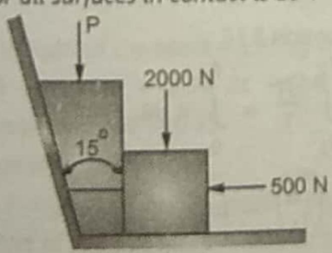


Fig. 3.39

Solution :

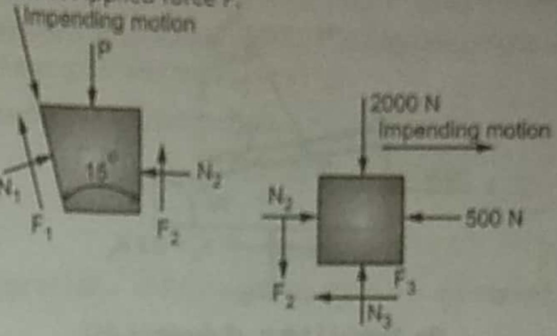
Given : Data : Forces acting on block : 2000 N and 500 N.

Angle of friction : 15°

$\therefore$ Coefficient of friction $= \mu_s = \tan 15^\circ = 0.268$

$$F_s = \mu_s N$$

To find : Applied force P.



(a) F.B.D. of wedge **(b) F.B.D. of block**

Fig. 3.39

(a) Consider F.B.D. of block. Block is in equilibrium under friction forces (F_2 and F_3). Normal reaction (N_2 and N_3) at contact surfaces as well as 2000 N and 500 N as shown in Fig. 3.39 (b).

By conditions of equilibrium,

$$\Sigma F_x = 0 \Rightarrow -500 + N_2 - F_3 = 0 \Rightarrow N_2 - \mu N_3 = 500$$

$$\Rightarrow N_2 - 0.268\, N_3 = 500 \quad \text{... (1)}$$

$$\Sigma F_y = 0 \Rightarrow -2000 - F_2 + N_3 = 0 \Rightarrow \quad -\mu N_2 + N_3 = 2000$$

$$\Rightarrow -0.268\, N_2 + N_3 = 2000 \quad \text{... (2)}$$

Solving equations (1) and (2),

$N_2 = 1116.16$ N $\Rightarrow F_2 = 0.268 \times 1116.16 = 299.13$ N

$N_3 = 2299.13$ N $\Rightarrow F_3 = 0.268 \times 2299.13 = 616.17$ N

(b) Consider F.B.D. of wedge. Wedge is in equilibrium under friction forces (F_1 and F_2), normal reactions (N_1 and N_2) and applied force P. Refer Fig. 3.39 (a).

By conditions of equilibrium,

$$\Sigma F_x = 0 \Rightarrow N_1 \cos 15^\circ - F_1 \sin 15^\circ - N_2 = 0$$

$$\Rightarrow N_1 \cos 15^\circ - \mu N_1 \sin 15^\circ - 1116.16 = 0$$

$$\Rightarrow N_1 \cos 15^\circ - 0.268\, N_1 \sin 15^\circ - 1116.16 = 0$$

$$\Rightarrow 0.896\, N_1 = 1116.16$$

$$\Rightarrow N_1 = 1245.7 \text{ N} \Rightarrow F_1 = 0.268 \times N_1 = 333.85 \text{ N}$$

$$\Sigma F_y = 0 \Rightarrow -P + F_1 \cos 15^\circ + N_1 \sin 15^\circ + F_2 = 0$$

$\Rightarrow$ P = **944 N** ... **Ans.**

Example 3.30 : *The 10° doorstop is inserted with a rightward horizontal force of 30 N. If the coefficient of static friction for all surfaces is $\mu_S = 0.20$, determine values N_U and N_L of normal forces on the upper and lower forces of the doorstop.*

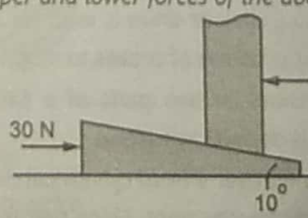


Fig. 3.40

Solution :

Data : Horizontal force required to insert doorstop = 30 N.

Coefficient of static friction = μ_S = 0.2 between contact surfaces.

To find : Normal forces acting at lower and upper surfaces, N_L and N_U = ?

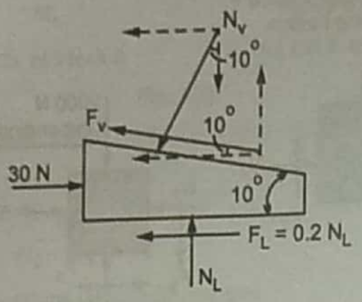


Fig. 3.40 (a) : F.B.D. of doorstop

(a) Forces acting on the doorstop are [Refer Fig. 3.45] :

(i) Frictional forces, $F_L = 0.2\ N_L$ and $F_U = 0.2\ N_U$ at contact surfaces.

(ii) Normal reactions N_L and N_U.

(iii) Horizontal force = 30 N.

Since doorstop is in equilibrium under forces

$$\sum F_x = 0 \quad \text{and} \quad \sum F_y = 0$$

(b) Resolving all forces along x-axis,

$$\sum F_x = 0 \Rightarrow 30 - N_U \sin 10° - 0.2\ N_L - P_U \cos 10° = 0$$

$$\therefore \quad 30 - N_U \sin 10° - 0.2\ N_L - 0.2\ N_U \cos 10° = 0$$

$$\therefore \quad -0.174\ N_U - 0.197\ N_U - 0.2\ N_L + 30 = 0$$

$$\therefore \quad -0.371\ N_U - 0.2\ N_L + 30 = 0$$

$$\therefore \quad 0.371\ N_U + 0.2\ N_L = 30 \qquad \text{... (1)}$$

(c) Resolving all forces along y-axis,

$$\sum F_y = 0 \Rightarrow -N_U \cos 10° + N_L + P_U \sin 10° = 0$$

$$\therefore \quad -N_U \cos 10° + N_L + 0.2\ N_U \sin 10° = 0$$

$$-0.985\ N_U + N_L + 0.035\ N_U = 0$$

$$-0.95\ N_U + N_L = 0 \qquad \text{... (2)}$$

(d) Substituting N_L in equation (i) ,

$$0.371\ N_U + 0.2\ (0.95\ N_U) = 30$$

$$\therefore \quad 0.371\ N_U + 0.19\ N_U = 30$$

$$\therefore \quad 0.561\ N_U = 30$$

$$\therefore \quad 0.561\ N_U = 30$$

$$\therefore \quad N_U = \mathbf{53.48\ N} \qquad \text{... Ans.}$$

$$N_L = \mathbf{50.806\ N} \qquad \text{... Ans.}$$

3.15 FLAT BELTS

A belt or rope alongwith pulley or drum is used for raising a load, transmitting power or application of brakes to stop the motion.

A relation between tensions in two parts of a flat belt passing over a cylindrical drum is derived as follows :

Consider a flat belt passing over a fixed cylindrical drum. Consider that belt is about to slip towards right. Since belt is about to slip rightward, tension in right side of the belt i.e. T_2 is greater than T_1 i.e. tension in left side of the belt. Refer Fig. 3.41 (a). Part of belt with tension T_2 is called tight side and that with tension T_1 is called slack of the side drum.

Let 'θ' be the lap angle subtended by the belt which is in contact with drum at the centre. Let AB be part of belt in contact with drum.

Let CD be the infinitesimally small part of belt of length 'ds' subtending an angle 'dθ' at centre. F.B.D. of this element CD is as shown in Fig. 3.46 (b).

Tensions at C and D are represented by T and (T + dT) respectively.

Let 'dN' be the normal reaction and dF = μdN be frictional force acting in opposite direction of impending motion.

Since belt is in equilibrium, applying conditions of equilibrium to the forces acting on the belt,

∑ Forces acting along the tangential direction = 0.

$$\sum F_T = 0 \Rightarrow -T\cos\left(\frac{d\theta}{2}\right) + (T + dT)\cos\left(\frac{d\theta}{2}\right) - \mu\, dN = 0$$

$$\Rightarrow dT \cos\left(\frac{d\theta}{2}\right) = \mu\, dN$$

When dθ is very small,

$$\cos\left(\frac{d\theta}{2}\right) = 1.$$

$$\therefore \quad dT = \mu\, dN \qquad \text{... (3.14)}$$

∑ Forces acting in the normal direction = 0

$$\sum F_N = 0 \Rightarrow -T\sin\left(\frac{d\theta}{2}\right) - (T + dT)\sin\frac{d\theta}{2} + dN = 0$$

$$-2T\sin\left(\frac{d\theta}{2}\right) - dT\sin\frac{d\theta}{2} + dN = 0$$

When dθ is very small, $\sin\frac{d\theta}{2}$ is $\frac{d\theta}{2}$.

$$\therefore \quad -2T\left(\frac{d\theta}{2}\right) - dT\left(\frac{d\theta}{2}\right) + dN = 0$$

Since $dT\left(\frac{d\theta}{2}\right)$ is a very small quantity, it is neglected.

$$\therefore \quad -2T\left(\frac{d\theta}{2}\right) + dN = 0$$

$$\therefore \quad T\, d\theta = dN \qquad \text{... (3.15)}$$

Dividing equation (3.14) by equation (3.15),

$$\frac{dT}{T} = \mu \cdot d\theta \qquad \text{... (3.16)}$$

Tension in the belt varies from T_1 to T_2. Lap angle varies from 0 to θ.

Integrating equation 3.16,

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\theta} \mu\, d\theta$$

$$\therefore \quad \ln\left(\frac{T_2}{T_1}\right) = \mu\theta$$

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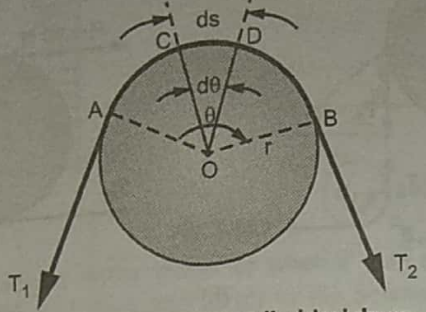
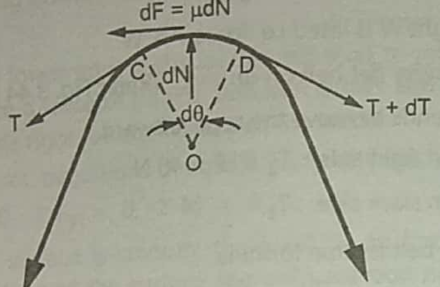
$$\therefore \quad \frac{T_2}{T_1} = e^{\mu\theta}$$

If r is the radius of the cylindrical drum, torque τ developed due to tensions in the belt is

$$\tau = T_2 \times r - T_1 \times r = (T_2 - T_1)\, r$$



(a) Belt passing over cylindrical drum



(b) F.B.D. of belt

Fig. 3.41

NUMERICAL EXAMPLES ON BELT FRICTION

Example 3.31: *A 137 kg block is supported by a rope which is wrapped $1\frac{1}{2}$ times around a horizontal rod. Knowing that the coefficient of static friction between the rope and the rod is 0.15, determine the range of values of P for which equilibrium is maintained.*

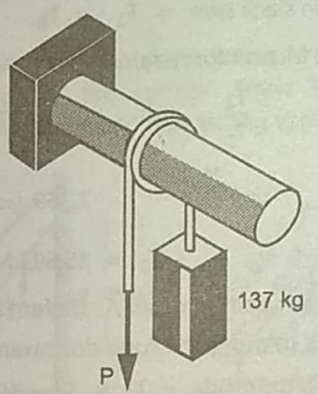


Fig. 3.42

Solution :

Given data : Weight of the block = 137 kg × 9.81 = 1343.97 N

Lap angle = $\beta = n \times (2\pi) = 1.5 \times 2\pi = (3\pi)$ radians.

μ between rope and rod = 0.15

To find : Range of force P.

(a) Case I : P > 1343.97 N i.e. force P tends to move the rope downward. Part of rope at force P side is tight side of rope (T_2) and other side is slack side (T_1).

From flat belt friction formula,

$$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{P}{1343.97} = e^{0.15 \times (3\pi)} = 4.111$$

$$\Rightarrow P = 1343.97 \times 4.11$$

$$\Rightarrow P = \mathbf{5525.34\ N} \qquad \textbf{... Ans.}$$

(b) Case II : P < 1343.97 N i.e. force P tends to move the rope downward. Part of rope at weight 137 kg side is tight side of rope (T_2) and other side is slack side (T_1).

From flat belt friction formula,

$$\frac{T_2}{T_1} = e^{\mu\beta} \Rightarrow \frac{1343.97}{P} = e^{0.15 \times 3\pi} = 4.111$$

$$\Rightarrow P = \mathbf{326.92\ N} \qquad \textbf{... Ans.}$$

Range of value of P for which equilibrium is maintained :

$$\mathbf{326.92\ N \le P \le 5525.34\ N} \qquad \textbf{... Ans.}$$

Example 3.32 : *Two cylinders are connected by a rope that passes over two fixed rods as shown. Knowing that the coefficient of static friction between the rope and the rods is 0.40, determine the range of the mass m of cylinder D for which equilibrium is maintained.*

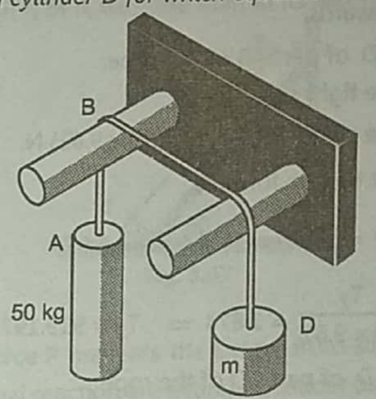


Fig. 3.43

Solution :

Given data :

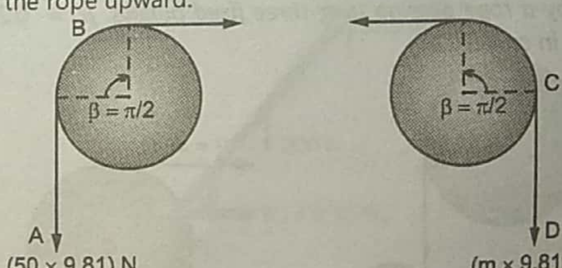
Weight of the cylinder A = (50 × 9.81) N

Lap angle = $\beta = \frac{\pi^c}{2}$ for both rods.

μ between rod and rope = 0.40

To find : Range of mass 'm'.

(a) Case I : (50 × 9.81) N > (m × 9.81) N i.e. cylinder of 50 kg tends to move the rope downward and cylinder of 'm' kg tends to move the rope upward.



(a) F.B.D. of AB (b) F.B.D. of CD

Fig. 3.43

Consider F.B.D. of part AB of rope. (Refer Fig. 3.43 (a)).

Tension at tight side : $T_2 = (50 \times 9.81)$ N

Tension at slack side : T_1.

From flat belt friction formula,

$$\frac{T_2}{T_1} = e^{\mu\beta} = e^{(0.4 \times \pi/2)} = 1.874$$

$$\Rightarrow \quad \frac{50 \times 9.81}{T_1} = 1.874 \Rightarrow T_1 = 261.67 \text{ N}$$

Consider F.B.D. of part CD of rope. Refer Fig. 3.43 (b).

Tension at tight side : $T_2 = 261.67$ N

Tension at slack side : $T_1 = (m \times 9.81)$ N

From flat belt friction formula,

$$\frac{T_2}{T_1} = e^{\mu\beta} = e^{(0.4 \times \pi/2)} = 1.874$$

$$\Rightarrow \quad \frac{261.67}{m \times 9.81} = 1.874 \Rightarrow m = \mathbf{14.23\ kg} \quad \text{... Ans.}$$

(b) Case II : (50×9.81) N < $(m \times 9.81)$ N i.e. cylinder of m kg tends to move the rope downward and cylinder of 50 kg tends to move the rope upwards.

Consider F.B.D. of part AB of the rope.

Tension in the tight side : T_2 N.

Tension in the slack side : $T_1 = (50 \times 9.81)$ N.

From flat belt friction formula,

$$\frac{T_2}{T_1} = e^{\mu\beta} = e^{(0.4 \times \pi/2)} = 1.874$$

$$\Rightarrow \quad \frac{T_2}{50 \times 9.81} = 1.874 \Rightarrow T_2 = 919.197 \text{ N}$$

Consider F.B.D. of part CD of the rope :

Tension in the tight side : $T_2 = (m \times 9.81)$ N

Tension in the slack side : $T_1 = 919.197$ N

From flat belt friction formula,

$$\frac{T_2}{T_1} = e^{\mu\beta} = e^{(0.4 \times \pi/2)} = 1.874$$

$$\Rightarrow \quad \frac{m \times 9.81}{919.197} = 1.874 \Rightarrow m = \mathbf{175.59\ kg} \quad \text{... Ans.}$$

Range of mass 'm' for which equilibrium is maintained :

$$\mathbf{14.23\ kg \le m \le 175.59\ kg} \quad \text{... Ans.}$$

Example 3.33 : *Find the weight W that can be (i) lifted by a force P = 40 N for the arrangement shown in Fig. 3.44. The weight is supported by a rope passing over three fixed pulleys. μ = 0.2 for all surfaces in contact.*

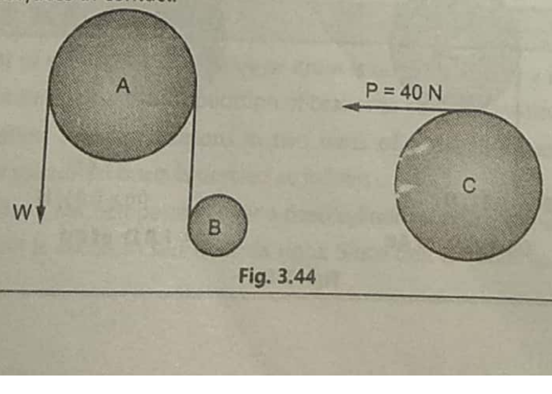


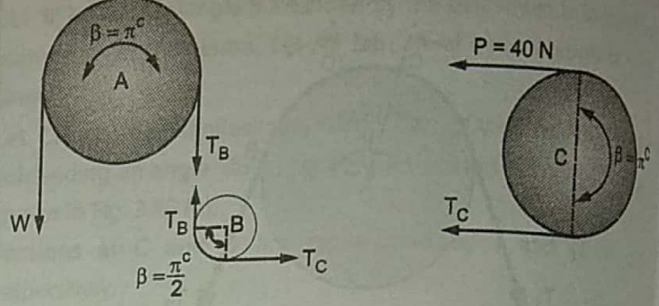
Fig. 3.44

Solution :

Given data : Force, P = 40 N

Coefficient of friction = μ = 0.2 for all surfaces in contact

To find : Weight 'W' when it is lifted.



(a) (b) (c)

Fig. 3.44

(a) Weight W is lifted i.e. force P > W.

(i) Consider flat belt on drum C. (Refer Fig. 3.44 (c)).

Force P tends to move the rope leftward.

Tension in tight side : $T_2 = P = 40$ N.

Tension in slack side : T_1.

From flat belt friction formula,

$$\frac{T_2}{T_1} = e^{\mu\beta} = e^{0.2 \times (\pi)^C} = 1.874$$

$$\Rightarrow \quad \frac{40}{T_1} = 1.874 \Rightarrow T_1 = 21.345 \text{ N}$$

$$\therefore \quad T_C = 21.345 \text{ N}$$

(ii) Consider flat belt on drum B (Refer Fig. 3.44 (b)).

Force T_C tends to move the rope towards rightward.

Tension in tight side = $T_2 = T_C = 21.345$ N

Tension in slack side = $T_1 = T_B$.

From flat belt friction formula,

$$\frac{T_2}{T_1} = e^{\mu\beta} = e^{0.2 \times (\pi/2)^C} = 1.369$$

$$\Rightarrow \quad \frac{21.345}{T_1} = 1.369 \Rightarrow T_1 = 15.592 \text{ N}$$

$$T_B = 15.592 \text{ N}$$

(iii) Consider flat belt on drum A. (Refer Fig. 3.44 (a)).

Force T_B tends to move the rope downward.

Tension in tight side = $T_2 = T_B = 15.592$ N

Tension in slack side = T_1 = W.

From flat belt friction formula,

$$\frac{T_2}{T_1} = e^{\mu\beta} = e^{0.2 \times \pi} = 1.874$$

$$\Rightarrow \quad \frac{15.592}{T_1} = 1.874$$

$$\Rightarrow \quad T_1 = 8.32 \text{ N}$$

Weight W that can be lifted = 8.32 N. ... Ans.

3.16 LADDER FRICTION

Consider a ladder AB resting on the ground and leading against a wall.

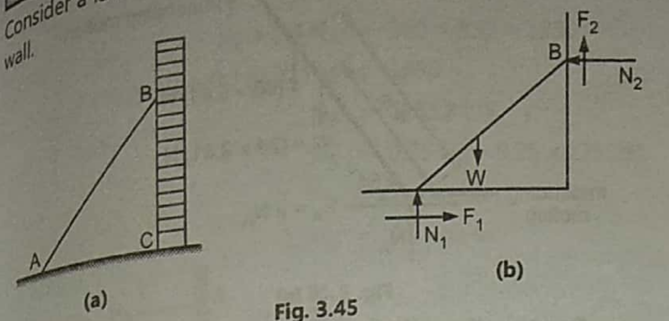


Fig. 3.45

Due to self weight of the ladder or when a man stands on the ladder, the end B of the ladder tends to slip downward, frictional force F_2 acting between ladder and the vertical wall will be acting upwards.

Similarly, the lower end of the ladder will tend to move towards left and hence a force of friction between ladder and floor will be acting towards right for equilibrium of the ladder.

Under all forces, conditions of equilibrium must satisfy

$$\Sigma F_x = 0, \ \Sigma F_y = 0, \ \Sigma M = 0$$

If any of the surface is smooth, there will be no force of friction between ladder and the surface and only, reaction normal to the surface is acting.

NUMERICAL EXAMPLES ON LADDER FRICTION

***Example 3.34 :** A ladder 5 m long and weighing 300 N is placed against a wall. Coefficient of friction between the wall and the ladder is 0.25. A man weighing 700 N climbs the ladder. A man finds that when he was 3.5 m from lower end of the ladder, it was about to slip away. What should be the coefficient of friction between ladder and floor ? Angle of inclination between the ladder and floor is 60°.*

Solution :

At the time of slip, frictional force acting in reverse direction and reaction normal to the surface. Draw F.B.D. of ladder considering weight of ladder and man acting vertically downward.

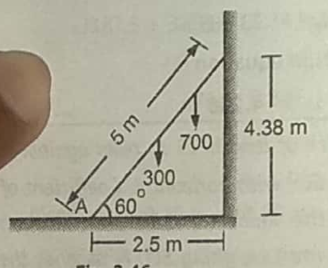


Fig. 3.46

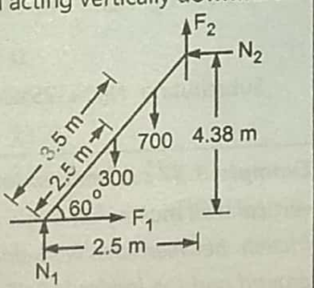


Fig. 3.46 (a)

Applying condition of equilibrium,

$$F_1 = N_2 \quad \text{... (1)}$$

$$N_1 + F_2 = 300 + 700 = 1000 \quad \text{... (2)}$$

Taking moment of forces about A,

$$4.33 \ N_2 + 2.5 \ F_2 = 300 \times 2.5 \cos 60^{\circ} + 700 \times 3.5 \cos 60^{\circ}$$

$$4.33 \ N_2 + 0.25 \ N_2 \times 2.5 = 375 + 1225$$

$$4.995 \ N_2 = 1600 \text{ or } N_2 = 322.91 \text{ N}$$

Using equations (1) and (2), we get,

$$F_1 = 322.91 \text{ N and } N_1 + F_2 = 1000$$

or $\quad N_1 + \mu N_2 = 1000$

$\therefore \quad N_1 = 919.27$ N and

$$\mu = \frac{F_1}{N_1} \quad (F_1 = \mu N_1)$$

$$= \frac{322.91}{919.27} = 0.351$$

The coefficient of friction between ladder and floor is **0.315.**

... Ans.

***Example 3.35 :** A uniform ladder 3 m long weighs 200 N. It is placed against a wall making an angle of 60° with the floor as shown in Fig. 3.47. The coefficient of friction between the wall and the ladder is 0.25 and that between floor and ladder is 0.35. The ladder in addition to its weight has to support a man of 1000 N at its top at B. Calculate :*

(i) The horizontal force P is to be applied to ladder at the floor level to prevent slipping.

(ii) If the force P is not applied, what should be the minimum inclination of the ladder with the horizontal so that there is no slipping of it, with the man at its top ?

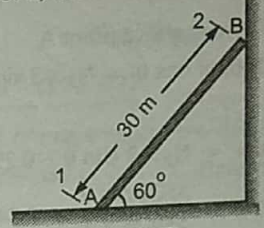


Fig. 3.47

Solution :

Case 1 : Force P prevents the ladder from slipping, frictional forces and normal reaction are acting on ladder as shown in F.B.D. of Fig. 3.47 (a) and (b).

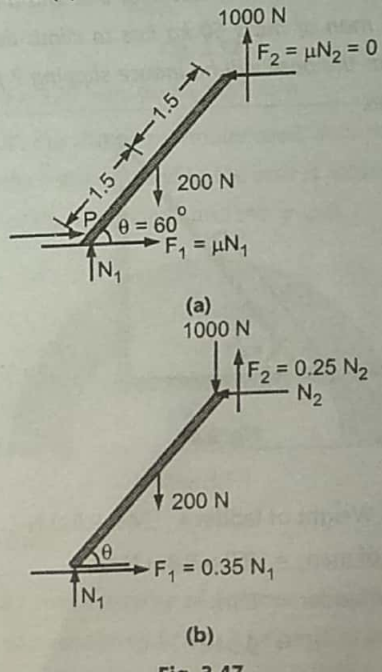


Fig. 3.47

Applying condition of equilibrium,

$$N_2 = P + F_1 \quad \text{... (1)}$$

$$N_1 + F_2 = 200 + 1000 = 1200 \text{ N} \quad \text{... (2)}$$

Taking moment about point A,

$$200 \times 1.5 \cos 60^\circ + 1000 \times 3 \cos 60^\circ = N_2 \times 3 \sin 60^\circ + F_2 \times 3 \cos 60^\circ$$

$$\therefore \quad 1650 = 2.973 \, N_2$$

$$\therefore \quad N_2 = 554.98 \text{ N}$$

Putting value of N_2 in equations (1) and (2), we get,

$$N_1 = 1061.25 \text{ N} \quad \text{and} \quad P = 183.54 \text{ N}$$

Case 2 : Minimum angle θ is to be found. For equilibrium, draw F.B.D. and applying condition of equilibrium,

$$F_1 = N_2 \quad \text{... (3)}$$

$$N_1 + F_2 = 200 + 1000 = 1200 \quad \text{... (4)}$$

Put $F_1 = 0.35 \, N_1$ and $F_2 = 0.25 \, N_2$.

Using equations (3) and (4), we get,

$$N_1 = 1103.44 \text{ N} \quad \text{and} \quad N_2 = 386.21 \text{ N}$$

Taking moment of forces about point A,

$$200 \times 1.5 \cos \theta + 1000 \times \cos \theta = N_2 \times 3 \sin \theta + 0.25 \, N_2 \times 3 \cos \theta$$

$$3300 \cos \theta = N_2 \times 3 (\sin \theta + 0.25 \cos \theta)$$

$$\sin \theta + 0.25 \cos \theta = \frac{3300 \cos \theta}{386.21 \times 3} = 2.846 \cos \theta$$

$$\therefore \quad \sin \theta = 2.598 \cos \theta$$

$$\tan \theta = 2.598 \quad \text{or} \quad \theta = \mathbf{68.95^\circ} \quad \text{... Ans.}$$

***Example 3.36 :** The uniform ladder AB has a length of 8 m and a mass of 24 kg. End A is on horizontal floor and end B rests against a vertical wall. A man of mass 60 kg has to climb this ladder. At what position from the base will he induce slipping ? $\mu = 0.34$ at all contact surfaces.*

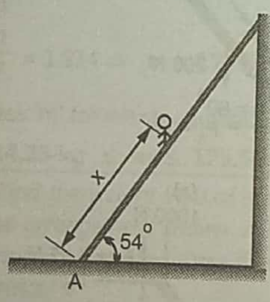


Fig. 3.48

Solution :

Given data : Weight of ladder = (24×9.81) N

Weight of man = (60×9.81) N

Length of ladder = 8 m

$\mu = 0.34$

To find : Distance x.

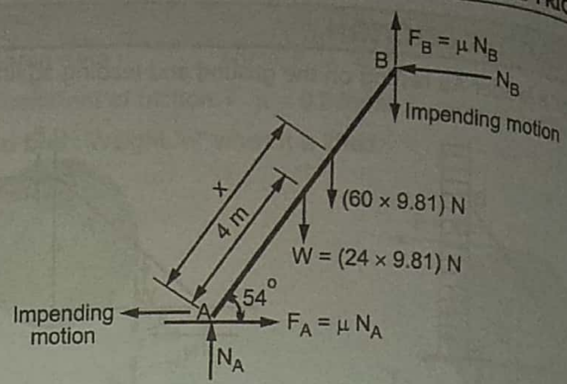


Fig. 3.48 (a)

(a) Due to self weight and weight of man, ladder tends to move leftward at end A and downward at end B. Refer Fig. 3.48 (a).

Ladder is in equilibrium under forces :

Self weight = $24 \times 9.81 = 235.44$ N

Weight of man = $60 \times 9.81 = 588.60$ N

Normal reaction at A and B : N_A and N_B.

Friction force : $F_A = \mu N_A$, $F_B = \mu N_B$.

(b) Applying conditions of equilibrium,

$$\Sigma M_A = 0$$

$$\Rightarrow \; -F_B \times 8 \cos 54^\circ - N_B \times 8 \sin 54^\circ + W \times 4 \cos 54^\circ + (60 \times 9.81) \times x \cos 54^\circ = 0$$

$$\Rightarrow \; 1.598 \, N_B - 6.472 \, N_B + 553.55 + 345.97 \, x = 0$$

$$\Rightarrow \; 345.97 \, x - 8.07 \, N_B = -553.55 \quad \text{... (1)}$$

$$\Sigma F_y = 0 \Rightarrow F_B + N_A - 824.04 = 0$$

$$\Rightarrow \mu N_B + N_A = 824.04$$

$$\Rightarrow 0.34 \, N_B + N_A = 824.04 \quad \text{... (2)}$$

$$\Sigma F_x = 0 \Rightarrow -N_B + F_A = 0$$

$$\Rightarrow -N_B + \mu N_A = 0$$

$$\Rightarrow -N_B + 0.34 \, N_A = 0 \quad \text{... (3)}$$

Solving equations (2) and (3) simultaneously,

$$N_B = 251.14 \text{ N}$$

$$N_A = 738.65 \text{ N}$$

Substituting $N_B = 251.14$ N in equation (1),

$$x = \mathbf{4.258 \text{ m}} \quad \text{... Ans.}$$

***Example 3.37 :** A uniform ladder of length 15 m rests against a vertical wall making an angle of 60° with horizontal. Coefficient of friction between the wall and the ladder is 0.30 and between ground and the ladder is 0.25. A man weighing 500 N ascends the ladder. How long will he be able to go before the ladder slips ? Find the weight that is necessary to put at the bottom of the ladder so as to be just sufficient to permit the man to go to the top. Assume weight of the ladder as 850 N.*

Solution :

Fig. 3.58 shows the given ladder alongwith the forces acting on it.

$$\Sigma P_x = 0 \quad \text{gives,}$$

$$N_A = 0.25\, N_B$$

$$\Sigma P_y = 0 \quad \text{gives,}$$

$$N_B + 0.3\, N_A = 500 + 850 = 1350$$

$$\therefore \quad N_B + 0.3\,(0.25\, N_B) = 1350$$

$$\therefore \quad N_B = 1255.81 \text{ N}$$

$$N_A = 0.25\, N_B = 0.25 \times 1255.81$$

$$= 313.95 \text{ N}$$

$$\Sigma M_B = 0 \quad \text{gives,}$$

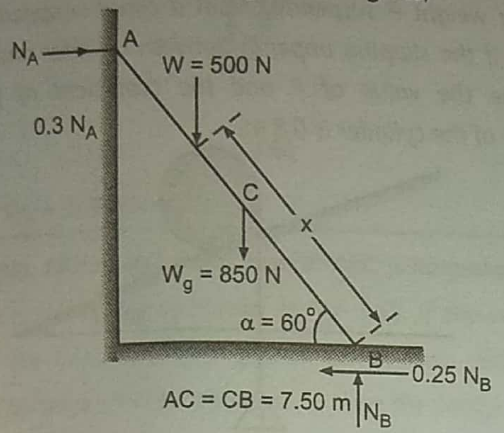


Fig. 3.49

$$850 \times 7.5 \cos 60^\circ + 500 \times x \cos 60^\circ - N_A\, 15 \sin 60^\circ - 0.3\, N_B\, 15 \cos 60^\circ = 0$$

$$\therefore \quad 850 \times \frac{7.5}{2} + \frac{500}{2} x - 313.95 \times 15 \times 0.866 - 0.3 \times 1255.81 \times \frac{15}{2} = 0$$

$$x = 6.39 \text{ m} \qquad \text{... Ans.}$$

Let W be the weight that is necessary to put at the bottom of the ladder so as to be just sufficient to permit the man to go to the top of ladder, $\mu W = 0.25\, W$ will be the frictional resistance.

Taking moment of forces about B,

$$850 \times \frac{7.5}{2} + 500 \times 7.5 - N_A \times 15 \sin 60^\circ - 0.3\, N_A \times 7.5 = 0$$

$$\therefore \quad 3187.5 + 3750 - 15.24\, N_A = 0$$

$$\therefore \quad N_A = 455.22 \text{ N}$$

But $\quad N_B + 0.3\, N_A = 1350$

$$\therefore \quad N_B = 1350 - 0.3 \times 455.22$$

$$= 1213.43 \text{ N}$$

Also, $\quad 0.25\, N_B + 0.25\, W = N_A = 455.22$

$$\therefore \quad 0.25 \times 1213.43 + 0.25\, W = 455.22$$

$$\therefore \quad W = 607.45 \text{ N} \qquad \text{... Ans.}$$

PROBLEMS FOR PRACTICE

Problem No. 1 : *Locate the centroid C of a cylindrical homogeneous wire of uniform cross-section is bent into shape as shown in Fig. 3.50. If dimension 'a' is fixed, find the dimension 'b' so that the centroid of wire will coincide with centre 'C' of the semicircular portion.*

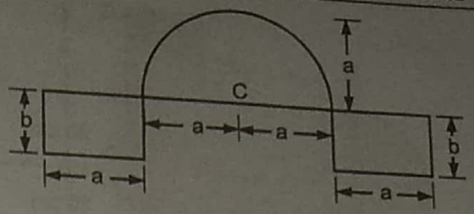


Fig. 3.50

Answer : b = 0.618 a

Problem No. 2 : *Locate the centroid of the shaded area shown in Fig. 3.51.*

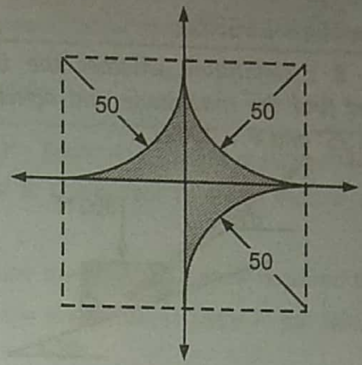


Fig. 3.51

Answer : Centroid = (3.72, 3.72)

Problem No. 3 : *A homogeneous wire ABCD is bent as shown in Fig. 3.61 and is suspended at point C. Determine the length "l" for which*

(a) portion BCD is horizontal, (b) portion AB is horizontal.

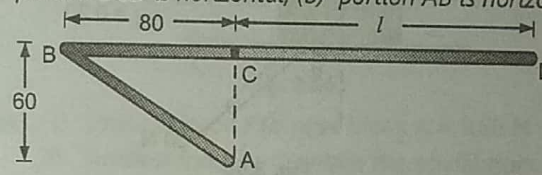


Fig. 3.52

Answer : (a) l = 120 mm, (b) l = 99.5 mm

Problem No. 4 : *For the semi-annular area, determine the ratio of "a" to "b" for which the centroid of the area is located at the point of intersection of the inner circle and the y-axis.*

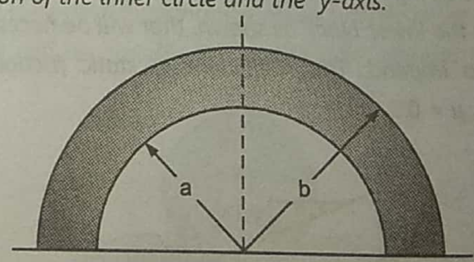


Fig. 3.53

Answer : $\frac{a}{b} = 0.495$

Problem No. 5 : *Find the value of distance 'a' so that the centroid of the uniform lamina shown in Fig. 3.54 remains at the centre of rectangle ABCD.*

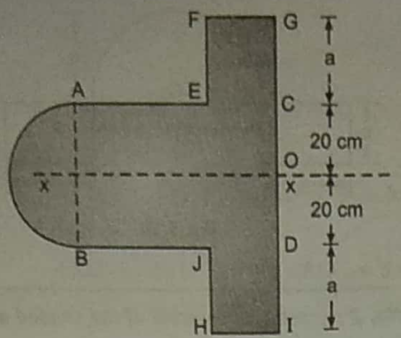


Fig. 3.54

Answer : Value of a = 21.58 cm

Problem No. 6 : *Determine whether the block shown is in equilibrium and find the magnitude and direction of the friction force when $\theta = 35°$ and $P = 200$ N.*

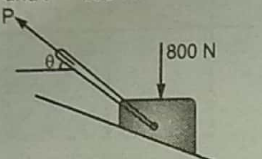


Fig. 3.55

Answer : $F_r = 141.13$ N and $F_m = 138.06$ N i.e. $F_r > F_m$ ∴ block is not in equilibrium. $F_{actual} = 103.55$ N

Problem No. 7 : *Considering only value of θ less than 90°, determine the smallest value of θ required to start the block moving to the right when W = 100 N.*

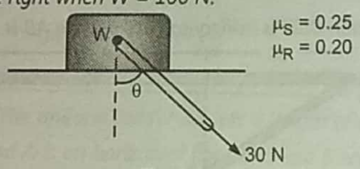


Fig. 3.56

Answer : θ = 67.73°

Problem No. 8 : *A block of weight 1000 N rests on a horizontal surface and supports on top of it another block of weight 250 N as shown in Fig. 3.57. The upper block is attached to a vertical wall by an inclined string AB. Find the magnitude of the horizontal force P applied to the lower block as shown, that will be necessary to cause slipping to impend. The coefficient of static friction for all the surfaces is μ = 0.3.*

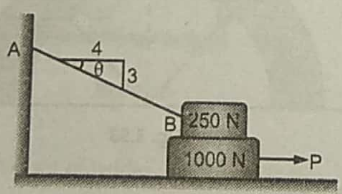


Fig. 3.57

Answer : P = 422.45 N

Problem No. 9 : *What force 'P' is required to move the block A of weight 0.5 kN as shown in Fig. 3.58. If the coefficient of friction for all contact surfaces is 0.2, also calculate the tension in the cord CD.*

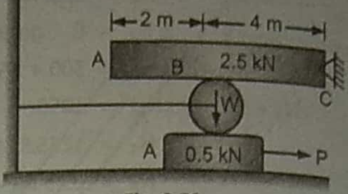


Fig. 3.58

Problem No. 10 : *A 500 N cylinder shown in Fig. 3.59 is held at rest by a weight P suspended from a chord wrapped around the cylinder. If the slipping impends between cylinder and the incline, determine the value of P and the coefficient of friction. The diameter of the cylinder is 0.8 m.*

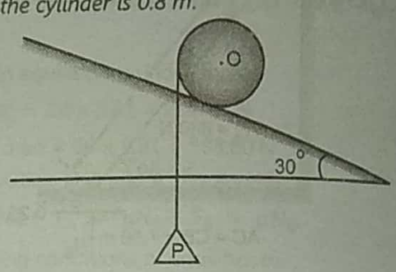


Fig. 3.59

Answer : P = 500 N and μ = 0.573

Problem No. 11 : *Determine the necessary force P acting parallel to the plane to cause motion to impend. Assume coefficient of friction is 0.25 and pulley is smooth.*

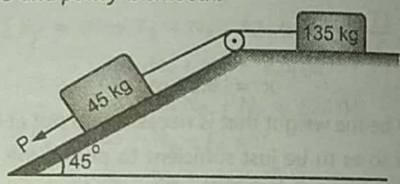


Fig. 3.60

Answer : P = 96.97 N

Problem No. 12 : *Two bodies of weight W_1 and W_2 rest on a rough inclined plane and are connected by a short piece of string as shown in Fig. 3.61. If $\mu_1 = 0.2$ and $\mu_2 = 0.3$, find the angle of inclination of the plane for which the sliding will impend. Assume $W_1 = W_2$.*

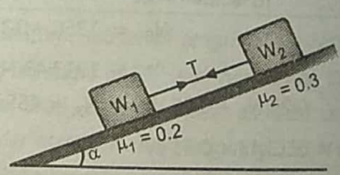


Fig. 3.61

Answer : α = 14.04°

Problem No. 13 : *Two blocks connected by a horizontal link AB are supported on two rough planes as shown in Fig. 3.62. μ_S for block A = 0.40, ϕ_S for block B is 15°. What is the smallest weight W_A of the block A for which the equilibrium of the system can exist ?*

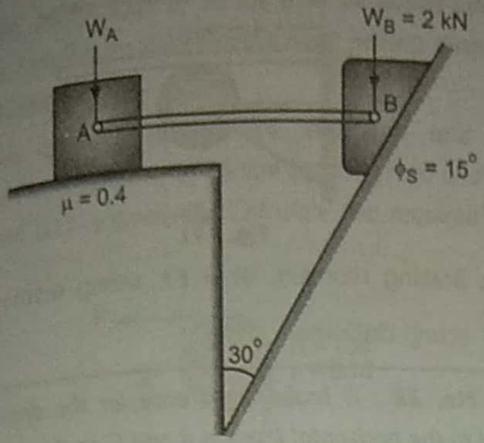


Fig. 3.62

Answer : $W_A = 5.00$ kN

Problem No. 14 : *A rigid U-pin bracket ABC is hinged at B. Its end C rests on a 15° wedge as shown in Fig. 3.63. If the vertical load acting on the bracket at A is 500 N downwards, determine the minimum horizontal force P required to push the wedge to the left. Neglect the weight of the wedge and the bracket. Assume coefficient of friction at all surfaces of contact as 0.2.*

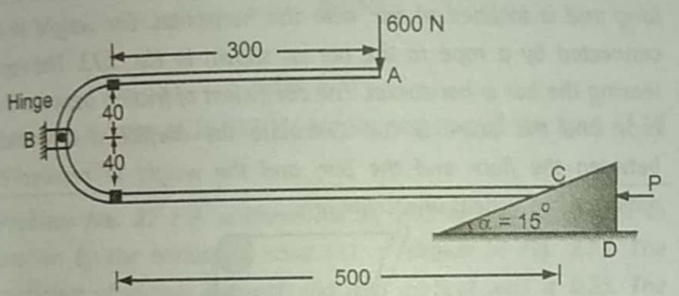


Fig. 3.63

Answer : $P_{min} = 217.19$ N

Problem No. 15 : *What force P must be applied to the wedges to start them under 1000 N block ? The angle of friction for all contact surfaces is 10°.*

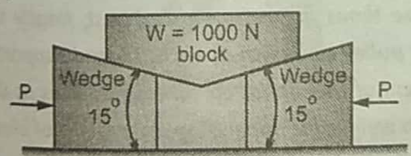


Fig. 3.64

Answer : P = 321 N

Problem No. 16 : *The two 5° wedges shown are used to adjust the position of the column under a vertical load of 5 kN. Determine the magnitude of the forces P required to lower the column if the coefficient of friction for all surfaces is 0.40.*

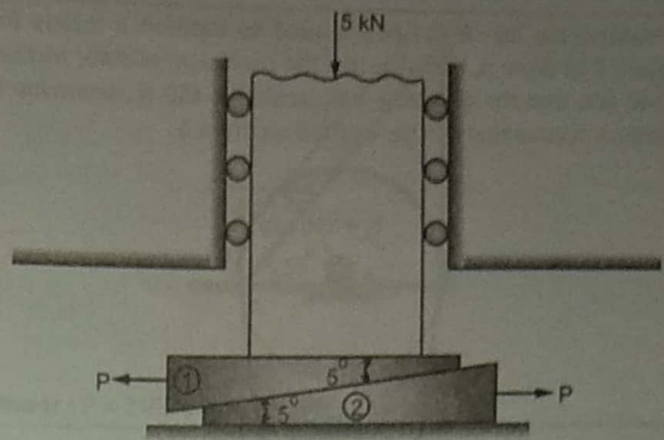


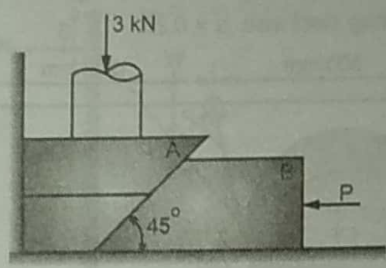
Fig. 3.65

Answer : P = 3.51 N

Problem No. 17 : *Block A supports a pipe column and rests as shown on wedge B. Coefficient of static friction at all surfaces of contact is 0.25.*

(1) *Determine the smallest force P required to raise block A.*

(2) *Determine the smallest force P for which equilibrium is maintained.*



Part (1) & (2)

Fig. 3.66

Answer : (1) Smallest force P to raise block A = 9.86 N

(2) Smallest force to maintain the equilibrium is P = 0.91 N

Problem No. 18 : *A smooth circular cylinder of weight W and radius r is supported by two semicircular cylinders, each of radius r and weight W/2. If μ between semicircular cylinders and the horizontal plane is 0.5 and friction between the cylinders themselves is neglected, determine maximum distance 'b' for which equilibrium will be possible without middle cylinder touching the ground.*

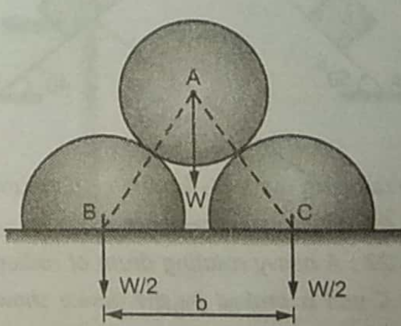


Fig. 3.67

Answer : b = 2.828 r

Problem No. 19 : A flat belt is used to transmit a torque from drum B to drum A. Knowing that the coefficient of static friction is 0.40 and that the allowable belt tension is 450 N, determine the largest torque that can be exerted on drum A.

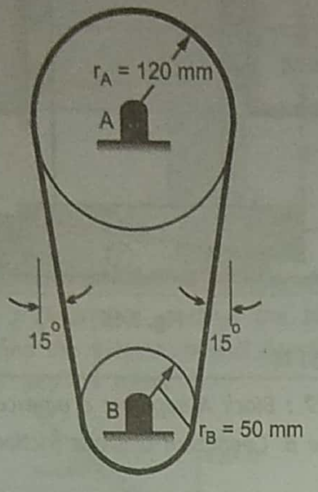


Fig. 3.68

Answer : Torque exerted on drum A = 35.04 Nm

Problem No. 20 : *Calculate the braking torque acting on the drum if drum is rotating clockwise. μ = 0.25.*

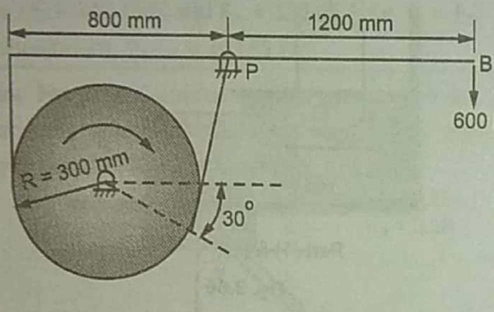


Fig. 3.69

Answer : Banking torque = 249.48 Nm (↻)

Problem No. 21 : *A mass M_1 of 150 kg is connected by flexible inextensible and massless cable passing over a fixed pulley to mass M_2 as shown in Fig. 3.70. If μ = 0.15 for plane and μ = 0.12 for pulley, find the range of mass M_2 for the equilibrium of the system.*

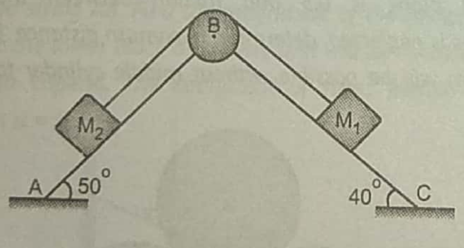


Fig. 3.70

Answer : The range of mass M_2 for the equilibrium of the system = 74.44 kg to 204.93 kg

Problem No. 22 : *A heavy rotating drum of radius r is supported in bearing at C and is braked by the device shown in Fig. 3.71. Calculate the braking moment w.r.t. point C, if the coefficient of friction between drum and brake shoe is μ.*

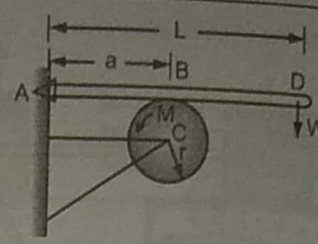


Fig. 3.71

Answer : Braking moment, M = F·r, taking moment at point $C = \frac{\mu WLr}{a}$ acting clockwise.

Problem No. 23 : *A brake band encircles the drum D and is connected to the horizontal lever at B and C as shown in Fig. 3.72. The coefficient of friction between the brake band and the drum is 0.3 and the force applied at P is 600 N. Calculate the braking torque if the drum is rotating (i) clockwise, (ii) anticlockwise.*

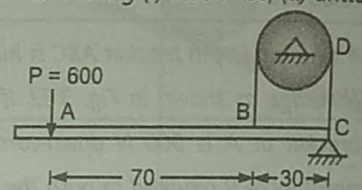


Fig. 3.72

Answer : (i) τ = 183.10 Nm, (ii) τ = 469.89 Nm

Problem No. 24 : *A homogeneous bar A of weight 125 N is 2 m long and is inclined at 60° with the horizontal. The weight W is connected by a rope to the bar as shown in Fig. 3.73. The rope leaving the bar is horizontal. The coefficient of friction between the rope and the drum is 0.2. Calculate the coefficient of friction between the floor and the bar, and the weight W required to maintain the system in equilibrium.*

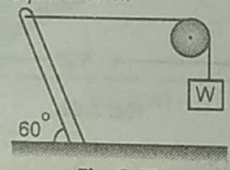


Fig. 3.73

Answer : Coefficient of friction (μ) = 0.29 and weight(W) = 49.43 N

Problem No. 25 : *A block of weight 10 kN rests on rough surface. Block is connected by cord which is passed over pulley B and then wrapped three times around a vertical post, finally the cord goes over another pulley as shown in Fig. 3.74 and supports a weight W. Take coefficient of friction for all contact surfaces as 0.15. Calculate the minimum weight for impending the motion of block.*

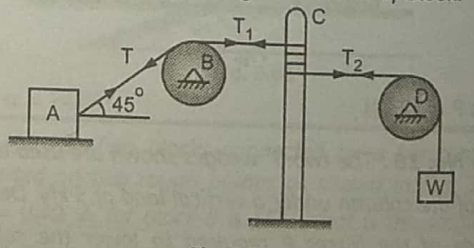


Fig. 3.74

Answer : W = T_3 = 44.412 kN

Problem No. 26 : *A load of 18 kN is to be pulled upward by a force P. The rope passes over three pulleys having diameters 150, 100 and 200 mm.*

The distance between pulleys are 400 mm and 500 mm respectively. The coefficients of friction between rope and pulley are 0.1, 0.12 and 0.15 respectively. Calculate the required minimum value of P.

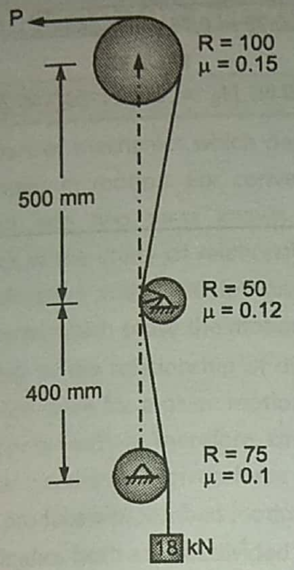


Fig. 3.75

Answer : A force of 26.532 kN is required to pull the load of 18 kN.

Problem No. 27 : *A uniform bar of 500 N weight is held in position by the horizontal cord CD as shown in Fig. 3.76. The coefficient of friction between rod and vertical wall is 0.25. The horizontal surface is frictionless. Calculate the tension taken by cord. If cord is cut, what will be change in equilibrium of the bar ?*

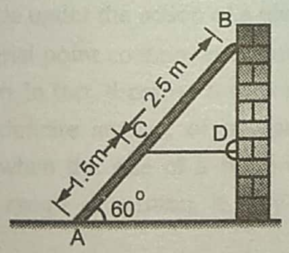


Fig. 3.76

Answer : (a) Tension in the cord is 187.62 N. (b) If cord is cut, bar will not remain in equilibrium and slide down the plane.

Problem No. 28 : *A ladder AB weighing 196 N is resting against a rough wall and a rough floor. Calculate the minimum horizontal force P required to be applied at C in order to push the ladder towards the wall. Assume μ_A = 0.3 and μ_B = 0.2.*

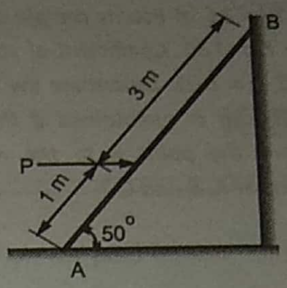


Fig. 3.77

Answer : P = 240.36 N

Problem No. 29 : *A ladder of length l is supported by a horizontal floor at A and by a vertical wall at B and makes an angle α with the horizontal as shown in Fig. 3.78. Find the maximum distance x up the ladder at which a man of weight W can stand without causing slipping to occur if the angle of friction between the floor and the ladder and between the wall and the ladder is φ. Neglect the weight of the ladder.*

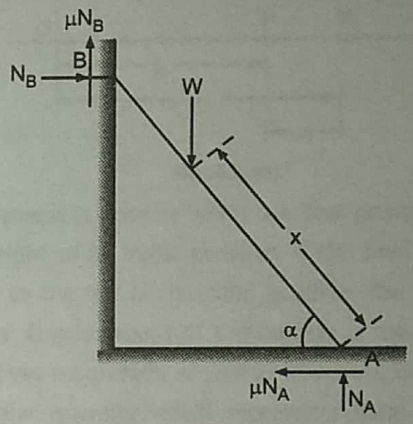


Fig. 3.78

Answer : x = $l \sin\phi \sin(\alpha + \phi) \sec\alpha$

Problem No. 30 : *A uniform plank of weight W and length l is supported at the ends A and B as shown in Fig. 3.79. If μ is the coefficient of static friction for all rubbing surfaces, determine the greatest angle θ to avoid slip.*

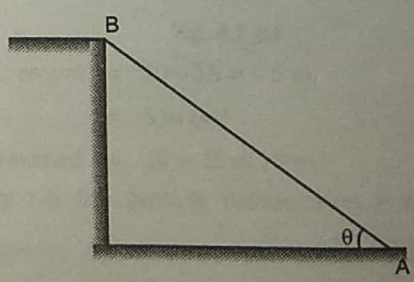


Fig. 3.79

Answer : $\theta = \left[\frac{1}{2}\sin^{-1}\left(\frac{4\mu}{1+\mu^2}\right)\right]$

Problem No. 31 : *Two 2.5 m beams are pin connected to D and loaded as shown in Fig. 3.80. Coefficient of static friction at A is zero and at B and C, μ = 0.25. Determine the smallest value of P for which the equilibrium is maintained if the given system of forces tends to move the point B to the right. What is the magnitude of reactions at A, B and C ?*

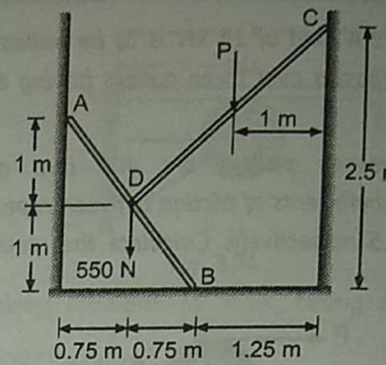


Fig. 3.80

Answer : N_A = 400 N; N_B = 800 N; N_C = 200 N; P = 200 N

UNIT - IV

KINEMATICS

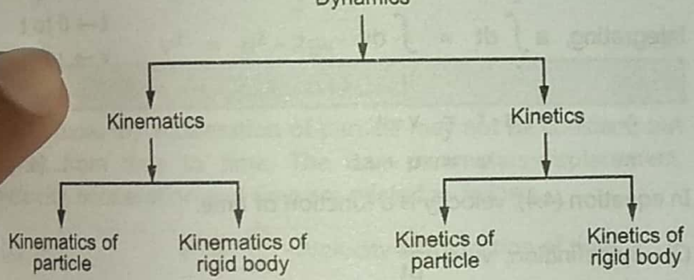
A – KINEMATICS OF PARTICLES (RECTILINEAR MOTION)

4.1 INTRODUCTION

Dynamics is the part of mechanics which deals with the analysis of particles or bodies in motion. For convenience, dynamics is commonly divided into two parts known as kinematics and kinetics. Kinematics is the study of relationship of displacement, velocity and acceleration with time for given motion, without considering the forces which cause the motion.

Kinetics is the study of the relationship of displacement, velocity and acceleration with time for a given motion by considering the force which causes the motion. Therefore, kinetics can be used to predict the motion caused by a given force or to determine the required force to produce a prescribed motion.

Kinematics and kinetics both are subdivided as particle and rigid body. The dynamics are classified as follows :



The dynamics is based on the Newton's laws governing the motion of a particle under the action of a given force.

A particle is material point containing definite quantity of matter without dimension. In fact, there is no such thing in the nature as a particle, since definite amount of matter must occupy some space. However, when the size of a body is extremely small as compared to its range of motion, it may be considered as a particle.

4.2 RECTILINEAR MOTION OF A PARTICLE

When a particle moves through space, it describes a curve which is known as path. The path of a particle may be either space curve or plane curve. In the simplest form, the path will be a straight line and the particle is said to be in rectilinear motion.

1. **Position :** The straight line path of the particle can be defined using a single coordinate axis s as shown in Fig. 4.1 (a).

The origin O on the path is fixed point and position vector $\bar{r}$ is used to specify the position of particle P at any given instant. For analytical calculation it is convenient to represent $\bar{r}$ by an algebraic scalar s, representing the position coordinate of the particle.

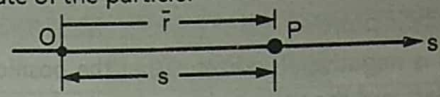


Fig. 4.1 (a)

2. **Displacement :** The change in position of a particle is known as displacement. If the particle moves from P to P_1, the displacement is $\Delta s = s_1 - s$, as shown in Fig. 4.1 (b).

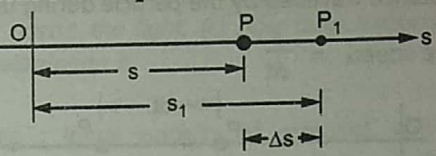


Fig. 4.1 (b)

Displacement is positive when the final position of particle is to the right of its initial position. If the final position of the particle to the left of its initial position, the displacement is negative. Displacement of the particle is the vector quantity which gives magnitude as well as direction. Distance travelled is a scalar quantity which represents total length of path travelled by the particle. For example, particle moves from O to A and A to B as shown in Fig. 4.1 (c).

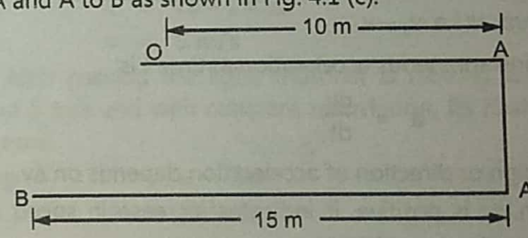


Fig. 4.1 (c)

$$\text{Displacement} = 10 - 15 = -5 \text{ m}$$
$$= 5 \text{ m } (\leftarrow)$$
$$\text{Distance travelled} = 10 + 15 = 25 \text{ m}$$

3. **Velocity :** If the particle moves from P to P_1 through a displacement $\Delta\bar{r}$ in time interval Δt, then the average velocity of the particle during the time interval Δt is

$$v_{avg.} = \frac{\Delta\bar{r}}{\Delta t}$$

For a smaller value of Δt, the magnitude of $\Delta\bar{r}$ becomes smaller, the instantaneous velocity is defined as,

$$v = \lim_{\Delta t \to 0} \left(\frac{\Delta\bar{r}}{\Delta t}\right)$$

or $$v = \frac{dr}{dt}$$

Considering v as a algebraic scalar, we can write,

$$\mathbf{v = \frac{ds}{dt}} \quad \text{... (4.1)}$$

Since, dt is always positive, the sign or direction of velocity depends on ds or Δs.

When Δs is positive, the particle is moving towards right and the velocity is positive.

When Δs is negative, it indicates that the position of particle towards left and the velocity is negative.

The magnitude of the velocity is known as speed and its SI unit is m/s.

The average speed is a scalar quantity which is given by the total distance travelled by the particle during time interval Δt.

$$\text{Average speed} = \frac{s}{\Delta t}$$

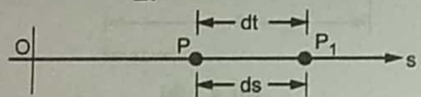


Fig. 4.1 (d)

4. **Acceleration :** The rate of change of velocity with respect to time is known as **Acceleration**. When the particle moves from P to P_1, if v is the velocity of particle at P and v_1 is the velocity of particle at P_1, then the average acceleration during the time Δt is given by

$$\mathbf{a_{avg} = \frac{\Delta v}{\Delta t}}$$

Δv represents the difference in the velocity during the time interval Δt i.e. $v_1 - v$.

The instantaneous acceleration at time t is

$$a = \frac{dv}{dt} \quad \text{... (4.2)}$$

The sign or direction of acceleration depends on Δv.

When Δv is positive, it indicates increase in speed and the acceleration is positive.

When Δv is negative, it indicates decrease in speed and the acceleration is negative which is known as deceleration or retardation.

Acceleration and deceleration is shown in Fig. 4.1 (e) and Fig. 4.1 (f).

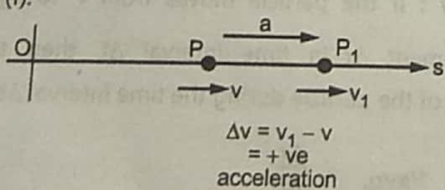


Fig. 4.1 (e)

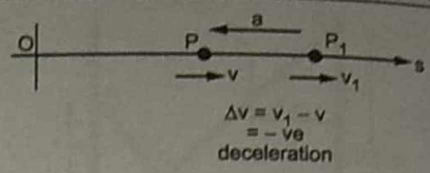


Fig. 4.1 (f)

A differential relation involving displacement, velocity and acceleration can be written as,

$$a = \frac{dv}{dt}$$

$$a = \frac{dv}{ds} \cdot \frac{ds}{dt} \quad \left(\text{where } \frac{ds}{dt} = v\right)$$

$$\therefore \quad a = v\frac{dv}{ds}$$

$$\therefore \quad \mathbf{a\,ds = v\,dv} \quad \text{... (4.3)}$$

4.3 EQUATION OF RECTILINEAR MOTION WITH UNIFORM ACCELERATION

When the acceleration is uniform, each of the three kinematic equations $a = \frac{dv}{dt}$, $v = \frac{ds}{dt}$ and $a\,ds = v\,dv$ may be integrated to obtain the equation of rectilinear motion with uniform acceleration.

(1) By definition, $a = \frac{dv}{dt}$

$$a\,dt = dv$$

Integrating, $$a\int_0^t dt = \int_u^v dv \qquad \begin{matrix} t \to 0 \text{ to } t \\ v \to u \text{ to } v \end{matrix}$$

$$\therefore \quad a \cdot t = v - u$$

$$\therefore \quad \mathbf{v = u + at} \quad \text{... (4.4)}$$

In equation (4.4), velocity is a function of time.

(2) By definition, $v = \frac{ds}{dt}$

$$\therefore \quad ds = v\,dt \qquad \begin{matrix} s \to 0 \text{ to } s \\ t \to 0 \text{ to } t \end{matrix}$$

Integrating, $$\int_0^s ds = \int_0^t v\,dt$$

Substituting, $$v = u + a \cdot t$$

$$\therefore \quad \int_0^s ds = \int_0^t (u + a \cdot t)\,dt$$

$$\therefore \quad \mathbf{s = ut + \frac{1}{2}at^2} \quad \text{... (4.5)}$$

In equation (4.5), position is a function of time.

(3) a $$ds = v\,dv \qquad \begin{matrix} s \to 0 \text{ to } s \\ v \to u \text{ to } v \end{matrix}$$

Integrating, $$a\int_0^s ds = \int_u^v v\,dv$$

$$as = \frac{v^2}{2} - \frac{u^2}{2}$$

$$\therefore \quad \frac{v^2 - u^2}{2} = 2as$$

$$v^2 - u^2 = 2as$$

$$v^2 = u^2 + 2as \qquad \text{... (4.6)}$$

In equation (4.6), velocity is a function of position.

4.4 MOTION UNDER GRAVITY

When the particle is projected vertically in the air, then its motion is under the action of gravitational force. The motion of particle under the action of gravitational force is known as **Motion under Gravity**.

Characteristics of Motion Under Gravity :

- Throughout the motion of particle, the gravitational acceleration is always in the downward direction. ($g = -9.81\ m/s^2$)
- At the maximum height, velocity must be zero ($v = 0$).
- The velocity of particle at any height must be same in magnitude in upward or downward direction.
- The time of motion for upward or downward journey must be same with reference to datum.
- The equation of motion with constant gravitational acceleration (g) becomes

> $$v = u - gt \qquad \text{... (4.7)}$$

> $$s = ut - \frac{1}{2} gt^2 \qquad \text{... (4.8)}$$

> $$v^2 = u^2 - 2gs \qquad \text{... (4.9)}$$

4.5 VARIABLE ACCELERATION

Many times the acceleration of particle may not be constant, but varies from time to time. The four parameters-displacement, velocity, acceleration and time are related as follows.

(1) $v = \frac{ds}{dt}$ – Velocity as a function of time.

$$v\,dt = ds$$

Integrating, $\int ds = \int v\,dt$

$$s = \int v\,dt$$

(2) $a = \frac{dv}{dt}$ – Acceleration as a function of time

$$a\,dt = dv$$

Integrating, $\int a\,dt = \int dv$

(3) $a = \frac{dv}{dt}$ – Acceleration as a function of position.

$$a = \frac{dv}{ds} \cdot \frac{ds}{dt} \qquad \text{... }\left(\text{where } \frac{ds}{dt} = v\right)$$

$$a = v\frac{dv}{ds}$$

Integrating, $\int a\,ds = \int v\,dv$

(4) $a = \frac{dv}{dt}$ – Acceleration as a function of time.

$$a = \frac{d}{dt}\frac{ds}{dt}$$

$$\therefore \quad a = \frac{d^2s}{dt^2}$$

$$a\,dt^2 = d^2s$$

Double integrating, $\int\int a\,dt^2 = \int\int d^2s$

When one or more of the above quantities are specified, the other can be obtained by differentiation or integration. In the process of integration, the constant of integration can be find by giving specified condition such as at $t = 0$, $s = 0$ and $v = 0$.

NUMERICAL EXAMPLES ON CONSTANT ACCELERATION

Example 4.1 : *A motorist is travelling at 72 kmph along a straight road when he observed a traffic light 187.5 m ahead of him, turn red. The traffic light is timed to stay for 15 seconds. If the motorist wishes to pass the light without stopping just as the signal turn green, determine : (a) uniform deceleration, (b) velocity of the motorist as he passes the light, (c) how long motorist will move with constant retardation ?*

Solution :

Given data : Initial velocity of motorist, u = 72 kmph = 20 m/s, distance between motorist and light, s = 187.5 m and time required to pass the light, t = 15 s.

(a) Using equation of kinematics for constant acceleration,

$$s = ut + \frac{1}{2}at^2$$

$$\therefore \quad 187.5 = 20 \times 15 + \frac{1}{2} \cdot a \times (15)^2$$

$$\therefore \quad a = -1.0\ m/s^2$$

or $\quad a = \mathbf{1.0\ m/s^2}$ **(retardation)** **... Ans.**

(b) $\quad v = u + at$

$$\therefore \quad v = 20 - 1 \times 15$$

$$\therefore \quad v = \mathbf{5\ m/s}$$ **... Ans.**

(c) After passing the light, motorist is moving with initial velocity of 5 m/s and with constant retardation. Its final velocity must be zero.

Using $\quad u^2 = u^2 + 2as$

$$\therefore \quad 0 = (5)^2 - 2 \times 1 \times s$$

$$\therefore \quad s = \mathbf{12.5\ m}$$ **... Ans.**

Example 4.2 : *A sprinter in 100 m race accelerates uniformly for the first 35 m and then runs with constant velocity. If the sprinter's time for the first 35 m is 5.4 s, determine his time for race.*

Solution :

Given data : Initial velocity of sprinter, $u_S = 0$.

Sprinter travelled 35 m in t = 5.4 s and then travelled with constant velocity, distance of race is 100 m.

Acceleration of sprinter is given by,

Using equation of kinematics,

$$s = ut + \frac{1}{2}at^2$$

$$35 = 0 + \frac{1}{2} \times a \times (5.4)^2$$

$$a = 2.4 \text{ m/s}^2$$

Maximum velocity attained by sprinter is given by,

$$v = u + at$$

$$v = 0 + 2.4 \times 5.4 = 12.96 \text{ m/s}$$

Distance travelled by sprinter with constant velocity is given by,

$$x = vt$$

$$(100 - 35) = 12.96 \text{ t}$$

$$t = 5.02 \text{ s}$$

$$\text{Total time of race} = 5.4 + 5.02$$

$$= \mathbf{10.42 \text{ s}}$$ ... **Ans.**

***Example 4.3 :** A bicyclist starts from rest and after travelling along a straight path a distance of 20 m reaches a speed of 30 kmph. Determine the constant acceleration and how long does take a reach the speed 30 kmph.*

Solution :

Given data : Initial velocity, u = 0; Final velocity, v = 30 kmph and Distance travelled, s = 20 m.

Using equation of kinematics, $v^2 = u^2 + 2as$

$$8.33^3 = 0 + 2 \times a \times 20$$

$$a = \mathbf{1.736 \text{ m/s}^2}$$ ... **Ans.**

Using equation of kinematics, v = u + at

$$8.33 = 0 + 1.736 \times t$$

$$t = \mathbf{4.8 \text{ s}}$$ ... **Ans.**

***Example 4.4 :** A car starts from rest and reaches a speed of 24 m/s after travelling 150 m along a straight road. Determine its constant acceleration and the time of travel.*

Solution :

Given data : Initial velocity, u = 0; Final velocity, v = 24 m/s and Distance travelled, s = 150 m.

Using equation of kinematics, $v^2 = u^2 + 2as$

$$24^2 = 0 + 2 \times a \times 150$$

$$a = \mathbf{1.92 \text{ m/s}^2}$$ ... **Ans.**

Using equation of kinematics, v = u + at

$$24 = 0 + 1.92 \times t$$

$$t = \mathbf{12.5 \text{ s}}$$ ... **Ans.**

***Example 4.5 :** A truck travel 164 m in 8 s while being decelerated at a constant rate of 0.5 m/s². Determine its initial velocity, find velocity and the distance travelled during first 0.6 s.*

Solution :

Given data : Distance travelled, s = 164 m, time, t = 8 s and acceleration, a = – 0.5 m/s²

Using equation of kinematics, $s = ut = \frac{1}{2} at^2$

$$164 = u \times 8 - 0.5 \times 0.5 \times 8^2$$

$$u = \mathbf{22.5 \text{ m/s}}$$ ... **Ans.**

Using equation of kinematics, v = u + at

$$v = 22.5 - 0.58$$

$$v = \mathbf{18.5 \text{ m/s}}$$ ... **Ans.**

Using equation of kinematics,

$$s = ut + \frac{1}{2} at^2$$

$$s = 22.5 \times 0.6 - 0.5 \times 0.5 \times 0.6^2$$

$$s = \mathbf{13.41 \text{ m}}$$ ... **Ans.**

NUMERICAL EXAMPLES ON MOTION UNDER GRAVITY

***Example 4.6 :** Ball A is released from rest at a height of 12 m at the same time that a second ball B is thrown upward 1.5 m from the ground. If the balls pass one another at a height of 6 m, determine the speed at which ball B was thrown upward. (Refer Fig. 4.2)*

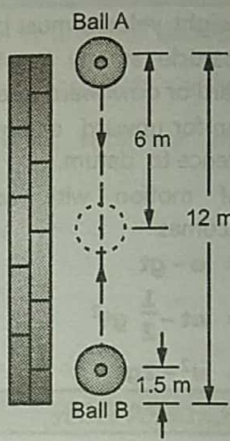


Fig. 4.2

Solution :

Given data : Initial velocity of ball A is $u_A = 0$, height, h = 12 m.

The balls pass one another at a height of 6 m. Downward distance travelled by ball A = 6 m. Time required to travel a downward distance of 6 m is given by,

$$s = ut + \frac{1}{2} gt^2$$

$$-6 = 0 - \frac{1}{2} \times 9.81 \times t^2$$

$$\therefore \quad t = 1.106 \text{ s}$$

Let 'u_B' be the initial velocity of ball B. Upward distance travelled by ball B is (6 – 1.5) = 4.5 m, in time t = 1.106 s.

$$s = ut - \frac{1}{2} gt^2$$

$$4.5 = u_B \times 1.106 - \frac{1}{2} \times 9.81 \times (1.106)^2$$

$$4.5 = 1.106 \, u_B - 6$$

$$\therefore \quad u_B = \mathbf{9.49 \text{ m/s}}$$ ... **Ans.**

***Example 4.7 :** At t = 0, bullet A is fired vertically with an initial velocity of 450 m/s. When t = 3 s, bullet B is fired upward with a muzzle velocity of 600 m/s. Determine the time t, after A is fired, as to when bullet B passes bullet A. At what altitude does this occurs ?*

Solution :

Given data :

Initial velocity of bullet A, u_A = 450 m/s.

Initial velocity of bullet B, u_B = 600 m/s.

Let 't' be the time required for bullet B to pass the bullet A.

Vertical distance travelled by bullet B in time t,

$$h_B = ut - \frac{1}{2}gt^2$$

$$h_B = 600\,t - \frac{1}{2} \times 9.81 \times t^2$$

$$= 600\,t - 4.905\,t^2 \quad \text{... (1)}$$

Vertical distance travelled by bullet A in time (t + 3),

$$h_A = ut - \frac{1}{2}gt^2 = 450\,(t+3) - \frac{1}{2} \times 9.81\,(t+3)^2$$

$$= 450\,t + 1350 - 4.905\,(t+3)^2 \quad \text{... (2)}$$

Equating (1) and (2),

$$600\,t - 4.905\,t^2 = 450 + 1350 - 4.905\,t^2 - 29.43\,t - 44.145$$

$$179.43\,t = 1305.855$$

$$\therefore \quad t = \mathbf{7.28\ s}$$

At t = 7.28 s bullet B pass the bullet A. ... Ans.

Maximum height attained by the bullet is given by,

$$s = ut - \frac{1}{2}\,gt^2$$

$$h_{max} = 600 \times 7.28 - \frac{1}{2} \times 9.81 \times (7.28)^2$$

$$= 4108\ \text{m} = \mathbf{4.11\ km} \quad \text{... Ans.}$$

***Example 4.8 :** The depth of well upto water surface is 'H' meters. A stone is dropped into the well from the ground. After 3 seconds the sound of splash is heard at the ground. If the velocity of the sound is 330 m/s, find the value of 'H'. (Refer Fig. 4.3).*

Solution :

Given data :

Initial velocity of stone, u_S = 0.

Depth of well upto water surface = H.

Velocity of sound, v_S = 330 m/s.

Let 't' be the time required to reach the stone at water surface.

Consider motion of stone and using equation of kinematics,

$$s = ut - \frac{1}{2}gt^2$$

$$-H = 0 - \frac{1}{2} \times 9.81 \times t^2$$

$$\therefore \quad H = 4.905\,t^2 \quad \text{... (1)}$$

(3 – t) be the time required to reach the sound of splash at ground with constant velocity. Using equation of kinematics,

$$s = ut + \frac{1}{2}\,at^2 \quad (a = 0)$$

$$H = 300\,(t-3) \quad \text{... (2)}$$

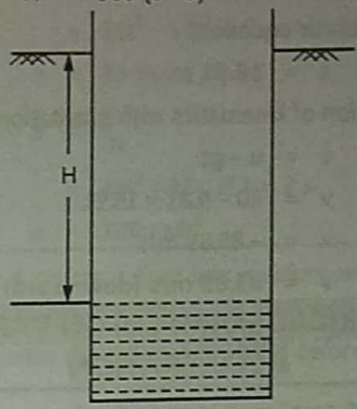


Fig. 4.3

Equating equations (1) and (2),

$$4.905\,t^2 = 330\,(3-t)$$

$$t^2 + 67.279\,t - 201.83 = 0$$

Solving quadratic equation,

$$t = 2.877\ \text{s}$$

From equation (1),

$$H = \mathbf{40.59\ m} \quad \text{... Ans.}$$

***Example 4.9 :** A baseball is thrown downward from a 15 m tower with an initial speed of 5 m/s. Determine the speed at which it hits the ground and the time of travel.*

Solution :

Given data : Initial velocity, u = 5 m/s, distance travelled, s = 15 m.

Using equation of motion with gravitational acceleration.

$$v^2 = u^2 - 2gh$$

$$v^2 = 5^2 - 2 \times 9.81 \times (-15)$$

$$v = \mathbf{17.9\ m/s\ (downward)} \quad \text{... Ans.}$$

$$v = u - gt$$

$$-17.9 = 5 - 9.81 \times t$$

$$t = \mathbf{1.31\ s} \quad \text{... Ans.}$$

***Example 4.10 :** A ball is thrown vertically upward with an initial speed of 80 m/s from the base of 50 m tower. Determine the distance h by which the ball clear the top of tower and the time t after release for the ball to land at base. Also calculate the velocity of impact with base.*

Solution :

Given data : Initial velocity, u = 80 m/s, height of tower = 50 m

Using equation of kinematics with gravitational acceleration, $v^2 = u^2 - 2gH$

$$0 = 80^2 - 2 \times 9.81 \times H$$

$$H = 326.2\ \text{m}$$

$$h = \mathbf{276.2\ m}$$

Using equation of kinematics with gravitational acceleration,

$$s = ut - \frac{1}{2}gt^2$$

$$50 = 80 \times t - 0.5 \times 9.81 \times t^2$$

$$t^2 - 16.31t + 10.19 = 0$$

Solving quadratic equation,

$$t = \mathbf{16.91\ s}$$

Using equation of kinematics with gravitational acceleration,

$$v = u - gt$$

$$v = 80 - 9.81 \times 16.91$$

$$v = -85.89\ m/s$$

$$v = \mathbf{85.89\ m/s\ (downward)}$$

NUMERICAL EXAMPLES ON VARIABLE ACCELERATION

Example 4.11 : *A small part in a mechanism travels on a straight line such that its position relative to a point O on the line is $x = t^4 - 10t^2 + 24$, where x is in mm and t is in seconds. Determine : (a) when the velocity is zero, (b) when the acceleration is zero, (c) the minimum speed reached by the particle, (d) distance travelled by the particle in 3 s, (e) an expression for x in terms of a.*

Solution :

Given : $x = t^4 - 10t^2 + 24$

(a) By definition, $v = \frac{dx}{dt}$

$$v = \frac{d}{dt}(t^4 - 10t^2 + 24)$$

$$v = 4t^3 - 20t$$

Substituting $v = 0$, $4t^3 - 20t = 0$

$$t = \mathbf{2.236\ s}$$ ... Ans.

(b) By definition, $a = \frac{dv}{dt} = \frac{d}{dt}(4t^3 - 20t)$

$$a = 12t^2 - 20$$

Substituting $a = 0$,

$$12t^2 - 20 = 0$$

$$t = \mathbf{1.29\ s}$$ ... Ans.

(c) At zero acceleration, velocity is minimum. Substituting t = 1.29 s in equation of velocity,

$$v_{min} = 4t^3 - 20t$$

$$= 4(1.29)^3 - 20 \times 1.29$$

$$= -17.213\ m/s$$

$$= \mathbf{17.213\ m/s\ (\leftarrow)}$$ ... Ans.

(d) Equation of displacement : $x = t^4 - 10t^2 + 24$.

At t = 0,	x = 24 m
At t = 1 s,	x = 15 m
At t = 2 s,	x = 0
At t = 2.236 s,	x = 1 m
At t = 3 s,	x = 15 m

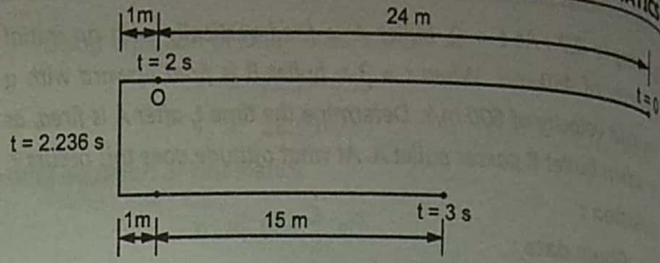


Fig. 4.4

Displacement with respect to time is shown in Fig. 4.4. Distance travelled by a particle in 3 s is given by,

$$s = 24 + 01 + 01 + 15$$

$$s = \mathbf{41\ m}$$ ... Ans.

(e) An expression for x in terms of a is,

$$a = 12t^2 - 20$$

$$t^2 = \frac{a + 20}{12}$$

$$t^4 = \frac{a^2 + 40a + 400}{144}$$

Substituting values of (t^2) and (t^4) in displacement equation,

$$x = \frac{a^2 + 40a + 400}{144} - 10\left(\frac{a + 20}{12}\right) + 24$$

$$144x = a^2 + 40a + 400 - 120a - 2400 + 3456$$

$$\mathbf{144x = a^2 - 80a + 1456}$$ ... Ans.

Example 4.12 : *The acceleration of an object moving along a straight path decreases uniformly from 10 m/s² to zero in 12 seconds, at which its velocity is 6 m/s. Find its initial velocity and the change in position during the 12 second interval. (Refer Fig. 4.5).*

Solution :

Given data :

At t = 0 s, $a = 10\ m/s^2$

At t = 12 s, $a = 0$

Equation of straight line is,

$$y = mx + C$$

$$m = \text{slope} = \frac{0 - 10}{12 - 0}$$

$$= \frac{-10}{12} \text{ and } C = 10$$

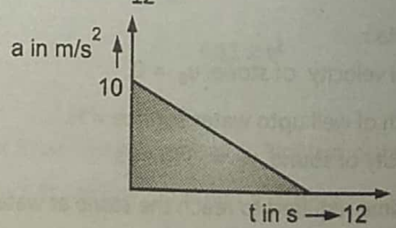


Fig. 4.5

Equation of acceleration is,

$$a = -\frac{10}{12}t + 10$$

$$a = \frac{dv}{dt}$$

$$dv = a\,dt$$

By integration, $\int dv = \int \left(-\frac{10}{12}t + 10\right) dt$

$$v = \frac{-10\,t^2}{24} + 10\,t + C, \quad \text{... C – Constant of integration.}$$

At $t = 12$ s, $v = 6$ m/s. Substituting values of t and v in velocity equation,

$$6 = -\frac{10 \times 12^2}{24} + 10 \times 12 + C_1$$

$$C_1 = -54$$

Equation of velocity,

$$v = -\frac{10\,t^2}{24} + 10\,t - 54$$

At $t = 0$, $v_0 = \mathbf{-54\ m/s}$ ... **Ans.**

By definition of velocity,

$$v = \frac{ds}{dt}$$

$$ds = v\,dt$$

$$\int ds = \int \left(\frac{-10\,t^2}{24} + 10\,t - 54\right) dt$$

$$s = \frac{-10\,t^3}{72} + 5\,t^2 - 54\,t$$

At $t = 0$, $s = 0$.

At $t = 12$s, $s = -168$ m.

Change in position during 12 s interval = 168 m. ... **Ans.**

Example 4.13 : *The acceleration of a particle is defined by $a = -0.4\ v$, where a is expressed in mm/s² and v in mm/s. If $v = 30$ mm/s at $t = 0$, determine the distance the particle will travel before coming to rest.*

Solution :

Given data : $a = -0.4\,v$, at $t = 0$, $v = 30$ mm/s

$$a\,ds = v\,dv$$

$$-0.4\,v\,ds = v\,dv$$

$$\frac{v\,dv}{-0.4\,v} = ds$$

By integration,

$$\int \frac{v\,dv}{-0.4\,v} = \int ds$$

$$\frac{1}{-0.4}\int dv = \int ds$$

$$\frac{1}{-0.4}[v]_{30}^{0} = s$$

$$s = \frac{-30}{-0.4}$$

$s = \mathbf{75\ mm}$ ... **Ans.**

Example 4.14 : *The motion of a particle is defined by the relation $x = 4t^4 - 6t^3 + 2t - 1$, where x and t are expressed in m and second respectively. Determine the position, the velocity and the acceleration of the particle when t = 2 s.*

Solution :

Given data : $x = 4t^4 - 6t^3 + 2t - 1$, at t = 2 s

$\therefore \quad x = \mathbf{19\ m}$ ... **Ans.**

By definition, $v = \frac{dx}{dt}$

$= 16t^3 - 18t^2 + 2$, at t = 2 s

$v = \mathbf{58\ m/s}$ ... **Ans.**

By definition, $a = \frac{dv}{dt}$

$= 48t^2 - 36t$, at t = 2 s

$\therefore \quad a = \mathbf{120\ m/s^2}$ ... **Ans.**

Example 4.15 : *A sphere is fired into medium with a initial speed of 27 m/s. If it experience a deceleration $a = (-6t)$ m/s², where t is in seconds, determine the distance travelled before it stop.*

Solution :

Given data : Initial velocity, u = 27 m/s and deceleration, $a = -6t$

By definition, $a = \frac{dv}{dt} = -6t$

$$dv = (-6t)\,dt$$

$$v = -3t^2 + 27$$

The time t at which the sphere is come to rest is given by equation v = 0

$$0 = -3t^2 + 27$$

$$t = 3\ \text{s}$$

By definition, $v = \frac{ds}{dt}$

$$ds = v\,dt$$

$$s = -t^3 + 27\,t$$

Substituting, t = 3 s

$s = \mathbf{54\ m}$ ... **Ans.**

4.6 MOTION CURVE

The graphical representation of displacement, velocity and acceleration with time is known as motion curve.

1. Displacement-Time Curve (s-t Curve) :

If the position of the particle is known during the time interval 't', the s-t curve for the particle can be plotted as shown in Fig. 4.6 (a). The slope of s-t curve at any instant gives the particle velocity as a function of time t. Therefore v-t curve is drawn by measuring the slope (ds/dt) of the s-t curve at various times and plotting the result.

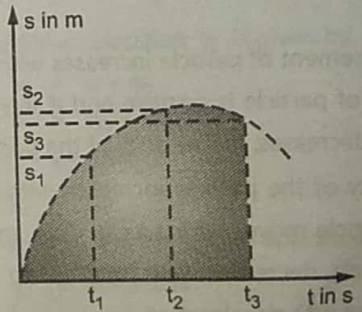


Fig. 4.6 (a)

2. Velocity-Time Curve (v-t Curve) :

If the velocity of the particle is known during the time interval 't', the v-t curve for the particle can be plotted as shown in Fig. 4.6 (b). The slope of the v-t curve at any instant gives the particle acceleration at that instant as a function of time. Therefore a-t curve is drawn by measuring a slope (dv/dt) of the v-t curve and plotting the result. The area under the v-t curve at any instant gives the displacement of the particle at that instant.

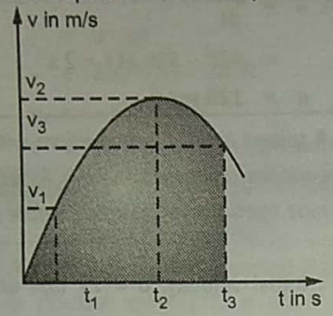


Fig. 4.6 (b)

3. Acceleration-Time Curve (a-t Curve) :

If the acceleration of the particle is known during the time period 't', the a-t curve can be plotted as shown in Fig. 4.6 (c). The area under the a-t curve at any instant gives the velocity of the particle at that instant. The moment of area of a-t curve at any time t gives the displacement of the particle at that time.

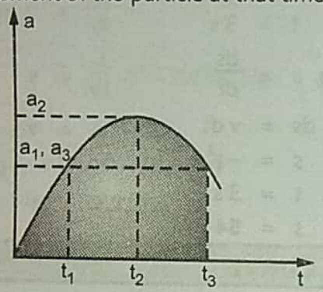


Fig. 4.6 (c)

Characteristics of Motion Curve :

- When the motion curve passes through the origin that represents initial displacement, velocity and acceleration must be zero (i.e. at t = 0, s = 0, v = 0, and a = 0).
- When the motion curve makes intercept on y-axis, the particle initial displacement, velocity and acceleration should not be zero.
- If the displacement of particle increases with respect to time, the velocity of particle is positive and if the displacement of the particle decreases, the velocity of the particle is negative.
- If the velocity of the particle increases with respect to time, then the particle moves with acceleration and if the velocity of the particle decreases with respect to time, then the particle moves with deceleration.

The nature of motion curves are summarised in the following table :

Sr. No.	a-t Curve	v-t Curve	s-t Curve
1.	Zero degree curve or rectangular.	I-degree curve or triangular.	II-degree curve or parabolic.
2.	I-degree curve or triangular.	II-degree curve or parabolic.	III-degree curve or cubic parabolic.
3.	II-degree curve or parabolic.	III-degree curve or cubic parabolic.	IV-degree curve.

NUMERICAL EXAMPLES ON MOTION CURVE

Example 4.16 : *An auto starts from rest and reaches a speed of 54 km/h in 15 seconds. The acceleration increases uniformly from zero for the first nine seconds after which the acceleration reduces uniformly to zero in the next six seconds. Find the displacement in first 15 seconds interval.*

Solution :

Given data :

Time for acceleration = 9s.

Time for deceleration = 6s.

The velocity at t = 15s.

$$v = 54 \text{ km/h}$$

$$= 15 \text{ m/s}$$

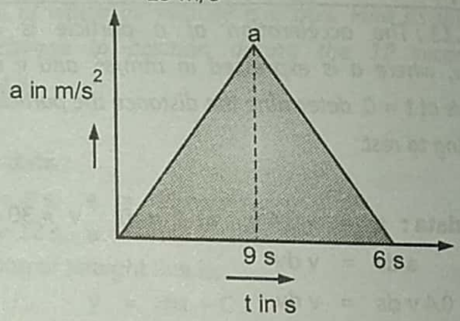


Fig. 4.7

As per the property of a-t curve, Area of a-t curve at t = 15s, gives velocity at t = 15s.

$$15 = \frac{1}{2} \times 9 \times a + \frac{1}{2} \times 6 \times a$$

$$a = 2 \text{ m/s}^2$$

Displacement of a-t curve at t = 15s is given by taking moment of area of a-t curve at t = 15s,

$$s = \frac{1}{2} \times 9 \times 2 \times (6 + 3) + \frac{1}{2} \times 6 \times 2 \times 4$$

$$= \mathbf{105\ m}$$ ... **Ans.**

Example 4.17 : *A jet plane starts from rest at x = 0 and is subjected to the acceleration shown in Fig. 4.8. Determine the speed of the plane when it has travelled 60 m.*

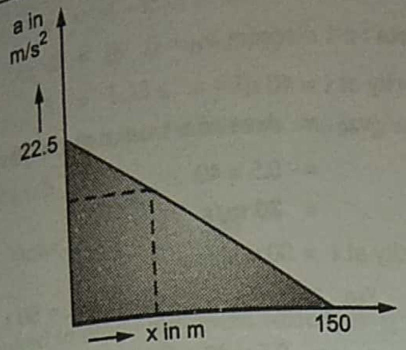


Fig. 4.8

Solution :

Given data :

Initial velocity, $v_o = 0$, initial displacement, $x_o = 0$.

$$v = ? \quad \text{at} \quad x = 60 \text{ m}$$

$$\frac{v\,dv}{ds} = a$$

$$v\,dv = a\,ds$$

Integrating on both sides,

$$\frac{1}{2}[v^2] = \int a \cdot ds$$

$$\frac{v^2}{2} = \text{Area of a-x curve} \quad \text{... (1)}$$

From Fig. 4.8,

Equation of acceleration is given by

$$a = 22.5 - 0.15\,x$$

$$\text{At } x = 60 \text{ m}$$

$$a = 13.5 \text{ m/s}^2$$

Area of a-x curve upto x = 60 m

$$= 13.5 \times 60 + \frac{1}{2} \times 60 \times (22.5 - 13.5)$$

$$= 1080$$

From equation (1),

$$\frac{v^2}{2} = 1080$$

$$v = \mathbf{46.48 \text{ m/s}} \quad \text{... Ans.}$$

Example 4.18 : *A car starting from rest moves along a straight track with an acceleration shown in Fig. 4.9 (a). Determine time t for the car to reach a speed of 50 m/s and construct the v-t diagram that describes the motion until the time 't'*

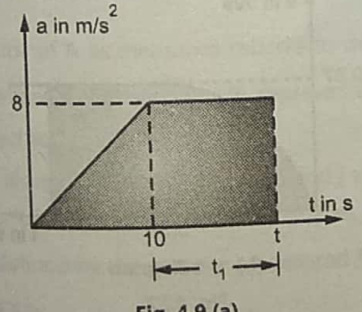


Fig. 4.9 (a)

Solution :

Given data : a-t curve is shown in Fig. 4.9 (a).

$$v = 50 \text{ m/s}, \quad t = ?$$

From the property of a-t curve, the area of a-t curve at time t, gives the velocity at that time.

$$50 = \text{Area of a-t curve}$$

$$50 = \frac{1}{2} \times 10 \times 8 + 8 \times t_1$$

$$t_1 = 1.25 \text{ s}$$

$\therefore$ Time to reach the car at a speed of 50 m/s is,

$$t = 10 + 1.25 = \mathbf{11.25 \text{ s}} \quad \text{... Ans.}$$

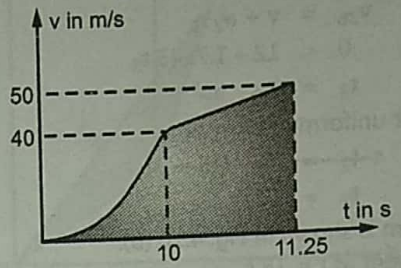


Fig. 4.9 (b)

Velocity at t = 10s is given by area of a-t curve at t = 10s

$$v_{10} = \frac{1}{2} \times 10 \times 8$$

$$= 40 \text{ m/s}$$

$$v_{11.25} = \mathbf{50 \text{ m/s}} \quad \text{... Ans.}$$

v-t diagram is shown in Fig. 4.9 (b).

Example 4.19 : *A bus starts from rest at point A and accelerates at the rate of 0.8 m/s till it reaches a speed of 12 m/s. It then proceeds at 12 m/s till the brakes are applied. It comes to rest at point B, 42 m beyond the point where brakes are applied. Assuming the uniform deceleration and that the total time of travel from A to B is 36 s, determine the distance between A and B using v-t diagram. Also draw v-t diagram.*

Solution :

Given data :

Initial velocity of car at t_o, $v_o = 0$.

Uniform velocity, v = 12 m/s.

Final velocity at t_{36}, $v_{36} = 0$.

Acceleration of car, $a_1 = 0.8$ m/s^2.

Distance travelled by the car after the brakes are applied, s = 42 m.

Time required for acceleration 't_1' is given by,

$$v = u + at$$

$$v = v_o + a_1 t_1$$

$$12 = 0.8\,t_1$$

$$t_1 = 15 \text{ s}$$

Deceleration a_2 after the brakes are applied is given by,

$$v^2 = u^2 + 2as$$

$$v_{36}^2 = v^2 + 2a_2\,s$$

$$0 = (12)^2 + 2 \times a_2 \times 42$$

$$a_2 = -1.7143 \text{ m/s}^2$$

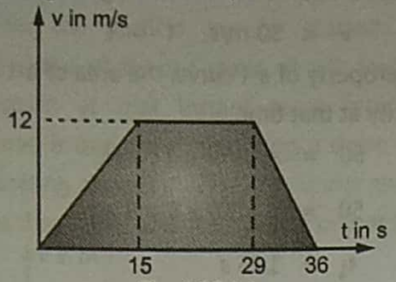


Fig. 4.10 (a)

Time of deceleration is given by,

$$v = u + at$$

$$v_{36} = v + a_2 t_3$$

$$0 = 12 - 1.7143\, t_3$$

$$t_3 = 7 \text{ s}$$

Time t_2 for uniform velocity is,

$$t_2 = 36 - (15 + 7)$$

$$t_2 = 14 \text{ s}$$

v-t diagram is shown in Fig. 4.10 (b).

