UNIT III

ELECTRON OPTICS, NUCLEAR PHYSICS AND QUANTUM MECHANICS

3.1 INTRODUCTION TO ELECTRON OPTICS

- It is a known fact that **Cathode Rays** consist of electrons moving with a high speed. Electrons enter into the constitution of any kind of matter. Therefore, before commencing the study of any electronic device, it is imperative to understand the behaviour or motion of the electron under the action of electric and magnetic fields. The first part of the chapter is devoted to this.
- The properties of the electrons of being deflected by electric and magnetic fields and of producing scintillations on a fluorescent screen are made use of in the construction and action of a CRO and an electron microscope.
- The electron microscope has gained a place as an invaluable device to professionals dealing with the ultra small in a number of spheres. The second part of the chapter deals with these instruments and the principles of focusing of electrons required for the functioning of an electron microscope.
- **Microscopy** is now an invaluable tool for the study of the finer and smaller details of matter. A preliminary discussion of scanning electron microscopy and scanning tunneling microscopy is given here.
- The last part of the unit concentrates on positive rays and their analysis, which led to the discovery of isotopes with the help of mass spectroscopy. Details of the most elegant of the mass spectrographs, the Bainbridge mass spectrograph are given in this section.

3.2 MEASUREMENT OF 'e/m' BY THOMSON'S METHOD [Dec. 18]

In 1897, J. J. Thomson succeeded in determining the e/m ratio of electrons.

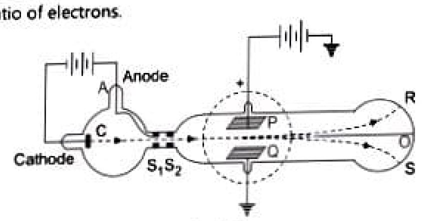

Fig. 3.1

1. **Description and Working :**

- Cathode rays are produced in the discharge tube, when a high potential difference is applied between the cathode C and the anode A.
- The rays then pass through two slits or metal diaphragms S_1 and S_2 maintained at anode potential. The purpose of the slits is to obtain a fine beam of electrons.
- The beam strikes the surface of the discharge tube normally at O. As it is coated with a fluorescent material, fluorescence is produced.
- P and Q are two plates between which electric field is produced by applying a suitable potential difference. This field is perpendicular to the plates and is directed from the positive plate to the negative.
- Due to the electric field, the electronic beam is deflected upwards (P being positive). Now, the fluorescent spot is obtained at R.
- By placing the tube between the pole pieces of a powerful electromagnet, a magnetic field is applied in a direction perpendicular to the plane of the plates and the direction of motion of electrons.
- The direction of the field is adjusted so that the electron beam is deflected downwards. The fluorescent spot is now obtained at S.
- The direction of deflection of electrons under the influence of a magnetic field is obtained by Fleming's left hand rule.

2. **Theory :**

- **Deflection of Electron Beam by Magnetic Field :**
- The electric and magnetic fields are firstly switched off. The position of the spot O on the screen for the undeflected beam is noted. The magnetic field is now applied in a direction perpendicular to the direction of motion of electrons.
- If 'B' is the intensity of the magnetic field, 'e' the charge of the electron, 'v' the velocity of the electron, then BeV is the magnetic force acting on the electron. This force is directed perpendicular to the motion of the electrons and magnetic field. (Fleming's law). Under the effect of

a constant uniform magnetic force, the electrons take up a circular path.

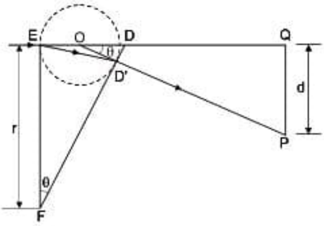

Fig. 3.2

- Beyond the region of influence of the magnetic field (dotted region), the electron beam emerges out in a straight line, tangential to the arc at the point of emergence as shown in Fig. 3.2.
- If r is the radius of the circular path, then the centripetal force acting on the electron beam is $\frac{mv^2}{r}$, where m is the mass of the electron and B the strength of the magnetic field. Under the influence of the magnetic field, the magnetic force supplies the centripetal force.

Hence, $$B e v = \frac{mv^2}{r}$$

or $$\frac{e}{m} = \frac{v}{Br} \qquad \text{... (3.1)}$$

- Thus, if r and v are known then the specific charge ratio (e/m) can be determined.
- **Calculation of r :** From Δ EDF, ∠ EDF = 90° − θ

As OQ is tangent to FD, hence ∠ OD'D = 90°

Thus from Δ ODD',

$$\angle DOD' = \theta$$

From Δ POQ, $$\tan\theta = \frac{PQ}{OQ} = \frac{d}{OQ} \qquad \text{... (i)}$$

From Δ EDF, $$\tan\theta = \frac{\text{arc } ED'}{r} \approx \frac{ED}{r} \qquad \text{... (ii)}$$

Hence, from (i) and (ii), we get

$$\frac{d}{OQ} = \frac{ED}{r}$$

or $$r = \frac{ED \times OQ}{d} \qquad \text{... (3.2)}$$

ED is the region of influence of the field. This is taken equal to the length of the plate P or Q in Fig. 3.1.

The value of OQ is given on the discharge tube, r can therefore be calculated from relation (3.2).

- **Determination of v :** Under the influence of crossed electric and magnetic fields, the beam strikes the screen at the same spot O as that of the undeflected beam. In such a case, the force on the electron due to the electric field (Ee) is balanced by the deflecting force (Bev) due to the magnetic field.

Hence, $$B e v = E e$$

i.e., $$v = \frac{E}{B} \qquad \text{... (3.3)}$$

Substituting values of v and r in (3.1), we have,

$$\frac{e}{m} = \frac{E}{B^2} \cdot \frac{d}{(ED \times OQ)} \qquad \text{... (3.4)}$$

Substituting values of E, d, B, ED and OQ, e/m can be calculated. Value of e/m of an electron is 1.7589×10^{11} coulomb/kg.

3. **Results :**

- Value of e/m is found to be a constant, independent of the material of the cathode and the nature of the gas in the tube. So, nature of particles in the cathode rays are of the same kind irrespective of their origin and velocity acquired.
- Comparing the value of e/m of the electron with that of e/m of the hydrogen ion, it is seen that the first is 1840 times that of the latter. This implies that either charge of electron is 1840 times charge of hydrogen ion, mass being same or mass of electron is $\frac{1}{1840}$ of mass of hydrogen ion, charge being the same. To ensure this, e is measured separately.

3.3 DETERMINATION OF ELECTRONIC CHARGE BY MILLIKAN'S OIL DROP METHOD

- The set up was developed by American scientist R. A. Millikan in 1917 to measure charge of an electron. The Fig. 3.3 shows the set-up of the instrument.

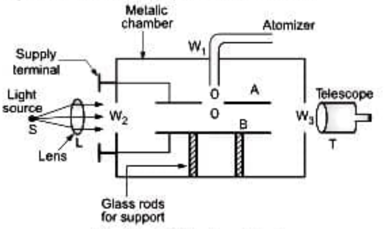

Fig. 3.3 : Millikan's method

- **Set-up**
  - A and B are plane metallic disc of about 20 cm in diameter placed at a distance pf 1.6 cm. The discs are clamped by insulating rods of glass or ebonite so that they remain perfectly parallel to each other.
  - The discs are placed in metallic chamber provided with three windows W_1, W_2 and W_3. A variable voltage is applied between the plates A and B. The upper plate A is provided with a small hole through which small droplets of a heavy non-violate oil can be introduced between two plates through a spring atomizer.
  - The droplets come slowly which gets charged due to friction in the spray process. The space between two plates is illuminated by high intensity light beam. This illuminates the oil droplets which can be seen by a telescope which is connected to a scale for measurement.
  - When illuminated the oil drops appear as brilliant spots on a dark background.

Procedure :

- The experiment is performed in two stages. In first stage the experiment is performed in the absence of electric field between the plates. When oil is dropped, the oil drop moves down under the influence of gravitational force. Due to air friction, soon electron will reach terminal velocity. i.e. its velocity will not increase further.
- At this stage the net downward gravitational force equals the force offered by air resistance. If 'ρ' is the density of oil, 'g' is acceleration due to gravity and 'a' the radius of the oil drop then,

Weight, $$W = \left(\frac{4}{3}\pi a^3\right)\rho h \qquad \text{... (3.5)}$$

- By Archimede's principle the upward thrust experience by oil drop due to displaced air is,

$$T = \left(\frac{4}{3}\pi a^3\right)\sigma g \qquad \text{... (3.6)}$$

where, 'σ' is density of air.

- If a spherical body of radius a falls under gravity in fluid having a coefficient of viscosity 'η', then by Stroke's law the resistive force due to medium

$$f = \sigma \pi \eta a v_1 \qquad \text{... (3.7)}$$

where, v_1 is the velocity of the drop.

- When the equilibrium is reached and the velocity of the drop becomes uniform the resultant force on the drop is zero, that is.

$$W = T + f$$

i.e. $$\frac{4}{3}\pi a^3 (\rho - \sigma) g = \sigma \pi \eta a v_1 \qquad \text{... (3.8)}$$

- In the second part of the experiment, an electric field E is applied between the plates. Now an additional upward electric force qE acts on the oil drop. Due to which the terminal velocity reduces to v_1. Therefore, the equation of motion for drop becomes.

$$\frac{4}{3}\pi a^3 (\rho - \sigma) g - qE = \sigma \pi \eta a v_2 \qquad \text{... (3.9)}$$

where, 'q' is charge on oil drop.

From equation (3.8) and (3.9),

$$qE = \sigma \pi \eta a (v_1 - v_2)$$

or $$q = \frac{\sigma \pi \eta a}{E}(v_1 - v_2) \qquad \text{... (3.10)}$$

Again from equation (3.8),

$$a^2 = \frac{g \eta v_1}{2 g (\rho - \sigma)}$$

or $$a = \left[\frac{g \eta v_1}{2 g (\rho - \sigma)}\right]^{1/2} \qquad \text{... (3.11)}$$

Substituting value of a in equation (3.10),

$$q = \frac{\sigma \pi \eta}{E}\left[\frac{g \eta v_1}{2 g (\rho - \sigma)}\right]^{1/2}(v_1 - v_2) \qquad \text{... (3.12)}$$

- The value of charge of an electron as worked out by Millikan's is 1.59×10^{-19} C., which agrees with recent experimental values.

3.4 MASS SPECTROGRAPH

- Isotopes are elements having the same atomic number but different atomic weights.

  For Example : ${}_1H^1$ and ${}_1D^2$; ${}_{92}U^{233}$, ${}_{92}U^{235}$ and ${}_{92}U^{238}$; ${}_{10}Ne^{20}$ and ${}_{10}Ne^{22}$
- As the atomic number of isotopes is the same, they have the same electronic configuration and hence the same chemical properties. They can therefore be separated by physical methods and not by chemical methods.
- To detect the presence of isotopes, to find an accurate value of isotopic masses and their abundance, F.W. Aston, an English Physicist, devised a mass spectrograph. This spectrograph brought about a

separation between the isotopes on the basis of their masses. This was followed by Dempster's which has recently been superseded by Bainbridge's magnetic deflection mass spectrograph.

3.5 BAINBRIDGE MASS SPECTROGRAPH

[Dec. 17, May 19]

- Bainbridge used a power electromagnet and a velocity selector in his spectrograph and was able to obtain, a high resolving power, precise symmetric images and a linear mass scale which could not be obtained in Aston's or Dempster's spectrographs.

Principle :

- Whatever be the velocities of the ions in the process of their generation, they are made perfectly homogeneous in velocity by the use of a special device called the **Velocity Selector**. They are then subjected to an extensive, transverse magnetic field and are brought to focus on a photographic plate.