Displacement at t = 15 s,

$$s_{15} = \frac{1}{2} \times 12 \times 15 = 90 \text{ m}$$

Displacement at t = 29 s,

$$s_{29} = \frac{1}{2} \times 12 \times 15 + 12 \times 14 = 258 \text{ m}$$

Displacement at t = 36 s,

$$s_{36} = \frac{1}{2} \times 12 \times 15 + 12 \times 14 + \frac{1}{2} \times 7 \times 12$$

$$= 300 \text{ m}$$

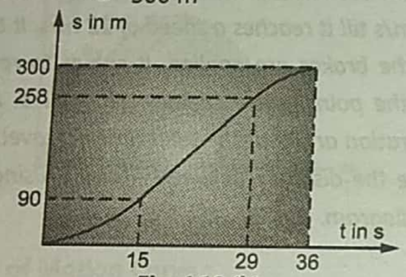


Fig. 4.10 (b)

s-t diagram is shown in Fig. 4.10 (b).

Distance between A and B is 300 m. **... Ans.**

Example 4.20 : *Fig. 4.11 shows an a-t diagram for a car which starts from rest and come to halt after time (60 + t) seconds. Find the value of 't' and 'f' shown in the diagram. Find also (i) velocity at 40 seconds, (ii) velocity at 60 seconds, (iii) distance travelled during first 40 seconds.*

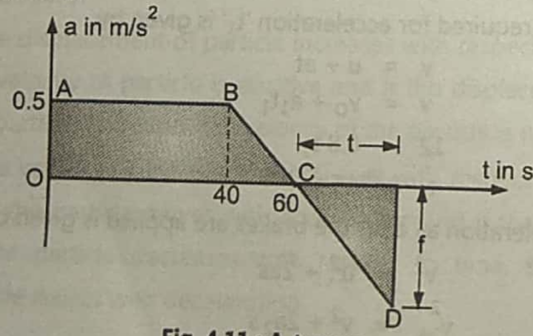


Fig. 4.11 : A-t curve

Solution :

Given data : a-t diagram, $v_O = 0$, $v_D = 0$.

(a) Velocity at t = 40 s,

$$v_{40} = \text{Area of a-t curve at t = 40 s}$$

$$= 0.5 \times 40$$

$$= \mathbf{20 \text{ m/s}}$$ **... Ans.**

(b) Velocity at t = 60 s,

$$v_{60} = \text{Area of a-t curve at t = 60 s}$$

$$= 0.5 \times 40 + \frac{1}{2} \times 0.5 \times 20$$

$$= \mathbf{25 \text{ m/s}}$$ **... Ans.**

(c) Distance travelled during first 40 seconds,

$$s_{40} = \text{Moment of area of a-t curve @ B}$$

$$= 0.5 \times 40 \times 20$$

$$= \mathbf{400 \text{ m}}$$ **... Ans.**

From similar triangles, $\frac{0.5}{f} = \frac{20}{t}$

$$\therefore \quad t = 40\, f \quad \text{... (1)}$$

$$v_D = 0.5 \times 40 + \frac{1}{2} \times 20 \times 0.5 - \frac{1}{2} \times t \cdot f = 0$$

$$25 - 0.5\, tf = 0$$

$$tf = 50 \quad \text{... (2)}$$

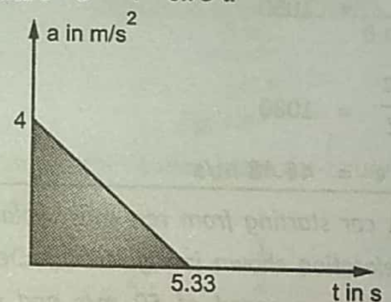
Solving equations (1) and (2),

$$f = 1.12 \text{ m/s}^2, \quad t = \mathbf{44.72 \text{ s}}$$ **... Ans.**

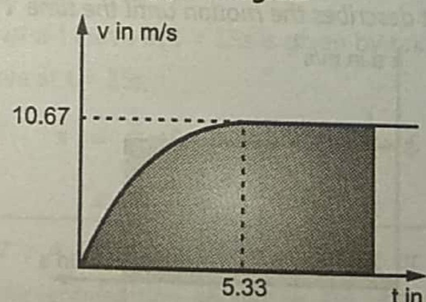
Example 4.21 : *A car starts from rest with an acceleration given by a = 4 – 0.75 t. When the car reaches maximum velocity, it maintains that velocity constant till it has gone a total distance of 200 m. Draw a-t and v-t curves for the motion and hence calculate the time t required for the car to travel 100 m from starting point.*

Solution :

Given data : $a = 4 - 0.75\, t$.



(a) a-t diagram



(b) v-t diagram

Fig. 4.12

$$a = 4 - 0.75\,t$$

$$\text{At } t = 0, \quad a = 4 \text{ m/s}^2$$

$$t = 5.33 \text{ s}, \quad a = 0$$

When a = 0, velocity must be maximum and is given by area of a-t diagram at t = 5.33 s.

$$v_{max} = \frac{1}{2} \times 4 \times 5.33$$

$$= 10.67 \text{ m/s}$$

Time required to attain maximum velocity is $t_1 = 5.33$ s.

Distance travelled in 5.33 s is given by area of v-t diagram.

$$s_{5.33} = \frac{2}{3} \times 5.33 \times 10.67 = 37.9 \text{ m}$$

Time required to travel remaining distance of (100 – 37.9) with constant velocity of 10.67 m/s is given by,

$$100 - 37.9 = 10.67\, t_2$$

$$\therefore \quad t_2 = 5.82 \text{ s}$$

Total time required to travel 100 m, $t = t_1 + t_2$

$$t = 5.33 + 5.82 = \mathbf{11.15 \text{ s}} \quad \text{... Ans.}$$

4.7 RELATIVE MOTION

We have described motion using co-ordinate axis. The displacement, velocity and acceleration determined for co-ordinate axis are termed as absolute. It is not always possible to use a fixed set of axes to describe the motion.

There are many engineering problems in which the analysis of motion is simplified by using measurements with respect to moving reference system. These measurements, when combined with the absolute motion of the moving co-ordinate system, help us to determine the absolute motion of the problem. This approach is called a **Relative Motion Analysis**.

The motion of the moving co-ordinate system is specified with respect to fixed co-ordinate system. For engineering purpose, the fixed system may be taken as any system whose absolute motion is negligible for the problem.

Representation of Relative Motion :

Consider two particles A and B which may have separate curvilinear motion in a given plane as shown in Fig. 4.13.

We will arbitrarily attach the origin of a set of translating axes x-y to particle B and observe the motion of A from our moving position on B.

The position vector of A as measured relative to the frame x-y is $\bar{r}_{A/B} = xi + yj$, where the subscript A/B represents A relative to B or A with respect to B.

The unit vectors along the x and y axes are i and j and x and y are the co-ordinates of A measured in the x-y axes. The absolute position of B is defined by the vector $\bar{r}_B$ measured from the origin of the fixed axes x-y.

The absolute position of A is determined by the vector,

$$\bar{r}_A = \bar{r}_B + \bar{r}_{A/B} \quad \text{... (4.10)}$$

Differentiating vector equation with respect to time t,

$$\bar{v}_A = \bar{v}_B + \bar{v}_{A/B}$$

$$\text{or} \quad \bar{v}_{A/B} = \bar{v}_A - \bar{v}_B \quad \text{... (4.11)}$$

Again differentiating vector equation with respect to time t,

$$\bar{a}_A = \bar{a}_B + \bar{a}_{A/B}$$

$$\text{or} \quad \bar{a}_{A/B} = \bar{a}_A - \bar{a}_B \quad \text{... (4.12)}$$

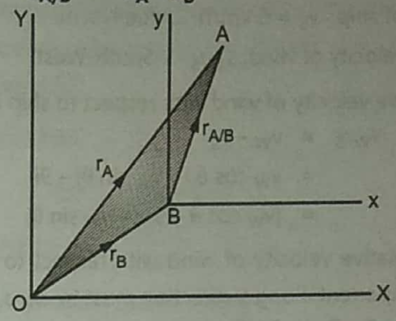


Fig. 4.13

Example 4.22 : *A body 'P' has a relative velocity of 20 km/h from West to East with respect to body 'Q'. The body 'Q' has a relative velocity of 48 km/h from North-East with respect to body 'R'. Determine the velocity of body 'P' relative to body 'R'.*

Solution :

Given data : $v_{P/Q}$ = 20 km/h along West-East

$v_{Q/R}$ = 48 km/h along North-East

Relative velocity of 'P' with respect to 'R',

$$v_{P/R} = v_P - v_R = v_{P/Q} + v_{Q/R} \quad \text{... (1)}$$

$$v_{P/Q} = 20\,i + 0\,j$$

$$v_{Q/R} = -48 \cos 45\, i - 48 \sin 45\, j$$

Substituting value of $v_{P/Q}$ and $v_{Q/R}$ in equation (1),

$$v_{P/R} = 20\,i + (-48 \cos 45\, i - 48 \sin 45\, j)$$

$$= 20\,i - 33.94\,i - 33.94\,j$$

$$= -13.94\,i - 33.94\,j$$

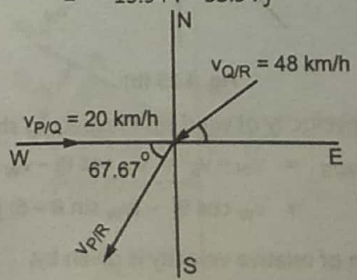


Fig. 4.14

$$v_{P/R} = \sqrt{(-13.94)^2 + (-33.94)^2}$$

$$= \mathbf{36.69 \text{ km/h}} \quad \text{... Ans.}$$

$$\theta = \tan^{-1} \frac{33.94}{13.94}$$

$$\theta = \mathbf{67.67°} \quad \text{... Ans.}$$

***Example 4.23 :** As observed from a ship moving due East at 9 km/h, the wind appears to blow from South. After the ship has changed course and speed moving due North at 6 km/h, the wind appears to blow from the South-West. Assuming that the direction and velocity of the wind is constant during the entire period of observation, compute the magnitude and direction of true wind velocity.*

Solution :

Given data : Velocity of ship, v_S = 9 km/h ... towards East.

Relative velocity of wind, $v_{W/S}$ ... from South.

Velocity of ship, v_S = 6 km/h ... due North.

Relative velocity of wind, $v_{W/S}$ ... South-West.

Case I : Relative velocity of wind with respect to ship is given by

$$\begin{aligned} v_{W/S} &= v_W - v_S \\ &= v_W \cos\theta\, i - v_W \sin\theta j - 9i \\ &= (v_W \cos\theta - 9)\, i - v_W \sin\theta j \end{aligned}$$

As the relative velocity of wind with respect to ship is from South, its component along x-direction must be zero.

$$v_W \cos\theta - 9 = 0$$

$$v_W \cos\theta = 9 \qquad \text{... (1)}$$

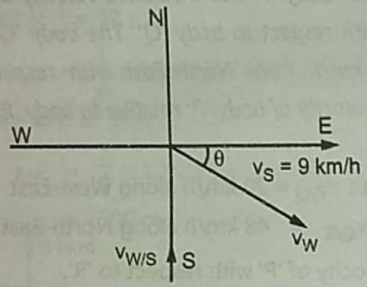


Fig. 4.15 (a)

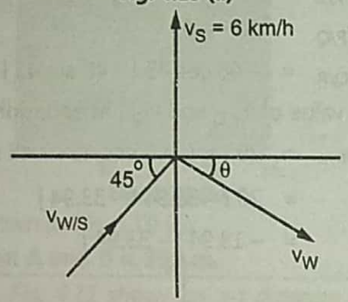


Fig. 4.15 (b)

Case II : Relative velocity of wind with respect to ship is given by,

$$\begin{aligned} v_{W/S} &= v_W - v_S = v_W \cos\theta j - v_W \sin\theta j - 6j \\ &= v_W \cos\theta j - (v_W \sin\theta - 6)\, j \end{aligned}$$

The direction of relative velocity is given by,

$$\tan 45 = \frac{-(v_W \sin\theta - 6)}{v_W \cos\theta}$$

From equation (1), $v_W \cos\theta = 9$

$$\tan 45 \times 9 = -v_W \sin\theta - 6$$

$$-v_W \sin\theta = 15 \qquad \text{... (2)}$$

From equations (1) and (2),

$$\frac{-v_W \sin\theta}{v_W \cos\theta} = \frac{15}{6}$$

$$\theta = -59.03°$$

(position of v_W in fourth quadrant). ... Ans.

From equation (1),

$$v_W = \frac{9}{\cos 59.03}$$

$$v_W = \mathbf{17.49\ km/h} \qquad \text{... Ans.}$$

***Example 4.24 :** A passenger travelling in a train tries to hit the pole nearby the track by throwing a stone with a horizontal velocity of 20 m/s relative to the train. Knowing that the speed of train is 36 km/h, determine :*

(1) The direction in which the passenger must throw the stone.

(2) The horizontal velocity of stone with respect to ground.

Solution :

Given data : Relative velocity of stone with respect to train is $v_{S/T}$ = 20 m/s, velocity of train, v_T = 36 km/h = 10 m/s. Vector diagram of velocity is shown in Fig. 4.16, where 'O' is the position of passenger, which throws the stone in horizontal direction and velocity of train along North direction.

Relative velocity of stone with respect to train is given by,

$$v_{S/T} = v_S - v_T$$

Component of velocity along x-direction,

$$v_{S/T} = v_S$$

$$20 \cos\theta = v_S \qquad \text{... (1)}$$

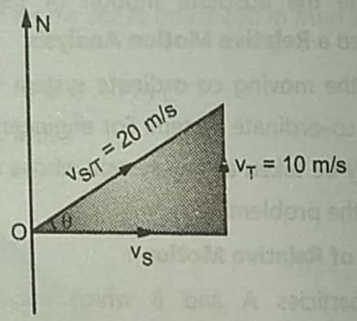


Fig. 4.16

Component of velocity along y-direction,

$$v_{S/T} = v_T$$

$$20 \sin\theta = 10 \qquad \text{... (2)}$$

From equation (2),

$$\sin\theta = \frac{10}{20}$$

$$\theta = \mathbf{30°} \qquad \text{... Ans.}$$

From equation (1),

$$v_S = 20 \cos 30$$

$$= \mathbf{17.32\ m/s} \qquad \text{... Ans.}$$

Example 4.25 : Two ships leave a port at the same time. The first streams N-W at 32 km/h and second streams 40° South of West at 24 km/h. (i) Determine the relative velocity of the second ship with respect to first ship. (ii) After what time they will be 160 km apart ?

Solution :

Given data :

Velocity of first ship N-W at 32 km/h.

Velocity of second ship 40° South of West at 24 km/h.

Distance between two ships, s = 160 km/h.

Relative velocity of second ship with respect to first ship.

(a) $v_{S_2/S_1} = v_{S_2} - v_{S_1}$

$= (-24 \cos 40\, i - 24 \sin 40\, j) - (-32 \cos 45\, i + 32 \sin 45\, j)$

$= (-24 \cos 40 + 32 \cos 45)\, i + (-24 \sin 40 - 32 \sin 45)\, j$

$= 4.24\, i - 38.05\, j$

$v_{S_2/S_1} = \mathbf{38.3\ km/h}$ ... **Ans.**

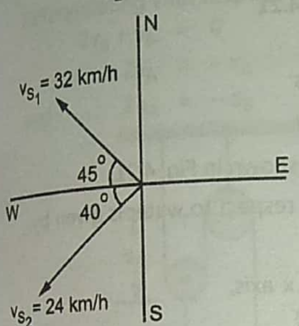


Fig. 4.17 (a)

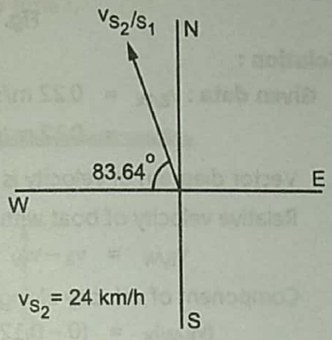


Fig. 4.17 (b)

$$\theta = \tan^{-1}\frac{38.05}{4.24} = 83.64°$$

(b) Relative velocity of second ship with respect to first ship is shown in Fig. 4.17 (b).

At time t, distance between two ships is 160 km,

$\therefore \quad t = \frac{160}{38.3} = \mathbf{4.18\ hours}$ ... **Ans.**

Example 4.26 : At an instant ship A is streaming due East at 20 km/h and ship B at that instant is 80 km due South and is streaming at 16 km/h. Determine : (1) $v_{B/A}$, (2) shortest distance between them, (3) time to attain the shortest distance.

Solution :

Given data :

Velocity of ship A due East at 20 km/h.

Velocity of ship B along N-W at 16 km/h.

At t = 0, distance between them = 30 km.

(a) Relative velocity of ship B with respect to ship A is,

$v_{B/A} = v_B - v_A = (-16 \cos 45\, i + 16 \sin 45\, j) - 20\, i$

$= (-31.31)\, i + 11.31\, j = \mathbf{33.3\ km/h}$... **Ans.**

$$\theta = \tan^{-1}\frac{11.31}{31.31} = 19.86°$$

(b) Shortest distance between them : AC is the shortest distance between two ships. From Fig. 4.18 (b),

$\cos 19.86 = \frac{AC}{30}$

$AC = \mathbf{28.22\ km}$ ... **Ans.**

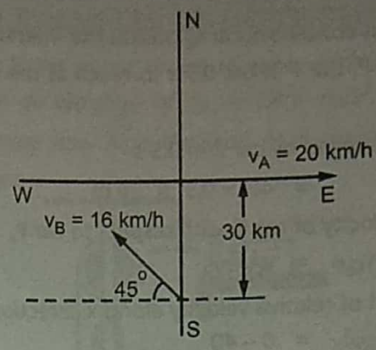


Fig. 4.18 (a)

(c) Time required to attain the shortest distance of 28.22 km, ship B travelled from point B to C relative to ship A. Distance BC is given by,

$\sin 19.86 = \frac{BC}{30}$

$BC = 10.19$ km

Required time $= \frac{10.19}{33.3}$

$\therefore$ Required time = **0.306 hours** ... **Ans.**

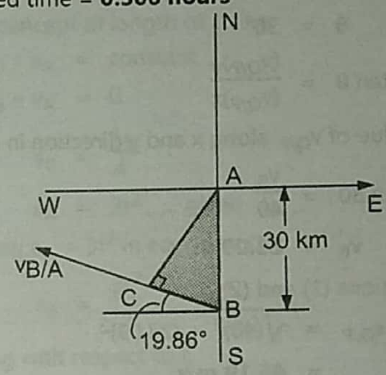


Fig. 4.18 (b)

Example 4.27 : Two straight roads cross at right angles at an intersection. A car 'P' is moving on one road and 'Q' on other. If car 'P' is traveling at 40 m/s and passes the intersection in 0.5 s after car 'Q' and the least distance between P and Q is 10 m, determine the velocity of Q and relative velocity of Q with respect to P.

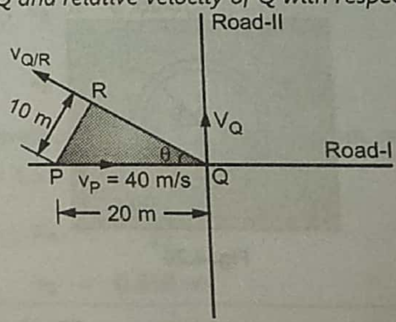


Fig. 4.19

Solution :

Given data : Velocity of car P, v_P = 40 m/s and passes intersection in 0.5 s. The least distance between P and Q is 10 m as shown in Fig. 4.19.

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As per the given condition, car Q passes the intersection of roads earlier than car P. Car P takes 0.5 s to reach at the intersection of roads.

Distance travelled by car P in 0.5 s

$$= 40 \times 0.5 = 20 \text{ m}$$

Relative velocity of car Q with respect to car P,

$$v_{Q/P} = v_Q - v_P$$

Component of relative velocity along x-direction,

$$(v_{Q/P})_x = 0 - 40 \qquad \text{... (1)}$$

Component of relative velocity along y-direction,

$$(v_{Q/P})_y = v_Q - 0 \qquad \text{... (2)}$$

From equations (1) and (2), line of relative velocity of Q with respect to P lies in second quadrant at an angle θ with x-axis.

Shortest distance of 10 m is a line perpendicular to line of relative velocity of Q with respect to P from point P (car P is stationary and car Q is travelling with relative velocity).

$$\sin\theta = \frac{10}{20}$$

$$\theta = 30°$$

$$\tan\theta = \frac{(v_{Q/P})_y}{(v_{Q/P})_x} \qquad \text{... (3)}$$

Substituting value of $v_{Q/P}$ along x and y direction in equation (3),

$$\tan 30 = \frac{v_B}{40}$$

$$v_B = \mathbf{23.09\ m/s} \qquad \text{... Ans.}$$

From equations (1) and (2),

$$v_{Q/P} = \sqrt{(40)^2 + (23.09)^2}$$

$$= \mathbf{46.19\ m/s} \qquad \text{... Ans.}$$

Example 4.28 : *Pin P moves with constant speed of 100 mm/s in counter clockwise sense along a circular slot which has been travelled in the slider block A shown. If the slider block travels upward at a constant speed of 75 mm/s, determine the velocity of the pin with respect to the slot when (a) $\theta = 90°$, (b) $\theta = 180°$.*

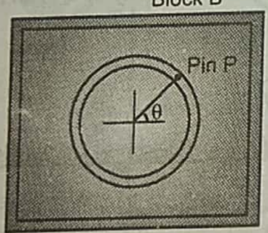


Fig. 4.20

Solution :

Case (a) : $\theta = 90°$, $v_P = -100$ mm/s, $v_B = 75$ mm/s

$$v_{P/B} = v_P - v_B = -100\,i - 75\,j = \mathbf{125\ mm/s} \qquad \text{... Ans.}$$

$$\theta = \tan^{-1}\frac{75}{100} = 36.87°$$

Case (b) : $\theta = 180°$, $v_P = -100$ mm/s, $v_B = 75$ mm/s

$$v_{P/B} = (-100 + 75)\,j = -25 \text{ mm/s} = \mathbf{25\ mm/s\ (\downarrow)} \qquad \text{... Ans.}$$

Example 4.29 : *A man can row a boat at a speed of 0.22 m/s in still water. He wants to cross a river and reach a point on the other bank, exactly opposite to his starting point. Speed of the water current is 0.12 m/s. Find the direction in which the boat should be headed, with reference to the direction of the current. If the width of river is 100 m, how long will it take for the man to reach the other bank ?*

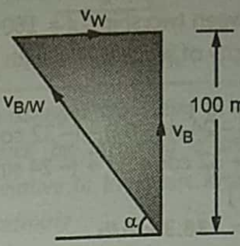


Fig. 4.21

Solution :

Given data : $v_{B/W} = 0.22$ m/s

$v_W = 0.12$ m/s

Vector diagram of velocity is shown in Fig. 4.21.

Relative velocity of boat with respect to water is given by,

$$v_{B/W} = v_B - v_W$$

Component of velocity along x-axis,

$$(v_{B/W})_x = (0 - 0.12)$$

$$-0.22\cos\alpha = -0.12 \qquad \text{... (1)}$$

Component of velocity along y-axis,

$$(v_{B/W})_y = v_B$$

$$0.22\sin\alpha = v_B \qquad \text{... (2)}$$

From equation (1),

$$\cos\alpha = \frac{0.12}{0.22}$$

$$\alpha = 56.94° \text{ (or } 123.06°)$$

From equation (2), $v_B = 0.22 \sin 56.94$

$$= \mathbf{0.184\ m/s} \qquad \text{... Ans.}$$

The time required for the man to reach at other bank is given by,

$$s = vt$$

$$t = \frac{100}{0.184} = 543.48 \text{ s}$$

$$= \mathbf{9\ hours\ 3.48\ s} \qquad \text{... Ans.}$$

4.8 DEPENDENT MOTION

The motion of one particle depends on the corresponding motion of another particle. This dependency commonly occurs if the particles are interconnected by an inextensible string which is wrapped around the pulley. For example, the movement of block A downward along the inclined plane will cause a corresponding movement of block B up the other inclined plane as shown in Fig. 4.22.

If the total length of the string is constant i.e. L,

$$s_A + s_B = L$$

Differentiating with respect to time,

$$v_A + v_B = 0$$

$$v_B = -v_A \text{ or } v_A = -v_B$$

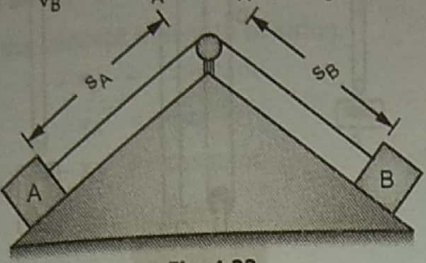


Fig. 4.22

Another example involving dependent motion of two blocks is shown in Fig. 4.23 (a).

Consider upper pulley as a reference.

Total length of the string must be constant.

$$2s_A + h + s_B = \text{constant}$$

Differentiating with respect to time t,

$$2v_A + v_B = 0$$

$$2v_A = -v_B$$

and $$2a_A = -a_B$$

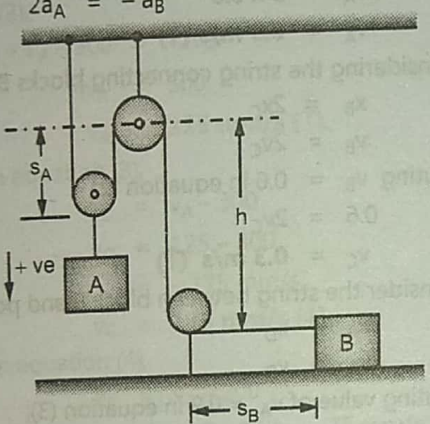


Fig. 4.23 (a)

Consider lower pulley as a reference. Total length of the string must be constant.

$$2(h - s_A) + h + s_B = \text{constant}$$

$$2h - 2s_A + h + s_B = \text{constant}$$

$$h - 2s_A + s_B = \text{constant.}$$

Differentiating with respect to time,

$$-2v_A + v_B = 0$$

$$2v_A = v_B$$

$$2a_A = a_B$$

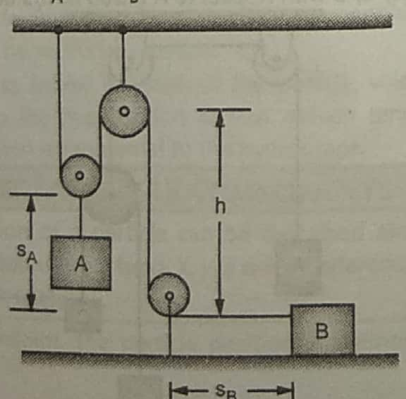


Fig. 4.23 (b)

NUMERICAL EXAMPLES ON DEPENDENT MOTION

Example 4.30 : *If the motor draws in cable such that point A is subjected to an acceleration of $a_A = (3t^2)\ m/s^2$, where t is in seconds, determine how high the load at B travels in t = 1.5 s starting from rest.*

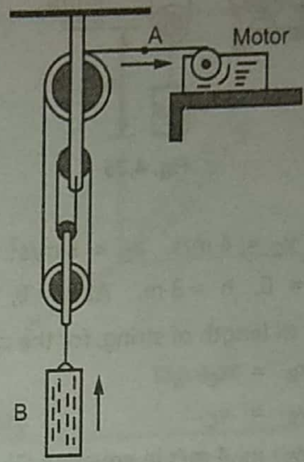


Fig. 4.24

Solution :

From the concept of length of string,

$$-4x_B + x_A = \text{constant}$$

$$-4v_B + v_A = 0$$

$$a_B = \frac{a_A}{4} \quad \text{... (1)}$$

$$a_A = 3t^2 \quad \text{... given}$$

Substituting $a_A = 3t^2$ in equation (1),

$$a_B = \frac{3t^2}{4}$$

Integrating with respect to t,

$$\int a_B = \int \frac{3t^2}{4} \, dt \ (t \to 0 \text{ to } t)$$

$$v_B = \frac{3t^3}{12}$$

Integrating with respect to t,

$$\int v_B = \int \frac{3t^3}{12} \, dt \ (t \to 0 \text{ to } t)$$

$$x_B = \frac{3t^4}{48} \quad \text{... (2)}$$

Load B travelled in t = 1.5 s, starting from rest is given by substituting t = 1.5 s in equation (2).

$$x_B = \frac{3 \times (1.5)^4}{48}$$

$$x_B = \mathbf{0.316\ m} \quad \text{... Ans.}$$

Example 4.31 : *The motor draws in the cable at C with a constant velocity of v_C = 4 m/s. The motor draws in the cable at D with a constant acceleration of $a_D = 8\ m/s^2$. If v_D = 0 and h = 3 m when t = 0, determine the time needed for h = 0. Also find the relative velocity of block A with respect to block B when this occurs.*

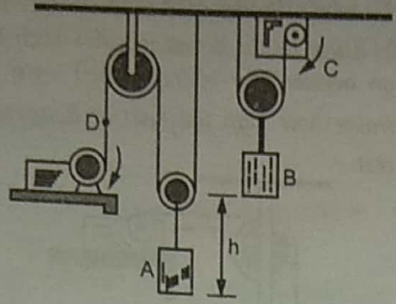


Fig. 4.25

Solution :

Given data : $v_C = 4$ m/s, $a_D = 8$ m/s^2.

At t = 0, $v_D = 0$, h = 3 m. At h = 0, t = ?, $v_{A/B}$ = ?

From concept of length of string, for the cable at C,

$$2x_B = x_C$$

$$2v_B = v_C \quad \text{... (1)}$$

Substituting v_C = 4 m/s in equation (1),

$$2v_B = 4$$

$$v_B = 2 \text{ m/s}$$

Similarly, for the cable at D,

$$2x_A = x_D$$

$$2v_A = v_D$$

$$2a_A = a_D \quad \text{... (2)}$$

Substituting a_D = 8 m/s^2 in equation (2),

$$2a_A = 8$$

$$a_A = 4 \text{ m/s}^2$$

Considering motion of block B and using equation of kinematics,

$$s = ut + \frac{1}{2}at^2$$

$$3 = 2t + \frac{1}{2} \times 4 \times t^2$$

$$3 = 2t + 2t^2$$

ing quadratic equation,

$$t = \mathbf{1.82 \text{ s}} \quad \text{... Ans.}$$

Considering motion of block A and using equation of kinematics,

$$v = u + at$$

$$v_A = 0 + 4 \times 1.82$$

$$v_A = 7.28 \text{ m/s}$$

Relative velocity of block A with respect to block B is given by,

$$v_{A/B} = v_A - v_B$$

$$v_{A/B} = 7.28 - 2$$

$$= \mathbf{5.28 \text{ m/s } (\uparrow)} \quad \text{... Ans.}$$

Example 4.32 : *Block B moves downward with a constant velocity of 0.6 m/s. Determine (a) the velocity of block A, (b) the velocity of block C, (c) the velocity of portion D of the cable, (d) the relative velocity of portion D of the cable with respect to block B.*

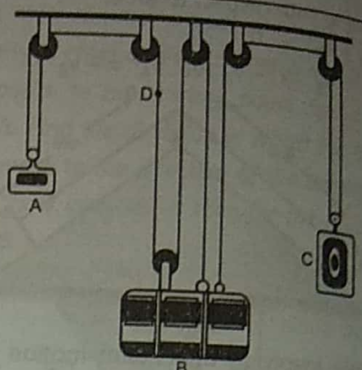


Fig. 4.26

Solution :

Given data : Constant velocity of block B = 0.6 m/s (↓)

(a) Considering the string connecting block B and block A,

$$2x_A = 3x_B$$

$$2v_A = 3v_B \quad \text{... (1)}$$

Substituting v_B = 0.6 m/s in equation (1),

$$2v_A = 3 \times 0.6$$

$$v_A = \mathbf{0.9 \text{ m/s } (\uparrow)} \quad \text{... Ans.}$$

(b) Considering the string connecting blocks B and C,

$$x_B = 2x_C$$

$$v_B = 2v_C \quad \text{... (2)}$$

Substituting v_B = 0.6 in equation (2),

$$0.6 = 2v_C$$

$$v_C = \mathbf{0.3 \text{ m/s } (\uparrow)} \quad \text{... Ans.}$$

(c) Consider the string between block B and point 'D',

$$2x_A = x_D$$

$$2v_A = v_D \quad \text{... (3)}$$

Substituting value of v_A = 0.9 in equation (3),

$$2 \times 0.9 = v_D$$

$$v_D = \mathbf{1.8 \text{ m/s } (\downarrow)} \quad \text{... Ans.}$$

(d) Relative velocity of portion D of cable with respect to block B is given by,

$$v_{D/B} = v_D - v_B$$

$$v_{D/B} = -1.8 - (-0.6)$$

$$= -1.2 \text{ m/s} = \mathbf{1.2 \text{ m/s } (\downarrow)} \quad \text{... Ans.}$$

Example 4.33 : *The three blocks shown move with constant velocities. Find the velocity of each block at the instant when the relative velocity of A with respect to C is 300 mm/s upward and the relative velocity of B with respect to A is 200 mm/s downward.*

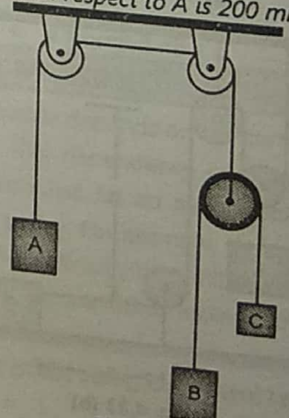


Fig. 4.27

Solution :

Given data : $v_A - v_C = 300$ and $v_B - v_A = -200$.

From the concept of length of string,

$$x_A + x_O = \text{constant}$$

$$v_A = -v_O \quad \text{... (1)}$$

$$x_B - x_O + x_C - x_O = \text{constant}$$

$$v_B + v_C = 2v_O$$

From equation (1),

$$v_B + v_C = -2v_A \quad \text{... (2)}$$

From the given condition,

$$v_A - v_C = 300$$

$$v_C = v_A - 300 \quad \text{... (3)}$$

and $$v_B - v_A = -200$$

$$v_B = v_A - 200 \quad \text{... (4)}$$

Substituting the value of $v_C = v_A - 300$, and $v_B = v_A - 200$ in equation (2),

$$v_A - 200 + v_A - 300 = -2v_A$$

$$4v_A = 500$$

$$v_A = \textbf{125 mm/s } (\uparrow) \quad \textbf{... Ans.}$$

From equation (3),

$$v_C = v_A - 300$$

$$v_C = 125 - 300$$

$$= -175 \text{ mm/s}$$

$$v_C = \textbf{175 mm/s } (\downarrow) \quad \textbf{... Ans.}$$

From equation (4),

$$v_B = v_A - 200$$

$$v_B = 125 - 200 = -75 \text{ mm/s}$$

$$v_B = \textbf{75 mm/s } (\downarrow) \quad \textbf{... Ans.}$$

B – KINEMATICS OF PARTICLE (CURVILINEAR MOTION)

4.9 GENERAL

When a particle is moving along a curve, the motion is said to be curvilinear motion.

In curvilinear motion, velocity is always tangential to the path of the particle. But if acceleration is also tangential, then the motion is said to be rectilinear motion.

In order to follow the path of the particle, velocity vector swing inside, so the acceleration cannot remain tangent to the path. Acceleration is tangential to the hydrograph.

4.10 RECTANGULAR COMPONENTS

The motion of a particle can be described along a path that is represented using a fixed x, y, z axis of reference.

4.10.1 Position

At any instant, the particle position is defined by the position vector

$$\bar{r} = xi + yj + zk$$

$$\therefore \quad \boxed{|\bar{r}| = \sqrt{x^2 + y^2 + z^2}}$$

$$\boxed{\text{Direction vector along } \bar{r} = \frac{\bar{r}}{|\bar{r}|}}$$

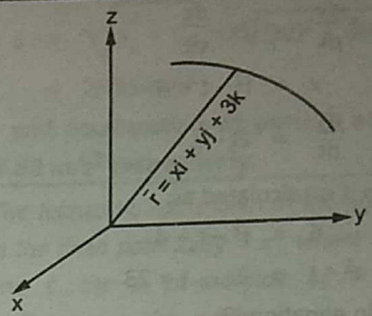


Fig. 4.28

4.10.2 Velocity

Differentiating position vector with time,

$$\bar{v} = \frac{d\bar{r}}{dt} = \frac{d}{dt}(xi + yj + zk)$$

$$= \frac{dx}{dt} i + \frac{dy}{dt} j + \frac{dz}{dt} k$$

$$\therefore \quad v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt} \text{ and } v_z = \frac{dz}{dt}$$

$$\boxed{|\bar{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}}$$

$\therefore$ Direction vector of velocity

$$v = \frac{\bar{v}}{|\bar{v}|} = \frac{v_x i + v_y j + v_z k}{\sqrt{v_x^2 + v_y^2 + v_z^2}}$$

4.10.3 Acceleration

Differentiating velocity vector with time,

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d}{dt}(v_x i + v_y j + v_z k)$$

$$\bar{a} = \frac{dv_x}{dt} i + \frac{dv_y}{dt} j + \frac{dv_z}{dt} k$$

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad a_z = \frac{dv_z}{dt}$$

$$\boxed{|\bar{a}| = \sqrt{a_x^2 + a_y^2 + a_z^2}}$$

$\therefore$ Direction vector of acceleration

$$= \frac{\bar{a}}{|\bar{a}|} = \frac{a_x i + a_y j + a_z k}{\sqrt{a_x^2 + a_y^2 + a_z^2}}$$

***Example 4.34 :** A particle is constrained to move upward to the right along the path $2y = x^2 + 26$, where x and y are in meters. The x co-ordinate of the particle at any time is $x = t^2 - t + 4$. Determine the y components of velocity and acceleration at x = 6 m.*

Solution :

Given data : Path of particle : $2y = x^2 + 26$

x co-ordinate : $x = t^2 - t + 4$

To find : v_y and a_y at x = 6 m

$$2y = x^2 + 26$$

$$2\frac{dy}{dt} = 2x \cdot \frac{dx}{dt} \quad \text{... (1)}$$

$$x = t^2 - t + 4$$

$$\frac{dx}{dt} = 2t - 1 \quad \text{... (2)}$$

At x = 6 m, t is calculated as,

$$6 = t^2 - t + 4$$

$$\therefore \quad t^2 - t = 2 \quad \therefore \; t = 23$$

Put t = 2 in equation (2).

$$\therefore \quad \frac{dx}{dt} = 2t - 1 = 2 \times 2 - 1 = 3 \text{ m/s}$$

Put $\frac{dx}{dt}$ = 3 m/s and x = 6 m in equation (1).

$$\therefore \quad 2 \cdot \frac{dy}{dt} = 2 \times 6 \times 3$$

$$\therefore \quad \frac{dy}{dt} = 18 \text{ m/s}$$

$$2\frac{d^2y}{dt^2} = 2\frac{dx}{dt} \cdot \frac{dx}{dt} + 2x\frac{d^2x}{dt^2} \quad \text{... (3)}$$

From equation (2),

$$\frac{dx}{dt} = 2t - 1$$

$$\therefore \quad \frac{d^2x}{dt^2} = 2 \quad \text{... (4)}$$

Put $\frac{d^2x}{dt^2}, \frac{dx}{dt}$ and x in equation (3).

$$2 \cdot \frac{d^2y}{dt^2} = 2 \times 3 \times 3 + 2 \times 6 \times 2$$

$$\frac{d^2y}{dt^2} = a_y = 21 \text{ m/s}^2$$

$\therefore$ **y-component of velocity and acceleration of particle is 18 m/s and 21 m/s² respectively.** **... Ans.**

Example 4.35 : *A particle moves along a hyperbolic path $\frac{x^2}{16} - y^2 = 28$. If the x component of velocity is v_x = 4 m/s and remains constant, determine the magnitudes of velocity and acceleration of the particle when it is at point (32 m, 6 m).*

Solution :

Given data : Path of the particle, $\frac{x^2}{16} - y^2 = 28$

v_x = 4 m/s (Constant velocity : $a_x = 0$)

To find : Velocity and acceleration at (32 m, 6 m).

$$\frac{x^2}{16} - y^2 = 28$$

Differentiate w.r.t. t,

$$\therefore \quad \frac{2x \cdot \frac{dx}{dt}}{16} - 2y\frac{dy}{dt} = 0 \quad \text{... (1)}$$

$$v_x = \frac{dx}{dt} = 4 \text{ m/s}, \; x = 32 \text{ m}, \; y = 6 \text{ m}$$

$$\therefore \quad \frac{2 \times 32 \times 4}{16} - 2 \times 6 \times \frac{dy}{dt} = 0$$

$$\therefore \quad \frac{dy}{dt} = v_y = 1.33 \text{ m/s}$$

$$\therefore \quad \text{Velocity} = \sqrt{v_x^2 + v_y^2}$$

$$= \sqrt{(4)^2 + (1.33)^2} = 4.22 \text{ m/s}$$

Again differentiating equation (1) w.r.t. t,

$$\frac{2x}{16} \cdot \frac{d^2x}{dt^2} + \frac{2}{16} \cdot \frac{dx}{dt} \cdot \frac{dx}{dt} - 2\frac{dy}{dt} \cdot \frac{dy}{dt} - 2y\frac{d^2y}{dt^2} = 0$$

$$\frac{2 \times x}{16} \cdot \ddot{x} + \frac{1}{8}\dot{x}^2 - 2 \times \dot{y}^2 - 2y \cdot \ddot{y} = 0$$

$$\ddot{x} = a_x = 0, \quad \dot{x} = v_x = 4 \text{ m/s}$$

$$x = 32 \text{ m}, \quad v_y = 1.33 \text{ m/s}, \; y = 6 \text{ m}$$

$$\frac{2 \times 32}{16} \times 0 + \frac{1}{8} \times (4)^2 - 2 \times (1.33)^2 - 2 \times 6 \times \ddot{y} = 0$$

$$\therefore \quad \ddot{y} = a_y = -0.13 \text{ m/s}^2$$

$\therefore$ **Magnitudes of velocity and acceleration are 4.22 m/s and 0.13 m/s² at point (32 m, 6 m).** **... Ans.**

Example 4.36 : *A particle has an initial velocity of 100 m/s upto the right at 30° with the horizontal. The components of acceleration are constant at a_x = – 4 m/s² and a_y = – 20 m/s². Find the horizontal distance covered until the particle reaches a point 60 m below its original elevation.*

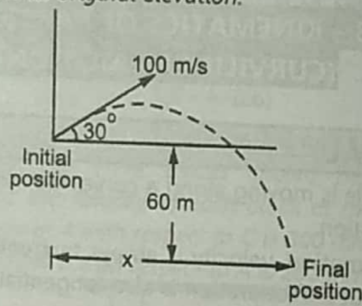


Fig. 4.29

Solution :

Given data : As shown in Fig. 4.29.

To find : Horizontal distance covered by the particle.

Let horizontal distance covered be x and

Total vertical distance covered = – 60 m

v = 100 m/s, θ be 30°

$$\therefore \quad u_x = v \cos 30 = 100 \cos 30 = 86.60 \text{ m/s}$$

$$u_y = v \sin 30 = 100 \sin 30 = 50 \text{ m/s}$$

$$\therefore \quad x = u_x t + \frac{1}{2} a_x \cdot t^2$$

$$x = 86.6\,t + \frac{1}{2}(-4)\,t^2 \quad \text{... (1)}$$

$$y = u_y t + \frac{1}{2} a_y t^2$$

$$-60 = 50\,t + \frac{1}{2}(-20)\,t^2 \quad \text{... (2)}$$

$$10\,t^2 - 50\,t - 60 = 0$$

$$\therefore \quad t = 6\text{ s}$$

Put t = 6 s in equation (1).

$$x = 86.6 \times 6 - \frac{1}{2} \times 4 \times (6)^2$$

$$= 447.6\text{ m}$$

∴ Horizontal distance covered by the particle is 447.6 m. **... Ans.**

Example 4.37 : *The y co-ordinate of a particle in curvilinear motion is given by $y = 4t^3 - 3t$, where y is in meters and t in seconds. Also the particle has an acceleration in the x-direction given by $a_x = 12t\ m/s^2$. If the velocity of the particle in the x-direction is 4 m/s when t = 0, calculate the magnitudes of the velocity 'v' and acceleration 'a' of the particle when t = 1 s.*

Solution :

Given data : Curvilinear motion, $y = 4t^3 - 3t$

Acceleration in x direction, $a_x = 12t\ m/s^2$, $u_x = 4$ m/s

To find : Magnitude and acceleration of the particle at t = 1 s.

$$y = 4t^3 - 3t$$

$$\frac{dy}{dt} = 12t^2 - 3$$

$$\text{At } t = 1\text{ s}, \quad \frac{dy}{dt} = v_y = 9\text{ m/s}$$

$$\frac{d^2y}{dt^2} = 24t$$

$$\therefore \quad \frac{d^2y}{dt^2} = a_y = 24\text{ m/s}^2$$

$$a_x = 12t$$

$$\frac{d^2x}{dt^2} = 12t$$

Integrating the above equation,

$$\frac{dx}{dt} = \frac{12t^2}{2} + C$$

$$\text{At } t = 0, \quad \frac{dx}{dt} = 4\text{ m/s}$$

$$\therefore \quad C = 4$$

$$\therefore \quad \frac{dx}{dt} = \frac{12t^2}{2} + C$$

$$\text{At } t = 1\text{ s}, \quad \frac{dx}{dt} = v_x = \frac{12t^2}{2} + 4$$

$$v_x = 10\text{ m/s}$$

$$\frac{d^2x}{dt^2} = a_x = 12\text{ m/s}^2$$

$$\therefore \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{(10)^2 + (9)^2}$$

$$= 13.45\text{ m/s}$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(12)^2 + (24)^2}$$

$$= 26.83\text{ m/s}^2$$

∴ Velocity and acceleration of particle at t = 1 sec is 13.45 m/s and 26.83 m/s² respectively. **... Ans.**

Example 4.38 : *The telescopic rod shown in the figure, forces the pin to move along the fixed path $225y = x^2$, where x and y are in mm. At any time t, the x co-ordinate of P is given by $x = 25t^2 - 125t$. Determine the y component of velocity and acceleration of P at x = 150 mm.*

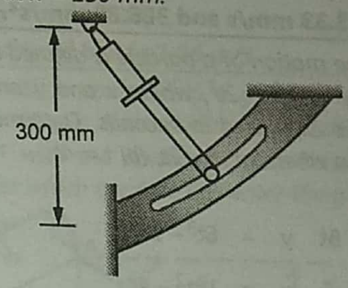


Fig. 4.30

Solution :

Given data : Path of the particle, $225y = x^2$

x co-ordinate of P, $x = 25t^2 - 125$ t

To find : y component of velocity and acceleration of P at x = 150 mm.

$$x = 25t^2 - 125t \quad \text{... (1)}$$

$$x = 150\text{ mm}$$

$$\therefore \quad 150 = 25t^2 - 125t$$

$$\therefore \quad t = 6\text{ s}$$

Differentiating equation (1),

$$\frac{dx}{dt} = 50t - 125 \quad \text{... (2)}$$

Put t = 6 s in equation (2).

$$\therefore \quad v_x = \frac{dx}{dt} = 50 \times 6 - 125 = 175\text{ mm/s}$$

Differentiating equation (2) w.r.t. t,

$$\frac{d^2x}{dt^2} = 50$$

$$a_x = \frac{d^2x}{dt^2} = 50$$

We have path of the particle as,

$$225y = x^2$$

Differentiating w.r.t. t,

$$225\frac{dy}{dt} = 2x\cdot\frac{dx}{dt} \quad \text{... (3)}$$

Put x = 150 mm and $\frac{dx}{dt}$ = 175 m/s in equation (3).

$$\therefore \quad 225 \cdot \frac{dy}{dt} = 2 \times 150 \times 175$$

$$\therefore \quad v_y = \frac{dy}{dt} = 233.33 \text{ mm/s}$$

Differentiating equation (3) w.r.t. t,

$$225 \cdot \frac{d^2y}{dt^2} = 2x \cdot \frac{d^2x}{dt^2} + 2 \cdot \frac{dx}{dt} \cdot \frac{dx}{dt} \quad \text{... (4)}$$

Put $\frac{d^2x}{dt^2} = 50 \text{ mm/s}^2$, $\frac{dx}{dt} = 175$ mm/s and $x = 150$ mm in equation (4).

$$\therefore \quad 225 \cdot \frac{d^2y}{dt^2} = 2 \times 150 \times 50 + 2 \times 175 \times 175$$

$$\therefore \quad a_y = \frac{d^2y}{dt^2} = 388.89 \text{ mm/s}^2$$

∴ y – component of velocity and acceleration at x = 150 mm is 233.33 mm/s and 388.89 mm/s². **... Ans.**

Example 4.39 : *The motion of a particle is defined by the equation $x = 4t^4 - 6t$ and $y = 6t^3 - 2t^2$, where x and y are expressed in millimeters and t is expressed in seconds. Determine the velocity and the acceleration when (a) t = 2s, (b) t = 4s.*

Solution :

$$x = 4t^4 - 6t \qquad y = 6t^3 - 2t^2$$

$$\dot{x} = 16t^3 - 6 \qquad \dot{y} = 18t^2 - 4t$$

$$\ddot{x} = 48t^2 \qquad \ddot{y} = 36t - 2$$

At t = 2s, $v_x = 122$ mm/s, $v_y = 64$ mm/s,

v = **139.67 mm/s, θ = 29.13°.** **... Ans.**

$a_x = 192 \text{ mm/s}^2$, $a_y = 70 \text{ mm/s}^2$,

a = **204.36 mm/s², θ = 20.03°** **... Ans.**

At t = 4s, $v_x = 1018$ mm/s, $v_y = 280$ mm,

v = **1055 mm/s, θ = 15.38°** **... Ans.**

$a_x = 768$ mm/s, $a_y = 142$ mm/s,

a = **781 mm/s², θ = 10.48°** **... Ans.**

4.11 MOTION OF PROJECTILE

We have been studied the motion of particles in unidirection only i.e. only vertical or only horizontal direction. But when particle is projected in the air at a certain angle, then particle falls on the ground other than the point of projection. Thus the particle, with combined effect of both vertical and horizontal direction, is called as a projectile.

In the motion of projectile, factors such as wind effect, air resistance effect and rotation of earth will be neglected, so projectile have constant downward acceleration (g).

Time of Flight : The total time of motion of the particle is known as time of flight.

Range : The distance between the point of projection and the point where it strikes the surface or ground.

The motion of projectile can be studied using rectangular components. Scalar analysis can be used because motion along x and y direction is rectilinear.

Motion of Projectile in Horizontal Direction :

Acceleration, $a_x = 0$

$$v_x = v_{ox} + a_x t = v_{ox}$$

Velocity along x direction at any instant is constant, during the motion.

$$x = v_{ox} t + \frac{1}{2} a_x t^2 = v_{ox} t$$

$$v_x^2 = v_{ox}^2 + 2a_x x = v_{ox}^2$$

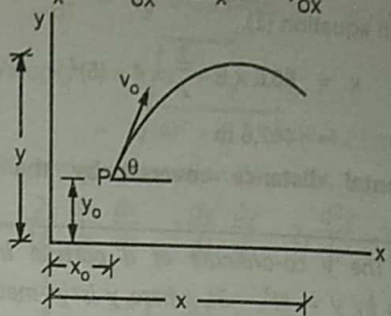


Fig. 4.31

Motion of Projectile in Vertical Direction :

Acceleration, $a_y = -g \text{ m/s}^2$

$$v_y = v_{oy} - gt$$

$$\boxed{y = v_{oy} t - \frac{1}{2} gt^2}$$

$$\boxed{v_y^2 = v_{oy}^2 - 2gy}$$

Example 4.40 : *A rocket is released at point A from a jet aircraft flying horizontally at 1000 km/hr at an altitude of 800 m. If horizontal rocket thrust gives the rocket a horizontal acceleration of 0.5 g, determine the angle θ from the horizontal to the line of sight.*

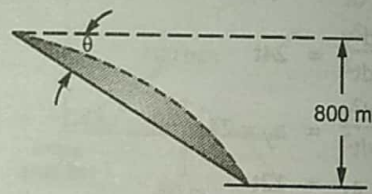


Fig. 4.32

Solution :

Given data : As shown in Fig. 4.32.

To find : Angle θ from horizontal to the line of sight.

$$u_x = 277.78 \text{ m/s}, \quad u_y = 0$$

$$x = u_x t + \frac{1}{2} a_x t^2 \quad \text{... (1)}$$

$$x = 277.78\, t + \frac{0.5 \times 9.81 \times t^2}{2}$$

$$x = 277.78\, t + 2.45\, t^2 \quad \text{... (2)}$$

$$y = u_y t - \frac{1}{2} \times 9.81 \times t^2$$

$$-800 = -\frac{1}{2} \times 9.81 \times t^2$$

$$\therefore \quad t = 12.77 \text{ s}$$

Put t = 12.77 s in equation (2).

$$x = 277.78 + 2.45\,t^2$$

$$\therefore \quad = 277.78 \times 12.77 + 2.45 \times (12.77)^2$$

$$= 3946.75 \text{ m}$$

$$\tan\theta = \frac{y}{x} = \frac{800}{3946.75}$$

$$\therefore \quad \theta = 11.46^\circ$$

$\therefore$ **Angle 'θ' from horizontal to the line of sight is 11.46°.** **... Ans.**

Example 4.41 : *During a race, the dirt bike was observed to leap up off the small hill at A at an angle of 60° with the horizontal. If the point of landing is 6 m away, determine the approximate speed at which the bike was traveling just before it left the ground. Neglect the size of the bike for the calculation.*

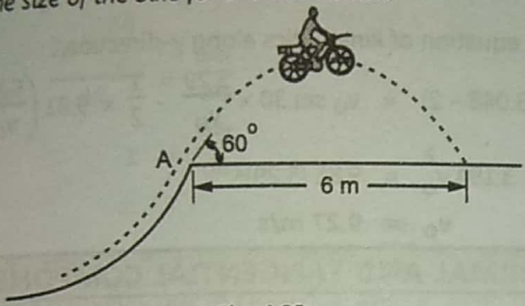


Fig. 4.33

Solution :

Given data : As shown in Fig. 4.33.

To find : Speed of the vehicle.

Let v_o be the velocity at A.

$$u_x = v_x = v_o \cos 60 = 0.5\,v_o$$

$$u_y = v_o \sin 60 = 0.866\,v_o$$

$$x = v_x \times t$$

$$6 = 0.5\,v_o \times t$$

$$t = \frac{12}{v_o}$$

$$y = u_y t - \frac{1}{2} g t^2$$

$$0 = 0.866\,v_o \times \frac{12}{v_o} - \frac{1}{2} \times 9.81 \times \left(\frac{12}{v_o}\right)^2$$

$$\therefore \quad v_o = 8.24 \text{ m/s}$$

$\therefore$ **Approximate speed of vehicle is 8.24 m/s at 60° with horizontal.** **... Ans.**

Example 4.42 : *A ball is dropped onto a step at point A and rebounds with a velocity v_o at an angle of 15° with the vertical. Determine the value of v_o knowing that just before the ball bounces at point B, its velocity forms an angle of 12° with the vertical.*

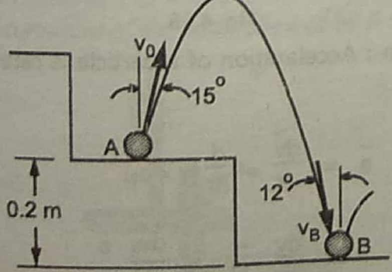


Fig. 4.34

Solution :

Given data : As shown in Fig. 4.34.

To find : v_o just before ball bounces at B.

$$v_y^2 = u_y^2 - 2gh$$

$$\therefore \quad v_y^2 = (v_o \cos 15)^2 - 2 \times 9.81 \times (-0.2)$$

$$v_y^2 = 0.93\,v_o^2 + 3.92$$

$$v_x = v_o \sin 15$$

$$\therefore \quad v_x^2 = 0.067\,v_o^2$$

$$\tan^2 12 = \frac{v_x^2}{v_y^2} = \frac{0.067\,v_o^2}{0.93\,v_o^2 + 3.92}$$

$$0.0452\,(0.93\,v_o^2 + 3.92) = 0.067\,v_o^2$$

$$v_o = 2.66 \text{ m/s}$$

Rebound velocity of ball is 2.66 m/s. **... Ans.**

Example 4.43 : *A milk is poured into a glass of height 140 mm and inside diameter 66 mm. If the initial velocity of milk is 1.2 m/s at an angle of 40° with the horizontal, determine the range of values of height h for which the milk will enter the glass.*

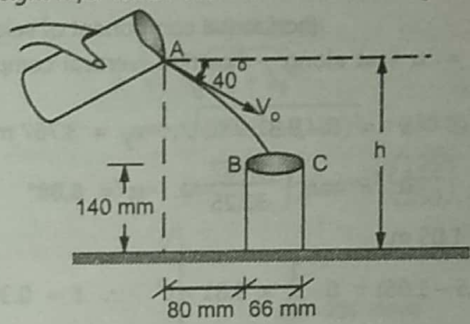


Fig. 4.35

Solution :

Given data : As shown in Fig. 4.35.

To find : Range of height 'h'.

u_x = 919.25 mm/s, u_y = − 771.34 mm/s

At point B : x = 80 mm, y = − (h − 140) mm

$$x = u_x \times t$$

$$\therefore \quad t = \frac{80}{919.25} = 0.087 \text{ s}$$

$$\therefore \quad y = u_y t - \frac{1}{2} g t^2$$

$$-(h - 140) = -771.34 \times 0.087 - \frac{1}{2} \times 9810 \times (0.087)^2$$

$$\therefore \quad h = 244.23 \text{ mm}$$

At point C : x = 146 mm, y = − (h − 140) mm

$$\therefore \quad t = \frac{146}{919.25} = 0.159 \text{ s}$$

$$\therefore \quad -(h - 140) = -771.34 \times 0.159 - \frac{1}{2} \times 9810 \times (0.159)^2$$

$$\therefore \quad h = 386.65 \text{ mm}$$

So range of height h is 244.23 ≤ h ≤ 386.65. **... Ans.**

***Example 4.44 :** A baseball player throws base ball with a horizontal velocity v_o. Knowing that the height varies between 775 mm and 1050 mm, determine (a) the range of value of u_o, (b) the values of α corresponding to h = 775 mm and h = 1050 mm.*

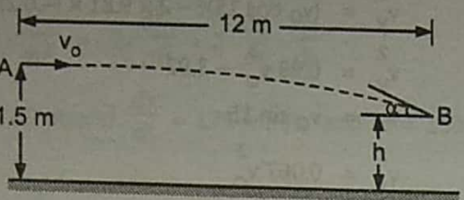


Fig. 4.36

Solution :

(a) $h = 0.775$ m,

using $s = ut + \frac{1}{2}at^2$ along y-direction

$$-(1.5 - 0.775) = 0 - \frac{1}{2} \times 9.81 \times t^2$$

$$t = 0.384 \text{ s}$$

Using same equation along x-direction,

$$12 = v_o \times 0.384$$

$\therefore$ $\mathbf{v_o = 31.25}$ **m/s**

(horizontal component of velocity at B)

Using $v = u + at$ along y-direction (vertical component of velocity at B)

$$v = 0 - 9.81 \times t \qquad \therefore v_y = 3.767 \text{ m/s}$$

$$\alpha = \tan^{-1}\frac{3.767}{31.25} \quad \therefore \alpha = 6.88° \quad \text{... Ans.}$$

(b) $h = 1.05$ m.

$$-(1.5 - 1.05) = 0 - \frac{1}{2} \times 9.81 \times t^2 \qquad \therefore t = 0.303 \text{ s}$$

$$12 = v_o \times 0.303 \qquad \therefore \mathbf{v_o = 39.6 \text{ m/s}}$$

$$v = 0 - 9.81 \times 0.303 \qquad \therefore v = 2.972 \text{ m/s}$$

$$\alpha = \tan^{-1}\frac{2.9724}{39.6} \quad \therefore \alpha = 4.29° \quad \text{... Ans.}$$

***Example 4.45 :** A basket ball player shoots when she is 5 m from the backboard. Knowing that the ball has an initial velocity v_o at an angle of 30° with the horizontal, determine the value of v_o when d is equal to (a) 220 mm, (b) 420 mm.*

Solution :

(a) When d = 220 mm : Using equation of kinematics along x-direction,

$$(5 - 0.22) = v_o \cos 30t$$

$$\therefore \quad t = \frac{5.5195}{v_o}$$

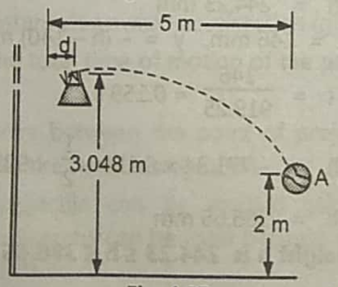


Fig. 4.37

Using equation of kinematics along y-direction,

$$(3.048 - 2) = v_o \sin 30 \times \frac{5.5195}{v_o} - \frac{1}{2} \times 9.81 \left(\frac{5.5195}{v_o}\right)^2$$

$$3.424\, v_o^2 = 9.81 \times 5.5195^2$$

$$v_o = \mathbf{9.34 \text{ m/s}}$$

... Ans.

(b) When d = 420 mm : Using equation of kinematics along x-direction,

$$(5 - 0.42) = v_o \cos 30 \times t \qquad \therefore t = \frac{5.290}{v_o}$$

Using equation of kinematics along y-direction,

$$(3.048 - 2) = v_o \sin 30 \times \frac{5.29}{v_o} - \frac{1}{2} \times 9.81 \left(\frac{5.29}{v_o}\right)^2$$

$$3.194\, v_o^2 = 9.81\,(5.29)^2$$

$$\mathbf{v_o = 9.27 \text{ m/s}}$$

... Ans.

4.12 NORMAL AND TANGENTIAL COMPONENTS

When the path of particle is known, it is convenient to describe the motion along tangent and normal to the path. The tangential axis is tangent to the curve at that instant and is positive in the direction of increasing 's'. Let $\bar{e}_t$ be the unit vector along tangential direction. The normal axis is perpendicular to the tangential axis and is directed towards centre of curvature at that instant. Let $\bar{e}_n$ be the unit vector along normal axis.

1. **Velocity :** The velocity of particle is always tangential to the path. Velocity is derivative of path function.

$$\bar{s} = f(t)$$

$$\bar{v} = \frac{d\bar{s}}{dt}$$

$$= v \times \bar{e}_t$$

where, v is magnitude of velocity.

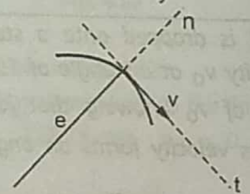


Fig. 4.38

2. **Acceleration :** Acceleration of a particle is rate of change of velocity.

$$\therefore \quad \bar{a} = \frac{d\bar{v}}{dt} = \frac{d}{dt}(v \times \bar{e}_t)$$

$$= \frac{dv}{dt} \cdot \bar{e}_t + v \cdot \frac{d\bar{e}_t}{dt}$$

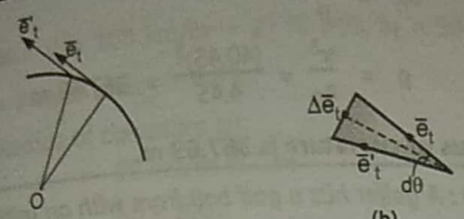


Fig. 4.39

Let $d\theta$ be the angle made by $\bar{e}_t$ and $\bar{e}_t'$.

$$\bar{e}_t' = \bar{e}_t + \Delta\bar{e}_t$$

$\bar{e}_t$ has magnitude 1. $\sin\frac{d\theta}{2} = \frac{\Delta e_t/2}{e_t} = \frac{\Delta e_t}{2}$

$$\therefore \quad \frac{2\sin\frac{\Delta\theta}{2}}{\Delta\theta} = \frac{\Delta e_t}{\Delta\theta}$$

$$1 = \frac{de_t}{d\theta}$$

As $d\theta$ approaches to zero, the vector $\frac{d\bar{e}_t}{d\theta}$ has unit magnitude and acting perpendicular to tangential axis i.e. along normal axis.

$$\therefore \quad \frac{d\bar{e}_t}{d\theta} = \bar{e}_n$$

$$\therefore \quad \bar{a} = a_t\,\bar{e}_t + v\cdot\frac{d\bar{e}_t}{d\theta}\cdot\frac{d\theta}{ds}\cdot\frac{ds}{dt}$$

$$= a_t\cdot\bar{e}_t + v\cdot\bar{e}_n\cdot\frac{1}{\rho}\cdot v$$

$$= a_t\cdot\bar{e}_t + \frac{v^2}{\rho}\cdot\bar{e}_n = a_t\,\bar{e}_t + a_n\,\bar{e}_n$$

$$\therefore \quad a_n = \frac{v^2}{\rho}$$

$\therefore$ Resultant acceleration,

$$a = \sqrt{a_t^2 + a_n^2}$$

$\therefore$ Tangential acceleration $= \frac{dv}{dt}$

Normal acceleration

$= \frac{v^2}{\rho}$, where ρ is radius of curvature.

Example 4.46 : *The pin P moves along a curved path which is determined by the motions of two slotted links A and B. At the instant shown, A has velocity of 300 mm/s and an acceleration of 250 mm/s² both towards right. While B has velocity of 400 mm/s and an acceleration of 125 mm/s² both vertically downward. Find the radius of curvature of the path followed by P at this instant.*

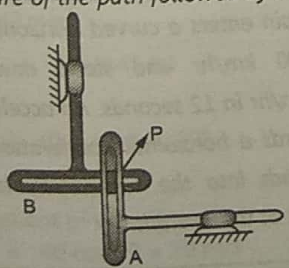


Fig. 4.40

Solution :

Given data : v_A = 300 mm/s (→), a_A = 250 mm/s² (→), v_B = 400 mm/s (↓), a_B = 125 mm/s² (↓)

To find : Radius of curvature of the path.

$$v = \sqrt{v_A^2 + v_B^2}$$
$$= \sqrt{(300)^2 + (400)^2}$$
$$= 500 \text{ mm/s}$$
$$\theta = \tan^{-1}\left(\frac{v_B}{v_A}\right)$$
$$= \tan^{-1}\left(\frac{400}{300}\right)$$
$$= 53.13^\circ$$

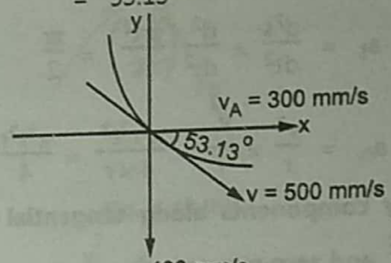


Fig. 4.41 (a)

$$a = \sqrt{a_A^2 + a_B^2}$$
$$= \sqrt{(250)^2 + (125)^2} = 279.51 \text{ mm/s}^2$$
$$\theta = \tan^{-1}\left(\frac{a_B}{a_A}\right) = \tan^{-1}\left(\frac{125}{250}\right) = 26.56^\circ$$

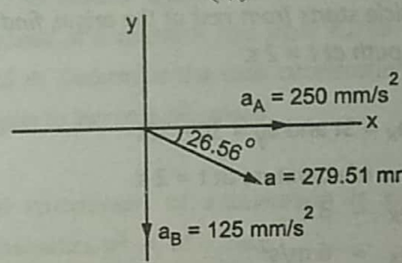


Fig. 4.41 (b)

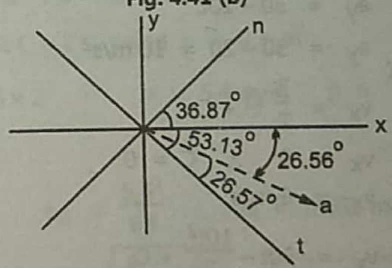


Fig. 4.41 (c)

$$\therefore \quad a_n = a\sin 26.57 = 279.51\sin 26.57$$
$$= 125.0 \text{ mm/s}^2$$
$$\therefore \quad a_n = \frac{v^2}{\rho}$$
$$125 = \frac{(500)^2}{\rho}$$
$$\therefore \quad \rho = 2000 \text{ mm}$$
$$\therefore \quad \rho = 2 \text{ m}$$

$\therefore$ **Radius of curvature of path is 2 m.** **... Ans.**

Example 4.47 : *A particle moves along a circular path of radius r. The length s of the arc traversed in time t is $s = \frac{\pi r t^2}{4}$. Determine the magnitudes of the tangential and normal components of the velocity and acceleration of the particle as a function of t.*

Solution :

Given data : $s = \frac{\pi r t^2}{4}$

To find : v_t, v_n, a_t, a_n

$$s = \frac{\pi r t^2}{4}$$

$$\frac{ds}{dt} = v_t = \frac{2\pi r t}{4} = \frac{\pi r t}{2}$$

$$v_n = 0 \text{ (always)}$$

$$a_t = \frac{d^2 s}{dt^2} = \frac{d^2}{dt^2}\left(\frac{\pi r t^2}{4}\right) = \frac{\pi r}{2}$$

$$a_n = \frac{v^2}{r} = \frac{v_t^2}{r} = \frac{\pi^2 r^2 t^2}{4 \times r} = \frac{\pi^2 t^2 r}{4}$$

∴ **Velocity components along tangential and normal direction are $\frac{\pi r t}{2}$ and zero respectively.** **... Ans.**

∴ **Acceleration components along tangential and normal direction are $\frac{\pi r}{2}$ and $\frac{\pi^2 t^2 r}{4}$ respectively.** **... Ans.**

Example 4.48 : *The rectangular components of acceleration for a particle are $a_x = 3t$ and $a_y = 30 - 10t$, where a is in m/s² and t is in seconds. If the particle starts from rest at the origin, find the radius of curvature of the path at t = 2 s.*

Solution :

Given data : $a_x = 3t$ and $a_y = 30 - 10t$

To find : Radius of curvature at t = 2 s.

$$a_x = 3t$$

At t = 2 s, $a_x = 6 \text{ m/s}^2$

$$a_y = 30 - 10t$$

At t = 2 s, $a_y = 30 - 20 = 10 \text{ m/s}^2$

$$v_x = \frac{3}{2} t^2 + C$$

At t = 0, $v_x = 0 \quad \therefore \quad C = 0$

At t = 2 s, $v_x = 6 \text{ m/s}$

$$v_y = 30t - \frac{10t^2}{2} + C_1$$

At t = 0, $v_y = 0 \quad \therefore \quad C_1 = 0$

∴ At t = 2 s, $v_y = 40 \text{ m/s}$

$$\therefore \quad v = \sqrt{v_x^2 + v_y^2} = \sqrt{(6)^2 + (40)^2}$$

$$= 40.45 \text{ m/s}$$

$$\therefore \quad \theta = 81.47^\circ$$

$$\therefore \quad a_n = 6 \cos 8.53 - 10 \sin 8.53$$

$$= 4.45 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho}$$

$$\therefore \quad \rho = \frac{v^2}{a_n} = \frac{(40.45)^2}{4.45} = 367.69 \text{ m}$$

∴ **Radius of curvature is 367.69 m.** **... Ans.**

Example 4.49 : *A golfer hits a golf ball from with an initial velocity of 50 m/s at an angle of 25° with the horizontal. Determine the radius of curvature of the trajectory described by the ball (a) at the start of the trajectory, (b) at the highest point of trajectory.*

Solution :

Given data : v = 50 m/s, θ = 25°

To find : Radius of curvature at start and at highest point.

$$a_n = g \cos 25$$

$$= 9.81 \times \cos 25 = 8.89 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho}$$

$$\therefore \quad \rho = \frac{v^2}{a_n} = \frac{(50)^2}{8.89} = 281.21 \text{ m}$$

Radius of curvature at start is 281.21 m.

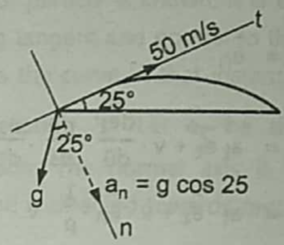


Fig. 4.42 (a) : At start

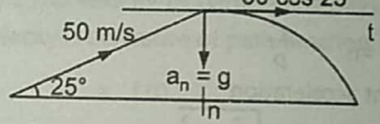


Fig. 4.42 : (b) At highest point

Concept : Velocity at highest point is only in horizontal direction.

$$\therefore \quad v = 50 \cos 25 = 45.32 \text{ m/s}$$

$$\therefore \quad a_n = \frac{v^2}{\rho}$$

$$\therefore \quad \rho = \frac{v^2}{a_n} = \frac{(45.32)^2}{g} = \frac{(45.32)^2}{9.81} = 209.37 \text{ m}$$

∴ **Radius of curvature at highest point is 209.37 m.** **... Ans.**

Example 4.50 : *A train enters a curved horizontal section of track at a speed of 100 km/hr and slows down with constant deceleration of 50 km/hr in 12 seconds. An accelerometer mounted inside the train records a horizontal acceleration of 2 m/s² when the train is 6 seconds into the curve. Calculate the radius of curvature of the track at that instant.*

Solution :

Given data : $u = 100$ km/hr = 27.78 m/s, a_t = 50 km/hr in 12 seconds, $a = 2\ m/s^2$ after $t = 6$ s.

To find : Radius of curvature (ρ).

$$a_t = 50 \text{ km/hr in } 12 \text{ s}$$

$$= \frac{50 \times 1000}{3600 \times 12} = 1.16\ m/s^2$$

$$a = 2\ m/s^2$$

$$\therefore \quad a = \sqrt{a_t^2 + a_n^2}$$

$$2 = \sqrt{(1.16)^2 + a_n^2}$$

$$\therefore \quad a_n = 1.63\ m/s^2$$

$$v = u + a_t \cdot t$$

$$v = 27.78 - 1.16 \times 6 = 20.82 \text{ m/s}$$

$$a_n = \frac{v^2}{\rho}$$

$$\therefore \quad \rho = \frac{(20.82)^2}{1.63} = 265.94 \text{ m}$$

Radius of curvature of track after t = 6 sec is 265.94 m. **... Ans.**

Example 4.51 : *The earth makes one complete revolution around the sun in 365.24 days. Assuming that the orbit of the earth is circular and has a radius of 150 × 10⁶ km, determine the velocity of the earth.*

Solution :

Given data : $\theta = 2\pi$, $t = 365.24$ days, $r = 150 \times 10^6$ km.

To find : Velocity and acceleration.

$$s = r\theta = 150 \times 10^6 \times 2\pi = 942.48 \times 10^6 \text{ km}$$

$$v = \frac{s}{t} = \frac{942.48 \times 10^6}{365.24 \times 24}$$

$$\therefore \quad v = 107.52 \text{ km/hr}$$

Velocity of the earth is 107.52 km/hr. **... Ans.**

Example 4.52 : *A projectile is projected with an initial velocity of 40 m/s, at an angle of 60° with horizontal. Determine radius of curvature of path of particle at an horizontal distance of 40 m from point A and also at the highest point of the trajectory.*

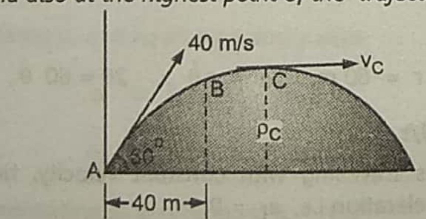


Fig. 4.43 (a)

Solution :

Given data : $u = 40$ m/s, $\theta = 60°$

At B, $x = 40$ m

Horizontal component of velocity,

$$v_x = 40 \cos 60 = 20 \text{ m/s}$$

Time required to travel,

$$x = 40 \text{ m}$$

$$40 = 40 \cos 60\ t$$

$$t = 2\text{s}$$

Vertical component of velocity at B,

$$v_y = u_y - gt$$

$$v_y = 40 \sin 60 - 9.81 \times 2$$

$$= 15.02 \text{ m/s}$$

$$v_B = 25 \text{ m/s}, \ \theta = 36.907°$$

$$a_n = 9.81 \cos 36.907$$

$$= 7.8442\ m/s^2$$

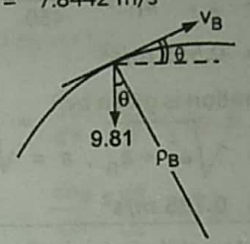


Fig. 4.43 (b)

$$a_n = \frac{v_B^2}{\rho_B}$$

$$\therefore \quad \rho_B = \frac{25^2}{7.8442} \quad \therefore \quad \rho_B = 79.68 \text{ m} \quad \text{... Ans.}$$

$$\rho_C = \frac{20^2}{9.81} \quad \therefore \quad \rho_C = 40.77 \text{ m} \quad \text{... Ans.}$$

Example 4.53 : *An outdoor track is 126 m in diameter. A runner increases her speed at a constant rate from 4.2 to 7.2 m/s over a distance of 28.5 m. Determine the total acceleration of the runner 2s after she began to increase her speed.*

Solution :

Tangential component of acceleration is given by using equation of kinematics, $v^2 = u^2 + 2as$

$$\therefore \quad 7.5^2 = 4.2^2 + 2 \times a_t \times 28.5 \quad \therefore \quad a = 0.6\ m/s^2$$

The velocity at t = 2 s, using $v = u + at$

$$v = 4.2 + 0.6 \times 2 \quad \therefore \quad v = 5.4 \text{ m/s}, \quad \rho = \frac{126}{2},$$

$$\rho = 63 \text{ m}$$

$$a_n = \frac{v^2}{\rho} \quad \therefore \quad a_n = \frac{5.4^2}{63} \quad \therefore \quad a_n = 0.463\ m/s^2$$

$$a = \sqrt{a_n^2 + a_t^2} = \sqrt{0.6^2 + 0.463^2}$$

$$a = \mathbf{0.758\ m/s^2} \quad \text{... Ans.}$$

Example 4.54 : *The driver of an automobile decreases her speed at a constant rate from 72 kmph to 48 kmph over a distance of 230 m along a curve of 460 m radius of curvature. Determine the magnitude of the total acceleration of the automobile after the automobile has travelled 150 m along the curve.*

Solution :

$u = 72$ kmph $= 20$ m/s, $v = 48$ kmph $= 13.33$ m/s, $s = 230$ m

$$v^2 = u^2 + 2as \quad \therefore \quad 13.33^2 = 20^2 + 2 \times a \times 230$$

$$\therefore \quad a_t = -0.483 \text{ m/s}^2$$

Velocity of automobile when it travels 150 m along the curve,

$$v^2 = 20^2 - 2 \times 0.483 \times 150$$

$$\therefore \quad \mathbf{v = 15.972 \text{ m/s}} \quad \text{... Ans.}$$

Normal component of acceleration is given by,

$$a_n = \frac{v^2}{\rho}, \quad a_n = \frac{15.972^2}{460}$$

$$\therefore \quad \mathbf{a_n = 0.555 \text{ m/s}^2} \quad \text{... Ans.}$$

Hence total acceleration is given by,

$$a = \sqrt{a_t^2 + a_n^2}, \quad a = \sqrt{0.483^2 + 0.555^2}$$

$$\therefore \quad \mathbf{a = 0.735 \text{ m/s}^2} \quad \text{... Ans.}$$

Example 4.55 : *The automobile is originally at rest at s = 0. If it then starts to increase his speed at $\dot{v} = 0.02t^2$ m/s², where t is in seconds, determine the magnitudes of its velocity and acceleration at s = 180 m.*

Solution : $\dot{v} = a = 0.02\, t^2$

At $t = 0$, $v = 0$, $s = 0$

$$\frac{dv}{dt} = 0.02\, t^2 \quad \text{... (1)}$$

$$\int_0^v dv = \int_0^t 0.02\, t^2\, dt$$

$$v = \frac{0.02\, t^3}{3} \quad \text{... (2)}$$

$$v = \frac{ds}{dt}$$

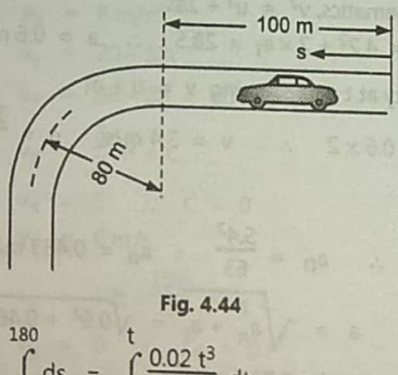


Fig. 4.44

$$\therefore \quad \int_0^{180} ds = \int_0^t \frac{0.02\, t^3}{3}\, dt$$

$$180 = \frac{0.02\, t^4}{12} \quad \therefore \quad t = 18.13 \text{ s}$$

From equation (2),

$$v = 39.72 \text{ m/s}$$

From equation (1),

$$a_t = 6.574 \text{ m/s}^2$$

Normal component of acceleration is given by

$$a_n = \frac{v^2}{\rho}, \quad a_n = \frac{39.72^2}{80}$$

$$\therefore \quad a_n = 19.72 \text{ m/s}^2$$

$$\text{Total acceleration, } a = \sqrt{6.574^2 + 18.72^2}$$

$$a = 20.79 \text{ m/s}^2, \quad \theta = \tan^{-1} \frac{6.574}{19.72}$$

$$= \mathbf{18.4°} \quad \text{... Ans.}$$

Example 4.56 : *A boy sits on a merry-go-round so that he is always located at r = 3 m from the centre of rotation. The merry-go-round is originally at rest and due to rotation, the boy's speed increased at constant rate of 0.6 m/s². Determine the time needed for his acceleration to become 1.2 m/s².*

Solution :

In curvilinear motion, magnitude of acceleration is given by,

$$a = \sqrt{a_t^2 + a_n^2},$$

where $a_t = 0.6$ m/s² which is constant and $a_n = \frac{v^2}{\rho}$, where v is the velocity of boy at time t. Initial velocity of boy is zero.

$$\therefore \quad v = u + at \quad \therefore \quad v = 0 + a_t\, t$$

$$u = 0.6\, t, \quad a_n = \frac{(0.6\, t)^2}{3} \quad \therefore \quad a_n = 0.12\, t^2$$

$$a = \sqrt{a_t^2 + a_n^2}$$

$$\therefore \quad 1.2 = \sqrt{0.36 + (0.12\, t^2)^2}$$

$$1.44 = 0.36 + 0.0144\, t^4 \quad \therefore \quad 1.08 = 0.0144\, t^4$$

$$t^4 = 75, \quad \mathbf{t = 2.94 \text{ s}} \quad \text{... Ans.}$$

Velocity at time $t = 2.94$ s is $v = 0 + 0.6 \times 2.94$,

$$\mathbf{v = 1.77 \text{ m/s}} \quad \text{... Ans.}$$

Example 4.57 : *A truck is travelling along the horizontal circular curve of radius r = 60 m with a constant speed v = 20 m/s. (a) Determine the angular rate of rotation $\dot{\theta}$ of the radial line r and the magnitude of track acceleration and (b) A speed of 20 m/s which is increasing at 3 m/s². Determine the track radial and transverse components of acceleration.*

Solution :

(a) $v = 20$ m/s, $r = 60$ m, $v_\theta = r\dot{\theta}$ $\therefore$ $20 = 60\, \dot{\theta}$,

$$\dot{\theta} = \mathbf{0.33 \text{ rad/s}}$$

Since the truck is travelling with constant velocity, tangential component of acceleration i.e. $a_t = 0$.

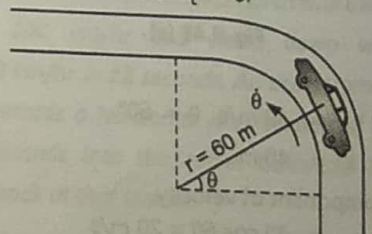


Fig. 4.45 (a)

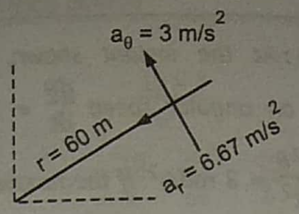


Fig. 4.45 (b)

Normal component of acceleration is given by

$$a_n = \frac{v^2}{\rho} \quad \therefore \quad a_n = \frac{20^2}{60} \quad \therefore \quad a_n = 6.67\ m/s^2$$

Track acceleration, a = 6.67 m/s² **... Ans.**

(b) $a_t = 3\ m/s^2$, $a_n = 6.67\ m/s^2$, $a_\theta = 3\ m/s^2$, $a_r = -6.67\ m/s^2$.

4.13 CYLINDRICAL COMPONENTS

In some engineering problems, it is convenient to describe the path of motion of a particle in terms of cylindrical components. In case of plane motion, the polar co-ordinates r and θ are used. In case of space, r, θ and z are used.

4.13.1 Polar Co-Ordinates

In certain problems of plane motion, the position of particle P is defined by its polar co-ordinates r, θ as shown in Fig. 4.46 (a).

In this type of motion, it is convenient to resolve the velocity and acceleration along OP and perpendicular to OP. These components are known as radial and transverse components.

Let, $\bar{e}_r$ = Unit vector along OP

$\bar{e}_\theta$ = Unit vector along the direction perpendicular to OP.

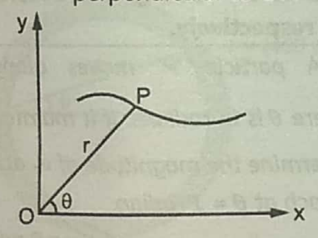


Fig. 4.46 (a)

These unit vectors are shown in Fig. 4.46 (b).

Resolving $\bar{e}_r$ and $\bar{e}_\theta$ along x and y axes,

$$\bar{e}_r = (\cos\theta\, i + \sin\theta\, j)$$

$$\bar{e}_\theta = (-\sin\theta\, i + \cos\theta\, j)$$

Differentiating $\bar{e}_r$ and $\bar{e}_\theta$ w.r.t. θ,

$$\therefore \quad \frac{d\bar{e}_r}{d\theta} = (-\sin\theta\, i + \cos\theta\, j) = \bar{e}_\theta$$

$$\frac{d\bar{e}_\theta}{d\theta} = (-\cos\theta\, i - \sin\theta\, j) = -\bar{e}_r$$

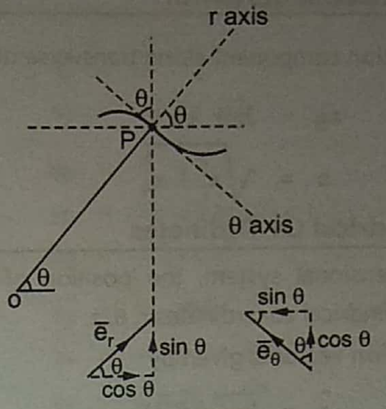


Fig. 4.46 (b)

Differentiating $\bar{e}_r$ and $\bar{e}_\theta$ w.r.t. t,

$$\frac{d\bar{e}_r}{dt} = \frac{d\bar{e}_r}{d\theta} \cdot \frac{d\theta}{dt} = \bar{e}_\theta \cdot \dot{\theta}$$

$$\frac{d\bar{e}_\theta}{dt} = \frac{d\bar{e}_\theta}{d\theta} \cdot \frac{d\theta}{dt} = -\bar{e}_r \cdot \dot{\theta}$$

We have,

Position : $\bar{r} = r \times \bar{e}_r$

Velocity : $$\bar{v} = \frac{d\bar{r}}{dt} = \frac{dr}{dt} \times \bar{e}_r + r \cdot \frac{d\bar{e}_r}{dt}$$

$$= \dot{r}\,\bar{e}_r + r\,\dot{\theta}\,\bar{e}_\theta$$

$$\therefore \quad \bar{v} = v_r\,\bar{e}_r + v_\theta\,\bar{e}_\theta$$

$$\therefore \quad v = v_r \quad \text{and} \quad v_\theta = r\,\dot{\theta}$$

∴ Velocity component along radial direction $= \frac{dr}{dt}$

$$= v_r = \dot{r}$$

Velocity component along transverse direction $= v_\theta = r\dot{\theta}$

$$\therefore \quad v = \sqrt{v_r^2 + v_\theta^2}$$

Acceleration : $$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d}{dt}\left[\dot{r}\,\bar{e}_r + r\,\dot{\theta}\,\bar{e}_\theta\right]$$

$$= \frac{d\dot{r}}{dt} \cdot \bar{e}_r + \dot{r} \cdot \frac{d\bar{e}_r}{dt} + \frac{dr}{dt} \cdot \dot{\theta}\,\bar{e}_\theta + r\frac{d\dot{\theta}}{dt}\,\bar{e}_\theta + r\,\dot{\theta}\,\frac{d\bar{e}_\theta}{dt}$$

$$= \ddot{r}\,\bar{e}_r + \dot{r}\,\bar{e}_\theta\,\dot{\theta} + \dot{r}\,\dot{\theta}\,\bar{e}_\theta + r\,\ddot{\theta}\,\bar{e}_\theta + r\,\dot{\theta}\,(-\bar{e}_r \times \dot{\theta})$$

$$\bar{a} = (\ddot{r} - r\,\dot{\theta}^2)\,\bar{e}_r + (2\dot{r}\,\dot{\theta} + r\,\ddot{\theta})\,\bar{e}_\theta$$

∴ Acceleration component along radial direction, $a_r = \ddot{r} - r\,\dot{\theta}^2$

Acceleration component along transverse direction,

$$a_\theta = 2\dot{r}\dot{\theta} + r\ddot{\theta}$$

$$\therefore \quad a = \sqrt{a_r^2 + a_\theta^2}$$

4.13.2 Cylindrical Co-Ordinates

In three dimensional system, the position of particle may be defined by cylindrical co-ordinates r, θ, z.

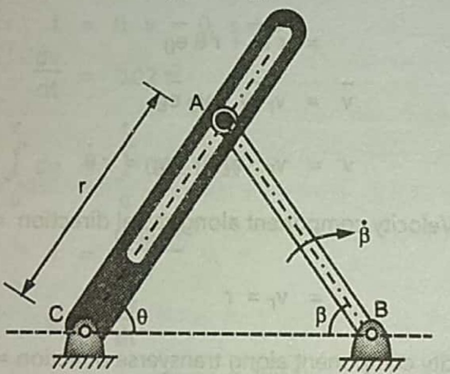
The position vector is given by

$$\bar{r} = (r\bar{e}_r + z\bar{k})$$

$$\bar{v} = \frac{d\bar{r}}{dt} = \left[\dot{r}\,\bar{e}_r + r\,\dot{\theta}\,\bar{e}_\theta\right] + \dot{z}\,\bar{k}$$

$$\bar{a} = \frac{d\bar{v}}{dt} = (\ddot{r} - r\dot{\theta}^2)\,\bar{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\,\bar{e}_\theta + \ddot{z}\,\bar{k}$$

***Example 4.58 :** A link AB rotates through a limited range of the angle β and its end A causes the slotted link AC to rotate. For the instant represented where β = 60° and $\frac{d\beta}{dt}$ = 0.6 rad/s, determine the corresponding values of $\frac{dr}{dt}$ and $\frac{d\theta}{dt}$.*



CA = BA = 150 mm

Fig. 4.47

Solution :

Given data : β = 60°, $\frac{d\beta}{dt}$ = 0.6 rad/s

To find : $\frac{dr}{dt}$ and $\frac{d\theta}{dt}$.

$$v = r\omega$$

$$= 150 \times 0.6 = 90 \text{ mm/s}$$

$$\therefore \quad v_r = \frac{dr}{dt} = v \sin 60$$

$$= 90 \sin 60 = 77.94 \text{ mm/s}$$

$$v_\theta = \frac{d\theta}{dt} = v \cos 60$$

$$= 90 \cos 60 = 30 \text{ mm/s}$$

The values of $\frac{dr}{dt}$ and $\frac{d\theta}{dt}$ is 77.94 mm/s and 30 mm/s respectively. ... Ans.

***Example 4.59 :** At the instant shown, the water sprinkler is rotating with an angular speed $\frac{d\theta}{dt}$ = 2 rad/s and an angular acceleration $\frac{d^2\theta}{dt^2}$ = 3 rad/s². If the nozzle lies in the vertical plane and water is flowing through it at a constant rate of 3 m/s, determine the magnitudes of the velocity and acceleration of a water particle as it leaves the open end A.*

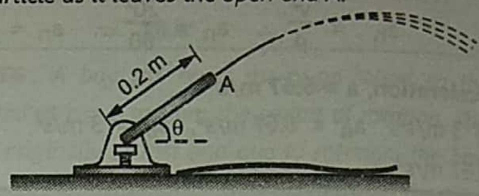


Fig. 4.48

Solution :

Given data : $\dot{\theta}$ = 2 rad/s, $\ddot{\theta}$ = 3 rad/s², r = 0.2 m, $\dot{r}$ = 3 m/s, $\ddot{r}$ = 0

To find : Magnitude of velocity and acceleration.

$$v = v_r e_r + v_\theta e_\theta = v_r e_r + r\dot{\theta}\, e_\theta$$

$$= 3e_r + 0.2 \times 2e_\theta = 3e_r + 0.4\, e_\theta$$

$$\therefore \quad v = 3.03 \text{ m/s}$$

$$a = a_r e_r + a_\theta e_\theta$$

$$= (\ddot{r} - r\dot{\theta}^2)\, e_r + (r\ddot{\theta} + 2\,\dot{r}\dot{\theta})\, e_\theta$$

$$= [0 - 0.2 \times (2)^2]\, e_r + (0.2 \times 3 + 2 \times 3 \times 2)$$

$$e_\theta = -0.8\, e_r + 12.6\, e_\theta$$

$$\therefore \quad a = 12.63 \text{ m/s}^2$$

∴ **Magnitude of velocity and acceleration is 3.03 m/s and 12.63 m/s² respectively. ... Ans.**

***Example 4.60 :** A particle 'P' moves along the spiral path $r = \left(\frac{10}{\theta}\right)$ m, where θ is in radians. If it maintains a constant speed of v = 6 m/s, determine the magnitude of v_r and v_θ as a function of θ and evaluate each at θ = 1 radian.*

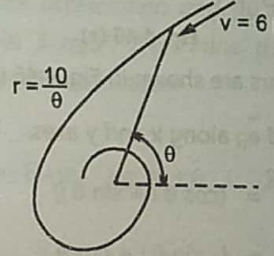


Fig. 4.49

Solution :

Given data : As shown in Fig. 4.49.

To find : v_r and v_θ.

$$r = \frac{10}{\theta}$$

$$\therefore \quad \frac{dr}{dt} = \frac{-10 \times \frac{d\theta}{dt}}{\theta^2}$$

$$v_r = \frac{dr}{dt} = \frac{-10 \times \dot{\theta}}{\theta^2}$$

$$v_\theta = r\dot{\theta} = \frac{10}{\theta} \times \frac{d\theta}{dt} = \frac{10}{\theta} \times \dot{\theta}$$

$$v = \sqrt{v_r^2 + v_\theta^2}$$

$$v^2 = v_r^2 + v_\theta^2 = \left(\frac{-10 \times \dot{\theta}}{\theta^2}\right)^2 + \left(\frac{10}{\theta} \times \dot{\theta}\right)^2$$

$$= \frac{100 \times \dot{\theta}^2}{\theta^4} + \frac{100 \times \dot{\theta}^2}{\theta^2}$$

At $\theta = 1$ radian, $v = 6$ m/s

$$\therefore \quad 6^2 = 100\,\dot{\theta}^2 + 100\,\dot{\theta}^2$$

$$\therefore \quad 36 = 200\,\dot{\theta}^2$$

$$\therefore \quad \dot{\theta} = 0.424 \text{ rad/s}$$

$$\therefore \quad v_r = \frac{-10 \times \dot{\theta}}{\theta^2} = \frac{-10 \times 0.424}{1^2} = -4.24 \text{ m/s}$$

$$v_\theta = \frac{10}{\theta} \times \dot{\theta} = \frac{10 \times 0.424}{1} = 4.24 \text{ m/s}$$

Values of v_r and v_θ at $\theta = 1$ is – 4.24 m/s and 4.24 m/s respectively. **... Ans.**

Example 4.61 : *The car travels around the circular track such that its transverse component is $\theta = (0.006\,t^2)$ rad, where t is in seconds. Determine the car's radial and transverse components of velocity and acceleration at the instant t = 4s.*

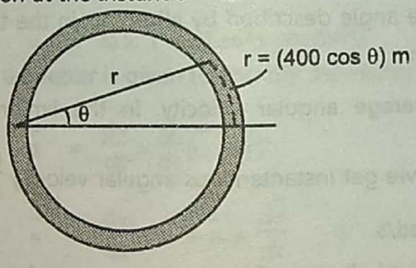


Fig. 4.50

Solution :

Given data : $\theta = (0.006\,t^2)$ rad at t = 4s, $\theta = 5.5°$

$$\dot{\theta} = 2 \times 0.006\,t, \qquad \dot{\theta} = 0.048 \text{ rad/s}$$

$$\ddot{\theta} = 2 \times 0.006, \qquad \ddot{\theta} = 0.012 \text{ rad/s}^2$$

$$r = (400 \cos\theta) \text{ at } \theta = 5.5°, \; r = 398.16 \text{ m}$$

$$\dot{r} = (-400 \sin\theta)\,\dot{\theta}, \qquad \dot{r} = -1.84 \text{ m/s}$$

$$\ddot{r} = (-400 \cos\theta)\,\dot{\theta}\,\dot{\theta} + (-400 \sin\theta)\,\ddot{\theta}$$

$$\ddot{r} = -0.917 + (-4.6), \qquad \ddot{r} = -1.367 \text{ m/s}^2$$

$$v_\theta = r\dot{\theta} \qquad \therefore \; v_\theta = 398.16 \times 0.048$$

$$\mathbf{v_\theta = 19.11 \text{ m/s}}$$

$$v_r = \dot{r}$$

$$\mathbf{v_r = -1.84 \text{ m/s}}$$

$$a_\theta = r\ddot{\theta} + 2\,\dot{r}\dot{\theta}$$

$$a_\theta = 398.16 \times 0.012 + 2\,(-1.84) \times 0.048,$$

$$\mathbf{a_\theta = 4.6 \text{ m/s}^2}$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$a_r = -1.367 - 398.17 \times 0.048^2,$$

$$\mathbf{a_r = -2.28 \text{ m/s}^2}$$

Example 4.62 : *For a short distance the train travels along a track having the shape of spiral, $r = \left(\frac{1000}{\theta}\right)$, where θ is in radian. If the angular rate is constant, $\dot{\theta} = 0.2$ rad/s, determine the radial and transverse components of its velocity and acceleration when $\theta = \frac{9\pi}{4}$ rad.*

Solution :

$$\theta = \frac{9\pi}{4} = 7.0686, \quad \dot{\theta} = 0.2 \text{ rad/s}, \quad \ddot{\theta} = 0$$

$$r = \frac{1000}{\theta} \text{ when } \theta = 7.0686 \text{ rad},$$

$$r = 141.47 \text{ m}$$

$$\dot{r} = -1000\,\theta^{-2} \cdot \dot{\theta}, \quad \dot{r} = -\frac{1000\,\dot{\theta}}{\theta^2},$$

at $\theta = 7.0686$ rad, $\dot{r} = -4$ m/s^2

$$\ddot{r} = -1000\,[-2\theta^{-3}\,\dot{\theta}\,\dot{\theta} + \theta^{-2}\,\ddot{\theta}],$$

$$\ddot{r} = 1000\left[\frac{2\dot{\theta}^2}{\theta^3} - \frac{\ddot{\theta}}{\theta^2}\right], \qquad \ddot{r} = 0.2265 \text{ m/s}^2$$

$$v_r = \dot{r} \qquad \therefore \; \mathbf{v_r = -4 \text{ m/s}^2}$$

$$v_\theta = r\dot{\theta}, \; v_\theta = 141.47 \times 0.2,$$

$$\mathbf{v_\theta = 28.29 \text{ m/s}^2} \qquad \textbf{... Ans.}$$

$$a_r = \ddot{r} - r\,\dot{\theta}^2, \; a_r = 0.2265 - 141.47 \times 0.2^2$$

$$\therefore \quad \mathbf{a_r = -5.43 \text{ m/s}^2}$$

$$a_\theta = r\ddot{\theta} + 2\,\dot{r}\dot{\theta}, \; a_\theta = 0 + 2 \times (-4) \times 0.2$$

$$\therefore \quad \mathbf{a_\theta = -1.6 \text{ m/s}^2} \qquad \textbf{... Ans.}$$

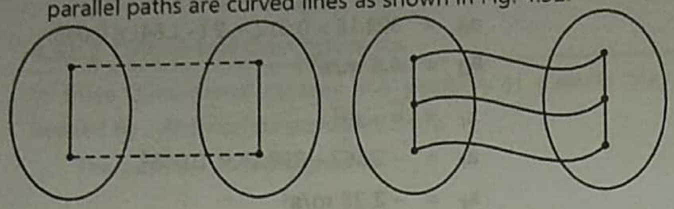
C – KINEMATICS RIGID BODIES

4.14 INTRODUCTION

The various types of the rigid body motion may be classified as below :

1. **Translation :** A motion is said to be a translation if any straight line inside the body keeps the same direction during the motion. In a translation all the particles forming the rigid

body move along parallel paths. In translation all points of the body have the same velocity and the same acceleration at any given instant. The translation may be rectilinear if the parallel paths are straight lines or may be curvilinear if these parallel paths are curved lines as shown in Fig. 4.51.



(a) Rectilinear **(b) Curvilinear**

Fig. 4.51 : Translation

2. **Rotation about a Fixed Axis :** In this motion the particles forming the rigid body move in parallel planes along circles centred on the same fixed axis called as axis of rotation. If the axis of rotation intersects the rigid body, the particles located on the axis have zero velocity and zero acceleration. Rotation about the fixed axis is shown in Fig. 4.52.

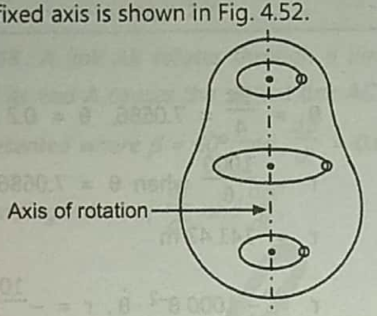


Fig. 4.52 : Rotation about the fixed axis.

3. **General Plane Motion :** Any plane motion, which is neither a translation nor a rotation, is referred to as a general plane motion. The motion of a rolling wheel, as shown in Fig. 4.53 (a), is an example of general plane motion.

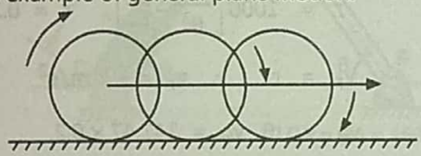


Fig. 4.53 (a) : A general plane motion

4. **Motion about a Fixed Point :** The three dimensional motion of a rigid body, such as a top attached at a fixed point O, is an example of the motion about a fixed point. Fig. 4.53 (b) shows such a motion.

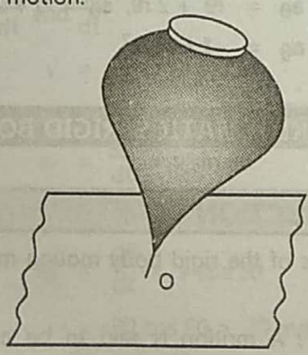


Fig. 4.53 (b) : Motion about a fixed point

5. **General Motion :** Any motion of a rigid body, which does not fall in any of the catagories mentioned in (1) to (4) above, is referred to as general motion.

4.15 ROTATION ABOUT A FIXED AXIS

Consider a body in xy plane rotating about a fixed z-axis passing through O as shown in Fig. 4.54. Any point P in the body at r from O will come at P' after rotating about z-axis from O in time δt describing an angle $\delta\theta$, PP' may be assumed perpendicular to OP and will have a length equal to $r\,\delta\theta$.

The angle described by all points, such as P in the rigid body in a given duration of time, is same. However, the length such as PP', which is a function of r, will differ in each case depending upon the distance of the point from O. The angular quantities such as angle described, angular velocity, and angular acceleration will have the same magnitude for all points of the rigid body and all angular quantities will be directed along z-axis if the rigid body is in xy plane.

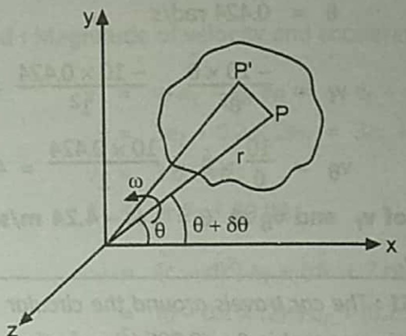


Fig. 4.54

Note :

- $\delta\theta$ is the angle described by all points in the time δt. Its unit is radian.
- $\frac{\delta\theta}{\delta t}$ is average angular velocity. In the limiting case when $\delta t \to 0$, we get instantaneous angular velocity $\frac{d\theta}{dt} = \dot{\theta} = \omega$. Its unit is rad/s.
- The rate of change of instantaneous angular velocity is the instantaneous angular acceleration, α.

Thus $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \ddot{\theta} = \frac{d\omega}{d\theta}\frac{d\theta}{dt} = \omega\frac{d\omega}{d\theta}$

Its unit is rad/s^2.

Equations Governing the Rotation :

- **Uniform Rotation :** In uniform rotation, the angular velocity ω remains constant and therefore there is no angular acceleration. θ, the angle described in time t, is found by the equation

$$\theta = \omega t \quad \text{... (4.13)}$$

This equation is similar to the equation which is for a particle undergoing a uniform rectilinear motion.

- **Uniformly Accelerated Rotation :** In this case, the angular acceleration α is constant. It is $-\alpha$ in case of uniformly decelerated rotation.

Following equations are applicable :

$$\omega = \omega_0 + \alpha t \quad \text{... (4.14)}$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2 \quad \text{... (4.15)}$$

and

$$\omega^2 = \omega_0^2 + 2\alpha\theta \quad \text{... (4.16)}$$

$$\theta_{nth} = \omega_0 + \alpha\left(n - \frac{1}{2}\right) \quad \text{... (4.17)}$$

These equations are similar to equations derived for a particle undergoing a rectilinear motion with uniform acceleration.

- **Relation between Rectilinear and Rotational Quantities :**

1. **Scalars :** Referring to Fig. 4.54 again, when P describes an angle $\delta\theta$ and comes to P' in time δt, the point P moves a linear distance

$$PP' = r\,\delta\theta \text{ perpendicular to r.}$$

If

$$PP' = \delta S,$$

following scalar relation can be written :

$$\delta S = r\,\delta\theta \quad \text{... (4.18)}$$

$$v = \frac{dS}{dt} = r\frac{d\theta}{dt} = r\omega \quad \text{... (4.19)}$$

$$a = \frac{dv}{dt} = \frac{d}{dt}(r\omega) = r\frac{d\omega}{dt} = r\alpha \quad \text{... (4.20)}$$

The equations given in (4.18), (4.19) and (4.20) relate different rectilinear quantities to the corresponding rotational quantities. It should be noted from the above equations that any rectilinear quantities such as δS, v, and a are equal to r times the corresponding rotational quantities such as θ, ω and α.

2. **Vectors :** We start with the basic relation :

$$\bar{v} = \bar{\omega} \times \bar{r} \text{ ... (Cross product)} \quad \text{... (4.21)}$$

A bar over the letter is given to identify the vector

$$\bar{a} = \frac{d\bar{v}}{dt} = \frac{d}{dt}(\bar{\omega} \times \bar{r})$$

$$\therefore \quad \bar{a} = \frac{d\bar{\omega}}{dt} \times \bar{r} + \bar{\omega} \times \frac{d\bar{r}}{dt}$$

$$\therefore \quad \bar{a} = \bar{\alpha} \times \bar{r} + \bar{\omega} \times \bar{v}$$

$$\therefore \quad \bar{a} = \bar{\alpha} \times \bar{r} + \bar{\omega} \times (\bar{\omega} \times \bar{r}) \quad \text{... (4.22)}$$

$\bar{\alpha} \times \bar{r}$ is the tangential component of $\bar{a}$ i.e. a_t and has a magnitude $r\alpha$ as $\bar{r}$ and $\bar{\alpha}$ are perpendicular to each other.

$\bar{\omega} \times (\bar{\omega} \times \bar{r})$ is the normal component of $\bar{a}$ i.e. a_n and has a magnitude $r\omega^2$ directed towards O. The expression for the acceleration is also written in the following form,

$$\bar{a} = (\bar{\alpha} \times \bar{r})\, e_t + \bar{\omega} \times (\bar{\omega} \times \bar{r})\, e_n \quad \text{... (4.23)}$$

where, e_t and e_n are unit vectors along the tangent and the normal respectively.

4.16 INSTANTANEOUS CENTRE OF ROTATION

The general plane motion is always the combination of translation and rotation. The general plane motion, however, can be proved consisting only of motion of rotation about a fixed axis normal to the plane of the body passing through a fixed point inside or outside the body. This point, about which the general plane motion is reduced to pure rotation, is called the centre of rotation. At every instant, the position of the centre of rotation changes and hence, it is called as instantaneous centre of rotation. (I. C. R.)

The locations of I. C. R. is called the centroid.

Instantaneous Centre of Rotation :

Many times, a body may be subjected to translation as well as rotation. Consider a link AB. Due to translation and rotation, let its position be changed to A'B'.

A has changed to A' and velocity as v_A'. B moves to B' having velocity v_B'. So I.C.R. is the intersection of the perpendicular to the velocities v_A' and v_B' as O'.

We know that velocity of any point in a circular motion is always tangent to the path or perpendicular to the radius.

where, v_A = Linear velocity of A (perpendicular to OA)

= Radius × Angular velocity of A

= $OA \times \omega_A$

v_B = Linear velocity of B (perpendicular to OB)

= Radius × Angular velocity of B

= $OB \times \omega_B$

$$\omega_A = \frac{v_A}{OA} \qquad \omega_B = \frac{v_B}{OB}$$

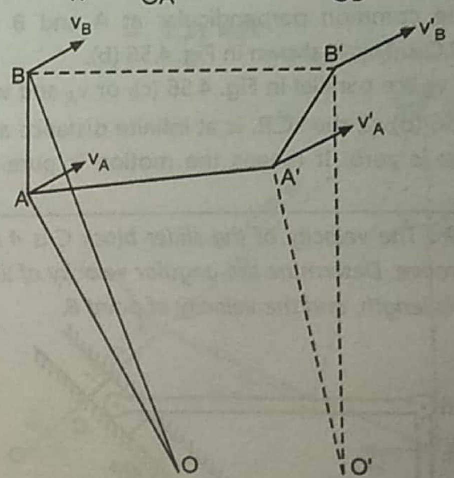


Fig. 4.55

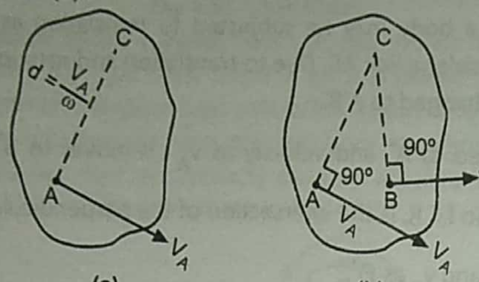
But as A and B are two points on the same rod, their angular velocities are same.

$$\omega_A = \omega_B$$

$$\frac{v_A}{OA} = \frac{v_B}{OB}$$

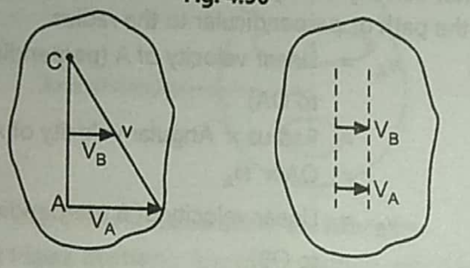
4.17 LOCATION OF I.C.R.

- If v_A, the linear velocity of the point A and ω, the angular velocity of the body are known, then C, the instantaneous centre of rotation is located on the line perpendicular to v_A at A at a distance d given by $\frac{v_A}{\omega}$ as shown in Fig. 4.56 (a).
- If v_A and v_B, the linear velocities at A and B respectively are known, we draw perpendiculars at A and B to the line of action of respective velocities. The intersection point of the two perpendiculars give I.C.R. at 'C'. This is shown in Fig. 4.56 (b).



(a) (b)

Fig. 4.56



(c) (c)

Fig. 4.56

- If points A and B are on the perpendicular to v_A and v_B, we join the extremities of these velocities v_A and v_B. This line meets the common perpendicular at A and B in point C which is I.C.R. This is shown in Fig. 4.56 (b).

  If v_A and v_B are parallel in Fig. 4.56 (c), or v_A and v_B are equal in Fig. 4.56 (d), C, the I.C.R. is at infinite distance and angular velocity ω is zero. It means the motion is pure translatory motion.

Example 4.63 : *The velocity of the slider block C is 4 m/s up the 45° inclined groove. Determine the angular velocity of links AB and BC, each of 1 m length, and the velocity of point B.*

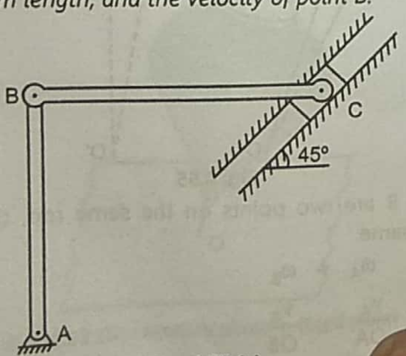


Fig. 4.57 (a)

Solution :

Given data : As shown in Fig. 4.57 (a).

To find : Angular velocities of rod and velocity at point B.

$$OC = 1.42 \text{ m}, OB = 1 \text{ m}$$

$$\omega_{ICR} = \frac{v_C}{OC} = \frac{4}{1.42} = 2.82 \text{ rad/s}$$

$$\omega_{BC} = \omega_{ICR} = \mathbf{2.82 \text{ rad/s}} \ (\circlearrowright) \quad \text{... Ans.}$$

$$v_B = OB \times \omega_{ICR}$$

$$= 1 \times 2.82 = \mathbf{2.82 \text{ m/s}} \ (\rightarrow) \quad \text{... Ans.}$$

$$\omega_{AB} = \frac{v_B}{AB} = \frac{2.82}{1}$$

$$= \mathbf{2.82 \text{ rad/s}} \ (\circlearrowright) \quad \text{... Ans.}$$

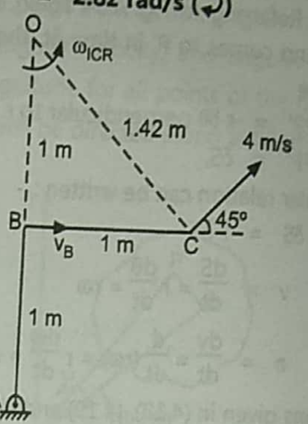


Fig. 4.57 (b)

Example 4.64 : *At the instant shown, the 350 mm long link BC is horizontal, whereas 500 mm long link AB is rotating counterclockwise with an angular velocity ω_{AB} = 4 rad/s. Determine the angular velocity of link BC and the velocity of collar at C.*

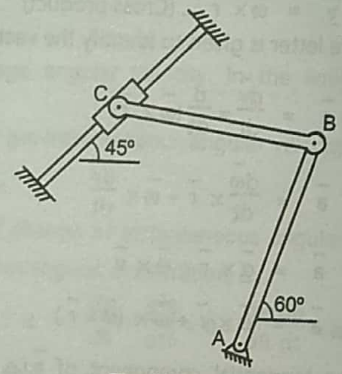


Fig. 4.58 (a)

Solution :

Given data : As shown in Fig. 4.58 (a).

To find : Angular velocity of BC and velocity of collar 'C'.

$$\omega_{ICR} = \frac{v_B}{OB} = \frac{2}{0.255}$$

$$= 7.84 \text{ rad/s}$$

$$\therefore \quad \omega_{BC} = \omega_{ICR} = \mathbf{7.84 \text{ rad/s}} \ (\circlearrowright) \quad \text{... Ans.}$$

$$\therefore \quad v_C = OC \times \omega_{ICR}$$

$$= 0.31 \times 7.84$$

$$= \mathbf{2.43 \text{ m/s}} \quad \text{... Ans.}$$

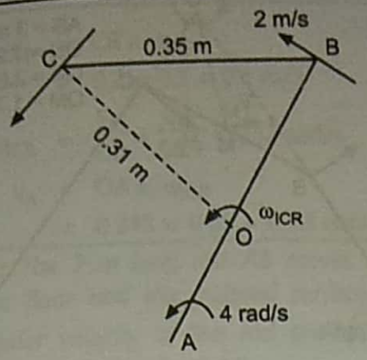


Fig. 4.58 (b)

***Example 4.65 :** If rod CD is rotating with an angular velocity ω_{DC} = 8 rad/s, counterclockwise, determine the angular velocity of AB.*

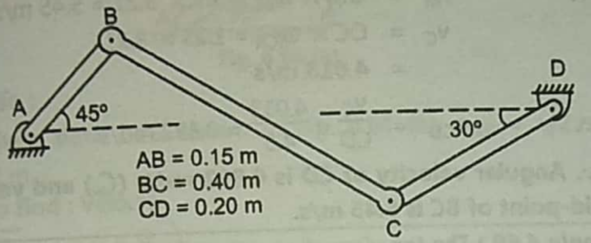


Fig. 4.59 (a)

Solution :

Given data : ω_{DC} = 8 rad/s, as shown in Fig. 4.59 (a).

To find : Angular velocity of AB.

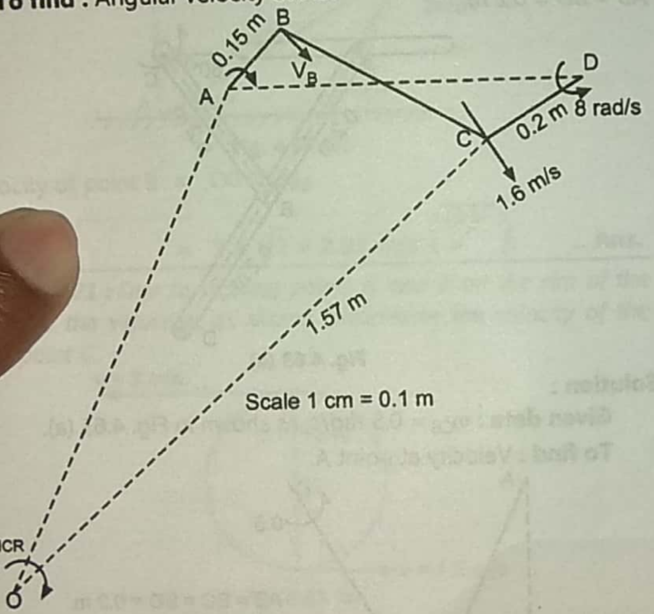


Fig. 4.59 (b)

$$v_C = 8 \times 0.2 = 1.6 \text{ m/s}$$

$$\omega_{ICR} = \frac{v_C}{OC} = \frac{1.6}{1.57} = 1.02 \text{ rad/s}$$

$$\therefore \quad v_B = \omega_{ICR} \times OB = 1.02 \times 1.43 = 1.46 \text{ m/s}$$

$$\therefore \quad \omega_{AB} = \frac{v_B}{AB} = \frac{1.46}{0.15} = 9.73 \text{ rad/s}$$

∴ **Angular velocity of rod AB is 9.73 rad/s (↻). ... Ans.**

***Example 4.66 :** The crank OA rotates at a constant rate of 80 rpm counterclockwise. At the instant shown, determine the angular velocity of rod BC and the velocity of midpoint of rod AB.*

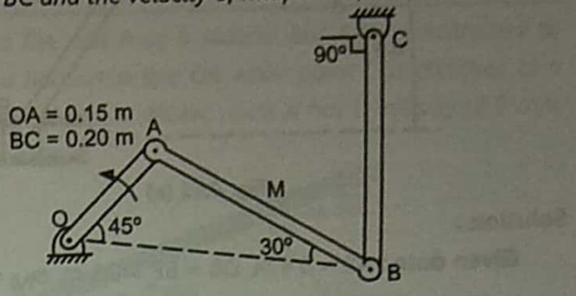


Fig. 4.60 (a)

Solution :

Given data : ω_{OA} = 80 rpm, as shown in Fig. 4.60 (a).

To find : Angular velocity of BC and velocity of midpoint of rod AB.

By sine rule,

$$\frac{OA}{\sin 30} = \frac{AB}{\sin 45}$$

$$\therefore \quad AB = 0.21 \text{ m}$$

$$\omega_{OA} = 80 \text{ rpm} = 8.38 \text{ rad/s}$$

$$v_A = OA \times \omega_{OA} = 0.15 \times 8.38 = 1.257 \text{ m/s}$$

$$\omega_{ICR} = \frac{v_A}{O'A} = \frac{1.257}{0.26} = 4.83 \text{ rad/s}$$

$$v_B = O'B \times \omega_{ICR} = 0.29 \times 4.83 = 1.4 \text{ m/s}$$

$$\therefore \quad \omega_{BC} = \frac{v_B}{BC} = \frac{1.4}{0.2} = \mathbf{7 \text{ rad/s}} \text{ (↻)} \quad \text{... Ans.}$$

$$\therefore \quad \mathbf{v_m = O'M \times \omega_{ICR} = 0.25 \times 4.83}$$

$$= \mathbf{1.21 \text{ m/s}} \quad \text{... Ans.}$$

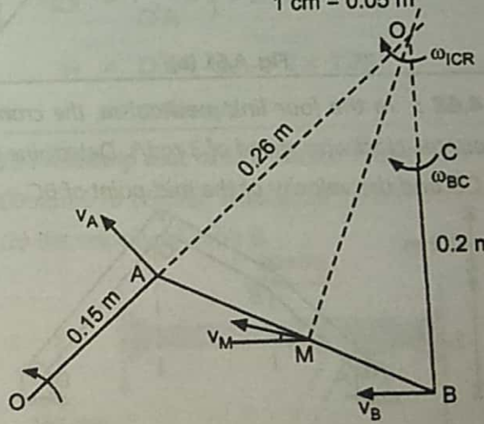


Fig. 4.60 (b)

***Example 4.67 :** At the position shown, the 0.4 m long crack AB has a clockwise angular velocity of 3 rad/s. For rod DBE, DB = BE = 0.5 m. E moves on a horizontal groove 0.4 m below A. Determine the velocity of point B.*

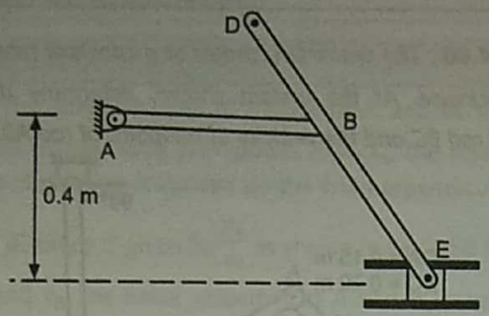


Fig. 4.61 (a)

Solution :

Given data : AB = 0.4 m, DB = BE = 0.5 m, ω_{AB} = 3 rad/s.

To find : Velocity at point B.

$$v_B = AB \times \omega_{AB} = 0.4 \times 3 = \mathbf{1.2\ m/s} \quad \text{... Ans.}$$

$$\omega_{ICR} = \frac{v_B}{OB} = \frac{1.2}{0.305} = 3.93 \text{ rad/s}$$

$$\mathbf{v_D = OD \times \omega_{ICR} = 0.73 \times 3.93}$$

$$= \mathbf{2.87\ m/s} \left(\angle 56.3^\circ\right) \quad \text{... Ans.}$$

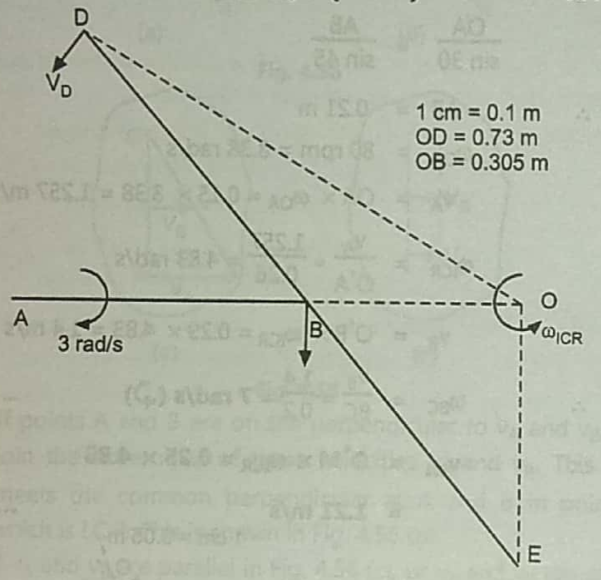


Fig. 4.61 (b)

Example 4.68 : *In the four link mechanism, the crank AB has a constant counter-clockwise speed of 3 rad/s. Determine the angular velocity of CD and the velocity of the mid-point of BC.*

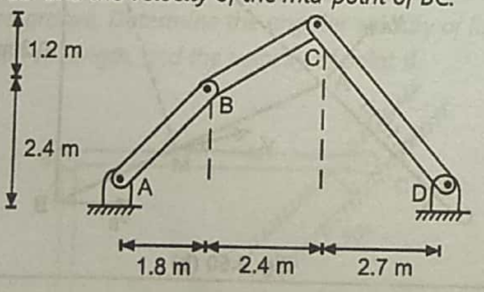


Fig. 4.62 (a)

Solution :

Given data : As shown in Fig. 4.62 (a).

To find : Angular velocity of CD, velocity of midpoint of BC.

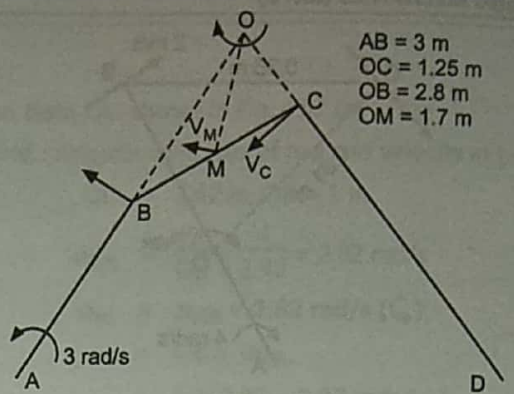


Fig. 4.62 (b)

$$v_B = AB \times \omega_{AB} = 3 \times 3 = 9 \text{ m/s}$$

$$\omega_{BC} = \omega_{ICR} = \frac{v_B}{OB} = \frac{9}{2.8} = 3.21 \text{ rad/s}$$

$$\therefore \quad v_M = OM \times \omega_{BC} = 1.7 \times 3.21 = 5.45 \text{ m/s}$$

$$v_C = OC \times \omega_{ICR} = 1.25 \times 3.21$$

$$= \mathbf{4.013\ m/s} \quad \text{... Ans.}$$

$$\therefore \quad \omega_{CD} = \frac{v_C}{CD} = \frac{4.013}{4.5} = 0.892 \text{ rad/s}$$

∴ Angular velocity of CD is 0.892 rad/s (↻) and velocity of mid-point of BC is 5.45 m/s. ... Ans.

Example 4.69 : *The top view of an automatic service window is as shown in Fig. 4.63 (a). During operation, a motor drives are connected link CB, which is 0.2 m long, with an angular velocity ω_{CB} = 0.5 rad/s. Determine the velocity at the instant shown of the end A, which moves along the slotted guide. For ABD, AB = BD = 0.2 m.*

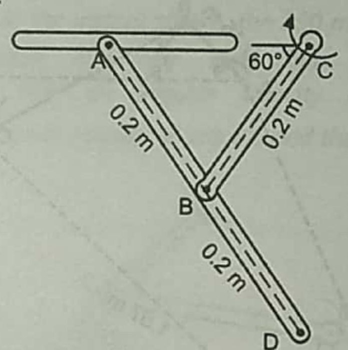


Fig. 4.63 (a)

Solution :

Given data : ω_{CB} = 0.5 rad/s, as shown in Fig. 4.63 (a).

To find : Velocity at point A.

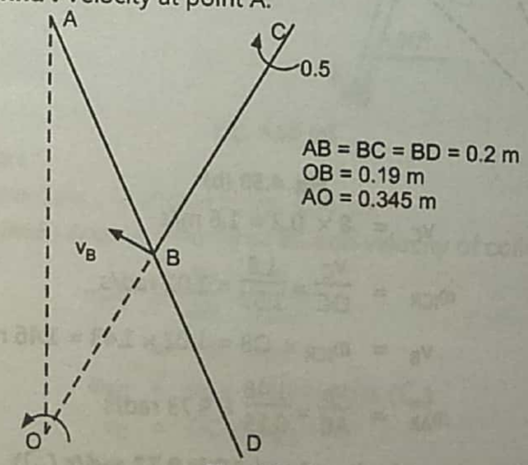


Fig. 4.63 (b)

$$v_B = CB \times \omega_{CB}$$
$$= 0.2 \times 0.5 = 0.1 \text{ m/s}$$
$$\omega_{ICR} = \frac{v_B}{OB} = \frac{0.1}{0.19} = 0.53 \text{ rad/s}$$
$$v_A = OA \times \omega_{ICR}$$
$$= 0.345 \times 0.53 = \mathbf{0.18 \text{ m/s } (\leftarrow)} \quad \text{... Ans.}$$

***Example 4.70 :** The 2 m long rod AB moves with its ends in contact with the floor and the inclined surface. At the instant shown, the angular velocity of the rod are $\sqrt{2}$ rad/s clockwise respectively. Find the velocity of point B.*

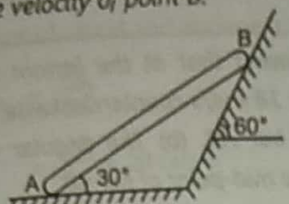


Fig. 4.64 (a)

Solution :

Given data : As shown in Fig. 4.64 (a), $\omega_{AB} = \sqrt{2}$ rad/s, AB = 2 m.

To find : Velocity.

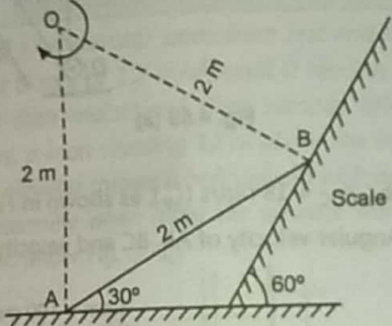


Fig. 4.64 (b)

$$\text{Velocity of point B} = OB \times \omega_{AB}$$
$$= 2 \times \sqrt{2} = \mathbf{2.83 \text{ m/s}} \quad (60°) \quad \text{... Ans.}$$

***Example 4.71 :** Due to slipping points A and B on the rim of the disk, have the velocities as shown. Determine the velocity of the centre point C.*

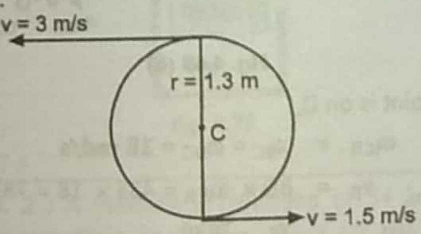


Fig. 4.65 (a)

Solution :

Given data : As shown in Fig. 4.65 (a).

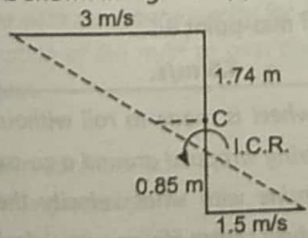
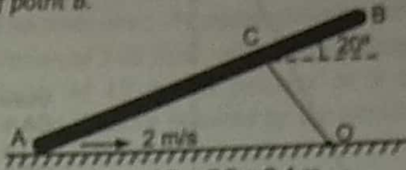


Fig. 4.65 (b)

$$\omega_{ICR} = \frac{3}{1.74} = 1.73 \text{ rad/s}$$
$$\therefore \quad v_C = \mathbf{0.75 \text{ m/s}} \quad \text{... Ans.}$$

***Example 4.72 :** The end A of a slender bar AB is constrained to move along the horizontal line OA while point C is attached to a crank OC. For the position shown, point A has a velocity of 2 m/s. Find velocity of point B.*



AC = 0.8 m, CB = OC = 0.4 m

Fig. 4.66 (a)

Solution :

Given data : v_A = 2 m/s.

To find : v_B.

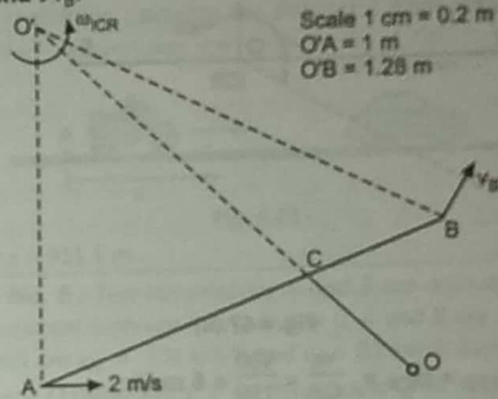


Fig. 4.66 (b)

$$\text{Using sine rule : } \frac{0.8}{\sin\theta} = \frac{0.4}{\sin 20}$$
$$\therefore \quad \theta = 43.16°$$
$$\omega_{ICR} = \frac{v_A}{O'A} = \frac{2}{1} = 2 \text{ rad/s}$$
$$v_B = O'B \times \omega_{ICR} = 2 \times 1.28$$
$$= \mathbf{2.56 \text{ m/s}} \quad \text{... Ans.}$$

***Example 4.73 :** Knowing that at the instant shown, the velocity of collar A is 900 mm/s to the left, determine (a) the angular velocity of rod ADB, (b) the velocity of point B.*

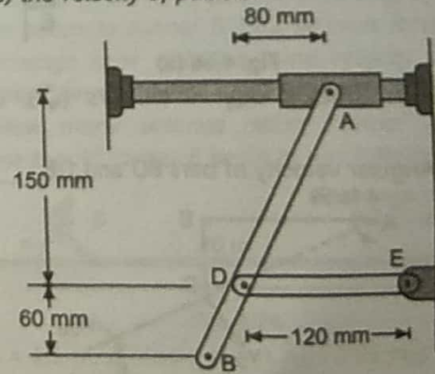


Fig. 4.67 (a)

Solution :

Given data : v_A = 900 mm/s, as shown in Fig. 4.67 (a).

To find : Angular velocity of ADB and velocity at point B.

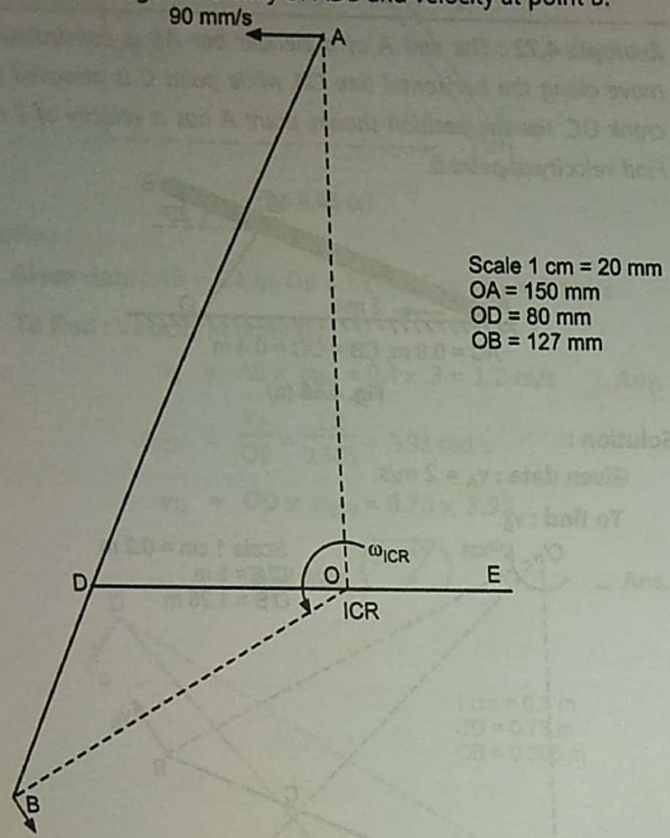


Fig. 4.67 (b)

$$\omega_{ABD} = \omega_{ICR} = \frac{v_A}{OA} = \frac{900}{150} = \mathbf{6\ rad/s} \quad \text{... Ans.}$$

$$v_B = OB \times \omega_{ICR} = 127 \times 6$$

$$= \mathbf{762\ mm/s} \left(62.2^\circ\right) \quad \text{... Ans.}$$

Example 4.74 : *In the position shown, bar AB has an angular velocity 4 rad/s, clockwise. Determine the angular velocity of bars BD and DE.*

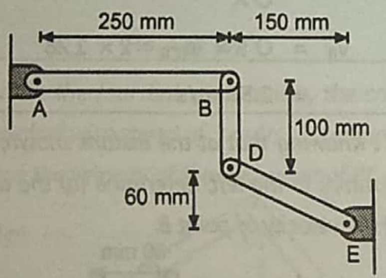


Fig. 4.68 (a)

Solution : Given data : ω_{AB} = 4 rad/s (↻), as shown in Fig. 4.68 (a).

To find : Angular velocity of bars BD and DE.

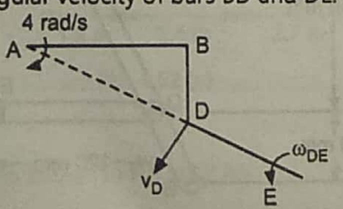


Fig. 4.68 (b)

$$\omega_{AB} = \omega_{ICR} = \omega_{BD} = 4 \text{ rad/s}$$

$$\therefore \quad v_B = 250 \times 4 = 1000 \text{ mm/s } (\downarrow)$$

$$v_D = AD \times \omega_{ICR} = 270 \times 4$$

$$= 1080 \text{ mm/s } \left(79^\circ\right)$$

$$v_D = DE \times \omega_{DE}$$

$$1080 = 161.55 \times \omega_{DE}$$

$$\therefore \quad \omega_{DE} = 6.68 \text{ rad/s}$$

$\therefore$ **Angular velocity of BD is 4 rad/s (↻) and of rod DE is 6.68 rad/s (↺).** **... Ans.**

Example 4.75 : *Knowing that at the instant shown the angular velocity of bar DC is 18 rad/s counterclockwise, determine : (a) The angular velocity of bar AB, (b) The angular velocity of bar BC, (c) The velocity of the mid-point of bar BC.*

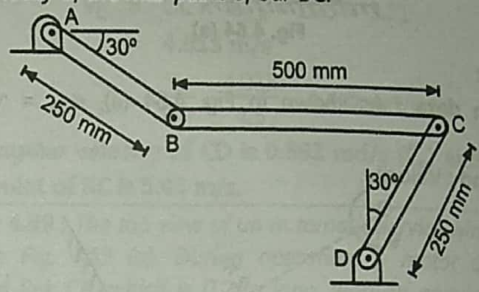


Fig. 4.69 (a)

Solution :

Given data : ω_{DC} = 18 rad/s (↺), as shown in Fig. 4.69 (a).

To find : Angular velocity of AB, BC and velocity of mid-point of bar BC.

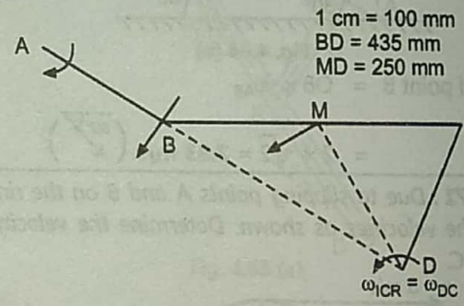


Fig. 4.69 (b)

As ICR point is on D,

$$\therefore \quad \omega_{ICR} = \omega_{BC} = \omega_{DC} = \mathbf{18\ rad/s} \quad \text{... Ans.}$$

$$v_B = BD \times \omega_{ICR} = 435 \times 18 = 7830 \text{ mm/s}$$

$$\omega_{AB} = \frac{v_B}{AB} = \frac{7830}{250} = \mathbf{31.32\ rad/s}\ (↻) \quad \text{... Ans.}$$

$$v_M = DM \times \omega_{ICR} = 250 \times 18 = 4500 \text{ mm/s}$$

$\therefore$ Velocity of mid-point BC

$$= \mathbf{4.5\ m/s.} \quad \text{... Ans.}$$

Example 4.76 : *A wheel is made to roll without slipping, towards right by pulling a string wrapped around a co-axial spool as shown in Fig. 4.70. Determine with what velocity the string should be pulled so that the wheel centre moves with velocity 10 m/s.*

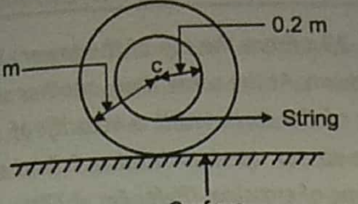


Fig. 4.70

Solution :

Given data : As shown in Fig. 4.70.

To find : Velocity of string.

The contact point of wheel and surface is I.C.R.

$$\therefore \quad \omega_{ICR} = \frac{v_C}{OC}$$

$$= \frac{10}{0.5} = 20 \text{ rad/s}$$

$$v_{string} = OS \times \omega_{ICR}$$

$$= 0.3 \times 20$$

$$= \mathbf{6\ m/s} \quad \text{... Ans.}$$

String should be pulled with 6 m/s. **... Ans.**

PROBLEMS FOR PRACTICE

Problem No. 1 : *An elevator starts from rest and moves upwards, accelerating at a rate of 1.2 m/s², until it reaches a speed of 7.8 m/s, which is then maintained. Two seconds after the elevator begins to move, a man standing 12 m above the initial position of the top of the elevator throws a ball upward with an initial velocity of 20 m/s. Determine when (after the elevator starts) the ball will hit the elevator. (Refer Fig. 4.71).*

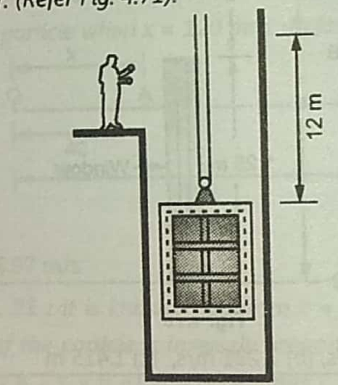


Fig. 4.71

Answer : 5.67 s

Problem No. 2 : *A motorist is travelling at 54 kmph when he observes that a traffic light 280 m ahead his car turns red. He knows that the signal is timed to stay red for 28 s. At the beginning, the motorist travelling with retardation, after attending minimum speed he will move with constant velocity for 8 s and then he will travel with acceleration of 0.6 m/s² to pass the signal at 54 kmph just as to green again.*

Determine :

(a) The uniform deceleration.

(b) The minimum velocity.

Answer : -1.105 m/s², $v_{min} = 7.22$ m/s

Problem No. 3 : *A body travels 40 m during its 6^th^ second and 38 m during its 10^th^ second. Determine :*

(a) Uniform acceleration and initial velocity.

(b) Distance travelled by the body from t = 0 to 8 seconds.

(c) Velocity of the body after 10 seconds.

Answer : u = 42.75 m/s, s = 326 m, v = 37.75 m/s

Problem No. 4 : *A freight elevator F moving upwards with constant velocity of 5 m/s, passes a passenger elevator P which is stopped. Three seconds later, the passenger elevator starts upwards with an acceleration of 1.25 m/s², when the passenger elevator has reached a velocity of 10 m/s it proceed at a constant speed. Determine time and distance required by the passenger elevator to overtake the freight elevator.*

Answer : t = 14 s, s = 70 m

Problem No. 5 : *A motorcycle patrolman starts from rest at A, two seconds after a car, speeding at the constant rate of 120 kmph, passed point A. If the patrolman accelerates at the rate of 6 m/s² until he reaches his maximum permissible speed of 150 km/h, which he maintains, calculate the distance s from point A to the point at which he overtakes the car.(Refer Fig. 4.72)*

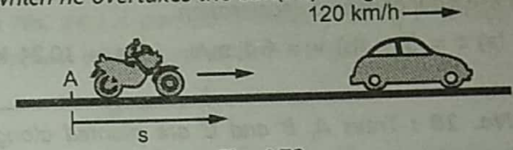


Fig. 4.72

Answer : s = 911.5 m

Problem No. 6 : *Two automobiles A and B are approaching each other in adjacent highway lanes. At t = 0, A and B are 1 km apart, their speeds are v_A = 108 km/h and v_B = 63 km/h and they are at points P and Q respectively. If A passes point Q 40 seconds after B was there and B passes point P 42 seconds after A was there, (a) determine uniform acceleration of A and B, and (b) when the vehicles pass each other, determine the speed of B at that time. (Refer Fig. 4.73).*

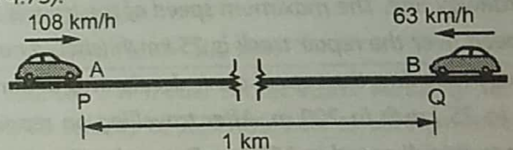


Fig. 4.73

Answer : a_A = 0.25 m/s² (retardation), a_B = 0.3 m/s², v = 85.5 km/h.

Problem No. 7 : *As a relay runner A enters the 20 m long exchange zone with a speed of 12.9 m/s, he begins to slow down. He hands the baton to runner B 1.82 seconds latter just as they leave the exchange zone with the same velocity. Determine the uniform deceleration of runner A and the uniform acceleration of runner B. How many seconds before runner A reaches the exchange zone should runner B begin to run ? (Refer Fig. 4.74).*

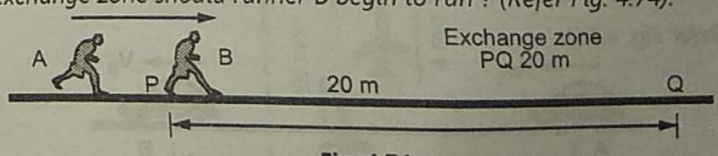


Fig. 4.74

Answer : a_A = 2.1 m/s² (decleration), a_B = 2.06 m/s², t = 2.59 s

Problem No. 8 : *Boxes are placed on a chute at uniform interval of time t_R and slide down the chute with uniform acceleration. Knowing that as any box B is released, the preceding box A has already travelled 6 m and that 1 second later they are 10 m apart, determine the value of t_R and the acceleration of the boxes. (Refer Fig. 4.75).*

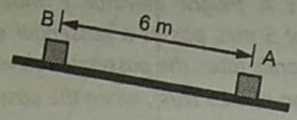


Fig. 4.75

Answer : a = 1.333 m/s^2, t_R = 3s

Problem No. 9 : *A commuter train travelling at 64 km/h is 4.8 km from a station. The train then decelerates so that its speed is 32 km/h when it is 0.8 km away from the station. If the train arrives at the station 7.5 minutes after beginning to decelerate and assuming constant deceleration, determine (a) the time required for the train to travel the first 4 km, (b) the speed of the train as it arrives at the station, (c) the final constant deceleration.*

Answer : (a) t = 5 s, (b) v = 6.4 m/s, (c) a = 10.24 km/h per minute

Problem No. 10 : *Trees A, B and C are planted alongside the straight road, at 50 m interval. A car traveling with uniform retardation passes tree A, takes 7 seconds to reach B, and takes 8 seconds to reach C. Calculate the velocities of the car at A, B and C. Also find the distance travelled by the car from C before coming to rest.*

Answer : s = 104.13 m, u_A = 7.56 m/s, v_B = 6.727 m/s, v_C = 5.775 m/s.

Problem No. 11 : *Track repairs are being taking place over 2 km length of railway line. The maximum speed of the train is 100 km/h and the speed over the repair track is 25 km/h (should not exceed). The train approaching the repair track decelerates uniformly from 100 km/h to 25 km/h in 200 m. After travelling on repair track, it accelerates to its full speed in 1600 m. Determine the time lost due to track repair.*

Answer : 255.11 s

Problem No. 12 : *Car A is travelling at a constant speed v_A. It approaches car B which is travelling in the same direction at constant speed of 72 km/h. The driver of car B notice car A when it is 60 m behind him and then accelerates at the constant rate of 0.75 m/s^2 to avoid being overtaken by car A. Knowing that the closest that A comes to B is 6 m, determine the speed v_A of car A. (Refer Fig. 4.76).*

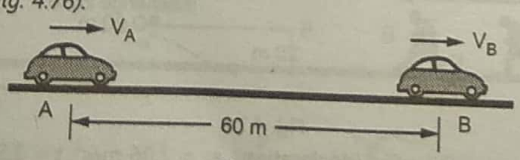


Fig. 4.76

Answer : v_A = 29 m/s

Problem No. 13 : *From the top of the tower, 100 m high, a stone was dropped down. At the same time, another stone was thrown up from the foot of the tower with a velocity of 30 m/s. When and where the two stones cross each other ? Find the velocity of each stone at the time of crossing. (Refer Fig. 4.77).*

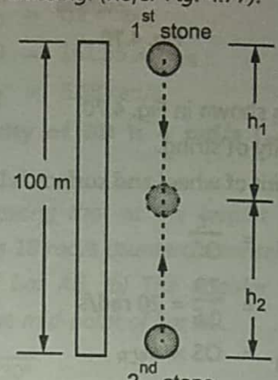


Fig. 4.77

Answer : h_1 = 54.5 m, h_2 = 45.5 m, 32.7 m/s (Downward direction), 2.7 m/s (Downward direction)

Problem No. 14 : *A stone is observed to fall past a 1.25 m high window in 0.2 s. Determine : (a) the average speed of the stone while it is in view, (b) the velocity of the stone when it reaches the bottom level of the window, and (c) the height above the top of the window from which it falls from rest. (Refer Fig. 4.78).*

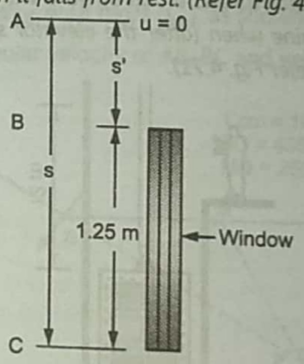


Fig. 4.78

Answer : (a) 6.25 m/s, (b) 7.231 m/s, (c) 1.415 m

Problem No. 15 : *In a flood relief area, a helicopter, going up with a constant velocity, drops first batch of food packets which takes 4 s to reach the ground. No sooner than this batch reaches the ground, second batch of food packets is released which takes 5 s to reach the ground. From what height the first batch of packets is released ? What is the velocity with which helicopter is moving ?*

Answer : s_1 = 43.16 m (↓), s_2 = 78.48 m (↓), v_1 = 8.83 m/s

Problem No. 16 : *A stone is released from top of cliff 196 m high. After two seconds, another stone is projected vertically downwards with such a velocity that both stones strike the ground at the same instant. Determine the velocity of projection of the second stone and the velocity with which each stone strikes the ground.*

Answer : v_{02} = 24.18 m/s (↓), v_1 = 62 m/s (↓), v_2 = 66.6 m/s (↓)

Problem No. 17 : *A rocket is launched vertically from rest and its thrust is programmed to give the rocket a constant upward acceleration of 6 m/s². If the fuel is exhausted in 20 s, after launching, calculate the maximum velocity attained by the rocket and the maximum altitude reached. Also determine the total time of flight and the velocity at which the rocket strikes the ground.*

Answer : v_{max} = 120 m/s, 1933.34 m, t = 52.09 s, v_D = – 194.8 m/s = 194.8 m/s (↓)

Problem No. 18 : *The acceleration of a particle is defined by the relation $a = -60\,x^{-1.5}$, where a and x are expressed in m/s² and m respectively. If the particle starts at x = 4m with no initial velocity, find the velocity of the particle when x = 2 m and the position when it has a velocity of 9 m/s.*

Answer : v = 7.05 m/s, x = 1.425 m

Problem No. 19 : *A projectile enters a resisting medium at x = 0 with an initial velocity v_0 = 270 m/s and travels 100 mm before coming to rest. Assuming that the velocity of the projectile is defined by the relation $v = v_0 - kx$, where v and x are expressed in m/s and m respectively, determine the initial acceleration of the projectile and the time required for the projectile to penetrate 97.5 mm into the resisting medium.*

Answer : a = 729 km/s² (deceleration), t = 1.366×10^{-3} s

Problem No. 20 : *A particle oscillates between the points A and B with an acceleration a = K (100 – x), where K is a constant. If the speed of the particle is 18 mm/s when x = 100 mm, determine the velocity of the particle when x = 120 mm. (Refer Fig. 4.79).*

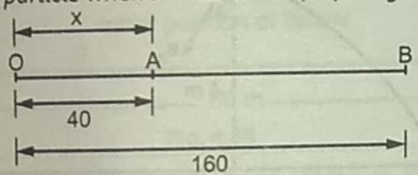


Fig. 4.79

Answer : ± 16.97 m/s

Problem No. 21 : *It is known that, from t = 2 s to t = 10 s, the acceleration of the particle is inversely proportional to the cube of time t. When t = 2 s, v = –15 m/s and when t = 10 s, v = 0.36 m/s. If the particle is twice as far from the origin when t = 2 s as it is when t = 10 s, determine the total distance travelled by the particle from t = 2 s to t = 10 s. (Refer Fig. 4.80).*

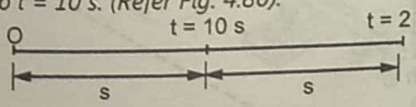


Fig. 4.80

Answer : 17.6 m

Problem No. 22 : *On its take of roll, the air plane starts from rest and accelerates according to $a = a_0 - kv^2$, where a_0 is the constant acceleration resulting from the engine thrust and $-kv^2$ is the acceleration due to aerodynamic drag. If a_0 = 2 m/s², k = 0.00004 m⁻¹ and v is in m/s, determine the design length of runway required for the air plane to reach the take off speed of 250 km/h. What would be the design length of runway if the drag were neglected ?*

Answer : 1268 m, 1205.6 m

Problem No. 23 : *The acceleration of a particle in rectilinear motion varies linearly from 2 m/s² to 4 m/s² as its position changes from x = 40 mm to x = 120 mm. If the velocity of the particle at x = 40 mm is 0.4 m/s, determine the velocity at x = 120 mm. Find the value of x for which the velocity is 0.6 m/s. (Refer Fig. 4.81).*

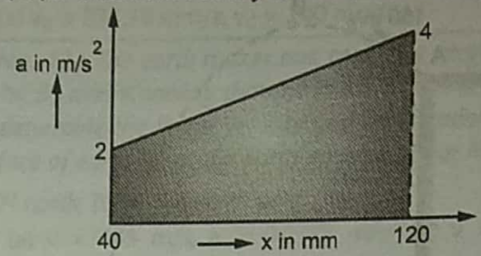


Fig. 4.81

Answer : v = 0.8 m/s, x = 80 mm

Problem No. 24 : *A particle is moving along a straight line in a viscous medium where acceleration a is given by*

$$a = -\frac{2}{x^2},$$

where a is in m/s² and x is in meters. The particle is at x = 1 m when time t = 1 second and has a velocity of 2 m/s. Calculate the velocity 'v' and position 'x' of the particle at t = 4s.

Answer : x = 4.677 m, v = 0.925 m/s

Problem No. 25 : *The acceleration of a particle is defined by the relation $a = -\frac{k}{x}$. It is known that the velocity 'v' is 5 m/s when displacement x is 200 mm and v = 3 m/s at x = 400 mm. Find velocity of particle at x = 500 mm. Also find position of the particle at which velocity is zero.*

Answer : v = 1.96 m/s, x = 590 mm

Problem No. 26 : *A model rocket starts vertically up from the ground with a velocity of 120 m/s. Due to aerodynamic drag and the gravitational acceleration, the net downward acceleration of the rocket is given by $a = 0.0005\,v^2$, where v is instantaneous velocity upwards. At the topmost point, a parachute ejects out of the rocket's nose-cone and the rocket quickly acquires a constant downward velocity of 4 m/s. Find the total flight time of the rocket, i.e. time elapsed from take off till landing. Refer Fig. 4.82.*

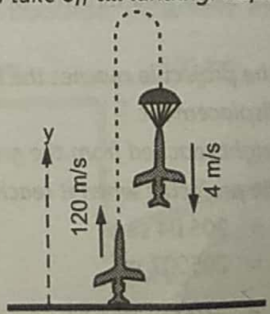


Fig. 4.82

Answer : 147.71 s

***Problem No. 27 :** A projectile is fired from the edge of a cliff 120 m above the ground level, with initial velocity 150 m/s at an angle of 30^o above the horizontal.*

Determine :

(i) *The horizontal distance from the foot of the vertical cliff to the point where projectile strikes the level ground.*

(ii) *The greatest height reached by the projectile above the ground level.*

(iii) *The velocity vector of the projectile at 10 s after firing.*

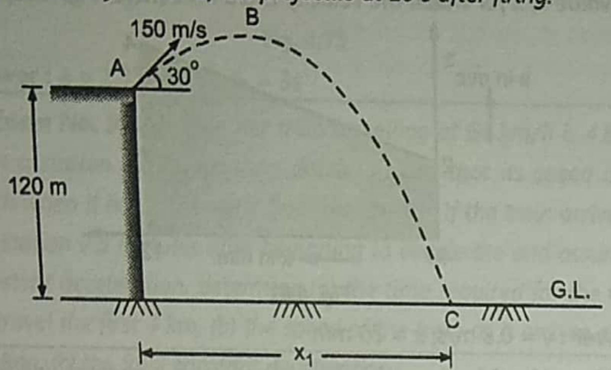


Fig. 4.83

Answer : Horizontal distance = 2175.89 m,

Greatest height = 406.7 m from ground,

Velocity vector at t = 10 s = 129.9 i – 23.1 j.

***Problem No. 28 :** The velocity of a projectile, when at its greatest height is $\sqrt{\frac{2}{5}}$ times of its velocity when half of its greatest height.*

Determine the angle of projection.

Answer : θ = 60°.

***Problem No. 29 :** A golfer hits a golf ball with initial velocity of 50 m/s at an angle of 25^o with the horizontal. Knowing that the fairway slopes downward at an average angle of 5^o, determine the distance between the golfer and point B where the ball first lands.*

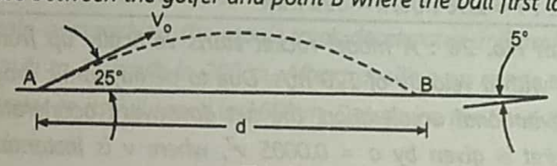


Fig. 4.84

Answer : Golf will land at 232.96 m from the golfer.

***Problem No. 30 :** A projectile is fixed from top of a cliff 1000 m high with a velocity of 1414 m/s directed at 45° with horizontal.*

Find :

(a) *Time when the projectile reaches the bottom of the cliff.*

(b) *Horizontal displacement.*

(c) *Maximum height reached from the ground.*

(d) *Velocity of the projectile when it reaches the ground.*

Answer : (a) t = 205.04 sec,

(b) s_x = 205007 m

(c) h = 52004.1 m

(d) v = 1420.86 m/s

***Problem No. 31 :** A car starts from rest on a curve of radius 250 m and accelerates at constant tangential acceleration $a_t = 1.2\ m/s^2$. Determine the distance travelled and time taken when the magnitude of total acceleration is $1.5\ m/s^2$.*

Answer : Distance travelled = 93.75 m.

***Problem No. 32 :** An automobile enters a curved road in the form of quarter of a circle and of length 360 m at a speed of 24 km/h and then leaves the curve at 48 km/h. If the car is travelling at constant acceleration along the curve, determine the resultant acceleration at both the ends of the curve.*

Answer : a = 10330.65 km/m^2 at 76.57 (first quadrant)

***Problem No. 33 :** The motion of a particle is given in polar coordinates by $r = ce^{bt}$ and $\theta = bt$ where c and b are constants, t in seconds and θ and r in radians and metre respectively. Determine the equation of path, velocity and acceleration of the particle.*

Answer : Path : $r = ce^{\theta}$, velocity : $\sqrt{2}\ bce^{bt}$, acceleration : $2b^2\ ce^{bt}$.

***Problem No. 34 :** A particle is projected to move along a parabola $y^2 = 4x$. At a certain instant when passing through a point P (4, 4), its speed is 5 m/s and the rate of increase of speed is $3\ m/s^2$ along the path. Express the velocity and acceleration of the particle in terms of rectangular coordinates and calculate acceleration.*

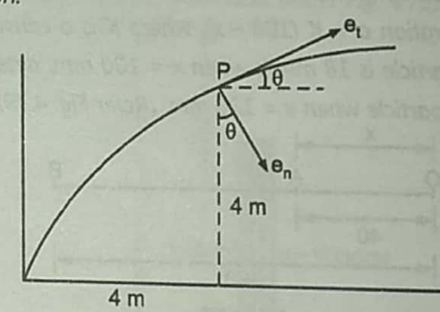


Fig. 4.85

***Problem No. 35 :** A jet plane flying at a constant speed v at an altitude h = 10 km is being traced by radar located at O directly below the line of flight. If angle θ is decreasing at the rate of 0.02 rad/s when $\theta = 60^o$, determine the value of component of acceleration.*

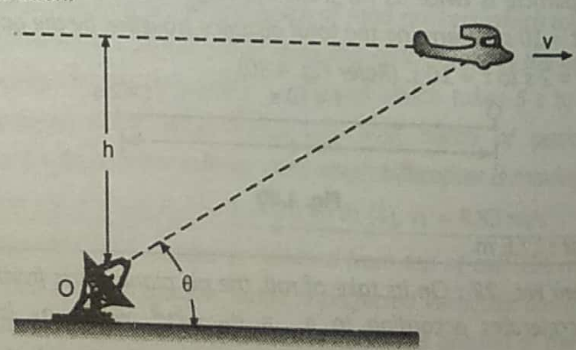


Fig. 4.86

Answer : $a = 6.16\ m/s^2$.

Problem No. 36 : *A rocket is fired vertically traced by the radar station as shown. At the instant when $\theta = 60^{\circ}$, measurement gives $\dot{\theta} = 0.03$ rad/s and r = 7500 m and the vertical acceleration of the rocket is found to be 20 m/s^2. For this instant, determine the values of $\ddot{r}$ and $\ddot{\theta}$.*

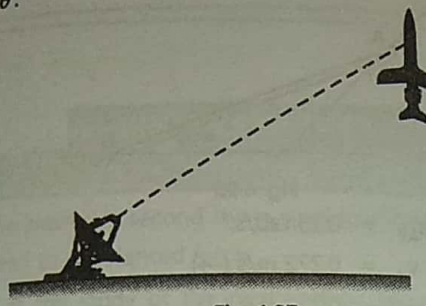


Fig. 4.87

Answer : $\ddot{r}$ = 24.07 m/s^2, $\ddot{\theta}$ = – 1.784 × 10^{-3} rad/s^2.

Problem No. 37 : *A batsman hits the ball A with an initial velocity of 30 m/s at an angle of 30° to the horizontal as shown in Fig. 4.88. The initial position of the ball is 0.9 m above the ground level. Fielder B requires 0.25 s to judge where the ball should be caught and begins moving to that position. If the catch position is the field location at which the ball altitude is 2.1 m, determine the velocity of the ball relative to the fielder at the instant the catch is made.*

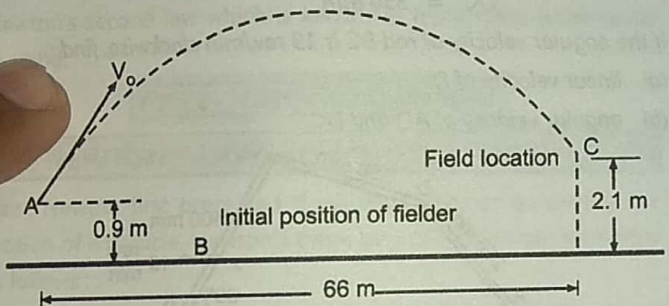


Fig. 4.88

Problem No. 38 : *The circular plate shown is initially at rest. Knowing that r = 200 mm and that the plate has a constant angular acceleration of 0.3 rad/s^2, determine the magnitude of the total acceleration of point B when (a) t = 0, (b) t = 2 s, (c) t = 4 s.*

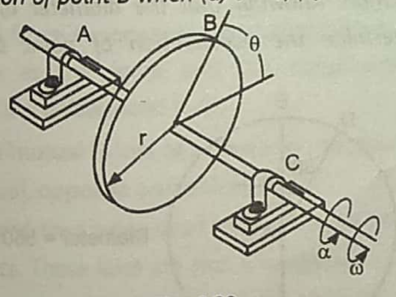


Fig. 4.89

Answer : (a) t = 0, a = 60 mm/s^2, (b) t = 2 s, a = 93.72 mm/s^2, (c) t = 4 s, a = 294.18 mm/s^2.

Problem No. 39 : *A rotor 25 mm in diameter is spinning at 200 rps. Find normal component of acceleration of a point on the rim.*

Answer : a_n = 19700 m/s^2.

Problem No. 40 : *A flywheel of an engine starting from rest makes 25 revolutions in 11th second. If the angular acceleration is uniform, calculate its angular velocity, when it has made 420 revolutions. Derive the formula of nth second if you use it.*

Answer : ω = 44.71 rev/s.

Problem No. 41 : *The motion of an oscillating crank is defined by the relation $\theta = \theta_0 \sin(\pi t/T) - 0.5\ \theta_0 \sin(2\pi t/T)$, where θ is expressed in radians and t in seconds. Knowing that θ_0 = 6 rad and T = 4 s, determine the angular co-ordinates, the angular velocity and the angular acceleration of the crank when (i) t = 0, (b) t = 2 s.*

Answer : (a) v_B = 274.76 mm/s, v_C = 240 mm/s.

Problem No. 42 : *The earth makes one complete revolution on its axis in 23 hr. 56 min. Knowing that the mean radius of the earth is 6370 km, determine the linear velocity and acceleration of a point on the surface of earth (a) at the earth equator, (b) at Philadelphia, latitude 40^{o} north, (c) at the north pole.*

Answer : (a) v = 465 m/s, a = 0.0338 m/s^2 (b) v = 356 m/s, a = 0.0259 m/s^2. (c) v = a = 0.

Problem No. 43 : *In the planetary gear system shown, the radius of the central gear A is a, the radius of each of the planetary gears is b, and the radius of the outer gear E is a + 2b. The angular velocity of gear A is ω_A clockwise, and the outer gear is stationary. If the angular velocity of the spider BCD is to be $\omega_A/5$ clockwise, determine (a) the required value of the ratio b/a, (b) the corresponding angular velocity of each planetary gear.*

Answer : (a) 1.5, (b) $\omega_A/3$ (↻)

Problem No. 44 : *Gear A rotates with an angular velocity of 120 r/min clockwise. Knowing that the angular velocity of arm AB is 90 r/min clockwise, determine the corresponding angular velocity of gear B.*

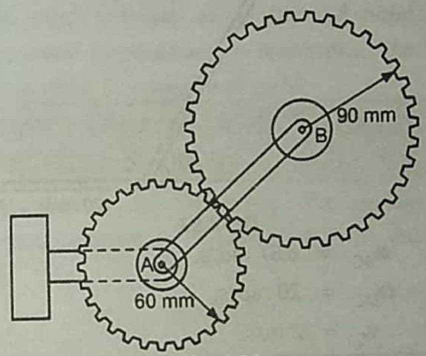


Fig. 4.90

Answer : 70 r/min (↻)

Problem No. 45 : *Bar AB has constant angular velocity of ω = 3 rad/s counterclockwise. Determine the angular velocity of bar DE.*

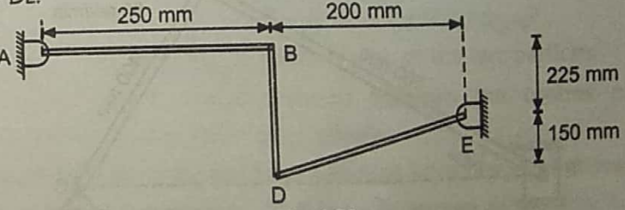


Fig. 4.91

Answer : ω_{DE} = 3.75 rad/s.

v_D = 0.9375 m/s.

***Problem No. 46 :** In the mechanism shown, crank OA rotates at 100 rpm clockwise. Links AB and AC are pin jointed at A and the pin ends B and C are attached to blocks sliding in horizontal and vertical guides. For the position shown when C is vertically below A, find the velocity of B and C and the angular velocity of links AB and AC.*

$$OA = AB = AC = 150 \text{ mm}$$

$$\angle AOB = \angle ABO = 30°$$

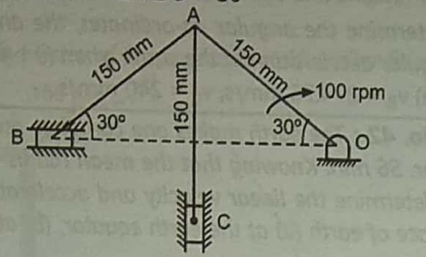


Fig. 4.92

Answer : $v_B = 0.785$ m/s $\quad \omega_{AC} = 5.233$ rad/s

$v_C = 1.36$ m/s $\quad \omega_{AB} = 5.233$ rad/s

***Problem No. 47 :** In a linkage shown, link AB at an instant is rotating in clockwise direction and point B has a linear velocity of 2 m/s. Find the angular velocities of bars BC and CD and linear velocity of pin P.*

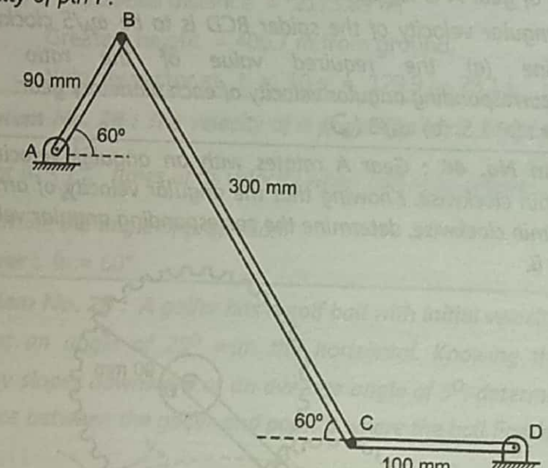


Fig. 4.93

Answer : (1) $\omega_{BC} = 6.67$ rad/s.

(2) $\omega_{DC} = 20$ rad/s.

(3) $v_C = 2$ m/s.

***Problem No. 48 :** In a crank and rotating rod mechanism, the crank is rotating at 80 rev./min. The connecting rod makes an angle of 30° with horizontal at that instant. Determine velocity of cross head A.*

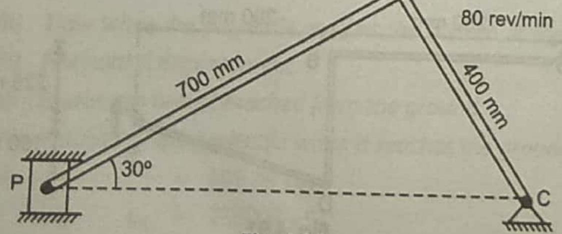


Fig. 4.94

Answer : $v_P = 3.86$ m/s.

***Problem No. 49 :** Bar AB having length of 2 m slides so that point B has a velocity of 0.4 m/s to the right. (a) What is the angular velocity of the bar at the instant shown ? (b) Also determine components of velocity of A in x and y directions.*

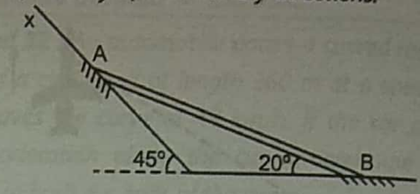


Fig. 4.95

Answer : (a) $\omega_{AB} = 0.15$ rad/s,

(b) $v_x = 0.272$ m/s (→)

$v_y = 0.272$ m/s (↓)

***Problem No. 50 :** A and B are two fixed points on the same level on a fixed base. Cranks BC and AD can rotate about B and A respectively. These cranks are connected by bar DC hinged at D and C.*

$$AB = 650 \text{ mm}$$

$$AD = 250 \text{ mm}$$

$$BC = 400 \text{ mm}$$

$$DC = 350 \text{ mm}$$

If the angular velocity of rod BC is 19 rev/min clockwise, find :

(a) linear velocity of D.

(b) angular velocity of AD and DC.

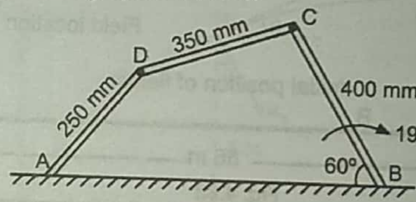


Fig. 4.96

Answer : (a) $v_D = 1.28$ m/s, (b) $\omega_{AD} = 5.12$ rad/s, $\omega_{DC} = 3.2$ rad/s

***Problem No. 51 :** An automobile travels to the right at a constant speed of 90 kmph. Knowing that the diameter of the wheel is 550 mm, determine the acceleration of point D. Use vector approach.*

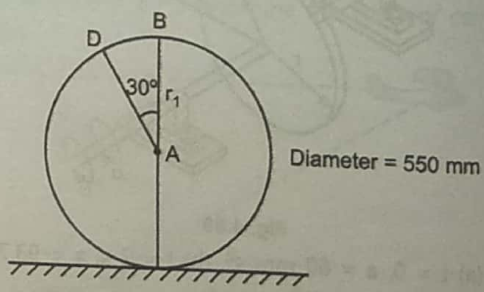


Fig. 4.97

Answer : $a_D = 2271.65$ mm/s^2.

KINETICS

A – KINETICS OF PARTICLE

5.1 INTRODUCTION

As per the Newton's second law, a particle will accelerate when it is subjected to unbalanced force.

Kinetics is the study of the relationships between unbalanced force and the resulting changes in motion.

In this chapter, we will study kinetics of particle of rectilinear motion. This chapter requires combined knowledge of properties of forces, which we have developed in statics and the kinematics of particle of rectilinear motion covered in previous chapter.

With the help of Newton's second law, we can combine these two topics and solve engineering problems involving force, mass and motion. In this chapter, we will study direct application of Newton's second law which is known as force-mass-acceleration method.

FORCE - MASS - ACCELERATION

5.2 NEWTON'S LAWS OF MOTION

Isaac Newton first presented three basic laws of governing the motion of a particle. Newton's three laws of motion can be stated as follows :

First Law : A particle remains in its state of rest or of uniform motion in a straight line with a constant velocity, will remain in its state unless it is not subjected to an unbalanced force.

Second Law : When an unbalanced force $\bar{F}$ acted upon a particle, the unbalanced force experiences an acceleration $\bar{a}$ that has the same direction as the force and the magnitude is directly proportional to the unbalanced force.

Third Law : The mutual forces of action and reaction between the particles are equal, opposite and collinear.

The first and third laws were used extensively in developing the concept of statics. These laws are also considered in dynamics.

Newton's second law of motion forms the basis for the study of kinetics because this law relates the accelerated motion of a particle to the unbalanced force acting on it.

Statics is the special case of dynamics, since Newton's second law yields the result of his first law when the unbalanced force is equal to zero.

According to Newton's second law, an unbalanced force is directly proportional to the acceleration or rate of change of momentum.

Consider a particle with mass 'm' moving with velocity v, under the action of unbalanced force 'F'.

$$F \propto \text{Rate of change of momentum}$$

$$F \propto \frac{d}{dt}(mv)$$

$$F = K\frac{d}{dt}(mv)$$

$$F = Km\frac{dv}{dt} \qquad \ldots\left(\because \frac{dv}{dt} = a\right)$$

$$\therefore \quad F = Kma$$

where, K is a dimensionless constant to be determined in order to preserve the equality. The unit force is defined as a force which should be applied on a unit mass to produce unit acceleration.

Hence, the Newton's law of motion may be written in mathematical form as

$$F = ma$$

This equation, which is known as equation of motion, is one of the most important formulations in mechanics. The S.I. unit of force is newton, which is represented by N.

5.3 NEWTON'S LAW OF GRAVITATIONAL ATTRACTION

Newton postulated a law governing the mutual attraction between any two particles. In mathematical form, this law can be expressed as,

$$F = \frac{Gm_1m_2}{r^2} \qquad \ldots (5.1)$$

where,

F = Force of attraction between the two particles

G = Universal constant of gravitation, according to experimental result, $G = 66.73 \times 10^{-12}\ m^3/kg\text{-}s^2$

m_1, m_2 = Mass of each of the two particles

r = Distance between the centres of two particles

Any two particles or bodies have a mutual attractive gravitational force acting between them. This force is termed as the weight. Weight will be the only gravitational force considered in mechanics.

5.4 MASS AND WEIGHT

Mass is the property of matter by which we can compare the action of one body with respect to another.

This property indicates itself as a gravitational attraction between two bodies and provide a quantitative measure of the resistance of matter to a change in velocity. It is an absolute quantity.

Therefore the measurement of mass can be made at any location; while as weight of the body is not a absolute, since it is measured in a gravitational field and hence its magnitude on the location of measurement.

From equation (5.1), we can write a general expression to find the weight W of a particle having mass $m_1 = m$.

If m_2 is the mass of the earth and r is the distance between the earth centre and the particle, then, $g = \frac{Gm_2}{r^2}$. We have,

$$W = mg \qquad \text{... (5.2)}$$

In comparison with F = ma, g is termed as acceleration due to gravity.

5.5 EQUATION OF RECTILINEAR MOTION

If more than one force acts on a particle, the resultant force may be determined by a vector summation of all the forces. i.e. $\bar{F}_R = \Sigma \bar{F}$. In general, the equation of motion may be written as,

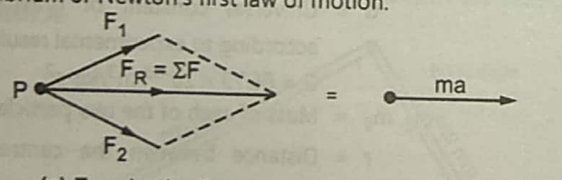
$$\Sigma F = ma \qquad \text{... (5.3)}$$

Consider a particle P of mass m subjected to action of more than one forces i.e. F_1 and F_2, having an acceleration 'a' as shown in Fig. 5.1 (a).

We can represent forces F_1 and F_2 graphically as shown in Fig. 5.1 (b). If F_R is the resultant of two forces F_1 and F_2, as per Newton's second law, resultant force $F_R = \Sigma F$ produces the vector ma, its magnitude and direction can be represented graphically on the kinetic diagram as shown in Fig. 5.1 (b).

The equal sign between the free body and the kinetic diagram represents graphical equivalency i.e. $\Sigma F = ma$.

In a particular case, when $F_R = \Sigma F = 0$, then the acceleration is zero, so that the particle will either remain at rest or move along a straight line with constant velocity. This is the condition of static equilibrium or Newton's first law of motion.



(a) Free body diagram (b) Kinetic diagram

Fig. 5.1

Rectangular Co-ordinates : When a particle is moving in the x-y plane, the forces acting on the particle as well as its acceleration, may be expressed in terms of i and j components as shown in Fig. 5.2.

Applying the equation of motion, we have,

$$\Sigma F = ma$$

$$\Sigma F_x i + \Sigma F_y j = m(a_x i + a_y j)$$

To satisfy the above equation, the i and j components of the left side must be equal to the corresponding components on the right side. Hence, we can write the following two equations of motion along x and y directions.

$$\left.\begin{aligned} \Sigma F_x &= ma_x \\ \Sigma F_y &= ma_y \end{aligned}\right\} \qquad \text{... (5.4)}$$

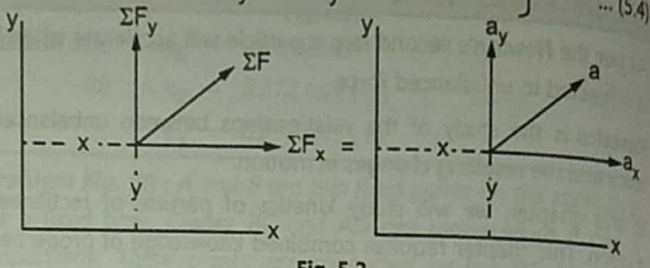


Fig. 5.2

5.6 D'ALEMBERT'S PRINCIPLE

The equation of motion, $\Sigma F = ma$, can also be written as $\Sigma F - ma = 0$. The term (– ma) is called as inertia force or the D'Alembert force and is defined as the resistance to the change in condition of rest or of uniform motion of a body. After applying D'Alembert force to a particle in motion, it comes under dynamic equilibrium and we can write equations of dynamic equilibrium as follows :

$$\Sigma F_x - ma_x = 0$$

$$\Sigma F_y - ma_y = 0 \qquad \text{... (5.5)}$$

D'Alembert was the first to point out that the equation of motion can be written as equilibrium equation by introducing inertia or D'Alembert force in addition to the force acting on the system.

It should be clearly understood that the equation of motion of a particle and the equation of dynamic equilibrium of a particle are the two concepts of expression which differ only in the manner of writing the equation. However, the final result will be the same.

D'Alembert's principle can be explained in the following example.

Consider a block of mass m resting on horizontal surface subjected to force F. Let μ_k be the coefficient of kinetic friction between the block and the horizontal surface. The F.B.D. of the block is shown in Fig. 5.3.

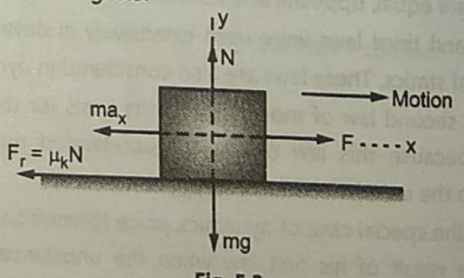


Fig. 5.3

According to D'Alembert's principle, equation of dynamic equilibrium is

$$\Sigma F_y - ma_y = 0 \quad \text{... } (\because a_y = 0)$$
$$N - mg = 0$$
$$N = mg$$
$$\Sigma F_x - ma_x = 0$$
$$F - \mu_k N - ma_x = 0$$
$$F - \mu_k mg - ma_x = 0$$
$$F = m(\mu_k g + a_x)$$

Note : In this text, all numerical examples are solved by using equation of motion.

NUMERICAL EXAMPLES ON EQUATIONS OF MOTION

Example 5.1 : *The 50 kg crate is travelled along the floor with an initial velocity 7 m/s at x = 0. The coefficient of kinetic friction is μ_k = 0.40. Calculate the time required for the crate to come to rest and the corresponding distance x travelled. [Refer Fig. 5.4]*

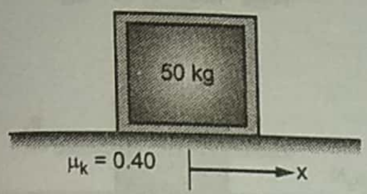


Fig. 5.4

Solution :

Given data : Initial velocity, u = 7 m/s

Final velocity, v = 0

Mass of the crate, m = 50 kg

Coefficient of kinetic friction, μ_k = 0.40.

As the crate travelled with initial velocity, u = 7 m/s (at x = 0) along the floor and come to rest, it decelerates and covered a distance x in time t.

F.B.D. of crate is as shown in Fig. 5.4 (a).

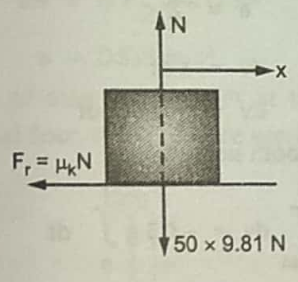


Fig. 5.4 (a) : F.B.D. of crate

Using equations of motion normal to the plane and along the floor,

$$\Sigma F_y = ma_y \quad \text{... } (\because a_y = 0)$$
$$-50 \times 9.81 + N = 50 \times 0$$
$$N = 490.5 \text{ N}$$
$$\Sigma F_x = ma_x$$
$$\mu_k \cdot N = ma_x$$
$$-0.4 \times 490.5 = 50 \times a_x$$
$$a_x = -3.924 \text{ m/s}^2$$
$$= 3.924 \text{ m/s}^2 \text{ (deceleration)}$$

Using equation of kinematics,

$$v^2 = u^2 + 2a_x \cdot x$$
$$0 = 49 + 2 \times (-3.924)\, x$$
$$x = 6.24 \text{ m} \quad \text{... Ans.}$$
$$v = u + a_x t$$
$$0 = 7 + (-3.924) \times t$$
$$\therefore \quad t = 1.784 \text{ s} \quad \text{... Ans.}$$

Example 5.2 : *100 N block is carefully placed with zero velocity on the inclined plane as shown in Fig. 5.5. If μ_s = 0.30 and μ_k = 0.25, determine the acceleration of the block if **(a)** θ = 15°, **(b)** θ = 20°.*

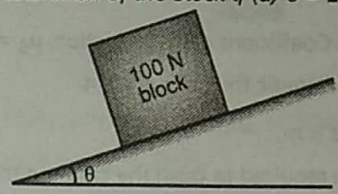


Fig. 5.5

Solution :

Given data : Initial velocity, u = 0

Weight of block, W = 100 N

As the initial velocity of the block is zero, possible motion of block is downward along the inclined plane and it is only possible when θ is more than $\phi = \tan^{-1} \mu_s$.

F.B.D. of block is as shown in Fig. 5.5 (a).

(a) When

$$\theta = 15^\circ$$
$$\mu_k = 0.25$$
$$\phi = \tan^{-1}(0.30)$$
$$= 16.69^\circ$$
$$\theta < \phi \text{, hence no motion}$$
$$\therefore \quad a = 0 \quad \text{... Ans.}$$

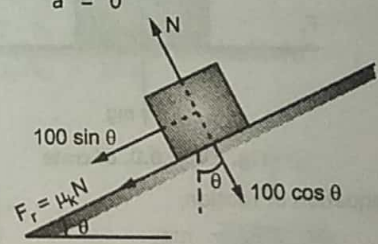


Fig. 5.5 (a) : F.B.D. of block

(b) When

$$\theta = 20^\circ$$
$$\phi = \tan^{-1}(0.30)$$
$$= 16.69^\circ$$

θ > ϕ, hence the block is moving downward along the plane.

Using equation of motion normal to the plane and along the plane,

$$\Sigma F_y = ma_y,$$

(normal to the inclined plane, a_y = 0)

$$-100\cos 20 + N = 0$$

$$N = 93.97\text{ N}$$

$$\Sigma F_x = ma_x, \quad \text{(along the inclined plane)}$$

$$100\sin 20 - 0.25\times 93.97 = \frac{100}{9.81}\cdot a_x$$

$$34.20 - 23.493 = 10.194\, a_x$$

$$a_x = \mathbf{1.05\ m/s^2} \qquad \text{... Ans.}$$

***Example 5.3 :** A man moves a crate by pushing horizontally against until it slides on the floor. If $\mu_s = 0.5$ and $\mu_k = 0.4$, with what acceleration does the crate begin to move ? Assume that the force exerted by the man at impending motion is maintained when sliding begins.*

Solution :

Given data : Coefficient of static friction, $\mu_s = 0.5$.

Coefficient of kinetic friction, $\mu_k = 0.4$

Mass of crate is m.

Let P be the force required to push the crate on the floor.

Initially the crate is at rest condition.

Using equation of statics,

$$\Sigma F_y = 0$$

$$-mg + N = 0$$

$$N = mg$$

$$\Sigma F_x = 0$$

$$P - F_r = 0$$

$$P - 0.5\,N = 0$$

$$P = 0.5\,mg$$

At the limiting condition, crate will impend the motion.

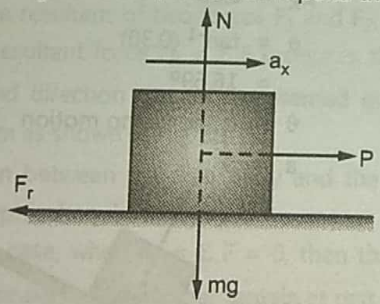


Fig. 5.6 : F.B.D. of crate

Using equation of motion,

$$\Sigma F_x = ma_x$$

$$0.5\,mg - 0.4\,mg = ma_x$$

$$a_x = 0.1\,g$$

$$= \mathbf{0.981\ m/s^2} \qquad \text{... Ans.}$$

***Example 5.4 :** Determine the minimum stopping distance s and the corresponding time t required by the truck, if the crate is not to slip forward. Take $\mu_s = 0.3$ and $\mu_k = 0.25$ between the crate and the flat bed of the truck which has a speed of 70 kmph.*

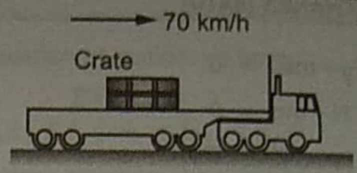


Fig. 5.7

Solution :

Given data : Initial velocity of truck, u = 70 km/h = 19.44 m/s

Final velocity of truck, v = 0.

Coefficient of static friction, $\mu_s = 0.3$

Coefficient of kinetic friction, $\mu_k = 0.25$

F.B.D. of crate is as shown in Fig. 5.7 (a).

Using equation of kinetics,

$$\Sigma F_y = ma_y, \qquad \text{... } (\because a_y = 0)$$

$$-mg + N = 0$$

$$N = mg$$

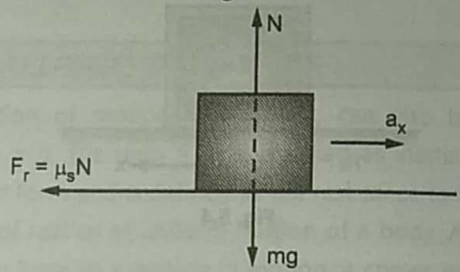


Fig. 5.7 (a) : F.B.D. of crate

$$\Sigma F_x = ma_x$$

$$-F = ma_x$$

$$-\mu_s N = ma_x$$

$$-\mu_s mg = ma_x$$

$$a_x = -0.3\,g$$

Using equation of kinematics,

$$a = \frac{dv}{dt}$$

$$dv = a\,dt$$

$$dv = (-0.3\,g)\,dt$$

Integrating on both sides,

$$\int_{19.44}^{0} dv = -0.3\,g\int_0^t dt \qquad \text{... (1)}$$

$$19.44 = 0.3\times g\times t$$

$$\therefore \quad t = \frac{19.44}{0.3\times 9.81}$$

$$\therefore \quad t = \mathbf{6.606\ s} \qquad \text{... Ans.}$$

From equation (1), $v = 0.3\times gt$

$$\frac{ds}{dt} = 0.3\times gt$$

$$ds = 0.3\times gt\,dt$$

Integrating on both sides,

$$\int_0^s ds = 0.3 \times 9.81 \int_0^t t\,dt$$

$$s = 0.3 \times 9.81 \times \frac{t^2}{2}$$

Substituting t = 6.606 s, we get,

$$s = 0.3 \times 9.81 \times \frac{(6.606)^2}{2}$$

$\therefore$ $s = \mathbf{64.2\ m}$ ... **Ans.**

***Example 5.5 :** A man weighs 700 N and supports a barbells which have a weight of 500 N. If he lift them 0.6 m in the air in 1.5 s, with uniform acceleration and starting from rest, determine the force exerted on his feet by the ground during the lift.*

Fig. 5.8

Solution :

Given data :

Weight of man, W_m = 700 N

Weight of barbells, W_b = 500 N

Lift of barbells, h = 0.6 m

Initial velocity of barbells, u = 0 and time, t = 1.5 s.

Using equation of kinematics along y-direction,

$$s = ut + \frac{1}{2} at^2$$

$$0.6 = 0 + \frac{1}{2}\, a \times (1.5)^2$$

$$a = 0.533 \text{ m/s}^2$$

Consider F.B.D. of man and barbells at the floor and use equation of motion at floor. R be the force exerted by the ground on the feet of man.

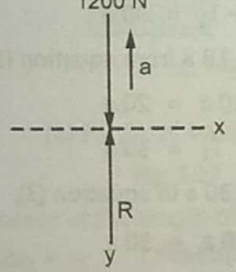


Fig. 5.8 (a) : F.B.D.

$$\Sigma F_y = ma_y$$

$$R - 1200 = \frac{500}{9.81} \times 0.533$$

$$R - 1200 = 27.18$$

$R = 27.18 + 1200 = \mathbf{1227.18\ N}$ ... **Ans.**

***Example 5.6 :** Determine the acceleration of the 5 kg cylinder A. Neglect the mass of pulleys and chords. The block B has a mass of 10 kg. The coefficient of kinetic friction between block B and surface is $\mu_K = 0.1$.*

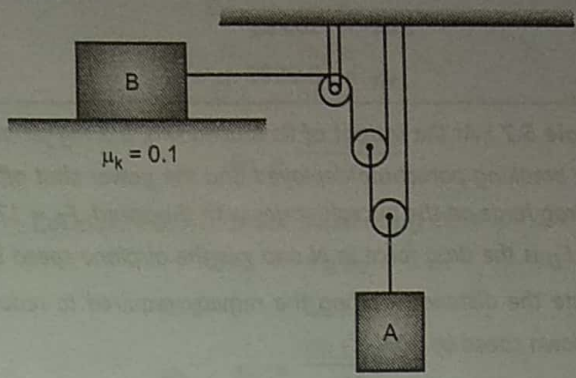


Fig. 5.9

Solution :

Given data : Mass of the cylinder, m_A = 5 kg

Mass of the block B, m_B = 10 kg

Coefficient of kinetic friction, μ_K = 0.1

From the concept of length of the string,

$$s_B = 4\, s_A$$

$$v_B = 4\, v_A$$

$$a_B = 4\, a_A \quad \text{... (1)}$$

Consider F.B.D. of cylinder A and using equation of motion along y-direction,

$$\Sigma F_y = m_A \cdot a_A$$

$$5 \times 9.81 - 4T = 5\, a_A \quad \text{... (2)}$$

Consider F.B.D. of block B and using equation of motion along x and y directions,

$$\Sigma F_y = m_B \cdot a_y, \quad \text{... } (\because a_y = 0)$$

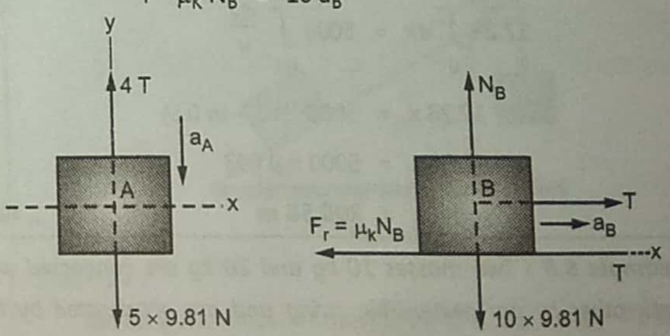
$$N_B - 10 \times 9.81 = 0$$

$$N_B = 98.1 \text{ N}$$

$$\Sigma F_x = m_B \cdot a_B$$

$$T - F_r = 10\, a_B$$

$$T - \mu_K N_B = 10\, a_B$$



(a) F.B.D. of cylinder A **(b) F.B.D. of block B**

Fig. 5.9

$$T - 0.1 \times 98.1 = 10\, a_B$$

Substituting $a_B = 4\, a_A$ from equation (1),

$$T - 9.81 = 40\, a_A$$

Multiplying by 4 on both sides,

$$4T - 4 \times 9.81 = 160\, a_A \qquad \text{... (3)}$$

Solving equations (2) and (3),

$$9.81 = 165\, a_A$$

$$\therefore \quad a_A = \mathbf{0.0595\ m/s^2} \qquad \text{... Ans.}$$

***Example 5.7 :** At the instant of its touchdown, a 5 Mg jet airplane has its breaking parachute deployed and the power shut off. If the total drag force on the aircraft varies with the speed $F_D = 17.28\, v^2$, where F_D is the drag force in N and v is the airplane speed in m/s, calculate the distance x along the runway required to reduce the touchdown speed by 50%.*

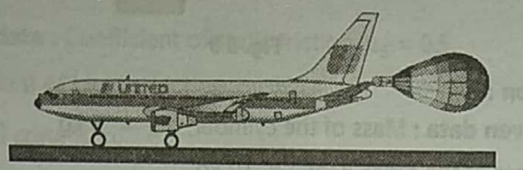


Fig. 5.10

Solution :

Given data : Mass of airplane, m = 5 Mg

Drag force, $F_D = 17.28\, v^2$ N

Using equation of kinetics along the runway and equation of kinematics,

$$\Sigma F = ma$$

$$17.28\, v^2 = 5 \times 10^3\, a$$

$$17.28\, v^2 = 5000\, v \cdot \frac{dv}{dx}$$

$$17.28\, dx = 5000 \frac{v\, dv}{v^2}$$

$$17.28\, dx = 5000 \frac{dv}{v}$$

Using definite integral,

$$17.28 \int_0^x dx = 5000 \int_{0.5}^{1} \frac{dv}{v}$$

$$17.28\, x = 5000\, (\ln 1 - \ln 0.5)$$

$$17.28\, x = 5000 \times 0.693$$

$$x = \mathbf{200.56\ m} \qquad \text{... Ans.}$$

***Example 5.8 :** Two masses 10 kg and 20 kg are connected with each other by an inextensible string and are accelerated by the force of gravity of mass 30 kg as shown in Fig. 5.11. Mass of pulleys, strings and friction at all contact surfaces are negligible. Determine the acceleration of the system and tensions T_1 and T_2 in the strings.*

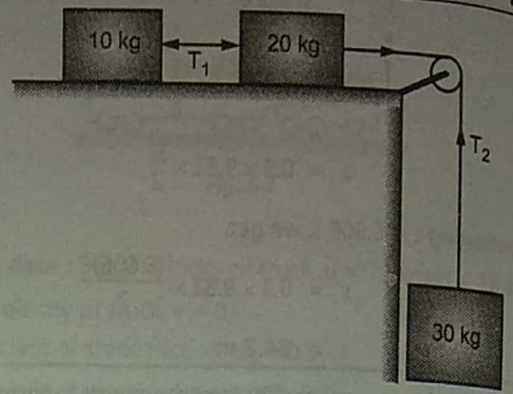
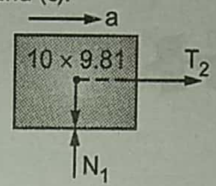


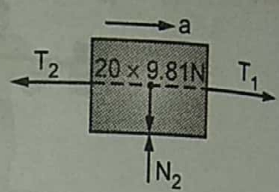
Fig. 5.11

Solution :

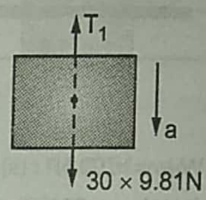
F.B.D.s of mass 10 kg, 20 kg and 30 kg are shown in Fig. 5.11 (a), (b) and (c).