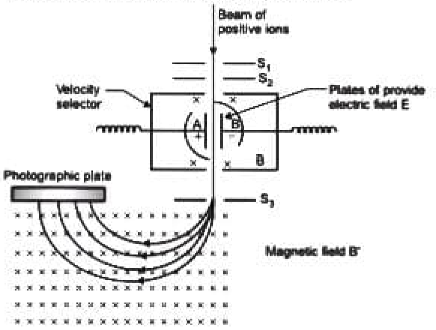

Fig. 3.4 : Bainbridge spectrograph

Apparatus :

- The given beam of ions is collimated by two narrow parallel slits S_1 and S_2. It is then passed through a velocity selector, which consists of a transverse electric field E which is produced by maintaining plates A and B at a suitable p.d.
- Simultaneously a magnetic field B is applied perpendicular to both E and the motion of the ions. The magnetic field is obtained by an electromagnet represented by the dotted circle.
- The velocity selector allows only those ions to pass undeviated which possess the same velocity 'v' given by the following relation.
- Inside the selector,

$$\text{Electric force} = \text{Magnetic force}$$

$$E q = B q v$$

(q is the charge of the ion moving with a velocity v).

$$\therefore \quad v = \frac{E}{B} \qquad \text{... (3.13)}$$

- All other ions bend away the straight path due to the unbalanced effect of one of the two opposing fields.
- The ions emerging from slit S_3, at the exit of the selector, are introduced into a uniform magnetic field of intensity B' acting at right angles to the plane of the paper.
- Under the influence of this field they travel along circular paths which are governed by the following relation.

Magnetic force = Centripetal force

$$\text{i.e.,} \quad B' q v = \frac{m v^2}{r} \qquad \text{... (3.14)}$$

m is the mass of the ion whose circular path has a radius r.

From (3.13) and (3.14), we get

$$B' q = \frac{m}{r}\frac{E}{B}$$

$$\frac{q}{m} = \frac{E}{B \cdot B'} \cdot \frac{1}{r} \qquad \text{... (3.15)}$$

- As E, B, B' are constants, q/m (specific charge ratio) is directly proportional to 1/r or $m \propto r$ and the mass scale is linear. Hence, after describing semi-circles, if the ions are made to fall on a photographic plate, they will strike it at different points depending on the value of mass. Lighter particles will trace small semi-circles while heavier ones will trace larger semi-circles. Traces are obtained on the photographic plate with the mass scale being linear.
- Presence of isotopes is therefore detected by the production of spots on the photographic plate. Mass numbers of the isotopes can be found by comparing the plate with a standard calibrated plate. Relative abundance of the isotopes in a given beam of ions can be found by studying the relative intensity of the spots on the photographic plate.

Hence a Bainbridge mass spectrograph is used :

- To detect the presence of isotopes in a given beam of positive ions,
- To determine the mass number of the isotopes,
- To find out the relative abundance of the isotopes in the given beam of positive ions.

SOLVED PROBLEMS

***Problem 3.1** : In a Bainbridge mass spectrometer, if the magnetic field in the velocity selector is 1 wb/m² and ions having a velocity of 0.4 × 10⁷ m/sec pass undeflected, find the electric field in the velocity selector.*

Data : $v = 0.4 \times 10^7$ m/sec., $B = 1$ wb/m²

Formula : $E = v \cdot B$

Solution : $E = 0.4 \times 10^7 \times 1 = \boxed{4 \times 10^6 \text{ V/m}}$

***Problem 3.2** : Singly ionised magnesium atoms enter a Bainbridge mass spectrograph with a velocity of 3 × 10⁵ m/sec. Calculate the radii of the paths followed by the three most abundant isotopes of masses 24, 25, 26 when the magnetic flux density is 0.5 wb/m².*

Data : $v = 3 \times 10^5$ m/sec., $B = 0.5$ wb/m², $q = 1.6 \times 10^{-19}$

Formula : $R = \dfrac{mv}{Bq}$

Solution : Mass of single ionised atom of Mg i.e.

$$m_{24} = \frac{24}{6.02 \times 10^{26}} \text{ kg} = 3.987 \times 10^{-26} \text{ kg}$$

$$R_{24} = \frac{M_{24} \cdot v}{B \cdot q} = \frac{3.987 \times 10^{-26} \times 3 \times 10^5}{0.5 \times 1.6 \times 10^{-19}}$$

$$= 14.95 \times 10^{-2} = 0.1495 \text{ m}$$

As $R \propto m$

$\therefore \quad \dfrac{R_{24}}{R_{25}} = \dfrac{m_{24}}{m_{25}}$

i.e., $R_{25} = \dfrac{m_{25}}{m_{24}} \cdot R_{24}$

$$= \frac{25}{24} \times 0.1495 = \boxed{0.1557 \text{ m}}$$

Similarly, $R_{26} = \dfrac{26}{24} \times 0.1495 = \boxed{0.1619 \text{ m}}$

***Problem 3.3** : A mixture of neon isotopes (Ne²⁰ and Ne²¹) is analysed using a Bainbridge mass spectrometer. Calculate the linear separation of isotopes when the field acting on the velocity selector is 80 kV/meter and the magnetic flux density is 0.55 weber/m².*

Data : $E = 80$ kV/meter; $B = 0.55$ weber/m²

Formulae : $v = \dfrac{E}{B}$, $R = \dfrac{mv}{Bq}$

Solution : $R = \dfrac{m\,(E/B)}{B\,q} = \dfrac{m\,E}{q\,B^2}$

$$m_{20} = \frac{20}{6.025 \times 10^{26}} \text{ kg}$$

$$= 3.3195 \times 10^{-26} \text{ kg}$$

$$R_{20} = \frac{3.3195 \times 10^{-26}}{1.6 \times 10^{-19}} \times \frac{80 \times 10^3}{(0.55)^2}$$

$$= 548.678 \times 10^{-4}$$

$$= 0.0549 \text{ m}$$

As $R \propto m$

$$\frac{R_{21}}{R_{20}} = \frac{m_{21}}{m_{20}}$$

$\therefore \quad R_{21} = \dfrac{m_{21}}{m_{20}} \cdot R_{20}$

$\therefore \quad R_{21} - R_{20} = \dfrac{m_{21}}{m_{20}} \cdot R_{20} - R_{20}$

$$= \left(\frac{m_{21}}{m_{20}} - 1\right) R_{20}$$

$$= \left(\frac{21}{20} - 1\right) \times 0.0549 = \frac{0.0549}{20}$$

$$R_{21} - R_{20} = 0.00275 \text{ m}$$

The separation on the photographic plate is double that of the radii difference.

∴ Linear separation of isotopes on the photographic plate

$$= 2 \times 0.00275 = \boxed{0.0055 \text{ m}}$$

3.6 INTRODUCTION TO NUCLEAR PHYSICS

- Rutherford, from his experiment on scattering of α-particles, suggested that an atom consists of a central nucleus surrounded by extra-nuclear electrons.
- The nucleus is positively charged and this charge is due to the protons present in the nucleus. The number of protons in the nucleus of an atom gives the atomic number 'Z' of the element to which the atom belongs.
- The nucleus also contains neutrons, which are electrically neutral. The number of neutrons in the nucleus is given by A – Z, where, 'A' is the mass number of the element.
- Nuclear size is about 10^{-15} m and the atomic size is about 10^{-11} m.
- An atom is electrically neutral, so the number of extra-nuclear electrons in an atom is always equal to the number of protons in the nucleus.
- A nucleus of an element is characterized by the mass number 'A' and the atomic number 'Z' of the element.
- Nuclei of mass number upto 25 are called light nuclei. Nuclei of mass number between 25 and 85 are called intermediate nuclei and those of mass number above 85 are called as heavy nuclei.

- For elements of low mass number, the number of protons is nearly equal to the number of neutrons in the nucleus. e.g. for sodium, Z = 11, A = 23. So number of protons in sodium nucleus is 11 and number of neutrons is 12.
- Hence the neutron-proton ratio (n : p) for sodium is nearly equal to unity. Such nuclei, for which n : p ratio is nearly unity, are called stable nuclei. Light nuclei are stable. But with increase of mass number, the number of neutrons exceeds that of protons and n : p ratio exceeds unity. Such nuclei, for which n : p ratio exceeds 1.5, are called unstable nuclei.
- Heavy nuclei are unstable. e.g. for ${}_{92}U^{238}$, A = 238, Z = 92. Hence, n : p ratio is more than 1.5. So uranium is unstable. It exhibits radioactivity and disintegrates till stable end products are formed.
- Henry Becquerel discovered that uranium gave out some type of radiations that could affect a photographic plate wrapped in a thick black paper. It was found that these radiations are highly penetrating, they ionize gases and cause scintillations on a fluorescent screen.
- The substances which emit these radiations are said to be radioactive. e.g. uranium, radium, polonium, radon, etc. The phenomenon of spontaneous emission of radiation from a substance is called as **Radioactivity**. All naturally occurring elements with atomic numbers greater than 82 are found to be radioactive because their $\frac{n}{p}$ ratio exceeds 1.5.
- The nuclear mass is the weight of the nucleus and it is equal to the sum of the masses of the neutrons and protons present in the nucleus.
- As the size of the nucleus is extremely small and its mass is very large, the nuclear density is enormously high, about 10^{17} kg/m^3.
- As mass of electron is negligible, the whole mass of an atom can be taken to be concentrated in its nucleus.
- The ordinary chemical and physical properties of elements are to be attributed to peripheral electrons in their atoms. An atom can be singly ionized by removing one electron and it will then have one excess positive charge.
- When all the electrons of an atom are removed, the bare nucleus will be left behind with only the positive charge, and even then the atom still retains its individuality and intrinsic nature.
- When the nucleus itself is tampered with and its constituent particles are altered in kind and manner, the original atom ceases to exist giving birth to a new one. This phenomenon of conversion of one element into another is known as disintegration or transmutation of elements.
- When transmutation is provoked by artificial means, it is called as artificial transmutation. It seemed possible that if atoms were bombarded with energetic particles, one of the latter might penetrate into a nucleus and cause transmutation.
- Alpha particles from natural radio nuclides were found to be effective for causing transmutation because of their relatively large energy and momentum. To reduce the probability of scattering of bombarding alpha particles and to increase the probability of disintegration, lighter elements were used as targets.
- The first artificial transmutation reaction observed by Rutherford was, when nitrogen was bombarded with α-particles. This transmutation can be represented as,

$${}_{7}N^{14} + {}_{2}He^{4} \rightarrow [{}_{9}F^{18}] \rightarrow {}_{8}O^{17} + {}_{1}H^{1}$$

Some of the artificially transmuted elements were found to be radioactive.

$$\text{e.g.}\ {}_{13}Al^{27} + {}_{2}He^{4} \rightarrow [{}_{15}P^{31}] \rightarrow {}_{15}\overset{*}{P}{}^{30} + {}_{0}n^{1}$$

${}_{15}\overset{*}{P}{}^{30}$ is radioactive and it disintegrates as

$${}_{15}\overset{*}{P}{}^{30} \rightarrow {}_{14}Si^{30} + {}_{1}e^{0} + \upsilon \text{ (neutrino)}$$

(positron)

Such reactions led to the discovery of artificial radioactivity.

3.7 PARTICLE DETECTOR – G.M. COUNTER

- A radioactive material keeps on radiating particles like α, β and γ, which cannot be sensed by humans directly. But their presence affects humans directly as well as indirectly, so their measurement becomes essential.
- For detecting and measuring intensity of these radiations some indirect method must be employed.
- When these radiations are passed through gases they have ability to ionise them. This property of the radiations can be employed to detect them.
- The commonly used radiation detectors are ionisation chamber, proportion counter, Geiger-Muller (G.M.) counter, cloud chamber etc.
- A G.M. counter uses a glass tube called G.M. tube along with electrical circuits, needed to amplify the current and display it. The tube consists of a rugged metal case enclosed in the glass tube.

- The hollow metal case acts as cathode. A fine metal wire passing through the centre of the tube acts as anode. The tube is evacuated and then filled with mixture of Argon (90%) at 10 cm pressure and ethyl alcohol vapour (10%) at 1 cm pressure. One end of the tube is enclosed with a thin sheet of mica which serves as window for radiations.
- A d.c. potential difference of about 1200 V is applied between metal case (cathode) and the wire (anode). The voltage is adjusted below the break down voltage of the gas. Fig. 3.5 shows schematic arrangement of G-M counter.

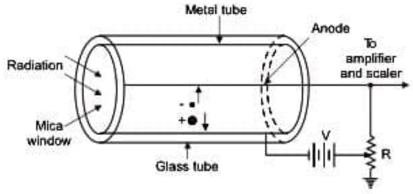

Fig. 3.5 : G-M counter

- When a high energy particle enters the G-M tube it inoises one or more argon atoms. The electron will be attracted by anode wire whereas positive ions will be attracted by negative of the supply. These moving charges further ionises the argon atoms.
- As a result of this a current pulse will pass. This current pulse through resistance R produces a voltage pulse of the order of 10 μV. An electronic pulse amplifier amplifies this weak voltage pulse to a voltage value between 5 to 50 V.
- These amplified pulses are applied to a counter. As each incoming radiation produces a pulse of current, the number of incoming radiations can be counted.
- The Fig. 3.6 shows a graph of counts per minute as a function of voltage for voltages less than 1000 V. There is no discharge and hence will show zero count. Between 1000 V to 1200 V the number of pulses are proportional to the voltage.
- Above 1200 V the number of counts remain constant for certain range of voltage. This is known as **Plateau Region**. The plateau region is used for normal operation of G-M counter.