(a) F.B.D. of mass 10 kg



(b) F.B.D. of mass 20 kg



(c) F.B.D. of mass 30 kg

Fig. 5.11

Consider F.B.D. of mass 10 kg and using equation of motion,

$$\Sigma F = ma$$

$$T_2 = 10\, a \qquad \text{... (1)}$$

Consider F.B.D. of mass 20 kg and using equation of motion,

$$\Sigma F = ma$$

$$T_1 - T_2 = 20\, a \qquad \text{... (2)}$$

Consider F.B.D. of mass 30 kg and using equation of motion,

$$\Sigma F = m\, a$$

$$30 \times 9.81 - T_1 = 30\, a$$

$$294.3 - T_1 = 30\, a \qquad \text{... (3)}$$

Substituting $T_2 = 10$ a from equation (1) in equation (2),

$$T_1 - 10\, a = 20\, a$$

$$T_1 = 30\, a$$

Substituting $T_1 = 30$ a in equation (3),

$$294.3 - 30\, a = 30\, a$$

$$294.3 = 60\, a$$

$$a = \mathbf{4.905\ m/s^2} \qquad \text{... Ans.}$$

From equation (1), $T_2 = 10$ a

$$T_2 = 10 \times 4.905$$

$$T_2 = \mathbf{49.05\ N} \qquad \text{... Ans.}$$

From equation (2),

$$T_1 - T_2 = 20\ a$$

$$T_1 = 20 \times 4.905 + 49.05$$

$$T_1 = \mathbf{147.15\ N} \quad \text{... Ans.}$$

Example 5.9 : *The coefficients of friction between blocks A and C and the horizontal surfaces are $\mu_S = 0.24$ and $\mu_K = 0.20$. If m_A = 5 kg, m_B = 10 kg and m_C = 10 kg, determine the tension in the chord and the acceleration of each block.*

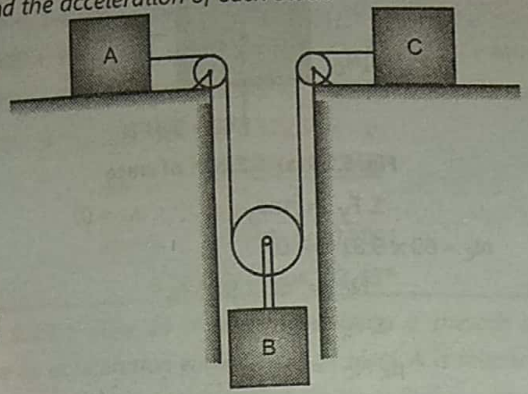


Fig. 5.12

Solution :

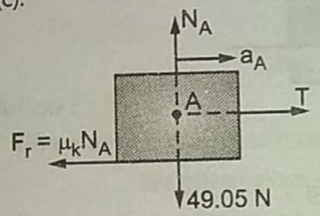
Given data :

Mass of block A, m_A = 5 kg

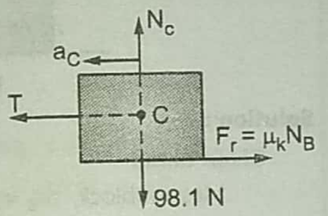
Mass of block B, m_B = 10 kg

Mass of block C, m_C = 10 kg

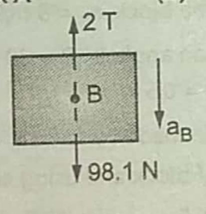
F.B.D.s of blocks A, B and C are shown in Fig. 5.12 (a), (b) and (c).



(a) F.B.D. of block A



(b) F.B.D. of block C



(c) F.B.D. of block B

Fig. 5.12

From the concept of the length of string,

$$x_A + 2x_B + x_C = \text{Constant}$$

$$v_A + 2v_B + v_C = 0$$

$$a_A + 2a_B + a_C = 0 \quad \text{... (1)}$$

Consider F.B.D. of block A and using equation of motion,

$$\Sigma F_x = m_A \cdot a_A$$

$$T - \mu_K N_A = m_A \cdot a_A$$

$$T - 0.2 \times 49.05 = 5\ a_A$$

$$a_A = \frac{T - 9.81}{5} \quad \text{... (2)}$$

Consider F.B.D. of block C and using equation of motion,

$$\Sigma F_x = m_C\, a_C$$

$$T - \mu_K N_C = m_C\, a_C$$

$$T - 0.2 \times 98.1 = 10\ a_C$$

$$a_C = \frac{T - 19.62}{10} \quad \text{... (3)}$$

Consider F.B.D. of block B and using equation of motion,

$$\Sigma F_y = m_B\, a_B$$

$$98.1 - 2T = 10\ a_B$$

$$a_B = \frac{98.1 - 2T}{10} \quad \text{... (4)}$$

Substituting value of a_A, a_C and a_B (↓) in equation (1),

$$\frac{T - 9.81}{5} - \frac{2\,(98.1 - 2T)}{10} + \frac{T - 19.62}{10} = 0$$

$$2T - 19.62 - 196.2 - 4T + T - 19.62 = 0$$

$$7T = 235.44$$

$$T = \mathbf{33.63\ N} \quad \text{... Ans.}$$

From equation (1), $a_A = \dfrac{33.63 - 9.81}{5}$

$$a_A = \mathbf{4.76\ m/s^2} \quad \text{... Ans.}$$

From equation (3), $a_B = \dfrac{98.1 - 2 \times 33.63}{10}$

$$a_B = \mathbf{3.08\ m/s^2\ (\downarrow)} \quad \text{... Ans.}$$

From equation (2), $a_C = \dfrac{33.63 - 19.62}{10}$

$$a_C = \mathbf{1.4\ m/s^2} \quad \text{... Ans.}$$

Example 5.10 : *Masses A and B, 30 kg each are connected by light inextensible rope passing over a smooth light pulley as shown in Fig. 5.13. Mass A slides over the smooth inclined plane making an angle of 30° with the horizontal. If the system is released from rest, find the distance moved by mass B in 2 s.*

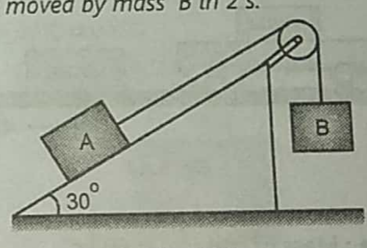
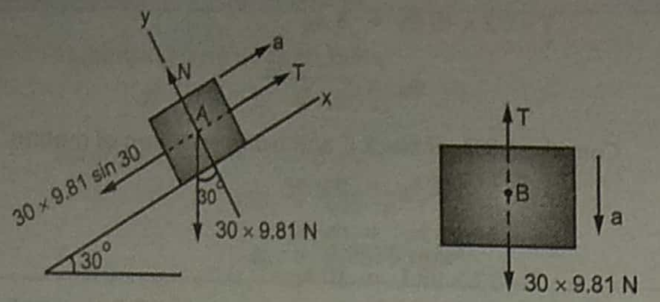


Fig. 5.13

Solution :

Given data : Mass of blocks A and B, $m_A = m_B$ = 30 kg

Let s be the distance travelled by block B in t = 2 s and a be the acceleration of the system. F.B.D.s of blocks A and B are shown in Fig. 5.13 (a) and (b).



(a) F.B.D. of block A (b) F.B.D. of block B

Fig. 5.13

Consider F.B.D. of block A and using equation of kinetics along the inclined plane,

$$\Sigma F = m_A \cdot a$$

$$T - 30 \times 9.81 \sin 30 = 30\ a$$

$$T - 147.15 = 30\ a \quad \text{... (1)}$$

Consider F.B.D. of block B and using equation of motion,

$$\Sigma F_y = m_B \cdot a$$

$$30 \times 9.81 - T = 30\ a$$

$$294.3 - T = 30\ a \quad \text{... (2)}$$

Solving equations (1) and (2),

$$147.15 = 60\ a$$

$$a = 2.4525\ m/s^2$$

Using equation of kinematics, distance travelled by block B in 2 s is given by,

$$s = ut + \frac{1}{2}\ at^2 \quad \text{... } (\because\ u = 0)$$

$$s = 0 + \frac{1}{2} \times 2.4525 \times (2)^2$$

$$s = \mathbf{4.905\ m} \quad \text{... Ans.}$$

***Example 5.11 :** A crate having mass of 60 kg falls horizontally of the back of truck which is traveling at 80 km/h. Determine the coefficient of kinetic friction between the road and the crate, if the crate slides 45 m on the ground with no tumbling along the road before coming to rest. Assume the initial velocity of the crate along the road be 80 km/h. Refer Fig. 5.14.*

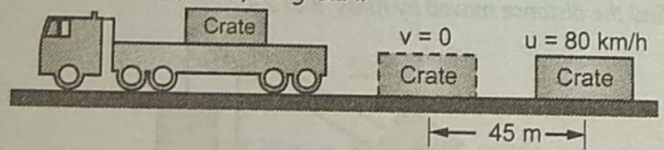


Fig. 5.14

Solution :

Given data : Mass of crate, m = 60 kg

Initial velocity of crate, u = 80 km/h = 22.222 m/s

Final velocity of crate, v = 0.

Distance travelled by the crate, s = 45 m

Let μ_K be the coefficient of kinetic friction and a be the acceleration of the crate. Using equation of kinematics, acceleration of the car is given by,

$$v^2 = u^2 + 2as$$

$$0 = (22.222)^2 + 2 \times a \times 45$$

$$\therefore \quad a = -5.487\ m/s^2$$

$$a = 5.487\ m/s^2 \text{ (deceleration)}$$

Consider F.B.D. of crate and using equation of kinetics along the road,

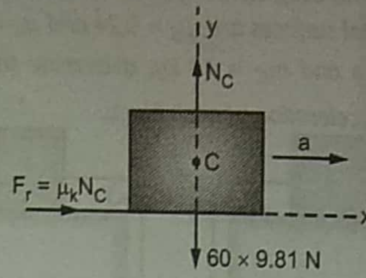


Fig. 5.14 (a) : F.B.D. of crate

$$\Sigma F_y = ma_y, \quad \text{... } (\because\ a_y = 0)$$

$$N_C - 60 \times 9.81 = 0$$

$$N_C = 60 \times 9.81\ N$$

$$\Sigma F_x = ma_x$$

$$\mu_K N_C = 60\ a$$

$$\mu_K \times 60 \times 9.81 = 60 \times 5.487$$

$$\mu_K = \mathbf{0.559} \quad \text{... Ans.}$$

***Example 5.12 :** For what value(s) of the angle θ will the acceleration of the 2.5 kg block be 8 m/s² to the right when subjected to a force of 30 N. μ_S = 0.6 and μ_K = 0.5 between the block and the floor.*

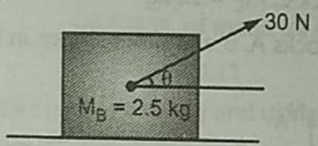


Fig. 5.15

Solution :

Given data :

Mass of block, m_B = 2.5 kg

Acceleration of the block, a_B = 8 m/s²

Applied force at an angle θ, P = 30 N

μ_S = 0.6 and μ_K = 0.5

Let θ be the angle made by force P = 30 N.

Consider F.B.D. of block and using equation of kinetics,

$$\Sigma F_y = m_B \cdot a_{By} \quad \text{... } (\because\ a_{By} = 0)$$

$$N - 2.5 \times 9.81 + 30 \sin \theta = 0$$

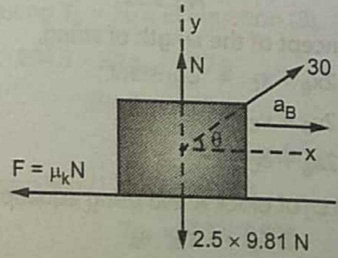


Fig. 5.15 (a) : F.B.D. of block

$$N = 24.53 - 30 \sin\theta \quad \text{... (1)}$$

$$\Sigma F_X = m_B a_{Bx}$$

$$30 \cos\theta - \mu_K N = 2.5 \times a_{Bx}$$

$$30 \cos\theta - 0.5 (24.53 - 30 \sin\theta) = 2.5 \times 8$$

$$30 \cos\theta + 15 \sin\theta = 32.265 \quad \text{... (2)}$$

Dividing both sides of equation (2) by cos θ,

$$30 + 15 \tan\theta = 32.263 \sec\theta$$

Squaring on both sides,

$$900 + 900 \tan\theta + 225 \tan^2\theta = 32.263 (1 + \tan^2\theta)$$

$$900 + 900 \tan\theta + 225 \tan^2\theta = 1041 + 1041 \tan^2\theta$$

$$\tan^2\theta - 1.1029 \tan\theta + 0.1728 = 0$$

$$\tan\theta = \frac{1.1029 \pm 0.7247}{2}$$

$$\tan\theta = 0.9138 \text{ or } 0.1891$$

$$\theta = \mathbf{10.7° \text{ or } 42.43°} \quad \text{... Ans.}$$

Example 5.13 : *The 25 N block B rests on a smooth surface. Determine its acceleration when the 15 N block A is released from rest. What would be the acceleration of block B if the block at A was replaced by a 15 N force acting on the attached cord ?*

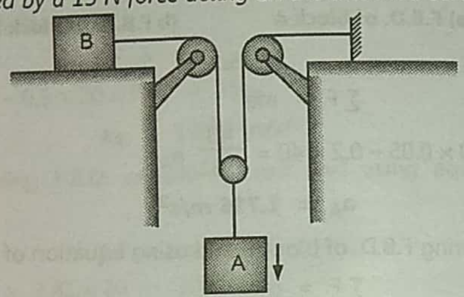


Fig. 5.16

Solution :

Given data : Weight of block B, W_B = 25 N

Weight of block A, W_A = 15 N

From equation of kinematics,

$$a_B = 2a_A \quad \text{... (1)}$$

Let T be the tension in the string. Considering F.B.D. of block B and using equation of motion,

$$\sum F = ma_B$$

$$\therefore \quad T = \frac{25}{9.81} a_B$$

$$\therefore \quad 9.81\, T = 25a_B$$

$$\therefore \quad 9.81\, T = 50a_A \quad \text{... (2)}$$

Considering F.B.D. of block A,

$$\sum F = ma_A$$

$$15 - 2T = \frac{15}{9.81} a_A$$

$$147.15 - 19.62\, T = 15a_A \quad \text{... (3)}$$

Solving equations (2) and (3),

$$a_A = 1.2796 \text{ m/s}^2$$

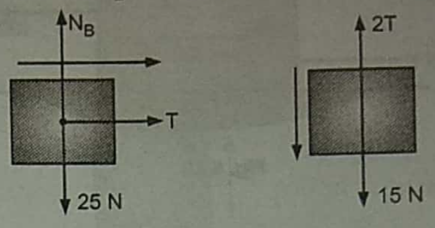
From equation (1), a_B = **2.56 m/s²**

When 15 N block is replaced with 15 N force, acceleration a_A becomes zero and T = 7.5 N.

From F.B.D. of block B,

$$9.81 \times 7.5 = 25a_B$$

$$a_B = \mathbf{2.943 \text{ m/s}^2}$$



(a) F.B.D. of B **(b) F.B.D. of A**

Fig. 5.16

Example 5.14 : *The conveyor belt is designed to transport packages of various weights. Each 10 kg package has a coefficient of kinetic friction μ_k = 0.15. If the speed of the conveyor is 5 m/s, and then it suddenly stops, determine the distance the package will slide on the belt before coming to rest.*

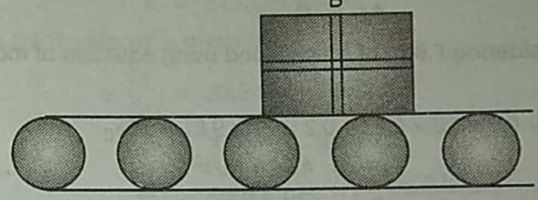


Fig. 5.17

Solution :

Given data :

Initial velocity of block, u = 5 m/s.

Final velocity of block, v = 0.

Mass of the block, m = 10 kg.

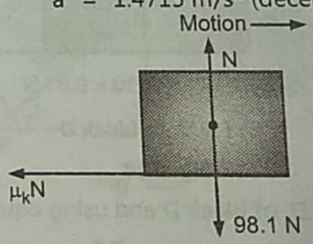
Coefficient of kinetic friction, μ_k = 0.15.

Considering F.B.D. of block and using equation of motion,

$$\sum F = ma$$

$$0.15 \times 10 \times 9.81 = 10a$$

$$\therefore \quad a = 1.4715 \text{ m/s}^2 \text{ (deceleration)}$$



(a) F.B.D. of block

Fig. 5.17

Using equation of kinematics,

$$v^2 = u^2 + 2as$$

$$0 = 5^2 - 2 \times 1.4715 \times s$$

$$\therefore \quad s = \mathbf{8.495 \text{ m}} \quad \text{... Ans.}$$

Example 5.15 : *Each of the three plates has a mass of 10 kg. If the coefficient of friction (static and kinetic) at each surface of contact are μ_s = 0.3 and μ_k = 0.2 respectively, determine the acceleration of each plate when the three horizontal forces are applied.*

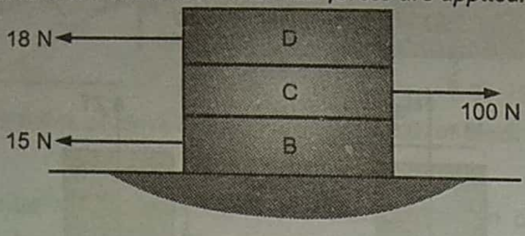


Fig. 5.18

Solution :

Given data :

Mass of each plate, m = 10 kg

Coefficient of static friction, μ_s = 0.3

Coefficient of kinetic friction, μ_k = 0.2

Considering F.B.D. of block B : At the limiting condition, frictional force is more than the external force. Hence block B must be in static condition.

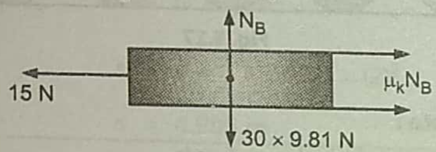
i.e. $a_B = 0$ ...**Ans.**

Considering F.B.D. of block C and using equation of motion,

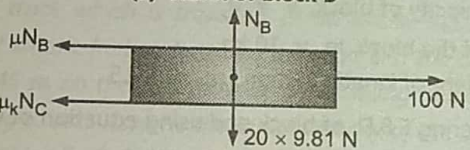
$$\sum F = ma$$

$$100 - 0.2 \times 10 \times 9.81 - 0.2 \times 20 \times 9.81 = 10a_C$$

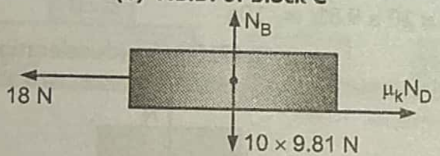
$\therefore$ $\mathbf{a_C = 4.114\ m/s^2}$ ... **Ans.**



(a) F.B.D. of block B



(b) F.B.D. of block C



(c) F.B.D. of block D

Fig. 5.18

Considering F.B.D. of block D and using equation of motion,

$$\sum F = ma$$

$$0.2 \times 10 \times 9.81 - 18 = 10a_D$$

$$\mathbf{a_D = 0.162\ m/s^2} \text{ ... Ans.}$$

Example 5.16 : *Block A has a weight of 40 N and block B has a weight of 30 N. They rest on a surface for which the coefficient of kinetic friction is μ_k = 0.2. If the spring has a stiffness of k = 300 N/m and it is compressed to 0.05 m, determine the acceleration of each block just after they are replaced.*

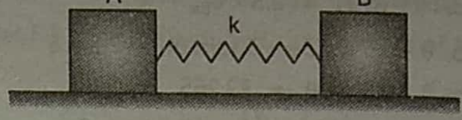


Fig. 5.19

Solution :

Given data :

Weight of block A, W_A = 40 N

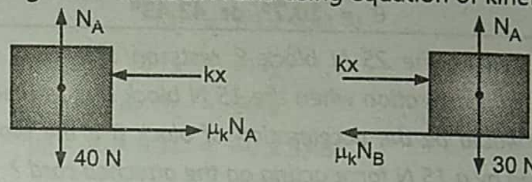
Weight of block B, W_B = 30 N

Coefficient of kinetic friction, μ_k = 0.2

Spring constant, k = 300 N/m

Compression of spring, x = 0.05

Considering F.B.D. of block A and using equation of kinetics,



(a) F.B.D. of block A **(b F.B.D. of block B**

Fig. 5.19

$$\sum F = ma$$

$$300 \times 0.05 - 0.2 \times 40 = \frac{40}{9.81} a_A$$

$\therefore$ $\mathbf{a_A = 1.716\ m/s^2}$ ... **Ans.**

Considering F.B.D. of block B and using equation of motion,

$$\sum F = ma$$

$$300 \times 0.05 - 0.2 \times 30 = \frac{30}{9.81} a_B$$

$\therefore$ $\mathbf{a_B = 2.94\ m/s^2}$ ...**Ans.**

Example 5.17 : *If the coefficient of static and kinetic friction between the 20 kg block A and the 100 kg cart B are both essentially the same value of 0.5, determine the acceleration of each part for (a) P = 40 N and (b) P = 60 N.*

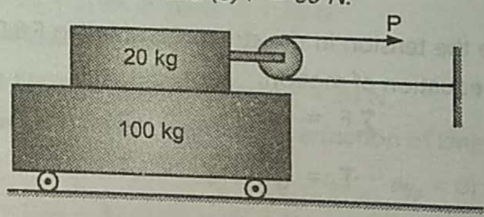


Fig. 5.20

Solution :

Given data : Mass of block, m_B = 20 kg

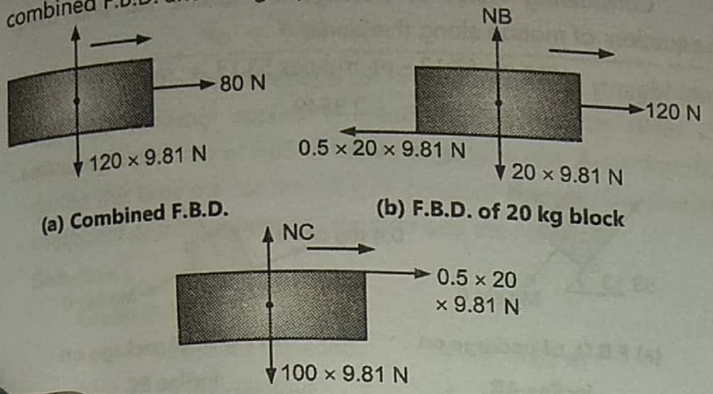
Mass of cart, m_C = 100 kg

Coefficient of static and kinetic friction,

$$\mu_k = \mu_s = 0.5$$

(a) P = 40 N : When P = 40 N, total force to pull the block is less than the frictional force induced in between block and cart.

Hence, the motion of block is not possible. But the block impend the motion with car having same acceleration. Considering combined F.B.D. and using equation of motion,



(a) Combined F.B.D. (b) F.B.D. of 20 kg block

(c) F.B.D. of 100 kg cart

Fig. 5.20

$$\sum F = ma$$

$$80 = 120a$$

$$\therefore \quad a_B \text{ or } a_C = \mathbf{0.67\ m/s^2} \quad \text{...Ans.}$$

(b) P = 60 N : Considering F.B.D. of 20 kg block and using equation of motion,

$$\sum F = ma$$

$$120 - 0.5 \times 20 \times 9.81 = 20a_B$$

$$\therefore \quad a_B = \mathbf{1.095\ m/s^2}$$

Considering F.B.D. of 100 kg cart and using equation of motion,

$$\sum F = ma$$

$$0.5 \times 9.81 \times 20 = 100\, a_C$$

$$\therefore \quad a_C = \mathbf{0.981\ m/s^2} \quad \text{... Ans.}$$

Example 5.18 : *The 4 lb collar is released from rest against the light elastic spring, which has a stiffness of 10 lb/in and has been compressed a distance of 6 in. Determine the acceleration a of the collar as a function of vertical displacement x of the collar measured in feet from the point of release. Find the velocity v of the collar when x = 0.5 ft. Friction is negligible.*

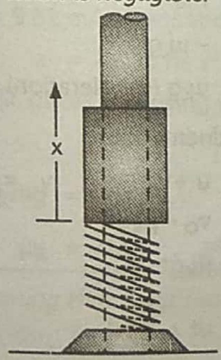


Fig. 5.21

Solution :

Given data :

Weight of collar, $W_C = 4$ lb

Spring constant, $k = 10$ lb/in

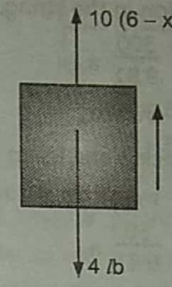
Maximum compression of spring, $x_{max} = 6$ in

Considering F.B.D. of collar and using equation of motion,

$$\sum F = ma$$

$$10\,(6 - x) - 4 = \frac{4}{32.2}\, a$$

$$2.5\,(6 - x) - 1 = \frac{a}{32.2}$$



(a) F.B.D. of collar

Fig. 5.21

$$15 - 30x - 1 = \frac{a}{32.2}$$

$$a = 32.2\,(14 - 30x),$$

at x = 0.6 ft, velocity is given by

$$v\,\frac{dv}{dx} = 32.2\,(14 - 30x)$$

$$\therefore \quad vdv = 32.2\,(14 - 30x)\, dx$$

$$\therefore \quad \frac{v^2}{2} = 32.2\left(14 \times 0.5 - \frac{30 \times 0.5^2}{2}\right)$$

$$= \mathbf{14.47\ \ ft/sec} \quad \text{... Ans.}$$

Example 5.19 : *Block A has a weight of 300 N and block B has a weight of 50 N. Determine the speed of block A after it moves 1.5 m down the plane, starting from rest. Neglect friction and mass of cord and pulleys.*

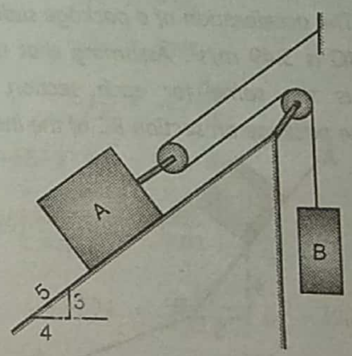


Fig. 5.22

Solution :

Given data :

Weight of block A, $W_A = 300$ N

Weight of block B, $W_B = 50$ N

From the principle of kinematics,

$$2x_A + x_B = \text{constant}$$

$$2v_A + v_B = 0$$

$$\therefore \quad 2a_A + a_B = 0$$

$$a_B (\uparrow) = 2a_A$$

Considering F.B.D. of block A and using equation of motion along the inclined plane,

$$\sum F = ma$$

Let a_A be the acceleration of block A, a_B be the acceleration of block B and T be the tension in the string.

$$300 \times 0.6 - 2T = \frac{300}{9.81} a_A$$

$$180 - 2T = 30.58 a_A \quad \text{... (1)}$$

Considering F.B.D. of block B and using equation of motion,

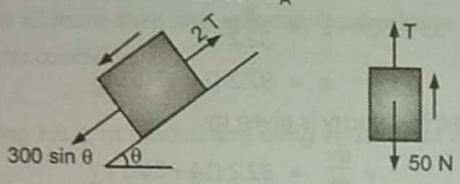
$$\sum F = ma \text{ along y-direction}$$

$$T - 50 = \frac{50}{9.81} a_B$$

Substituting $\quad a_B = 2a_A$

$$T - 50 = 10.19 a_A$$

$$2T - 100 = 20.387\ a_A \quad \text{...(2)}$$



(a) F.B.D. of block A (b) F.B.D. of block B

Fig. 5.22

Solving equations (1) and (2),

$$a_A = 1.5696 \text{ m/s}^2$$

Using equation of kinematics,

$$v^2 = u^2 + 2as$$

$$v^2 = 0 + 2 \times 1.5696 \times 1.5$$

$$\mathbf{v = 2.17 \text{ m/s}} \quad \text{... Ans.}$$

***Example 5.20 :** The acceleration of a package sliding down section AB of incline ABC is 5.49 m/s². Assuming that the coefficient of kinetic friction is the same for each section, determine the acceleration of the package on section BC of the incline.*

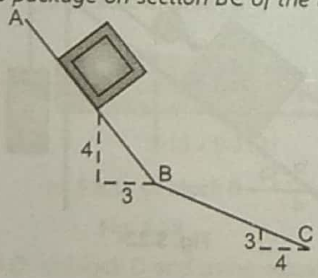


Fig. 5.23

Solution :

Given data :

Mass of block be m.

a_{AB} be acceleration of package on incline AB.

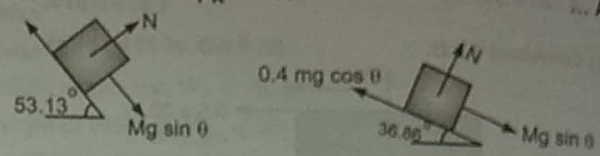
Let μ_k be the coefficient of friction and a_{BC} be the acceleration of package on incline BC.

Considering F.B.D. of package on incline AB and using equation of motion along the plane,

$$mg \sin 53.13 - \mu_k\, mg \cos 53.13 = m \times 5.49$$

$$\therefore \quad -\mu_k \times 5.866 = -2.3549$$

$$\therefore \quad \mu_k = 0.4 \quad \text{... Ans.}$$



(a) F.B.D. of package on incline AB **(b) F.B.D. of package on incline BC**

Fig. 5.23

Considering F.B.D. of package on incline BC and using equation of motion along the incline,

$$mg \sin 38.86 - 0.4\ mg \cos 36.86 = ma_{BC}$$

$$\mathbf{a_{BC} = 2.745 \text{ m/s}^2} \quad \text{...Ans.}$$

***Example 5.21 :** A hockey player hits a puck so that it comes to rest in 9s after sliding 30 m on the ice. Determine (a) the initial velocity of puck, (b) the coefficient of friction between the puck and the ice.*

Solution :

Given data :

Mass of hockey puck – m

Distance travelled, $x = 30$ m

Required time, $t = 9$s

Let v_o be the initial velocity of puck and μ_k be the coefficient of kinetic friction between hockey puck and ice.

Considering F.B.D. of hockey puck and using equation of motion along horizontal direction,

$$\sum F = ma$$

$$-\mu_k\, mg = ma$$

$$\therefore \quad a = -\mu_k g$$

$$\therefore \quad a = \mu_k g \text{ (deceleration)}$$

Using equation of kinematics,

$$v = u + at \quad \therefore \quad v = 0$$

$$0 = v_o - \mu_k g t$$

$$\therefore \quad v_o = \mu_k g t \quad \text{...(1)}$$

$$s = ut + \frac{1}{2} at^2$$

$$\therefore \quad 30 = \mu_k \times 9.81 \times 9 \times 9 - \frac{1}{2} \times \mu_k \times 9.81 \times 9^2$$

$$30 = 79.46\ \mu_k - 397.305\ \mu_k$$

$$\therefore \quad 30 = 397.305\ \mu_k$$

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$\therefore \quad \mu_k = 0.0755$

From equation (1),

$$v_o = 0.0755 \times 9.81 \times 9$$

$$\mathbf{v_o = 6.67\ m/s} \quad \text{... Ans.}$$

***Example 5.22 :** The driver of a car, travelling along a straight level highway, suddenly applies a break so that the car slides 2s, covering a distance of 9.81 m before coming to rest. Assuming that during this time the car moved with constant deceleration, find the coefficient of friction between the tires and the highway.*

Solution :

Given data :

Distance travelled, $s = 9.81$ m

Time required, $t = 2$s

Let a be the constant deceleration and μ be the coefficient of kinetic friction.

Using equation of kinematics,

$$v = u + at \quad (\because v = 0)$$

$$a = -\frac{u}{t} \quad \text{... (1)}$$

Using $v^2 = u^2 + 2as$

$$0 = u^2 - 2as$$

Substituting $a = -\frac{u}{t}$

$$0 = u^2 - 2\frac{u}{2} \times 9.81$$

$$u^2 = 9.81\ u$$

$$\therefore \quad u = 9.81\ m/s$$

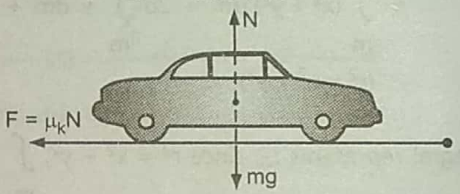


Fig. 5.24 : F.B.D. of car

Substituting u = 9.81 m/s in equation (1),

$$a = 4.905\ m/s^2$$

Considering F.B.D. of car and using equation of motion along the highway,

$$\mu_k mg = ma \quad \therefore \quad \mu = \frac{4.905}{9.81}$$

$$\mathbf{\mu_k = 0.5} \quad \text{... Ans.}$$

***Example 5.23 :** Assuming that a car has sufficient power and that there is sufficient friction, find the maximum acceleration that it would be able to develop without tipping over backward.*

Solution :

Considering F.B.D. of car and taking moment about B,

$$W \times b = \frac{W}{g} a \times h$$

$$\therefore \quad a = \frac{bg}{h}$$

Substituting the values of b = 0.8 m, h = 0.4 m,

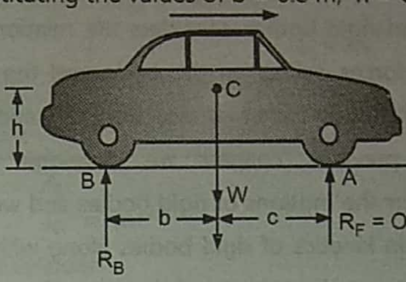


Fig. 5.25

$$a = \frac{0.8 \times 9.81}{0.4} = 19.82\ m/s^2$$

***Example 5.24 :** A smooth 10 N collar C fits loosely on the horizontal shaft. If the spring is unextended when s = 0, determine the velocity of the collar when s = 1 m if the collar is given an initial horizontal velocity of 4.5 m/s when s = 0.*

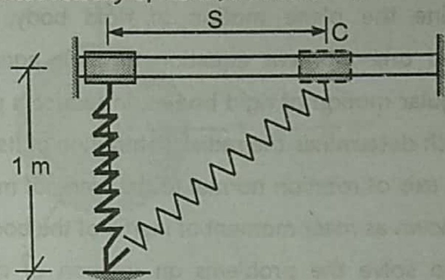


Fig. 5.26

Solution :

Considering F.B.D. of collar at s = 1 m. The acceleration is not constant, it varies with deformation of spring. The deformation of spring is zero at s = 0 and 0.414 m at s = 1 m.

Using equation of motion along the shaft,

$$\Sigma F_x = ma_x$$

$$-F_s = ma_t$$

$$-F_s = mv\frac{dv}{dx}$$

$$-65 \cdot x \cdot dx = m\,v\,dv$$

$$-65\left[\frac{x^2}{2}\right]_0^{0.414} = \frac{10}{9.81}\left[\frac{v^2}{2}\right]_{4.5}^{v}$$

$$-5.5704 = \frac{10}{9.81 \times 2}\,[v^2 - 20.25$$

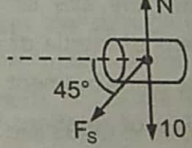


Fig. 5.26 (a)

$$-10.93 = v^2 - 20.25$$

$$\mathbf{v = 3.05\ m/s} \quad \text{... Ans.}$$

B – KINETICS OF RIGID BODIES

5.7 INTRODUCTION

The kinetics of rigid bodies considers the relationships between the external forces acting on the body and the corresponding motions of the body i.e. translation, rotation and general plane motion. In previous chapter we develope the kinematic relationship for the motions of rigid bodies and we will use these relationships in kinetics of rigid bodies along with the effects of external forces on the two dimensional motion of rigid bodies. The plane of motion will contain the mass centre of rigid body and all the external forces acting on the body.

Kinetics of particle require two force equations to define the motion of particle whereas kinetics of rigid bodies require an additional equation to specify the state of rotation of the body. Thus to define the plane motion of rigid body, two force equations and one moment equation or their equivalent are required. Angular motion of rigid bodies, introduce a property of the body which determines the radial distribution of its mass with respect to an axis of rotation normal to the plane of motion. This property is known as mass moment of inertia of the body and it is very useful to solve the problems on rotation of rigid body. Kinetics of rigid body consists translation, rotation about a fixed axis and general plane motion.

5.8 BASIC CONCEPTS

5.8.1 Mass Moment of Inertia

An external force system may cause the body to translate and rotate. The translational effects of the motion were studied in kinetics of particles which are represented by the equation, $\sum F = ma$. The rotational effects caused by the moment M are represented by the equation, $\sum M = I\alpha$, where, I is mass moment of inertia. The mass moment of inertia is a resistance of a body to angular acceleration similar to mass is a resistance of a body to linear acceleration ($\sum F = ma$).

A rigid body has a definite size and shape, consisting of a number of particles or elements. Consider an element having mass dm, at a shortest distance r from the axis of rotation as shown in Fig. 5.27. The mass moment of inertia of an element about the axis of rotation, $I = r^2$ dm, since moment of inertia is a second moment of mass about axis of rotation. The mass moment of inertia of a rigid body about the axis of rotation, $I = \int_m r^2\, dm$. If the axis of rotation passes through the centroidal axis, then mass of inertia is denoted by I_G.

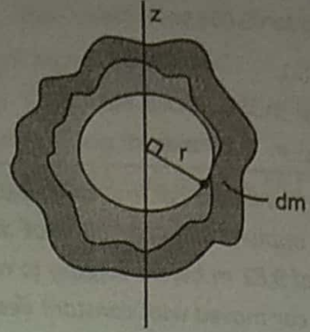


Fig. 5.27

5.8.2 Parallel Axis Theorem

If the moment of inertia of body about an axis passing through the mass centre is known, then the moment of inertia about any parallel axis may be determined by using the parallel axis theorem. This theorem can be derived by considering an element of a body having mass dm, as shown in Fig. 5.28. Let z be the axis passing through the mass centre and z' be the parallel axis which lies at constant distance d from mass centre. An element of mass dm is at (x, y) from the z-axis. The position of element of mass dm with respect to z' axis is r' = (d + x) + y. The mass moment of inertia of the body about z'-axis is :

$$I_{z}' = \int_m (r')^2\, dm = \int_m [(d + x)^2 + y^2]\, dm$$

$$= \int_m (d^2 + 2dx + x^2 + y^2)\, dm$$

$$= \int_m (x^2 + y^2)\, dm + 2d \int_m x\, dm + d^2 \int_m dm;$$

$$(r^2 = x^2 + y^2)$$

This first integral represents I_G, since $r^2 = x^2 + y^2$, $\int_m r^2\, dm = I_G$

$\int_m x\, dm = m\bar{x}$; ($\bar{x}$ -position of z-axis from mass centre = 0).

The second integral is equal to zero. Third integral represents the total mass of the body i.e. d^2m. Hence, the moment of inertia about the axis parallel to axis passing through the mass centre can be written as

$$I_z' = I_G + md^2$$

where, I_G = Mass moment of inertia about the axis passing through the mass centre.

m = Mass of the body

and d = Shortest distance between the parallel axis

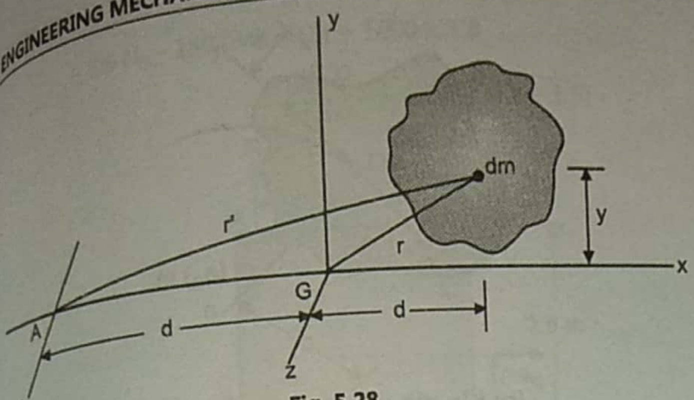


Fig. 5.28

5.8.3 Radius of Gyration

Sometimes, the moment of inertia of a body about a axis is expressed using radius of gyration k.

$$I = mk^2 \text{ or } k = \sqrt{I/A}$$

where,

I = Mass moment of inertia

m – Mass of body

k – Radius of gyration

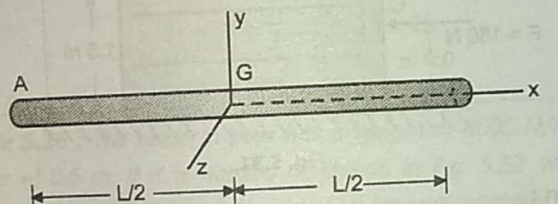
5.8.4 Mass Moment of Inertia of Some Basic Bodies

The mass moment of inertia of some basic rigid bodies are as follows :

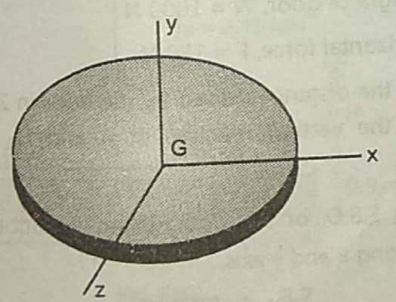
(a) Slender rod or bar of length L.

$$I_G = \frac{mL^2}{12}$$

$$I_A = \frac{mL^2}{3}$$

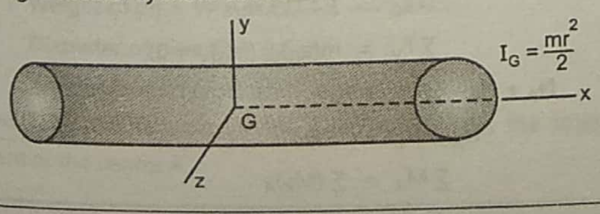


(b) Thin circular disc of radius r.



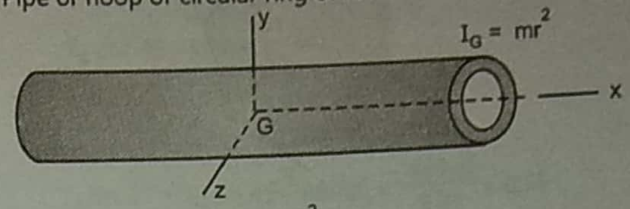
$$I_G = \frac{mr^2}{2}$$

(c) Rigid circular cylinder of radius r.



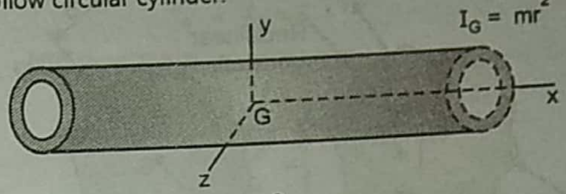
$$I_G = \frac{mr^2}{2}$$

(d) Pipe or hoop or circular ring of radius r.



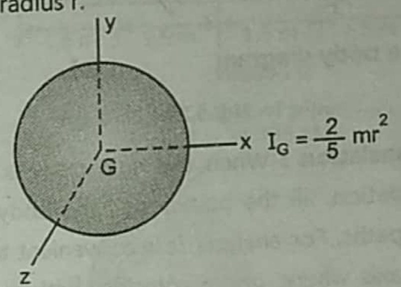
$$I_G = mr^2$$

(e) Hollow circular cylinder.



$$I_G = mr^2$$

(f) Sphere of radius r.



$$I_G = \frac{2}{5} mr^2$$

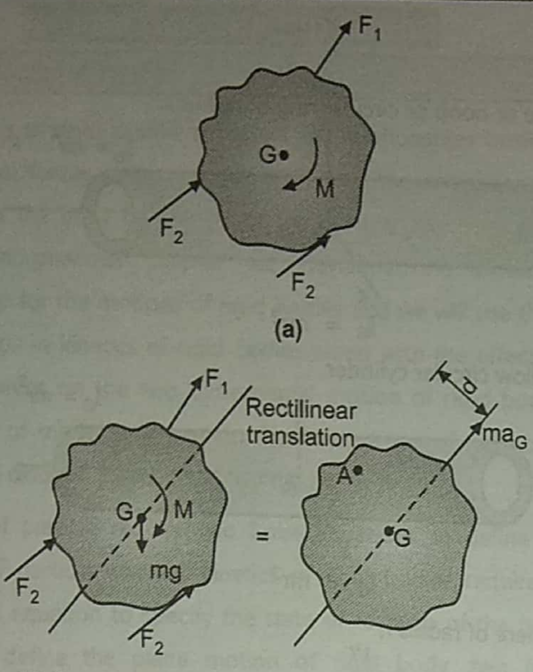
5.9 KINETICS OF RIGID BODY : TRANSLATIONAL MOTION

When the rigid body is subjected to resultant force, it undergoes a translation and all the particles of the body are moving in a translational motion with same acceleration, $a_G = a$. The translational motion of the body may be rectilinear or curvilinear.

Rectilinear Translation : When the body is subjected to rectilinear translation, all the particles of the body move along the parallel straight line path which is shown in the free body and kinetic diagram. The angular acceleration of the body (α) is zero since the motion of the body is rectilinear translation. Hence, the equations of motion for rectilinear translation are,

$$\Sigma F_x = m(a_G)_x$$
$$\Sigma F_y = m(a_G)_y \qquad \text{... (5.6)}$$
$$\Sigma M_G = 0$$

The last equation indicates the sum of the moment of all the external forces about the mass centre equal to zero. If the summation of the moments of all the forces about the other point (on or outside of the body) is taken, then the force ma_G must be considered. The point A is at a perpendicular distance d from the line of action of ma_G. The moment equation becomes $\Sigma M_A = ma_G \cdot d$ or $\Sigma M_A = \Sigma (M_k)_A$.



(b) Free body diagram (c)

Fig. 5.29

Curvilinear Translation : When the rigid body is subjected to curvilinear translation, all the particles of the body move along parallel curved paths. For analysis, it is convenient to use normal and tangential axis where, origin coincides with mass centre as shown in Fig. 5.30. Hence, equations of motion for curvilinear translation are,

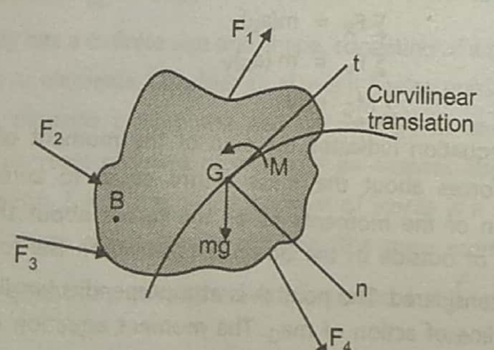
$$\Sigma F_n = m(a_G)_n$$

$$\Sigma F_t = m(a_G)_t \qquad \text{... (5.7)}$$

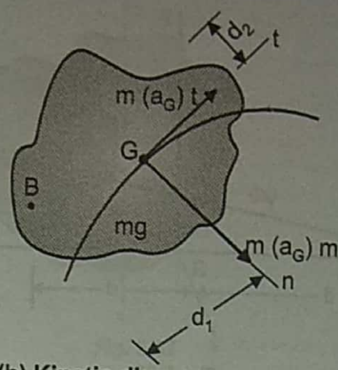
$$\Sigma M_G = 0$$

where, $(a_G)_n$ and $(a_G)_t$ represent magnitude of the normal and tangential components of acceleration of point G.

If the moment equation $\Sigma M_G = 0$ is replaced by ΣM about any other point B, then the force $m(a_G)_n$ and $m(a_G)_t$ must be considered. The required moment equation becomes $\Sigma M_B = \Sigma (M_k)_B$ or $\Sigma M_B = d_1 [m(a_G)_n] - d_2 [m(a_G)_t]$.



(a) Free body diagram



(b) Kinetic diagram

Fig. 5.30

NUMERICAL EXAMPLES ON TRANSLATIONAL MOTION OF RIGID BODY

Example 5.25 : *The door has a weight of 1000 N and a centre of gravity at G. Determine how far the door moves in 2 s, starting from rest, if a man pushes on it at C with a horizontal force F = 150 N. Also find the vertical reactions at the rollers A and B. (Refer Fig. 5.31)*

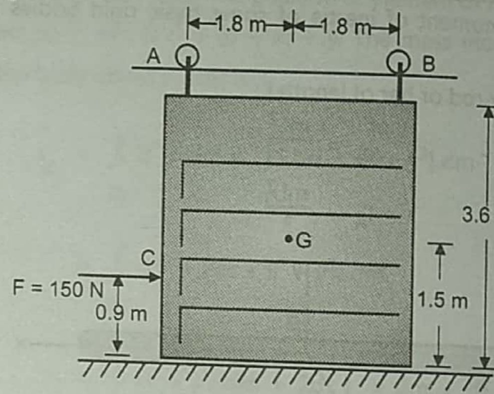


Fig. 5.31

Solution :

Given data :

Weight of door, W = 1000 N

Horizontal force, F = 150 N

Let s be the distance moved by the door in 2 s, starting from rest, N_A be the vertical reaction at A and N_B be the vertical reaction at B.

Consider F.B.D. of the door. Using equations of rectilinear translation along x and y axis,

$$\Sigma F_x = m(a_G)_x$$

$$150 = \frac{1000}{9.81} a_G$$

$$a_G = 1.4715 \text{ m/s}^2$$

$$\Sigma F_y = m(a_G)_y; \; ([a_G]_y = 0)$$

$$N_A + N_B - 1000 = 0$$

$$N_A + N_B = 1000 \qquad \text{... (1)}$$

$$\Sigma M_A = \Sigma (M_k)_A$$

$$-3.6\,N_B - 150\,(3.6-0.9) + 1000 \times 1.8 = -\frac{1000}{9.81} \times 1.4715 \times (3.6-1.5)$$

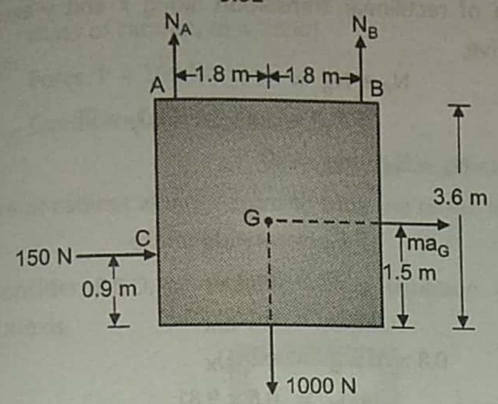


Fig. 5.31 (a) : F.B.D. of door

$$-3.6\,N_B - 405 + 1800 = -315$$

$$-3.6\,N_B = -1710$$

$$N_B = \mathbf{475\ N} \qquad \text{... Ans.}$$

From equation (1),

$$N_A = \mathbf{525\ N} \qquad \text{... Ans.}$$

Distance moved by door in 2 s with uniform acceleration of $a_G = 1.4715$ m/s^2 is determined by equation of kinematics,

$$s = ut + \frac{1}{2}at^2;$$

(u = 0, since door starts from rest)

$$s = 0 + \frac{1}{2} \times 1.4715 \times 2^2$$

$$s = \mathbf{2.943\ m} \qquad \text{... Ans.}$$

Example 5.26 : *The uniform pipe has a weight of 8000 N/m and diameter of 0.6 m. If it is hoisted as shown in Fig. 5.32. with an acceleration of 0.15 m/s², determine the internal moment at the centre A of the pipe due to the lift.*

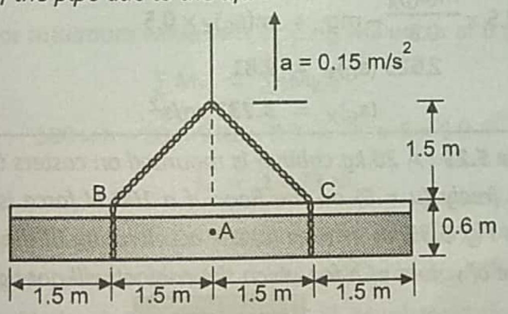


Fig. 5.32

Solution :

Given data :

Weight of pipe, W = 8000 × 6 = 48000 N

Diameter of pipe, D = 0.6 m

Acceleration of pipe, a = 0.15 m/s² (↑)

Let T be the tension in the rope and M_A be the internal moment at the centre A.

Consider F.B.D. of pipe. Using equation of rectilinear translation along x and y-axis,

$$\Sigma F_x = ma_x; \ (a_x = 0)$$

$$T_B \cos 45 - T_C \cos 45 = 0$$

$$T_B = T_C \qquad \text{... (1)}$$

$$\Sigma F_y = ma_y$$

$$T_B \sin 45 + T_C \sin 45 - 48000 = \frac{48000}{9.81} \times 0.15$$

From equation (1), substituting,

$$T_B = T_C = T$$

$$2 \times 0.707\,T = 48733.945$$

$$T = 34.465\ \text{kN}$$

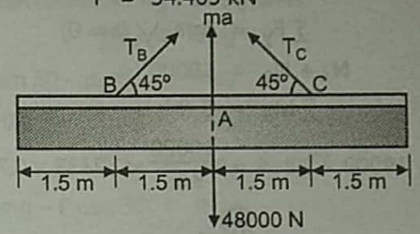


Fig. 5.32 (a) : F.B.D. of pipe

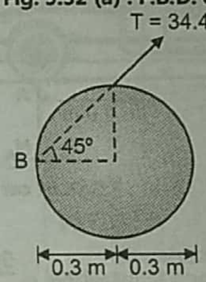


Fig. 5.32 (b) : c/s of pipe at B

Consider cross-section of pipe. Using equation of statics,

$$M_A = 34.465 \times \sin 45 \times 0.3$$

$$M_A = \mathbf{7.31\ kNm} \qquad \text{... Ans.}$$

Example 5.27 : *The car accelerates uniformly from rest to 26 m/s in 15 seconds. If it is having a weight of 19 kN and a centre of gravity of G, determine the normal reaction of each wheel on the pavement during the motion. Power is developed at the front wheels, whereas the rear wheels are free to roll. Neglect the mass of wheels and take the coefficient of static and kinetic friction to be* $\mu_s = 0.4$ *and* $\mu_k = 0.2$*, respectively.*

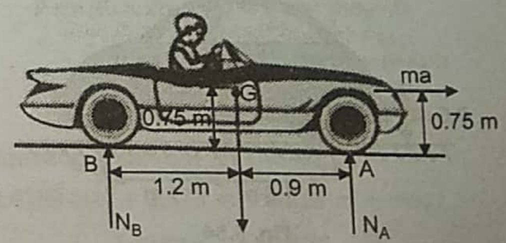


Fig. 5.33

Solution :

Given data :

Initial velocity of car, u = 0

Final velocity of car, v = 26 m/s

Required time, t = 15 s

Weight of car, W = 19 kN.

Coefficient of friction, $\mu_s = 0.4$ and $\mu_k = 0.2$

Let N_A be the normal reaction at front wheels and N_B be the normal reaction at rear wheels.

Consider F.B.D. of car. Using equation of rectilinear translation along x and y axes acceleration of mass centre of car is given by,

$$a_G = \frac{26-0}{15}$$

$$a_G = 1.7333 \text{ m/s}^2$$

$$\Sigma F_y = m(a_G)_y; \; (a = 0)$$

$$N_A + N_B = 19000 \quad \text{... (1)}$$

$$\Sigma M_B = \Sigma (M_k)_B$$

$$-2.1\, N_A + 19000 \times 2.2 = \frac{19000}{9.81} \times 1.7333 \times 0.75$$

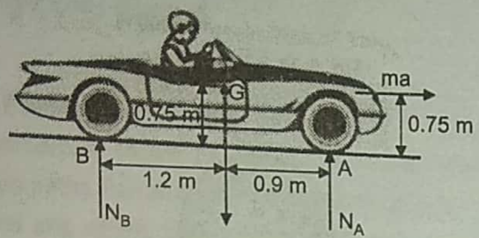


Fig. 5.33 (a) : F.B.D. of car

$$N_A = 9658.17 \text{ N}$$

$$\text{Normal reaction on each front wheel} = \frac{9658.17}{2}$$

$$= \mathbf{4.83\ kN} \quad \text{... Ans.}$$

From equation (1),

$$N_B = 19000 - 9658.17$$

$$N_B = 9341.83 \text{ N}$$

$$\text{Normal reaction on each rear wheel} = \frac{9341.83}{2}$$

$$= \mathbf{4.67\ kN} \quad \text{... Ans.}$$

Example 5.28 : *Knowing that the coefficient of static friction between the tyres and the road is 0.8 for the automobile shown in Fig. 5.34, determine the maximum possible acceleration on a level road, assuming (a) four wheel drive, (b) rear wheel drive.*

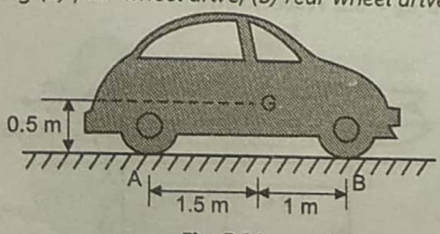


Fig. 5.34

Solution : Given data :

Coefficient of static friction, $\mu_s = 0.8$

Mass of automobile - m.

Let a_G be the acceleration of automobile.

(a) Four wheel drive : Consider F.B.D. of automobile. Using equation of rectilinear translation along x and y-axes. For four wheel drive,

$$N_A + N_B = N$$

$$\Sigma F_y = m(a_G)_y; \; [(a_G)_y = 0]$$

$$N_A + N_B - mg = 0$$

$$N = mg$$

$$\Sigma F_x = m(a_G)_x$$

$$F = m(a_G)_x$$

$$0.8\, N = m(a_G)_x$$

$$0.8 \times m \times g = m(a_G)_x$$

$$(a_G)_x = 0.8 \times 9.81$$

$$(a_G)_x = \mathbf{7.848\ m/s^2} \quad \text{... Ans.}$$

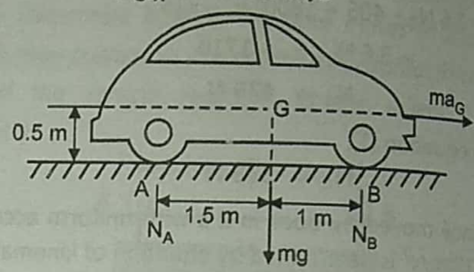


Fig. 5.34 (a) : F.B.D. of automobile

(b) Rear wheel drive :

$$\Sigma F_x = m(a_G)_x$$

$$0.8\, N_A = m(a_G)_x$$

$$N_A = \frac{m(a_G)_x}{0.8} \quad \text{... (2)}$$

$$\Sigma M_B = (M_k)_B$$

$$2.5\, N_A - mg \times 1 = m(a_G)_x \times 0.5$$

Substituting $N_A = \dfrac{m(a_G)_x}{0.8}$, we get,

$$2.5 \times \frac{m(a_G)_x}{0.8} - mg = m(a_G)_x \times 0.5$$

$$2.625\, (a_G)_x = 9.81$$

$$(a_G)_x = \mathbf{3.737\ m/s^2} \quad \text{... Ans.}$$

Example 5.29 : *A 20 kg cabinet is mounted on casters that allow it to move freely ($\mu = 0$) on the floor. If a 100 N force is applied as shown in Fig. 5.35, determine (a) the acceleration of the cabinet, (b) the range of values of h for which the cabinet will not tip.*

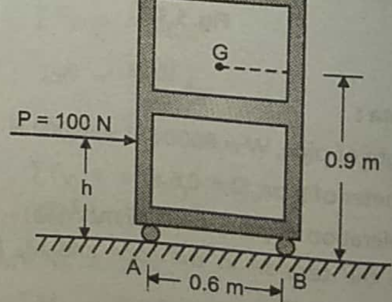


Fig. 5.35

Solution :

Given data :

Mass of cabinet, $m = 20$ kg

Force, $P = 100$ N

Coefficient of friction, $\mu = 0$

Let a_G be the acceleration of mass centre of cabinet along x-axis and h be the range for which the cabinet will not tip.

Consider F.B.D. of cabinet. Using equation of translation along x-axis,

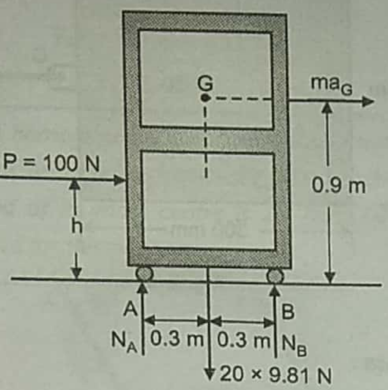


Fig. 5.35 (a) : F.B.D. of cabinet

$$\Sigma F_x = m(a_G)_x$$

$$100 = 20\,(a_G)_x$$

$$(a_G)_x = \mathbf{5\ m/s^2} \qquad \text{... Ans.}$$

For minimum value of h, tipping will occur at A. ($N_B = 0$)

$$\Sigma M_A = \Sigma (M_k)_A$$

$$100 \times h + 20 \times 9.81 \times 0.3 = 20 \times 5 \times 0.9$$

$$h = \mathbf{0.3114\ m} \qquad \text{... Ans.}$$

For maximum value of h, tipping will occur at B ($N_A = 0$).

$$\Sigma M_B = \Sigma (M_k)_B$$

$$100 \times h - 20 \times 9.81 \times 0.3 = 20 \times 5 \times 0.9$$

$$h = \mathbf{1.489\ m} \qquad \text{... Ans.}$$

Example 5.30 : *A rectangular crate, with its mass centre and geometric centre coinciding, is dragged by a rope. The coefficient of sliding friction is 0.10 and b/h = 1/2. At what acceleration will the crate start to tip ?*

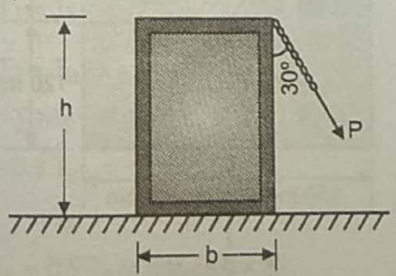


Fig. 5.36

Solution :

Given data :

Coefficient of friction, $\mu_s = 0.1$

Weight of crate, $W = mg$

Width of crate = b.

Height of crate, $h = 2b$.

Let a_x be the acceleration of the crate and P be the drag force.

Normal reaction acts at A is zero, since the crate starts to tip about point B. Consider F.B.D. of crate. Using equation of rectilinear translation along x and y axes,

$$\Sigma F_x = ma_x$$

$$P \sin 30 - \mu_s N_B = ma_x$$

$$0.5\,P - 0.1\,N_B = ma_x \qquad \text{... (1)}$$

$$\Sigma F_y = ma_y;\ (a_y = 0)$$

$$N_B - mg - P\cos 30 = 0$$

$$N_B = mg + P\cos 30$$

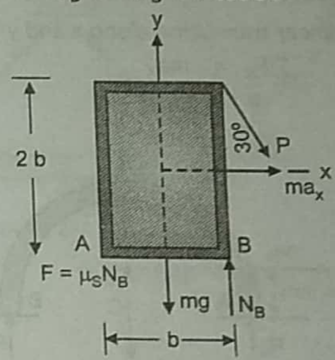


Fig. 5.36 (a) : F.B.D. of crate

Substituting, $N_B = (mg + P\cos 30)$ in equation (1),

$$0.5P - 0.1\,(mg + P\cos 30) = ma_x$$

$$0.5P = ma_x + 0.1\,mg + 0.1\,P\cos 30$$

$$P\,(0.5 - 0.1\cos 30) = m\,(a_x + 0.19)$$

$$0.4134\,P = m\,(a_x + 0.981)$$

$$P = 2.4189\,m\,(a_x + 0.981) \qquad \text{... (2)}$$

$$\Sigma M_B = \Sigma (M_k)_B$$

$$P \sin 30 \times 2b - mg \times \frac{b}{2} = ma_x \cdot b$$

$$2b \times 0.5P = 0.5\,mgb + ma_x\,b$$

$$P = 0.5\,mg + ma_x \qquad \text{... (3)}$$

Equating equations (2) and (3),

$$2.4189\,ma_x + 2.373\,m = 4.905\,m + ma_x$$

$$1.4189\,a_x = 2.532$$

$$a_x = \frac{2.532}{1.4189}$$

$$a_x = \mathbf{1.784\ m/s^2} \qquad \text{... Ans.}$$

Example 5.31 : *The motion of 1.4 kg rod AB is guided by two small wheels that roll freely in a horizontal slot cut in a vertical plate. If a force P of magnitude 22 N is applied at B, determine (a) the acceleration of the rod, (b) the reaction at A and B.*

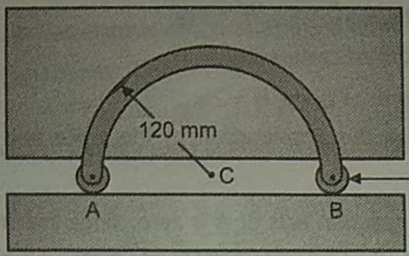


Fig. 5.37

Solution :

Given data :

Mass of rod AB, m = 1.4 kg

External force, P = 22 N

Radius of rod, r = 120 mm

Let a_x be the acceleration of the rod, R_A be the reaction at A and R_B be the reaction at B. Consider F.B.D. of rod AB. Using equation of rectilinear translation along x and y-axes,

$$\sum F_x = ma_x$$

$$P = 1.4\, a_x$$

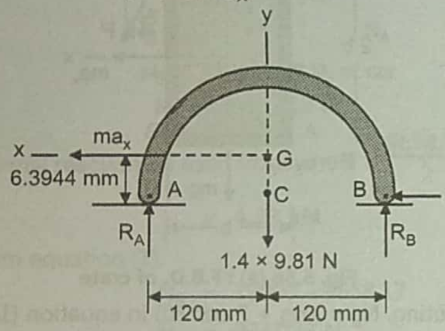


Fig. 5.37(a) : F.B.D. of rod AB

$$22 = 1.4\, a_x$$

$$a_x = \mathbf{15.714\ m/s^2} \qquad \text{... Ans.}$$

$$\sum F_y = ma_y;\ (a_y = 0)$$

$$R_A + R_B - 1.4 \times 9.81 = 0$$

$$R_A + R_B = 13.734 \qquad \text{... (1)}$$

Centroid of semicircular rod AB is at $\frac{r \sin \alpha}{\alpha}$ from centre C,

$$\bar{y} = \frac{120 \times \sin 90}{90 \times \frac{\pi}{180}}$$

$$\bar{y} = 76.3944 \text{ mm}$$

$$\sum M_A = \sum (M_k)_A$$

$$240 \times R_B - 1.4 \times 9.81 \times 120 = 1.4 \times 15.714 \times 76 \cdot 3944$$

$$R_B = \mathbf{13.87\ N\ (\uparrow)} \qquad \text{... Ans.}$$

From equation (1),

$$R_A = 13.734 - 13.87$$

$$R_A = -0.136 \text{ N}$$

$$R_A = \mathbf{0.136\ N\ (\downarrow)} \qquad \text{... Ans.}$$

Example 5.32 : *A uniform rectangular plate has a mass of 5 kg and is held in position by three ropes as shown in Fig. 5.38 immediately after rope CF has been cut. Determine : (a) the acceleration of the plate, (b) the tension in ropes AD and BE.*

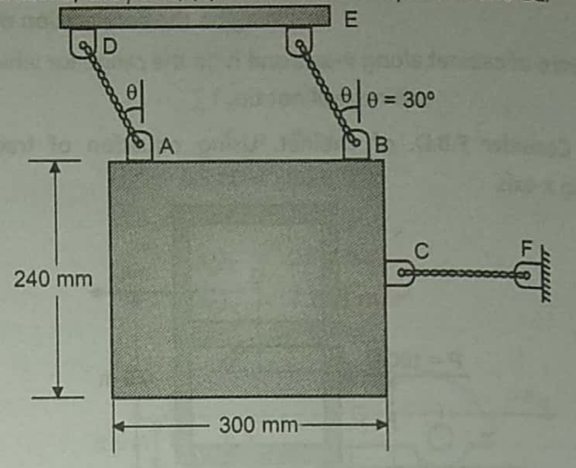


Fig. 5.38

Solution :

Given data :

Mass of plate, m = 5 kg

Position of ropes with vertical, θ = 30°

Let a_t be the tangential component of acceleration, T_A be the tension in rope AD and T_B be the tension in rope BE. Consider F.B.D. of plate. Positive sense of normal and tangential axes is shown in F.B.D. Using equation of curvilinear translation along normal and tangential axes,

$$\sum F_t = ma_t$$

$$5 \times 9.81 \times \cos 60 = 5\, a_t$$

$$a_t = \mathbf{4.905\ m/s^2} \qquad \text{... Ans.}$$

$$\sum F_n = ma_n;$$

(a_n = 0, since initial velocity of plate is zero)

$$\sum F_n = 0$$

$$T_A + T_B - 5 \times 9.81 \times \sin 60 = 0$$

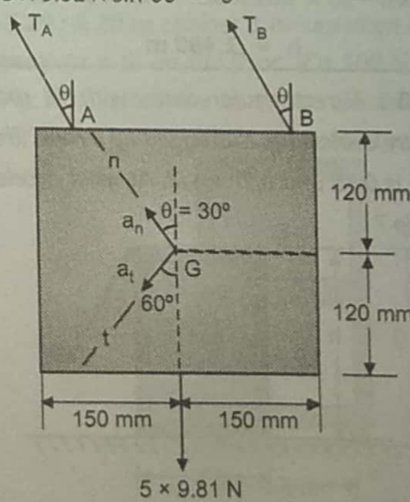


Fig. 5.38 (a) : F.B.D. of plate

$$T_A + T_B = 42.479 \quad \text{... (1)}$$

$$\sum M_G = 0$$

$T_A \cos 30 \times 150 - T_A \sin 30 \times 120 - T_B \sin 30 \times 120 - T_B \cos 30 \times 150 = 0$

$$129.9\ T_A - 60\ T_A - 60\ T_B - 129.9\ T_B = 0$$

$$T_A = 2.7167\ T_B$$

Substituting $T_A = 2.7167\ T_B$ in equation (1),

$$2.7167\ T_B + T_B = 42.479$$

$$T_B = \mathbf{11.43\ N} \quad \text{... Ans.}$$

From equation (1),

$$T_A = 42.479 - 11.43$$

$$T_A = \mathbf{31.05\ N} \quad \text{... Ans.}$$

Example 5.33 : *A homogeneous triangular plate weighing 18 N is suspended by cords AC and BD. When the plate reaches its lowest position, the speed of its mass centre is 2.4 m/s. Determine the tension in each cord for this position.*

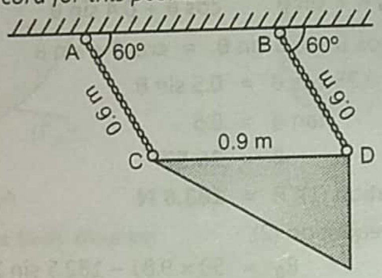


Fig. 5.39

Solution :

Given data :

Weight of plate, W = 18 N

Velocity of plate at lowest position (θ = 90°), v = 2.4 m/s.

Let T_A be the tension in cord AC and T_B be the tension in cord CD.

Consider F.B.D. of plate. At the lowest position of plate, cords AC and BD become vertical. Using equation of curvilinear translation along normal and tangential axes,

$$\sum F_n = ma_n$$

$$T_A + T_B - 18 = \frac{m \cdot v^2}{\rho}$$

$$T_A + T_B = \frac{18}{9.81} \times \frac{(2.4)^2}{0.6} + 18$$

$$T_A + T_B = 35.615 \quad \text{... (1)}$$

$$\sum M_G = 0$$

$$0.6\ T_A - 0.3\ T_B = 0$$

$$T_A = 0.5\ T_B$$

Substituting $T_A = 0.5\ T_B$ in equation (1),

$$1.5\ T_B = 35.615$$

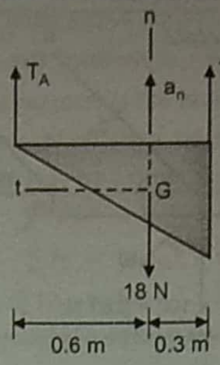


Fig. 5.39 (a) : F.B.D. of plate

$$T_B = \mathbf{23.74\ N} \quad \text{... Ans.}$$

From equation (1),

$$T_A = 35.615 - 23.74$$

$$T_A = \mathbf{11.87\ N} \quad \text{... Ans.}$$

Example 5.34 : *The uniform pole of mass 100 kg is suspended in the horizontal position by 3 wires as shown in Fig. 5.40. If the wire CB breaks, calculate the tension in the wire BD immediately after break. Also find the acceleration of the pole.*

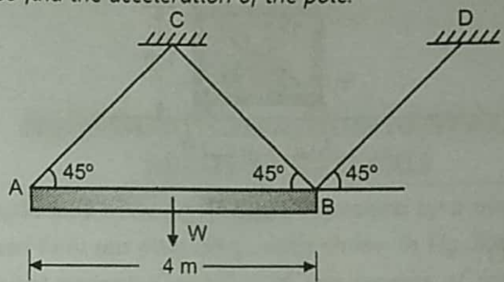


Fig. 5.40

Solution :

Given data :

Mass of pole, m = 100 kg

Length of pole, L = 4 m

Let T_A be the tension in wire AC and T_B be the tension in wire BD.

Consider F.B.D. of pole. Using equation of curvilinear translation along normal and tangential axes,

$$\sum F_n = ma_n;$$

$$\left(a_n = \frac{v^2}{\rho}, \text{ initial velocity is zero, hence } a_n = 0\right)$$

$$T_A + T_B - 100 \times 9.81 \times \cos 45 = 0$$

$$T_A + T_B = 693.67 \quad \text{... (1)}$$

$$\sum M_G = 0$$

$$T_A \sin 45 \times 2 - T_B \sin 45 \times 2 = 0$$

$$T_A = T_B \quad \text{... (2)}$$

Solving equations (1) and (2),

$$T_A = T_B = \mathbf{346.84\ N} \quad \text{... Ans.}$$

$$\sum F_t = ma_t$$

$$100 \times 9.81 \times \sin 45 = 100\ a_t$$

$$a_t = \mathbf{6.94\ m/s^2} \quad \text{...Ans.}$$

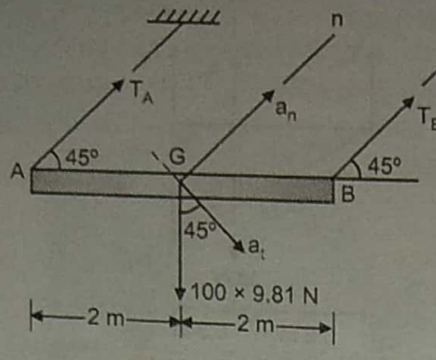


Fig. 5.40 (a) : F.B.D. of pole

Example 5.35 : *The 50 kg packing crate shown in Fig. 5.41 is pulled by a rope attached at corner D. Knowing that μ_s = 0.40 and μ_k = 0.30 between the crate and floor, determine (a) the value of θ and P for which sliding the tipping of the crate are both impending, (b) the acceleration of the crate if P is then slightly increased.*

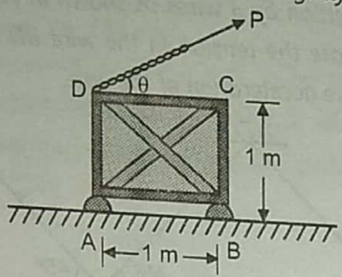


Fig. 5.41

Solution :

Given data :

Mass of crate, m = 50 kg

Coefficient of friction, $\mu_s = 0.4$ and $\mu_k = 0.3$

Let a_x be the acceleration of the crate when P is slightly increased, P be the force and θ be the position of force for which sliding and tipping are both impending.

(a) Tipping and sliding both are impending. Tipping will occur at point B, hence reaction at A should be zero i.e. $R_A = 0$. Using equations of statics,

$$\Sigma M_B = 0$$

$$P\cos\theta \times 1 + P\sin\theta \times 1 - mg \times 0.5 = 0$$

$$P(\cos\theta + \sin\theta) = 50 \times 9.81 \times 0.5$$

$$P = \frac{245.25}{\cos\theta + \sin\theta} \quad \text{... (1)}$$

$$\Sigma F_y = 0$$

$$R_B - mg + P\sin\theta = 0$$

$$R_B = mg - P\sin\theta \quad \text{... (2)}$$

$$\Sigma F_x = 0$$

$$P\cos\theta - \mu_s R_B = 0$$

Substituting $R_B = (mg - P\sin\theta)$,

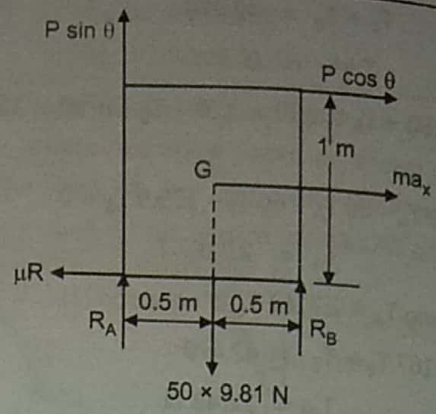


Fig. 5.41 (a)

$$P\cos\theta - 0.4(mg - P\sin\theta) = 0$$

$$P\cos\theta - 0.4 \times 50 \times 9.81 + 0.4 P\sin\theta = 0$$

$$P(\cos\theta + 0.4\sin\theta) = 196.2$$

$$P = \frac{196.2}{(\cos\theta + 0.4\sin\theta)} \quad \text{... (3)}$$

Equating equations (1) and (3),

$$\frac{245.25}{\cos\theta + \sin\theta} = \frac{196.2}{\cos\theta + 0.4\sin\theta}$$

$$1.25\cos\theta + 0.5\sin\theta = \cos\theta + \sin\theta$$

$$0.25\cos\theta = 0.5\sin\theta$$

$$\tan\theta = 0.5$$

$$\theta = \mathbf{26.57°} \quad \text{... Ans.}$$

From equation (1), $P = \mathbf{182.8\ N}$ ... Ans.

(b) From equation (2),

$$R_B = 50 \times 9.81 - 182.5 \sin 26.57$$

$$R_B = 408.74$$

$$\Sigma F_x = ma_x$$

$$P\cos\theta - \mu_k R_B = ma_x$$

$$182.8\cos 26.57 - 0.3 \times 408.74 = 50\, a_x$$

$$a_x = \mathbf{0.818\ m/s^2} \quad \text{... Ans.}$$

5.10 ROTATION ABOUT A FIXED AXIS

Consider a rigid body shown in Fig. 5.42 which is rotating in the vertical plane about a fixed axis perpendicular to the paper and passing through point P. The angular velocity and acceleration are caused by the external forces acting on the body. The mass centre (G) of body travels in a circular path. The acceleration of the mass centre is represented by its normal and tangential components. The magnitude of tangential component of acceleration is $(a_G)_t = r\alpha$ and it must act in a direction which is consistent with the angular acceleration of body. The magnitude of normal component of acceleration is $(a_G)_n = r\omega^2$. The normal component of acceleration is always directed towards the centre of rotation irrespective of angular velocity of rigid body.

The free body diagram and kinetic diagram of the rigid body are shown in Fig. 5.42 (a) and (b). The weight of the body, W = mg and component of pin reaction along normal $(F_p)_n$ and tangential direction $(F_p)_t$ are shown in F.B.D., since these are the external forces.

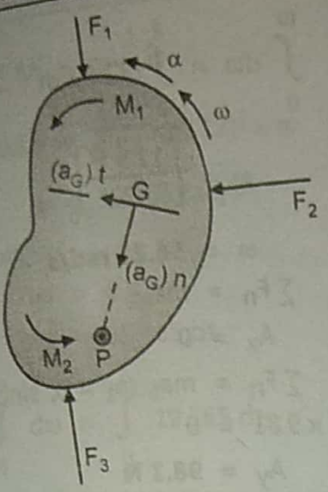
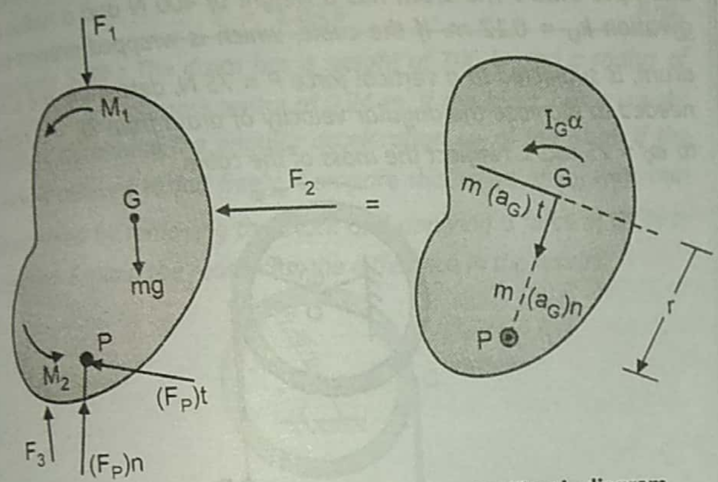


Fig. 5.42



(a) Free body diagram

(b) Kinetic diagram

Fig. 5.42

The two components of force i.e. $m(a_G)_n$ and $m(a_G)_t$ shown in kinetic diagram are associated with normal and tangential components of acceleration of the rigid body. The components of force are acting in the direction of acceleration and have a magnitude of $m(a_G)_n$ and $m(a_G)_t$. The vector $I_G\alpha$ acts in the direction of α and has a magnitude of $I_G\alpha$, where I_G is the mass moment of inertia of the body about the axis perpendicular to paper through mass centre G and α is the angular acceleration of the rigid body. The equation of motion of the rigid body for rotation about a fixed axis may be written in the following form :

$$\sum F_n = m(a_G)_n = mr\omega^2$$
$$\sum F_t = m(a_G)_t = mr\alpha \qquad \text{... (5.8)}$$
$$\sum M_G = I_G\alpha$$

The moment equation may be replaced by moment summation about any other point (on or off the body) provided it must consider the moment $\sum (M_k)_p$ produced by $I_p\alpha$, $m(a_G)_t$ and $m(a_G)_n$ about the point. In most of the problems, it is convenient to sum moments about the pin at p in order to eliminate the unknown force F_P. From the kinetic diagram,

$$\sum M_P = \sum (M_k)_P$$
$$\sum M_P = r\, m(a_G)_t + I_G\alpha$$

The moment of $m(a_G)_n$ is not carried in the summation, since the line of action of this force passes through P. Substituting $(a_G)_t = r\alpha$, we may write above equation as $\sum \mathbf{M_O} = (\mathbf{I_G} + \mathbf{mr^2})\,\alpha$.

As per the parallel axis theorem, $I_P = I_G + md^2$, and therefore the term in the bracket represents the moment of inertia of the body about the fixed axis of rotation passing through P. The three equations of motion may be written as

$$\sum F_n = m(a_G)_n = mr\omega^2$$
$$\sum F_t = m(a_G)_t = mr\alpha$$
$$\sum M_p = I_p\alpha$$

$I_P\alpha$ represents moment of both $m(a_G)_t$ and $I_G\alpha$ about point P, Fig. 5.42 (b). In other words,

$$\sum M_P = \sum (M_k)_P = I_P\alpha$$

If the complete solution cannot be obtained by the equation of motion, then we can use following relations.

For variable angular acceleration,

$$\alpha = \frac{d\omega}{dt},\ \omega = \frac{d\theta}{dt} \text{ and } \alpha\, d\theta = \omega\, d\omega$$

For constant angular acceleration,

$$\omega = \omega_0 + \alpha t$$
$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

and

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

NUMERICAL EXAMPLES ON ROTATION ABOUT A FIXED AXIS

Example 5.36 : *The 80 kg disk is supported by a pin at A. If it is released from rest from the position shown in Fig. 5.43, determine the initial horizontal and vertical components of reaction at the pin.*

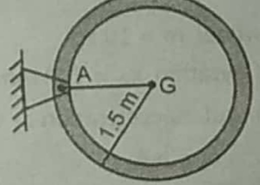


Fig. 5.43

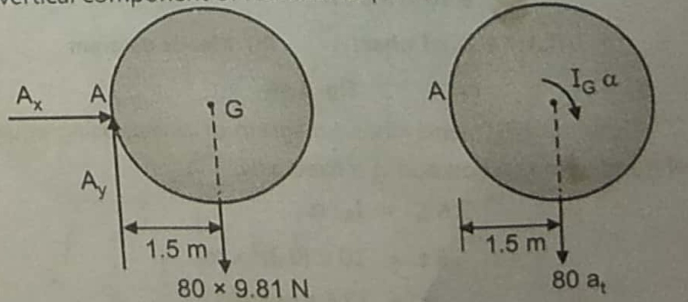
Solution :

Given data :

Mass of disk, m = 80 kg

Radius of disk, r = 1.5 m.

Let A_x be the horizontal component of reaction and A_y be the vertical component of reaction at pin A.



(a) F.B.D. of disk

(b) Kinetic diagram

Fig. 5.43

Considering F.B.D. and kinetic diagram of disk. Using equation of motion for rotation about a fixed axis.

$$\sum M_A = \sum (M_k)_A$$

$$90 \times 9.81 \times 1.5 = \left[\frac{80 \times (1.5)^2}{2} + 80 \times (1.5)^2\right] \alpha$$

$$\alpha = 4.36 \text{ rad/s}^2$$

$$\sum F_n = m(a_G)_n = mr\omega^2; \ (\omega = 0)$$

$$A_x = \mathbf{0} \quad \text{... Ans.}$$

$$\sum F_y = m(a_G)_t = mr\alpha$$

$$-A_y + 80 \times 9.81 = 80 \times 1.5 \times 4.36$$

$$-A_y = 80 \times 1.5 \times 4.36 - 80 \times 9.81$$

$$-A_y = -261.6$$

$$A_y = \mathbf{261.6\ N} \quad \text{... Ans.}$$

Example 5.37 : *The 10 kg wheel has a radius of gyration k_A = 200 mm. If the wheel is subjected to a moment M = (5t) N.m, where t is in seconds, determine its angular velocity when t = 3 s starting from rest. Also, compute the reaction which the fixed pin A exerts on the wheel during the motion.*

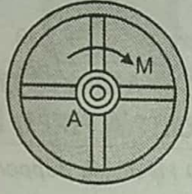


Fig. 5.44

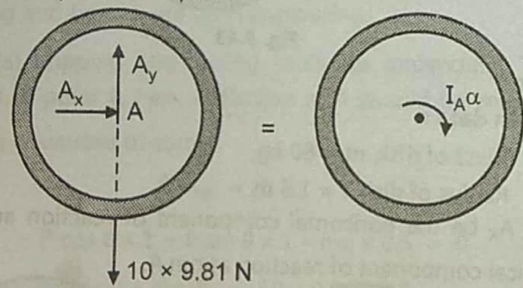
Solution :

Given data :

Mass of wheel, m = 10 kg

Radius of gyration, k_A = 200 mm

Let α be the angular acceleration at t = 3 s, A_x and A_y be the reaction components at pin A.



(a) F.B.D. of wheel **(b) Kinetic diagram**

Fig. 5.44

Consider F.B.D. and kinetic diagram of wheel. Using equation of motion for rotation about a fixed axis,

$$\sum M_A = I_A \cdot \alpha$$

$$5t = 10 \times (0.2)^2 \times \alpha$$

$$\alpha = 12.5\ t \text{ rad/s}^2$$

Using $d\omega = \alpha\, dt$,

$$\int_0^{\omega} d\omega = \int_0^{3} 12.5\ t\ dt$$

$$\omega = \left[\frac{12.5\ t^2}{2}\right]_0^3$$

$$\omega = \mathbf{56.25\ rad/s} \quad \text{... Ans.}$$

$$\sum F_n = ma_n; \quad (a_n = 0)$$

$$A_x = \mathbf{0} \quad \text{... Ans.}$$

$\sum F_n = ma_t$; (a_t = 0, since wheel is rotating about A)

$$A_y - 10 \times 9.81 = 0$$

$$A_y = \mathbf{98.1\ N} \quad \text{... Ans.}$$

Example 5.38 : *The drum has a weight of 400 N and a radius of gyration k_O = 0.12 m. If the cable, which is wrapped around the drum, is subjected to a vertical force P = 75 N, determine the time needed to increase the angular velocity of drum from ω_1 = 5 rad/s to ω_2 = 25 rad/s. Neglect the mass of the cable.*

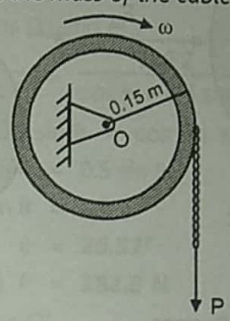


Fig. 5.45

Solution :

Given data :

Weight of drum, W = 400 N

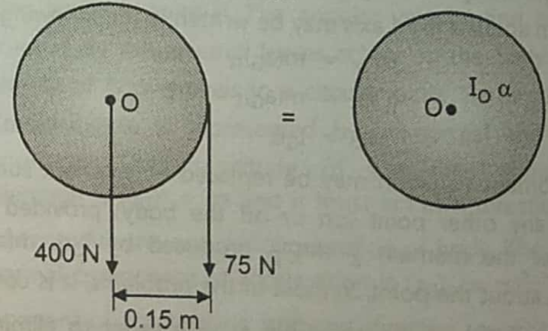
Radius of gyration, k_O = 0.12 m

Vertical force, P = 75 N

Initial angular velocity, ω_1 = 5 rad/s

Final angular velocity, ω_2 = 25 rad/s.

Let t be the required time to increase the angular velocity of drum from ω = 5 rad/s to 25 rad/s.



(a) F.B.D. of drum **(b) Kinetic diagram**

Fig. 5.45 (a)

Consider free body and kinetic diagram of drum. Using equation of moment for rotation about a fixed axis,

$$\sum M = I_O \alpha$$

$$\Sigma M = mk_o^2 \alpha$$

$$75 \times 0.15 = \frac{400}{9.81} \times (0.12)^2 \times \alpha$$

$$\alpha = 19.165 \text{ rad/s}^2$$

Using the relation,

$$d\omega = \alpha \, dt$$

$$d\omega = 19.165 \, dt$$

Integrating, $$\int_5^{25} d\omega = \int_0^t 19.165 \, dt$$

$$20 = 19.165 \, t$$

$$t = \mathbf{1.04 \ s} \qquad \text{... Ans.}$$

Example 5.39 : *The drum has a weight of 100 N and a radius of gyration about its mass centre of 0.24 m. If the block has a weight of 60 N, determine the angular acceleration α_D of the drum if the block is allowed to fall freely. Compare this value of α_D with that determined by removing the block and applying a force of 60 N to the cord. Explain the reason for the difference in the results.*

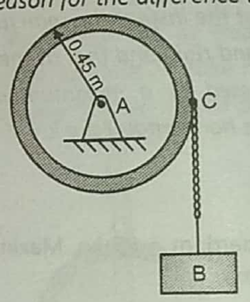


Fig. 5.46

Solution :

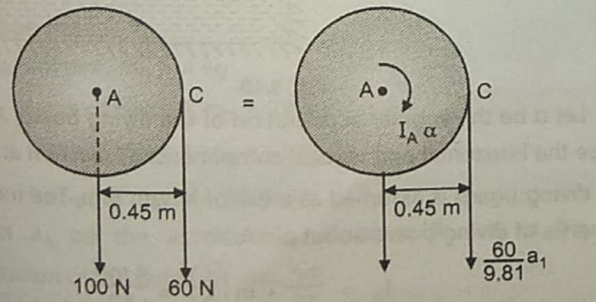
Given data :

Weight of drum, W = 100 N

Radius of gyration, k_A = 0.24 m

Weight of block B, W_B = 60 N.

Let α_D be the angular acceleration of drum when the block is allowed to fall freely. Consider free body and kinetic diagram of drum.



(a) F.B.D. of drum **(b) Kinetic diagram of drum**

Fig. 5.46

Using equation of moment about A,

$$\Sigma M_A = \Sigma (M_k)_A = (I_A \alpha + m a_t \times r)$$

$$60 \times 0.45 = \frac{100 \times (0.24)^2}{9.81} \alpha_D + \frac{60}{9.81} a_t \times 0.45;$$

(substituting $a_t = r\alpha$)

$$60 \times 0.45 = 0.587 \, \alpha_D + 6.11621 \times 0.45 \, \alpha_D \times 0.45$$

$$27 = 0.587 \, \alpha_D + 1.2385 \, \alpha_D$$

$$\alpha_D = \mathbf{14.8 \ rad/s^2} \qquad \text{... Ans.}$$

Angular acceleration of drum is obtained by replacing a block with a force of 60 N. When the block is replaced by a force of 60 N, the tangential component of acceleration of force at point C is zero. Using equation of moment about A,

$$\Sigma M_A = I_A \alpha_D$$

$$60 \times 0.45 = \frac{100}{9.81} \times (0.24)^2 \times \alpha_D$$

$$\alpha_D = \mathbf{46 \ rad/s^2} \qquad \text{... Ans.}$$

Example 5.40 : *The spool is supported on small rollers at A and B. Determine the angular acceleration of the spool and the normal forces at A and B if a vertical force P = 80 N is applied to the cable. The spool has a mass of 60 kg and a radius of gyration k_O = 0.65 m. Neglect mass of the cable and the mass of the rollers.*

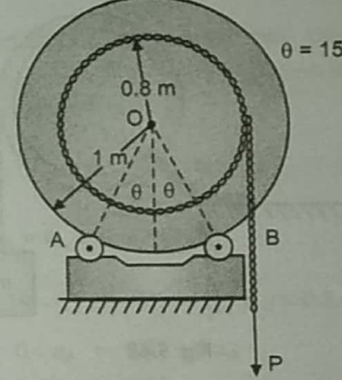
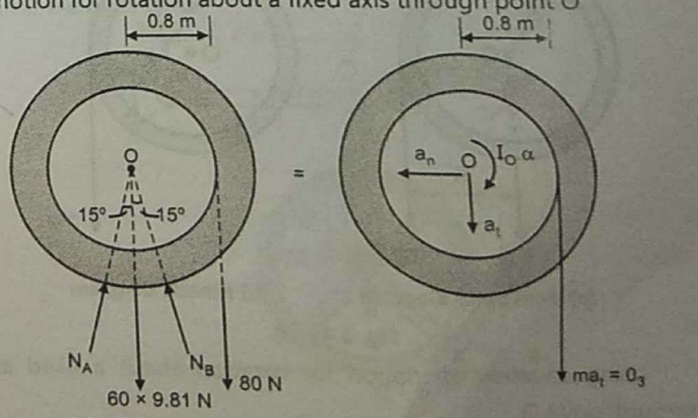


Fig. 5.47

Solution :

Given data : Vertical force, P = 80 N, Mass of spool, M = 60 kg, Radius of gyration of spool, k_O = 0.65 m.