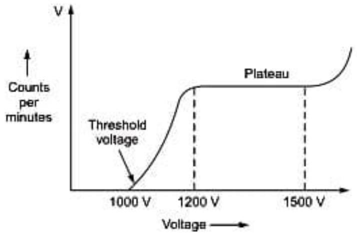

Fig. 3.6 : Characteristics of G-M counter

- Ethyl alcohol vapour is used to prevent undesirable avalanching. A G-M counter can count upto 500 particles per second.
- The main drawback of G-M counter is the **Dead Time**, the time taken by the tube to recover between counts.
- The **Dead Time** of G-M counter is about 200 μs. If the radiation density is high, the tube will not have time to recover and hence some of the particles may not be counted.
- The G-M counter can be used to count β and γ-radiations and α-particles with some modifications in the tube.

3.8 INTRODUCTION TO QUANTUM MECHANICS

- The most outstanding development in modern science is the conception of quantum mechanics. The quantum mechanics is better than Newtonian classical mechanics in explaining the fundamental physics. There was big development in physics between the time of Newton and the time of quantum mechanics.
- Newton showed that the motion of planets and the free fall of an object on earth is governed by the same law. Thus, he unified terrestrial and celestial mechanics. This was in contrast to ancient belief that the world of the earth and heaven is governed by different laws.
- It was earlier believed that the heat is some peculiar substance called **Caloric**, which flows from a hot object to a cold object. But latter it was proved that the heat is the random motion or vibration of constituents of matter. Thus, thermodynamics and mechanics were unified.
- For a long time, the phenomena of electricity, magnetism and light were treated as independent branches and were unconnected. But in nineteenth century, Faraday and Maxwell along with others unified

these independent branches of physics. They proved that all three phenomena are manifestations of electromagnetic field.

- The simplest example is the electric field of an electric charge that exerts a force an another charge when it comes in the range. An electric current produces a magnetic field that exerts a force on magnetic materials.
- Such fields can travel through space, independent of charge and magnet, in the form of electromagnetic wave. The best example of electromagnetic wave is light. Finally, Einstein unified space, time and gravity in his theory of relativity.
- Quantum mechanics also unified two branches of science : physics and chemistry.
- In previous developments in physics, fundamental concepts were not different from those of everyday experience, such as particle, position, speed, mass, force, energy and even field. These concepts are referred as **Classical**.
- The world of atoms cannot be described and understood with these concepts. For atoms and molecules, the ideas and concepts used in dealing with objects in day to day life is not sufficient. Thus, it needed new concepts to understand the properties of atoms.
- A group of scientists W. Heisenberg, E. Schrodinger, P.A.M. Dirac, W. Pauli, M. Born and Neils Bohr, conceived and formulated these new ideas in the beginning of 20^{th} century. This new formulation, a branch of physics, was named as **Quantum Mechanics**.

3.8.1 Limitations of Classical Mechanics

- The classical physics is complete and beautiful in explaining daily experiences where big bodies are involved. But it breaks down severely at subatomic level and failed to explain some of the phenomenon totally.
- The phenomena which classical physics failed to explain are black body radiation, photoelectric effect, emission of X-rays, etc.
- In classical physics, a body which is very small in comparison with other body is termed as **Particle**. Whereas in quantum mechanics, the body which cannot be divided further is termed as **Particle**.
- The other main difference is the quantized energy state. In classical physics, an oscillating body can assume any possible energy. On the contrary, quantum mechanics says that it can have only discrete non-zero energy.

3.8.2 Need of Quantum Mechanics

- Classical mechanics successfully explained the motions of object which are observable directly or by instruments like microscope. But when classical mechanics is applied to the particles of atomic levels, it fails to explain actual behaviour. Therefore, the classical mechanics cannot be applied to atomic level, e.g. motion of an electron in an atom.
- Other phenomena which classical mechanics failed to explain are black body radiation, photoelectric effect, emission of X-rays, etc.
- The above problems were solved by Max Planck in 1900 by the introduction of the formula

$$E = nh\nu$$

where, $n = 0, 1, 2, \ldots,$

h = Planck's constant = 6.63×10^{-34} J/s

- This is known as **Quantum Hypothesis** and marked the beginning of modern physics. The whole microscopic world obeys the above formula.

3.9 HEISENBERG'S UNCERTAINTY PRINCIPLE

[May 18]

- One of the tacit assumptions of classical physics, that the position of a mechanical system can be uniquely determined without disturbing its motion, is valid only for the motion of a body of ordinary size, like a cricket ball.
- But if one is considering the motion of an atomic particle, like an electron, a certain uncertainty is unavoidably introduced into the experimental measurement of its position and momentum.
- This uncertainty is not due to the imperfection of the measuring instruments but is something inherent in the nature of a moving body.
- The fact that a moving body must be regarded as a De Broglie wave group (packet) rather than as a localised entity suggests that, there is a fundamental limit to the accuracy with which we can measure its particle properties.

- A De Broglie wave group is shown in Fig. 3.7 (a). The particle may be anywhere within the group. For a very narrow wave group, as in Fig. 3.7 (b), the position of the particle can be readily found, but the wavelength λ, and hence the momentum $p = \frac{h}{\lambda}$, is impossible to establish.
- For a wide wave group, as in Fig. 3.7 (c), the wavelength and hence momentum estimate is satisfactory, but then the location of the position of the particle becomes uncertain.

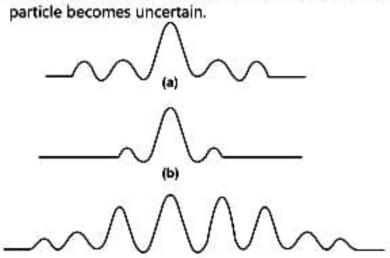

Fig. 3.7

- The answer to this question was given by Heisenberg in 1927, when he put forth the uncertainty principle.