Let α be the angular acceleration of the spool, N_A and N_B be the normal force at A and B. (Initial normal reaction). Consider free body diagram and kinetic diagram of the spool. Use equation of motion for rotation about a fixed axis through point O



(a) F.B.D. of spool **(b) Kinetic diagram of spool**

Fig. 5.47

A force P = 80 N is not having tangential component of acceleration.

$$\Sigma M_O = I_O \alpha$$

$$80 \times 0.8 = 60 \times (0.65)^2 \times \alpha$$

$$\alpha = \mathbf{2.52\ rad/s^2} \quad \text{... Ans.}$$

$$\Sigma F_n = ma_n \quad (a_n = 0)$$

$$N_A \sin 15 - N_B \sin 15 = 0$$

$$N_A = N_B \quad \text{... (1)}$$

$$\Sigma F_t = ma_t; \ (a_t = 0)$$

$$-N_A \cos 15 - N_B \cos 15 + 60 \times 9.81 + 80 = 0$$

$$N_A + N_B = 692.18 \quad \text{... (2)}$$

Solving equations (1) and (2),

$$N_A = N_B = \mathbf{346.1\ N} \quad \text{... Ans.}$$

Example 5.41 : *The disk has a mass M and a radius R. If a block of mass m is attached to the cord, determine the angular acceleration of the disk when the block is released from rest. Also through what distance the block falls from rest in time t ?*

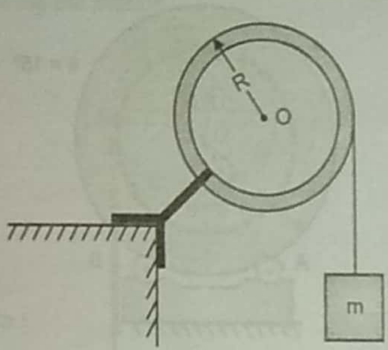


Fig. 5.48

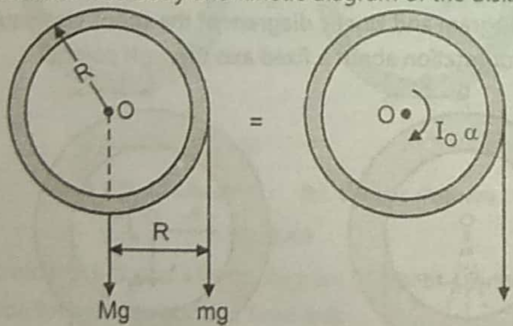
Solution :

Given data :

M - mass of disk, R - radius of disk and m – mass of block.

Let α be the angular acceleration of block and s be the distance through which the block falls from rest in time t.

Consider free body and kinetic diagram of the disk.



(a) Free body diagram **(b) Kinetic diagram**

Fig. 5.48 (a)

Using equation of motion for rotation about a fixed axis through point O,

$$\Sigma M_O = \Sigma (M_k)_O$$

$$mgR = \frac{MR^2}{2}\alpha + mR \times R\alpha$$

$$2mgR = MR^2\alpha + 2\,mR^2\alpha$$

$$2\,mg = (M + 2m)\,R\alpha$$

$$\alpha = \mathbf{\frac{2mg}{(M + 2m)\,R}} \quad \text{... Ans.}$$

Using equation of kinematics,

$$s = ut + \frac{1}{2}at^2; \ \text{... } (u = 0)$$

$$s = 0 + \frac{1}{2}a_t\,t^2;$$

$$\text{...}\left[a_t = R\alpha = \left(\frac{2mg}{M + 2m}\right)\right]$$

$$s = \frac{1}{2} \times \frac{2mg}{(M + 2m)}\,t^2$$

$$s = \mathbf{\frac{mgt^2}{(M + 2m)}} \quad \text{... Ans.}$$

Example 5.42 : *Determine the angular acceleration of the 25 kg diving board and the horizontal and vertical components of reaction at the pin A at the instant the man jumps off. Assume that the board is uniform and rigid and that at the instant he jumps off, the spring is compressed by a maximum amount of 200 mm, ω = 0, and the board is horizontal. Take k = 7 kN/m.*

Solution :

Given data :

Mass of diving board, m = 25 kg, Maximum compression of spring, x_{max} = 200 mm.

Initial angular velocity, ω = 0. Spring constant, k = 7 kN/m.

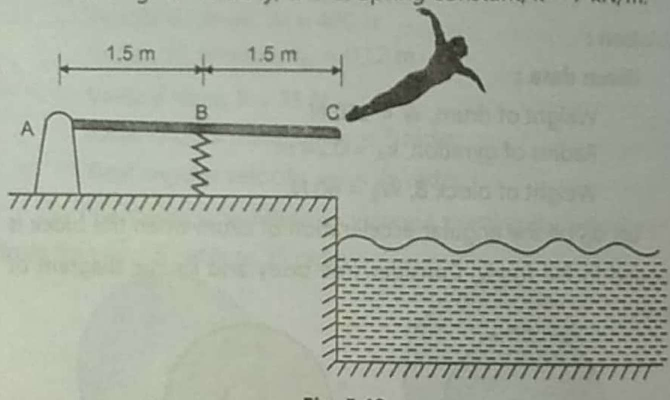


Fig. 5.49

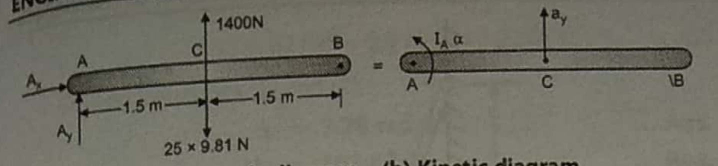
Let α be the angular acceleration of the diving board, A_x and A_y be the horizontal and vertical components of reaction at pin A. The diving board is assumed as a bar of length 3 m. The moment of inertia of diving board about pin A,

$$I_A = \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2 = \frac{4\,ml^2}{12}$$

$$I_A = \frac{ml^2}{3} = \frac{25 \times 3^2}{3} = 75\ \text{N.m}^2$$

$$\text{Spring force, } k_S = 7 \times 0.2 \times 10^3 = 1400\ \text{N}$$

Consider F.B.D. and kinetic diagram of diving board.



(a) Free body diagram (b) Kinetic diagram

Fig. 5.49

Using equation of motion for rotation about a fixed axis through point A,

$$\Sigma M_A = I_A\alpha$$

$$1400 \times 1.5 - 25 \times 9.81 \times 1.5 = 75 \times \alpha$$

$$\alpha = 23.095 \text{ rad/s}^2 \qquad \textbf{... Ans.}$$

$$\Sigma F_y = ma_y$$

$$A_y - 25 \times 9.81 + 1400 = 25 \times 1.5 \times 23.095;$$

$$\ldots \left(a_y = r \text{ or } \frac{l}{2} \times \alpha\right)$$

$$A_y = 288.69 \text{ N} \qquad \textbf{... Ans.}$$

$$\Sigma F_x = ma_x;$$

($a_x = 0$, since initial angular velocity $\omega = 0$)

$$A_x = \mathbf{0} \qquad \text{... } \textbf{Ans.}$$

***Example 5.43 :** The two blocks A and B have a mass of 10 kg and 15 kg respectively. If the pulley can be treated as a 4 kg disk, determine the acceleration of block A. Take r = 200 mm. Neglect the mass of the cord and any slipping on the pulley.*

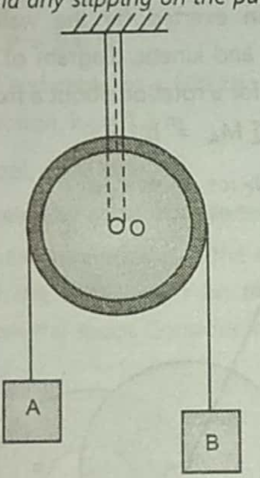


Fig. 5.50

Solution :

Given data :

Mass of disk, $m_D = 4$ kg

Mass of block A, $m_A = 10$ kg

Mass of block B, $m_B = 15$ kg

Radius of disk, r = 200 mm

Let a_A be the acceleration of block A and α be the acceleration of the pulley or disk.

Consider F.B.D. of block B. Using equation of motion (↓ positive),

$$\Sigma F_y = ma_y$$

$$15 \times 9.81 - T_2 = 15\, a_B$$

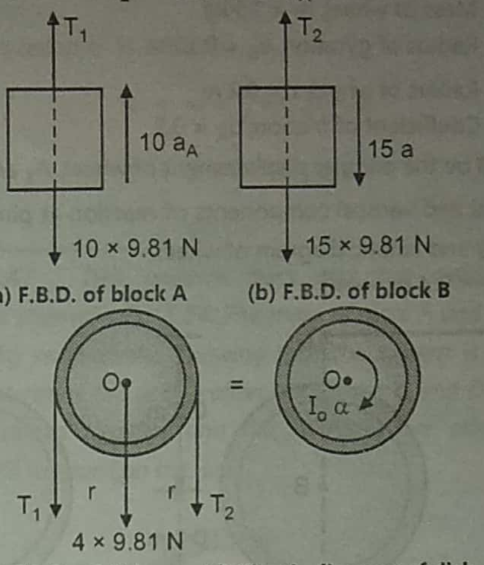
$$T_2 = 147.15 - 15\, a_B \qquad \text{... (1)}$$

Consider F.B.D. of block A. Using equation of motion,

$$\Sigma F_y = ma_y$$

$$10 \times 9.81 - T_1 = -10\, a_A$$

$$T_1 = 98.1 + 10\, a_A \qquad \text{... (2)}$$



(a) F.B.D. of block A (b) F.B.D. of block B

(c) F.B.D. of disk (d) Kinetic diagram of disk

Fig. 5.50

Consider F.B.D. and kinetic diagram of disk. Using equation of motion for rotation about a fixed axis of rotation through O,

$$\Sigma M_O = I_O\alpha$$

$$T_2 \times r - T_1 \times r = \frac{m_D r^2}{2} \cdot \alpha$$

Substituting $a_B = a_A$ and $\alpha = \dfrac{a_A}{0.2}$

$$(147.15 - 15\, a_A - 98.1 - 10\, a_A) \times 0.2 = \frac{4 \times (0.2)^2}{2} \cdot \frac{a_A}{0.2}$$

$$0.4\, a_A = 9.81 - 5\, a_A$$

$$5.4\, a_A = 9.81$$

$$a_A = \mathbf{1.82\ m/s^2} \qquad \textbf{... Ans.}$$

***Example 5.44 :** The wheel of mass 25 kg and a radius of gyration $k_B = 0.15$ m. It is spinning at 40 rad/s. If it is placed on the ground, for which the coefficient of kinetic friction is $\mu_k = 0.5$, determine the angular displacement of the wheel required to stop the motion. Also find the components of reaction at pin A during this time. Neglect the mass of AB.*

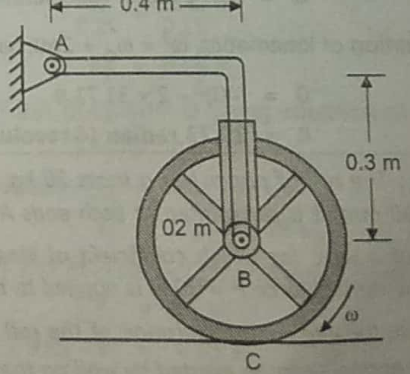


Fig. 5.51

Solution :

Given data :

Initial angular velocity, $\omega_0 = 40$ rad/s

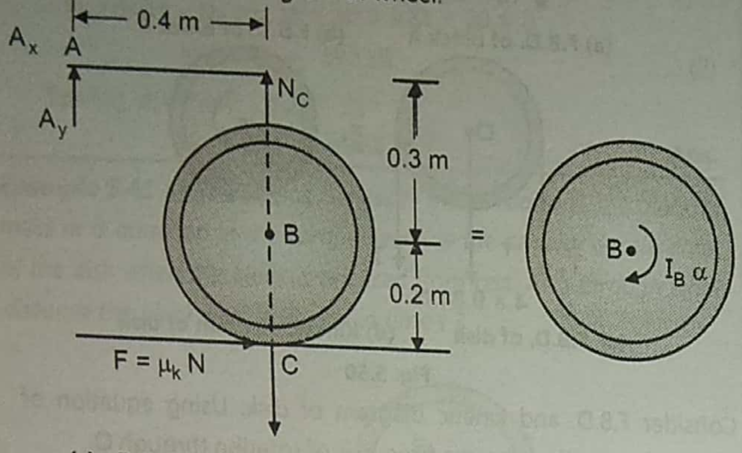
Mass of wheel, m = 25 kg

Radius of gyration, $k_B = 0.15$ m

Radius of wheel, r = 0.2 m

Coefficient of friction, $\mu_k = 0.5$.

Let θ be the angular displacement of wheel, A_x and A_y be the horizontal and vertical components of reaction at pin A. Consider free body and kinetic diagram of wheel.



(a) Free body diagram of wheel (b) Kinetic diagram of wheel

Fig. 5.51

Using equation of motion for rotation about a fixed axis through point B,

$$\sum M_B = I_B \cdot \alpha$$

$$-\mu_k N_C \times 0.2 = 25 \times (0.15)^2 \times \alpha$$

$$-0.5\, N_C \times 0.2 = 5.625\, \alpha$$

$$N_C = -5.625\, \alpha \qquad \ldots (1)$$

$$\sum M_A = \sum (M_k)_A$$

$$25 \times 9.81 \times 0.4 - N_C \times 0.4 - 0.5\, N_C \times 0.5 = 25 \times (0.15)^2 \times \alpha$$

$$98.1 - 0.65\, N_C = 0.5625\, \alpha$$

Substituting, $N_C = -5.625\, \alpha$,

$$98.1 + 3.656\, \alpha = 0.5625\, \alpha$$

$$3.09375\, \alpha = -98.1$$

$$\alpha = -31.71 \text{ rad/s}^2$$

$$\alpha = 31.71 \text{ rad/s}^2 \text{ (deceleration)}$$

Using equation of kinematics, $\omega^2 = \omega_0^2 + 2\alpha\theta$; $(\omega = 0)$

$$0 = (40)^2 - 2 \times 31.71\, \theta$$

$$\theta = \mathbf{25.23 \text{ radian (4 revolutions)}} \ldots \textbf{Ans.}$$

Example 5.45 : *The roll of paper has a mass 20 kg and radius of gyration k_A = 90 mm. It is pin jointed at both ends A and B. If the roll rests against a wall, for which coefficient of kinetic friction is μ_k = 0.2 and the vertical force P = 30 N is applied to the end of the paper, determine the angular acceleration of the roll as the paper unrolls and the normal reaction exerted by wall on the roll.*

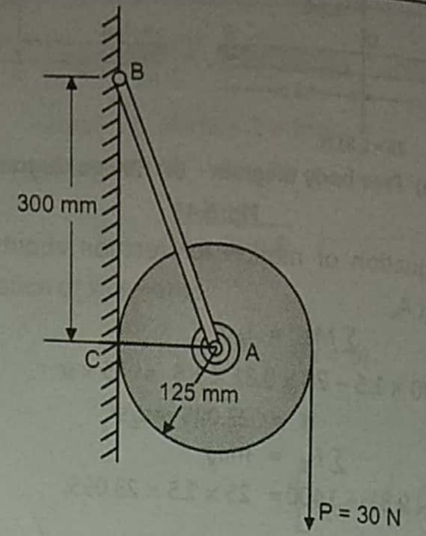


Fig. 5.52

Solution :

Given data :

Mass of roll, m = 20 kg

Radius of gyration, $k_A = 0.09$ m

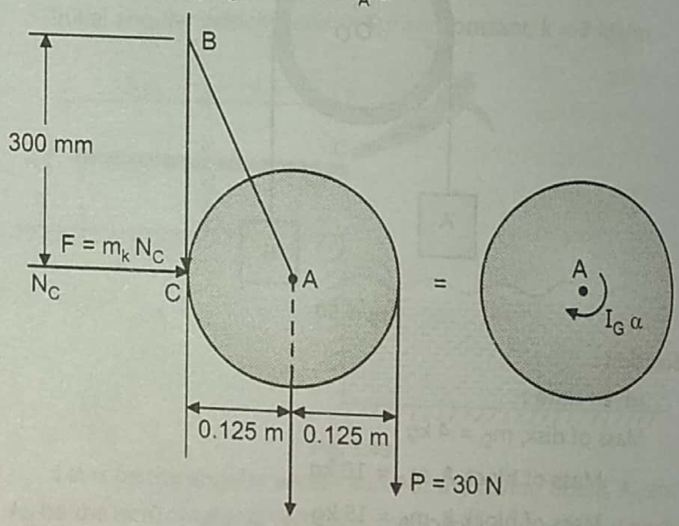
Coefficient of kinetic friction, $\mu_k = 0.2$

Vertical force, P = 30 N.

Let α be the angular acceleration of the paper roll and N_C be the normal reaction exerted by the wall on the paper roll. Consider free body and kinetic diagram of the paper roll. Using equation of motion for a rotation about a fixed axis,

$$\sum M_A = I_A \cdot \alpha$$

$$P \times r - \mu_k N_C \times r = m\, k_A^2\, \alpha$$



(a) Free body diagram of paper roll (b) Kinetic diagram of paper roll

Fig. 5.52

$$30 \times 0.125 - 0.2\, N_C \times 0.125 = 20 \times (0.09)^2 \times \alpha$$

$$3.75 - 0.025\, N_C = 0.162\, \alpha \qquad \ldots (1)$$

$$\sum M_B = \sum (M_k)_B$$

$$20 \times 9.81 \times 0.125 + 30 \times 0.250 - N_C \times 0.3 = 20 \times (0.09)^2 \times \alpha$$

$$32.025 - 0.3\ N_C = 0.162\ \alpha \quad \text{... (2)}$$

Solving equations (1) and (2),

$$\alpha = \mathbf{7.28\ rad/s^2} \quad \text{... Ans.}$$

$$N_C = \mathbf{102.82\ N} \quad \text{... Ans.}$$

***Example 5.46 :** Cable is unwound from a spool supported on small rollers at A and B by exerting a force P = 300 N on the cable. Compute the time needed to move 5 m of cable from the spool if the spool and cable have a total mass of 600 kg and radius of gyration k_O = 1.2 m. Neglect the mass of cable being unwound and mass of rollers at A and B.*

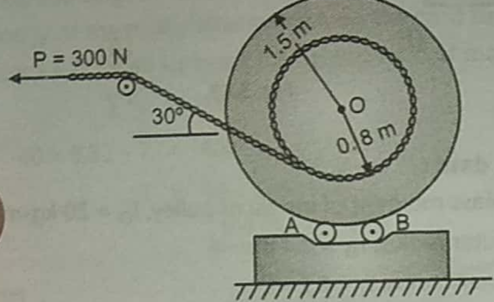


Fig. 5.53

Solution :

Given data :

Pulling force, P = 300 N

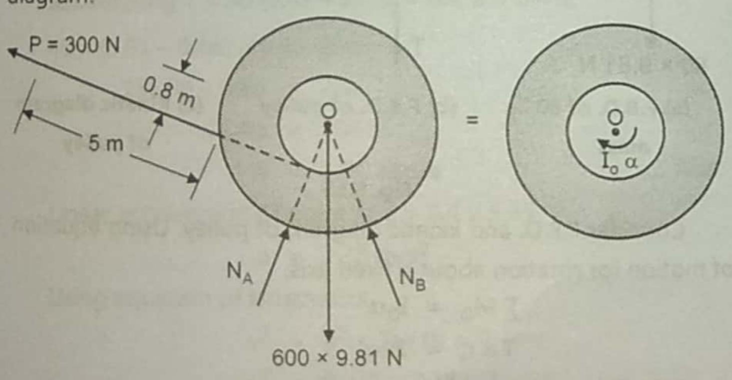
Mass of pool and cable, m = 600 kg

Radius of gyration, k_O = 1.2 m

Radius of spool, r = 0.8 m.

Distance traveled by cable from spool, s = 5 m.

Let α be the angular acceleration of the spool, $a = r\alpha$ be the linear acceleration of the cable and t be the time required to move 5 m of cable from the spool. Consider free body and kinetic diagram.



(a) Free body diagram of spool (b) Kinetic diagram of spool

Fig. 5.53

Using equation of motion for a rotation about a fixed axis,

$$\sum M_O = I_O \alpha$$

$$300 \times 0.8 = 600 \times (1.2)^2 \times \alpha$$

$$\alpha = \frac{300 \times 0.8}{600 \times (1.2)^2}\ \text{rad/s}^2$$

Linear velocity of cable,

$$a = r\alpha$$

$$a = \frac{0.8 \times 300 \times 0.8}{600 \times (1.2)^2} = 0.22222\ \text{m/s}^2$$

Using equation of kinematics,

$$s = ut + \frac{1}{2}at^2,\ (u = 0)$$

$$5 = 0 + \frac{1}{2} \times 0.22222\ t^2$$

$$t = \mathbf{6.71\ s} \quad \text{... Ans.}$$

***Example 5.47 :** Two uniform discs and two cylinders are assembled as shown in Fig. 5.54. The mass of discs A and B are 20 kg and 12 kg respectively. Knowing that the system is released from rest, determine the acceleration of cylinder C and D. Discs A and B are bolted together and the cylinders are attached to separate cords wrapped on the discs.*

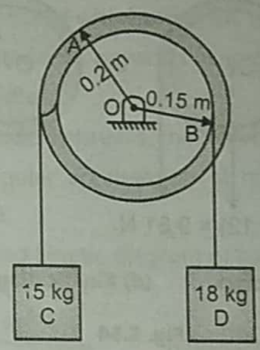


Fig. 5.54

Solution :

Given data :

Mass of disc A, m_A = 20 kg, Radius of disc A, r_A = 0.2 m

Mass of disc B, m_B = 12 kg, Radius of disc B, r_B = 0.15 m

Mass of cylinder C, m_C = 15 kg, mass of cylinder D, m_D = 1.8 kg.

Let a_C be the linear acceleration of cylinder C, a_D be the linear acceleration of cylinder D and α be the angular acceleration of discs.

$$a = r\alpha \text{ or } \alpha = \frac{a}{r}$$

$$\frac{a_C}{0.2} = \frac{a_D}{0.15}$$

$$a_D = 0.75\ a_C \quad \text{... (1)}$$

Consider F.B.D. of cylinder D. Using equation of motion,

$$\sum F_y = m_D\ a_D$$

$$18 \times 9.81 - T_1 = 18\ a_D$$

$$T_1 = 176.58 - 18\ a_D \quad \text{... (2)}$$

Consider F.B.D. of cylinder C. Using equation of motion,

$$\sum F_y = m_C\ a_C$$

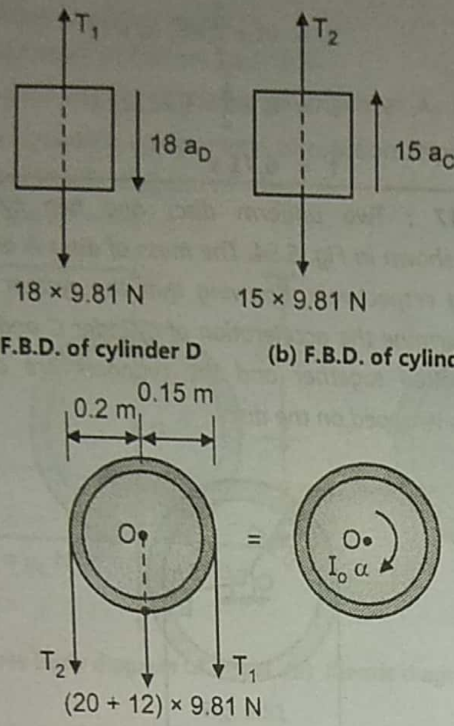
$$15 \times 9.81 - T_2 = -15\ a_C$$

$$T_2 = 147.15 + 15\ a_C \quad \text{... (3)}$$

Consider F.B.D. and kinetic diagram of discs. Using equation of motion for rotation about a fixed axis,

$$\Sigma M_O = \Sigma I_O \alpha$$

$$T_1 \times r_B - T_2 \times r_A = \frac{m_B r_B^2}{2}\alpha + \frac{m_A r_A^2}{2}\alpha$$



(a) F.B.D. of cylinder D (b) F.B.D. of cylinder C

(c) F.B.D. of discs (d) Kinetic diagram of discs

Fig. 5.54

$$(176.58 - 18a_D) \times 0.15 - (147.15 + 15a_C) \times 0.2 = \frac{12 \times (0.15)^2}{2}$$

$$\alpha_B + \frac{20 \times (0.2)^2}{2}\alpha_A$$

Substituting $a_D = 0.75\, a_C$, $\alpha_B = \frac{a_D}{0.15} = \frac{0.75\, a_C}{0.15}$, $\alpha_A = \frac{a_C}{0.2}$,

$$26.487 - 2.025\, a_C - 29.43 - 3a_C = \frac{12 \times (0.15)^2}{2} \times \frac{0.75\, a_C}{0.15} + \frac{20 \times (0.2)^2}{2} \times \frac{a_C}{0.2}$$

$$-2.943 - 5.025\, a_C = 0.675\, a_C + 2a_C$$

$$7.7a_C = -2.943$$

$$a_C = -0.382 \text{ m/s}^2$$

$$a_C = \mathbf{0.382 \text{ m/s}^2 (\uparrow)} \quad \text{... Ans.}$$

From equation (1), $a_D = 0.75 \times 0.382$

$$a_D = \mathbf{0.287 \text{ m/s}^2 (\downarrow)} \quad \text{... Ans.}$$

Example 5.48 : *Each of the double pulleys shown in Fig. 5.55 has a mass moment of inertia of 20 kg.m² and is initially at rest. The outside radius is 0.4 m and the inner radius is 0.2 m. Find (a) the angular acceleration of each pulley, (b) the angular velocity of each pulley after point A on the cord has moved 3 m.*

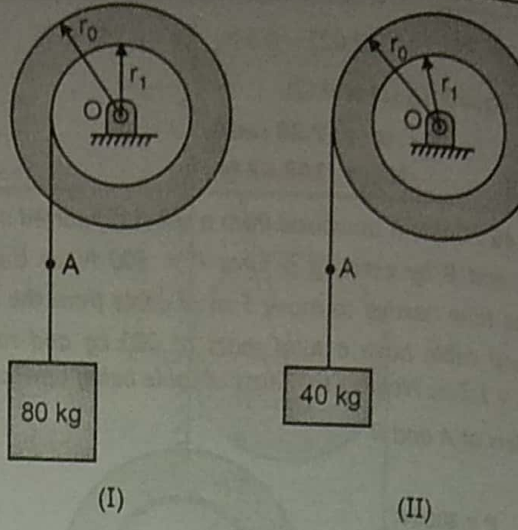


Fig. 5.55

Solution :

Given data :

Mass moment of inertia of pulley, $I_O = 20$ kg-m²

Outer radius, $r_O = 0.4$ m.

Inner radius, $r_i = 0.2$ m.

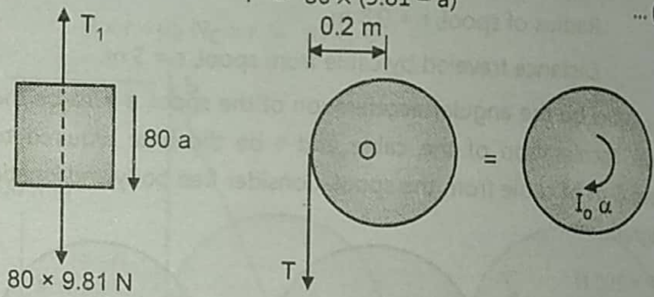
Case I : Mass of block, m = 80 kg.

Let α be the angular acceleration of pulley and ω be the angular velocity of the pulley after point A on the cord has moved 3 m. Consider F.B.D. of 80 kg block. Using equation of motion,

$$\Sigma F_y = ma$$

$$80 \times 9.81 - T = 80a$$

$$T = 80 \times (9.81 - a) \quad \text{... (1)}$$



(a) F.B.D. of 80 kg mass (b) F.B.D. of pulley (c) Kinetic diagram of pulley

Fig. 5.55

Consider F.B.D. and kinetic diagram of pulley. Using equation of motion for rotation about a fixed axis,

$$\Sigma M_O = I_O \alpha$$

$$T \times r_i = 20\alpha$$

Substituting T = 80 (9.81 − a) and $r_1 = 0.2$,

$$80 \times (9.81 - a)\, 0.2 = 20\alpha$$

Substituting $a = 0.2\alpha$,

$$80 \times (9.81 - 0.2\alpha) \times 0.2 = 20\alpha$$

$$156.96 = 23.2\alpha$$

$$\alpha = \mathbf{6.77 \text{ rad/s}^2} \quad \text{... Ans.}$$

Linear acceleration of point A,

$$a = 0.2 \times 6.77$$
$$a = 1.354 \text{ m/s}^2$$

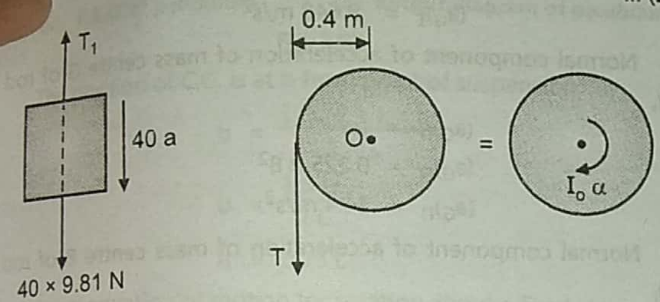
Using equation of kinematics, $v^2 = u^2 + 2as$

$$v^2 = 0 + 2 \times 1.354 \times 3; \ (u = 0)$$
$$v = 2.85 \text{ m/s}$$
$$\omega = \frac{2.85}{0.2}; \text{ (using } \omega = r\omega)$$
$$\omega = \mathbf{14.25 \ rad/s} \qquad \text{... Ans.}$$

Case II : Mass of block, m = 40 kg

Let α be the angular acceleration of pulley and ω be the angular velocity of the pulley after point A on the cord has moved 3 m. Consider F.B.D. of 40 kg block. Using equation of motion,

$$\Sigma F_y = ma$$
$$40 \times 9.81 - T = 40a$$
$$T = 40 \times (9.81 - a) \qquad \text{... (2)}$$



(d) F.B.D. of 40 kg mass (e) F.B.D. of pulley (f) Kinetic diagram of pulley

Fig. 5.55

Consider free body and kinetic diagram of pulley. Using equation of motion for rotation about a fixed axis,

$$\Sigma M_O = I_O\alpha$$
$$Tr_O = I_O\alpha$$

Substituting $T = 40\ (9.81 - a)$, $r_O = 0.4$, $a = 0.4\ \alpha$,

$$40 \times (9.81 - 0.4\alpha) \times 0.4 = 20\alpha$$
$$156.96 - 6.4\alpha = 20\alpha$$
$$26.4\alpha = 156.96$$
$$\alpha = \mathbf{5.96 \ rad/s} \qquad \text{... Ans.}$$

Linear acceleration of point A, $a = 0.4 \times 5.95$

$$a = 2.38 \text{ m/s}^2$$

Using equation of kinematics,

$$v^2 = u^2 + 2as \ (u = 0)$$
$$v^2 = 0 + 2 \times 2.38 \times 3$$
$$v = 3.78 \text{ m/s}$$

Angular velocity of point A,

$$\omega = \frac{v}{r_o}$$
$$\omega = \frac{3.78}{0.4}$$
$$\omega = \mathbf{9.45 \ rad/s^2} \qquad \text{... Ans.}$$

Example 5.49 : *The slender rod weighing 400 N is supported by cords AB and BC. If cord AC suddenly breaks, determine the initial angular acceleration of the bar and tension in the cord AB. (Refer Fig. 5.56).*

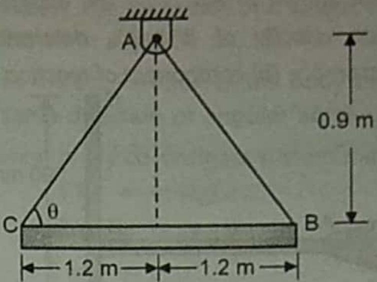


Fig. 5.56

Solution :

Given data :

Weight of rod, W = 400 N

Length of rod, L = 2.4

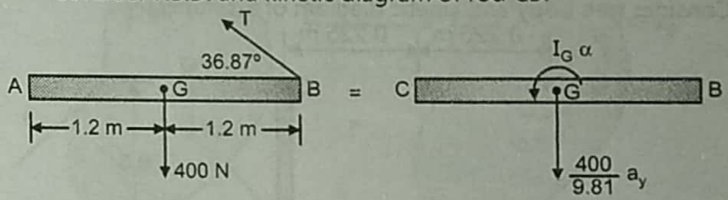
Orientation of cord with horizontal, $\theta = 36.87°$

The example is similar to previous example. The approach of solution is slightly different.

If the cord AC suddenly breaks, the bar rotates about point B.

Let α be the angular acceleration of rod AB and T be the tension in the wire AB.

Consider F.B.D. and kinetic diagram of rod CB.



(a) F.B.D. of rod **(b) Kinetic diagram of rod**

Fig. 5.56 (a)

Using equation of motion for a rotation about a fixed axis,

$$\Sigma F_y = ma_y$$
$$400 - T \sin 36.87 = \frac{400}{9.81} a_y$$
$$0.6\ T = 400 - 40.774\ a_y$$

Substituting $a_y = 1.2\ \alpha$, $0.6\ T = 400 - 40.774 \times 1.2\ \alpha$

$$0.6\ T = 400 - 48.93\ \alpha \qquad \text{... (1)}$$
$$\Sigma M_B = \Sigma (M_k)_B$$
$$400 \times 1.2 = I_G\alpha + \frac{400}{9.81} a_y \times 1.2; \left(I_G = \frac{ML^2}{12}\right)$$
$$480 = \frac{400 \times (2.4)^2}{12} \alpha + \frac{400}{9.81} \times 1.2\ \alpha \times 1.2$$
$$480 = 78.2875\ \alpha$$
$$\alpha = \mathbf{6.13 \ rad/s^2} \qquad \text{... Ans.}$$

From equation (1),

$$0.6\ T = 400 - 48.93 \times 6.13$$
$$T = \mathbf{166.7 \ N} \qquad \text{... Ans.}$$

Example 5.50 : *Two uniform slender rods AB with mass of 3 kg and CD with mass of 4 kg, are welded together to form 'T' shaped assembly which swings freely about A in vertical plane. Knowing that at the instant shown in Fig. 5.57, the assembly has counter clockwise angular velocity of 8 rad/s, determine (a) angular acceleration of assembly, (b) component of reaction at A.*

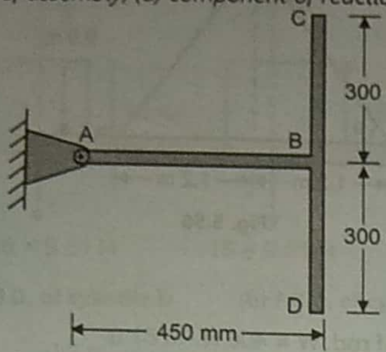


Fig. 5.57

Solution :

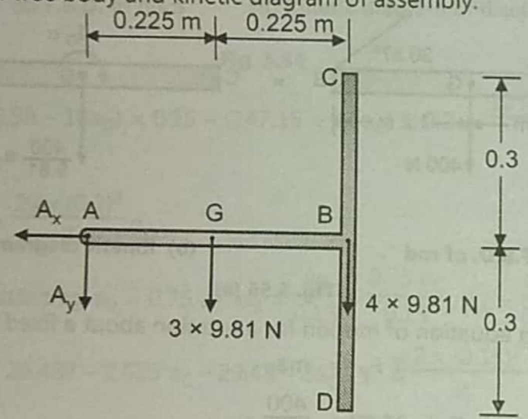
Given data :

Mass of rod AB, $m_A = 3$ kg

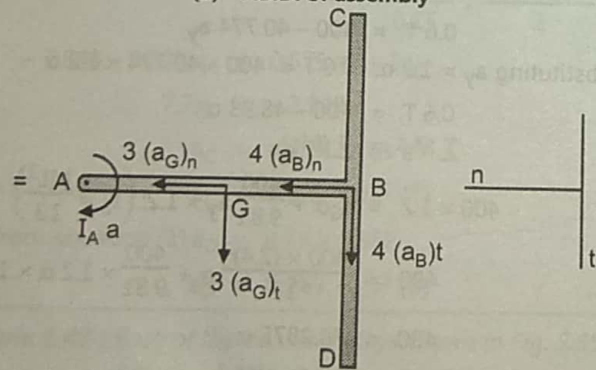
Mass of rod CD, $m_C = 4$ kg

Angular velocity of assembly, $\omega = 8$ rad/s

Let α be the angular acceleration of assembly, A_x and A_y be the horizontal and vertical components of reaction at hinge A. Consider free body and kinetic diagram of assembly.



(a) F.B.D. of assembly



(b) Kinetic diagram of assembly

Fig. 5.57

Using equation of motion of a rotation about a fixed axis,

$$\Sigma M_A = I_A \alpha$$

$$3 \times 9.81 \times 0.225 + 4 \times 9.81 \times 0.45 = (\text{M.I. AB} + \text{M.I. of CD}) \text{ about A} \times \alpha$$

$$24.28 = \left(\frac{3 \times (0.45)^2}{3} + \frac{4 \times (0.6)^2}{12} + 4 \times (0.45)^2\right) \alpha$$

$$24.28 = 1.1325\, \alpha$$

$$\alpha = \mathbf{21.44\ rad/s^2} \qquad \text{... Ans.}$$

Tangential component of acceleration of mass centre G of rod AB,

$$(a_G)_t = r\alpha;\ (r = 0.45/2)$$

$$(a_G)_t = \frac{0.45}{2} \times 21.44$$

$$(a_G)_t = 4.824 \text{ m/s}^2$$

Tangential component of acceleration of mass centre of B of rod CD,

$$(a_G)_t = r\alpha;\ (\alpha = 0.45)$$

$$(a_G)_t = 0.45 \times 21.44$$

$$(a_G)_t = 9.648 \text{ m/s}^2$$

Normal component of acceleration of mass centre G of rod AB,

$$(a_G)_n = r\omega^2$$

$$(a_G)_n = 0.225 \times 8^2$$

$$(a_G)_n = 14.4 \text{ m/s}^2$$

Normal component of acceleration of mass centre B of rod CD,

$$(a_G)_n = r\omega^2 = 0.45 \times 8^2$$

$$= 28.8 \text{ m/s}^2$$

Now using equation of motion along normal and tangential axis,

$$\Sigma F_n = ma_n$$

$$A_x = 3 \times 14.4 + 4 \times 28.8$$

$$A_x = \mathbf{158.4\ N} \qquad \text{... Ans.}$$

$$\Sigma F_t = ma_t$$

$$-A_y + 3 \times 9.81 + 4 \times 9.81 = 3 \times 4.824 + 4 \times 9.648$$

$$-A_y = -15.606$$

$$A_y = \mathbf{15.606\ N} \qquad \text{... Ans.}$$

Example 5.51 : *Develop a formula for the period t for small oscillation of the compound pendulum shown in Fig. 5.58. Treat OD and AB as identical slender bars of uniform cross-section.*

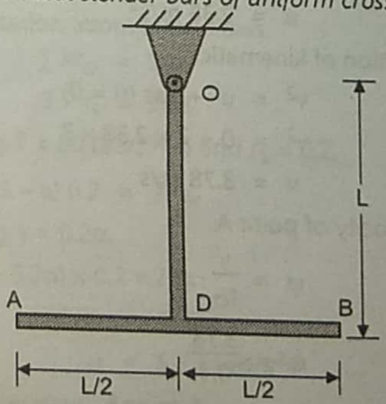
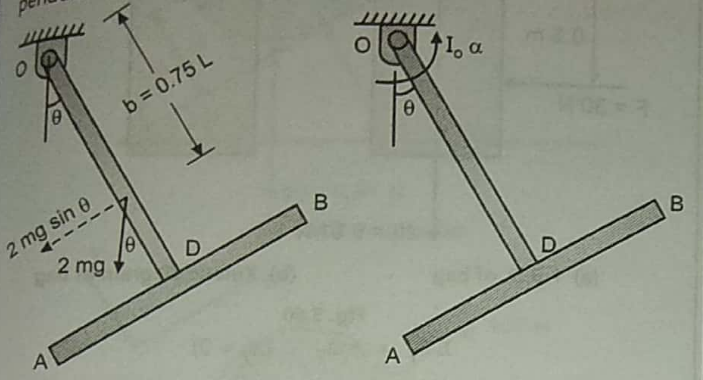


Fig. 5.58

Solution : Let I_O be the moment of inertia of the body about axis of rotation O, α be the angular acceleration of the body and b be the distance between the point of suspension and centre of gravity. Consider free body and kinetic diagram of the compound pendulum as shown in Fig. 5.58 (a), (b).



(a) F.B.D. of pendulum **(b) Kinetic diagram of pendulum**

Fig. 5.58

The position of C.G. is at b from point of suspension

$$b = \frac{L \cdot L/2 + L \times L}{L + L}$$

$$b = \frac{1.5\, L^2}{2\, L}$$

$$b = 0.75\, L$$

Using equation of motion for rotation about a fixed axis,

$$\Sigma M_O = I_O \alpha$$

$$-2mg \sin\theta \times b = I_O\alpha; (\sin\theta = \theta, \text{ since angular displacement is small})$$

$$-2\, mg\, \theta\, b = I_O \alpha$$

$$\alpha + \frac{2\, mg\, \theta\, b}{I_O} = 0 \qquad \text{... (1)}$$

Comparing with equation of simple pendulum, $\ddot{\theta} + P^2\theta = 0$,

$$P^2 = \frac{2\, mg\, \theta\, b}{I_O}$$

$$\text{Periodic time, } t = \frac{2\pi}{P}$$

$$t = 2\pi\sqrt{\frac{I_O}{2mg\, b}} \qquad \text{... (2)}$$

Moment of inertia @ O,

$$I_O = \frac{mL^2}{3} + \frac{mL^2}{12} + mL^2$$

$$I_O = \frac{17\, mL^2}{12}$$

Substituting $I_O = \frac{17\, mL^2}{12}$ and $b = \frac{3}{4}$ L in equation (2),

$$t = 2\pi\sqrt{\frac{\frac{17}{12} mL^2}{2mg \cdot \frac{3}{4} L}}$$

$$t = 2\pi\sqrt{\frac{17}{18} \times \frac{L}{g}} \qquad \text{... Ans.}$$

5.11 GENERAL PLANE MOTION

When the rigid body shown in Fig. 5.59 is subjected to external applied force and moment system, then a rigid body undergoes a general plane motion. The free body and kinetic diagrams for the rigid body are shown in Fig. 5.59 (a), (b). The force ma_G has the same direction as the acceleration of the body's mass centre and $I_G\alpha$ acts in the same direction of angular acceleration. The three equations of motion in x-y co-ordinate system may be written as :

$$\Sigma F_x = m\, (a_G)\, x$$
$$\Sigma F_y = m\, (a_G)\, y \qquad \text{... (5.9)}$$
$$\Sigma M_G = I_G\, \alpha$$

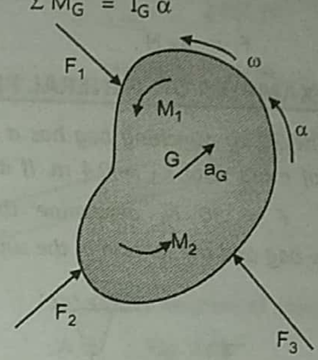
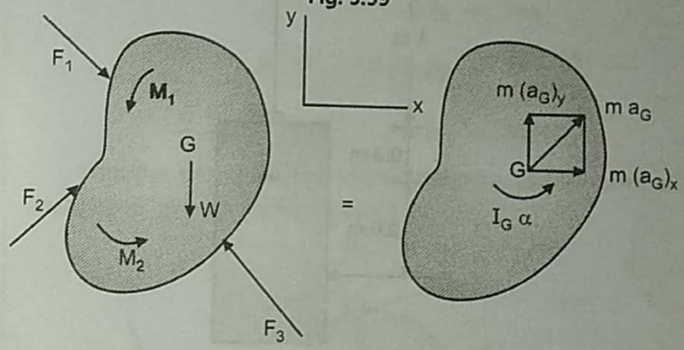


Fig. 5.59



(a) Free body diagram **(b) Kinetic diagram**

Fig. 5.59

In some of the problems, it may be convenient to sum the moments about any point P other than G, in order to eliminate unknown forces from the moment summation. The three equations of motion may be written as :

$$\Sigma F_x = m\, (a_G)\, x$$
$$\Sigma F_y = m\, (a_G)\, y \qquad \text{... (5.10)}$$
$$\Sigma M_P = \Sigma\, (M_K)_P$$

where $\Sigma\, (M_K)_P$ is moment summation of $I_G\alpha$ and ma_G about P as determined from the kinetic diagram. If a complete solution cannot be obtained from the equation of motion, then the complete solution can be obtained by using equation of kinematics.

Rolling Without Slipping : If the frictional force F is enough to allow the body to roll without slipping, then a_G may be related to α by the equation of kinematics,

$$a_G = r\alpha \qquad \text{... (5.11)}$$

The unknowns are determined by using three equations of motion and one equation of kinematics. For rolling without slipping condition, $F \le \mu_S N$, where μ_S is coefficient of static friction. If $F > \mu_S N$, then the solution may be obtained, considering rolling with slipping.

Rolling With Slipping : In this case, α and a_G are independent of one another so that equation (5.11) does not apply. In rolling with slipping condition, the magnitude of frictional force is related to magnitude of normal reaction using the coefficient of kinetic friction μ_K

i.e. $$F = \mu_K N \quad \text{... (5.12)}$$

NUMERICAL EXAMPLES ON GENERAL PLANE MOTION

***Example 5.52 :** The 20 kg punching bag has a radius of gyration about its centre of mass G of k_G = 0.4 m. If it is subjected to a horizontal force F = 30 N, determine the initial angular acceleration of the bag and the tension in the supporting cable AB.*

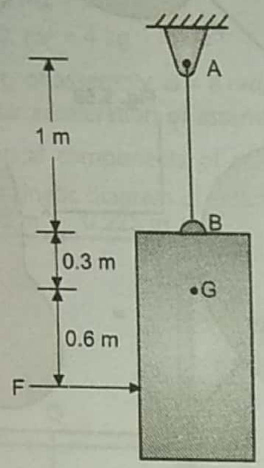


Fig. 5.60

Solution :

Given data :

Mass of punching bag, m = 20 kg

Radius of gyration, k_G = 0.4 m

Horizontal force, F = 30 N

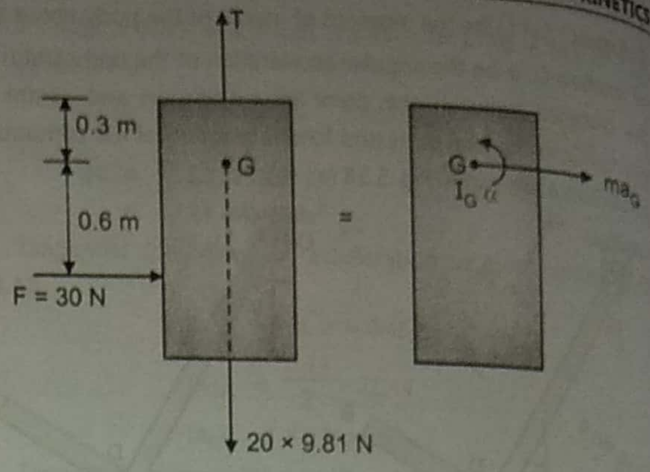
Let α be the angular acceleration of the bag and T be the tension in the supporting cable AB.

Consider the free body and kinetic diagram of the bag. Using equation of general plane motion,

$$M_G = I_G \alpha$$

$$00 \times 0.6 = 20 \times (0.4)^2 \times \alpha$$

$$\therefore \quad \alpha = \mathbf{5.625\ rad/s^2} \quad \text{... Ans.}$$



(a) F.B.D. of bag **(b) Kinetic diagram of bag**

Fig. 5.60

$$\Sigma F_y = ma_y \quad \text{... } (a_y = 0)$$

$$T - 20 \times 9.81 = 0$$

$$\therefore \quad T = \mathbf{196.2\ N} \quad \text{... Ans.}$$

***Example 5.53 :** A uniform cylinder of mass 10 kg and diameter 1 m is being pulled along an inclined plane by a force of 100 N, acting through the mass centre as shown in Fig. 5.61. Find the frictional force necessary to have rolling without slipping. Also find corresponding angular acceleration of the cylinder and linear acceleration of its mass centre.*

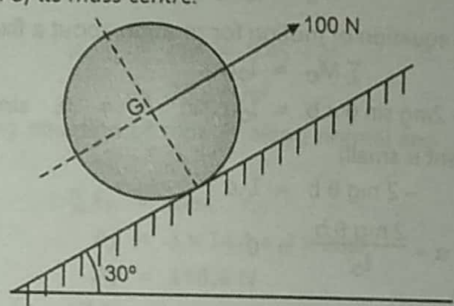


Fig. 5.61

Solution :

Given data : Mass of cylinder, m = 10 kg

Diameter of cylinder, D = 1 m

Force, P = 100 N

Let α be the angular acceleration of the cylinder, a_G be the linear acceleration of mass centre of cylinder and F be the frictional force between the cylinder and inclined plane.

Consider the free body and kinetic diagram of cylinder. Using equation of general plane motion,

$$\Sigma M_C = I_C \alpha$$

$$100 \times 0.5 - 10 \times 9.81 \times \sin 30 \times 0.5 = \left(\frac{10 \times (0.5)^2}{2} + 10 \times (0.5)^2\right) \alpha$$

$$50 - 24.525 = 3.75\ \alpha$$

$$\therefore \quad \alpha = \mathbf{6.793\ rad/s^2} \quad \text{... Ans.}$$

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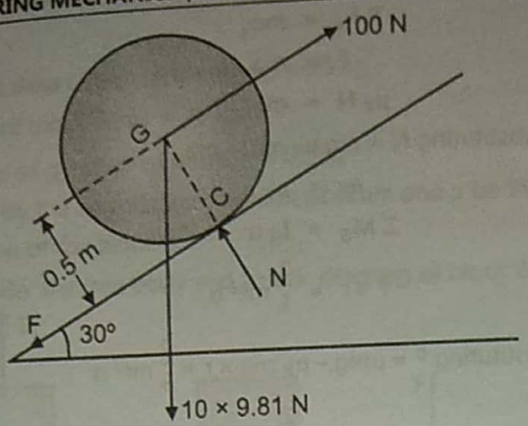
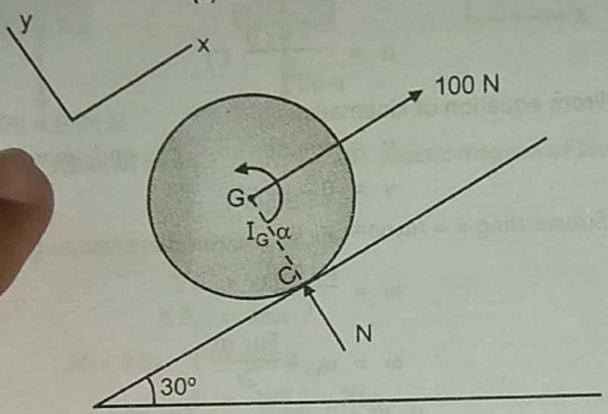


(a) F.B.D. of cylinder



(b) Kinetic diagram of cylinder

Fig. 5.61

$$a_G = r\alpha$$

$$a_G = 0.5 \times 6.793$$

$$\therefore \quad a_G = \mathbf{3.397\ m/s^2} \quad \text{... Ans.}$$

$$\Sigma F_x = ma_x$$

$$100 - 10 \times 9.81 \times \sin 30 - F = 10 \times 3.397$$

$$\therefore \quad -F = 33.97 - 100 + 49.05$$

$$\therefore \quad F = \mathbf{16.98\ N} \quad \text{... Ans.}$$

Example 5.54 : *The 150 N plank rests on two identical cylindrical rollers, each weighing 50 N. Determine the force P that will accelerate the plank up the plane at 2 m/s². Assume no slipping occurs either at the plank or at the incline. Refer Fig. 5.62.*

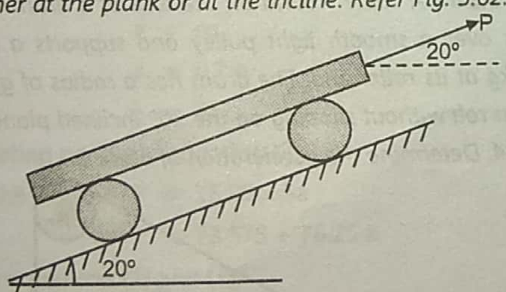


Fig. 5.62

Solution :

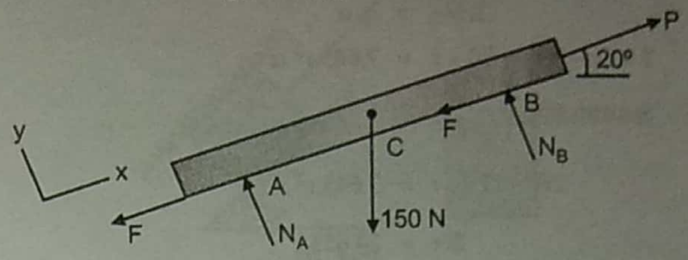
Given data : Weight of plank, $W_P = 150$ N

Weight of roller, $W_R = 50$ N each

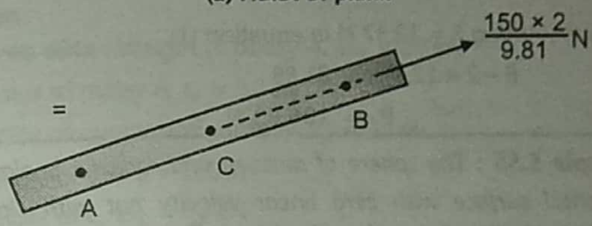
Linear acceleration of plank, $a_C = 2$ m/s²

Let α be the angular acceleration of roller and P be the force that will accelerate the plank up the plane at 2 m/s². Consider the free body and kinetic diagram of plank. Using equation of motion,

$$\Sigma F = ma$$



(a) F.B.D. of plank



(b) Kinetic diagram of plank

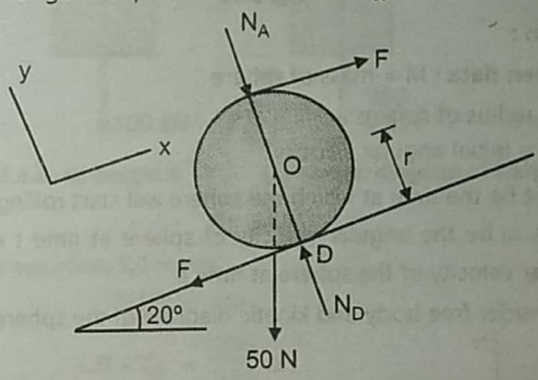
Fig. 5.62

$$\Sigma F_x = ma_C$$

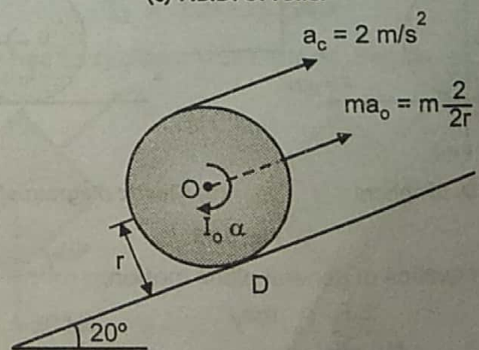
$$P - 2F - 150 \sin 20 = \frac{150 \times 2}{9.81}$$

$$P - 2F = 81.88 \quad \text{... (i)}$$

Consider free body and kinetic diagram of the roller. Using equation of general plane motion, $\Sigma M = I_\alpha$.



(c) F.B.D. of roller



(d) Kinetic diagram of roller

Fig. 5.62

Moment of inertia of roller about D.

$$I_D = I_G + mr^2$$

$$I_D = \frac{50\,r^2}{9.81 \times 2} + \frac{50}{9.81} \times r^2$$

$$I_D = 7.645\,r^2$$

$$\Sigma M_D = I_D\,\alpha$$

$$F \times 2r - 50 \sin 20 \times r = 7.645\,r^2\,\alpha$$

Substituting $\alpha = \frac{2}{2r}$,

$$2Fr - 17.10\,r = 7.645\,r^2 \times \frac{2}{2r}$$

$$\therefore \quad 2Fr = 24.655\,r$$

$$\therefore \quad F = 12.37 \text{ N}$$

Substituting F = 12.37 N in equation (1),

$$P - 2 \times 12.37 = 81.88$$

$$\therefore \quad P = \mathbf{106.63 \text{ N}} \quad \text{... Ans.}$$

***Example 5.55 :** The sphere of mass m and radius r is placed on horizontal surface with zero linear velocity but with clockwise angular velocity of ω_0. If μ_K is the coefficient of kinetic friction between the sphere and the floor, determine (a) time t at which sphere will start rolling without slipping, (b) linear and angular velocity of sphere at time t.*

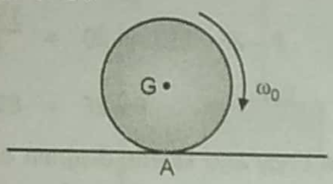


Fig. 5.63

Solution :

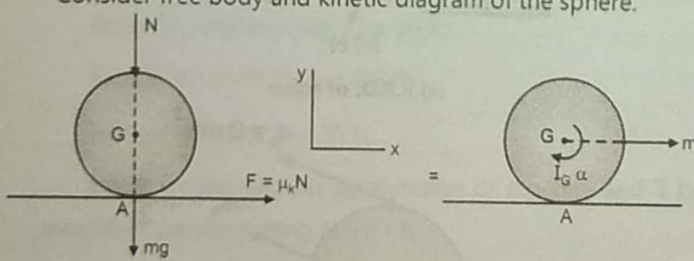
Given data : M = mass of sphere

r = radius of sphere

ω_0 = initial angular velocity

Let t be the time at which the sphere will start rolling without slipping, ω be the angular velocity of sphere at time t and v be the linear velocity of the sphere at time t.

Consider free body and kinetic diagram of the sphere.



(a) F.B.D. of sphere **(b) Kinetic diagram of sphere**

Fig. 5.63

Using equation of general plane motion,

$$\Sigma F_y = ma_y$$

$$N - mg = ma_y \quad \text{... } (a_y = 0)$$

$$N = mg$$

$$\Sigma F_x = ma_x$$

$$F = ma_x$$

$$\mu_K N = ma_x$$

SSubstituting $N = mg, \mu_K mg = ma_x$

$$\therefore \quad a_x = \mu_K g$$

$$\Sigma M_G = I_G\,\alpha \quad \text{...(1)}$$

$$-F \times r = \frac{2}{5} mr^2\,\alpha$$

Substituting $F = \mu mg, -\mu_K mg \times r = \frac{2}{5} mr^2\,\alpha$

$$-\mu_K g = \frac{2}{5} r\alpha$$

$$\alpha = \frac{-5\,\mu_K g}{2r} \quad (^*)$$

From equation of kinematics,

$$v = u + at \qquad \text{and} \quad \omega = \omega_0 + \alpha t$$

$$v = 0 + \mu_K gt$$

Substituting $v = r\omega, r\omega = \mu_K g$

$$\omega = \frac{\mu_K gt}{r} \quad \text{...(2)}$$

$$\omega = \omega_0 - \frac{5\mu_K gt}{2r}$$

Substituting $\omega = \frac{\mu_K gt}{r}, \ \frac{\mu_K gt}{r} = \omega_0 - \frac{5\mu_K gt}{2r}$

$$\frac{7\mu_K gt}{2r} = \omega_0$$

$$\therefore \quad t = \frac{2\omega_0 r}{7\mu g}\text{ s} \quad \text{... Ans.}$$

From equation (2), $\omega = \frac{\mu_K g}{r} \times \frac{2\omega_0 r}{7\mu_K g}$

$$\therefore \quad \omega = \frac{2}{7}\omega_0 \quad \text{... Ans.}$$

$$v = \omega r$$

$$\therefore \quad v = \frac{2}{7}\omega_0 r \text{ m/s} \quad \text{... Ans.}$$

***Example 5.56 :** A cable wound on the axle of a drum of mass 25 kg passes over a smooth light pulley and supports a block B of mass 20 kg at its roller end. The drum has a radius of gyration 0.5 m and can roll without slipping on the 30° inclined plane as shown in Fig. 5.64. Determine the acceleration of block B.*

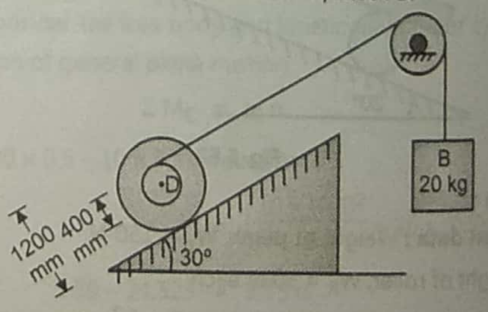


Fig. 5.64

Solution :

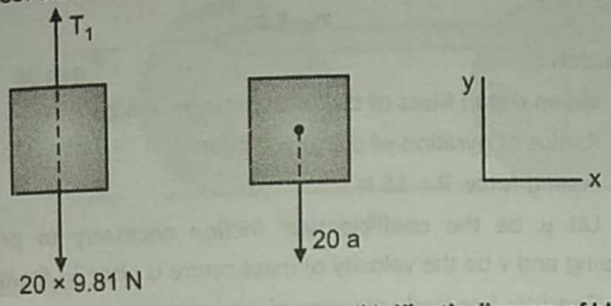
Given data : Mass of drum, $m_D = 25$ kg

Mass of block B, $m_B = 20$ kg

Radius of gyration of drum, $k = 0.5$ m

Let α be the angular acceleration of drum and a be the linear acceleration of the block B.

Consider the free body and kinetic diagram of block B.



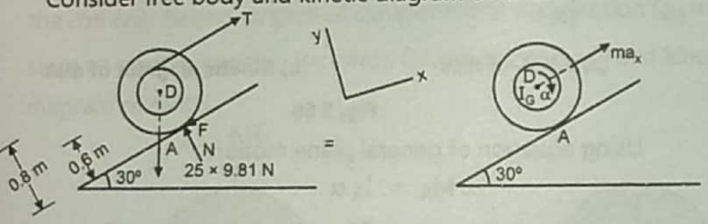
(a) F.B.D. of block **(b) Kinetic diagram of block**

Fig. 5.64

Using equation of motion,

$$\Sigma F = ma,$$
$$\Sigma F_y = ma$$
$$20 \times 9.81 - T = 20\,a$$
$$T = 196.2 - 20\,a \qquad \text{... (1)}$$

Consider free body and kinetic diagram of drum.



(c) F.B.D. of drum **(d) Kinetic diagram of drum**

Fig. 5.64

Using equation of general plane motion,

$$\Sigma M_A = I_A\,\alpha$$
$$T \times 0.8 - 25 \times 9.81 \times \sin 30 \times 0.6 = \left[25 \times (0.5)^2 + 25 \times (0.6)^2\right]\alpha$$
$$0.8\,T - 73.575 = 15.25\,\alpha \qquad \text{... (2)}$$
$$a = r\alpha$$
$$a = 0.2\,\alpha$$
$$\alpha = 5a$$

Substituting $\alpha = 5a$ in equation (2),

$$0.8\,T - 73.575 = 15.25 \times 5a$$
$$0.8\,T = 73.575 + 76.25\,a \qquad \text{... (3)}$$

Solving equations (1) and (3),

$$a = \mathbf{0.9\ m/s^2} \qquad \text{... Ans.}$$

Example 5.57 : *The system shown in Fig. 5.65 consists of pulley A and a rope connecting two weights B and C. For pulley A, weight = 20 kN, radius = 200 mm and radius of gyration = 100 mm. Weight of B = 100 kN, weight of C = 30 kN, inclination of a supporting plane is 45° with horizontal. If the system is released from rest, determine (a) a distance traveled downward by weight B to attain a velocity of 3 m/s, (b) the acceleration of weight B and C.*

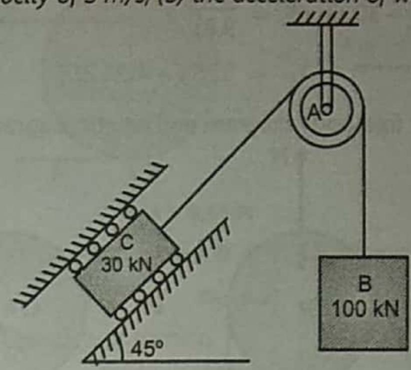


Fig. 5.65

Solution :

Given data : Weight of pulley A, $W_A = 20$ kN

Radius of pulley A, $r_A = 0.2$ m

Radius of gyration of pulley A, $k_A = 0.1$ m

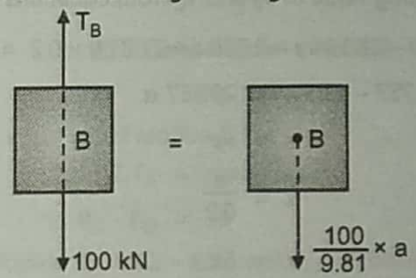
Weight of B, $W_B = 100$ kN

Weight of C, $W_C = 30$ kN

Initial velocity of weight B, $u_B = 0$

Final velocity of weight B, $v_B = 3$ m/s

Let a be the acceleration of weights and s be the distance traveled by weight B to attain a velocity of 3 m/s. Consider free body diagram and kinetic diagram weight B.



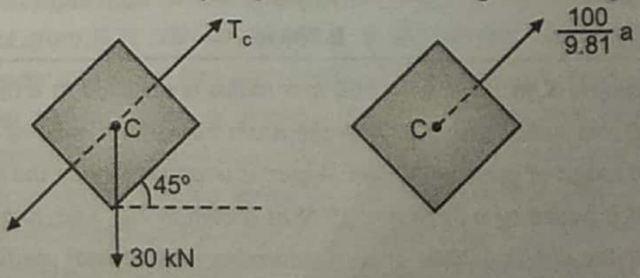
(a) F.B.D. of weight B **(b) Kinetic diagram of weight B**

Fig. 5.65

Using equation, $\Sigma F = ma$,

$$\Sigma F_y = ma$$
$$100 - T_B = \frac{100}{9.81}\,a$$
$$T_B = 100 - 10.194\,a \qquad \text{... (1)}$$

Consider free body diagram and kinetic diagram of weight C.



(c) F.B.D. of weight C **(d) Kinetic diagram of weight C**

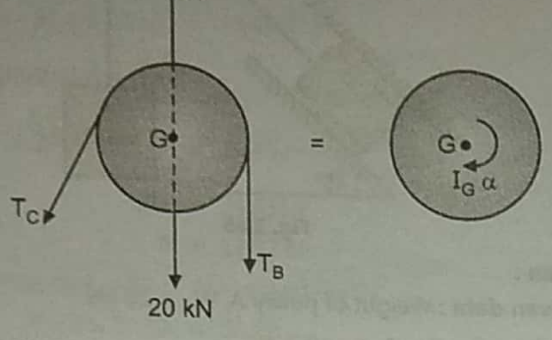
Fig. 5.65

Using equation, $\Sigma F = ma$,

$$T_C - 30 \sin 45 = \frac{30}{9.81} a$$

$$T_C = 3.058 a + 21.213 \quad \text{... (2)}$$

Consider free body diagram and kinetic diagram of pulley.



(e) F.B.D. of pulley **(f) Kinetic diagram of pulley**

Fig. 5.65

Using equation of motion, $\Sigma M = I\alpha$,

$$\Sigma M_G = I_G \alpha$$

$$T_B r - T_C r = m_A \times k_A^2$$

$$(T_B - T_C) r = \frac{20 \times 10^3 \times (0.1)^2}{9.81}$$

Substituting value of T_B and T_C from equations (1) and (2),

$$(100 - 10.194 a - 3.058 a - 21.213) \times 0.2 = 20.387 \alpha$$

$$15.757 - 2.65 a = 20.387 \alpha \quad \text{... (3)}$$

$$a = r\alpha$$

$$\alpha = \frac{a}{0.2}$$

$$\alpha = 5a$$

Substituting $\alpha = 5a$ in equation (3),

$$15.757 - 2.65 a = \frac{20.387 \times 5a}{1000}$$

$$2.75 a = 15.757$$

$$\therefore \quad a = \mathbf{5.726 \ m/s^2} \quad \text{... Ans.}$$

From equation of kinematics,

$$v^2 = u^2 + 2as \quad \text{... (u = 0)}$$

$$(3)^2 = 0 + 2 \times 5.726 \times s$$

$$\therefore \quad s = \mathbf{0.786 \ m} \quad \text{... Ans.}$$

***Example 5.58 :** A drum of 60 mm radius is attached to a disk of 120 mm radius. The disk with the drum has a total mass of 4 kg and radius of gyration 90 mm. A cord is wrapped around the drum and is pulled by a force P = 15 N as shown in Fig. 5.66. Knowing that the disk is initially at rest, determine (a) frictional coefficient necessary to prevent slipping, (b) the velocity of centre G after 3 s from rest.*

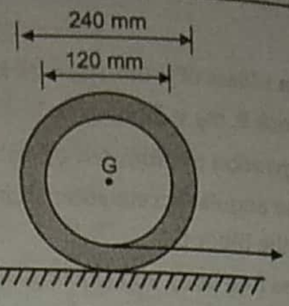


Fig. 5.66

Solution :

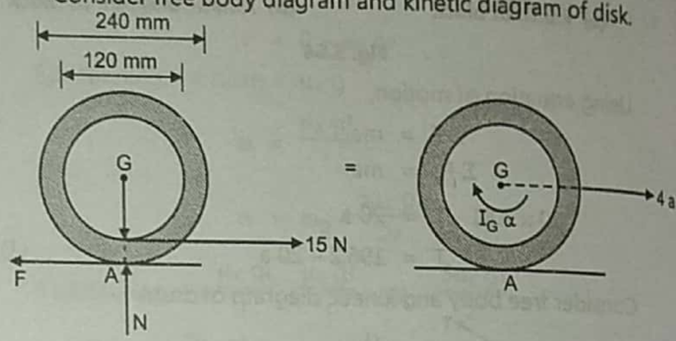
Given data : Mass of disk with drum, m = 4 kg

Radius of gyration of disk, k = 0.09 m

Pulling force, P = 15 N

Let μ be the coefficient of friction necessary to prevent slipping and v be the velocity of mass centre G after 3 s from rest.

Consider free body diagram and kinetic diagram of disk.



(a) F.B.D. of disk **(b) Kinetic diagram of disk**

Fig. 5.66

Using equation of general plane motion,

$$\Sigma M_A = I_A \alpha$$

$$15 \times (0.12 - 0.06) = [4 \times (0.09)^2 + 4 \times (0.12)^2] \alpha$$

$$0.9 = 0.09 \alpha$$

$$\therefore \quad \alpha = 10 \ rad/s^2$$

$$a = r\alpha$$

$$a = 0.12 \times 10$$

$$\therefore \quad a = 1.2 \ m/s^2$$

$$\Sigma F_y = m a_y \quad \text{... } (a_y = 0)$$

$$N - mg = 0$$

$$N = 4 \times 9.81$$

$$N = 39.24 \ N$$

$$\Sigma F_x = m a_x$$

$$15 - F = 4a$$

$$15 - \mu N = 4 \times 1.2$$

$$-\mu \times 39.24 = 4.8 - 15$$

$$\therefore \quad \mu = \mathbf{0.26} \quad \text{... Ans.}$$

From equation of kinematics,

$$v = u + at \quad \text{... (u = 0)}$$

$$v = 0 + 1.2 \times 3$$

$$\therefore \quad v = \mathbf{3.6 \ m/s} \quad \text{... Ans.}$$

Example 5.59 : Two uniform disks A and B, each of mass 4 kg, are connected by a 3 kg uniform rod CD by pin connections as shown in Fig. 5.67. A counter-clockwise couple M of magnitude 0.5 Nm is applied to disk A. Knowing that the disks roll without sliding, determine the acceleration of the centre of each disk and the pin reactions at C and D for the starting position of the system, shown in Fig. 5.67.

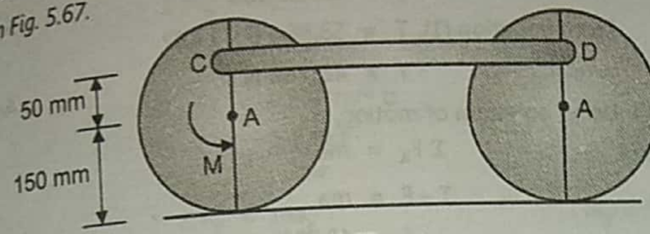


Fig. 5.67

Solution :

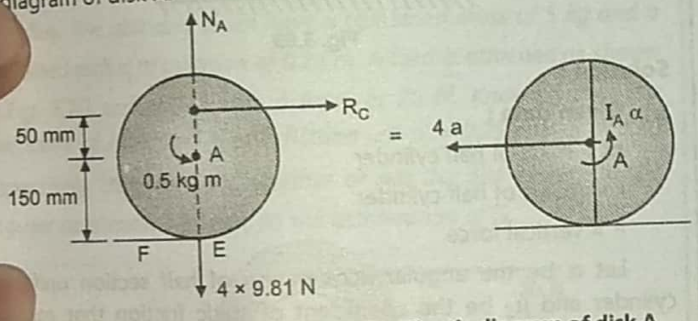
Given data : Mass of rod CD, m_{CD} = 3 kg

Mass of disks, m = 4 kg each

Magnitude of couple, M = 0.5 Nm

Radius of disk, r = 0.15 m

Let a be the acceleration of centre of each disk, R_C and R_D be the reactions at pin. Both the disks have general plane motion, while rod CD has curvilinear translation. For the starting position, the rod only having tangential component of acceleration (a_n = 0, since at starting position v is zero). Consider free body and kinetic diagram of disk A.



(a) F. B. D. of disk A **(b) Kinetic diagram of disk A**

Fig. 5.67

Using equation of motion,

$$\Sigma M_E = I_E \alpha$$

$$0.5 - 0.2\,R_C = \left[\frac{4 \times (0.15)^2}{2} + 4 \times (0.15)^2\right] \times \alpha$$

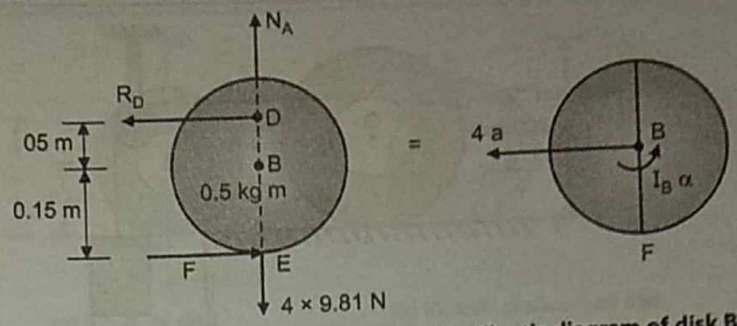
$$0.5 - 0.2\,R_C = 0.135\,\alpha$$

Substituting $\alpha = \dfrac{1}{0.15}$,

$$0.5 - 0.2\,R_C = 0.135 \times \frac{a}{0.15}$$

$$R_C = 2.5 - 4.5\,a \qquad \text{... (i)}$$

Consider free body and kinetic diagram of disk B.



(c) F.B.D. of disk B **(d) Kinetic diagram of disk B**

Fig. 5.67

Using equation of motion,

$$\Sigma M_F = I_F \alpha$$

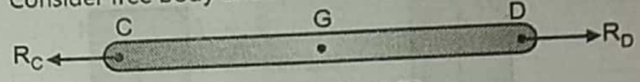
$$0.2\,R_D = \left[\frac{4 \times (0.15)^2}{2} + 4 \times (0.15)^2\right] \times \alpha$$

$$0.2\,R_D = 0.135\,\alpha$$

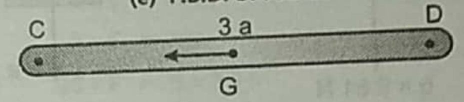
Substituting $\alpha = \dfrac{a}{0.15}$, $0.2\,R_D = 0.135 \times \dfrac{a}{0.15}$

$$\therefore \quad R_D = 4.5\,a \qquad \text{... (2)}$$

Consider free body and kinetic diagram of rod CD.



(e) F.B.D. of rod CD



(f) Kinetic diagram of rod CD

Fig. 5.67

Using equation of motion $\Sigma F = ma$,

$$\Sigma F_x = ma_x$$

$$R_C - R_D = 3a \qquad \text{... (3)}$$

Substituting $R_C = 2.5 - 4.5\,a$ and $R_D = 4.5\,a$ in equation (3),

$$2.5 - 4.5\,a - 4.5\,a = 3a$$

$$2.5 = 12\,a$$

$$\therefore \quad a = \mathbf{0.208\ m/s^2} \qquad \text{... Ans.}$$

From equation (1), $R_C = 2.5 - 4.5 \times 0.208$

$$\therefore \quad R_C = \mathbf{1.564\ N} \qquad \text{... Ans.}$$

From equation (2), $R_D = 4.5 \times 0.208$

$$\therefore \quad R_D = \mathbf{0.936\ N} \qquad \text{... Ans.}$$

Example 5.60 : *In the arrangement shown in Fig. 5.68, the radius of drum D is 100 mm and that of the wheel C mounted on the same axle is 200 mm. Combined mass of wheel and the drum is 16 kg and the radius of gyration is 120 mm. The wheel rests on a horizontal floor and adequate friction between the floor and wheel is available. A string wound round the drum and passing over a small smooth pulley, supports a block of 6 kg at its other end. Find the angular acceleration of wheel and acceleration of 6 kg mass. Also find frictional force and string tension.*

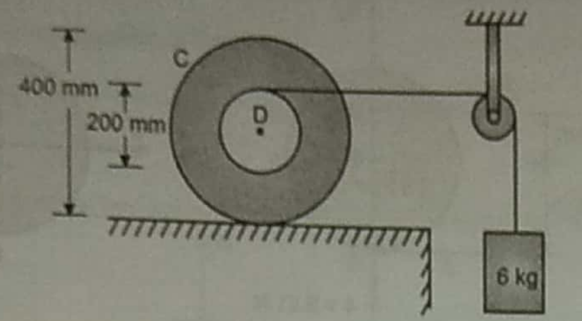


Fig. 5.68

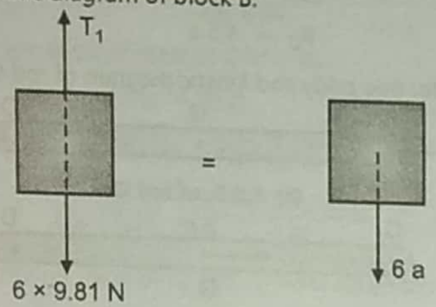
Solution :

Given data : Mass of drum with wheel, $m_D = 16$ kg

Radius of gyration, $k = 0.12$ m

Mass of block, $m_B = 6$ kg

Let α be the angular acceleration of drum, a be the acceleration of 6 kg block, F be the frictional force between the wheel and floor and T be the tension in the string. Consider free body and kinetic diagram of block B.



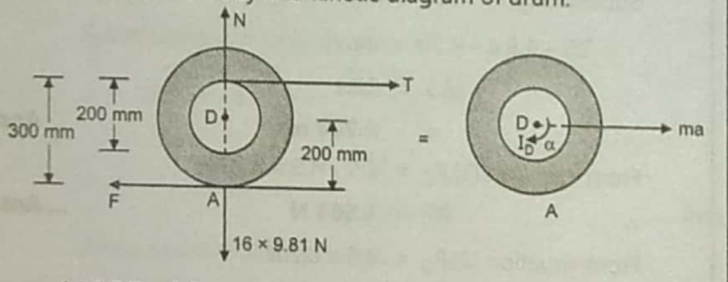
(a) F.B.D. of block B **(b) Kinetic diagram of block B**

Fig. 5.68

Using equation of motion,

$$\Sigma F = ma,$$
$$6 \times 9.81 - T = 6a$$
$$T = 58.86 - 6a \qquad \dots (1)$$

Consider free body and kinetic diagram of drum.



(c) F.B.D. of drum **(d) Kinetic diagram of drum**

Fig. 5.68

Using equation of general plane motion,

$$\Sigma M_A = I_A \alpha$$
$$T \times 0.3 = \left[16 \times (0.12)^2 + 16 \times (0.2)^2\right] \times \alpha$$
$$0.3\,T = 0.87\,\alpha \qquad \dots (2)$$

Substituting T = 56.86 – 6a in equation (2),

$$0.3 \times (58.86 - 6a) = 0.87\,\alpha \qquad \dots (3)$$
$$a = r\alpha$$
$$a = 0.1\,\alpha$$

Substituting $a = 0.1\,\alpha$ in equation (3),

$$0.3 \times (58.86 - 6 \times 0.1\,\alpha) = 0.87\,\alpha$$
$$17.658 - 0.18\,\alpha = 0.87\,\alpha$$
$$17.658 = 1.05\,\alpha$$
$$\therefore \quad \alpha = \mathbf{16.82\ rad/s^2} \qquad \text{... Ans.}$$

From equation (2), $a = 0.1 \times 16.82$

$$\therefore \quad a = \mathbf{1.682\ m/s^2} \qquad \text{... Ans.}$$

From equation (1), $T = 58.86 - 6 \times 1.682$

$$\therefore \quad T = \mathbf{48.768\ N} \qquad \text{... Ans.}$$

Using equation of motion,

$$\Sigma F_x = ma_x$$
$$T - F = ma$$
$$F = 48.768 - 16 \times 1.682$$
$$\therefore \quad F = \mathbf{21.86\ N} \qquad \text{... Ans.}$$

Example 5.61 : *A half section of a uniform cylinder of mass m and radius r is at rest when a vertical force P is applied as shown in Fig. 5.69. Assuming that the section rolls without sliding, determine, in terms of m, r and P, (a) its angular acceleration, (b) the minimum value of the coefficient of static friction that makes such a motion possible.*

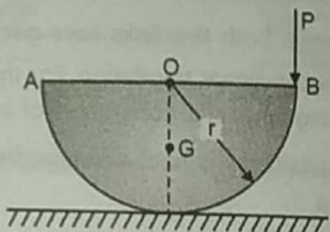


Fig. 5.69

Solution :

Given data :

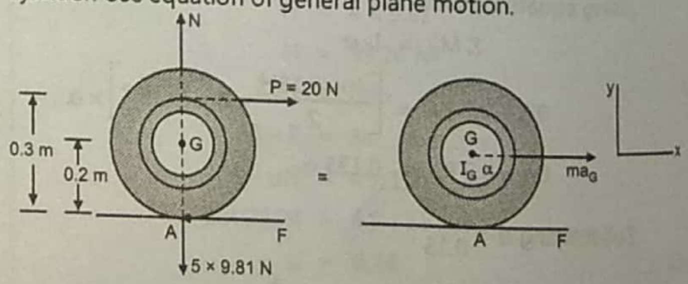
m = mass of half cylinder

r = radius of half cylinder

P = vertical force

Let α be the angular acceleration of half section uniform cylinder and μ_s be the coefficient of static friction that makes such a motion possible.

Consider free body diagram and kinetic diagram of half cylinder. Use equation of general plane motion.



(a) F.B.D. of half cylinder **(b) Kinetic diagram of half cylinder**

Fig. 5.69

Moment of inertia about mass centre G is determined by using parallel axis theorem,

$$I_O = I_G + mh^2$$

$$\frac{mr^2}{2} = I_G + m \times \left(\frac{4r}{3\pi}\right)^2$$

$$\frac{mr^2}{2} = I_G + 0.18\ mr^2$$

$$I_G = 0.32\ mr^2$$

$$\Sigma M_C = I_C\ \alpha$$

$$P \times r = \left[0.32\ mr^2 + m \times (0.576\ r)^2\right] \times \alpha$$

$$Pr = 0.652\ mr^2\ \alpha$$

$$\therefore \quad \alpha = \mathbf{\frac{1.534\ P}{mr}} \quad \text{... Ans.}$$

$$\Sigma F_y = ma_y \quad \text{... } (a_y = 0)$$

$$N - P - mg = 0$$

$$N = mg + P \quad \text{... (1)}$$

$$\Sigma F_x = ma_x$$

$$F = ma$$

Substituting $a = 0.576, \mu_S\ N = m \times 0.576\ r\alpha$

Substituting $N = mg + P$ and $\alpha = \frac{0.1534\ P}{mr}$,

$$\mu_S\ (mg + P) = \frac{mr \times 0.576 \times 0.1534\ P}{mr}$$

$$\therefore \quad \mu_S = \mathbf{\frac{0.884\ P}{mg + P}} \quad \text{... Ans.}$$

Example 5.62 : *A drum of 0.1 m radius is attached to a disk of 0.2 m radius. The disk and drum have a combined mass of 5 kg and a combined radius of gyration of 0.15 m. A cord is attached as shown in Fig. 5.70 and pulled with a force of 20 N. Knowing that the coefficient of static and kinetic friction are $\mu_s = 0.25$ and $\mu_k = 0.20$ respectively, determine (a) whether or not the disk slides, (b) the angular acceleration of disk, (c) the acceleration of G.*

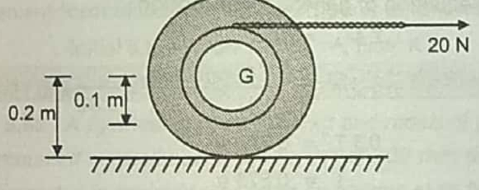


Fig. 5.70

Solution :

Given data : Mass of disk with drum, m = 5 kg

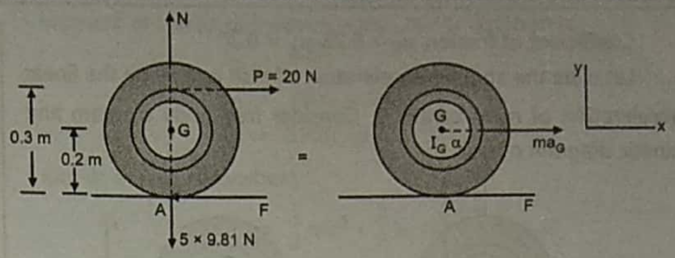
Combined radius of gyration, k = 0.15 m

Pulling force, P = 20 N

Coefficient of friction, $\mu_S = 0.25$, $\mu_K = 0.2$.

Let α be the angular acceleration of disk and a_G be the linear acceleration of mass centre G.

Consider free body and kinetic diagram of disk.



(a) F.B.D. of disk **(b) Kinetic diagram of disk**

Fig. 5.71

By using equation of general plane motion,

$$\Sigma M_A = I_A\ \alpha$$

$$20 \times 0.3 = \left[5 \times (0.15)^2 + 5 \times (0.2)^2\right] \times \alpha$$

$$6 = 0.3125\ \alpha$$

$$\therefore \quad \alpha = \mathbf{19.2\ rad/s^2} \quad \text{... Ans.}$$

Using equation, $a = r\alpha$

$$a_G = 0.2 \times 19.2$$

$$\therefore \quad a_G = \mathbf{3.84\ m/s^2} \quad \text{... Ans.}$$

$$\Sigma F_y = ma_y \quad \text{... } (a_y = 0)$$

$$N - 5 \times 9.81 = 0$$

$$N = 49.05\ N$$

$$\Sigma F_x = ma_x$$

$$P - F = ma_G$$

$$20 - F = 5 \times 3.84$$

$$\therefore \quad F = \mathbf{0.8\ N} \quad \text{... Ans.}$$

$$\mu_S\ N = 0.25 \times 49.05$$

$$\mu_S\ N = 12.263\ N$$

$$\mathbf{F < \mu_S N.}$$

Hence disk does not slide **... Ans.**

Example 5.63 : *A drum of 0.1 m radius is attached to a disk of 0.2 m radius. The disk and drum have a combined mass of 5 kg and a combined radius of gyration of 0.15 m. A cord is attached as shown in Fig. 5.72 and pulled with a force of 20 N. Knowing that the coefficient of static and kinetic friction are $\mu_s = 0.25$ and $\mu_k = 0.20$ respectively, determine (a) whether or not the disk slides, (b) the angular acceleration of the disk, (c) the acceleration of G.*

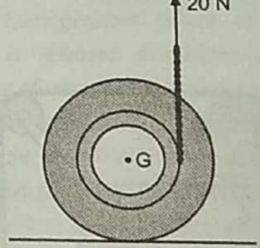


Fig. 5.72

Solution :

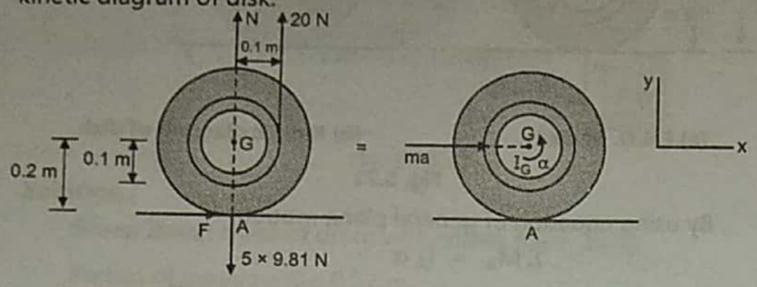
Given data : Combined mass of disk and drum, m = 5 kg

Combined radius of gyration, k = 0.15 m

Pulling force, P = 20 N

Coefficient of friction, $\mu_s = 0.25$, $\mu_k = 0.20$

Let α be the angular acceleration of disk and a_G be the linear acceleration of mass centre G. Consider free body diagram and kinetic diagram of disk.



(a) F.B.D. of disk **(b) Kinetic diagram of disk**

Fig. 5.72

Using equation of general plane motion,

$$\Sigma M_A = I_A \alpha$$

$$20 \times 0.1 = \left[5 \times (0.15)^2 + 5 \times (0.2)^2\right] \times \alpha$$

$$2 = 0.3125\, \alpha$$

$$\therefore \quad \alpha = \mathbf{6.4\ rad/s^2} \qquad \text{... Ans.}$$

Using equation, $a = r\alpha$

$$a_G = 0.2 \times 6.4$$

$$\therefore \quad a_G = \mathbf{1.28\ m/s^2} \qquad \text{... Ans.}$$

$$\Sigma F_y = ma_y \quad \text{... } (a_y = 0)$$

$$N + 20 - 5 \times 9.81 = 0$$

$$N = 29.05\ N$$

$$\Sigma F_x = ma_x$$

$$-F = 5 \times 1.28$$

$$\therefore \quad F = -6.4\ N$$

$$\therefore \quad F = 6.4\ N\ (\leftarrow)$$

$$\mu_s N = 0.25 \times 29.04$$

$$\therefore \quad \mu_s N = 7.263\ N$$

$$F < \mu_s N,$$

Hence the disk does not slide ... **Ans.**

Example 5.64 : *The 50 N disk and 20 N block are released from rest. Determine the velocity of the block when it has descended 1 m. The coefficient of static friction at A is $\mu_A = 0.2$. Neglect the mass of the cord and pulleys.*

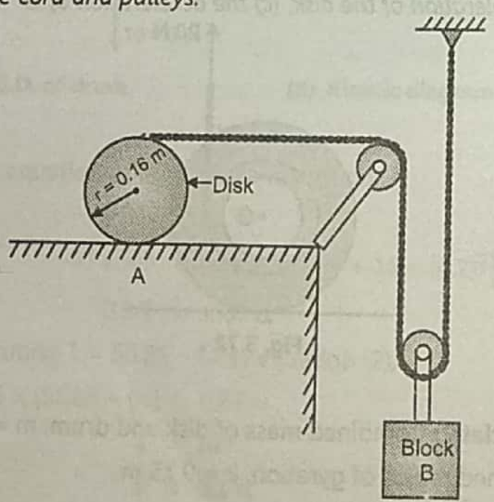


Fig. 5.73

Solution :

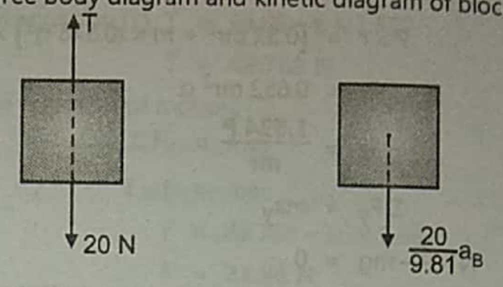
Given data : Weight of disk, $W_D = 50$ N

Weight of block, $W_B = 20$ N

Distance traveled by block, s = 1 m

Coefficient of friction, $\mu_A = 0.2$

Let v be the velocity of block B when it has descended 1 m. Consider free body diagram and kinetic diagram of block.



(a) F.B.D. of block **(b) Kinetic diagram of block**

Fig. 5.73

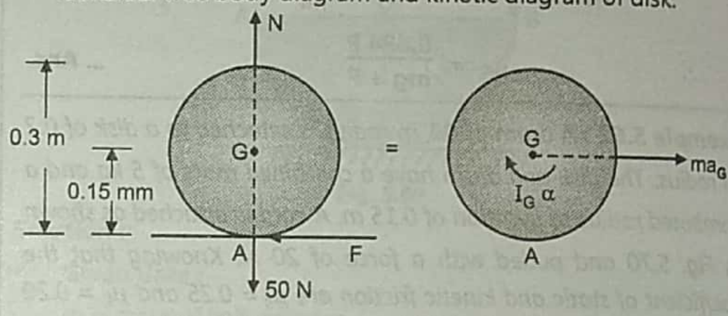
By using equation,

$$\Sigma F = ma,$$

$$20 - 2T = \frac{20}{9.81} a_B$$

$$20 - 2T = 2.039\, a_B \qquad \text{... (1)}$$

Consider free body diagram and kinetic diagram of disk.



(c) F.B.D. of disk **(d) Kinetic diagram of disk**

Fig. 5.73

Using equation of general plane motion,

$$\Sigma M_A = I_A \alpha$$

$$0.3 \times T = \left[\frac{50}{9.81} \times \frac{(0.15)^2}{2} + \frac{50}{9.81} \times (0.15)^2\right] \times \alpha$$

$$0.3\,T = 0.172\, \alpha$$

$$T = 0.573\, \alpha \qquad \text{... (2)}$$

$$a_A = r\alpha$$

$$\alpha = \frac{a_A}{0.15}$$

Substituting in equation (2),

$$T = 0.573 \times \frac{a_A}{0.15}$$

$$T = 3.82\, a_A \qquad \text{... (3)}$$

From equation of kinematics, $a_A = a_B$, substituting in equation (3),

$$T = 3.82 \times a_B$$

$$\therefore \quad T = 3.82\, a_B$$

Substituting in equation (1),

$$20 - 7.645\, a_B = 2.039\, a_B$$
$$9.684\, a_B = 20$$
$$\therefore \quad a_B = 2.065 \text{ m/s}^2$$

From equation of kinematics,

$$v^2 = u^2 + 2as,$$

(u = 0, since block released from rest),

$$v^2 = 0 + 2 \times 2.065 \times 1$$
$$\therefore \quad v = \mathbf{2.03 \text{ m/s}} \quad \text{... Ans.}$$

5.12 WORK-ENERGY PRINCIPLE

The work done by the force is studied in the chapter, Work, Power, Energy. The work done by the moment or couple is given by,

$$\text{Work} = M \cdot d\theta$$

When the body rotates in the plane through a finite angle θ measured in radian, from θ_1 to θ_2, the work done by the couple is

$$\text{Work done} = \int_{\theta_1}^{\theta_2} M \cdot d\theta$$

If the couple M has a constant magnitude, then

$$\text{Work done} = M(\theta_2 - \theta_1)$$

The kinetic energy of a rigid body is consisting in two forms in general plane motion.

1. **Translation :** The kinetic energy of rigid body of mass m moving with velocity v is given by

$$\text{K.E.} = \frac{1}{2} mv^2$$

2. **Rotation :** The kinetic energy of rigid body having moment of inertia I_O moving with angular velocity ω is given by

$$\text{K.E.} = \frac{1}{2} I_O\, \omega^2$$

The convenient form of work-energy principle is as follows :

$$\text{Initial K.E.} + \text{Work done} = \text{Final K.E.}$$

NUMERICAL EXAMPLES ON WORK-ENERGY PRINCIPLE

Example 5.65 : *A flywheel of mass 3000 kg and radius of gyration 0.8 m is mounted centrally on a solid shaft of 100 mm diameter, which is supported in horizontal position by bearing at its two ends. Find the kinetic energy of the flywheel when it is rotating at 150 r.p.m. If the bearing friction is 1 percent of normal reaction, find the number of revolutions the wheel will make before coming to rest. Neglect the M.I. of the shaft.*

Solution :

Given data : Mass of flywheel, m = 3000 kg

Radius of gyration, k = 0.8 m

Diameter of shaft, d = 100 mm

Angular velocity, ω = 150 r.p.m.