Statement :

- Heisenberg's uncertainty principle states that, it is impossible to determine accurately and simultaneously the values of both the members of a pair of physical variables which describe the motion of an atomic system. Such pairs of variables, like position x and momentum p; or energy E and time t, are called canonically conjugate variables.
- To examine the uncertainty principle, consider an electron of mass m associated with matter waves of wavelength λ. This electron can be found somewhere within this wave and therefore, the uncertainty in its position measurement Δx is equal to its wavelength λ.

$$\therefore \quad \Delta x = \lambda \qquad \text{... (3.16)}$$

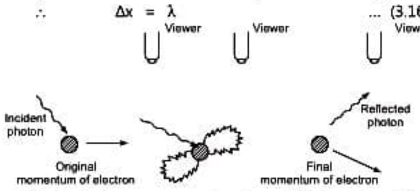

Fig. 3.8 : An electron cannot be observed without changing its momentum by an indeterminate amount

- To observe the electron we have to illuminate it with light, say of wavelength λ, as in Fig. 3.8. In this process, photons of light strike the electron and bounce off it. Each photon possesses the momentum $\frac{h}{\lambda}$ and when it collides with the electron, original momentum p of the electron is changed.
- The precise change of the momentum of the electron cannot be predicted, but it is likely to be of the same order of magnitude as the photon momentum $\frac{h}{\lambda}$. Thus, the electron cannot be observed without changing its momentum by an indeterminate amount.
- The act of measurement of its position introduces an uncertainty in its momentum Δp and this uncertainty in the momentum is at least equal to the momentum of incident photon.

$$\therefore \quad \Delta p = \frac{h}{\lambda} \qquad \text{... (3.17)}$$

From equations (3.16) and (3.17),

$$\Delta x \cdot \Delta p = \lambda \cdot \frac{h}{\lambda} = h \qquad \text{... (3.18)}$$

- It is clear from equation (3.18) that if the position of the electron is known exactly at any given instant, i.e. if $\Delta x = 0$, then the momentum becomes indeterminate and vice-versa. Thus, both the position and the momentum cannot be determined accurately and simultaneously.
- The product of uncertainty in position measurement Δx of a body at some instant and the uncertainty in its momentum measurement Δp at the same instant is at best equal to the Planck's constant h (more correctly $\frac{h}{4\pi}$).

i.e. $\quad \Delta x \cdot \Delta p \geq \frac{h}{4\pi}$

This is Heisenberg's uncertainty principle.

3.9.1 Uncertainty Principle Applied to the Pair of Variables

Energy and Time

- Kinetic energy and time form another pair of canonically conjugate variables. Consider again the Problem of an electron of mass m moving with velocity v. We can write the K.E. of the electron as

$$E = \frac{1}{2} mv^2 \qquad \text{... (3.19)}$$

- The uncertainty in the energy measurement ΔE can be found by differentiating equation (3.19), assuming mass to be constant.

$$\therefore \quad \Delta E = \frac{1}{2} m \cdot 2 v \Delta v$$

$$\Delta E = v (m \cdot \Delta v)$$

$$\Delta E = v \cdot \Delta p$$

$$\Delta E = \frac{\Delta x}{\Delta t} \cdot \Delta p \qquad (\because v = \frac{\Delta x}{\Delta t})$$

Hence $\Delta E \cdot \Delta t = \Delta x \cdot \Delta p$

- But by Heisenberg's uncertainty principle,

$$\Delta x \cdot \Delta p \geq \frac{h}{4\pi}$$

$$\therefore \quad \Delta E \cdot \Delta t \geq \frac{h}{4\pi}$$

- This means that the product of uncertainties in energy and time measurements is of the order of Planck's constant.

3.9.2 Illustration of Uncertainty Principle [May 19]

(1) Diffraction at a Single Slit

- Consider a narrow beam of electrons passing normally through a single vertical narrow slit of width Δy and producing a diffraction pattern on the screen. (See Fig. 3.9)

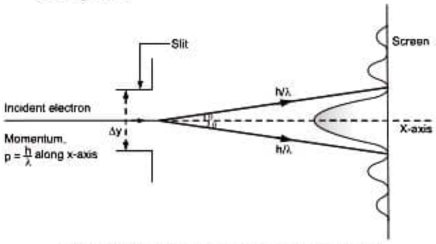

Fig. 3.9 : Diffraction of electrons at a single slit

- We know that the positions of minima in the diffraction pattern due to a slit of width a formed by incident light of wavelength λ are given by

$$a \sin\theta = n\lambda \qquad \text{... (3.20)}$$

where θ is the angle of deviation of n^{th} order minimum in the diffraction pattern.

- So if the first order minimum in the diffraction pattern due to a slit of width Δy is formed for an angle θ, when electron waves of wavelength λ are diffracted by it, we shall have

$$\Delta y \sin\theta = 1 \cdot \lambda \qquad \text{... (3.21)}$$

- All the electrons producing the diffraction pattern on the screen have passed through the slit, but we cannot say definitely at what position of the slit. So the uncertainty in position determination of electrons is equal to the width Δy of the slit, and from equation (3.21), we have

$$\Delta y = \frac{\lambda}{\sin\theta} \qquad \text{... (3.22)}$$

- Electrons are initially moving along positive x-axis. Their momentum along x-axis is $\frac{h}{\lambda}$ and they do not have any component of momentum along y-axis.
- But after diffraction at the slit, electrons are deviated from their initial path to form the diffraction pattern, and the y-component of their momentum may be between $\frac{h}{\lambda}\sin\theta$ and $-\frac{h}{\lambda}\sin\theta$ (See Fig. 3.9).
- So the uncertainty in momentum measurement along y - direction is given by

$$\Delta p_y = \frac{h}{\lambda}\sin\theta - \left(-\frac{h}{\lambda}\sin\theta\right)$$

$$\Delta p_y = \frac{2h}{\lambda}\sin\theta \qquad \text{... (3.23)}$$

From equations (3.21) and (3.22), we have

$$\Delta y \cdot \Delta p_y = \frac{\lambda}{\sin\theta} \cdot \frac{2h}{\lambda}\sin\theta = 2h$$

i.e. $\Delta y \cdot \Delta p_y \geq h$

- Thus, the product of uncertainties in position and momentum measurements of the electron is of the order of Planck's constant, which is Heisenberg's uncertainty principle.