Bearing friction, F_r = 1% of normal reaction

Moment of inertia of flywheel about axis of rotation is

$$I_O = mk^2$$
$$I_O = 3000 \times (0.8)^2$$
$$I_O = 1920 \text{ kg-m}^2$$

Kinetic energy of flywheel,

$$\text{K.E.} = \frac{1}{2} mv^2 + \frac{1}{2} I_O\, \omega_O^2 \quad \text{... } (v = 0)$$
$$= \frac{1}{2} \times 1920 \times \left(2\pi \times \frac{150}{60}\right)^2$$
$$= \mathbf{236870.5 \text{ J}} \quad \text{...Ans.}$$

Bearing friction, F_r = 1% of normal reaction

$$= \frac{3000 \times 9.81}{100}$$
$$= 294.3 \text{ N}$$

$$\text{Frictional torque} = F_r \times r$$
$$= 294.3 \times 0.05$$
$$= 14.715 \text{ N-m}$$
$$\text{Frictional torque} = I_O\, \alpha$$
$$\therefore \quad \alpha = \frac{14.715}{1920}$$
$$\therefore \quad \alpha = 0.00766 \text{ rad/s}^2$$

Using equation of kinematics, $\omega = \omega_O + \alpha t$; ... $(\omega = 0)$

$$t = \frac{2\pi \times 150}{60 \times 0.0076}$$
$$\therefore \quad t = 2050.65 \text{ s}$$

Again using equation of kinematics,

$$\theta = \omega_O t - \frac{1}{2} \alpha t^2$$
$$= \frac{2\pi \times 150}{60} \times 2050.65 - \frac{1}{2} \times 0.00766 \times (2050.65)^2$$
$$\therefore \quad \theta = 16105.72 \text{ rad}$$

No. of revolutions,

$$N = \frac{\theta}{2\pi}$$
$$N = \frac{16105.72}{2\pi}$$
$$\therefore \quad N = \mathbf{2563.3} \quad \text{... Ans.}$$

Example 5.66 : *A homogeneous sphere of mass 10 kg and diameter 300 mm is released simultaneously along with a homogeneous cylinder of mass 30 kg and diameter 400 mm. Both are released from the same height h meter to roll down an inclined plane making an angle of 30° with the horizontal. Assume that both bodies roll without slipping. Compute the velocity with which sphere and cylinder will reach the bottom of incline. Which body will reach the bottom first ?*

Solution :

Given data : Mass of sphere, m_S = 10 kg

Radius of sphere, r_S = 150 mm

Mass of cylinder, m_C = 30 kg

Radius of cylinder, r_C = 200 m

Position of inclined plane, $\theta = 30^\circ$

Height is 'h'

Let v be the velocity with which sphere and cylinder will reach the bottom of incline.

Consider free body diagram of sphere.

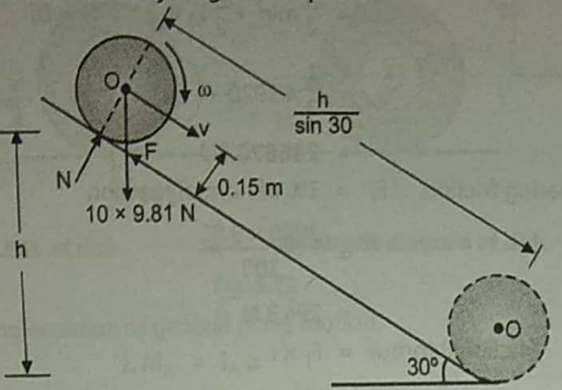


Fig. 5.74 (a) : F.B.D. of sphere

Using work-energy principle,

Initial K.E. (rotation + translation) + Work done = Final K.E. (rotation + translation)

$$0 + 0 + 10 \times 9.81 \times \sin 30 \times \frac{h}{\sin 30} = \frac{1}{2} \times 10 \times v^2 + \frac{1}{2} I_O \cdot \omega^2$$

Substituting $I_O = \frac{2}{5} \times 10 \times (0.15)^2$ and $\omega = \frac{v}{0.15}$,

$$98.1\,h = \frac{1}{2} \times 10 \times v^2 + \frac{1}{2} \times \frac{2}{5} \times 10 \times (0.15)^2 \times \left(\frac{v}{0.15}\right)^2$$

$$98.1\,h = 5v^2 + 2v^2$$

$$\therefore \quad v = \mathbf{3.74\sqrt{h}} \qquad \text{... Ans.}$$

Consider free body diagram of cylinder.

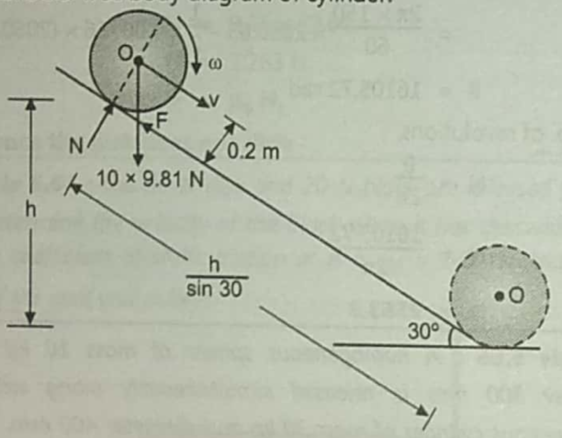


Fig. 5.74 (b) : F.B.D. of cylinder

Using work-energy principle,

Initial K.E. (rotation + translation) + Work done = Final K.E. (rotation + translation)

$$0 + 0 + 30 \times 9.81 \times \sin 30 \times \frac{h}{\sin 30} = \frac{1}{2} \times 30 \times v^2 + \frac{1}{2} \times I_O \times \omega^2$$

Substituting $I_O = \frac{30 \times (0.2)^2}{2}$, and $\omega = \frac{v}{0.2}$,

$$30 \times 9.81 \times h = 15\,v^2 + \frac{1}{2} \times \frac{30 \times (0.2)^2}{2} \times \frac{v^2}{(0.2)^2}$$

$$294.3\,h = 22.5\,v^2$$

$$\therefore \quad v = \mathbf{3.62\sqrt{h}} \qquad \text{... Ans.}$$

The sphere will reach the bottom of incline first, since it is having larger velocity than cylinder.

***Example 5.67 :** A uniform solid disk of mass m, starting from rest rolls down an inclined plane without slipping. Find its linear velocity v at the time when it descends through a vertical height h. Also obtain expression for K.E. of translation and rotation for the disk at this position.*

Solution :

Given data : M = mass of disk

r = radius of disk

θ = position of plane

h = vertical distance traveled

Let v be the velocity when it descends through a vertical height h. Consider free body diagram of disk.

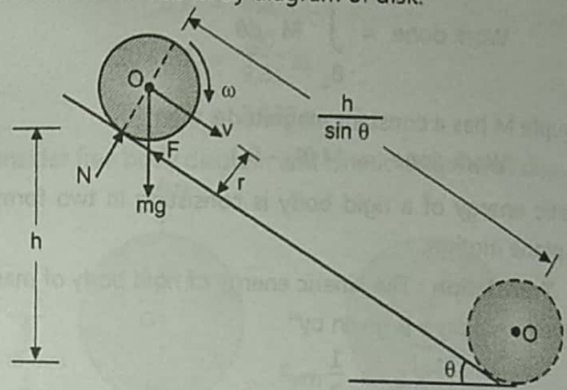


Fig. 5.75 : F.B.D. of disk

Using work-energy principle,

Initial K.E. (rotation + translation) + Work done = Final K.E. (rotation + translation)

$$0 + 0 + mg \sin\theta \times \frac{h}{\sin\theta} = \frac{1}{2} mv^2 + \frac{1}{2} I_O\,\omega^2$$

$$mgh = \frac{1}{2} mv^2 + \frac{1}{2} \cdot \frac{mr^2}{2} \times \frac{v^2}{r^2}$$

$$mgh = \frac{1}{2} mv^2 + \frac{mv^2}{4}$$

$$mgh = \frac{3}{4} mv^2$$

$$v^2 = \frac{4}{3} gh$$

$$\therefore \quad v = \sqrt{\frac{4}{3} gh} \qquad \text{... Ans.}$$

$$\text{K.E. (translation)} = \frac{1}{2} mv^2$$

$$= \frac{1}{2} \times m \times \frac{4}{3} gh$$

$$\therefore \quad \text{K.E. (translation)} = \mathbf{\frac{2}{3} mgh} \qquad \text{... Ans.}$$

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$$\text{K.E. (rotation)} = \frac{1}{2} I_O \omega^2 = \frac{1}{2} \cdot \frac{mr^2}{2} \times \frac{4}{3} \times \frac{gh}{r^2}$$

$$\therefore \quad \text{K.E. (rotation)} = \frac{mgh}{3} \quad \text{... Ans.}$$

***Example 5.68 :** A uniform bar of mass m and length l hangs from a frictionless hinge. It is released from the horizontal position AB. Determine the angular velocity and the linear velocity of the centre of mass of the bar when it is in vertical position in terms of g and L.*

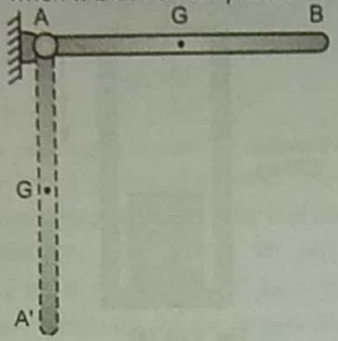

Fig. 5.76

Solution :

Given data : M = mass of bar AB, l = length of bar AB

Let ω be the angular velocity of bar AB and v be the linear velocity of the mass centre of bar AB.

The kinetic energy due to translation is zero, since the motion of bar AB is angular.

Consider free body diagram of bar AB.

Using work-energy principle from A to A',

Initial K.E. + Work done = Final K.E.

$$0 + Wh = \frac{1}{2} I_A \omega^2$$

$$mg\frac{l}{2} = \frac{1}{2} \times \frac{ml^2}{3} \times \omega^2$$

$$3g = l\omega^2$$

$$\therefore \quad \omega = \sqrt{\frac{3g}{l}} \quad \text{... Ans.}$$

By using equation, $v = r\omega$

$$v = \frac{l}{2} \cdot \sqrt{\frac{3g}{l}} = \frac{1}{2}\sqrt{3gl}$$

$$\therefore \quad v = \sqrt{\frac{3}{4} gl} \quad \text{... Ans.}$$

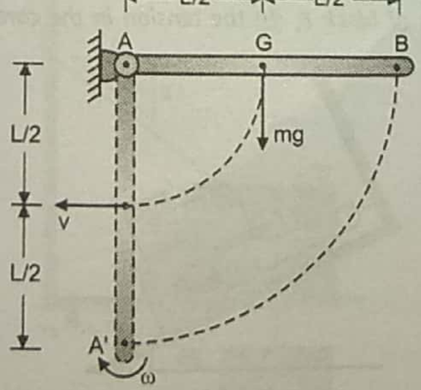

Fig. 5.76 (a) : F.B.D. of bar AB

***Example 5.69 :** A thin prismatic bar of length 1.25 m is hinged at one end. Its other end is lifted to a position above the hinge end and the bar is released to swing in a vertical plane. Show that at any instant, angular velocity of the bar is given by $\omega = \sqrt{2.4\, g\, (1 - \cos \theta)}$, where θ is the angle, the bar makes with vertical and g is the gravitational acceleration. Hence find the velocity of moving end of the bar at the time when the bar is in horizontal position.*

Solution :

Given data : Length of bar, l = 1.25 m, Mass of bar = m

Consider free body diagram of bar AB.

Potential energy of the bar when the other end is lifted to a position above to hinge end, i.e. A to A' = $mg\frac{l}{2}$.

Potential energy of the bar at any instant, i.e. A' to A" = $mg\frac{l}{2}\cos\theta$

Kinetic energy of the bar at any instant, i.e. A' to A" = $\frac{1}{2} I_O \omega^2$

Kinetic energy due to translation is zero, since the bar is hinged at O.

Using principle of conservation of energy, loss of potential energy is equal to gain in kinetic energy.

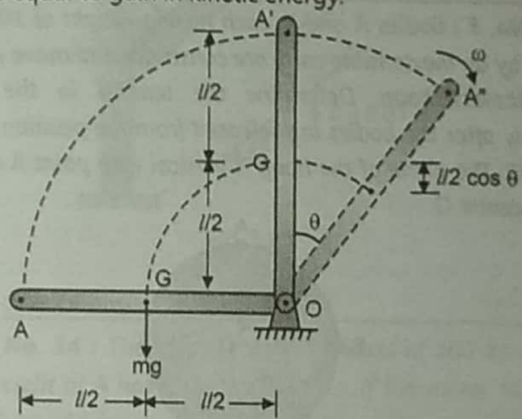

Fig. 5.77 : F.B.D. of bar

$$mg\frac{l}{2} - mg\frac{l}{2}\cos\theta = \frac{1}{2} I_O \omega^2$$

$$mg\frac{l}{2}(1 - \cos\theta) = \frac{1}{2} \cdot \frac{ml^2}{3} \times \omega^2$$

$$g(1 - \cos\theta) = \frac{l}{3}\omega^2$$

$$\omega^2 = \frac{3g}{l}(1 - \cos\theta)$$

Substituting l = 1.25, $\omega = \sqrt{2.4\, g\, (1 - \cos\theta)}$ ... **Ans.**

At θ = 90°, ω = 4.85 rad/s. ∴ v = rω = 1.25 × 4.85

= **6.06 m/s** ... **Ans**

PROBLEMS FOR PRACTICE

***Problem No. 1 :** A child having a mass of 22 kg sits on a swing and is held in the position shown by second child. Neglecting the mass of the swing, determine the tension in rope AB (a) while the second child holds the swing with his arms outstretched horizontally, (b) immediately after the swing is released. (Refer Fig. 5.78).*

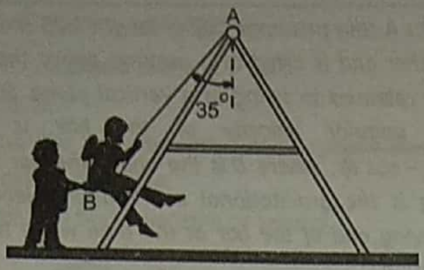

Fig. 5.78

Answer : T = 131.46 N, T = 84.39 N

Problem No. 2 : *The collar A is free to slide along the smooth shaft B mounted in the frame. The plane of the frame is vertical. Determine the necessary horizontal acceleration a of the frame to maintain the collar in a fixed position on the shaft. (Refer Fig. 5.79)*

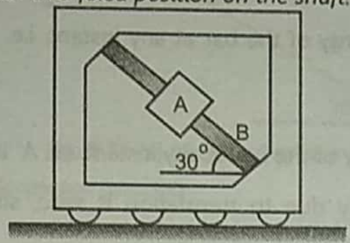

Fig. 5.79

Answer : a = 5.66 m/s^2

Problem No. 3 : *Bodies A and B, each having weight of 10 N and connected by an inextensible cord, are constrained to move along a fixed frictionless hoop. Determine the tension in the chord immediately after the bodies are released from the position shown in Fig. 5.80. The plane of the hoop is vertical with point A directly above the centre O.*

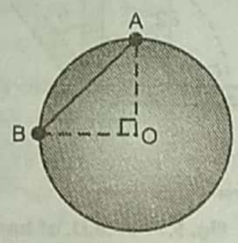

Fig. 5.80

Answer : T = 7.07 N

Problem No. 4 : *A 200 N suitcase slides from rest 6 m down the smooth ramp. Determine the point where it strikes the ground at C. How long does it take to go from A to C ? (Refer Fig. 5.81)*

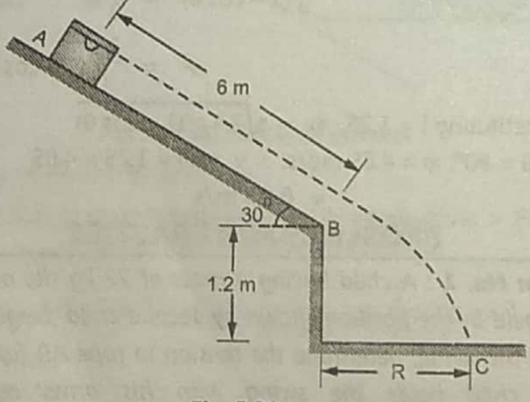

Fig. 5.81

Answer : t = 1.8 s, R = 1.59 m

Problem No. 5 : *An elevator being lowered into a mine shaft starts from rest and attains a speed of 10 m/s with a distance of 15 m. The elevator alone has a mass of 500 kg and it carries a box of mass 600 kg in it. Find the total tension in the cables supporting the elevator, during the accelerated motion. Also find the total pressure between the box and the floor of the elevator. (Refer Fig. 5.81 (a))*

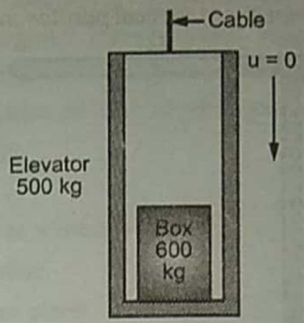

Fig. 5.81 (a)

Answer : T = 7124.3 N, P = 3886.02 N

Problem No. 6 : *To transport a series of bundles A to a roof, a contractor uses a motor driven lift consisting of horizontal platform BC that rides on rails attached to the side of a ladder. The lift starts from rest and initially moved with a constant acceleration of a_1. The lift then decelerates at a constant rate of a_2 and comes to rest at D, near the top of the ladder. If μ_S = 0.3 between the bundles and the horizontal platform, determine the largest allowable a_1 and a_2 if the bundles are not to slide on the platform. (Refer Fig. 5.82)*

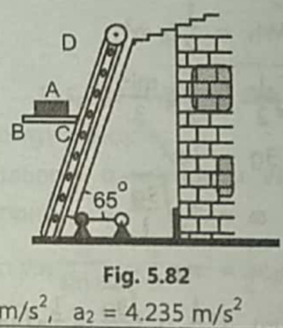

Fig. 5.82

Answer : a_1 = 19.5 m/s^2, a_2 = 4.235 m/s^2

Problem No. 7 : *Block A has a mass of 40 kg and block B has a mass of 8 kg. The coefficients of friction between all surfaces of contact are μ_S = 0.20 and μ_K = 0.15. If P = 4 N (→), determine (a) the acceleration of block B, (b) the tension in the cord. (Refer Fig. 5.83)*

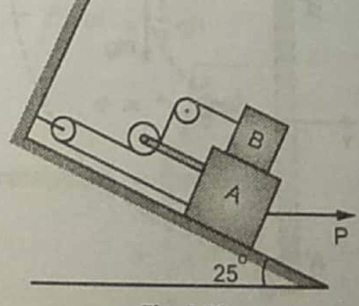

Fig. 5.83

Answer : a_B = 1.794 m/s^2, T = 58.2 N

Problem No. 8 : *Boxes A and B are at rest on a conveyor belt that is initially at rest. The belt is suddenly started in an upward direction so that slipping occurs between the belt and the boxes. If μ_K = 0.30 between the box A and the belt and 0.32 between the box B and the belt, determine the initial acceleration of each block. (Refer Fig. 5.84)*

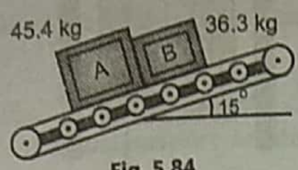

Fig. 5.84

Answer : a_B = 0.493 m/s², a_A = 0.304 m/s²

Problem No. 9 : *A package is at rest on a conveyor belt, which is initially at rest. The belt is started and moves to the right for 1.3 s with a constant acceleration of 2 m/s². The belt then moves with a constant deceleration a_2 and comes to rest after a total displacement at 2.2 m. If μ_S = 0.35 and μ_K = 0.25 between the package and the belt, determine (a) the largest allowable a_2 of the belt, (b) the displacement of the package relative to the belt as the belt comes to stop. (Refer Fig. 5.85)*

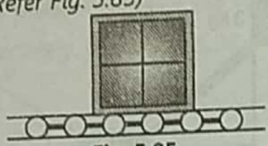

Fig. 5.85

Answer : a_2 = 6.63 m/s² (deceleration), s = 0.32 m

Problem No. 10 : *A shunting engine with two bogies is moving on a straight level track. The mass of the engine is 15000 kg and that of the each bogie is 10000 kg. The frictional resistance to motion is 1 kN for the engine and 0.75 kN for each bogie. If the acceleration is 2 m/s², compute the tractive force exerted by the engine and the tension in the two connecting couplings. If the speed is 18 km/h, find the h.p. developed by the engine. (Refer Fig. 5.86)*

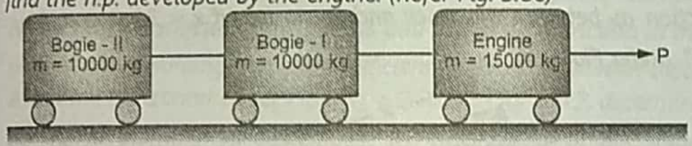

Fig. 5.86

Answer : T=20.75 kN, T_1=41.5 kN, P=72.5 kN, Power = 485.93 hp.

Problem No. 11 : *The system is released from rest in the position shown in Fig. 5.37. Calculate the tension T in the cord and the acceleration a_X of the 30 kg block A. The small pulley attached to the block has negligible mass and friction. (Refer Fig. 5.87)*

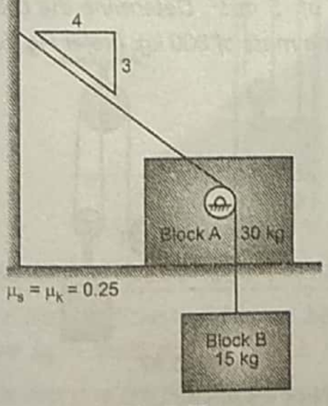

Fig. 5.87

Answer : a_x = 0.777 m/s², T = 138.4 N

Problem No. 12 : *The speed of 17.5 kN car is plotted over the 30 s time period. Determine the tractive force F acting on the car needed to cause the motion at t = 5s and at t = 20s. (Refer Fig. 5.88)*

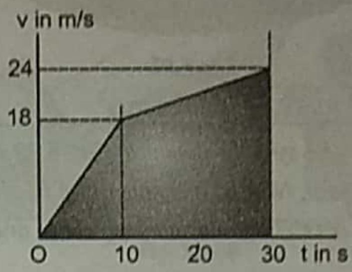

Fig. 5.88

Answer : F_{20} = 0.535 kN

Problem No. 13 : *The two kg collar C is free to slide along the smooth shaft AB. Determine the acceleration of the collar C if collar A is subjected to an upward acceleration of 4 m/s². (Refer Fig. 5.89)*

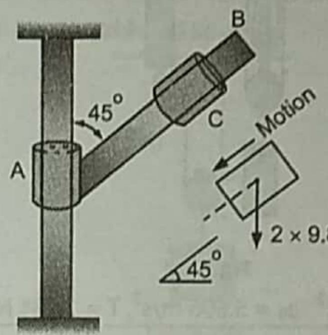

Fig. 5.89

Answer : a_c = 7.49 m/s² 45°

Problem No. 14 : *The elevator E has a mass of 500 kg and the counter weight at A has a mass of 150 kg. If the motor supplies a constant force of 5 kN on the cable at B, determine the speed of the elevator in t = 3s starting from rest. Neglect the mass of the pulleys and cable. (Refer Fig. 5.90)*

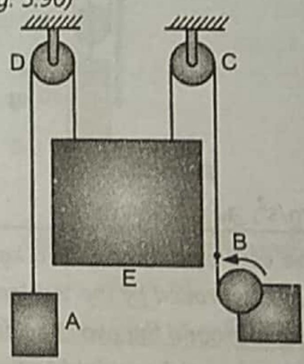

Fig. 5.90

Answer : v = 7.23 m/s

Problem No. 15 : *Block B rests on a smooth surface. If the coefficient of static friction, μ_S = 0.4 and kinetic friction μ_K = 0.3, determine the acceleration of each block if block A is pushed by horizontal force of (a) P = 30 N, (b) 250 N. (Refer Fig. 5.91)*

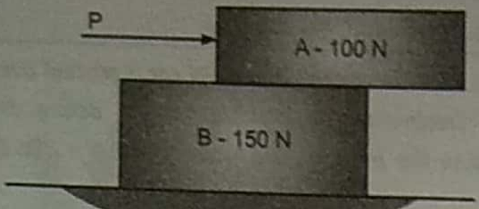

Fig. 5.91

Answer : a = 1.177 m/s^2, a_A = 21.58 m/s^2

Problem No. 16 : *The system shown in Fig. 5.92 is released from rest with all cables taut. Neglect the mass and friction of all pulleys and determine the acceleration of each cylinder and the tension in the cable.*

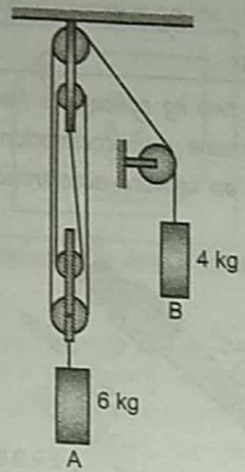

Fig. 5.92

Answer : a_A = 1.4 m/s^2 a_B = 5.606 m/s^2, T = 16.84 N

Problem No. 17 : *Neglect all friction and mass of the pulleys and determine the acceleration of bodies A and B released from rest as shown in Fig. 5.93.*

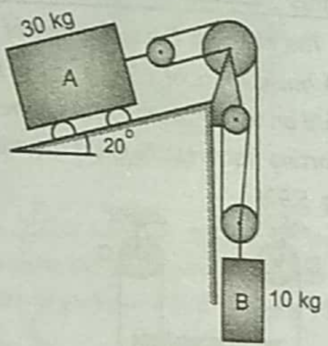

Fig. 5.93

Answer : a_B = 0.682 m/s^2, a_A = 1.024 m/s^2

Problem No. 18 : *The acceleration of the 50 kg carriage A in its smooth vertical guides is controlled by the tension T exerted on the control cable which passes around the two circular pegs fixed to the carriage. Determine the value of T required to limit the downward acceleration of the carriage to 1.2 m/s^2, if the coefficient of friction between the cable and the pegs is 0.2. (Use $T_1/T_2 = e^{\mu\beta}$). Refer Fig. 5.94.*

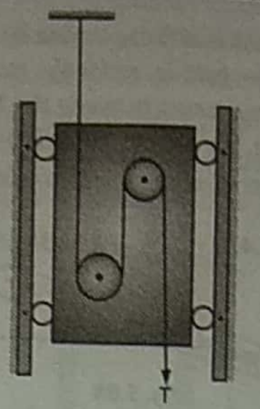

Fig. 5.94

Answer : T = 171.3 N

Problem No. 19 : The sliders A and B connected by a light rigid bar of length l = 0.5 m and move with negligible friction in the horizontal slot shown. For the position where x_A = 0.4 m, the velocity of A is v_A = 0.9 m/s to the right. Determine the acceleration of each slider and the force in the bar at this instant. (Refer Fig. 5.95)

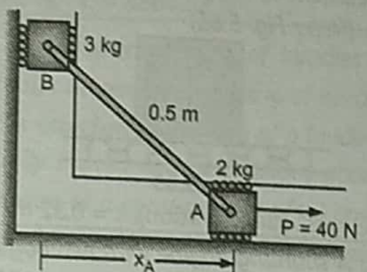

Fig. 5.95

Answer : a_B = 9.32 m/s^2, T = 46.6 N, a_A = 1.36 m/s^2

Problem No. 20 : *The block shown is observed to have a velocity v_1 = 20 ft/sec as it passes point A and a velocity v_2 = 10 ft/sec as it passes point B on the incline. Calculate the coefficient of kinetic friction μ_k between the block and the incline if x = 30 ft and θ = 15°. (Refer Fig. 5.96)*

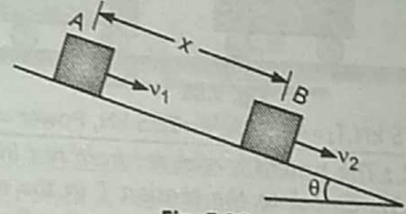

Fig. 5.96

Answer : μ_k = 0.429

Problem No. 21 : *The winding drum D is drawing in the cable at an accelerated rate of 5 m/s^2. Determine the cable tension if the suspended crate has a mass of 800 kg. (Refer Fig. 5.97).*

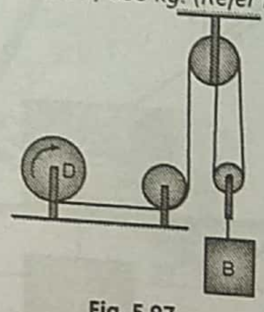

Fig. 5.97

Answer : T = 4924 N or 4.924 kN

Problem No. 22 : *In anticipation of long 7° upgrade, a bus driver accelerates at a constant rate of 1 m/s² while still on level section of the highway. Knowing that the speed of the bus is 100 kmph as it begins to climb the grade and that the driver does not change the setting of his throttle or shift gears, determine the distance travelled by the bus up the grade when his speed has decreased to 80 kmph.*

Answer : s = 711 m

Problem No. 23 : *If an automobile's breaking distance from 90 kmph is 50 m on level pavement, determine the automobile's breaking distance from 90 kmph when it is (a) going up 5° incline, (b) going down 3 percent incline.*

Answer : a = 6.25 m/s² (deceleration), F = – 6.25 mN, s = 44 m, s = 52.5 m

Problem No. 24 : *A 20 kg package is at rest on an incline when a force P is applied to it. Determine the magnitude of P if 10 s is required for the package to travel 5 m up the incline. The static and kinetic coefficient of friction between the package and the incline are 0.4 and 0.3 respectively. (Refer Fig. 5.98)*

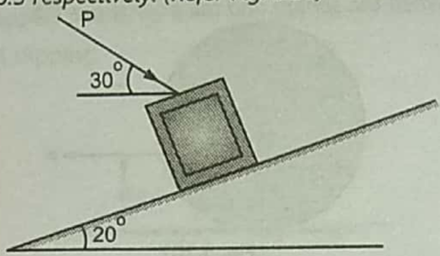

Fig. 5.98

Answer : 301.1 N

Problem No. 25 : *The two blocks shown are originally at rest. Neglecting the masses of the pulleys and the effect of friction in the pulleys and assuming that the coefficient of friction between block A and the horizontal surface are μ_s = 0.25 and μ_k = 0.2, determine (a) the acceleration of each block, (b) the tension in the cable. (Refer Fig. 5.99)*

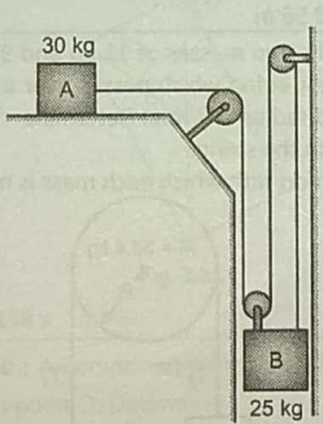

Fig. 5.99

Answer : a_B = 0.236 m/s², a_A = 0.71 m/s², T = 79.78 N

Problem No. 26 : *The coefficient of friction between package A and the incline are μ_s = 0.35 and μ_k = 0.3. Knowing that the system is initially at rest and the block B comes to rest on block C, determine (a) the maximum velocity reached by package A, (b) the distance up the incline through which package A travels before coming to rest. (Refer Fig. 5.100)*

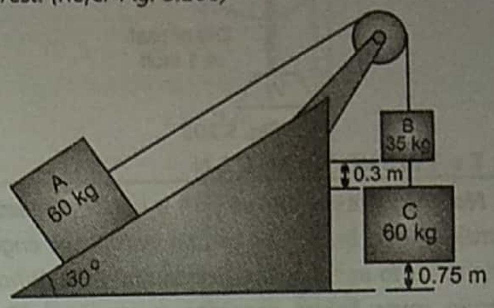

Fig. 5.100

Answer : v_{max} = 2.17 m/s, s_{max} = 1.36 m

Problem No. 27 : *A weight W = 1000 N is suspended in a vertical plane by a string and pulleys arranged as shown in Fig. 5.101. Neglecting friction in pulleys, find the tension T in the string, if free end A of string is pulled vertically downward (a) with constant acceleration a = 6 m/s², (b) what is the acceleration of A, if it is subjected to mass, m = 66.56 kg ?*

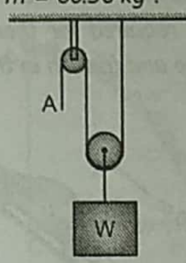

Fig. 5.101

Answer : T = 652.9 N, a_w = 3 m/s, a_A = 6 m/s

Problem No. 28 : *A small block starts from rest at point A and slides down the inclined plane AB as shown in Fig. 5.102. What distance s along the horizontal plane BC will it travel before coming to rest ? The coefficient of kinetic friction between the block and either plane is μ = 0.3. Assume that the initial velocity with which it starts to move along BC is of the same magnitude as that gained in sliding from A to B.*

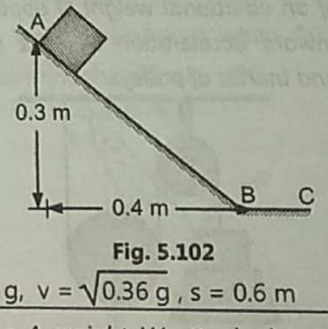

Fig. 5.102

Answer : a = 0.36 g, v = $\sqrt{0.36\,g}$, s = 0.6 m

Problem No. 29 : *A weight W attached to the end of a small flexible rope of diameter 1/4 in is raised vertically by winding the rope on a real as shown in Fig. 5.103. If the real is turned uniformly at the rate of 2 rps, what will be the tension T in the rope ? Neglect inertia of the rope and slight lateral motion of the suspended weight W.*

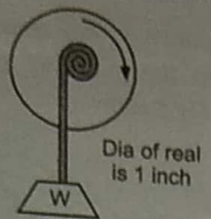

Fig. 5.103

Answer : T = 1.016 W, T = 1002.1 N

Problem No. 30 : *A train moves with a uniform speed of 36 mph along a straight level track. At a certain instant the engineer moves the throttle so as to increase the traction by 20%. What distance x will the train cover before acquiring a speed of 45 mph if the resistance to motion is constant and equal to $\frac{1}{200}$ of the weight of the train ?*

Answer : s = 7339.45 m

Problem No. 31 : *Two small cars of weights W_1 = 200 N and W_2 = 100 N are connected by a flexible but inextensible string overrunning a pulley C and are free to roll on an inclined plane as shown in Fig. 5.104. If the cars are released from rest in the position shown, find the time t required for them to exchange positions. Neglect rolling resistance and friction in the pulley.*

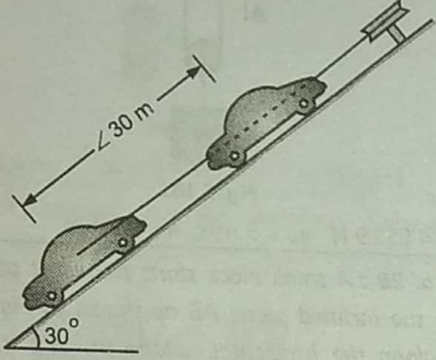

Fig. 5.104

Answer : t = 6.1 sec

Problem No. 32 : *Weights W and 2W are supported in a vertical plane by a string and pulleys arranged as shown in Fig. 5.105. Find the magnitude of an additional weight Q applied on the lift which will give a downward acceleration a = 0.1 g to the weight W. Neglect friction and inertia of pulleys.*

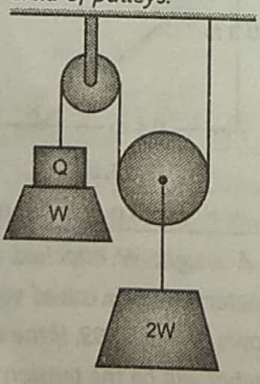

Fig. 5.105

Answer : $Q = \frac{W}{6}$

Problem No. 33 : *A block of weight W rests on a flat car which moves horizontally with constant acceleration a shown in Fig. 5.106. Determine (a) the value of the acceleration a at which slipping of the block on the car will impend, (b) the value of the acceleration at which tipping of the block about the edge A impend without slipping and (c) the shortest distance s in which it can be stopped with constant deceleration without disturbing the block. Take c = 20 mm, h = 30 mm and μ = 0.5, u = 20 m/s.*

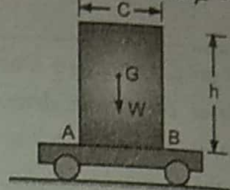

Fig. 5.106

Answer : a_1 = 4.905 m/s^2 (deceleration), a_2 = 6.54 m/s^2 (deceleration), s = 40.77 m

Problem No. 34 : *A homogeneous sphere of radius r = 100 mm and weight W = 100 N slides along the floor under the action of constant horizontal force P applied to a string and travelled a distance s = 9.82 m in time t = 2s starting from rest. Determine the height h and external force P during this motion if the coefficient of friction between the sphere and the floor is μ = 0.5.*

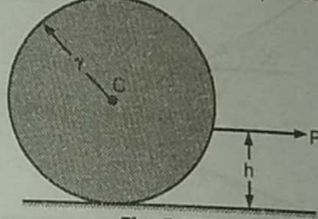

Fig. 5.107

Answer : a = 4.91 m/s^2, P = 100 N, h = 50 mm

Problem No. 35 : *A jet airplane with a mass of 5 mg has a touchdown speed of 300 kmph, at which instant the breaking parashute is deployed and the power shut-off. If the total drag on the aircraft varies with velocity 120 kN at v = 300 kmph, calculate the distance x along the runway required to reduce the speed to 150 kmph. The variation of the drag by an equation of D = kv^2, where k is a constant.*

Answer : x = 200.56 m

Problem No. 36 : Two masses of 11 kg and 9 kg are attached to the ends of a light string which passes over a solid disc pulley of mass 38.4 kg and radius 225 mm. Determine -

(a) the tension in the string.

(b) the acceleration with which each mass is moving.

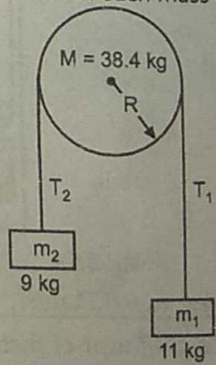

Fig. 5.108

Answer : a = 0.5 m/s^2, T_1 = 102.23 N, T_2 = 92.93 N.

Problem No. 37 : A right circular cylinder of mass m and radius r is suspended from a cord that is wound round its circumference. If the cylinder is allowed to fall freely, find the acceleration of the mass centre G and the tension in the cord.

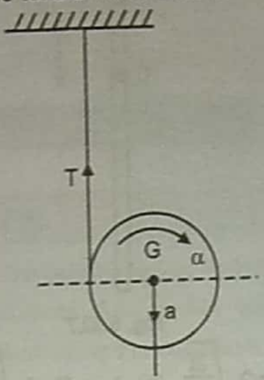

Fig. 5.109

Answer : $a = \frac{2}{3} g$, $T = \frac{mg}{3}$

Problem No. 38 : A solid cylinder having diameter 1.2 m weighs 270 N. It is acted upon by an upward force of 45 N applied by a chord wrapped round it. Find the coefficient of friction required to prevent slipping.

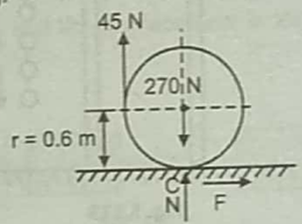

Fig. 5.110

Answer : $\mu = 0.133$

Problem No. 39 : A thin rectangular plate of size 300 mm × 400 mm is freely suspended from one of its corners and oscillates as a compound pendulum. Determine the time period of oscillation.

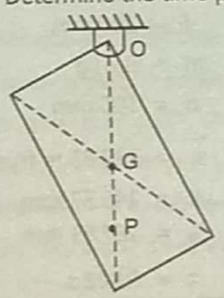

Fig. 5.111

Answer : t = 1.158 s.

Problem No. 40 : A compound pendulum shown in Fig. 5.112 is suspended from point O. Determine -

(a) equivalent length l_e of the compound pendulum.

(b) time period of oscillations.

$$m_{rod} = 5 \text{ kg}$$
$$m_{disc} = 2.5 \text{ kg (each)}$$

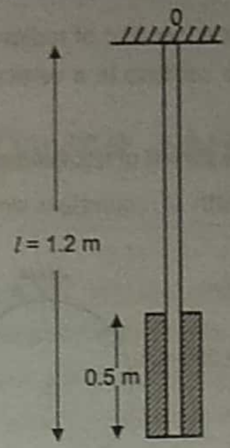

Fig. 5.112

Answer : $l_e = 0.9$ m, t = 1.91 ms

Problem No. 41 : A flexible chain of length (πr/2) is held on smooth cylindrical surface of radius 'r' as shown in following Fig. 5.113. Calculate the velocity v with which the chain will leave the point B if released from rest. The chain has a length 'w' per unit length.

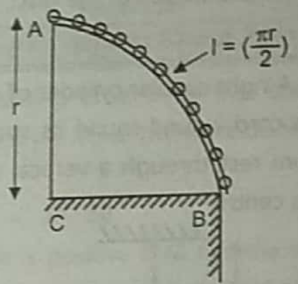

Fig. 5.113

Answer : $v = \sqrt{\frac{32\ gr + 4\pi^2 rg}{8\pi}}$.

Problem No. 42 : A compound pendulum of mass 48 kg is free to rotate about the pivot A. Distance of CG is 400 mm (AG). The radius of gyration about the axis of rotation is 240 mm. The pendulum is released from rest when θ = 50°. Compute the angular velocity of the pendulum when AG is vertical. Neglect friction. Also determine the velocity of G.

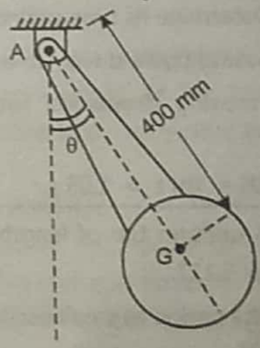

Fig. 5.114

Answer : ω = 6.973 rad/s, v= 2.789 m/s

Problem No. 43 : A circular disc of radius 200 mm has a uniform thickness. It is free to oscillate in a vertical plane about point A, on its rim.

(a) Determine the period of oscillation.

(b) Also find length of equivalent simple pendulum for the same period.

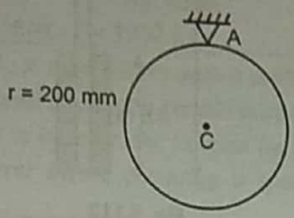

Fig. 5.115

Answers : $t = 1.099$ s, $l_e = 0.3$ m.

Problem No. 44 : A cord is wrapped around a homogeneous disc of radius $r = 0.5$ m and $m = 15$ kg. If the cord is pulled upward with a force 'T' of magnitude 180 N, determine :

(a) the acceleration of the centre of disc.

(b) the angular acceleration of the disc.

Answer : $a = 2.19$ m/s², $\alpha = 48$ rad/s².

Problem No. 45 : A right circular cylinder of mass m and radius r is suspended by a cord wound round its surface. The cylinder is allowed to fall from rest through a vertical distance h. Find the velocity of its mass centre.

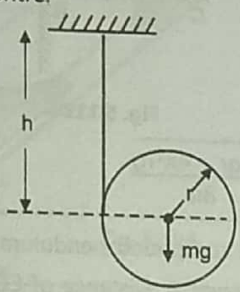

Fig. 5.116

Answer : $v = \sqrt{\frac{4\,gh}{3}}$

Problem No. 46 : A pendulum having a time period of 1 second is installed in a lift. Determine its time period when

(a) the lift is moving upward with an acceleration of (g/10),

(b) the lift is moving downward with an acceleration of (g/10).

Answer : (a) $t = 0.95$ s (b) $t = 1.05$ s.

Problem No. 47 : A uniform bar of length l is suspended from end A as shown in Fig. 5.117.

(a) Determine the period of small oscillations.

(b) If the bar is then suspended from point A' at one quarter of its length, find the period of small oscillations.

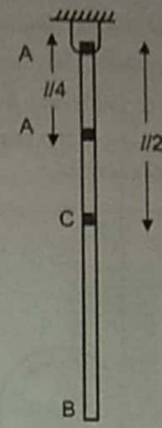

Fig. 5.117

Answer : (a) $t = 2\pi\sqrt{\frac{2l}{3g}}$ (b) $t = 2\pi\sqrt{\frac{7l}{12g}}$

Problem No. 48 : A flexible chain PQ of length l is held on a smooth table with portion "h" overhanging. Calculate the velocity with which the chain will leave the table if released from rest. The chain weighs "w" per unit length.

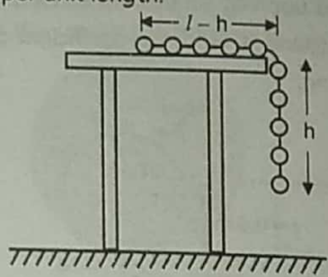

Fig. 5.119

Answer : $v = \sqrt{\frac{g}{l}(l^2 - h^2)}$

Problem No. 49 : For the compound pendulum shown, find the time period of oscillation.

Rod : $m = 0.4$ kg

$l = 200$ mm

Disc : $m = 0.2$ kg

$d = 600$ mm

Answer : $\bar{x} = 133.33$ m from O.

$K = 149.57$ mm

$l_e = 167.83$ mm

$t = 0.822$ s.

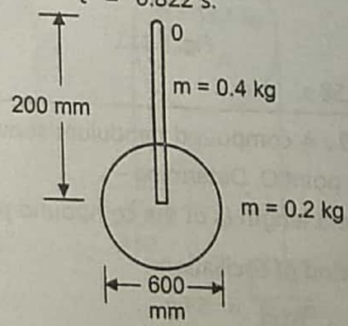

Fig. 5.120

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WORK, POWER AND ENERGY

A – VIRTUAL WORK

6.1 INTRODUCTION

In the earlier chapters the problems involving the equilibrium of a rigid body and a system of rigid bodies when acted upon by several forces were solved using the equations of equilibrium.

In this chapter we shall discuss another method of expressing the conditions of equilibrium which is based on the principle of virtual work.

6.2 WORK OF A FORCE

Consider a constant force F acting on a body whose movement along the plane from A to A' is represented by dx, called the displacement of the body. By definition, the work done is the product of the force and the displacement in the direction of the force.

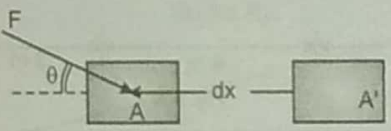


Fig. 6.1

$$U = (F \cos \theta)\, dx$$

The work done by a force is a scalar quantity which has a magnitude and sign but no direction. The unit of work (N-m) is called joule (J).

Note :

- If $\theta = 0$, then the work done by the force will be $U = (F)\, dx$
- Work done by the force is zero if either the displacement is zero or the force acts normal ($\theta = 90°$) to the displacement.
- Work done by the force is positive only when the direction of the force and displacement are in the same direction.

For example, in the figure, the work done in all the four cases is positive.

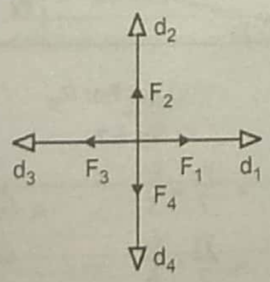


Fig. 6.2 (a)

- Work done by a force is negative when the force and the displacement are in opposite directions.

For example, in the figure, the displacements are in the opposite directions of the forces. Hence, in all the four cases the work done is negative.

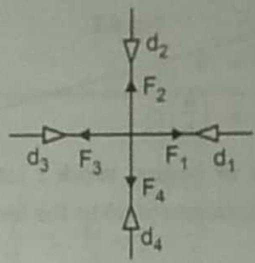


Fig. 6.2 (b)

6.3 WORK OF A COUPLE

Let M be a couple acting on a body that changes its angular position by an amount $d\theta$. Thus, the work done by the couple is defined as -

$$U = M \cdot d\theta$$

The work of a couple is positive if M has the same sense as $d\theta$ and negative if M has a sense opposite to that of $d\theta$.

6.4 VIRTUAL DISPLACEMENT AND VIRTUAL WORK

When a system of forces acting on a body are in equilibrium, then the displacement of the body would be zero, and no work is possible. But an imaginary infinite small displacement can be assumed to be given to the body in equilibrium. Such a displacement is called virtual displacement. The resulting work done by the forces acting on the body during the virtual displacement is called the virtual work.

Principle of Virtual Work :

"If a rigid body is in equilibrium, then the total work done by the active and reactive forces acting on the body is zero for any virtual displacement of the body consistent with the geometrical conditions of the body."

Forces Which do Not Work are :

- The internal forces of the nature of action and reaction (tension in the string and axial forces in the bars)
- Weight of the body when the c.g. moves in the horizontal direction.
- Reaction at a frictionless hinge when the body rotates about the hinge.

- Reaction at a frictionless surface when the body moves along the surface.

Application of Principle of Virtual Work :

Example 1 : Obtain the relation between Q and P by using –

(1) Equilibrium equations

(2) Principle of virtual work

(1) Using Equilibrium Equation :

$$\Sigma M_C = 0$$

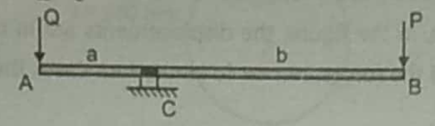


Fig. 6.3

$$-(Q \times a) + (P \times b) = 0$$

$$P = \left(\frac{a}{b}\right) Q$$

(2) Using Principle of Virtual Work : Let us give an infinite small angular displacement dθ to the lever about C as shown in Fig. 6.4.

End A moves from A to A' and end B moves from B to B'.

$$AA' = a\,d\theta$$

$$BB' = b\,d\theta$$

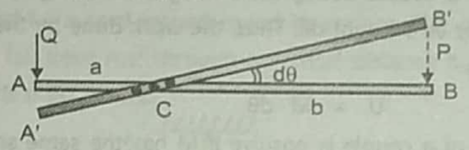


Fig. 6.4

The virtual displacements of end A and end B are (a dθ) and (b dθ) respectively. It is also seen that the virtual displacements of the ends A and B are consistent with the geometrical condition of the body.

Applying the principle of virtual work, we have,

$$Q\,(AA') - P\,(BB') = 0$$

$$Q\,(a\,d\theta) - P\,(b\,d\theta) = 0$$

$$P = \left(\frac{a}{b}\right) Q$$

Example 2 : Determine the force in member AD in terms of P for the frame shown in Fig. 6.5 (a)

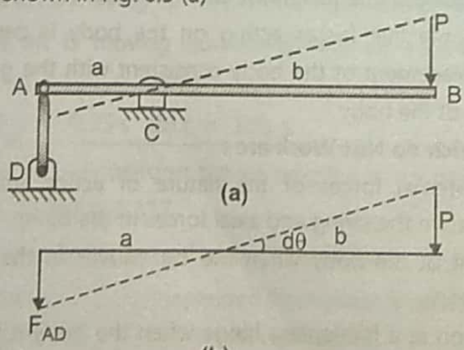


Fig. 6.5

In the previous example, the lever was movable about hinge C i.e. it was partially restrained. It was, therefore, possible to give virtual angular displacement. Here the system is completely restrained. It is therefore, not possible to give virtual displacement. To solve such problems we remove one of the constraints or supports and replace it by a suitable force acting at the support point.

Equation of virtual work can now be written as -

$$+ F_{AD}\,(a\,d\theta) - P\,(b\,d\theta) = 0$$

$$F_{AD} = \left(\frac{b}{a}\right) P$$

Important Note :

- Apply unit load at that reaction whose value, we have to calculate.
- For concentrated load multiply by the ordinate and for uniformly distributed load, multiply by the area below that uniformly distributed load i.e. (UDL).

SOLVED EXAMPLES

***Example 6.1 :** Using method of virtual work, determine reactions at A and B.*

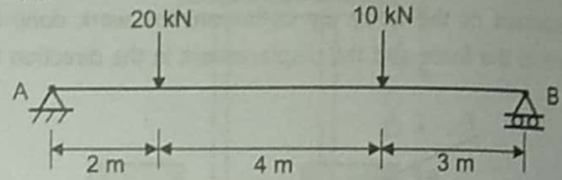


Fig. 6.6

Solution :

Given data : As shown in Fig. 6.6.

To find : Reactions at A and B.

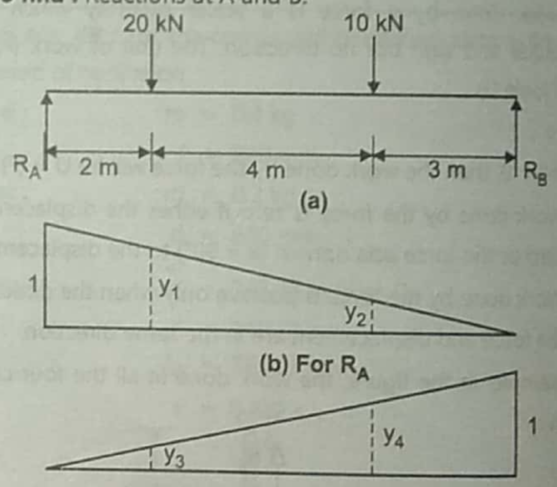


Fig. 6.7

$$\text{Fig. 6.7 (b)}: \quad \frac{1}{9} = \frac{y_1}{7} = \frac{y_2}{3}$$

$$\text{Fig. 6.7 (c)}: \quad \frac{1}{9} = \frac{y_3}{2} = \frac{y_4}{6}$$

From Fig. 6.7 (b),

$$R_B \times 0 + R_A \times 1 - 20 \times y_1 - 10 \times y_2 = 0$$

$$\therefore \quad R_A = 20 \times \frac{7}{9} + 10 \times \frac{3}{9} = \mathbf{18.89\ kN} \quad \text{... Ans.}$$

From Fig. 6.7 (c),

$$R_B \times 1 + R_A \times 0 - 20 \times y_3 - 10 \times y_4 = 0$$

$$R_B = 20 \times \frac{2}{9} + 10 \times \frac{6}{9} = \mathbf{11.11\ kN} \quad \text{... Ans.}$$

Example 6.2 : *Using virtual work method, determine reactions at A and B.*

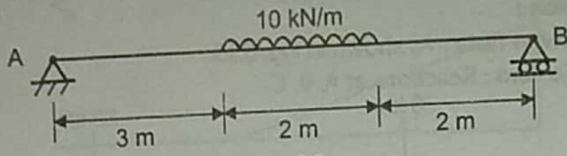
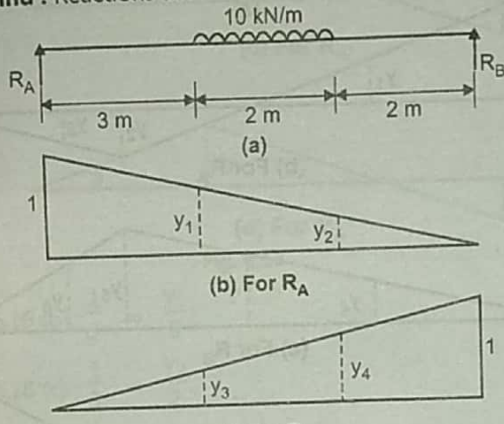


Fig. 6.8

Solution :

Given data : As shown in Fig. 6.8.

To find : Reactions at A and B.



(a) (b) For R_A (c) For R_B

Fig. 6.9

$$\text{Fig. 6.9 (b)}: \frac{1}{7} = \frac{y_1}{4} = \frac{y_2}{2}$$

$$\text{Fig. 6.9 (c)}: \frac{1}{7} = \frac{y_3}{3} = \frac{y_4}{5}$$

From Fig. 6.9 (b),

$$R_A \times 1 + R_B \times 0 - 10\left(\frac{y_1 + y_2}{2}\right) \times 2 = 0$$

$$\therefore \quad R_A = \mathbf{8.57\ kN} \quad \text{... Ans.}$$

From Fig. 6.9 (c),

$$R_A \times 0 + R_B \times 1 - 10\left(\frac{y_3 + y_4}{2}\right) \times 2 = 0$$

$$\therefore \quad R_B = \mathbf{11.43\ kN} \quad \text{... Ans.}$$

Example 6.3 : *Determine reactions at A and B. Use virtual work method.*

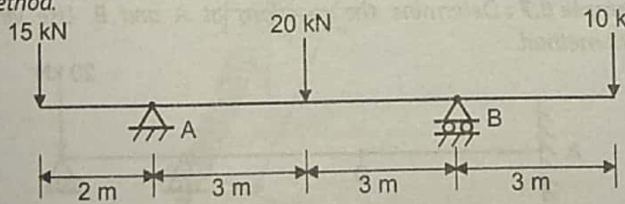
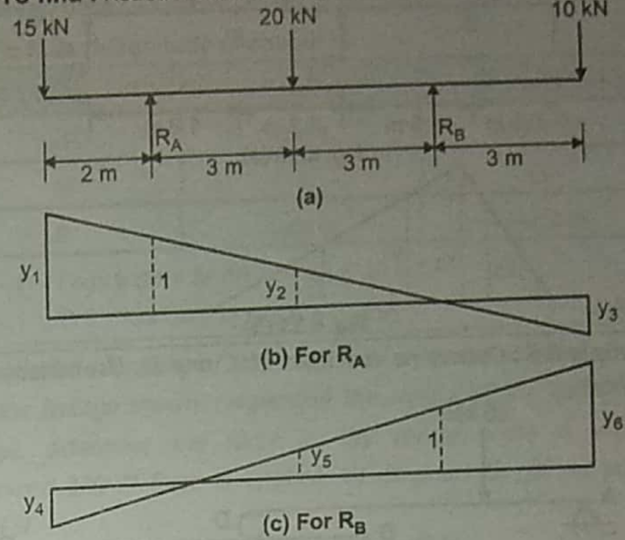


Fig. 6.10

Solution :

Given data : As shown in Fig. 6.10.

To find : Reactions at A and B.



(a) (b) For R_A (c) For R_B

Fig. 6.11

$$\text{Fig. 6.11 (b)}: \frac{1}{6} = \frac{y_1}{8} = \frac{y_2}{3} = \frac{y_3}{3}$$

$$\text{Fig. 6.11 (c)}: \frac{1}{6} = \frac{y_4}{2} = \frac{y_5}{3} = \frac{y_6}{9}$$

From Fig. 6.11 (b),

$$-5 \times y_1 + R_A \times 1 - 20 \times y_2 + R_B \times 0 + 10 \times y_3 = 0$$

$$\therefore \quad R_A = \mathbf{11.67\ kN} \quad \text{... Ans.}$$

From Fig. 6.11 (c),

$$5 \times y_4 + R_A \times 0 - 20 \times y_5 + R_B \times 1 - 10 \times y_6 = 0$$

$$\therefore \quad R_B = \mathbf{23.33\ kN} \quad \text{... Ans.}$$

Example 6.4 : *Using method of virtual work, determine the reaction at E for the compound beam AEH as shown in Fig. 6.12. The beam is having internal hinge at C and F.*

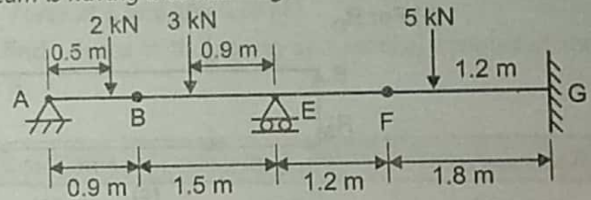


Fig. 6.12

Solution :

Given data : As shown in Fig. 6.12.

To find : Reaction at E.

$$\text{Fig. 6.13 (b)}: \frac{1}{1.2} = \frac{y_1}{2.1}$$

$$= \frac{y_2}{2.7}, \frac{y_2}{0.9} = \frac{y_3}{0.5}$$

From Fig. 6.13 (b), $-2 \times y_3 - 3 \times y_1 + R_E \times 1 = 0$

$$\therefore \quad R_E = \mathbf{7.75\ kN\ (\uparrow)} \quad \text{... Ans.}$$

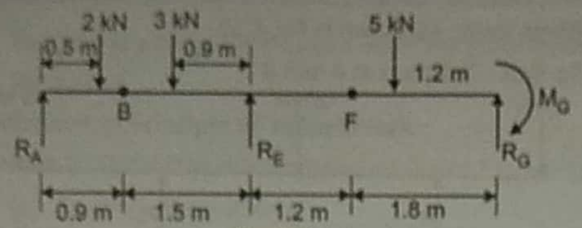


Fig. 6.13 (a)

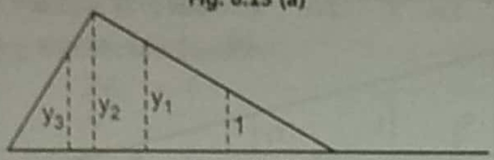


Fig. 6.13 (b)

Example 6.5 : *Determine reactions at C and D. Use virtual work method.*

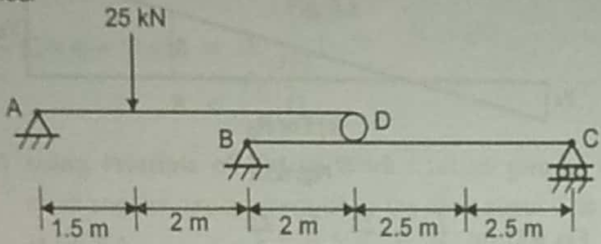


Fig. 6.14

Solution :

Given data : As shown in Fig. 6.14.

To find : Reactions at C and D.

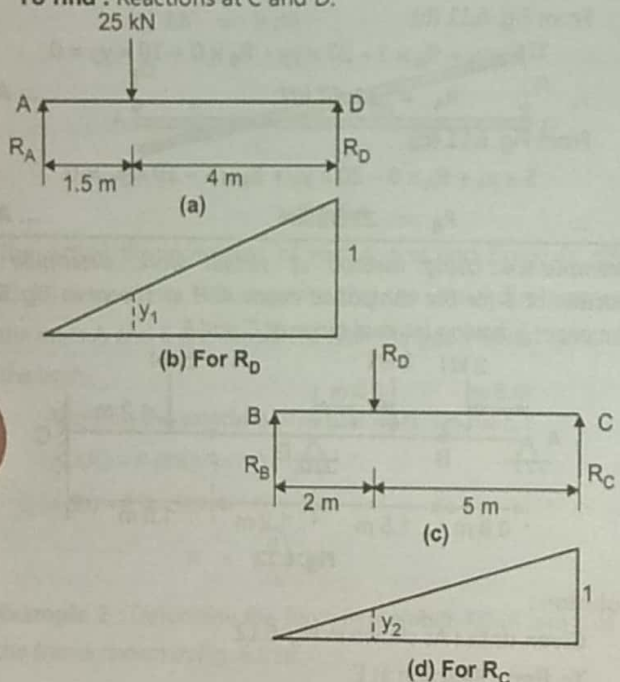


Fig. 6.14

Fig. 6.14 (b) : $\frac{1}{5.5} = \frac{y_1}{1.5}$

Fig. 6.14 (d) : $\frac{1}{7} = \frac{y_2}{2}$

From Fig. 6.14 (b),

$R_A \times 0 - 25 \times y_1 + R_D \times 1 = 0$

$\therefore \quad R_D = $ **6.82 kN** ... **Ans.**

From Fig. 6.14 (c),

$R_B \times 0 - 6.82 \times y_2 + R_C \times 1 = 0$

$\therefore \quad R_C = $ **1.95 kN** ... **Ans.**

Example 6.6 : *Determine the reactions at A, B and C. Use virtual work method.*

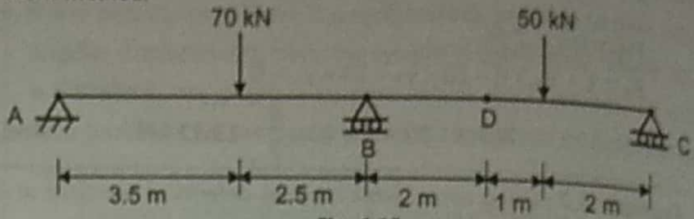


Fig. 6.15

Solution :

Given data : As shown in Fig. 6.15.

To find : Reactions at A, B, C.

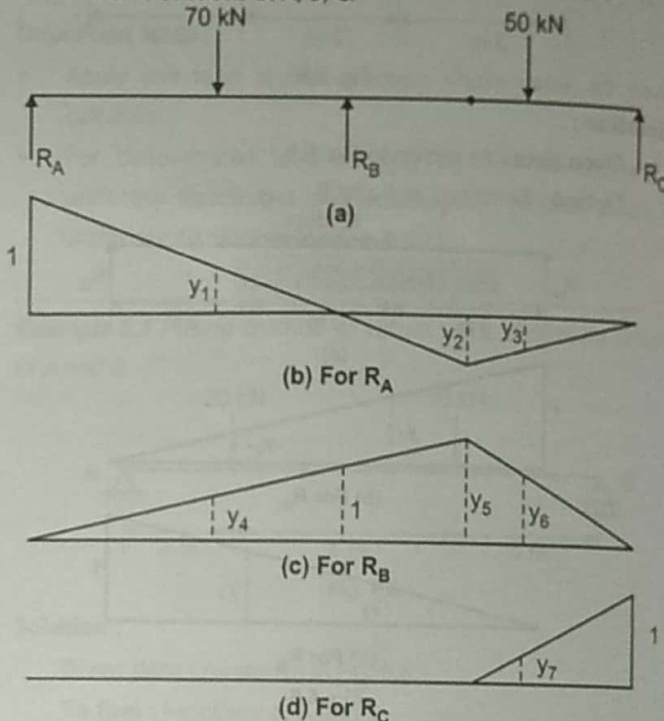


Fig. 6.15

Fig. 6.15 (b) : $\frac{1}{6} = \frac{y_1}{2.5}, \frac{y_1}{2.5} = \frac{y_2}{2}, \frac{y_2}{3} = \frac{y_3}{2}$

Fig. 6.15 (c) : $\frac{1}{6} = \frac{y_4}{3.5} = \frac{y_5}{8}, \frac{y_5}{3} = \frac{y_6}{2}$

Fig. 6.15 (d) : $\frac{1}{3} = \frac{y_7}{1}$

From Fig. 6.15 (b), $R_A \times 1 - 70 \times y_1 + 50 \times y_3 = 0$

$R_A = $ **18.06 kN** ... **Ans.**

From Fig. 6.15 (c),

$-70 \times y_4 + R_B \times 1 - 50 \times y_6 = 0$

$\therefore \quad R_B = $ **85.27 kN** ... **Ans.**

$R_C \times 1 - 50 \times y_7 = 0$

$\therefore \quad R_C = $ **16.67 kN** ... **Ans.**

Example 6.7 : *Determine the reactions at A and B. Use virtual work method.*

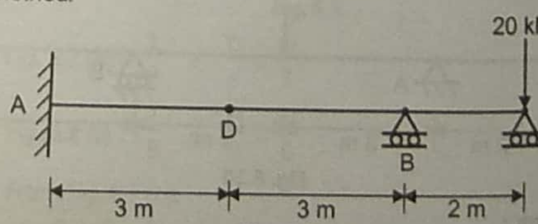
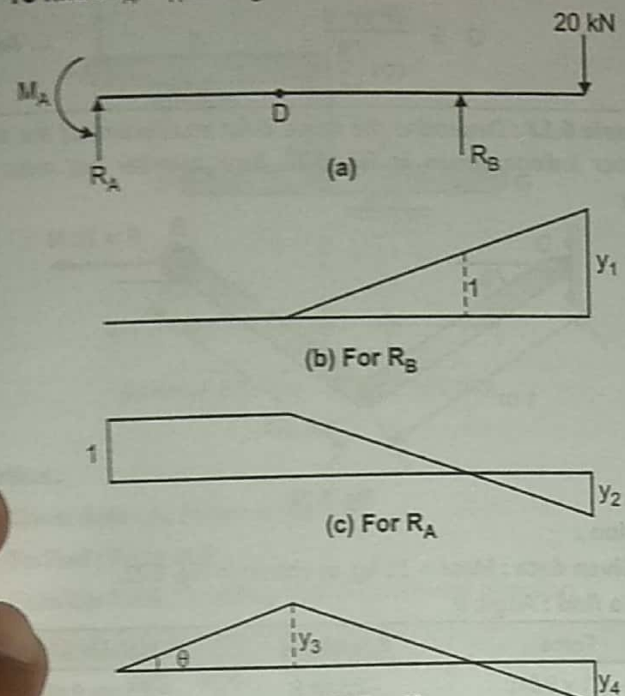


Fig. 6.16

Solution :

Given data : As shown in Fig. 6.16.

To find : R_A, M_A and R_B.



(b) For R_B

(c) For R_A

(d) For M_A

Fig. 6.16

Fig. 6.16 (b) : $\frac{1}{3} = \frac{y_1}{5}$

Fig. 6.16 (c) : $\frac{1}{3} = \frac{y_2}{2}$

Fig. 6.16 (d) : $\theta = \frac{y_3}{3}, \frac{y_3}{3} = \frac{y_4}{2}$

From Fig. 6.16 (b), $R_B \times 1 - 20 \times y_1 = 0$

$\therefore \quad R_B = \mathbf{33.33\ kN}$ **... Ans.**

From Fig. 6.16 (c), $R_A \times 1 + 20 \times y_2 = 0$

$\therefore \quad R_A = -13.33\ N = \mathbf{13.33\ kN\ (\downarrow)}$ **... Ans.**

From Fig. 6.16 (d), $M_A \cdot \theta - 20 \times y_4 = 0$

$\therefore \quad M_A = \mathbf{-40\ kN\text{-}m = 40\ kN\text{-}m\ (\circlearrowleft)}$ **... Ans.**

Example 6.8 : *Derive an expression for the magnitude of the couple M required to maintain the equilibrium of the linkage as shown in Fig. 6.17.*

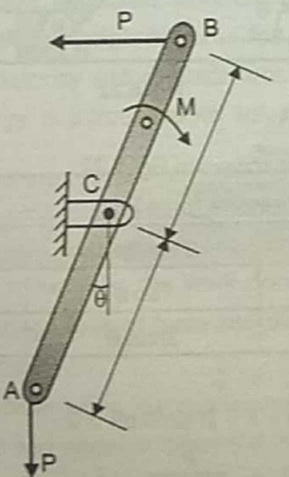


Fig. 6.17

Solution :

Given data : As shown in Fig. 6.17.

To find : Magnitude of couple M.

Force	Co-ordinate	Displacement
– P	$l \sin \theta$	$l \cos \theta\ d\theta$
M	θ	dθ
P	$l \cos \theta$	$-l \sin \theta\ d\theta$

$-P \times l \cos \theta\ d\theta + M\ d\theta - Pl \sin \theta\ d\theta = 0$

$\therefore \quad M = Pl (\sin \theta + \cos \theta)$ **... Ans.**

Example 6.9 : *A spring of stiffness 15 kN/m connects points C and F of the linkage shown. Neglecting the weight of the spring and linkage, determine the force in the spring when a vertical downward 120 N force is applied (a) At point C (b) At points C and H.*

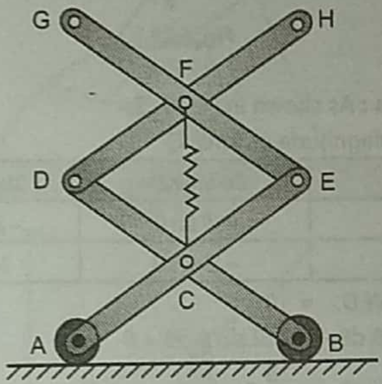


Fig. 6.18

Solution :

Given data : As shown in Fig. 6.18.

(a) Force at C = 120 N and

(b) Force at C and H = 120 N.

To find : Force in the spring and vertical downward motion of point G.

(a)

Force	Co-ordinate	Displacement
– 120	$l \sin \theta$	$l \cos \theta\ d\theta$
– F	$l \sin \theta$	$l \cos \theta\ d\theta$
F	$3l \sin \theta$	$3l \cos \theta\ d\theta$

$\therefore \quad -120 \times l \cos \theta\ d\theta - F \times l \cos \theta\ d\theta + F \times 3l \cos \theta\ d\theta = 0$

$F = 60\ N$

(b)

Force	Co-ordinate	Displacement
– 120	$l \sin \theta$	$l \cos \theta\ d\theta$
– F	$l \sin \theta$	$l \cos \theta\ d\theta$
F	$3l \sin \theta$	$3l \cos \theta\ d\theta$
– 120	$4l \sin \theta$	$4l \cos \theta\ d\theta$

$\therefore \quad -120 \times l \cos \theta\ d\theta - F \times l \cos \theta\ d\theta + F \times 3l \cos \theta\ d\theta - 120 \times 4l \cos \theta\ d\theta = 0$

$\therefore \quad F = \mathbf{300\ N}$ **... Ans.**

***Example 6.10 :** The mechanism shown is acted upon by the force P. Derive an expression for the magnitude of the force Q required for equilibrium.*

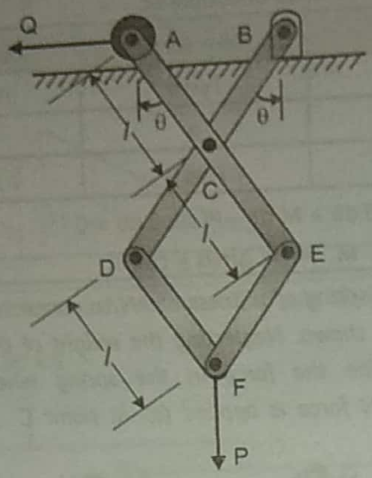


Fig. 6.19

Solution :

Given data : As shown in Fig. 6.19.

To find : Magnitude of force Q.

Force	Co-ordinate	Displacement
– Q	$-2l \sin \theta$	$-2l \cos \theta \, d\theta$
– P	$-3l \cos \theta$	$3l \sin \theta \, d\theta$

$$\text{W.D.} = 0$$

$$+ Q \times 2l \sin \theta \, d\theta - P \times 3l \sin \theta \, d\theta = 0$$

$$\therefore \quad Q = \frac{3}{2} \mathbf{P} \tan \theta \qquad \text{... Ans.}$$

***Example 6.11 :** Knowing that the line of action of the force Q passes through point C, derive an expression for the magnitude of Q required to maintain equilibrium.*

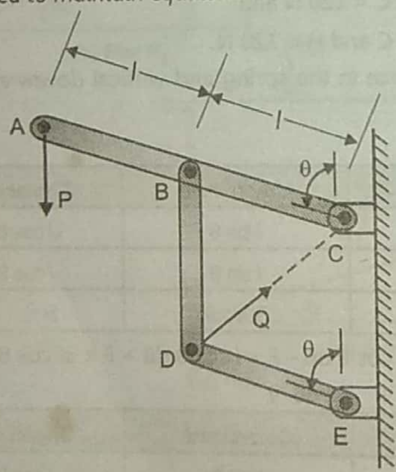


Fig. 6.20

Solution :

Given data : As shown in Fig. 6.20.

To find : Expression for force Q.

Force	Co-Ordinate	Displacement
– P	$2l \cos \theta$	$-2l \sin \theta \, d\theta$
Q sin θ	$(l - l \cos \theta)$	$l \sin \theta \, d\theta$
Q cos θ	$-l \sin \theta$	$-l \cos \theta \, d\theta$

$$\text{W.D.} = 0$$

$$-P \times (-2l \sin \theta) \, d\theta + Q \sin \theta \times l \sin \theta \, d\theta - Q \cos \theta \times l \cos \theta \, d\theta = 0$$

$$Q \cos^2 \theta - Q \sin^2 \theta = 2P \sin \theta$$

$$\therefore \quad Q = \frac{2P \sin \theta}{\cos \left(\frac{\theta}{2}\right)} \qquad \text{... Ans.}$$

***Example 6.12 :** Determine the angle θ for equilibrium of the two member linkage shown in Fig. 6.21. Each member has mass of 11 kg.*

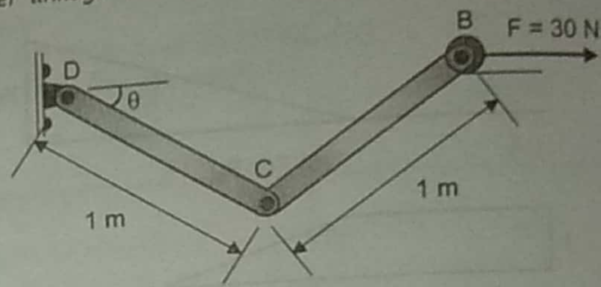


Fig. 6.21

Solution :

Given data : Mass = 11 kg, as shown in Fig. 6.21.

To find : Angle θ.

Force	Co-ordinate	Displacement
– (11 × 9.81)	– 0.5 sin θ	– 0.5 cos θ dθ
– (11 × 9.81)	– 0.5 sin θ	– 0.5 cos θ dθ
30	2 cos θ	– 2 sin θ dθ

$$\text{W.D.} = 0$$

$$(11 \times 9.81) \times 0.5 \cos \theta \, d\theta + 11 \times 9.81 \times 0.5 \cos \theta \, d\theta - 30 \times 2 \sin \theta \, d\theta = 0$$

$$107.91 \cos \theta = 60 \sin \theta$$

$$\therefore \quad \theta = \mathbf{60.93°} \qquad \text{... Ans.}$$

***Example 6.13 :** The toggle joint is subjected to the load P. Determine the compressive force F it creates on the cylinder at A as a function of θ.*

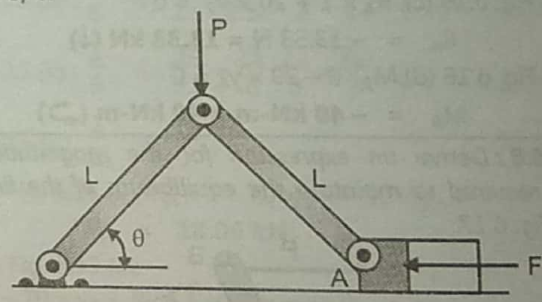


Fig. 6.22

Solution :

Given data : As shown in Fig. 6.22.

To find : Compressive force F.

Force	Co-ordinate	Displacement
– P	$l \sin \theta$	$l \cos \theta \, d\theta$
– F	$2l \cos \theta$	$-2l \sin \theta \, d\theta$

$$\text{W.D.} = 0$$

$$-P \times l \cos \theta \, d\theta + F \times 2l \sin \theta \, d\theta = 0$$

$$F = \frac{\mathbf{P}}{\mathbf{2 \tan \theta}} \qquad \text{... Ans.}$$

Example 6.14 : Determine the vertical force P which must be applied at G to maintain the equilibrium of the linkage.

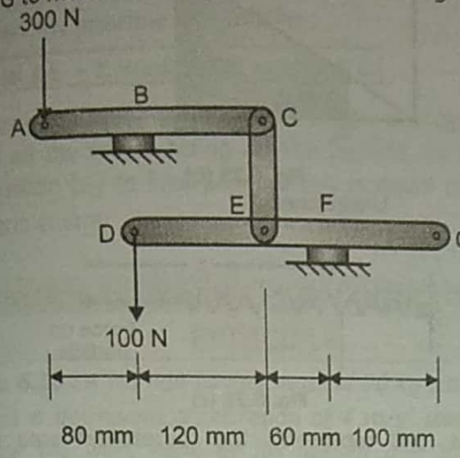


Fig. 6.23

Solution :

Given data : As shown in Fig. 6.23.

To find : Force at G.

Consider force P is acting at G in downward direction.

Applying δ displacement at D,

$$W.D. = 0$$

$$-100 \times \delta + 300 \times \frac{2\delta}{9} + P \times \frac{\delta}{1.8} = 0$$

$$\therefore \quad P = 60 \text{ N} \quad \text{... Ans.}$$

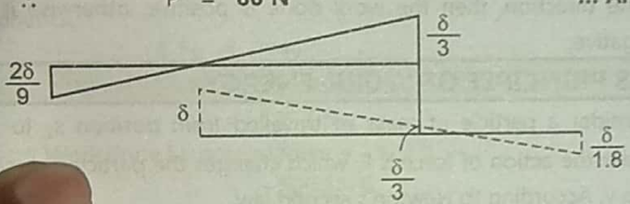


Fig. 6.24

B – WORK-ENERGY PRINCIPLE

6.5 INTRODUCTION

In this chapter, we will integrate the equation of motion with respect to displacement to obtain work-energy principle. The equation of work-energy principle is useful for solving problems which involve force, velocity and displacement. The theorem of conservation of energy is introduced to solve the problem of kinetics of a particle.

6.6 WORK

When the particle or rigid body undergoes a displacement along the line of action of force, then the work done by the force is given by product of magnitude of force and displacement along the line of action of force.

The work done is positive when the displacement takes place in the direction of force and negative when the displacement takes place in opposite direction of force.

Let F be the force and s be the displacement.

$$\text{Work done} = F \times s \text{ Nm}$$

$$= F \times s \text{ joule} \quad \text{... (1 Nm = 1 Joule)}$$

6.7 (I) WORK BY A VARIABLE FORCE

If the particle undergoes a finite displacement along its path from s_1 to s_2 shown in Fig. 6.25, the work is given by integration. If the force F is expressed as a function of position, F = F (s); we can write,

$$\boxed{\text{Work done} = \int_{s_1}^{s_2} F \cos\theta \, ds} \quad \text{... (6.1)}$$

If the component of force F cos θ is plotted against position s, Fig. 6.25 (a), the integral of equation (6.1) represents area under the curve from the position s_1 to s_2.

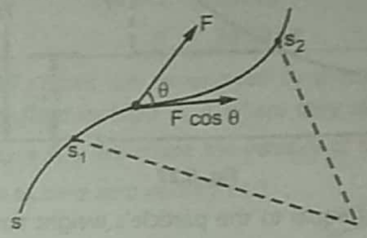


Fig. 6.25

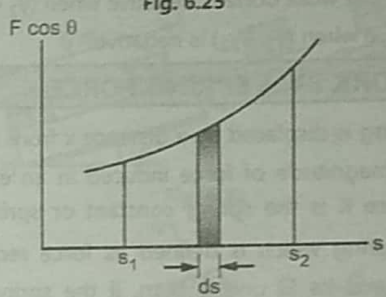


Fig. 6.25 (a)

6.7 (II) WORK BY A CONSTANT FORCE ALONG A STRAIGHT LINE

If the force of constant magnitude F acts on a particle along the straight line path shown in Fig. 6.26 (a), the workdone by the force F, when the particle is displaced from s_1 to s_2, workdone is given by,

$$\text{Work done} = F \cos\theta \int_{s_1}^{s_2} ds$$

$$\boxed{\text{Work done} = F \cos\theta \, (s_2 - s_1)} \quad \text{... (6.2)}$$

The work done by a constant force F is the area of rectangle shown in Fig. 6.26 (b).

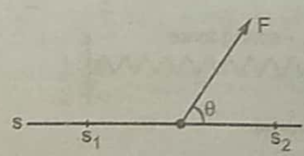


Fig. 6.26 (a)

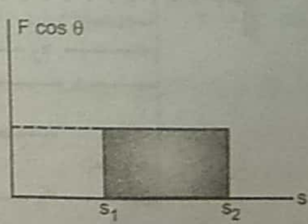


Fig. 6.26 (b)

6.7 (III) WORK BY A WEIGHT OR GRAVITY FORCE

Consider a particle which moves up along the paths from position s_1 to s_2 as shown in Fig. 6.27. At an intermediate position, the displacement ds = $y_2 - y_1$. The work done by the gravity force is given by,

$$\text{Work done} = \int_{y_1}^{y_2} W\, dy$$

$$\text{Work done} = -W(y_2 - y_1) \qquad \text{... (6.3)}$$

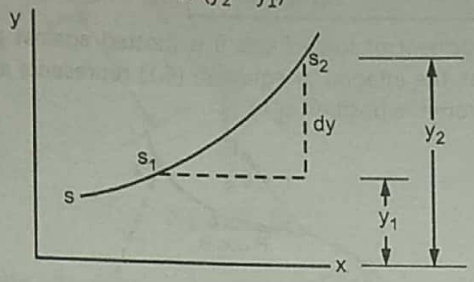


Fig. 6.27

The work done is equal to the particle's weight times its vertical displacement. The work done is negative when $(y_2 - y_1)$ is positive and it is positive when $(y_2 - y_1)$ is negative.

6.7 (IV) WORK BY A SPRING FORCE

When the spring is displaced by a distance x from its unstretched position, the magnitude of force induced in an elastic spring is $F_S = K \cdot x$, where K is the spring constant or spring stiffness or modulus of spring which is defined as force required for unit displacement and its SI unit is N/m. If the spring is deformed from a position x_1 to x_2, the work done on the spring by F_S is positive, since in both the cases force and displacement are in the same direction, as shown in Fig. 6.28 (a).

$$\text{Work done} = \int_{x_1}^{x_2} F_S\, dx = \int_{x_1}^{x_2} Kx\, dx$$

$$\boxed{\text{Work done} = \frac{1}{2} K\left(x_2^2 - x_1^2\right)} \qquad \text{... (6.4)}$$

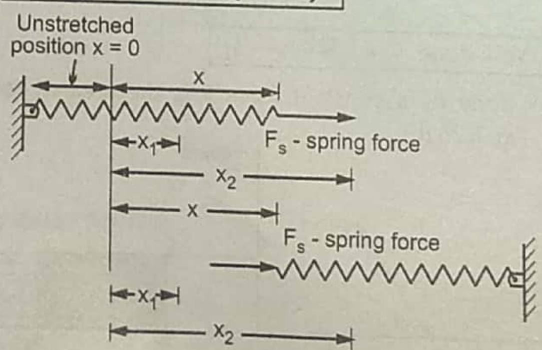


Fig. 6.28 (a)

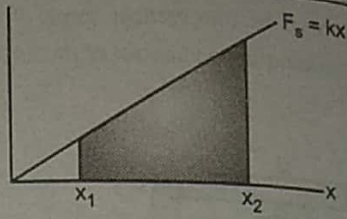


Fig. 6.28 (b)

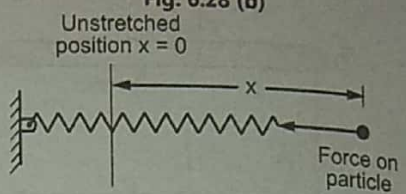


Fig. 6.28 (c)

The equation (6.4) represents the trapezoidal area shown in Fig. 6.28 (b). If the particle or body is attached to a spring, then the force F_S exerted on the particle is opposite to that of the spring, shown in Fig. 6.28 (c), hence the spring force will do the negative work on the particle when the particle is further moving so as to elongate or compress the spring. The equation (6.4) becomes,

$$\boxed{\text{Work done} = -\frac{1}{2} K\left(x_2^2 - x_1^2\right)} \qquad \text{... (6.5)}$$

If the spring force acting on a particle and displacement are in the same direction, then the work done is positive, otherwise it is negative.

6.8 PRINCIPLE OF WORK-ENERGY

Consider a particle of mass m travelled from position s_1 to s_2 under the action of force Σ F, which changes the particle velocity u to v. According to Newton's second law,

$$\Sigma F = ma$$

$$\Sigma F = m \cdot \frac{dv}{ds} \cdot \frac{ds}{dt} \qquad \ldots \left(\because \frac{ds}{dt} = v\right)$$

$$\Sigma F = m \cdot \frac{dv}{ds} \cdot v$$

$$\text{Integrating, } \Sigma F \int_{s_1}^{s_2} ds = m \int_u^v v\, dv$$

$$\boxed{\Sigma F (s_2 - s_1) = mv^2 - \frac{1}{2} mu^2} \qquad \text{... (6.6)}$$

$$\Sigma \text{ Work done} = \text{Final K.E.} - \text{Initial K.E.}$$

When a particle of mass m is traveled from position s_1 to s_2 with velocity u to v, the work done by all the forces is equal to the change in kinetic energy is known as work-energy principle.

Equation (6.6) represents work-energy principle for a particle. The term on the left hand side is the sum of the work done by all the forces acting on the particle from the position s_1 to s_2. The term on the right hand side, defines the particle's final and initial kinetic energy respectively. These terms are positive scalar quantities since they do not depend on the direction of the particle velocity. Equation (6.6) is dimensionally homogeneous, hence unit of kinetic energy is N-m or joule. The convenient form of work-energy principle is as follows :

$$\boxed{\text{Initial K.E.} + \Sigma \text{ Work done} = \text{Final K.E.}} \quad \text{... (6.7)}$$

which states that the particle's initial kinetic energy plus the work done by all the forces acting on the particle, as it moves from initial position (s_1) to final position (s_2), is equal to the particle's final kinetic energy.

NUMERICAL EXAMPLES ON WORK-ENERGY PRINCIPLE

Example 6.15 : *A woman having mass of 70 kg stands in elevator which has a downward acceleration of 4 m/s² starting from rest. Determine the work done by her weight and the work of the normal force which the floor exerts on her when the elevator descends 6 m. Explain why work of these forces is different.*

Solution :

Given data : Mass of woman, $m_W = 70$ kg

Acceleration of elevator, $a = 4 \text{ m/s}^2$ (↓)

Distance moved by elevator, $h = 6$ m (↓)

Work done by woman's weight $= m_W \cdot g \cdot h = 70 \times 9.81 \times 6$

$= 4120.2$ Nm $=$ **4.12 kJ** ... **Ans.**

Work Done by Normal Force : Normal force is determined by considering F.B.D. of floor of elevator and using equation of kinetics,

$$\Sigma F_y = m a_y$$

$$70 \times 9.81 - N = 70 \times 4$$

$$N = 506.7 \text{ N}$$

Work done by normal force $= -N \times h$

$= -506.7 \times 6$

$= -2440.2$ Nm

$=$ **– 2.44 kNm or kJ** ... **Ans.**

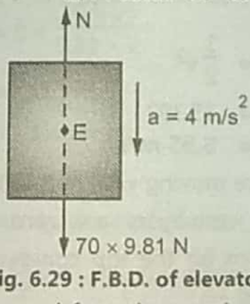


Fig. 6.29 : F.B.D. of elevator

Work done by the normal force is negative as the normal force and displacement are opposite direction.

Example 6.16 : *The car having mass of 2 Mg is originally travelling at 2 m/s. Determine the distance it must be travelled by a force F = 4 kN in order to attain a speed of 5 m/s. Neglect the friction.*

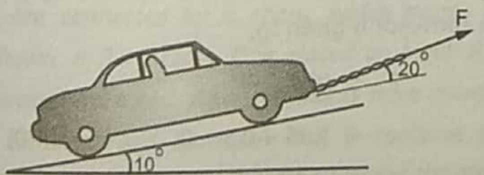


Fig. 6.30

Solution :

Given data :

Mass of car, $m = 2$ Mg $= 2000$ kg

Initial velocity of car, $u = 2$ m/s

Final velocity of car, $v = 5$ m/s

Let d be the distance travelled by the car along the inclined plane. Considering F.B.D. of car and using work-energy principle,

Initial K.E. + Σ Work done = Final K.E.

$$\frac{1}{2} \times 2000 \times 2^2 + 4000 \cos 20 \times d - 2000 \times 9.81 \sin 10 \times d = \frac{1}{2} \times 2000 \times 5^2$$

$$4000 + 3758.77\, d - 3406.97\, d = 25000$$

$$351.8\, d = 21000$$

$$\therefore \quad d = 59.69 \text{ m} \quad \text{... Ans.}$$

Example 6.17 : *Boxes are transported by a conveyor belt with a velocity v_o to fixed inclined at A where they slide and eventually fall at B. If $\mu_K = 0.4$, determine the velocity of the conveyor belt if the boxes are to have zero velocity at B.*

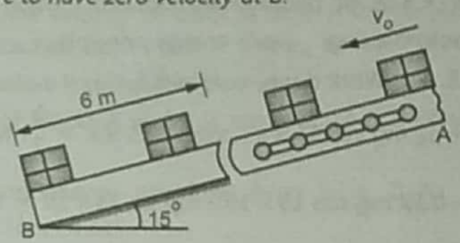


Fig. 6.31

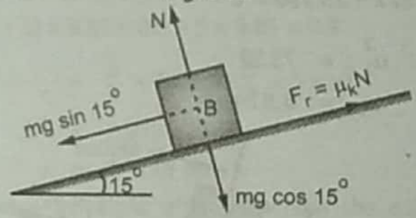


Fig. 6.31 (a) : F.B.D. of block

Solution :

Given data : Initial velocity of block at A, $v_A = v_O$

Final velocity of block at B, $v_B = 0$

Mass of block = m and $\mu_K = 0.4$

Distance travelled by block, $x = 6$ m

Let v_O be the velocity of belt. Considering F.B.D. of block and using work-energy principle,

Initial K.E. + Σ Work done = Final K.E.

$$\frac{1}{2} m v_A^2 + mg \sin 15 \times x - \mu_K\, mg \cos 15 \times x = \frac{1}{2} m v_B^2$$

$$\frac{1}{2} m v_O^2 + mg \sin 15 \times 6 - 0.4\, mg \cos 15 \times 6 = 0$$

$$\frac{1}{2} v_O^2 + 15.234 - 22.742 = 0$$

$$v_O^2 = 2 \times 7.508$$

$$\therefore \quad v_O = \mathbf{3.875 \text{ m/s}} \quad \text{... Ans.}$$

Example 6.18 : *A package is projected 10 m up an inclined plane so that it just reaches the top of the inclined plane with zero velocity. If μ_K = 0.12 between the package and the inclined plane, determine : (a) The initial velocity of the package at A and (b) The velocity of the package as it returns to its original position.*

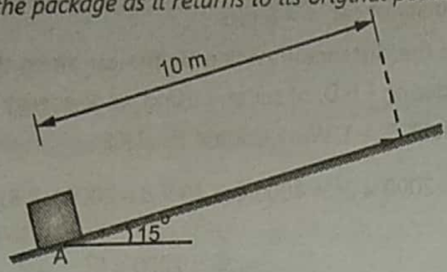


Fig. 6.32

Solution :

Given data : Distance travelled by the package, x = 10 m

Velocity of package at the top of inclined plane, v = 0

Coefficient of kinetic friction, μ_K = 0.12

Let u_1 be the initial velocity of package at A and u_2 be the velocity of the package as it returns to its original position.

Consider F.B.D. of package for upward motion along the inclined plane and using work-energy principle,

Initial K.E. + Σ Work done = Final K.E.

$$\frac{1}{2}mu_1^2 + \mu_K \, mg \cos 15 \times x - mg \sin 15 \times x = \frac{1}{2}mv^2$$

$$\frac{1}{2}mu_1^2 - 0.12 \, mg \cos 15 \times 10 - mg \sin 15 \times 10 = 0$$

$$\frac{1}{2}u_1^2 - 11.371 - 25.390 = 0$$

$$u_1^2 = 73.52$$

$$\therefore \quad u_1 = \mathbf{8.57\ m/s} \qquad \text{... Ans.}$$

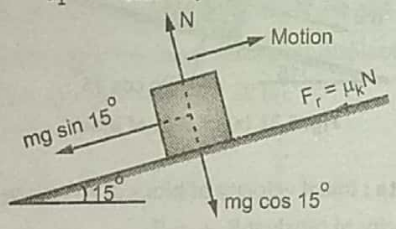


Fig. 6.33 (a) : F.B.D. of package

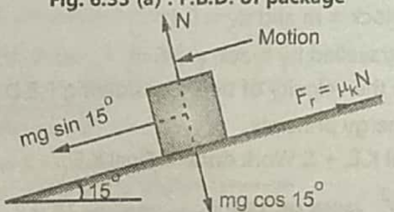


Fig. 6.33 (b) : F.B.D. of package

Consider F.B.D. of package for downward motion along the inclined plane and using work-energy principle,

Initial K.E. + Σ Work done = Final K.E.

$$\frac{1}{2}mv^2 + mg \sin 15 \times x - \mu_K \, mg \cos 15 \times x = \frac{1}{2}mu_2^2$$

$$0 + mg \sin 15 \times 10 - 0.12 \times mg \cos 15 \times 10 = \frac{1}{2}mu_2^2$$

$$25.39 - 11.37 = \frac{1}{2}mu_2^2$$

$$u_2^2 = 28.04$$

$$\therefore \quad u_2 = \mathbf{5.3\ m/s} \qquad \text{... Ans.}$$

Example 6.19 : *Packages are transferred horizontally from one conveyor to the next using a ramp for which μ_K = 0.15. The top conveyor is moving at 2 m/s and the packages are spaced 1 m apart. Determine the required speed of the bottom conveyor so that no sliding occurs when the packages come horizontally in contact with it. What is the spacing between the packages on the bottom conveyor ?*

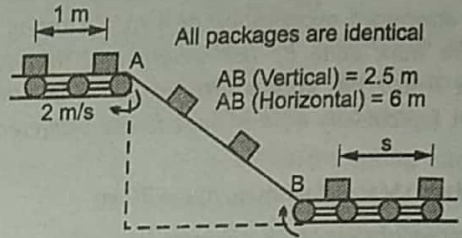


Fig. 6.34

Solution :

Given data : Initial velocity of packages at A, u = 2 m/s

Horizontal displacement, x = 6 m

Vertical displacement, y = 2.5 m

Coefficient of kinetic friction, μ_K = 0.15

Initial spacing of packages, s_O = 1 m

Let v be the speed of bottom conveyor and s be the spacing between the packages at bottom conveyor. Using work-energy principle,

K.E. at A + Σ Work done = K.E. at B

$$\frac{1}{2}mu^2 + mgy - \mu_K \, mgx = \frac{1}{2}mv^2$$

$$\frac{1}{2} \times m \times 2^2 + m \times 9.81 \times 2.5 - 0.15 \times m \times 9.81 \times 6 = \frac{1}{2}mv^2$$

$$2 + 24.525 - 8.829 = \frac{1}{2}v^2$$

$$v^2 = 35.392$$

$$v = \mathbf{5.95\ m/s} \qquad \text{... Ans.}$$

Both the conveyors are moving with constant velocity, hence the accelerations of the conveyors are zero. Time required for packages to travel 1 m on the top conveyor is given by using equation of kinematics,

$$s = ut + \frac{1}{2}at^2$$

$$1 = 2 \times t + 0 \qquad \text{... } (\because a = 0)$$

$$t = 0.5 \text{ s}$$

During the same time, distance 's' travelled by packages on the bottom conveyor is given by,

$$s = ut + \frac{1}{2}at^2$$

$$= 5.95 + 0.5 + 0 \qquad \text{... } (\because a = 0)$$

$$= \mathbf{2.97\ m} \qquad \text{... Ans.}$$

***Example 6.20 :** A force, which varies with x as shown, pulls a 10 N body that is originally at rest, along a horizontal floor. If μ_K = 0.2 between the body and the floor, determine (a) The work done by the force in moving the body from x = 0 to x = 8 m, (b) The speed of the body when it has travelled 3 m, (c) The speed of the body when it has travelled 8 m.*

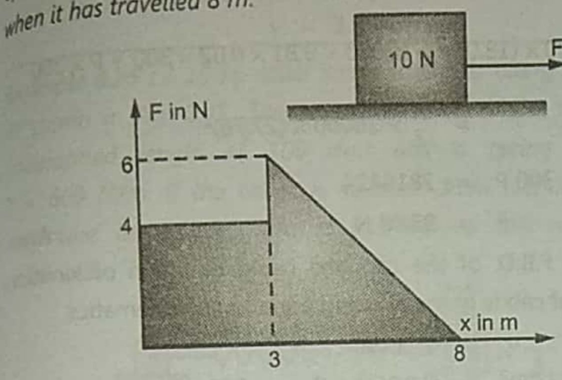


Fig. 6.35

Solution :

Given data : Initial velocity of body, u = 0

Weight of body, W = 10 N

Coefficient of kinetic friction, μ_K = 0.2

(a) Work done by the body from x = 0 to x = 8 m is given by,

$$\text{Work} = \Sigma F \cdot x$$

$$= 4 \times 3 + \frac{1}{2} \times 6 \times 5$$

$$= \mathbf{27\ Nm}$$ **... Ans.**

(b) Using work-energy principle from x = 0 to x = 3 m,

$$-\mu_K N \cdot x + F \cdot x + \frac{mu^2}{2} = \frac{mv^2}{2}$$

$$-0.2 \times 10 \times 3 + 4 \times 3 + 0 = \frac{10 \times v^2}{9.81 \times 2}$$

$$v^2 = 11.772$$

$$\therefore \quad v = \mathbf{3.43\ m/s}$$

- velocity of body at x = 3 m **... Ans.**

(c) By the work-energy principle, from x = 0 to x = 8 m,

$$-0.2 \times 10 \times 8 + 4 \times 3 + \frac{1}{2} \times 6 \times 5 + 0 = \frac{10 \times v^2}{9.81 \times 2}$$

$$v^2 = 21.585$$

$$\therefore \quad v = \mathbf{4.65\ m/s}$$

- velocity of body at x = 8 m **... Ans.**

***Example 6.21 :** Two blocks A and B of mass 4 kg and 5 kg respectively, are connected by a chord, which passes over the pulleys as shown. A 3 kg collar C is placed on block A and the system is released from rest. After the blocks have moved 0.9 m, collar C is removed and blocks A and B continue to move. Determine the speed of block A just before it strikes the ground.*

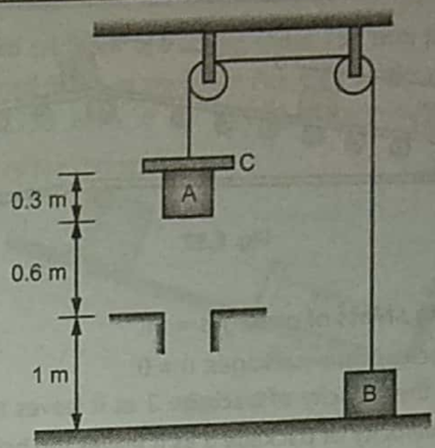


Fig. 6.36

Solution :

Given data :

Mass of block A, m_A = 4 kg

Mass of block B, m_B = 5 kg

Mass of block C, m_C = 3 kg

At h = 0.9, collar is removed.