(2) Why an Electron cannot Exist in the Nucleus [May 18]

- If the electrons had to exist inside the nucleus then its De-Broglie wavelength should be roughly of the order of nucleus diameter i.e. 10^{-14} m.

Therefore, the corresponding momentum will be

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{10^{-14}} = 6.63 \times 10^{-20} \text{ kg-m/sec}$$

$$\therefore \quad E = \frac{p^2}{2m} = \frac{(6.63 \times 10^{-34})^2}{2 \times 9.1 \times 10^{-31}} = 2.42 \times 10^{-9} \text{ J}$$

$$= \frac{2.42 \times 10^{-9}}{1.6 \times 10^{-19}} \text{ eV} = 15095 \text{ MeV}$$

- If the electron had to exist in the nucleus then its energy should be 15095 MeV. However, this is greater than the maximum binding energy of the nucleus. Thus, the electron cannot exist inside the nucleus.

***Problem 3.4 :** In an experiment, the wavelength of a photon is measured to an accuracy of one part per million. What is the uncertainty Δx in a simultaneous measurement of the position of the photon having a wavelength of 6000 A° ?*

Data : $\lambda = 6000\ A° = 6000 \times 10^{-10}$ m, $h = 6.6 \times 10^{-34}$ J-sec

$$\frac{\Delta\lambda}{\lambda} = \frac{1}{10^6}$$

Formulae : $\Delta p = \frac{h}{\Delta\lambda}, \quad \Delta x \cdot \Delta p \approx h$

Solution :

$$\Delta p = \frac{6.6 \times 10^{-34}}{6000 \times 10^{-16}}$$

$$= 1.1 \times 10^{-21} \text{ kg-m/sec}$$

$$\Delta x = \frac{h}{\Delta p} = \frac{6.6 \times 10^{-34}}{1.1 \times 10^{-21}} = \boxed{6 \times 10^{-13} \text{ m}}$$

***Problem 3.5 :** In order to locate the electron in an atom within a distance of 5×10^{-12} m using electromagnetic waves, the wavelength must be of the same order. Calculate the energy and momentum of the photon. What is the corresponding uncertainty in its momentum ?*

Data : $\lambda = \Delta x = 5 \times 10^{-12}$ m

Formulae : $p = \frac{h}{\lambda}, \quad E = \frac{hc}{\lambda}, \quad \Delta p_x = \frac{h}{\Delta x}$

Solution :

$$p = \frac{6.6 \times 10^{-34}}{5 \times 10^{-12}} = 1.32 \times 10^{-22} \text{ kg-m/sec}$$

$$E = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{5 \times 10^{-12}}$$

$$= 3.96 \times 10^{-14} \text{ J}$$

$$\Delta p_x = \frac{6.6 \times 10^{-34}}{5 \times 10^{-12}}$$

$$= \boxed{1.32 \times 10^{-22} \text{ kg-m/sec}}$$

***Problem 3.6 :** Compute the uncertainty in the location of a 2 gram mass moving with a speed of 1.5 m/sec. and the minimum uncertainty in the location of an electron moving with a speed of 0.5×10^8 m/sec. Given, $\Delta p = 10^{-3}$ p.*

Data : $v_e = 0.5 \times 10^8$ m/sec, $\Delta p = 10^{-3}$ p, m = 2 grams

$= 2 \times 10^{-3}$ kg,

v for the body = 1.5 m/sec

Formula : $\Delta x\ \Delta p = h$

Solution :

(i) For the body,

$$\Delta p = 10^{-3}\ p = 10^{-3}\ (mv)$$

$$\Delta x \cdot \Delta p = h$$

$$\Delta x = \frac{h}{\Delta p} = \frac{6.6 \times 10^{-34}}{10^{-3} \times 2 \times 1.5 \times 10^{-3}}$$

$$= \boxed{2.2 \times 10^{-28} \text{ m}}$$

(ii) For the electron,

$$\Delta x = \frac{6.6 \times 10^{-34}}{10^{-3} \times 9.1 \times 10^{-31} \times 0.5 \times 10^8}$$

$$= \boxed{1.45 \times 10^{-8} \text{ m}}$$

It can be seen that the uncertainty associated with a microscopic body is very large and therefore, it plays a significant role in measurements.

***Problem 3.7 :** Assume that the uncertainty in the location of a particle is equal to its De Broglie wavelength. Show that the uncertainty in its velocity is equal to its velocity.*

Data : $\Delta x = \lambda$

Formula : $\Delta x \cdot \Delta p = h$

Solution : $\Delta x \cdot m\ \Delta v_x = h$

$$\Delta v_x = \frac{h}{\Delta x \cdot m} = \frac{h}{m\lambda}$$

Using $\lambda = \frac{h}{mv}$

we have $\Delta v_x = \frac{h\,m\,v}{mh} = v$

***Problem 3.8 :** An electron is confined to a box of length 1 A°. Calculate the minimum uncertainty in its velocity, given mass of electron = 9.1×10^{-31} kg, $h = 6.6 \times 10^{-34}$ J-sec.*

Data : $\Delta x = 1\ A° = 10^{-10}$ m, $h = 6.6 \times 10^{-34}$ J-sec, $m = 9.1 \times 10^{-31}$ kg

Formula : $\Delta x\ \Delta p_x = h$

Solution : $(\Delta x)_{max}\ (\Delta p_x)_{min} = h$

$$(\Delta x)_{max}\ (\Delta v)_{min} = h$$

$$(\Delta v)_{min} = \frac{h}{m\ \Delta x} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{-10}}$$

$$= \boxed{0.725 \times 10^7 \text{ m/sec.}}$$

This is comparable to the speed of the electron and is therefore very large.