Initial velocity of system, u = 0

Let v be the velocity of block A at h = 0.9 m (↓), at which collar C is detached from block A and v_A be the velocity of the block A just before it strikes the ground.

Velocity of blocks A and B is same, since it is connected by single cord.

Using work-energy principle, from h = 0 to h = 0.9, displacement (↓) of block A :

$$\text{Initial K.E.} + \Sigma \text{ Work done} = \text{Final K.E.}$$

$$0 + 0 + (4 + 3) \times 9.81 \times 0.9 - 5 \times 9.81 \times 0.9 = \frac{1}{2} \times 7 \times v^2 + \frac{1}{2} \times 5 \times v^2$$

$$v^2 = 2.943$$

$$\therefore \quad v = 1.7155\ m/s$$

Using work-energy principle, after detaching the collar of 3 kg, downward displacement of block to strike the ground is (1 – 0.3) = 0.7 m.

Initial K.E. + Σ Work done = Final K.E.

$$\frac{1}{2} \times 4 \times (1.7155)^2 + \frac{1}{2} \times 5 \times (1.7155)^2 + 4 \times 9.81 \times 0.7 - 5 \times 9.81 \times 0.7 = \frac{4v_A^2}{2} + \frac{5v_A^2}{2}$$

$$v_A^2 = 1.4169$$

$$\therefore \quad v_A = \mathbf{1.19\ m/s}$$ **... Ans.**

***Example 6.22 :** Four identical packages are held in place by friction on a conveyor belt when its power supply is shut off. When the system is released from rest, package 1 leaves the belt at A just as package 4 comes into the inclined portion of the belt at B. Determine (a) The velocity of package 2 as it leaves the belt at A, (b) The velocity of package 3 as it leaves the belt at A. Neglect the mass of belt and rollers.*

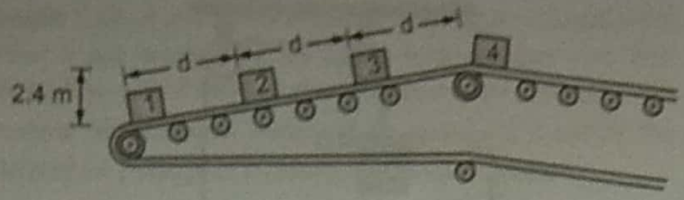


Fig. 6.37

Solution :

Given data : Mass of packages = m

Initial velocity of the packages, u = 0

Let v_2 be the velocity of package 2 as it leaves the belt at A and v_3 be the velocity of package 3 as it leaves the belt at A.

Vertical distance moved by package 2, $h_2 = \frac{2.4}{3} = 0.8$ m

Using work-energy principle,

Initial K.E. + Σ Work done = Final K.E.

$$\frac{1}{2}mu^2 + mgh_2 = \frac{1}{2}mv_2^2$$

$$0 + m \times 9.81 \times 0.8 = \frac{1}{2} \times m \times v_2^2$$

$$v_2^2 = 15.696$$

∴ v_2 = **3.96 m/s** ... **Ans.**

Vertical distance moved by package 3,

$$h_3 = \frac{2 \times 2.4}{3} = 1.6 \text{ m}$$

Using work-energy principle,

$$\frac{1}{2}mu^2 + mgh_3 = \frac{1}{2}mv_3^2$$

$$0 + m \times 9.81 \times 1.6 = \frac{1}{2} \times m \times v_3^2$$

$$v_3^2 = 31.392$$

∴ v_3 = **5.6 m/s** ... **Ans.**

Example 6.23 : *A trailer truck has a 2000 kg cab and 8000 trailer. If it enters a 2% uphill grade at 65 km/h and reaches a speed of 100 km/h in 300 m, determine (a) The average force of traction at the cab wheels, (b) The average force in the coupling between the cab and the trailer.*

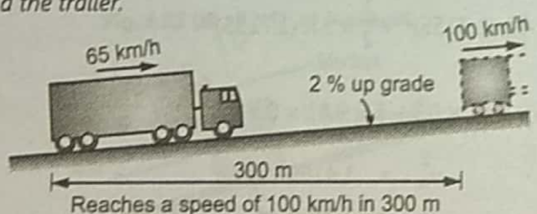


Fig. 6.38

Solution :

Given data : Mass of cab, m_C = 2000 kg

Mass of trailer, m_t = 8000 kg

Initial velocity of truck, u = 65 km/h = 18.06 m/s

Final velocity of truck, v = 100 km/h = 27.78 m/s

Distance traveled by truck, s = 300 m

Let P be the force of traction at the cab wheels and F be the average force in the coupling between the cab and trailer. Using work-energy principle,

Initial K.E. + Σ Work done = Final K.E.

$$\frac{1}{2} \times mu^2 - mgx + P \times s = \frac{1}{2} mv^2$$

$$\frac{1}{2} \times 10000 \times (18.06)^2 - 10000 \times 9.81 \times 0.02 \times 300 + P \times 300$$

$$= \frac{1}{2} \times 10000 \times (27.78)^2$$

$$300\,P = 2816424$$

∴ P = **9388 N** ... **Ans.**

Consider F.B.D. of the cab and using equation of kinetics acceleration of cab is given by using equation of kinematics.

$$v^2 = u^2 + 2as$$

$$(27.78)^2 = (18.06)^2 + 2 \times a \times 300$$

∴ $a = 0.743$ m/s²

$$\Sigma F = ma$$

$$P - F = ma$$

$$9388 - F = 2000 \times 0.743$$

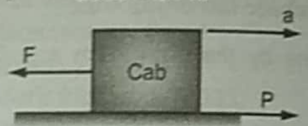


Fig. 6.39

∴ F = **7903 N (Tension)** ... **Ans.**

Example 6.24 : *The ball is released from the position A with a velocity of 3 m/s and swings in a vertical plane. At the bottom position, the cord strikes the fixed bar at B, and the ball continues to swing in the dashed arc. Determine the velocity of the ball as it passes position at C as shown in Fig. 6.40.*

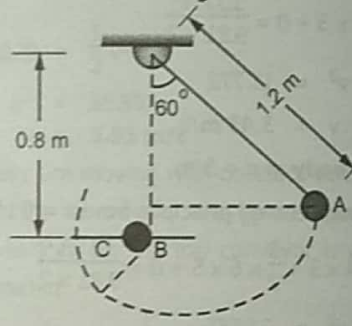


Fig. 6.40

Solution :

Given data : Initial velocity at A, u = 3 m/s

Mass of ball = m

Let v be the velocity of ball at C. Distance moved by ball in downward direction,

h = 0.8 − 1.2 cos 60, h = 0.2 m

Using work-energy principle,

Initial K.E. + Σ Work done = Final K.E.

$$\frac{1}{2}mu^2 + mgh = \frac{1}{2}mv^2$$

$$\frac{1}{2}m \times 3^2 + m \times 9.81 \times 0.2 = \frac{1}{2} \times m \times v^2$$

$$4.5 + 1.962 = 0.5\,v^2$$

$$v^2 = 12.924$$

$$v = \mathbf{3.59\ m/s} \quad \text{... Ans.}$$

Example 6.25 : *A 10 kg collar slides smoothly along a vertical rod as shown in Fig. 6.41. The spring attached to the collar has an undeformed length of 100 mm and a spring constant of k = 600 N/m. If the collar is released from rest in position A, determine its velocity after it has moved 150 mm down to position B.*

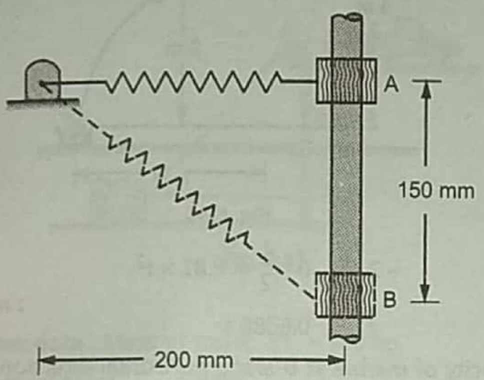


Fig. 6.41

Solution :

Given data :

Mass of collar, m = 10 kg

Undeformed length of spring, L_O = 100 mm

Length of spring at position A, L_A = 200 mm

Length of spring at position B, $L_B = \sqrt{(200)^2 + (150)^2}$ = 250 mm

Initial velocity of collar, u = 0

Initial deformation of spring at A, x_A = 100 mm

Final deformation of spring at B, x_B = 150 mm

Let v be the velocity of collar when it has moved 150 mm down to position B. Using work-energy principle,

Initial K.E. + Σ Work done = Final K.E.

$$\frac{1}{2}mu^2 + mgh - \frac{1}{2}k\left(x_B^2 - x_A^2\right) = \frac{1}{2}mv^2$$

$$0 + 10 \times 9.81 \times 0.15 - \frac{1}{2} \times 600 \times [(0.15)^2 - (0.10)^2]$$

$$= \frac{1}{2} \times 10 \times v^2$$

$$14.715 - 3.75 = 5v^2$$

$$\therefore \quad v^2 = 2.193$$

$$\therefore \quad v = \mathbf{1.48\ m/s} \quad \text{... Ans.}$$

Example 6.26 : *A block of 7.25 kg slides 150 mm from rest down the 20° inclined plane as shown in Fig. 6.42. It hits a spring whose modulus is 1750 N/m. If μ_K = 0.20, determine the maximum compression of the spring.*

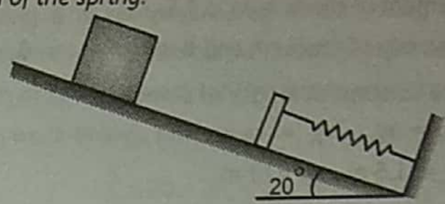


Fig. 6.42

Solution :

Given data :

Mass of block, m = 7.25 kg

Distance moved along the plane, d = 150 mm

Spring modulus, k = 1750 N/m

Coefficient of kinetic friction, μ_K = 0.20

Initial velocity of block, u = 0 and Final velocity of block,

v = 0

Let x be the maximum compression of spring.

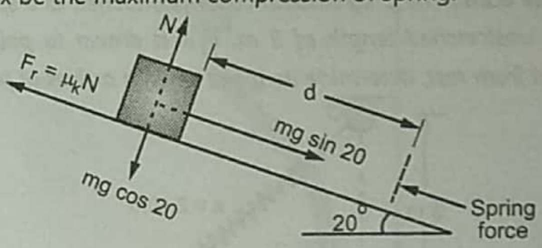


Fig. 6.42 (a) : F.B.D. of block

Considering F.B.D. of block and using work-energy principle,

$$\frac{1}{2}mu^2 + mg\sin 20 \times d - \mu_K\, mg\cos 20 \times d - \frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$0 + 7.25 \times 9.81 \times \sin 20 \times 0.15 - 0.2 \times 7.25 \times 9.81 \times \cos 20 \times 0.15 - \frac{1}{2} \times 1750 \times x^2 = 0$$

$$3.649 - 2.005 - 875x^2 = 0$$

$$\therefore \quad x^2 = 1.8788 \times 10^{-3}$$

$$\therefore \quad x = 0.04335\ \text{m}$$

$$\therefore \quad x = \mathbf{43.35\ mm} \quad \text{... Ans.}$$

Example 6.27 : *Block A has a weight of 300 N and block B has a weight of 50 N. Determine the speed of block A after it moves 1.5 m down the plane, starting from rest. Neglect friction and mass of the cord and pulleys.*

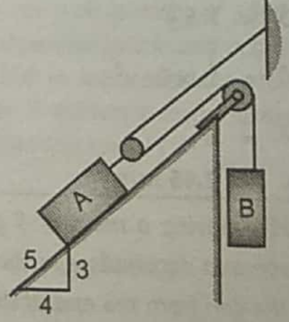


Fig. 6.43

Solution :

Given data : Weight of block A, w_A = 300 N.

Weight of block B, w_B = 50 N.

Displacement of block A, s_A = 1.5.

Initial velocity of blocks A and B is $v_A = v_B = 0$.

From the concept of length of string,

$2x_A = x_B, \; 2v_A = v_B$

At $x_A = 1.5$ m, $x_B = 3$ m

Let v be the velocity of block A.

Using work-energy principle,

$$K.E._1 + \sum W.D. = K.E._2$$

$$0 + 0 + 300 \sin 36.87 \times 1.5 - 50 \times 3 = \frac{1}{2} \times \frac{300}{9.81} v^2 + \frac{1}{2} \times \frac{50}{9.81} (2v)^2$$

$$2354.413 = 500 v^2$$

$$v^2 = 4.709$$

$$\therefore \quad v = \mathbf{2.17 \ m/s} \qquad \text{... Ans.}$$

Example 6.28 : *The 2 kg smooth collar is attached to a spring that has an unstretched length of 3 m. If it is drawn to point B and released from rest, determine its speed when it arrives at point A.*

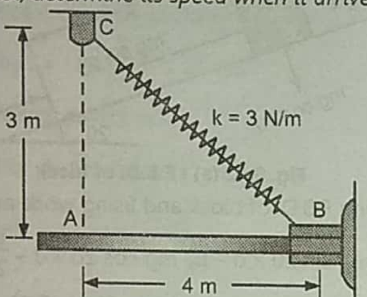


Fig. 6.44

Solution :

Given data : Mass of collar, m_C = 2 kg.

Unstretched length, s_0 = 3 m.

Initial velocity at B, v_B = 0.

Stretched length of spring is,

$$s_1 = \sqrt{3^2 + 4^2}, \; s_1 = 5 \text{ m},$$

Deformation of spring = (5 – 3), s = 2 m.

Using work-energy principle,

$$K.E._1 + \sum W.D. = K.E._2$$

$$0 + \frac{1}{2} \cdot 3 \times 2^2 = \frac{1}{2} \times 2 \cdot v^2$$

$$v^2 = 6$$

$$\therefore \quad v = \mathbf{2.45 \ m/s} \qquad \text{... Ans.}$$

Example 6.29 : *Marble having a mass of 5 g falls from rest at A through the glass tube and accumulate in the can at C. Determine the placement R of the can from the end of the tube and the speed at which the marble falls into the can. Neglect the size of the can.*

Solution :

Given data : Mass of marble, m = 5 g.

Let v be the velocity of marble at C. Using work-energy principle,

$$0 + \frac{5 \times 9.81 \times 3}{1000} = \frac{1}{2} \times \frac{5}{1000} \times v^2$$

$$58.86 = v^2$$

$$v = 7.672 \text{ m/s}$$

Using equation of kinematics, time required to travel 2 m.

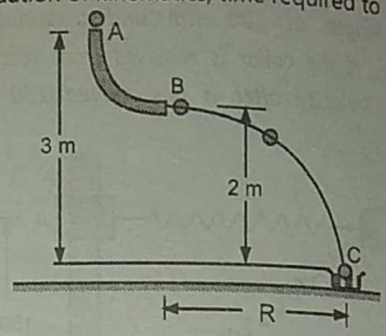


Fig. 6.45

$$-2 = 0 - \frac{1}{2} \times 9.81 \times t^2$$

$$t = 0.6386 \text{ s}$$

Velocity of marble at B along horizontal direction is given by using work-energy principle,

$$0 + \frac{5 \times 9.81 \times 1}{1000} = \frac{1}{2} \times \frac{5}{1000} v_B^2 \quad \therefore \quad v_B = 4.429 \text{ m/s}$$

Using equation of kinematics along x-direction ($a_x = 0$),

$$s_x = R = 4.429 \times 0.6386$$

$$\therefore \quad R = \mathbf{2.83 \ m} \qquad \text{... Ans.}$$

Example 6.30 : *The steel ingot has a mass of 1800 kg. It travels along the conveyor at a speed v = 0.5 m/s when it collides with the nested spring assembly. Determine the maximum deflection in each spring needed to stop the motion of the ingot. k_A = 5 kN/m, k_B = 3 kN/m.*

Solution :

Given data : Mass of steel ingot, m = 1800 kg. Velocity of ingot, v = 0.5 m/s.

Stiffness of spring A, k_A = 5 kN/m. Stiffness of spring B, k_B = 3 kN/m.

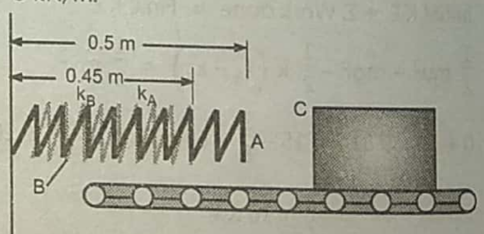


Fig. 6.46

Let x be the deflection of spring A, then deflection of spring B should be (x – 0.05) and at the maximum deflection of both the springs, velocity of ingot becomes zero.

Using work- energy principle,

$$K.E._1 + \sum W.D. = K.E._2$$

$$\frac{1}{2} \times 1800 \times 0.5^2 - \frac{1}{2} \cdot 5000\, x^2 - \frac{1}{2} \times 3000\, (x - 0.05)^2 = 0$$

$$225 - 2500\, x^2 - 1500\, x^2 + 150\, x - 3.75 = 0$$

$$x^2 - 0.0375x - 0.0553 = 0 \quad \therefore \quad x = 0.255 \text{ m}$$

x_A = **0.255 m** and x_B = **0.205 m.** ... **Ans.**

Example 6.31 : *The 100 kg stone is being dragged across the smooth surface by means of the truck T. If the towing cable passes over a small pulley at A, determine the speed of the stone when θ = 60°. The stone is at rest when θ = 30° and the truck exerts the constant force F = 500 N on the cable at B.*

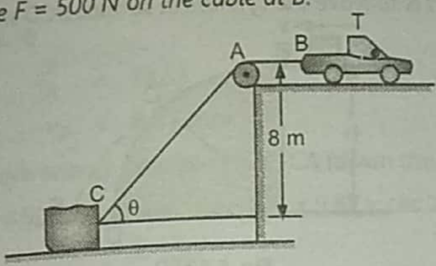


Fig. 6.47

Solution :

Given data : Mass of stone, m = 100 kg.

Constant force, F = 500 N.

At θ = 30°, u = 0. At θ = 60°, v = ?

Let v be the velocity of truck at θ = 60°.

At θ = 30°,

$$l(AC) = \sqrt{8^2 + (8/\tan 30)^2}$$

$$l(AC) = 16 \text{ m}$$

At θ = 60°,

$$l(AC) = \sqrt{8^2 + (8/\tan 60)^2}$$

$$l(AC) = 9.24 \text{ m}$$

Deflection of cable, s = 16 – 9.24, s = 6.76 m.

Using work-energy principle, $KE_1 + \sum WD = KE_2$

$$500 \times 6.76 = \frac{1}{2} \times 100\, v^2$$

$\therefore$ v = **8.22 m/s** ... **Ans.**

Example 6.32 : *The collar has a mass of 20 kg and slides along the smooth rod. Two springs are attached to it and ends of the rod as shown in Fig. 6.48. If each spring has an uncompressed length 1 m and the collar has a speed of 2 m/s when s = 0, determine the maximum compression of each spring due to the back and forth (oscillation) motion of the collar.*

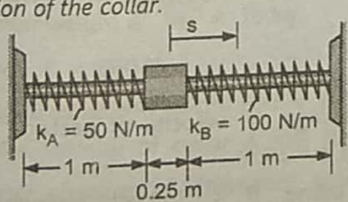


Fig. 6.48

Solution :

Given data : Mass of collar, m = 20 kg.

Initial velocity of collar at s = 0 is, u = 2 m/s.

When spring B is compressed by s, the spring A elongates by same amount of deformation. At the maximum compression and elongation of spring, its velocity becomes zero, v = 0.

Using work-energy principle,

$$\frac{1}{2} \times 20 \times 2^2 - \frac{1}{2} \times 50 \times s^2 - \frac{1}{2} \times 100\, s^2 = 0$$

$$80 = 150\, s^2, \quad s^2 = 0.533,$$

s = **0.73 m.** ... **Ans.**

Example 6.33 : *The small collar of mass m is released from rest at A and slides down the curved rod in the vertical plane. If m = 0.5 kg, b = 0.8 m and h = 1.5 m and if the velocity of the collar as it strikes the base B is 4.7 m/s after release of the collar from rest at A, calculate the work Q of friction. What happens to the energy which is lost ?*

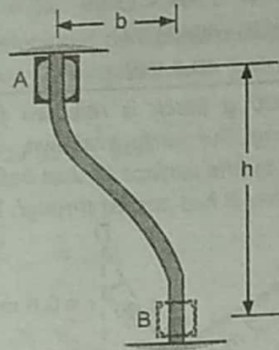


Fig. 6.49

Solution :

Given data : Mass of collar, m = 0.5 kg.

Height of rod, h = 1.5 m.

Initial velocity, v_A = 0.

Final velocity, v_B = 4.75 m/s.

Using work-energy principle, $K.E._1 + \sum WD = K.E._2$

$$0 + 0.5 \times 9.81 \times 1.5 - Q = \frac{1}{2} \times 0.5 \times 4.7^2$$

Q is the work done due to frictional force.

$$0 + 7.3575 - Q = 5.5225$$

Q = **1.835 N.m**

Example 6.34 : *A car with a mass of 1500 kg starts from rest at the bottom of a 10 percent grade and acquires a speed of 50 km/h in a distance of 100 m with constant acceleration up the grade. What is the power P delivered to the drive wheels by the engine when the car reaches this speed ?*

Solution :

Given data :

Mass of car, m = 1500 kg

Initial velocity of car,

u = 0

Distance travelled,

$$s = 100 \text{ m}$$

Final velocity of car,

$$v = 50 \text{ kmph}, \; v = 13.89 \text{ m/s}$$

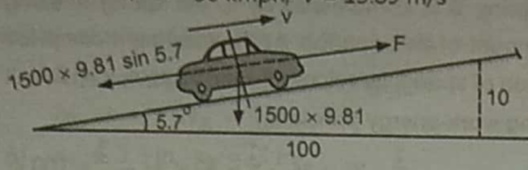


Fig. 6.50 : F.B.D. of car

Let a be the acceleration of the car considering equation of kinematics,

$$v^2 = u^2 + 2as$$

$$13.89^2 = 0 + 2 \times a \times 200$$

$$a = 0.965 \text{ m/s}^2$$

Considering F.B.D. of car and using equation of motion along the plane,

$$-1500 \times 9.81 \sin 5.71 - F = 1500 \times 0.965, \; F = 2909 \text{ N}$$

$$\text{Power} = F \cdot v = 2909 \times 13.89$$

$$P = 40405 \text{ W}$$

$$\mathbf{P = 40.4 \text{ kW}}$$

... Ans.

Example 6.35 : *A 300 g block is released from rest and slides without friction along the surface shown. Determine the force exerted on the block by the surface (a) Just before the block reaches B, (b) Immediately after it has passed through B.*

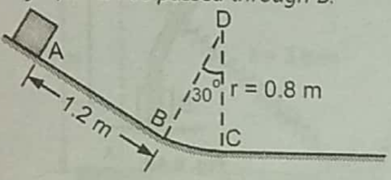


Fig. 6.51

Solution :

Given data : Mass of block = 0.3 kg.

Distance AB, $s = 1.2$ m.

Velocity of block at A, $v_A = 0$.

Let v be the velocity of block at B.

Using work-energy principle,

$$0 + 0.3 \times 9.81 \times 0.6 = \frac{1}{2} \times 0.3 \, v^2$$

$$v = \mathbf{3.431 \text{ m/s.}}$$

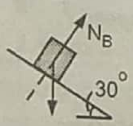
(a) Considering F.B.D. of block just before A and using equation of motion,

$$\Sigma F_y = ma_y \; (\because \; a_y = 0)$$

$$N_B - 0.3 \times 9.81 \times \cos 30 = 0$$

$$\mathbf{N_B = 2.55 \text{ N.}}$$

... Ans.



0.3 × 0.81 N

Fig. 6.51 (a) : F.B.D. of block just before B

(b) Considering F.B.D. of block just after point B on curve and using equation of motion along normal direction,

$$N_B - 0.3 \times 9.81 \cos 30 = 0.3 \times \frac{3.431^2}{0.8}$$

$$\mathbf{N_B = 6.96 \text{ N}}$$

... Ans.

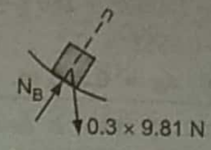


Fig. 6.51 (b) : F.B.D. of block just after B

Example 6.36 : *A small block slides at a speed v on a horizontal surface. Knowing that h = 2.5 m, determine the required speed of the block if it is to leave the cylindrical surface BCD when θ = 40°.*

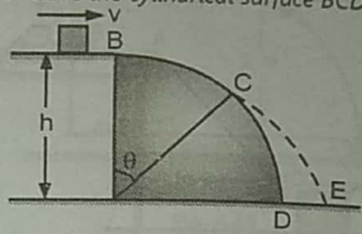


Fig. 6.52 (a)

Solution :

Given data : Mass of block, m.

Height, $h = 2.5$ m, $\theta = 45°$.

Let v be the velocity of the block on horizontal plane and v_C be the velocity of block at C. Considering F.B.D. of block at C and using equation of motion along normal direction,

$$mg \cos 40 = \frac{mv_C^2}{2.5}$$

$$v_C = 4334 \text{ m/s}$$

Using work-energy principle (from B to C),

$$\frac{1}{2} m \cdot v^2 + mg\,(2.5 - 2.5 \cos 40) = \frac{1}{2} \times m \times 4.334^2$$

$$v^2 = 2\,(9.39 - 5.74)$$

$$v = \mathbf{2.7 \text{ m/s}}$$

... Ans.

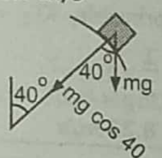


Fig. 6.52 (b) : F.B.D. of block at C

Example 6.37 : *A package is projected 10 m up a 15° incline so that it just reaches the top of the incline with zero velocity. Knowing that the coefficient of kinetic friction between the package and incline is 0.12, determine (a) The initial velocity of the package at A, (b) The velocity of the package as it returns to its original position.*

Solution :

Given data :

Mass of package, m, velocity of package at C, $v_C = 0$, distance traveled, $s = 10$ m.

Position of plane, $\theta = 15°$ with horizontal, coefficient of kinetic friction, $\mu_k = 0.12$

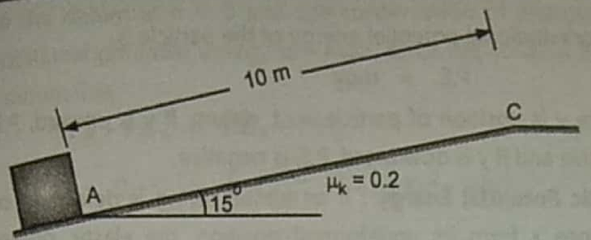


Fig. 6.53

Let v_A be the velocity of package at A and v be the velocity of package as it returns to its initial position.

Using work-energy principle along AC (up the plane),

$$\frac{1}{2} \times m \times v_A^2 - m \times 9.81 \times 10 \sin 15 - 0.12 \times m \times 9.81 \times \cos 15 \times 10 = 0$$

$$v_A^2 = 73.52$$

$$\therefore \quad \mathbf{v_A = 8.57\ m/s} \quad \text{... Ans.}$$

Using work-energy principle along CA (down the plane),

$$0 + m \times 9.81 \times 10 \sin 15 - 0.12 \times m \times 9.81 \times \cos 15 \times 10 = \frac{1}{2} mv^2$$

$$v^2 = 28.04$$

$$\mathbf{v = 5.3\ m/s.} \quad \text{... Ans.}$$

Example 6.38 : *The system shown consisting of 20 kg collar A and 10 kg counter weight B, is at rest when a constant 500 N force is applied to collar A. (a) Determine the velocity of A just before it hits the support at C, (b) Solve part (a) Assuming that counter weight B is replaced by a 98 N downward force. Ignore friction and the mass of the pulleys.*

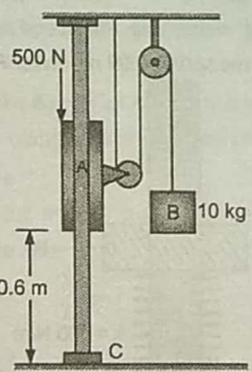


Fig. 6.54

Solution :

Given data : Mass of collar A, $m_A = 20$ kg.

Mass of counter weight B, $m_B = 10$ kg.

Constant force, F = 500 N.

Initial velocity of collar and weight B, u = 0.

Let v_A be the velocity of collar A just before it hits the support at C. From equation of kinematics, velocity of counter weight B is $v_B = 2v_A$.

Using work-energy principle,

(a) $0 + 0 + (500 + 20 \times 9.81)\ 0.6 - 10 \times 9.81 \times 1.2$

$$= \frac{1}{2} \times 20\ v_A^2 + \frac{1}{2} \times 10\ (2v_A)^2$$

$$300 = 30v_A^2 \quad \therefore \quad \mathbf{v_A = 3.16\ m/s} \quad \text{... Ans.}$$

(b) $0 + 0 + [(500 + 20 \times 9.81) - 2 \times 98.1] \times 0.6$

$$= \frac{1}{2} \times 20 \times v_A^2$$

$$300.12 = 10v_A^2 \quad \therefore \quad \mathbf{v_A = 5.48\ m/s} \quad \text{... Ans.}$$

Example 6.39 : *The particle of mass m moves rectilinearly along x-axis under the action of force F = kx, where k is constant. Find the velocity v as a function of displacement x if the initial conditions of motion are $x_0 = 0$ and $\dot{x}_0 = v_0$.*

Solution :

Given data : Mass of particle, m.

Initial displacement, x = 0, initial velocity, $v = v_0$.

Given : Force F = kx.

Let, v be the velocity of particle at a displacement x.

Using work-energy principle, $K.E._1 + \sum W.D. = K.E._2$.

$$\frac{1}{2} \cdot m \cdot v_0^2 + \int_0^x kx\, dx = \frac{1}{2} mv^2$$

$$\frac{mv_0^2}{2} + \frac{kx^2}{2} = \frac{1}{2} mv^2$$

$$\therefore \quad v^2 = v_0^2 + \frac{kx^2}{m}$$

$$\therefore \quad v = \sqrt{v_0^2 + \frac{kx^2}{m}} \quad \text{... Ans.}$$

Example 6.40 : *In the design of an inside loop for an amusement part ride, it is desired to maintain the same normal acceleration throughout the loop. Assuming negligible loss of energy during the motion, determine the radius of curvature ρ of the path as a function of height y above the low point A, where the velocity and radius of curvature are v_0 and ρ_0 respectively.*

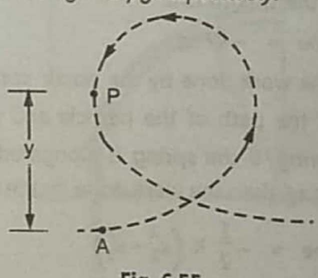


Fig. 6.55

Solution :

Given data :

Velocity at A = v_0.

Radius of curvature at A = ρ_0.

Radius of curvature at P = ρ.

Velocity at P = v_P.

$$\frac{v_o^2}{\rho_o} = \frac{v_p^2}{\rho};$$

(since the normal accelerations are same)

$$v_p^2 = \frac{v_o^2}{\rho_o}\cdot\rho$$

Using work-energy principle from A to P,

Initial K.E. + $\sum$ Work done = Final K.E.

$$\frac{mv_o^2}{2} - mgy = \frac{1}{2}mv_p^2$$

$$\frac{mv_o^2}{2} - mgy = \frac{1}{2}m\cdot\frac{v_o^2}{\rho_o}\rho$$

$$v_o^2 - 2gy = \frac{v_o^2}{\rho_o}\rho$$

$$\frac{v_o^2}{v_o^2}\cdot\rho_o - 2gy\frac{\rho_o}{v_o^2} = \rho$$

$$\rho = \rho_o\left(1 - \frac{2gy}{v_o^2}\right) \quad \textbf{... Ans.}$$

6.9 CONSERVATIVE FORCES

A unique type of force acting on a particle depends on the net change in the particle's position and independent of the particle's velocity and acceleration. If the work done by this force in moving the particle from one position to another is independent of path followed by the particle, this force is known as **Conservative Force**. The weight of the particle and spring force are two examples of conservative forces.

Weight : The work done by the weight of the particle is independent of the path of the particle and it depends only on the particle's vertical displacement. If Δy is positive upward, then the work done by the weight ism

$$\text{Work done} = -W\,\Delta y$$

Elastic Spring : The work done by the elastic spring on a particle is independent of the path of the particle and depends on the deformation of spring. If the spring is elongated or compressed from position x_1 to x_2, then the work done by the spring is

$$\text{Work done} = -\frac{1}{2}K\left(x_2^2 - x_1^2\right)$$

Friction : Work done by the friction on a particle depends on the path of the particle. Friction is non-conservative force. Longer the path, the work done is more.

Potential Energy : The energy which comes from the position of particle is known as potential energy. The potential energy is the work done on a particle by the conservative force when it moves from a given position to datum.

The gravitational potential energy of the particle is,

$$\text{P.E.} = mgy$$

where y is position of particle w.r.t. datum. If y is upward, P.E. is positive and if y is downward, P.E. is negative.

Elastic Potential Energy : If an elastic spring is deformed by a distance x from its undeformed position, the elastic potential energy is,

$$\text{P.E.} = \frac{1}{2}Kx^2$$

The potential energy is always positive, when the spring is returned to its undeformed position.

6.10 CONSERVATION OF ENERGY

When the particle is subjected to conservative and non-conservative forces, the work done by the conservative forces is written in the form of potential energy. According to work-energy principle,

$$\text{Initial (K.E. + P.E.)} + \Sigma\,\text{Work done} = \text{Final (K.E. + P.E.)}$$

Σ Work done is the work done by non-conservative force. If only conservative force is acting on a particle, then we have,

$$\text{K.E. (1) + P.E. (1)} = \text{K.E. (2) + P.E. (2)} \quad \text{... (6.8)}$$

Equation (6.8) is known as **Conservation of Energy**.

NUMERICAL EXAMPLES ON CONSERVATION OF ENERGY

Example 6.41 : *The collar has a weight of 40 N. If it is released from rest at a height of h = 0.6 m from the top of the uncompressed spring, determine the speed of the collar after it falls and compresses the spring 0.09 m. (Refer Fig. 6.56)*

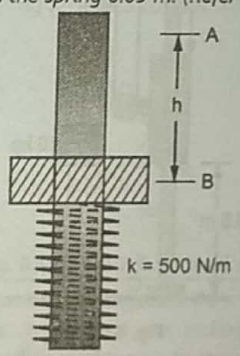


Fig. 6.56

Solution :

Given data :

Weight of collar, W = 40 N

Downward distance travelled by collar, h = 0.6 m

Compression of spring, x = 0.09 m

Spring constant, k = 500 N/m

Let v be the velocity of collar after it falls from h = 0.6 m and compress the spring 0.09 m.

Take the datum at h = 0 and use conservation of energy. The gravitational potential energy is – mgh, since the collar is below the datum line.

$$K.E._A + P.E._A = K.E._B + P.E._B$$

$$0 + 0 = \frac{1}{2} \times \frac{40}{9.81} v^2 + \left[\frac{1}{2} Kx^2 - Wh\right]$$

$$0 = \frac{1}{2} \times \frac{40}{9.81} v^2 + \frac{1}{2} \times 500 \times (0.09)^2 - 40\,(0.6 + 0.09)$$

$$0 = 2.038\, v^2 + 2.205 - 27.6$$

$$\therefore \quad v^2 = 12.461$$

$$\therefore \quad v = \mathbf{3.53\ m/s} \qquad \text{... Ans.}$$

Example 6.42 : *The 2 kg smooth collar is attached to a spring that has an unstretched length of 3 m. If it is drawn to point B and released from rest, determine its speed when it arrives at point A.*

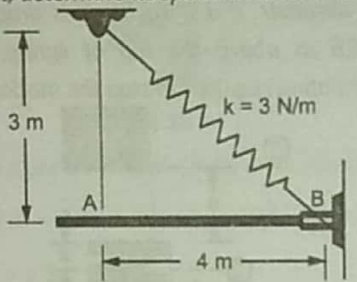


Fig. 6.57

Solution :

Given data : Mass of collar, m = 2 kg

Unstretched length, L_O = 3 m

Length after stretching, $L = \sqrt{3^2 + 4^2} = 5$ m, Spring constant, k = 3 N/m

Velocity of collar at B, u = 0, Elongation of the spring,

$x = L - L_O = 5 - 3 = 2$ m

For convenience, take datum at AB. Let v be the velocity of collar when it arrives at A. Using conservation of energy,

$$K.E._B + P.E._B = K.E._A + P.E._A$$

Gravitational potential energy is zero, since the collar is moving along the datum line AB.

$$0 + \frac{1}{2} Kx^2 = \frac{1}{2} mv^2 + 0$$

$$\frac{1}{2} \times 3 \times (2)^2 = \frac{1}{2} \times 2 \times v^2$$

$$v^2 = 6$$

$$\therefore \quad v = \mathbf{2.45\ m/s} \qquad \text{... Ans.}$$

Example 6.43 : *A smooth tube AB in the form of a quarter circle of mean radius r is fixed in a vertical plane and contain a flexible chain of length $\pi r/2$ and weight $\frac{w\pi r}{2}$ as shown in Fig. 6.58. If released from rest in position shown, find the velocity v with which the chain will move along the smooth horizontal plane BC after it emerges from the tube.*

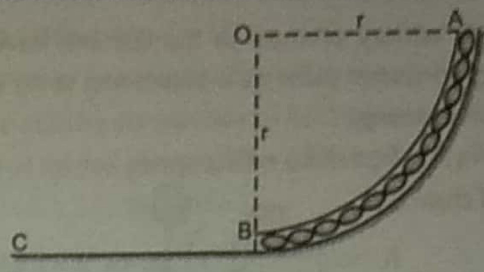


Fig. 6.58

Solution :

Given data : Radius of rube AB = r.

Length of chain, $l = \pi r/2$.

Weight of chain, $W = w\pi r/2$.

Initial velocity of chain, u = 0.

Let v be the velocity of chain after it exit from the tube.

Considering BC as a datum and using principle of conservation of energy,

$$PE_1 + KE_1 = PE_2 + KE_2$$

Centroid of the chair is lying at

$-\frac{\sin \alpha}{\alpha}$ from O.

Hence its centroid from B is,

$$h = r - \left(\frac{r \sin 45}{45 \times \pi/180}\right) \sin 45$$

$$\therefore \quad h = 0.364\, r$$

$$0 + \frac{w\pi r}{2} \times 0.364\, r = 0 + \frac{1}{2} \cdot \frac{w\pi r}{g \times 2} \cdot v^2$$

$$\therefore \quad v^2 = 0.364\, r \times 2g$$

$$\therefore \quad v = \sqrt{0.7289\, r}$$

At r = 1 m, $\mathbf{v = 2.67\ m/s}$ ... **Ans.**

Example 6.44 : *The flexible bicycle type chain of length $\pi r/2$ and mass per unit length ρ is released from rest with $\theta = 0$ in the smooth circular channel and falls through the hole in the supporting surface, determine the velocity v of the chain as the last link leaves the slot.*

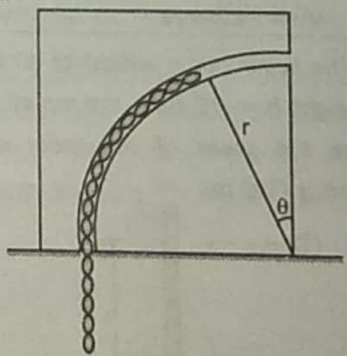


Fig. 6.59

Solution :

Given data : Length of chain, $l = \pi r/2$.

ρ - mass per unit length.

Initial velocity of chain, u = 0.

Let v be the velocity of chain as the last link leaves the slot. Considering horizontal plane as a datum and using principle of conservation of energy,

$$K.E._1 + P.E._1 = K.E._2 + P.E._2$$

Centroid of chain is at,

$$h = \left(\frac{\sin 45}{45 \times \frac{\pi}{180}} r\right) \sin 45, \quad h = 0.636\ r$$

$$0 + \frac{\rho \pi r}{2} \times g \times 0.636\ r = \frac{1}{2} \cdot \frac{\rho \pi r}{2} v^2 - \frac{\rho \pi r}{2} \times g \times \frac{\pi r}{4}$$

$$0.636 g\ r + \frac{\pi r g}{4} = \frac{1}{2} v^2$$

$$1.42\ gr = \frac{1}{2} v^2$$

$$\therefore \quad v^2 = 2.843\ gr$$

$$\therefore \quad v = \sqrt{2.843\ gr}$$

At $r = 1$ m, $v =$ **5.28 m/s** ... **Ans.**

Example 6.45 : *The collar has a weight of 40 N. If it is pushed down so as to compress the spring 0.6 m and then released from rest (h = 0), determine its speed when it is displaced h = 1.35 m. The spring is not attached to the collar. Neglect friction.*

Solution :

Given data : Weight of collar, W = 40 N. (Refer Fig. 6.60)

Compression of spring, x = 0.6 m.

Displacement of spring, x = 1.35 m.

v be the velocity of collar when it is displaced by h = 1.35 m.

Considering compressed position of spring is datum and using conservation of energy,

$$\frac{1}{2} \times \frac{40}{9.81} v^2 = \frac{1}{2} \times 500 \times 0.6^2 - \frac{40}{9.81} \times 9.81 \times 1.35$$

$$2.0387\ v^2 = 36$$

$$v^2 = 17.658$$

$$v = 4.2 \text{ m/s}$$ ... **Ans.**

Example 6.46 : *The collar has a weight of 40 N. If it is released from rest at a height h = 0.6 from the top of the uncompressed spring, determine the speed of the collar after it falls and compresses the spring 0.09 m.*

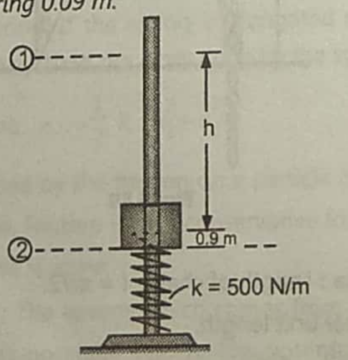


Fig. 6.60

Solution :

Given data : Weight of collar, w = 10 N.

Height, h = 0.6.

Spring constant, k = 500 N/m.

Compression of spring, x = 0.09 m.

Initial velocity of collar, u = 0.

Considering datum as compressed position of spring and using conservation of energy.

$$KE_1 + PE_1 = KE_2 + PE_2$$

$$0 + 40 \times (0.6 + 0.09) = \frac{1}{2} \times \frac{40}{9.81} v^2 + \frac{1}{2} \times 500 \times 0.09^2$$

$$v^2 = 12.545, \ \mathbf{v = 3.54\ m/s}$$ ... **Ans.**

Example 6.47 : *Two equal length springs having stiffness k_A = 300 N/m and k_B = 200 N/m are nested together in order to form a shock absorber. If a 2 kg block is dropped from at rest position s = 0.6 m above the top of spring, determine their maximum deformation required to stop the motion of the block.*

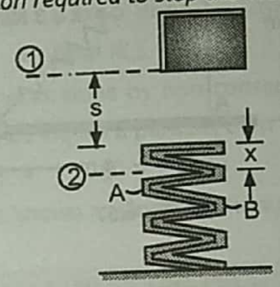


Fig. 6.61

Solution :

Given data : Stiffness of spring A, s_A = 300 N/m

Stiffness of spring B, s_B = 200 N/m.

Mass of block, m = 2 kg.

Height of drop, h = 0.6 m.

Let x be the maximum deformation of spring at which velocity of block, v = 0.

Considering position of maximum compression of spring datum and using conservation of energy.

$$K.E._1 + P.E._1 = K.E._2 + P.E._2$$

$$0 + 2 \times 9.81\ (0.6\ x) = \frac{1}{2} \cdot 300\ x^2 + \frac{1}{2} \times 200\ x^2$$

$$11.772 + 19.62\ x = 250\ x^2$$

$$x^2 - 0.078\ x - 0.047 = 0$$

$$x = \mathbf{0.259\ m}$$ ... **Ans.**

Example 6.48 : *Point P on the 2 kg cylinder has an initial velocity v_0 = 0.8 m/s as it passes position A. Neglect the mass of the pulleys and cable and determine the distance y of point P below A when the 3 kg cylinder has acquired an upward velocity of 0.6 m/s.*

Solution :

Given data : Mass of cylinder A, m_A = 2 kg.

Mass of cylinder B, m_B = 3 kg.

Initial velocity of cylinder A, $u_A = 0.8$ m/s

Final velocity of cylinder B, $v_B = 0.6$ m/s.

Let y be the distance below A when cylinder B acquired upward velocity of 0.6 m/s.

From concept of length of string,

$$x_A = 2x_B, \quad v_B = 2v_B$$

$$u_B = 0.4 \text{ m/s.}$$

$$\therefore \quad y = 2x_B \quad \therefore \quad x_B = y/2.$$

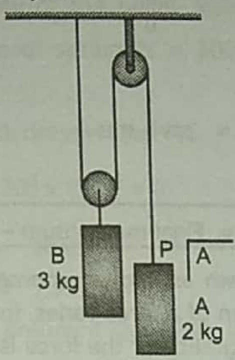


Fig. 6.62

Considering datum at A and using conservation of energy

$$\frac{1}{2} \times 2 \times 0.8^2 + \frac{1}{2} \times 3 \times 0.4^2 = \frac{1}{2} \times 2 \times 1.2^2 + \frac{1}{2} \times 3 \times 0.6^2 - 2 \times 9.81y + 3 \times 9.81 \times y/2$$

$$0.8 = 1.98 - 4.905\,y, \quad 4.905\,y = 1.98 - 0.88$$

$$\therefore \quad y = \mathbf{0.224\ m.} \quad \textbf{... Ans.}$$

***Example 6.49 :** The simple pendulum shown in Fig. 6.63 is released from rest at A with the string horizontal and swing downward under the influence of gravity. Express the velocity v of the bob as a function of angle θ.*

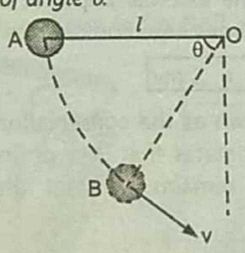


Fig. 6.63

Solution :

Given data : Weight of pendulum - W.

Velocity of pendulum at A, $v_A = 0$.

Length of pendulum - L.

Let v - be the velocity of pendulum at B.

Considering datum at B and using conservation of energy,

$$0 + Wl \sin\theta = \frac{1}{2} \cdot \frac{W}{g} v^2$$

$$v^2 = 2gl \sin\theta$$

$$\therefore \quad v = \sqrt{2gl \sin\theta} \quad \textbf{... Ans.}$$

***Example 6.50 :** If the pendulum shown in Fig. 6.64 is released from rest in its position of unstable equilibrium as shown, find the value of the angle φ defining the position in its downward fall at which the axial force in the rod changes from compression to tension.*

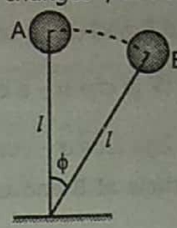


Fig. 6.64

Solution :

Given data : Weight of pendulum - W.

Initial velocity of pendulum at A, $v_A = 0$.

Length of pendulum - l.

v - be the velocity of pendulum at B.

Considering datum at B and using conservation of energy,

$$0 + W(l - l\cos\theta) = \frac{1}{2} \cdot \frac{W}{g} \cdot v^2$$

$$v^2 = 2gl(1 - \cos\theta)$$

Considering F.B.D. of pendulum bob at B and using equation of motion along normal axis,

$$\Sigma F_x = ma_n$$

$$W\cos\theta = \frac{W}{g} \cdot \frac{2gl(1-\cos\theta)}{l}$$

$$\cos\theta = 2 - 2\cos\theta$$

$$3\cos\theta = 2$$

$$\cos\theta = \frac{2}{3}$$

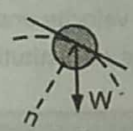


Fig. 6.64 (a) : F.B.D. of pendulum bob

$$\therefore \quad \theta = \mathbf{48.19°}$$

***Example 6.51 :** Find the angle φ defining the position of the point B as shown in Fig. 6.65, where the particle will jump clear to the cylindrical surface after the string OA bas been cut. Neglect friction.*

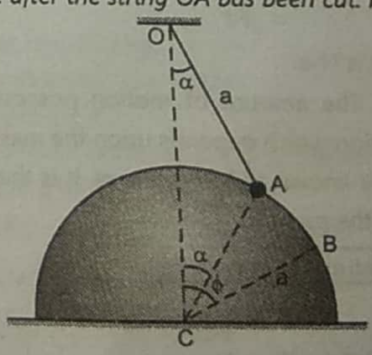


Fig. 6.65

Solution :

Given data : Weight of particle - W.

Radius of cylindrical surface, r = a.

Velocity of particle at B is v.

Considering datum at B and using conservation of energy.

$$v^2 = W[a\cos\alpha - a\cos\phi] = \frac{1}{2}\cdot\frac{W}{g}\cdot v^2$$

$$v^2 = 2ga[\cos\alpha - \cos\phi]$$

Considering F.B.D. of particle at B and using equation of motion along normal axis.

$$W\cos\phi - R = \frac{W}{g}\cdot\frac{2ga[\cos\alpha-\cos\phi]}{a}$$

$$\therefore \quad R = 0$$

$$W\cos\phi = 2W\cos\alpha - 2W\cos\phi$$

$$3W\cos\phi = 2W\cos\alpha$$

$$\cos\phi = \frac{2}{3}\cos\alpha$$

At $\alpha = 30$, $\phi = $ **54.74°**

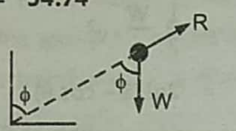


Fig. 6.65 (a) : F.B.D. of particle at B

C – IMPULSE-MOMENTUM PRINCIPLE

6.11 INTRODUCTION

In this chapter, we will integrate the equation of motion with respect to time to obtain principle of impulse and momentum. The equation of impulse-momentum is useful for solving problems involving force, velocity and time. The conservation of momentum and coefficient of restitution are introduced to solve the problems of impact.

6.12 IMPULSE AND MOMENTUM

Impulse : The change in momentum produced by a force on the particle within an infinite short interval of time is known as **Impulse**.

$$\boxed{\text{Impulse} = \text{Force} \times \text{Time}}$$

$$= F\cdot t$$

Its S.I. unit is N-s.

Momentum : The amount of motion possessed by a particle during the motion which depends upon the mass and the velocity of the particle is known as **Momentum**. It is the product of mass and velocity of the particle.

$$\boxed{\text{Momentum} = \text{Mass} \times \text{Velocity}}$$

$$= m \times v$$

Its S.I. unit is kg-m/s or N-s.

Impulse-Momentum Principle : According to Newton's second law of motion,

$$\Sigma F = ma$$

$$\Sigma F = m\cdot\frac{dv}{dt}$$

Integrating between the limits $v = u$ at $t = t_1$ and $v = v$ at $t = t_2$,

$$\Sigma\int_{t_1}^{t_2} F\cdot dt = m\int_u^v dv$$

$$\Sigma\int_{t_1}^{t_2} F\cdot dt = mv - mu \quad \text{... (6.9)}$$

$$\boxed{\Sigma\,\text{Impulse} = \text{Final momentum} - \text{Initial momentum}}$$

Equation (6.9) is known as impulse-momentum principle. If the magnitude or direction of a force varies, the impulse of the force is determined by integration. If the force is constant for the time interval $(t_2 - t_1)$, the impulse of the force is $F(t_2 - t_1)$.

The following equation represents the principle of linear impulse-momentum for the particle in the x and y directions respectively.

$$\left.\begin{aligned}\Sigma F_x(t_2-t_1) &= m(v_x - u_x)\\ \Sigma F_y(t_2-t_1) &= m(v_y - u_y)\end{aligned}\right\} \quad \text{... (6.10)}$$

While solving numerical problems, impulse in the direction of motion is considered positive.

6.13 CONSERVATION OF MOMENTUM FOR SYSTEM OF PARTICLES

Rewriting impulse-momentum principle in the following form,

$$mu + \Sigma F(t_2 - t_1) = mv$$

When the sum of the external impulses acting on a system of particle is zero, the above equation reduces to a simplified form

$$\boxed{\Sigma mu = \Sigma mv} \quad \text{... (6.11)}$$

This equation is known as the conservation of momentum for a system of particle. It states that sum of linear momentum for a system of particles remains constant throughout the period t_1 to t_2.

NUMERICAL EXAMPLES ON IMPULSE-MOMENTUM PRINCIPLE

Example 6.52 : *A 20 g bullet is fired horizontally into the 300 g block, which rests on the smooth surface. After the block becomes pregnant with the bullet, it moves 300 mm to the right before coming to rest. Determine the speed of the bullet as it strikes the block. The spring is originally unstretched and has a constant of 200 N/m.*

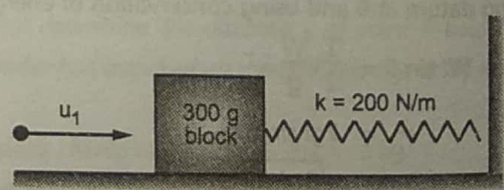


Fig. 6.66

Solution :

Given data : Mass of bullet, m_1 = 20 g

Mass of block, m_2 = 300 g

Displacement of block with bullet, x = 0.3 m

Spring constant, k = 200 N/m

Initial velocity of the block, $u_2 = 0$

Let v be the velocity of block when it was pregnant by bullet and u_1 be the initial velocity of the bullet. Velocity of the block at maximum compression of spring (x = 300 mm) is zero. Using work-energy principle,

Initial K.E. + Σ Work done = Final K.E.

$$\frac{1}{2}\times\frac{320}{1000}\,v^2 - \frac{1}{2}\times 200\times(0.3)^2 = 0$$

$$v^2 = 56.25$$

$$\therefore \quad v = 7.5 \text{ m/s}$$

When the block was pregnant by bullet, both are moving with velocity 7.5 m/s. Using conservation of momentum,

$$m_1u_1 + m_2u_2 = mv_1 + mv_2,$$

$$\dots (u_2 = 0,\ v_1 = v_2 = v = 7.5 \text{ m/s})$$

$$\frac{20}{1000}\,u_1 + \frac{300}{1000}\times 0 = \left(\frac{20+300}{1000}\right)\times 7.5$$

$$\therefore \quad u_1 = \mathbf{120\ m/s} \qquad \dots \textbf{Ans.}$$

Example 6.53 : *An estimate of the expected load on seat belt is to be made before designing prototype belts that will be evaluated in automobile crash tests. If an automobile travelling at 72 km/h, during a crush, is brought to stop in 110 milliseconds, determine (a) The average impulsive force exerted by a 100 kg man on the belt, (b) The maximum force F exerted on a belt if the force-time diagram has the shape as shown in Fig. 6.67.*

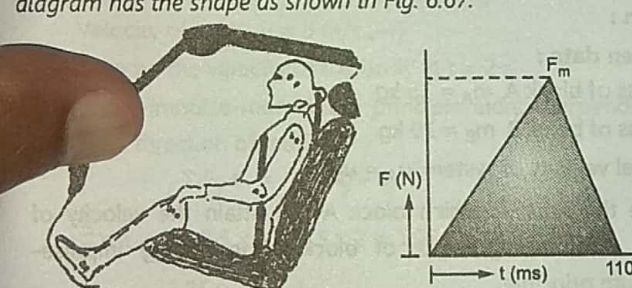


Fig. 6.67

Solution :

Given data : Mass of man, m = 100 kg

The velocity of an automobile changes from 72 km/h to 0 in t = 110 milliseconds.

Let F be the average impulsive force exerted by man on a belt in time t.

Using impulse-momentum principle,

$$F\cdot t = m(v-u)$$

$$F = \frac{m(v-u)}{t}$$

$$F = \frac{100\left(\frac{72}{3.6}-0\right)}{0.110}$$

$$\therefore \quad F = \mathbf{18.18\ kN} \qquad \dots \textbf{Ans.}$$

Let F_m be the maximum impulsive force from F-t diagram of Fig. 6.67. Using impulse-momentum principle,

$$\frac{1}{2}F_m\times t = m(v-u)$$

$$F_m = \frac{2m(v-u)}{t}$$

$$F_m = \frac{2\times 100\left(\frac{72}{3.6}-0\right)}{0.110} = \mathbf{36.36\ kN} \qquad \dots \textbf{Ans.}$$

Example 6.54 : *The 200 kg lunar-lander is descending into the moon's surface with a velocity of 6 m/s, when its engine is fired. If the engine produces a thrust T, for 4 s, which varies with time as shown in Fig. 6.68 and then cuts off, calculate the velocity of the lander when t = 5s. Assume that it has not yet landed. Gravitational acceleration at the moon's surface is 1.62 m/s².*

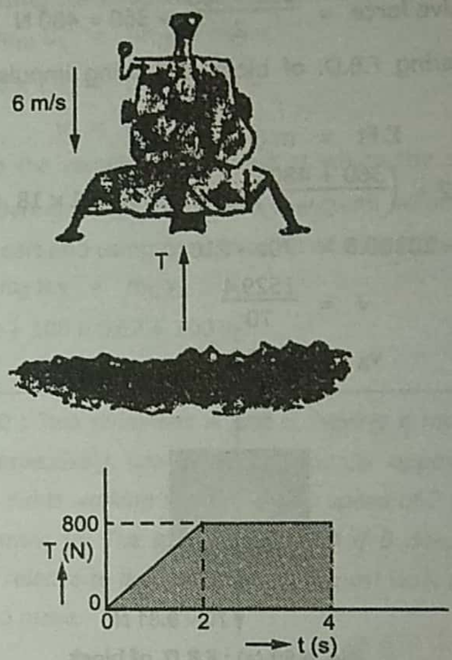


Fig. 6.68

Solution :

Given data :

Mass of lander, m = 200 kg

Initial velocity of lander, u = 6 m/s

F-t diagram upto t = 4 s.

Let v be the velocity of lander at t = 5 s, before landing. Using impulse-momentum principle,

$$\Sigma F\cdot t = \text{Change in momentum}$$

$$\frac{1}{2}\times 2\times 800 + 2\times 800 - 200\times 1.62\times 5 = 20(6-v)$$

$$2400 - 1620 = 1200 - 200v$$

$$\therefore \quad v = 2.1 \text{ m/s} \qquad \dots \textbf{Ans.}$$

Example 6.55 : *The winch delivers a horizontal force F, which varies as shown, to the cable at A. The pulley carries a 70 kg block B. If B is originally moving upwards at 3 m/s, determine the speed of the block at t = 18 s.*

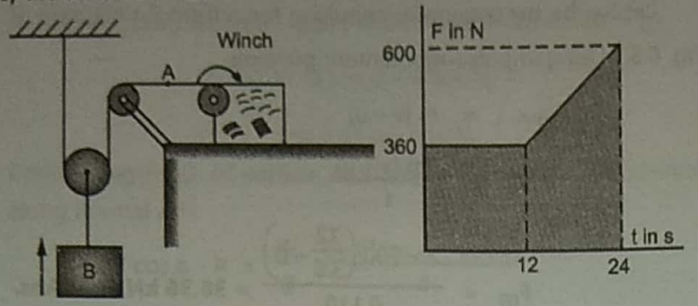


Fig. 6.69

Solution :

Given data : Mass of pulley, m = 70 kg

Initial velocity of block, u = 3 m/s

Let v be the velocity of the block at t = 18 s

From Fig. 6.69, at t = 18 s, impulsive force is given by

$$\text{Impulsive force} = \frac{600 - 360}{2} + 360 = 480 \text{ N}$$

Considering F.B.D. of block and using impulse-momentum principle,

$$\Sigma Ft = m (v - u)$$

$$2\left[360 \times 12 + \left(\frac{360 + 480}{2}\right) \times 6\right] - 70 \times 9.81 \times 18 = 70 (v - 3)$$

$$13680 - 12360.6 = 70v - 210$$

$$\therefore \quad v = \frac{1529.4}{70}$$

$$v_B = \mathbf{21.85 \ m/s} \qquad \text{... Ans.}$$

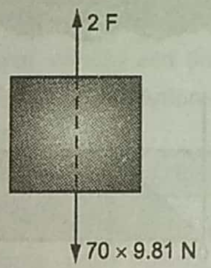


Fig. 6.69 (a) : F.B.D. of block

Example 6.56 : *An 8 kg cylinder C rests on a 4 kg platform A supported by a cord which passes over the pulleys D and E and is attached to a 4 kg block B. Knowing that the system is released from rest, determine : (a) the velocity of block B after 0.8 s, (b) the force exerted by the cylinder on the platform.*

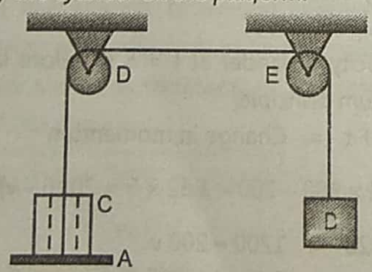


Fig. 6.70

Solution :

Given data :

Mass of cylinder, $m_C = 8$ kg

Mass of platform A, $m_A = 4$ kg

Mass of block B, $m_B = 4$ kg

Let v be the velocity of block B at t = 0.8 s.

From the equation of kinematics, $v_C (\uparrow) = v_B (\downarrow)$

Using impulse-momentum principle for the system,

$$\Sigma Ft = m (v - u)$$

$$12 \times 9.81 \times 0.8 - 4 \times 9.81 \times 0.8 = 12 (v_C - 0) + 4 (v_B - 0)$$

$$62.784 = 16 v$$

$$\therefore \quad v = \mathbf{3.924 \ m/s} \qquad \text{... Ans.}$$

Let R be the force exerted by the cylinder on the platform and using impulse-momentum principle to cylinder,

$$8 \times 9.81 \times 0.8 - R \times 0.8 = 8 (3.924 - 0)$$

$$-0.8 R = -31.392$$

$$\therefore \quad R = \mathbf{39.24 \ N} \qquad \text{... Ans.}$$

Example 6.57 : *The system is released from rest. Determine the time it takes for the velocity of A to reach 0.6 m/s. Neglect friction and mass of pulleys.*

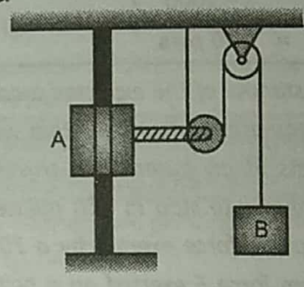


Fig. 6.71

Solution :

Given data :

Mass of block A, $m_A = 15$ kg

Mass of block B, $m_B = 10$ kg

Initial velocity of system, $u_A = u_B = 0$

Let t be the time at which block A will attain the velocity of 0.6 m/s. Considering F.B.D. of block A and using impulse-momentum principle,

$$\Sigma F \cdot t = m (v - u)$$

$$15 \times 9.81 \times t - 2T \cdot t = 15 (0.6 - 0)$$

$$147.15 \, t - 2T \cdot t = 9 \qquad \text{... (1)}$$

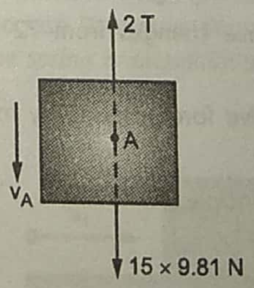


Fig. 6.71 (a) : F.B.D. of block A

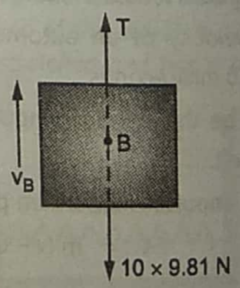


Fig. 6.71 (b) : F.B.D. of block B

Considering F.B.D. of block B and using impulse-momentum principle,

$$\Sigma Ft = m(v_B - u_B)$$

From equation of kinematics,

$$2x_A + x_B = \text{Constant}$$

$$2v_A + v_B = 0$$

$$v_B = 1.2 \text{ m/s } (\uparrow)$$

$$10 \times 9.81 \times t - T \cdot t = 10(-1.2 - 0)$$

$$98.1\,t - T \cdot t = -12 \qquad \text{... (2)}$$

Solving equations (1) and (2),

$$t = \mathbf{0.673 \text{ s}} \qquad \text{... Ans.}$$

Example 6.58 : *A 1.25 kg body is travelling in a horizontal straight line with a velocity of 3 m/s when a horizontal force P is applied to it at right angles to the initial direction of motion. The magnitude of P varies as shown in Fig. 6.72, but the direction remains constant. Assuming that P is the only force acting on the body, determine the velocity of the body at t = 2 s. Specify the direction with reference to the direction of P.*

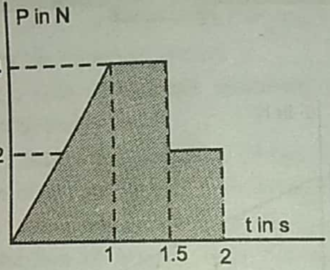


Fig. 6.72

Solution :

Given data : Mass of body, m = 1.25 kg

Velocity of body, u = 3 m/s (→)

Let v be the velocity of the body at t = 2 s

Using impulse-momentum principle along perpendicular to the initial direction of motion,

$$\Sigma F_y t = m(v_y - 0)$$

$$\frac{1}{2} \times 1 \times 4 + 0.5 \times 4 + 0.5 \times 2 = 1.25(v_y - 0)$$

$$1.25\,v_y = 5$$

$$v_y = 4 \text{ m/s}$$

$$v_x = 3 \text{ m/s} \qquad \text{... (Given)}$$

Resultant velocity of body at t = 2 s,

$$v = \sqrt{v_x^2 + v_y^2}$$

$$v = \sqrt{3^2 + 4^2}$$

$$\therefore \quad v = \mathbf{5 \text{ m/s}} \qquad \text{... Ans.}$$

$$\theta = \tan^{-1}\left(\frac{v_x}{v_y}\right) = \tan^{-1}\left(\frac{3}{4}\right)$$

$$\therefore \quad \theta = \mathbf{36.9^\circ} \qquad \text{... Ans.}$$

Example 6.59 : *Two 70 kg men stand on 100 kg cart, which is at rest. Determine the final speed of the cart, if one of the men runs at a speed of 1.5 m/s and jumps on the cart at one end and then the other runs at the same speed and jumps on the same end.*

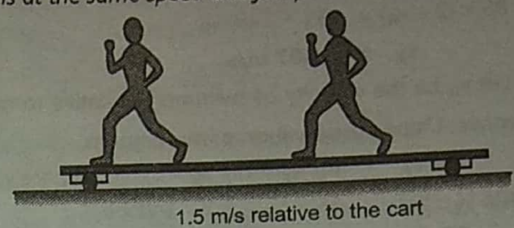


Fig. 6.73

Solution :

Given data : Mass of each man, m_m = 70 kg

Mass of cart, m_C = 100 kg

Initial velocity of men, u = 1.5 m/s

Let v be the common velocity of second man and cart. Considering jumps of first man with velocity u_1 = 1.5 m/s from the cart and using conservation of momentum,

$$m_m u_1 = (m_m + m_C)\, v$$

$$70 \times 1.5 = (70 + 100)\, v$$

$$\therefore \quad v = 0.62 \text{ m/s}$$

Let v_C be the velocity of the cart at which the second man jumps. Considering jumps of second man with velocity v = 0.62 m/s from the cart and using conservation of momentum,

$$m_m v + m_C \times v = m_C v_C$$

$$70 \times 0.62 + 100 \times 0.62 = 100\, v_C$$

$$\therefore \quad v_C = \mathbf{1.054 \text{ m/s}} \qquad \text{... Ans.}$$

Example 6.60 : *Two swimmers A and B, having a mass of 75 kg and 55 kg respectively, are at the diagonally opposite ends of 140 kg raft. A starts walking towards B at a speed of 2 m/s relative to raft. Determine (a) The speed of the raft if B does not move, (b) The speed relative to boat, with which B must walk towards A if the boat not to move.*

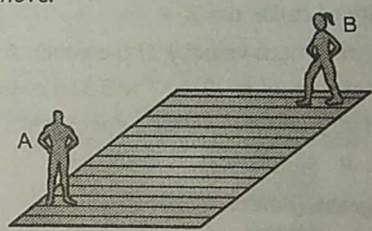


Fig. 6.74

Solution :

Given data : Mass of swimmer A, m_A = 75 kg

Mass of swimmer B, m_B = 55 kg

Mass of raft, m_R = 140 kg

Velocity of swimmer A relative to cart, u_A = 2 m/s

(a) A starts walking towards B, if B does not move ($u_B = 0$). Let v_R be the common velocity of raft and swimmer B. Using conservation of momentum,

$$m_A u_A = (m_B + m_R) v_R$$
$$85 \times (2 - v_R) = (55 + 140) v_R$$
$$\therefore \quad v_R = \mathbf{0.607\ m/s} \quad \text{... Ans.}$$

(b) Let v_B be the velocity of swimmer B relative to raft if raft is not to move. Using conservation of momentum,

$$m_A u_A = m_B v_B + m_R v_R \quad \text{... (where } v_R = 0\text{)}$$
$$\therefore \quad m_A (2 - v_R) = 55 \times (v_B - v_R) + 140 \times v_R$$
$$\therefore \quad 85 \times 2 = 55 \times v_B$$
$$\therefore \quad v_B = \mathbf{3.09\ m/s} \quad \text{... Ans.}$$

Example 6.61 : *A 2 kg collar, which can slide on a frictionless vertical rod, is acted upon by a force P that varies in magnitude as shown in Fig. 6.75. If the collar is initially at rest, determine the maximum velocity of the collar.*

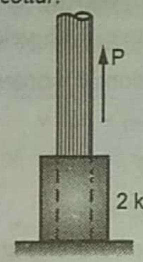


Fig. 6.75

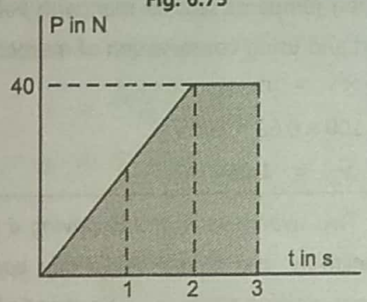


Fig. 6.75 (a)

Solution :

Given data : Mass of collar, $m_C = 2$ kg

Initial velocity of collar, u = 0

Let v be the maximum velocity of the collar. A 2 kg collar can slide on a vertical rod when P ≥ 2 × 9.81 i.e. weight of collar acting downward at time t. For limiting condition,

$$P = 2 \times 9.81$$

From P-t diagram, [Refer Fig. 6.75 (b)]

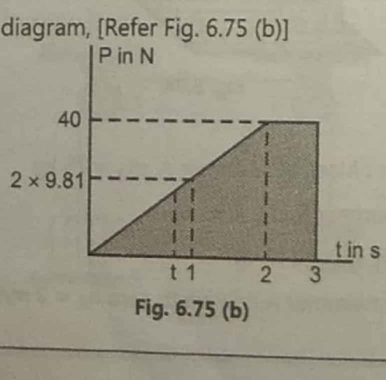


Fig. 6.75 (b)

$$\frac{2 \times 9.81}{t} = \frac{40}{2}$$
$$t = 0.981 \text{ s}$$

Force at t = 0.981 s is 2 × 9.81

$$P = 19.62 \text{ N}$$

Using impulse-momentum principle,

$$\Sigma F \cdot t = m (v - u)$$
$$-2 \times 9.81 \times (3 - 0.981) + \frac{19.62 + 40}{2} \times (2 - 0.981) + 40 \times 1$$
$$= 2 (v - 0)$$
$$-39.613 + 30.376 + 40 = 2v$$
$$\therefore \quad 2v = 30.763$$
$$\therefore \quad v = \mathbf{15.38\ m/s} \quad \text{... Ans.}$$

Example 6.62 : *The 3 kg collar is initially at rest and is acted upon by the force F which varies as shown in Fig. 6.76. If $\mu_K = 0.25$, determine (a) The maximum velocity reached by the collar, (b) The time at which the collar come to rest.*

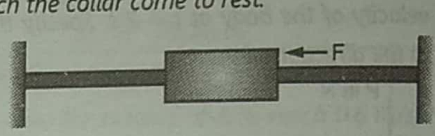


Fig. 6.76

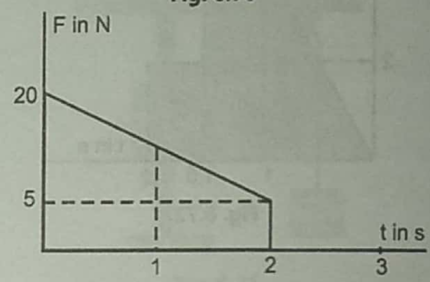


Fig. 6.76 (a)

Solution :

Given data : Mass of collar, $m_C = 3$ kg

Coefficient of kinetic friction, $\mu_K = 0.25$

The variation of force F is linear, hence the equation of force is

$$F = -7.5\,t + 20$$

The net force due to friction on the collar is given by

$$F = -7.5\,t + 20 - 0.25 \times 3 \times 9.81$$
$$F = -7.5\,t + 12.6425$$

Let v be the maximum velocity of the collar and t be the time at which the collar come to rest. Using impulse-momentum principle in integral form, since the force is variable,

$$\int_0^t F \cdot dt = m \int_0^v dv$$
$$\int_0^t (-7.5t + 12.6425)\ dt = 3 \int_0^v dv$$
$$\frac{-7.5\,t^2}{2} + 12.6425\,t = 3v \quad \text{... (1)}$$

When collar come to rest, v = 0, from equation (1),

$$\frac{-7.5t^2}{2} + 12.6425\,t = 0$$

$$t = \frac{12.6425 \times 2}{7.5}$$

$$\therefore \quad t = 3.37 \text{ s} \qquad \text{... Ans.}$$

For maximum velocity, $\frac{dv}{dt}$ should be zero.

Differentiating equation (1) w.r.to t,

$$\frac{-2 \times 7.5\,t}{2} + 12.6425 = 3\frac{dv}{dt} \qquad \cdots \left(\frac{dv}{dt} = 0\right)$$

$$\therefore \quad t = 1.686 \text{ s}$$

Substituting t = 1.686 in equation (6.14),

$$\frac{-7.5}{2} \times (1.686)^2 + 12.6425 \times 1.686 = 3v_{max}$$

$$\therefore \quad 3\,v_{max} = 10.655$$

$$\therefore \quad v_{max} = 3.55 \text{ m/s} \qquad \text{... Ans.}$$

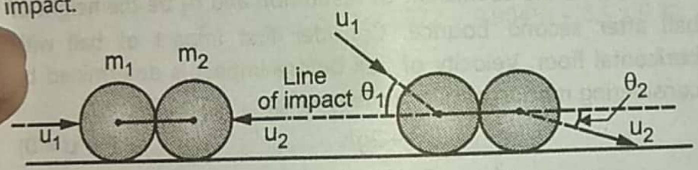
6.14 IMPACT

Impact : When two bodies collide with each other during a very short interval of time, causing impulsive force to be exerted between the bodies is known as impact.

Line of Impact : Line joining the centroids of two colliding particles is known as line of impact.

Central Impact : When the velocities of two colliding particles are along the line of impact, then it is known as central impact.

Oblique Impact : When the velocities of one or both the particles are not along the line of impact, then it is known as oblique impact.



(a) Central impact **(b) Oblique impact**

Fig. 6.77

Deformation : During impact, the two bodies undergo change in shape and size during a very short interval of time. This is known as deformation.

Restitution : After deformation, the two bodies try to regain their original shape and size during a short interval of time. This is known as restitution.

Coefficient of Restitution : Consider two particles having mass m_1 and m_2. Let u_1 and u_2 be the initial velocities of particles before impact and v_1 and v_2 be the final velocities of particles after impact ($u_1 > u_2$).

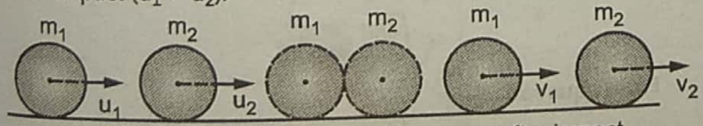


Fig. 6.78

Relative velocity of approach = $u_1 - u_2$

Relative velocity of separation = $v_2 - v_1$

The ratio of relative velocity of separation to relative velocity of approach is known as coefficient of restitution and it is denoted by e.

$$e = \frac{v_2 - v_1}{u_1 - u_2} \qquad \text{... (6.12)}$$

During the impact, momentum is conserved.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \qquad \text{... (6.13)}$$

During the impact, energy is not conserved and loss in kinetic energy = Initial K.E. – Final K.E.

Types of Impact :

1. **Perfectly Elastic Impact :** During the perfectly elastic impact, velocities are interchanged after impact.
   (a) Coefficient of restitution, e = 1.
   (b) Momentum is conserved.
   $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
   (c) Kinetic energy is conserved.
   $$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$
2. **Partially Elastic Impact :**
   (a) Coefficient of restitution varies in between 0 to 1.
   (b) Momentum is conserved.
   $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
   (c) Kinetic energy is not conserved.
   Loss of K.E. = Initial K.E. – Final K.E.
3. **Plastic Impact :** In the plastic impact, after impact, both the particles are moving with same velocity.
   (a) Coefficient of restitution, e = 0
   (b) Momentum is conserved.
   $$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$
   (c) Kinetic energy is not conserved.
   Loss of K.E. = Initial K.E. – Final K.E.

NUMERICAL EXAMPLES ON IMPACT

Example 6.63 : *The slider block B is at rest in a smooth track shown in Fig. 6.79. The block B moves after it is hitted by another block A traveling at a speed of 8 m/s. Take e = 0.4 and the mass of the block to be 1.5 kg. Find the average force exerted by the block A during the impact, which lasts for 5 milliseconds.*

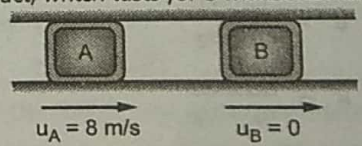


Fig. 6.79

Solution :

Given data : Mass of each block, $m_A = m_B = 1.5$ kg

Initial velocity of block A, $u_A = 8$ m/s

Initial velocity of block B, $u_B = 0$

Coefficient of restitution, e = 0.4

Let v_1 be the velocity of block A and v_2 be the velocity of block B after impact. Considering impact of blocks A and B and using conservation of momentum,

$$m_A u_A + m_B u_B = m_A v_1 + m_2 v_2$$

$$1.5 \times 8 + 1.5 \times 0 = 1.5 \times v_1 + 1.5 \times v_2$$

$$v_1 + v_2 = 8 \quad \text{... (1)}$$

Using coefficient of restitution,

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$0.4 = \frac{v_2 - v_1}{8 - 0}$$

$$v_2 - v_1 = 3.2 \quad \text{... (2)}$$

Solving equations (1) and (2),

$$v_1 = 2.4 \text{ m/s}$$

$$v_2 = 5.6 \text{ m/s}$$

Now using impulse-momentum principle for block A,

$$\Sigma Ft = m(v_1 - u_1)$$

$$F \times 0.005 = 1.5(8 - 2.4)$$

$$F = \mathbf{1.68 \text{ kN}} \quad \text{... Ans.}$$

Example 6.64 : *Two identical balls A and B move towards each other and make direct central impact. Ball B moves with an initial velocity u_B. After the impact, ball A is stopped. Derive a formula for the ratio of the initial velocity of ball A, u_A before the collision in terms of u_B and the coefficient of restitution e. Explain the motion of the balls after the impact when e = 0 and when e = 1.*

Solution :

Given data : Velocity of ball A before impact = u_A

Mass of each ball, $m_A = m_B = m$

Velocity of ball B before impact = u_B

Coefficient of restitution = e

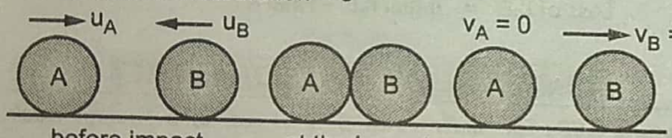


Fig. 6.80

Using coefficient of restitution,

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$e = \frac{v_B}{u_A + u_B}$$

$$e(u_A + u_B) = v_B \quad \text{... (1)}$$

Using conservation of momentum,

$$m_A u_A + m_B u_B = m_A v_A + m_B v_B$$

$$m u_A - m u_B = 0 + m v_B$$

$$u_A - u_B = v_B$$

$$u_A = v_B + u_B \quad \text{... (2)}$$

Substituting $v_B = e(u_A + u_B)$ in equation (2),

$$u_A = e(u_A + u_B) + u_B$$

$$u_A - e u_A = e u_B + u_B$$

$$u_A(1 - e) = u_B(1 + e)$$

$$\mathbf{u_A = \frac{(1+e)}{(1-e)} u_B} \quad \text{... Ans.}$$

When e = 0, then

$$\mathbf{u_A = u_B} \quad \text{... Ans.}$$

When e = 1, then

$$\mathbf{u_A = \infty} \quad \text{... Ans.}$$

Example 6.65 : *A small rubber ball is released from a height of 800 mm on a horizontal floor. After the first bounce it rises to a height of 480 mm. Compute the coefficient of restitution. Upto what height it will rise after the second bounce ?*

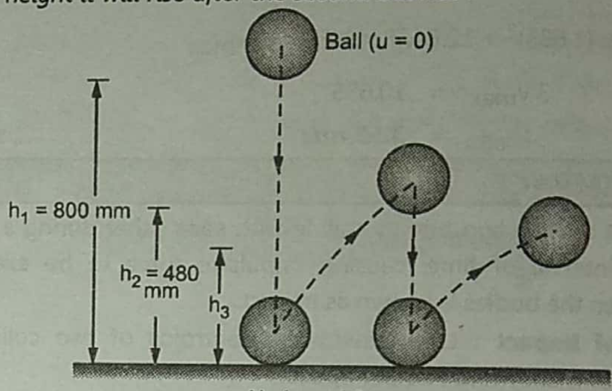


Fig. 6.81

Solution :

Given data : Height of drop of ball, h_1 = 800 mm

Height of first bounce, h_2 = 480 mm

Let e be the coefficient of restitution and h_3 be the height of ball after second bounce. Consider first impact of ball with horizontal floor. Velocity of ball before impact is determined by considering motion of ball under gravity.

$$v^2 = u^2 - 2gh \quad \text{... } (\because u = 0)$$

$$v^2 = 0 - 2 \times g(-h_1)$$

$$v = \sqrt{2gh_1} = u_1$$

Velocity of ball after impact,

$$v^2 = u^2 - 2gh \quad \text{... } (\because v = 0)$$

$$0 = u^2 - 2gh_2$$

$$u = \sqrt{2gh_2} = v_1$$

The velocity of horizontal floor before and after impact is zero i.e. $u_2 = v_2 = 0$

$$e = \frac{\text{Relative velocity of separation}}{\text{Relative velocity of approach}}$$

$$e = \frac{\sqrt{2gh_2}}{\sqrt{2gh_1}} = \sqrt{\frac{h_2}{h_1}} \quad \text{... (1)}$$

$$e = \sqrt{\frac{480}{800}} = \mathbf{0.77} \quad \text{... Ans.}$$

From equation (1),

$$0.77 = \sqrt{\frac{h_3}{480}}$$

$$\therefore \quad h_3 = \mathbf{288 \text{ mm}} \quad \text{... Ans.}$$

Example 6.66 : *A ball of mass 0.5 kg is dropped on the floor from a height of 5 m. After striking the floor, it rebounds to a height of 3 m. Find the impulse of the force acting on the ball during its contact with the floor. Assuming the contact to last for 1/50 second, find the average force exerted by the floor on the ball.*

Solution :

Given data : Mass of ball, $m_B = 0.5$ kg

Height of drop, $h_1 = 5$ m

Height of rebounce, $h_2 = 3$ m

Time of contact, $t = \frac{1}{50}$ s

Let F be the impulse and R be the force exerted by the floor on the ball.

Considering impact between ball and horizontal floor, velocity of ball before impact is,

$$u_1 = \sqrt{2gh_1}$$
$$u_1 = \sqrt{2 \times 9.81 \times 5}$$
$$u_1 = 9.9045 \text{ m/s } (\downarrow)$$

Velocity of ball after impact,

$$v = \sqrt{2gh_2}$$
$$v = \sqrt{2 \times 9.81 \times 3}$$
$$v = 7.672 \text{ m/s } (\uparrow)$$

Using impulse-momentum principle,

$$\Sigma Ft = m(v - u)$$
$$(R - 0.5 \times 9.81) \times \frac{1}{50} = 0.5\,[7.672 - (-9.9045)]$$
$$R = 50 \times 0.5 \times (7.672 + 9.9045) + 0.5 \times 9.81$$
$$R = \mathbf{444.32\ N} \quad \text{... Ans.}$$
$$\text{Impulse} = m(v - u)$$
$$\text{Impulse} = 0.5 \times (7.672 + 9.9045)$$
$$\text{Impulse} = \mathbf{8.788\ N\text{-}s} \quad \text{... Ans.}$$

Example 6.67 : *A simple pendulum, as shown in Fig. 6.82, is released from rest when it was in horizontal position OA and falls in a vertical plane under the influence of gravity. If it strikes a vertical wall at B and coefficient of restitution e = 0.5, find angle ϕ defining its total rebound.*

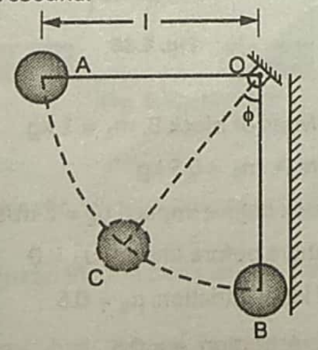


Fig. 6.82

Solution :

Given data :

Velocity of pendulum before impact = u_1

Velocity of pendulum after impact = v_1

As the wall is fixed, its velocity before and after the impact is zero i.e. $u_2 = v_2 = 0$

Coefficient of restitution, e = 0.5

Velocity of pendulum before impact,

$$v^2 = u^2 - 2gh \quad \text{... } (\because u = 0)$$
$$v^2 = 0 - 2g \times (-l)$$
$$v = u_1 = \sqrt{2gl}$$

Consider impact between pendulum and vertical wall. The coefficient of restitution,

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$
$$0.5 = \frac{0 - v_1}{\sqrt{2gl} - 0}$$
$$v_1 = -0.5\sqrt{2gl}$$
$$v_1 = 0.5\sqrt{2gl}\ (\uparrow)$$

Using conservation of energy after impact,

$$\text{K.E.} = \text{P.E.}$$
$$\frac{1}{2} m v_1^2 = mgh$$
$$\frac{1}{2} \times (0.5\sqrt{2gh})^2 = gh$$
$$h = 0.25\,l$$

From geometry of Fig. 6.83,

$$\cos\phi = \frac{(l - h)}{l}$$

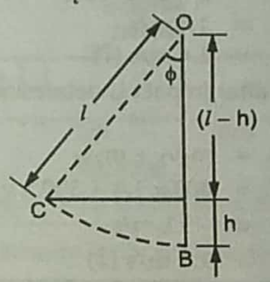


Fig. 6.83

$$\cos\phi = 0.75$$
$$\phi = \mathbf{41.4^\circ} \quad \text{... Ans.}$$

Example 6.68 : *The 400 kg ram R of a pile driver is designed to fall 1.5 m from rest and strikes the top of a 300 kg pile partially driven in the ground. The deeper the penetration, the greater is the tendency for the ram to rebound as a result of the impact. Calculate the velocity of the pile immediately after the impact if the resistance is high and a ram is found to rebound to a height of 100 mm above the point of impact.*

Solution :

Given data :

Mass of ram, m_1 = 400 kg

Mass of pile, m_2 = 300 kg

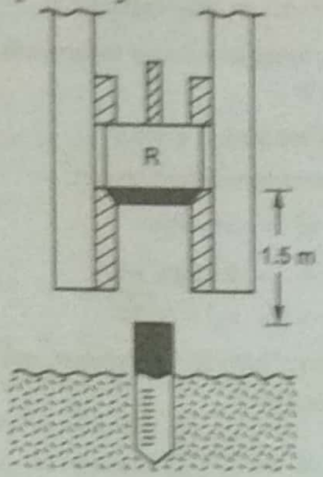


Fig. 6.84

Height of fall, h = 1.5 m

Height of rebound, h_1 = 0.1 m

Let u_1 and u_2 be the velocities of ram and pile before impact. v_1 and v_2 be the velocities of ram and pile after impact.

Velocity of ram before impact is determined by considering fall of ram.

Motion under gravity,

$$v^2 = u^2 - 2gh \quad \text{...}(\because u = 0)$$

$$v^2 = -2 \times 9.81 \times (-1.5)$$

$$v = 5.425 \text{ m/s} = u_1 (\downarrow)$$

Velocity of pile before impact,

$$u_2 = 0$$

Velocity of ram after impact,

$$v^2 = u^2 - 2gh$$

$$u^2 = 2 \times 9.81 \times 0.1$$

$$u = 1.4 \text{ m/s}$$

$$v_1 = 1.4 \text{ m/s} (\uparrow)$$

Velocity of pile after impact is determined by conservation of momentum.

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$-400 \times 5.425 + 0 = 400 \times 1.4 + 300 \times v_2$$

$$v_2 = -9.1 \text{ m/s}$$

$$v_2 = \mathbf{9.1 \text{ m/s} (\downarrow)} \quad \text{... Ans.}$$

Example 6.69 : *Block A of mass m is released from rest and falls a distance h to strike the plate B of mass 2 m which is attached to a spring. The coefficient of restitution between A and B is e. Determine the velocity of the plate, in terms of h and e, just after the collision.*

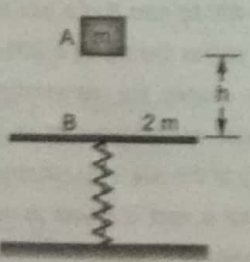


Fig. 6.85

Solution :

Given data : Mass of block A, $m_1 = m$

Mass of plate B, $m_2 = 2m$

Velocity of block A before impact, $v^2 = u^2 - 2gh$ ... $(\because u = 0)$

$u_1 = \sqrt{2gh}$

Velocity of plate B before impact, $u_2 = 0$.

Consider impact between block A and plate B. v_1 and v_2 be the velocities of block A and plate B after impact.

From coefficient of restitution,

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$e = \frac{v_2 - v_1}{\sqrt{2gh} - 0}$$

$$v_2 - v_1 = e\sqrt{2gh} \quad \text{...(1)}$$

Using conservation of momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$m \times \sqrt{2gh} + 0 = mv_1 + 2mv_2$$

$$v_1 + 2v_2 = \sqrt{2gh} \quad \text{...(2)}$$

Solving equations (1) and (2),

$$3v_2 = e\sqrt{2gh} + \sqrt{2gh}$$

$$3v_2 = (1 + e)\sqrt{2gh}$$

$$v_2 = \mathbf{\frac{1}{3}(1 + e)\sqrt{2gh}} \quad \text{... Ans.}$$

Example 6.70 : *A 1 kg block B is moving with a velocity of 2 m/s as it hits the 0.5 kg sphere A, which is at rest and hanging from a chord attached at O. If μ_K = 0.6 between the block and horizontal surface and e = 0.8 between the block and sphere, determine after impact, (a) the maximum height h reached by the sphere, (b) the distance x travelled by the block.*

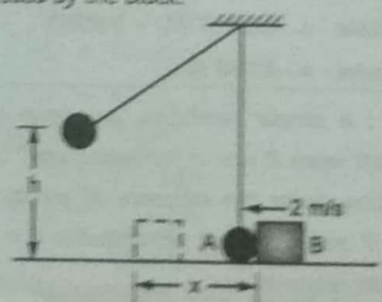


Fig. 6.86

Solution :

Given data : Mass of block B, m_1 = 1 kg

Mass of sphere A, m_2 = 0.5 kg

Velocity of block before impact, u_1 = 2 m/s

Velocity of sphere before impact, u_2 = 0

Coefficient of kinetic friction, μ_K = 0.6

Coefficient of restitution, e = 0.8

Velocity of block and sphere after impact is v_1 and v_2.

Let h be the maximum height reached by the sphere and x be the distance travelled by block.

Considering the impact between the block and sphere and using conservation of momentum,

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$1 \times 2 + 0.5 \times 0 = 1 \times v_1 + 0.5 \times v_2$$

$$v_1 + 0.5\, v_2 = 2 \quad \text{... (1)}$$

From coefficient of restitution,

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$0.8 = \frac{v_2 - v_1}{2 - 0}$$

$$v_2 - v_1 = 1.6 \quad \text{... (2)}$$

Solving equations (1) and (2),

$$v_1 = 0.8 \text{ m/s}$$

$$v_2 = 2.4 \text{ m/s}$$

Now using equation of kinematics for sphere,

$$v^2 = u^2 - 2gh \quad \text{... } (\because v = 0)$$

$$u^2 = 2gh$$

(Here u is the velocity of sphere after impact i.e. $v_2 = 2.4$ m/s)

$$h = \frac{(2.4)^2}{2 \times 9.81}$$

$\therefore$ $h = $ **294 mm** **... Ans.**

Using work-energy principle for the block,

$$\frac{m_1 v_1^2}{2} - \mu mgx = 0$$

$$\frac{1 \times (0.8)^2}{2} - 0.6 \times 1 \times 9.81\, x = 0$$

$\therefore$ $x = $ **54.4 mm** **... Ans.**

Example 6.71 : *Two identical steel balls are connected by a rigid bar of negligible mass and are dropped in the horizontal position from a height of 150 mm above the heavy steel and brass base plates. e = 0.6 between the ball and the steel base and e = 0.4 between the other ball and the brass base. Assuming that both the balls make the impact simultaneously, determine the angular velocity of the bar immediately after the impact.*

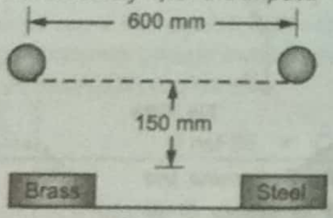


Fig. 6.87

Solution :

Given data :

Coefficient of kinetic friction between steel ball and steel base = 0.6

Coefficient of kinetic friction between the steel ball and brass plate = 0.4

Velocity of steel ball before impact is determined by considering motion of steel balls with rigid bar under gravity.

$$v^2 = u^2 - 2gh \quad \text{... } (\because u = 0)$$

$$v^2 = -2 \times 9.81 \times (-0.15)$$

$$u_1 = 1.72 \text{ m/s } (\downarrow)$$

Brass and steel plate rest on horizontal surface, hence its velocities before and after impact must be zero i.e. $u_2 = v_2 = 0$. Consider impact of left steel ball and brass plate. Let v_{1L} be the velocity of steel ball (left) after impact.

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$0.4 = \frac{0 - v_{1L}}{-1.72 - 0}$$

$$v_{1L} = 0.688 \text{ m/s } (\uparrow)$$

Consider impact of right steel ball and steel plate. Let v_{1R} be the velocity of steel ball after impact.

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

$$0.6 = \frac{0 - v_{1R}}{-1.72 - 0}$$

$$v_{1R} = 1.032 \text{ m/s } (\uparrow)$$

Now consider rotation of rigid bar.

$$v_{1L} + r\omega_{rod} = v_{1R}$$

$$0.688 + 0.6\, \omega_{rod} = 1.032$$

$\therefore$ $\omega_{rod} = $ **0.573 rad/s**

(angular velocity of rigid bar) ... Ans.

PROBLEMS FOR PRACTICE

Problem No. 1 : *Two beams AC and CD are hinged at C. Find the reaction at A, B and C.*

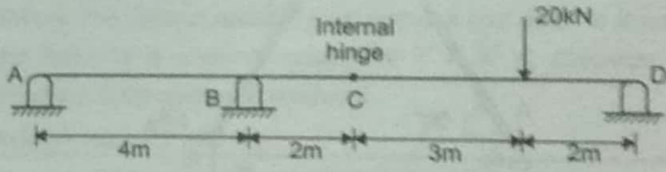


Fig. 6.88

Answer : $R_A = 4$ kN ($\downarrow$), $R_B = 12$ kN ($\uparrow$), $R_D = 12$ kN ($\uparrow$)

Problem No. 2 : *A compound beam with an internal hinge at 'C' is loaded as shown in Fig. 6.89. Using the principle of virtual work, determine the reactions at 'A', 'B', and 'D'.*

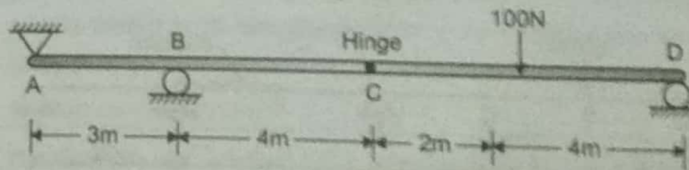


Fig. 6.89

Problem No. 3 : *A hexagonal frame is made up of six bars of identical length and cross-sections. Each bar has a weight of W newtons. The rod AB is fixed in a horizontal position. A string joints the mid points of the rods AB and DE. Find the tension in the string.*

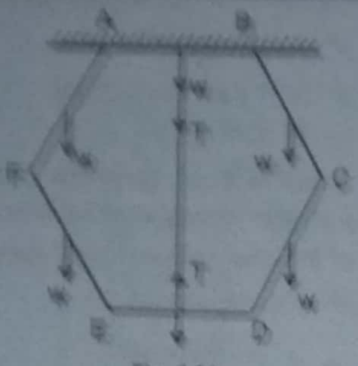


Fig. 6.90

Answer : $T = 3W$

Hint : $W\left(\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+\frac{3}{4}+1\right) = (T \times 1)$

Problem No. 4 : *Two beams AB and BC are supported and loaded as shown in the figure. Find the reactions at A, E and C.*

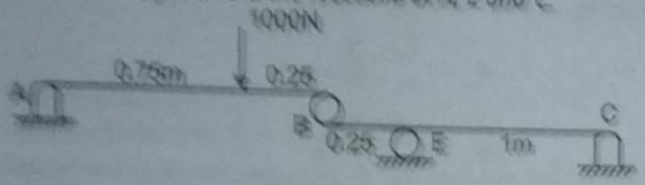


Fig. 6.91

Answer :

$R_A = 250$ N (↑)

$R_E = 937.5$ N (↑)

$R_C = 187.5$ N (↓)

Problem No. 5 : *Find the frictional force P (μR_B) required to keep the mechanism in equilibrium. $\mu = 0.2$*

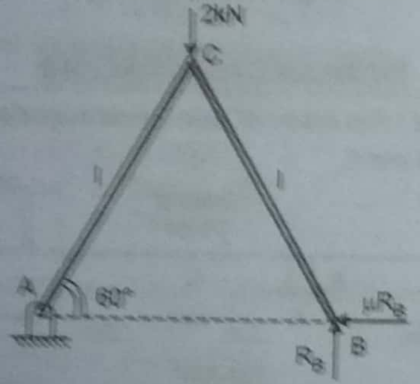


Fig. 6.92

Answer : $P = 0.777$ kN

Problem No. 6 : *Determine the support reaction at D for the beam shown. C and E are internal hinges.*

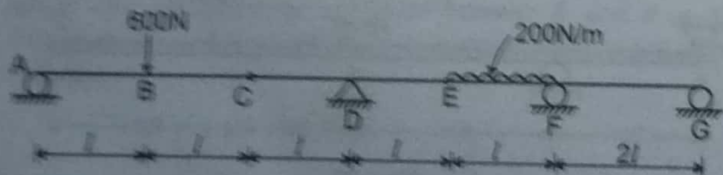


Fig. 6.93

Answer :

$R_A = 300$ N (↑)

$R_D = 500$ N (↑)

$R_F = 100$ N (↑)

$R_G = 200$ (↓)

Problem No. 7 : *For the mechanism shown, derive an expression for the magnitude of the force Q required to maintain equilibrium.*

$AC = CF = FD = DB = BC = CE = l$

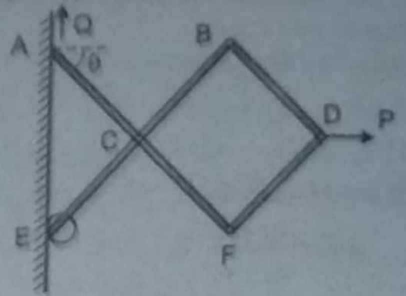


Fig. 6.94

Answer : $Q = \frac{3}{2} P \tan\theta$

Problem No. 8 : *Determine the magnitude of force P required to maintain the equilibrium of the linkage shown.*

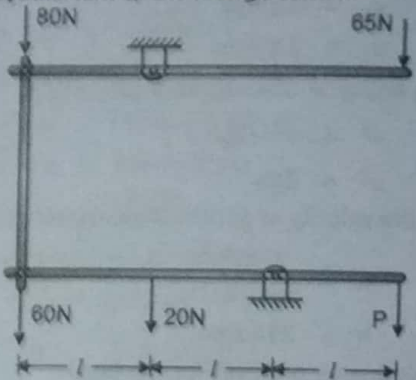


Fig. 6.95

Answer : $P = 40$ kN

Problem No. 9 : Determine the magnitude of Q to keep the system in equilibrium $AB = BC = CD = l$

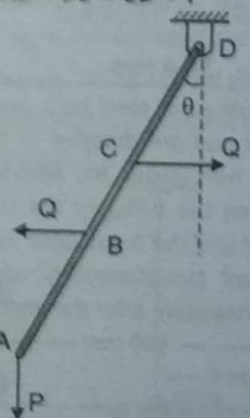


Fig. 6.96

Answer : $Q = 3P \tan\theta$

Problem No. 10 : *Determine the force P in terms of couple M to maintain the equilibrium of the system. (BC = 2 AB = l)*

Answer : $P = \frac{2M}{l} \tan\theta$

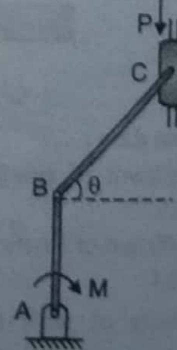


Fig. 6.97

Problem No. 11 : *Two bars AB and BC each of length l and mass m are hinged at B and supported as shown. Determine angle θ for equilibrium position in terms of P.*

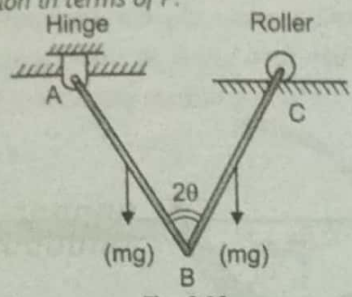


Fig. 6.98

Answer : $\tan\theta = \dfrac{2P}{mg}$

Problem No. 12 : *Determine the reactions at B and D for the beam shown.*

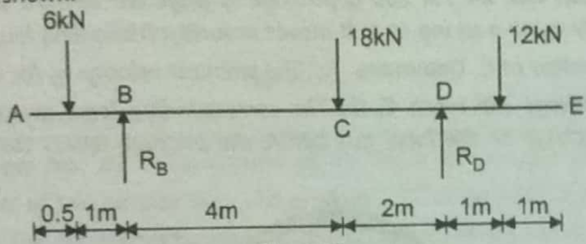


Fig. 6.99

Answer : $R_B = 11$ kN

$R_D = 25$ kN

Problem No. 13 : *Two spheres weighing W_1 and W_2 resting on smooth inclined planes are connected by a string passing over a smooth pulley as shown. Find the ratio of W_1 to W_2.*

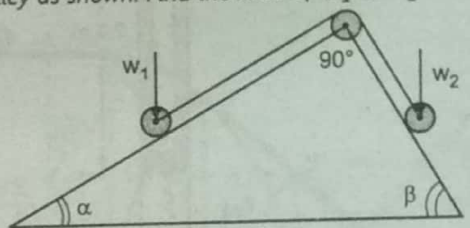


Fig. 6.100

Answer : $\dfrac{W_1}{W_2} = \dfrac{\sin\beta}{\sin\alpha}$

Problem No. 14 : *A uniform ladder weighing 250 N and having length l rests with its upper end against a smooth vertical wall and its foot on a rough horizontal ground making angle 45°. Determine the frictional force required for equilibrium.*

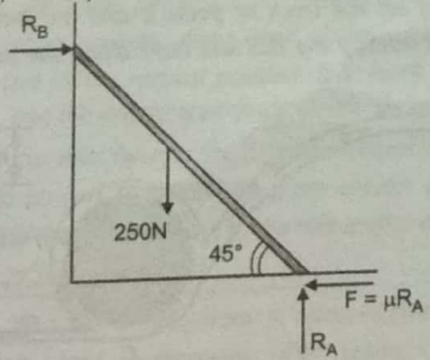


Fig. 6.101

Answer : F = 125 N

Problem No. 15 : *The spring bumper is used to arrest the motion of the 20 N block, which is sliding towards it at v = 2.7 m/s as shown, the spring is confined by the plate P and wall using cables so that its length is 0.45 m. If the stiffness of the spring is k = 800 N/m, determine the required unstretched length of the spring so that the plate is not displaced more than 0.06 m after the block collides into it. Neglect friction, the mass of the plate and spring, and the energy loss between the plate and block during the collision.*

Answer : 0.586 m

Problem No. 16 : *The collar has a mass of 20 kg and is supported on the smooth rod. The attached springs are both compressed 0.4 m when d = 0.5 m. Determine the speed of the collar after the applied force F = 100 N causes it to be displaced so that d = 0.3 m. When d = 0.5 m, the collar is at rest.*

Answer : v = 2.344 m/s

Problem No. 17 : *The 2 kg collar is released from rest at A and the slides down the inclined fixed rod in the vertical plane. The coefficient of kinetic friction is 0.4. Calculate (a) The velocity v of the collar as it strikes the spring and (b) The maximum deflection x of the spring.*

Answer : v = 2.56 m/s, x = 99.09 mm

Problem No. 18 : *The 0.5 kg collar slides with negligible friction along the fixed spiral rod, which lies in the vertical plane. The rod has the shape of spiral r = 0.3 θ, where r is in meters and θ is in radians. The collar is released from rest at A and slides to B under the action of a constant radial force T = 10 N. Calculate the velocity v of the slider as it reaches B.*

Answer : $v_B = 5.3$ m/s

Problem No. 19 : *The 7 kg collar A slides with negligible friction on the fixed vertical shaft. When the collar is released from rest at the bottom position shown, it moves up the shaft under the action of the constant force F = 200 N applied to the cable. Calculate the shiffness k which the spring must have if its maximum compression is to be limited to 75 mm. The position of the smaller pulley at B is fixed.*

Answer : k = 8790 N/m

Problem No. 20 : *A small block slides at a speed v = 3 m/s on a horizontal surface at a height h = 1 m above the ground. Determine the angle θ at which it will leave the cylindrical surface BCD.*

Answer : θ = 13.47°

Problem No. 21 : *A thin circular rod is supported in a vertical plane by a bracket at A, attached to the bracket and loosely wound around the rod is a spring of constant 40 N/m and undeformed length equal to the arc of circle AB. A 200 g collar C, not attached to the spring, can slide without friction along the rod. If the collar is released from rest when $\theta = 30°$, determine : (a) The maximum height above the point B, as defined by angle ϕ reached by the collar, (b) The maximum velocity of the collar.*

Answer : $\phi = 88.5°$, $v = 3.89$ m/s

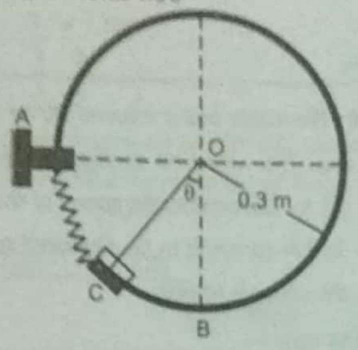


Fig. 6.102

Problem No. 22 : *The small slider of mass m is released from rest while in position A and then slides along the vertical plane track. The track is smooth from A to D and rough (coefficient of kinetic friction μ_k) from D onward. Determine (a) The distance s travelled along the inclined past point D before the slider stop, (b) The angle θ at which the normal reaction exerted by the track on the slider is 5 mg.*

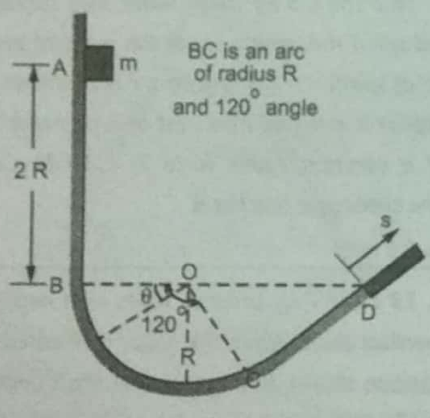


Fig. 6.103

Answer : $s = \dfrac{4R}{(1 + \sqrt{3}\,\mu_k)}$, $\theta = 19.47°$

Problem No. 23 : *The 1.2 kg slider of the system is released from rest in the position shown in Fig. 6.104 and slides without friction along the vertical plane guide. The portion ABC of the guide is circular with a radius of 1.5 m. Determine the normal force exerted by the guide on the slider (a) Just before it passes point C, (b) Just after it passes point C. What is the maximum compression of the spring ?*

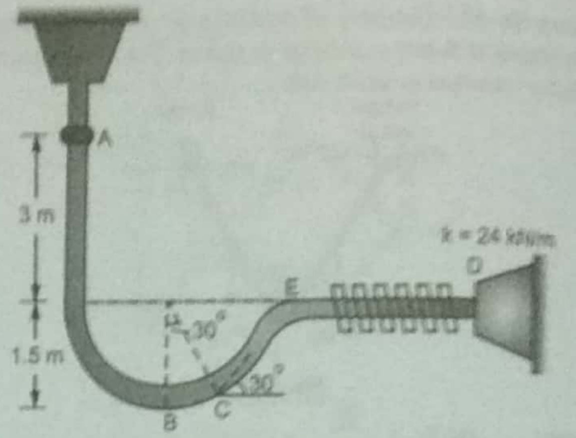


Fig. 6.104

Answer : $N_b = 77.67$ N, $N_s = 10.19$ N, $x_{max} = 54.25$ mm

Problem No. 24 : *A 200 g package is projected upward with a velocity v_0 by a spring at A. It moves around a frictionless loop and it deposited at C. Determine (a) The smallest velocity v_0 for which the package will reach C, (b) The corresponding force exerted by the package on the loop, just before the package leaves the loop at C.*

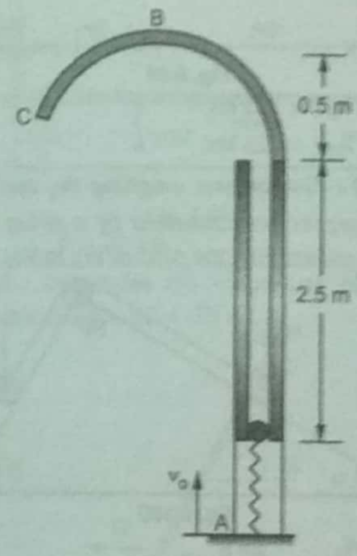


Fig. 6.105

Answer : $v_o = 7.99$ m/s, $N_C = 5.89$ N

Problem No. 25 : *A small ball of weight W starts from rest from point A and rolls without friction along the loop ABCD. What is the least height h above the top of the loop at which ball can start without falling off the track at point B and for such a starting position, what velocity the ball will have along the portion CD of the track ?*

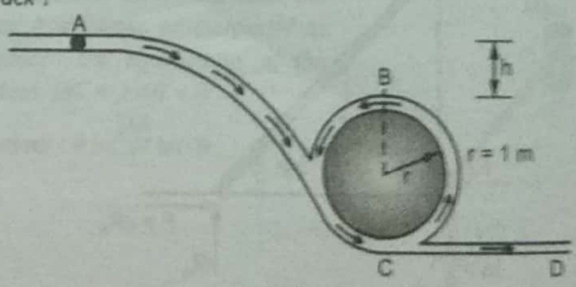


Fig. 6.106

Answer : h = 0.5 m, $v_C = 7$ m/s

Problem No. 26 : *The 0.5 kg ball of negligible size is fired up the vertical circular track using the spring plunger. The plunger keeps the spring compressed 80 mm when x = 0. Determine, by what distance x, the plunger must be pulled back and released, so that the ball will begin to leave the track at θ = 135°.*

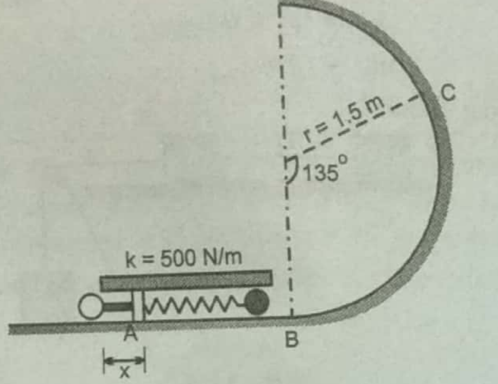


Fig. 6.107

Answer : s = 0.179 m

Problem No. 27 : *The section of the track for a roller coaster consists of two circular arcs, AB of radius 27 m and CD of radius 72 m. The car and its occupants, of total mass 250 kg, reach point A with zero velocity and then drop freely along the track. Neglecting air resistance and rolling resistance, determine the maximum and minimum values of the normal force exerted by the track on the car, along with their locations, as the car travels from A to D.*

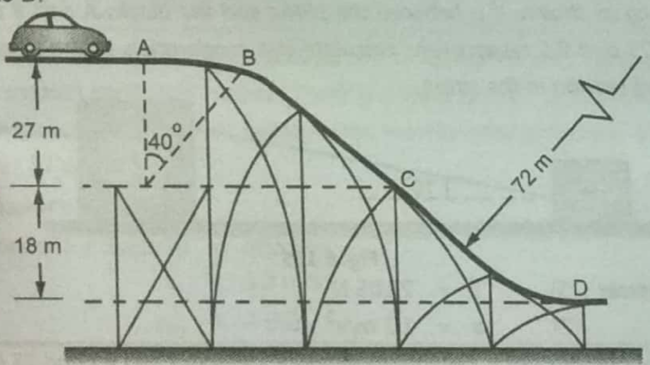


Fig. 6.108

Answer : N_B = 731.09 N, N_D = 5518.2 N

Problem No. 28 : *The pendulum A released from rest in the horizontal position swings down and strikes the pendulum B initially at rest in the vertical position. B is twice as heavy as A, $O_1A = O_2B$, and the impact is perfectly elastic. Determine (a) The angle ϕ where the axial force in the lower pendulum changes from compression to tension as it falls along the circular path, (b) The angle θ defining the total rebound of the ball A after the impact.*

Answer : θ = 27.26°, ϕ = 15.64°

Problem No. 29 : *The 5 kg cylinder is released from rest in the position shown and compresses the spring of stiffness K = 1.8 kN/m. Determine the maximum compression of the spring and maximum velocity of the cylinder. (Refer Fig. 6.109)*

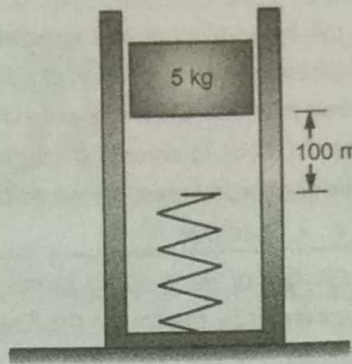


Fig. 6.109

Answer : x_{max} = 105.95 mm, v = 1.493 m/s

Problem No. 30 : *A 3 kg block rests on top of 2 kg block that is attached to spring of constant 40 N/m. The upper block is suddenly removed. Determine the maximum height and the maximum velocity reached by the 2 kg block. (Refer Fig. 6.110)*

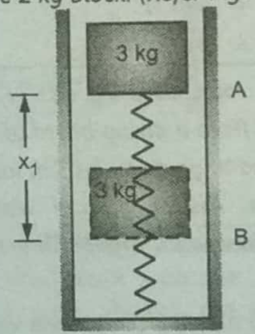


Fig. 6.110

Answer : x_{max} = 1.472 m, v = 3.29 m/s

Problem No. 31 : *The sphere at A is given a downward velocity v_0 and swings in a vertical circle of radius l and centre O. Determine the smallest velocity v_0 for which the sphere will reach point B as it swings about O (a) If AO is a rope, (b) If AO is a slender rod of negligible mass. (Refer Fig. 6.111)*

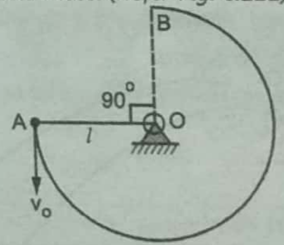


Fig. 6.111

Answer : (a) $v_o = \sqrt{3gl}$, (b) $v_o = \sqrt{2gl}$

Problem No. 32 : *A package block weighing 90 N is projected up a 25° incline with an initial velocity of 7.4 m/s. Determine :*

(1) *The maximum distance 'x' the block will move up the incline.*

(2) *Velocity of the block as it returns to its original position.*

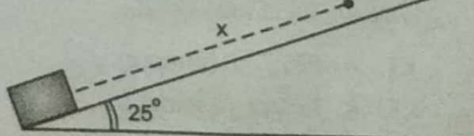


Fig. 6.112

Answer : (1) x = 4.63 m

(2) v = 4.67 m/s.

Problem No. 33 : *A body of mass 400 kg starts from rest and moves along a straight line under the action of a force which varies as square of the time from the start. The magnitude of the force was found out to be 400 N at 15 seconds from the start. Calculate the velocity of the body after 10 seconds from start.*

Answer : $v = 1.48$ m/s

Problem No. 34 : *A boy of mass 50 kg stands on an elevator. Determine the force exerted by the boy on the floor of the elevator when the elevator moves*

(1) *Upwards with constant acceleration of 2 m/s²*

(2) *Downwards with constant acceleration of 2 m/s².*

(3) *Downwards with constant acceleration of 9.81 m/s².*

Answer : (1) R = 590 N

(2) R = 390 N

(3) R = 0

Problem No. 35 : *A man weighing 637 N dives vertically down into a swimming pool from a diving board of height 6 m above the water level. He is found to go down by 3 m into the water and then starts rising upwards. Assuming total water resistance to be constant during the downward motion in water, find the resisting force.*

Answer : (1) v = 10.85 m/s (↓) (This is the velocity with which the man strikes the water)

(2) R = 1911 N

Problem No. 36 : *The world records for the shotput and discuss throw are 30 m and 80 m respectively. Assuming that their respective masses are 7 kg and 2 kg respectively, compare the work done by the champions in making their record throw if each trajectory starts at an elevation of 3 m and has an angle of projection of 45° w.r.t. horizontal.*

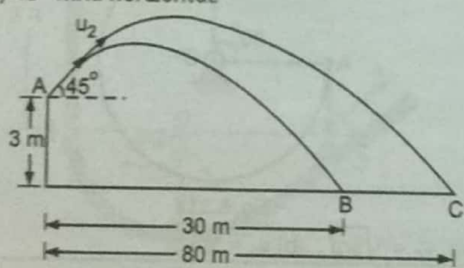


Fig. 6.113

Answer : (1) For AB,

$u_1 = 16.36$ m/s

(2) For AC,

$u_2 = 27.51$ m/s

(3) K.E. = 936.77 J (shotput)

K.E. = 756.8 J (discuss)

Note : As the work done is equal to the K.E. imparted to the object at the instant of projection, the champion throwing shotput does more work than the champion throwing discuss.

Problem No. 37 : *Three blocks of masses m_1, m_2 and m_3 are connected by two cords as shown.*

(a) Determine the acceleration of the system and the tensions in the cords.

(b) If $m_1 = m_2 = m$

$\mu_1 = \mu_2 = 0.3$

$m_3 = 2m$

Find a, T_1 and T_2

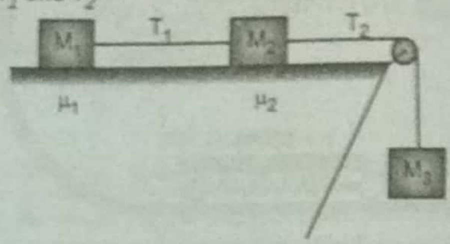


Fig. 6.114

Answer : (a)

$$a = \frac{(m_3 - \mu_1 m_1 - \mu_2 m_2)\, g}{(m_1 + m_2 + m_3)}$$

$T_1 = m_1 (\mu_1 g + a)$

$T_2 = m_3 (g - a)$

(b) $a = 4.578$ m/s²

$T_1 = 7.52$ N

$T_2 = 10.464$ N

Problem No. 38 : *A horizontal force of 100 N is exerted on block A of mass 20 kg which is tied by an inclined string to block B of mass 5 kg as shown. If μ between the plane and the blocks A and B are 0.25 and 0.5 respectively, calculate the acceleration of the system and tension in the string.*

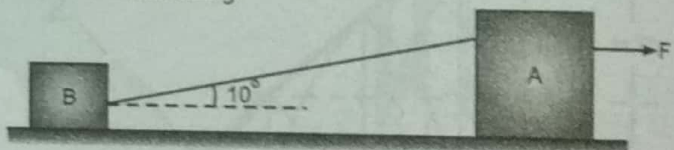


Fig. 6.115

Answer : (1) T = 28.05 N

(2) a = 1.1 m/s²

Problem No. 39 : *Two blocks A and B weighing 100 kg and 75 kg respectively are connected to each other as shown.*

(a) *Determine the acceleration of block B.*

(b) *Now, if instead of block A, a vertical downward force of (100 × 9.81) is applied, would the acceleration remain same ?*

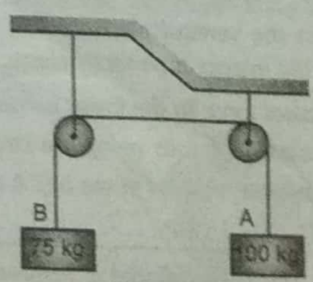


Fig. 6.116

Answer : (a) a = 1.4 m/s², T = 840.75 N

(b) a = 3.27 m/s², T = 981 N

Problem No. 40 : *An aeroplane having a mass of 320 mg has four engines each of which produces a constant thrust of 165 kN during the take off roll. Determine the length of the runway and the take-off time if the take-off speed is 200 kmph. μ between the tyres and the runway is 0.1.*

Answer : (a) $a = 1.0815$ m/s^2

(b) $s = 1426.63$ m

(c) $t = 51.36$ s.

Problem No. 41 : *Two equal masses m each are connected at the two ends of a smooth rope passing over a smooth pulley as shown. An additional mass 'm₁' is placed on one of the two masses. If the system moves with a constant acceleration, find the magnitude of mass "m₁".*

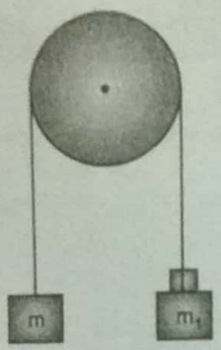


Fig. 6.117

Answer : $m_1 = \dfrac{2 \cdot m \cdot a}{(g - a)}$

Problem No. 42 : *A horizontal force F is applied to a 75 kg body on smooth horizontal surface. There is a variation of F with time. If the body starts from rest, compute the velocity after 6 seconds and after 13 seconds.*

Answer :

Area of F-t diagram = Impulse

$v_6 = 1.2$ m/s

$v_{13} = 4$ m/s

Problem No. 43 : *A 2 kg mass attached to a spring slides along a horizontal circular guide without friction. The undeformed length of the spring is 150 mm and stiffness is 200 N/m. Determine the velocity of the mass as it passes point D*

(1) directly from A to D,

(2) passing through B and C.

Answer : In both cases, the work done is same and hence, the velocity will be same.

$v = 1.732$ m/s.

Problem No. 44 : *A tennis player strikes the tennis ball with her racket while the ball is still rising. The ball speed before impact with the racket is $v_1 = 15$ m/s and after impact its speed is 22 m/s, with direction shown in Fig. 6.118. If the 60 g ball is in contact with the racket for 50 milliseconds, determine the average force exerted by the racket on the ball.*

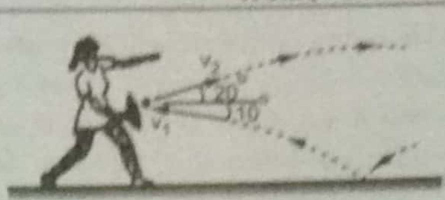


Fig. 6.118

Answer : F = 42.94 N, θ = 7.9°

Problem No. 45 : *At the instant shown in Fig. 6.119, the 2 kg block B is moving downward with a speed of 1 m/s. Determine the velocity of 4 kg block A when t = 1 s. Assume that $\mu_K = 0.15$ between the horizontal plane and the block A.*

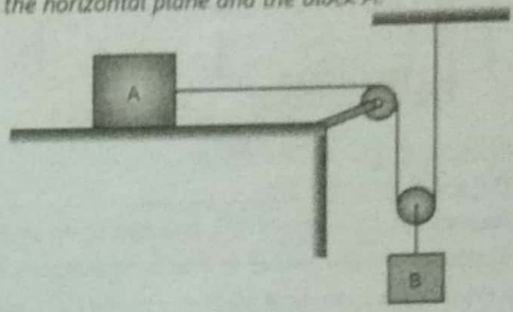


Fig. 6.119

Answer : v = 2.87 m/s

Problem No. 46 : *The 300 kg and 400 kg mine cars are rolling in opposite directions along track with the respective speeds of 0.6 m/s and 0.3 m/s. Upon impact, the cars become coupled together. Just before the impact, a 100 kg stone leaves the delivery chute with a velocity of 1.2 m/s in the direction shown in Fig. 6.120 and lands in the 300 kg car. Calculate the velocity v of the system after the stone has come to rest relative to the car.*

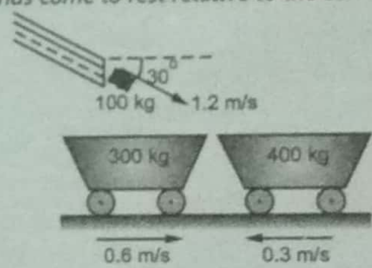


Fig. 6.120

Answer : v = 0.205 m/s

Problem No. 47 : *Two cars of same mass collide head on at C. After the collision, the cars skid on the road with their brakes locked and come to stop in the position shown in the lower part of Fig. 6.121. If the speed of car A just before the impact was 5 km/h and $\mu_K = 0.3$ between the tyre and road, determine (a) The speed of the car B just before impact, (b) The effective coefficient of restitution.*

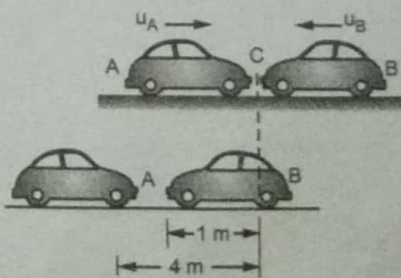


Fig. 6.121

Answer : $u_B = 8.67$ m/s (←), e = 0.241

Problem No. 48 : *A 70 g ball B dropped from a height h_0 = 1.5 m reaches a height h_2 = 0.25 m after bouncing twice from identical 210 g plates. Plate A rests directly on hard ground, while plate C rests on foam-rubber mat. Determine (a) 'e' between the plates and the ball, (b) The height h_1 of the ball's first bounce.*

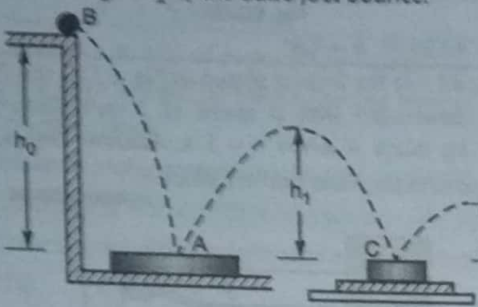


Fig. 6.122

Answer : h = 1.234 m, e = 0.93

Problem No. 49 : *A 1.5 kg block B is attached to an undeformed spring of constant 80 N/m and is resting on a horizontal surface when it is struck by an identical block A moving at a speed of 5 m/s. If e = 1 for the impact and μ_S = 0.5, μ_K = 0.3 between the blocks and the surface, determine the final position of (a) Block A and (b) Block B.*

Answer : (a) x_A = 2.98 m (→ from point of impact)

(b) x = 0.632 m (maximum compression of spring)

Problem No. 50 : *The three blocks shown in Fig. 6.123 are identical. Block B and block C are at rest when block B is hitted by block A, which is moving with a velocity of 3 m/s. After the impact, which is assumed to be perfectly plastic, the velocity of blocks A and B decreases due to friction; while block C picks up speed, until all the three blocks are moving with the same velocity v. If μ_K = 0.2 between all the surfaces, determine (a) The time required for the three blocks to reach the same velocity v, (b) The total distance travelled by each block during that time.*

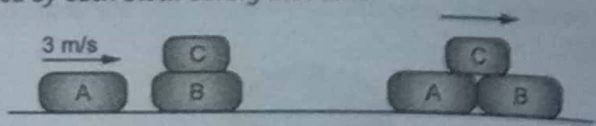


Fig. 6.123

Answer : t = 0.255 s, x_C = 63.79 mm, x_A or x_B = 255 mm

MID SEM. EXAM. OCTOBER 2017

Time : 1 Hour   Total Marks : 20

Instructions to the candidates :

(1) Assume the appropriate data if not given.

1. Fill in the blanks. **[6]**

(i) The free body diagram of a body we ____ all the supports and ____ them by the reactions which these supports exert on the body.

(Subtract, remove, add, replace, represent)

Ans. : **remove, represent**

(ii) Moment of a force about a point is equal to the ____ of the forces and ____ distance of the point from the line of action of the force.

(Addition, multiplication, product, parallel, perpendiculars, equal)

Ans. : **parallel, perpendicular**

(iii) Frame is a structure consisting of ____ bars or members pinned together and in which one or more than one of its members is subjected to more than ____ forces. (One, two, several, fix.)

Ans. : **several, one**

2. Attempt any one of the following **[6]**

(a) A uniform wheel 60 cm in diameter rests against a rigid rectangular block 15 cm thick as shown in the Fig.. Find the least pull force P through the centre of the wheel to just turn the wheel over the corner of the block. All surfaces are smooth. Find also the reaction of the block. The wheel weights 10,000 newtons. (Fig. 1)

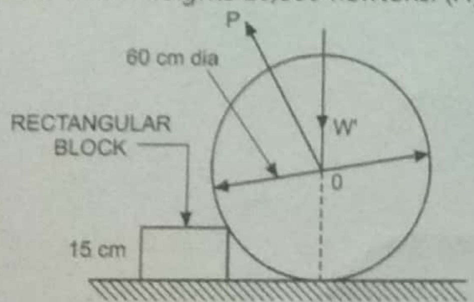


Fig. 1

Ans. : Considering FBD of wheel, for minimum value of P. It must be normal to normal reaction N

Using Lami's theorem

$$\frac{P}{\sin 120} = \frac{10{,}000}{\sin 90} = \frac{N}{\sin 150}$$

$$P = 8660.25 \text{ N}$$

$$N = 5000 \text{ N}$$

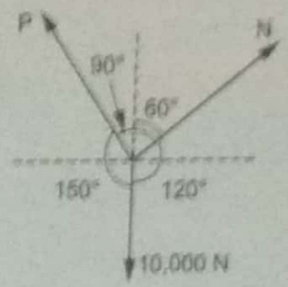


Fig. 2

(b) Explain and elaborate the following.

(i) Parallelogram Law (ii) Varignon's Theorem

(iii) Trusses and Frames.

Ans. : (i) Parallelogram Law : Please Refer Article 1.3.3 on Page No. 1.5.

(ii) Varignon's Theorem : Please Refer Article 1.6 on Page No. 1.14.

(iii) Trusses and Frames : Please Refer Article 2.17 and 2.20 on Page No. 2.31 and 2.44

3. Attempt any two of the following : **[8]**

(a) A beam is supported and loaded by hinged support at A and roller support at B as shown in the Fig. Find the reactions at A and B (Fig. 3)

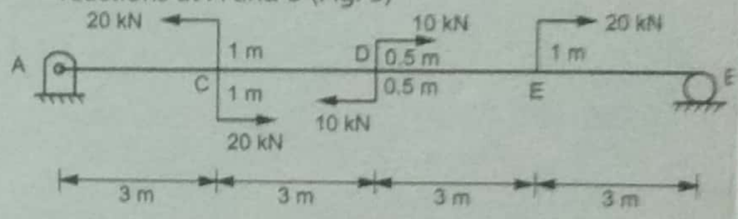


Fig. 3

Ans. :

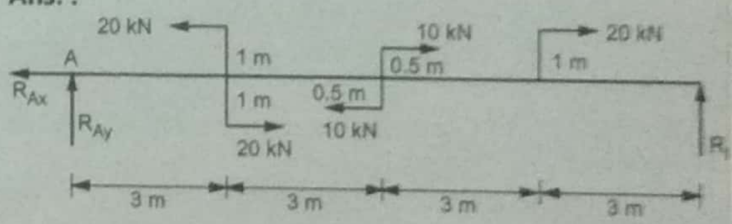


Fig. 4

Using equation of equilibrium,

$$\Sigma fx = 0, -R_{Ax} + 20 = 0, R_{Ax} = 20 \text{ kN}$$

$$\Sigma M@A = 0, -12R_B - 40 + 10 + 20 \times 1 = 0$$

$$R_B = -0.833 \text{ kN}, RB = 0.833 \text{ kN} (\downarrow)$$

$$\Sigma fy = 0$$

$$R_{AY} - 0.833 = 0$$

$$R_{AY} = 0.833 \text{ kN}$$

(b) Find the axial force in the member DE of the truss using the method of sections (Fig. 5).

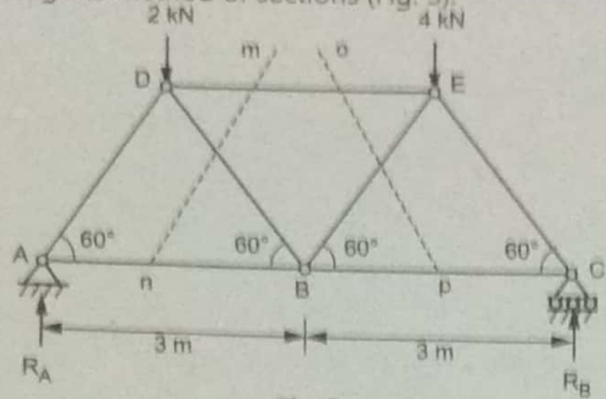


Fig. 5

Ans. :

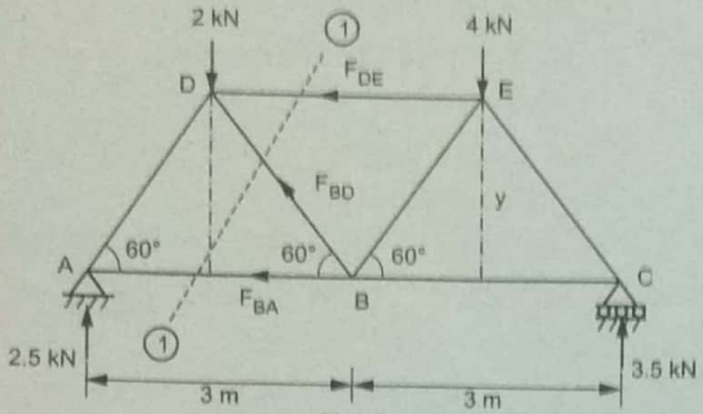


Fig. 6

Taking moment @A

$-6\,R_C + 2 \times 1.5 + 4 \times 4.5 = 0 \qquad \therefore R_C = 3.5$ kN

Taking section 1 – 1 and Moment @ B considering right hand side

$$\Sigma M_B = 0, -3.5 \times 3 + 4 \times 1.5 - FDE \times 2.6 = 0$$

$$FDE = -1.73 \text{ kN} = 1.73 \text{ kN (comp.)}$$

(c) How will you find out the resultant of two parallel forces acting in the same direction. Explain with neat diagram.

Ans. : Resultant of Parallel Force System :

- Parallel force can be in the same or in opposite directions.
- The sign can be chosen taking one direction as positive and other one is negative.
- The complete definition of resultant is according to its magnitude, direction and line of action.

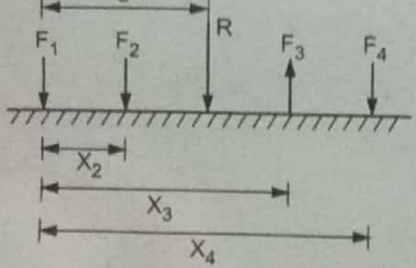


Fig. 7

$$R = \Sigma F = F_1 + F_2 + F_3 + \ldots + F_n$$

$$R_d = \Sigma F_x = F_1x_1 + F_2x_2 + F_3x_3 + \ldots F_nx_n$$

END SEM. EXAM. DECEMBER 2017

Time : 3 Hour **Total Marks : 60**

Instructions to the students :

(1) Each question carries 12 marks.

(2) Attempt any FIVE questions of the following.

(3) Illustrate your answers with neat sketches, diagram etc. wherever necessary.

(4) If some part of parameter is noticed to be missing, you may appropriately assume it and should mention it clearly.

1. (a) How will you add two forces? Explain the Parallelogram Law and Law of Triangle of forces. **[6]**

Ans. : Please Refer Article 1.3.2 on Page No. 1.5 and Article 1.3.3 and Refer Q. 2 (b) October 2017.

(b) Two ropes are tied together at C. If the maximum permissible tension in each rope is 3.5 kN, What is the maximum force P that can be applied and in what direction as shown in Fig. 1. **[6]**

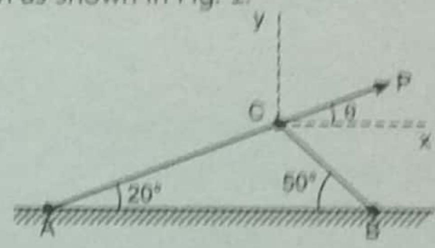


Fig. 1

Ans. : Considering FBD of Joint c and using equation of equilibrium

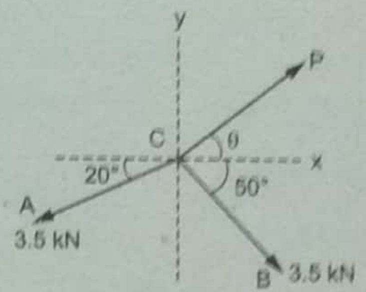


Fig. 2

$\Sigma fx = 0,\ 3.5 \cos 50 - 3.5 \cos 20 + p \cos 0 = 0$

$$P \cos \theta = 1.039 \qquad \ldots (1)$$

$\Sigma fy = 0$

$P \sin \theta - 3.5 \sin 50 - 3.5 \sin 20 = 0$

$$P \sin \theta = 3.878 \qquad \ldots (2)$$

From equation (1) and (2), $\tan \theta = 3.7326,\ \theta = 75°$

From equation (1) or (2), P = 4.014 kN

2. (a) What are various types of supports and support reactions? Explain with its free body diagram. **[4]**

Ans. : Please Refer Table 2.1 on Page no. 2.9.

Using the method of Joints, find the axial forces in all the members of a truss with the loading shown in the Fig. 3. **[8]**

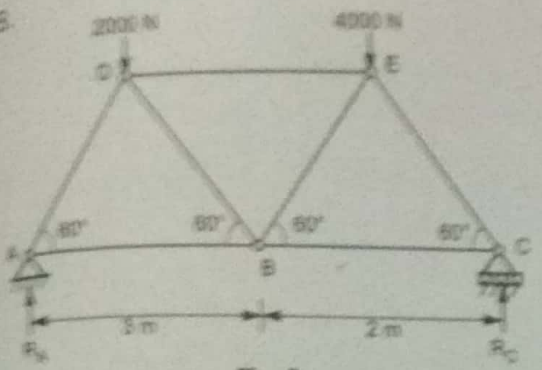


Fig. 3

Ans.: Considering whole truss in equilibrium and using equation of equilibrium

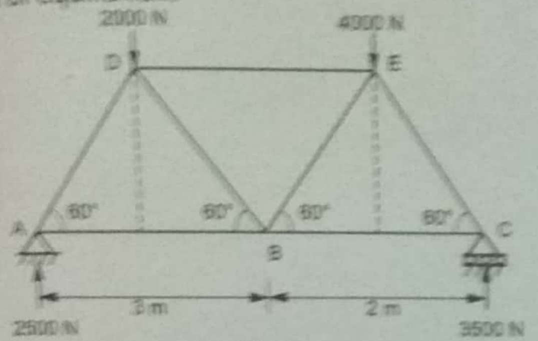


Fig. 4

$\Sigma M@A = 0$

$-6\,R_B + 2000 \times 1.5 + 4000 \times 4.5 = 0$

$R_B = 3500\text{ N}$

$\Sigma fy = 0,\quad R_A = 2500\text{ N}$

Considering FBD of Joint A

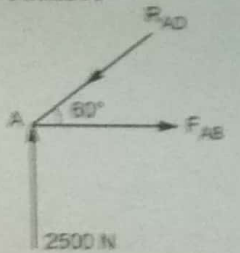


Fig. 5

$\Sigma fy = 0$

$2500 - F_{AD} \sin 60 = 0$

$F_{AD} = 2886.75\text{ N (Comp.)}$

$\Sigma fx = 0$

$2886.75 \cos 60 + F_{AB} = 0$

$F_{AB} = 1443.38\text{ kN (Tension)}$

Considering FBD of Joint C

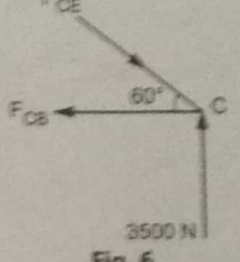


Fig. 6

$\Sigma fy = 0,\ 3500 - F_{CE} \sin 60 = 0$

$F_{CE} = 4041.45\text{ N (Comp.)}$

$\Sigma fx = 0,$

$4041.45 \cos 60 - F_{CB} = 0$

$F_{CB} = 2020.725\text{ N (Tension)}$

Considering FBD of Joint D

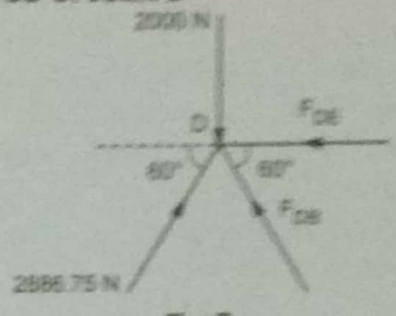


Fig. 7

$\Sigma fy = 0$

$-2000 + 2886.75 \sin 60 + F_{DB} \sin 60 = 0$

$F_{BD} = 577.35\text{ N (comp.)}$

$\Sigma fx = 0$

$2886.75 \cos 60 - 577.35 \cos 60 - F_{DE} = 0$

$F_{DE} = 1154.7\text{ N (comp.)}$

Considering RBD of Joint E

$F_{EB} = 577.36\text{ N (comp).}$

3. (a) Locate the centroid of the shaded are obtained by removing a semicircle of diameter a from a quadrant of a circle of radius a as shown in Fig. 8.

Ans. : $A_1 = \frac{\pi a^2}{4},\ x_1 = \frac{4a}{3\pi},\ y_1 = \frac{4a}{3\pi}$

$A_2 = \frac{\pi a^2}{8},\ x_2 = a/2,\ y_2 = \frac{4a}{6\pi}$

$$\bar{x} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2}$$

$$= \frac{\frac{\pi a^2}{\cancel{4}} \cdot \frac{\cancel{4}a}{3\pi} - \frac{\pi a^2}{8} \times \frac{a}{2}}{\frac{\pi a^2}{4} - \frac{\pi a^2}{8}} = \frac{\frac{\pi a^3}{3\pi} - \frac{\pi a^3}{16}}{\frac{\pi a^2}{4}}$$

$$= \frac{0.333\,a^3 - 0.196\,a^3}{0.785\,a^2}$$

$$\Rightarrow \quad \bar{x} = 0.175\,a$$

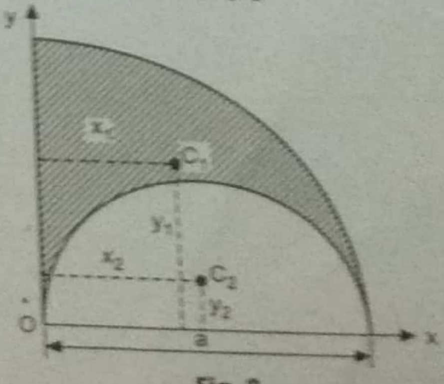


Fig. 8

$$\bar{y} = \frac{\frac{\pi a^2}{4} \cdot \frac{4a}{3\pi} - \frac{\pi a^2}{8} \times \frac{4a}{6\pi}}{0.785\, a^2}$$

$$= \frac{0.333\, a^3 - 0.0833\, a^3}{0.785 a^2}$$

$$= 0.318\, a$$

(b) A 7 m long ladder rests against a vertical wall, with which it makes an angle of 45°, and on a floor. If a man, whose weight is one half of that of the ladder, climbs it at the distance along the ladder will he be when the ladder is about to slip shown in Fig. 9? The coefficient of friction between the ladder and the wall is 1/3 and that between the ladder and the floor is 1/2. **[6]**

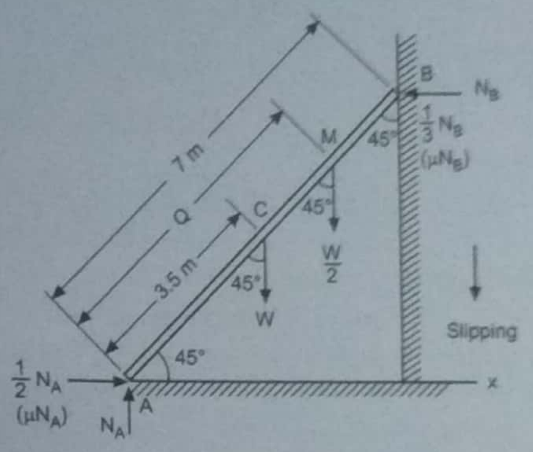


Fig. 9

Ans. :

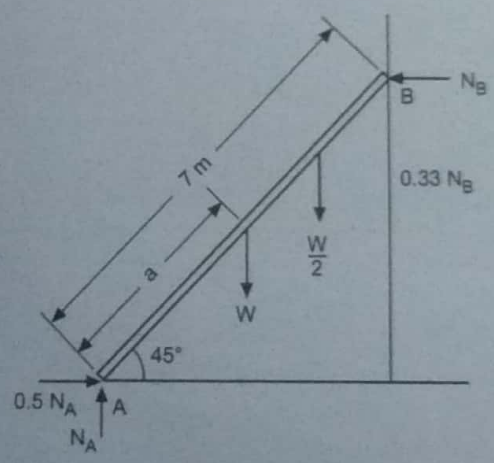


Fig. 10

From FBD of ladder and using equation of equilibrium

$\Sigma fx = 0$

$$0.5\ NA - NB = 0$$

$$NB = 0.5\ NA$$

$\Sigma fy = 0$

$$NA + 0.33\ NB - W - W/2 = 0$$

$$NA + 0.33\ NB = 1.5\ W$$

$$NA + 0.33 \times 0.5\ NA = 1.5\ W$$

$$1.165\ NA = 1.5\ W ; NA = 1.288\ W$$

$$NB = 0.644\ W$$

$\Sigma MA = 0$

$W \times 3.5 \cos 45 + W/2\ a \cos 45 - 0.644\ W \times 7 \sin 45 -$
$0.33 \times 0.644\ W \times 7 \times \cos 45 = 0$

$$2.47\ W + 0.354\ Wa - 3.188\ W - 1.52\ W = 0$$

$$a = 5\ m$$

4. (a) A trolley resting on a horizontal plane starts from rest and is moved to the right with a constant acceleration of 0.18m/s² Shown in Fig. 11.

Determine :

(i) acceleration of the block B connected to the trolley and

(ii) Velocities of the block B connected to the trolley and

(iii) Velocities of the trolley and the block after a time of 4 seconds and the distance moved by each of them. **[6]**

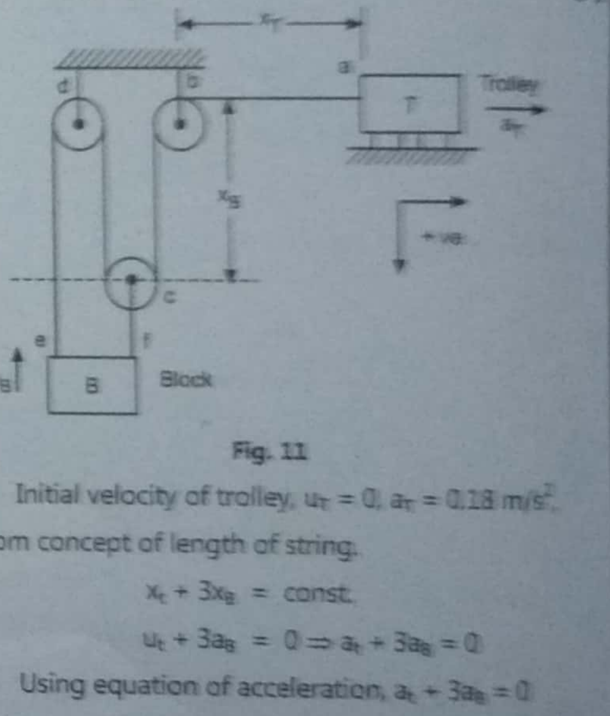


Fig. 11

Ans. : Initial velocity of trolley, $u_T = 0$, $a_T = 0.18\ m/s^2$.

From concept of length of string.

$$x_t + 3x_B = const.$$

$$u_t + 3a_B = 0 \Rightarrow a_t + 3a_B = 0$$

(i) Using equation of acceleration, $a_t + 3a_B = 0$

$$0.18 = 3a_B \Rightarrow a_B = 0.06\ m/s^2$$

(ii) Using equation of velocity, $\mu_t + 3\ \mu_B = 0$

$$\mu_t = 0$$

(iii) Velocity of trolley after 4 s, using equation of kinematics

$$\mu = v + at$$

$$\mu_t = 0 + 0.18 \times 4 \Rightarrow \mu_t = 0.72\ m/s$$

$$\mu_t + 3\ \mu_B = 0 \Rightarrow 0.72 = 3\ \mu_B \Rightarrow V_b = 0.24\ m/s$$

(b) A passenger train passes a certain station at 60 km/hr and covers a distance of 12 km with this speed and then stops at the next station 15 km from the first with uniform retardation. A local train starting from the first station covers the same distance in double this time and stops at the next station. Determine the maximum speed of the local train which covers a part of the distance with uniform acceleration and the rest with uniform retardation. **[6]**

Ans. :

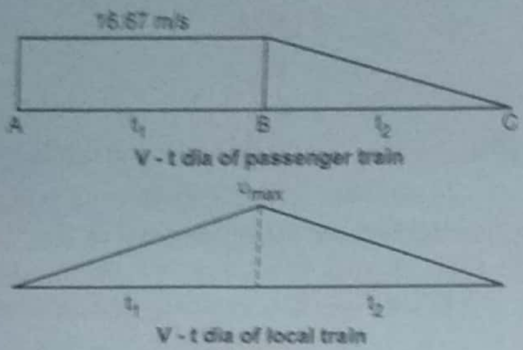


Fig. 12

From V-t dia. of passenger train

$$16.67\ t_1 = 12000 \therefore t_1 = 719.85\ S$$

$$\frac{1}{2} \times 16.67 \times t_2 = 15000 \therefore t_2 = 1799.64\ S$$

For local train time of accelerated motion is

$$2 \times 719.855 = 1439.64\ S = 3599.25\ S$$

1439.64 S and deaccelerated motion

$$\frac{1}{2} \times u_{max} \times 1439.64 + \frac{1}{2} \times v_{max} \times 3599.28\ S = 27000$$

$$2519.49\ v_{max} = 27000,\ v_{max} = 10.72\ m/s$$

5. (a) Explain in detail : D' Alemberts principle and write the equations of dynamic equilibrium of the particle. **[6]**

Ans. : Please Refer Article 5.6 on Page No. 5.2.

(b) Two blocks of masses M1 and M2 are connected by a flexible but inextensible string as shown in the Fig. 13. Assuming the coefficient of friction between block M1 and horizontal surface to be μ find the acceleration of the masses and tension in the string as per Fig. 13. Assume M_1 = 10 kg and M_2 = 5 kg, μ = 0.25 **[6]**

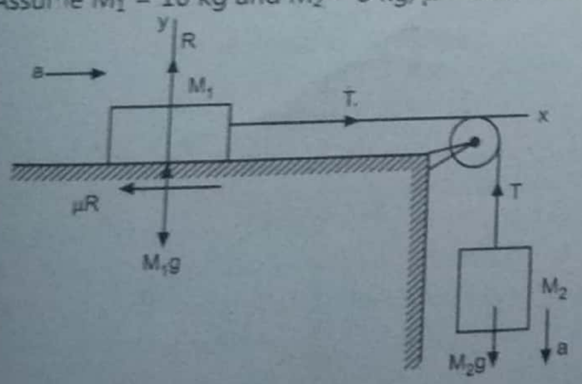


Fig. 13

Ans. : Considering FBD of mass m_2

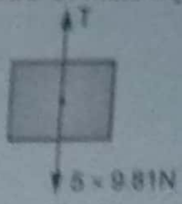


Fig. 14

Using Newton's law

$$5 \times 9.81 - T = 5a$$

$$5a + T = 49.05 \qquad (1)$$

Considering FBD of mass m_1

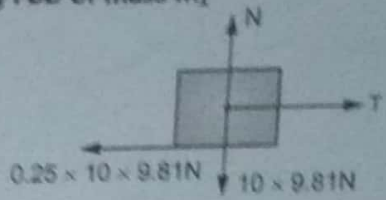


Fig. 15

$$T - 0.25 \times 10 \times 9.81 = 10\ a$$

$$T - 10\ a = 24.525 \qquad (2)$$

Solving equation (1) and (2), a = 1.635 m/s², T = 40.875 N

6. (a) A spring of stiffness 1000 N/m is stretched by 10 cm from the unreformed position. Find the work of the spring force. Also find the work required to stretch it by another 10 cm. **[6]**

Ans. : **Given :** K = 1000 N/m

$$\text{Work done} = -\frac{1}{2} K \left(x_2^2 - x_1^2\right)$$

$$= -\frac{1}{2} \times 1000\ (10^2 - 0^2) = -50\ kJ$$

∴ Work done by another 10 cm

$$\text{Work done} = \frac{-1}{2} K\ (20^2 - 0^2)$$

$$= \frac{-1}{2} \times 1000\ (400 - 0) = 200\ kJ$$

(b) What do you understand by direct central impact ? Also explain the coefficient of restitution. **[6]**

Ans. : Please Refer Article 6.14 on Page No. 6.27.

END SEM. EXAM. MAY 2018

Time : 3 Hrs **Total Marks : 60**

1. **Attempt the following.** **[6 × 2 = 12]**

(a) State and explain the Principle of Transmissibility. How it is useful as per engineering mechanics point of view? Explain with any example.

Ans. : Please Refer Article 1.3 on Page No. 1.4.

(b) How will you find out resultant of a several concurrent coplanar forces by summing rectangular components? Explain this method with resolution and projections of the forces with any example.

Ans. : Please Refer Article 1.3.3 on Page No. 1.6.

2. Attempt the following. **[6 × 2 = 12]**

(a) Two equal loads of 2500 N are supported by a flexible string ABCD at points B and C as shown in Fig. 1. Find the tensions in the portions AB, BC and CD of the string.

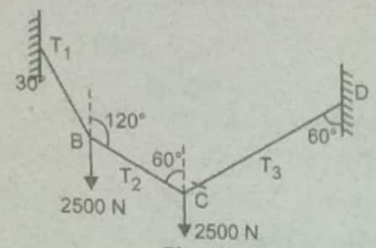


Fig. 1

Ans. : Considering FBD of joint C

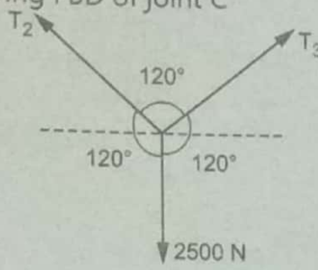


Fig. 2

$$T_3 = T_2 = 2500 \text{ N}$$

$\Sigma fx = 0$

$$2500 \cos 30 - T_1 \cos\theta = 0$$

$$T_1 \cos\theta = 2165.06 \quad \dots (1)$$

Considering FBD of joint B

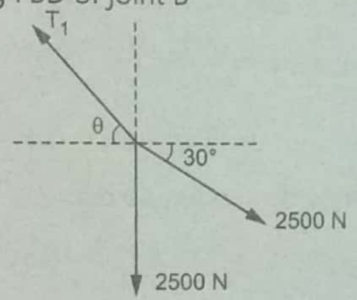


Fig. 3

$\Sigma fy = 0$

$$-2500 - 2500 \sin 30 + T_1 \sin\theta = 0$$

$$T_1 \sin\theta = 3750 \quad \dots (2)$$

$$\theta = 60°, \qquad T_1 = 4330.13 \text{ N}$$

(b) A truss is loaded and supported as shown in Fig. 3. Determine the axial forces in the member CE, CG and FG.

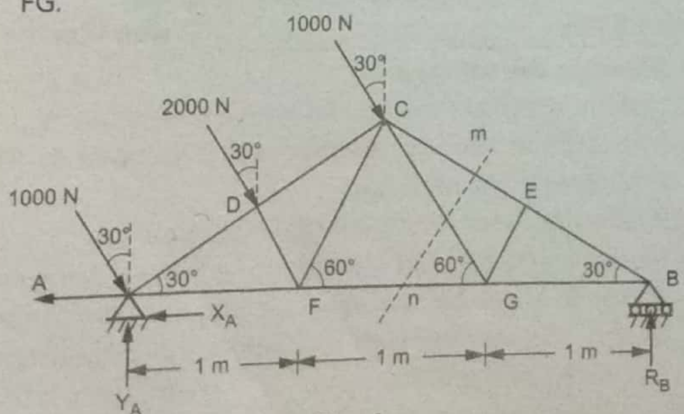


Fig. 4

Ans. :

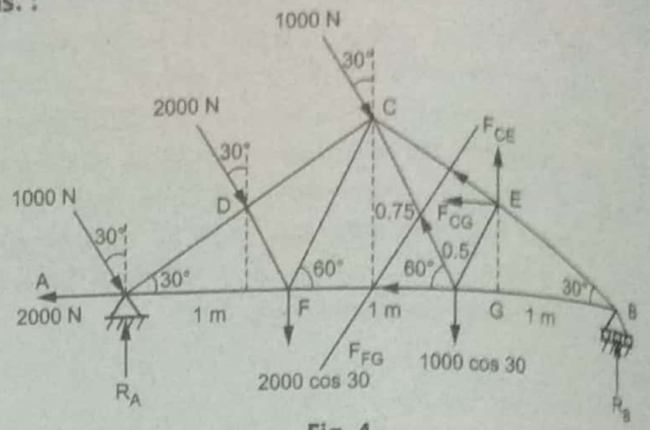


Fig. 4

$$\Sigma M@A = 0$$

$$2000 \cos 30 \times 1 + 1000 \cos 30 \times 2 - 3 R_B = 0$$

$$R_B = 1154, \qquad R_A = 1444 \text{ N}$$

Considering RHS and taking moment @ G

$$-1 \times 1154 - F_{CE} \cos 30 \times 0.433 - F_{CE} \sin 30 \times 0.25 = 0$$

$$-1 \times 1154 - F_{CE} \cos 30 \times 0.433 - F_{CE} \sin 30 \times 0.25 = 0$$

$$F_{CE} = -2308 \text{ N}$$

$$= 2308 \text{ N (comp.)}$$

$\Sigma fy = 0$

$$F_{CG} \sin 60 + 1154 - 2308 \sin 30 = 0$$

$$F_{CG} = 0$$

$\Sigma fx = 0$

$$-F_{FG} - 2308 \cos 30 = 0$$

$$F_{FG} = 1998 \text{ N (comp.)}$$

3. Attempt the following. **[6 × 2 = 12]**

(a) A block A weighing 1000 N is to be raised by means of a 15° wedge B weighing 500 N as shown in Fig. 5. Assuming the coefficient of friction between all contact surfaces to be 0.2, determine what minimum horizontal force P should be applied to raise the block. **[6]**

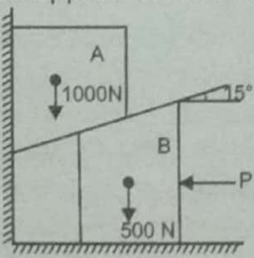


Fig. 5

Ans. : Considering FBD of block A.

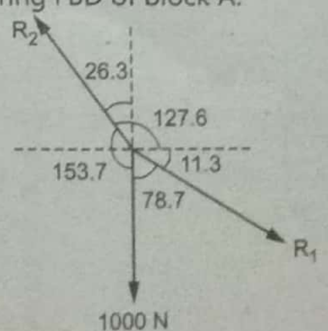


Fig. 6

Using Lami's theorem,

$$\frac{1000}{\sin 127.6} = \frac{R_1}{\sin 153.7} = \frac{R_2}{\sin 78.7}$$

$R_1 = 559.23$ N, $R_2 = 1237.7$ N

F_{BD} of wedge,

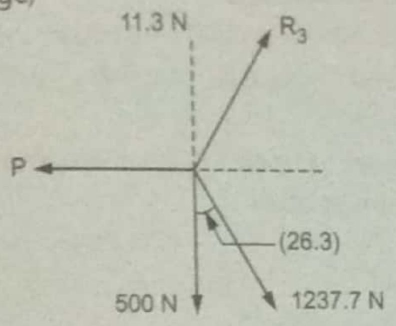


Fig. 7

$\Sigma fy = 0$

$R_3 \cos 11.3 - 500 - 1237.7 \cos 26.3 = 0$

$R_3 = 1609.6$ N

$\Sigma fx = 0$

$-P + 1609.6 \sin 11.3 + 1237.7 \sin 26.3 = 0$

$P = 863.78$ N

(b) A square hole is punched out of a circular lamina as shown in Fig. 8. The diagonal of the square which is punched out is equal to the radius of circle. Find the centroid of the remaining lamina? **[6]**

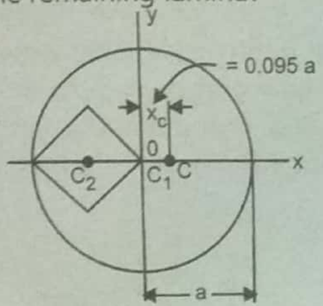


Fig. 8

Ans. : Area of circle, $A_1 = \pi a^2$, $x_1 = 0$

Area of square, $A_2 = \frac{a^2}{2}$, $x_2 = \frac{-a}{2}$

Section is symmetrical (g) x – axis hence the $\bar{y}$ is at on x – axis

$$\bar{x} = \frac{A_1x_1 - A_2x_2}{A_1 - A_2} = \frac{\pi a^2 \times 0 - \frac{a^2}{2} \times \left(\frac{-a}{2}\right)}{\pi a^2 - \frac{a^2}{2}} = \frac{\frac{a^3}{4}}{2.64\ a^2}$$

$$= \frac{0.25a^3}{2.64\ a^2} = 0.095\ a$$

4. **Attempt the following.** **[6 × 2 = 12]**

(a) Two cylinders A and B rest in a horizontal channel as shown in Fig. 9. The cylinder A has a weight of 1000 N and radius of 9 cm. The cylinder B has weight of 400 N and a radius of 5 cm. the channel is 18 cm wide at the bottom with one side vertical. The other side is inclined at an angle 60° with the horizontal. Find the reactions at the points L, N and P.

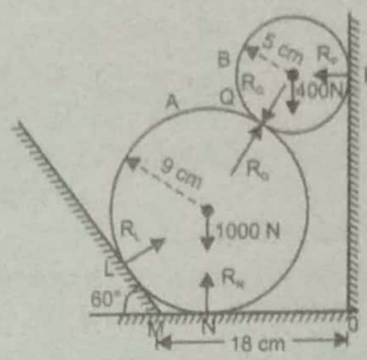


Fig. 9

Ans. : Considering FBD of upper cylinder

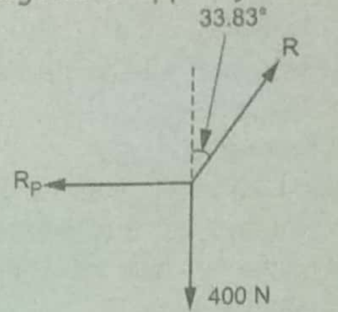


Fig. 10

$\Sigma fy = 0$

$R \cos 33.83 - 400 = 0$

$R = 481.52$ N

$\Sigma fx = 0$

$481.52 \sin 33.83 - R_P = 0$

$R_P = 268.08$ N

Considering FBD of cylinder A

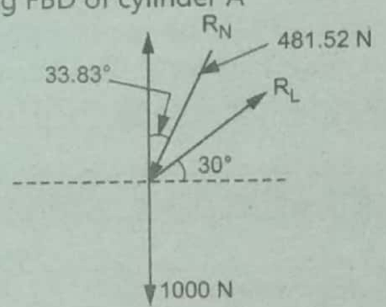


Fig. 11

$\Sigma fx = 0$

$-481.52 \sin 33.83 + R_L \cos 30 = 0$, $R_L = 309.55$ N

$\Sigma fy = 0$

$R_N - 1000 - 481.52 \cos 33.83 + 309.55 \sin 30 = 0$,

$R_N = 1245.22$ N

(b) A car weighing 4000 N is moving at a speed of 100 m/s as shown in Fig. 12. The resistance to the car is largely due to air drag which is equal to $0.004\ v^2$. What distance will it travel before its speed is reduced to 50 m/s? **[6 × 2 = 12]**

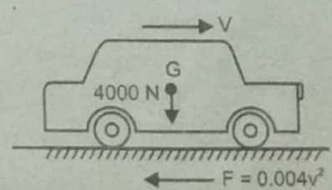


Fig. 12

Ans. : Using work energy principle

$$\frac{1}{2}\times\frac{4000}{9.81}\times 100^2 - 0.004\,(100^2 - 50^2)\,S = \frac{1}{2}\times\frac{4000}{9.81}\times 50^2$$

$$4000\times 100^2 - 2\times 9.81\times 0.004\,(100^2 - 50^2)\,S = 4000\times 50^2$$

$$S = 50.97 \text{ km}$$

5. Attempt the following.

(a) What is meant by impulse of a force and momentum? State and prove the principle of impulse and momentum.

Ans. : Please Refer Article 6.12 on Page No. 6.22.

(b) Explain the components of motion: rectangular components of velocity and acceleration.

Ans. : Please Refer Article 4.10 and 4.10.2 and 4.10.3 on Page No. 4.17.

6. Attempt the following. [6 × 2 = 12]

(a) Ball A of mass 1 kg moving with a velocity of 2 m/s, impinges directly on a ball B of mass 2 kg at rest. Find the velocities of the two balls after impact. Assume the coefficient of restitution e = ½.

Ans. : $MA = 1$ kg, $\mu_A = 2$ m/s, $MB = 2$ kg, $u_B = 0$, $e = 0.5$

$$m_1 u_A + m_2 u_B = m_1 v_A + m_2 v_B$$

$$\mu_A + 2 v_B = 2 \quad \dots (3)$$

$$e = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow 0.5 = \frac{v_B - v_A}{2 - 0}$$

$$v_B - v_A = 1 \quad \dots (4)$$

From equation (3) and (4), $3 v_B = 3 \Rightarrow v_B = 1$ m/s

$$v_A = 0$$

(b) Explain and prove D'Alembert's principle. How will you explain the concept of dynamic equilibrium?

Ans. : Please Refer Q. 5 (a) December 2017.

END SEM. EXAM. NOVEMBER 2018

Time : 3 Hours **Total Marks : 60**

1. Attempt the following. [12]

(a) What do you understand by resolution of forces and calculate the resultant of following forces shown in figure 1 ?

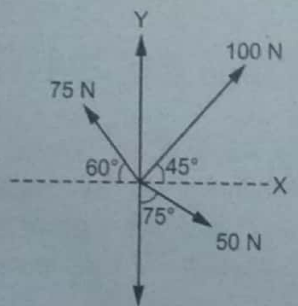


Fig. 1

Ans. : Please Refer Article 1.3.2 on Page No. 1.5.

$$\Sigma fx = 100\cos 45 - 75\cos 60 + 50\sin 75 = 81.50 \text{ N},$$

$$\Sigma fy = 100\sin 45 + 75\sin 60 - 50\cos 75 = 122.75$$

$$= \sqrt{81.50^2 + 122.75^2} = 147.34 \text{ N}, \theta = \tan^{-1}\frac{122.75}{81.50} = 56.41°$$

(b) What are the components of accelerations for the curvilinear motion? How will you calculate these components? Explain with some examples.

Ans. : Please Refer Article 4.9 on Page No. 4.10.3.

2. Attempt the following. [12]

(a) Define constraint, action, reaction and types of supports and support reactions with free body diagram.

Ans. : Constraint : Constraint on a system is a parameter that the system must obey.

Example :

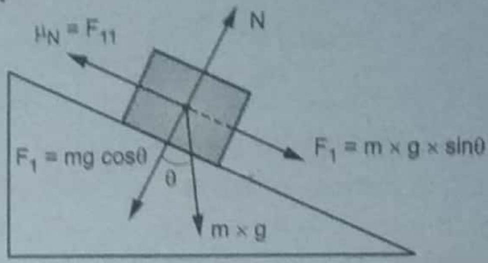


Fig. 2

- **Action :** It is a classical equations of motion of a system that can be derived by minimizing the value of integral.
- **Types of Support and Support Reactions :** Please Refer Article 2.4 and table 2.1 on Page No. 2.9.

(b) Three identical right circular cylinders A, B and C each weight W are arranged on smooth inclined surface as shown in figure 3. Determine the least value of angle that will prevent the arrangement from collapsing.

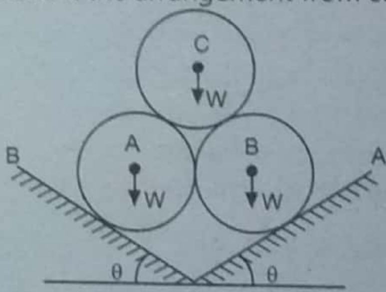


Fig. 3

Ans. :

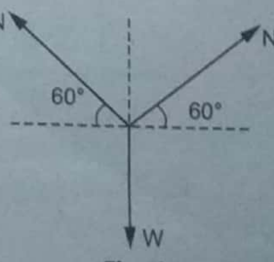


Fig. 4

Using Lami's theorem,

$$\frac{W}{\sin 60} = \frac{N}{\sin 150} \quad \therefore N = 0.577 \text{ kW}$$

Cylinder A or B is in equilibrium under three forces. It's own weight reaction from surface and reaction transformed by cylinder C.

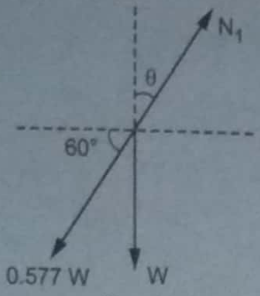


Fig. 5

From FBD of cylinder A

$\Sigma Fx = 0,$

$$N_1 \sin\theta - 0.577W \cos\theta = 0$$

$$N_1 \sin\theta = 0.288 \text{ W} \quad \dots (1)$$

$\Sigma Fy = 0$

$$N_1 \cos\theta - W - 0.577 \text{ W} \sin 60 = 0$$

$$N_1 \cos\theta = 1.5 \text{ W} \quad \dots (2)$$

From equation (1) and (2)

$$\theta = 10.87°$$

3. Attempt the following. [12]

(a) Three spherical balls of mass 2 kg, 6 kg and 12 kg are moving in the same direactions with velocities 12m/s, 4m/s and 2 m/s respectively. If the ball of mass 2 kg impinges with the ball of mass 6 kg which in turn impinges with the ball of mass 12 kg prove that the balls of masses 2 kg and 6 kg will be brought to rest by the impact. Assume the balls to be perfectly elastic.

Ans. : $m_1 = 2$ kg, $m_2 = 6$ kg, $m_3 = 12$ kg

$u_1 = 12$ m/s, $u_2 = 4$ m/s, $u_3 = 2$ m/s

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

$$2 \times 12 + 6 \times 4 = 2 \times u_1 + 6\, v_2$$

$$2\, u_1 + 6\, v_2 = 48 \quad \dots (1)$$

$$1 = \frac{v_2 - v_1}{u_1 - u_2} \Rightarrow 1 = \frac{v_2 - v_1}{12 - 4}$$

$$v_2 - v_1 = 8 \quad \dots (2)$$

$$2v_2 - 2u_1 = 16$$

$$8v_2 = 64 \Rightarrow v_2 = 8 \text{ m/s} \quad \dots \therefore v_1 = 0$$

Ball of mass 2 kg come to rest after impact

$u_2 = 8$ m/s, $u_3 = 2$ m/s

$$6 \times 8 + 12 \times 2 = 6 \times v_2 + 12 \times v_3$$

$$6\, v_2 + 12\, v_2 = 72 \quad \dots (3)$$

$$1 = \frac{v_3 - v_2}{u_2 - u_3} \Rightarrow 1 = \frac{v_3 - v_2}{8 - 2}$$

$$6 = v_3 - v_2 \quad \dots (4)$$

$$36 = 6v_3 - 6\, v_2$$

$$18\, v_3 = 108$$

$$v_3 = 6$$

$$v_2 = 0$$

Ball of mass 6 kg come in rest after impact

(b) What do you understand by trusses and frames? How will you determine the axial forces in the members? Explain method of Joints and method of sections.

Ans. : Please refer Article 2.17 and 2.17.1 ((1) and (2)) on Page No. 2.32 and Article 2.20 on Page No. 2.44.

4. Attempt the following. [12]

(a) What force P must be applied to the weightless wedges show in Fig. 6 to start them under the 1000 N block ? The angle of friction for all contact surfaces is 10 degree.

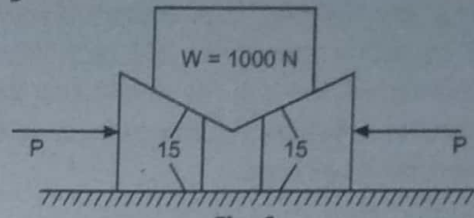


Fig. 6

Ans. : Considering FBD of block of 1000 N

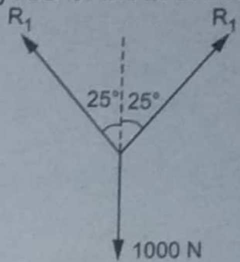


Fig. 7

Using $\Sigma fy = 0$

$$2 \times R_1 \cos 25 - 1000 = 0$$

$$R_1 = 551.69 \text{ N}$$

Considering FBD of wedge

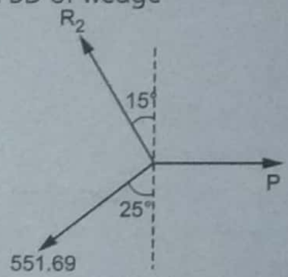


Fig. 8

Using $\Sigma fy = 0$

$$R_2 \cos 15 - 551.69 \cos 25 = 0 \qquad R_2 = 517.64 \text{ N}$$

$$\Sigma fx = P - 517.64 \sin 15 - 551.69 \sin 25 = 0$$

$$P = 367.13 \text{ N}$$

(b) Locate the centroid of the shaded area obtained by removing semicircle of diameter 'α' from a quadrant of a circle of radius 'a' as shown in figure 9.

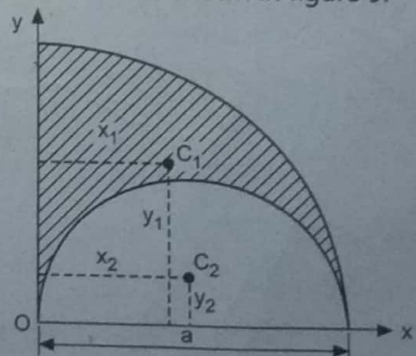


Fig. 9

Ans. : Please Refer Q. 3 (a) December 2017.

. Attempt the following. **[12]**

a) Explain the direct central impact, nature of impact and coefficient of restitution.

ns. : Please Refer Q. 6 (b) December 2017.

b) A gun of mass 3000 kg fires horizontally a shell of mass 50 kg with a velocity of 300 m/s. What is the velocity with which the gun will recoil? Also determine the uniform force required to stop the gun in 0.6 m. In how much time it will stop.

ns. : Using Conservation of Momentum

$$m_1 u_1 = m_2 u_2$$
$$3000 \times u_1 = 50 \times 300$$
$$u_1 = 5 \text{ m/s}$$

Using work energy principle

$$\frac{1}{2} mu^2 + WB = \frac{1}{2} mv^2$$
$$\frac{1}{2} \times 3000 \times 5^2 + F \times 0.6 = 0$$
$$F = \frac{1500 \times 25}{0.6} = 62.5 \text{ kN}$$

Using impulse Momentum principle

$$mu + \Sigma_{impulse} = mu$$
$$3000 \times 5 + F.t = 0$$
$$3000 \times 5 - 62500\, t = 0$$
$$t = 0.24 \text{ s}$$

Attempt the following.

(a) Define the explain the D' Alemberts principle. Write and elaborate the equation of this, for rectilinear sna curvilinear motion. **[4]**

Ans. : Please Refer Q. 6 (b) May 2018 and Refer Article 5.5 on Page No. 5.2 also.

(b) If the coefficient of kinetic friction is 0.25 under each body in the system shown in fig. 10, how far and in what direction will body B move in 5 sec. starting from rest. Pulleys are frictionless. **[8]**

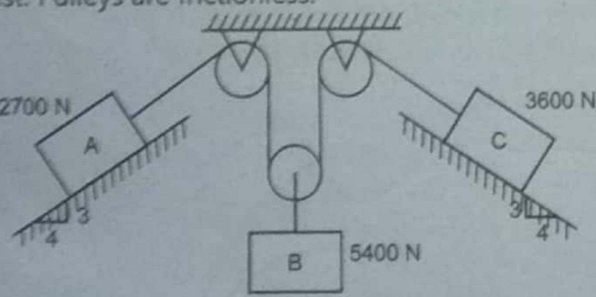


Fig. 10

Ans. : From concept of length of string

$$X_A + 2X_B + X_C = \text{constant} \Rightarrow U_A + 2\mu_B + \mu_C = 0$$
$$\Rightarrow a_A + 2a_B + a_C = 0 \qquad \ldots (5)$$

Considering block B is moving downward.

Considering FBD of block A

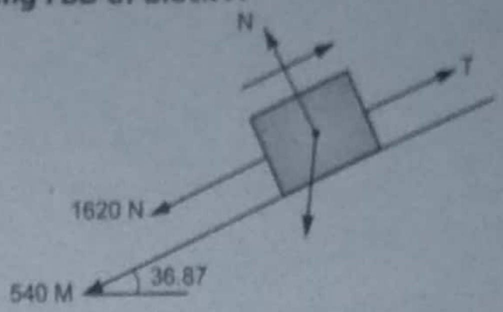


Fig. 11

$$\Sigma F = ma_A$$
$$T - 1620 - 540 = \frac{2700}{9.81} a_A \qquad \ldots (6)$$

Considering FBD of block C

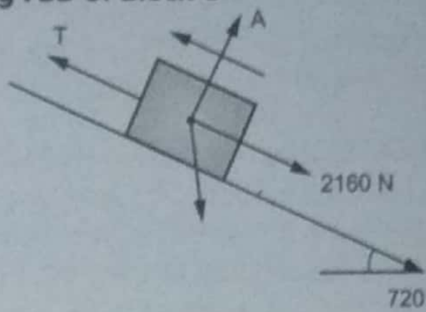


Fig. 12

$$\Sigma F = ma_C$$
$$T - 2160 - 720 = \frac{3600}{9.81} a_C \qquad \ldots (7)$$

Considering FBD of block B

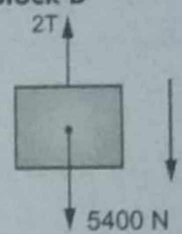


Fig. 13

$$\Sigma F = ma_B$$
$$5400 - 2T = \frac{5400}{9.81} a_B$$
$$\frac{(5400 - 2T) \times 9.81}{5400} = a_B \qquad \ldots (8)$$

Substituting values of a_A, a_B and a_C in equation (1)

$$\frac{(T - 2160) \times \cancel{9.81}}{27\cancel{00}} + \frac{(T - 2880) \times \cancel{9.81}}{36\cancel{00}} + \frac{2(5400 - 2T) \times \cancel{9.81}}{5400} = 0$$
$$4\,(T - 2160) + 3\,(T - 2880) + 4\,(5400 - 2T) = 0$$
$$\cancel{4T} + 3T - \cancel{8T} - 8640 - 8640 - 21600 = 0$$
$$T = 4320 \text{ N}$$
$$a_A = 7.848 \text{ m/s}^2,\ a_C = 3.924 \text{ m/s}^2,\ a_B = -5.886 \text{ m/s}^2$$

Dist. move by body B in 5 sec

$$S = ut + \frac{1}{2} at^2$$
$$S = 0 + \frac{1}{2} \times 5.886 \times 5^2$$
$$= 73.575 \text{ m } (\downarrow)$$

END SEM. EXAM. MAY 2019

Time : 3 Hours **Total Marks : 60**

1. Solve any three from the following.

(a) Define the following : Principle of Transmissibility of Forces, Equilibrant, and Dynamics. [4]

Ans. : Please Refer Q. 1 (1) May 2018.

(b) Compute the resultant in magnitude and direction of a parallel force system shown in Fig. [4]

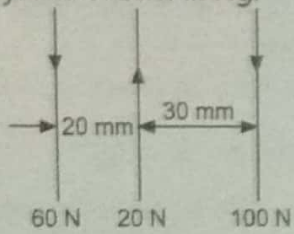


Fig. 1

Ans. :

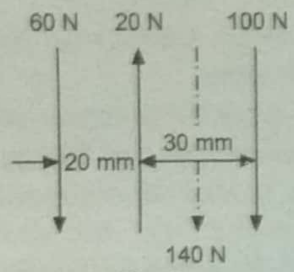


Fig. 2

$R = \Sigma fy = -60 + 20 - 100 = -140 \text{ N} = 140 \text{ N } (\downarrow)$

Using Varignon's theorem,

$$-20 \times 20 + 100 \times 50 = 140 \times x$$

$$x = 32.86 \text{ mm}$$

The resultant is acting at x = 32.86 mm from line of action of 60 N force.

(c) A uniform beam AB of weight 100 N and 6 m long had two bodies of weights 60 N and 80 N suspended from its two ends as shown in Fig. Find analytically at what point the beam should be supported, so that it may rest horizontally. [4]

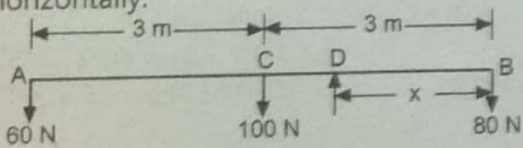


Fig. 3

Ans. :

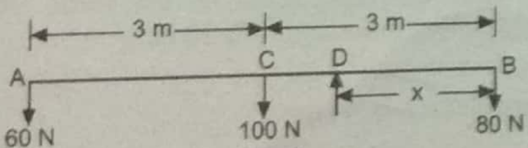


Fig. 4

Let beam AB is supported at D, x-meter from pt. B

The moment @ D must be zero

$$80 \times x - 100 (3 - x) 60 (6 - x) = 0$$

$$80x - 300 + 100 x - 360 + 60x = 0$$

$$240x = 660 \quad \therefore x = 2.75 \text{ m}$$

(d) Define a couple and write the characteristics of a couple. [4]

Ans. : Please Refer Article 1.7 on Page No. 1.15.

2. Solve any two from the following.

(a) Explain in brief with neat sketch the different types of loads studied in engineering mechanics. [6]

Ans. : Please Refer Article 2.5 on Page No. 2.10.

(b) Determine the support reactions and forces in all the members of the truss subjected an external force F as shown in Fig. [6]

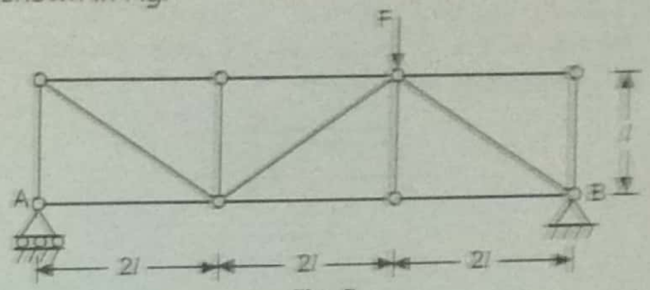


Fig. 5

Ans. :

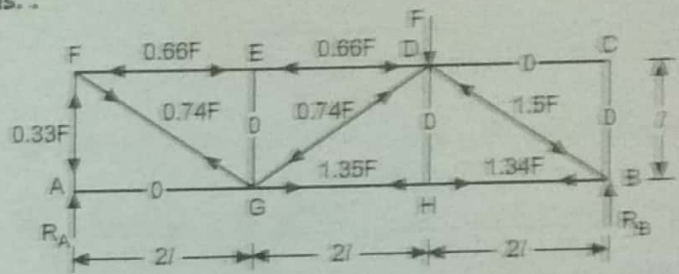


Fig. 6

Zero Force Member : CB, CD, EG, DH and AG

Taking moment @ A

$-6 \text{ RB} \times \cancel{l} + 4F \times \cancel{l} = 0 \qquad \therefore R_B = 0.67 \text{ F}$

$\Sigma fy = 0, R_A = 0.33F$

Considering FBD of joint B

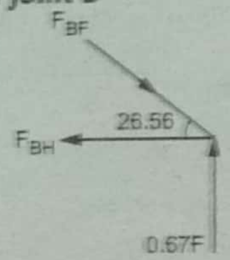


Fig. 7

$$\Sigma fy = 0, 0.67 \text{ F} - F_{BE} \sin 26.56 = 0$$

$$F_{BE} = 1.5 \text{ F (comp.)}$$

$$\Sigma fx = 0, 10.5 \cos 26.56 - F_{BH} = 0, F_{BH} = 1.34 \text{ F (Ten)}$$

Considering FBD of joint F

$\Sigma fy = 0$

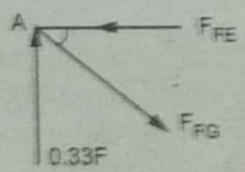


Fig. 8

$$0.33 \text{ F} - F_{FG} \sin 26.56 = 0$$

$$F_{FG} = 0.74 \text{ F (Ten)},$$

$$Fx = 0, 0.74F \cos 26.56 - F_{FE} = 0$$

$$F_{FE} = 0.66 \text{ F (comp.)}$$

From FBD of joint E, $F_{ED} = 0.66$ F (comp)

Considering FBD of Joint G

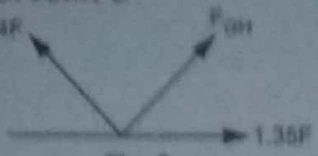


Fig. 9

$\Sigma fy = O$

$F \sin 2656 - F_{GH} \sin 26.56 = 0, \quad F_{GH} = 0.74\ F$

(c) Determine the centroid of the area shown in Fig. (All dimensions are in mm) **[6]**

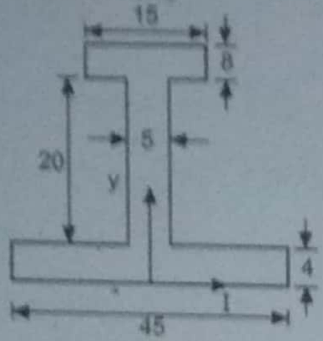


Fig. 10

Ans. : $A_1 = 15 \times 8 = 120, y_1 = 28, A_3 = 45 \times 4 = 180, y_3 = 2$

$A_2 = 20 \times 5 = 100, y_2 = 14$

$$\bar{Y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{A_1 + A_2 + A_3} = \frac{120 \times 28 + 100 \times 14 + 180 \times 3}{120 + 1100 + 180}$$

3. Solve the following.

(a) Obtain an expression for maximum height of a projectile projected from a horizontal plane. **[6]**

Ans. : Using equation of motion along y direction

$$v^2 = u^2 - 2.g.h$$

For maximum height v = 0

$$O = (u \sin 2)^2 - 2g\, h_{max}$$

$$h_{max} = \frac{u^2 \sin^2 \alpha}{2g}$$

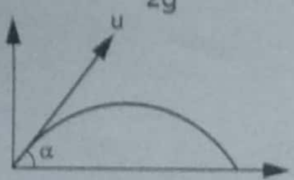


Fig. 11

(b) A stone is thrown vertically upwards, from the ground, with a velocity 49 m/s. After 2 seconds, another stone is thrown vertically upwards from the same place. If both the stone strike the ground at the same time, find the velocity, with which the second stone was thrown upwards. **[6]**

Ans. : Considering motion of first stone

Time required to attend max. height

$$v = u - gt$$

$$O = 49 - 9.81\ t$$

$$t = 4.99s$$

Total time of journey of first stone = $4.99 \times 2 = 9.98$ s.

Total time of journey of second stone = $9.98 \times 2 = 7.98$ s.

Time of upward journey of second stone = 7.98/2

= 3.995

Using

$$v = u - gt$$

$$0 = u - 9.81 \times 3.99$$

$$u = 39.14 \text{ m/s}$$

4. Solve the following.

(a) State and explain in brief D'Alembert's principle. **[6]**

Ans. : Please Refer Q. 6 (b) May 2018.

(b) A flywheel of mass 8 tonnes starts from rest, and gets up a speed if 180 r.p.m. in 3 minutes. Find the average torque exerted on it, if the radius of gyration of the flywheel is 60 cm. **[6]**

Ans. : Initial velocity of flywheel, $w_o = 0$

Final velocity of flywheel, $w_o = \frac{2\pi N}{60} = \frac{2\pi \times 180}{60} = 18.84$ m/s

Using equation of kinematics for angular motion

$w = w_o + \alpha t,\ 18.84 = 0 + \alpha \times 180,\ \alpha = 0.105 \text{ rad/s}^2$

Using equation of kinetics

$\Sigma m = I\alpha,\ T = (mk^2)\ \alpha = 8000 \times 0.6^2 \times 0.105 = 302.4$ N.m

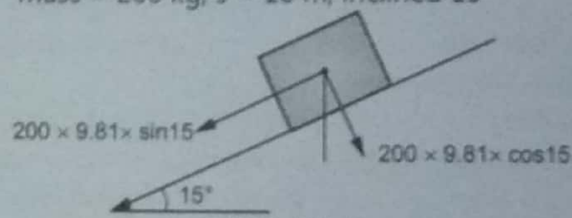
5. Solve the following.

(a) State and prove the law of conservation of energy. **[8]**

Ans. : **Law of Conservation of Energy :** "Energy that can neither be created not destroyed means it can only be transformed from one position to another". And also Refer Article 6.10 on Page No. 6.18.

(b) Calculate the work done in pulling up a block of mass 200 kg for 10 m on a smooth plane inclined at an angle of 15° with the horizontal. **[4]**

Ans. : Mass = 200 kg, s = 10 m, inclined 15°



FBD of block

Fig. 12

work done $= mg \sin 15 \times \rho$

$= 200 \times 9.81 \times \sin 15 \times 10$

$= 5078.03$ N.m or J = 5.078 kJ

OR

(b) A spring is stretched by 50 mm by the application of a force. Find the work done, if the force required to stretch 1 mm of the spring is 10 N. **[4]**

Ans. : Spring stretch = 50 mm

10N produce 1 mm stretch

Spring constant $k = \frac{10}{1} = 10$ N/mm

work done $= \frac{1}{2} \cdot K \cdot s^2 = \frac{1}{2} \times 10 \times 50^2$

$= 12500$ N.mm

$= 12.5$ Nm or Joule

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