***Problem 3.9 :** Calculate the minimum uncertainty in the velocity of an electron confined to a box of length 10 A°.*

Data : $L = 10\ A° = 10 \times 10^{-10}$ m

Formula : $\Delta x \cdot \Delta p_x = h$

Solution : $(\Delta x)_{max} (\Delta p_x)_{min} = h$

i.e. $(\Delta x)_{max}\, m(\Delta v_x)_{min} = h$

$$\Delta v_x = \frac{h}{m \cdot (\Delta x)_{max}} = \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31} \times 10^{-9}}$$

$$= \boxed{0.725 \times 10^6 \text{ m/sec.}}$$

***Problem 3.10 :** An electron has a speed of 600 m/sec with an accuracy of 0.005 %. Calculate the uncertainty with which we can locate the position of the electron.*

Data :

$$v = 600 \text{ m/sec}$$

$$\Delta v = 0.005 \text{ \% of } v$$

$$= \frac{0.005}{100} \times 600 \text{ m/sec}$$

Formula : $\Delta x \cdot \Delta p = h$

$$\Delta x = \frac{h}{\Delta p} = \frac{h}{m\,\Delta v}$$

$$= \frac{6.6 \times 10^{-34}}{9.1 \times 10^{-31}} \times \frac{0.005}{100} \times 600$$

$$= \boxed{0.024 \text{ m}}$$

3.10 PHYSICAL SIGNIFICANCE OF WAVE FUNCTION

3.10.1 Concept of Wave Function

- A wave motion appears in almost all branches of physics. A wave motion is defined as a periodic disturbance travelling with finite velocity through a medium or space.
- The simplest form of vibration is simple harmonic motion (S.H.M.) and a particle executing S.H.M. acts as a source which radiates waves.
- The wave motion provides a way for energy and momentum to move from one place to another without material particles making that journey.
- The waves can be classified according to their broad physical properties into mainly three categories :
  - Electromagnetic waves which need not require any medium to propagate.
  - Matter waves which give the probability amplitude of finding a particle at a given position and time.
  - **Mechanical Waves :** The mechanical waves are simplest one to understand because they are produced by some sort of mechanical vibrations which we can see.
- When a mechanical wave passes through a medium, the medium particles perform an S.H.M. given by equation

$$y = A \cos \omega t \qquad \text{... (3.24)}$$

where A is the amplitude of the oscillation and $\omega = 2\pi\upsilon$, where υ is the frequency.

  - This equation is applicable to all individual particles affected by the wave. Suppose the wave is progressing forward with velocity v. If P is the origin of the wave, then a particle at Q at a distance x from P will receive the wave x/v sec later than P did.

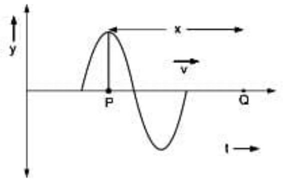

Fig. 3.10 : Progressive wave moving with velocity v

- Hence, its displacement at time t and distance x from the origin will be

$$y = A \cos \omega \left(t - \frac{x}{v}\right) \qquad \text{... (3.25)}$$

The wave equation of such a wave is

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2} \qquad \text{... (3.26)}$$

The solution of equation (3.26) is given by

$$y = Ae^{-i\omega(t - x/v)} \qquad \text{... (3.27)}$$

- In a string we can represent the wave disturbance by the transverse displacement of y. Similarly, for light waves the field vectors E and B vary in space and time, for sound waves, pressure P varies in space and time. In the same way, for matter waves, the wave function ψ varies in space and time.
- So ψ in wave mechanics is analogous to electric field E in electromagnetic waves or to pressure P in the sound waves. However, ψ itself unlike E and P has no direct physical significance, but gives a measure of the probability of finding a particle at a particular position. Hence, it is called **Probability Amplitude**.
- However, a probability is always real and positive, whereas ψ can be positive or negative. Therefore, ψ^2 is taken which is always positive. In general, ψ is complex,

therefore, one takes $|\psi|^2$ instead of ψ^2, where $|\psi|^2 = \psi^* \psi$, ψ^* denoting the complex conjugate of ψ. In any case, $\psi^* \psi$ is always real and positive.

- If dv is a volume element located at a point, then the probability of finding the particle in the volume element at time t is proportional to $\psi^*\psi dv$. By analogy with ordinary mass density, the square of the wave function $\psi^*\psi$ is called the **Probability Density** i.e. **Probability Per Unit Volume**.

3.10.2 Physical Significance of the Wave Function

- Schrodinger interpreted ψ in terms of charge density. If A is the amplitude of an electromagnetic wave, then the energy per unit volume, i.e. energy density is equal to A^2. Also, the photon energy $h\upsilon$ is constant. So the number of photons per unit volume, i.e. the photon density is equal to $\frac{A^2}{h\upsilon}$, and it is proportional to the amplitude square.
- Similarly, if ψ is the amplitude of matter waves at any point in space, the particle density at that point may be taken as proportional to $|\psi|^2$. So $|\psi|^2$ is a measure of particle density and on multiplying this by the charge of the particle, we shall get the charge density. Thus, $|\psi|^2$ is a measure of **Charge Density**.
- According to Max Born, the value of $|\psi|^2$ at a point at a given time is related to the probability of finding the body described by its wave function ψ at that point at that instant. A large value of $|\psi|^2$ means a strong possibility of the presence of the body, while a small value of $|\psi|^2$ means a slight possibility of its presence. As long as $|\psi|^2$ is not actually zero somewhere, there is a definite chance, however small, of detecting the body there.
- Although the wave function ψ of a particle is spread out in space, this does not mean that the particle itself is also thus spread out. When an experiment is performed to detect a particle, an electron for instance, a whole electron is either found at a certain place and time, or it is not. There is nothing like 20 % of an electron. However, it is certainly possible that there is 20 % chance that the electron be found at that place and time, and it's likelihood that is specified by $|\psi|^2$ or $\psi\psi^*$, ψ^* being the complex conjugate of ψ.
- $|\psi|^2$ or $\psi\psi^*$ is taken as the probability density, i.e. the probability of finding the particle in unit volume. So the probability of the particle being present in a volume element dx·dy·dz is $|\psi|^2$ dx dy dz. Then, the wave function ψ is called the **Probability Density Amplitude**.
- Since the particle is certainly to be found somewhere in space, we must have,

$$\iiint |\psi|^2 \, dx\, dy\, dz = 1 \qquad \text{... (3.28)}$$

the triple integral extending over all possible values of x, y, z.

- A function ψ satisfying this relation is called a **Normalised Wave Function** and equation (3.28) is known as the **Normalisation Condition**. Thus, ψ has to be a normalisable function.

  Besides being normalisable, ψ must also satisfy the following conditions :
  - Must be a single valued function, because ψ is related to the probability of finding the particle at a given place and time, and the probability can have only one value at a given point and time.
  - Must be finite, because the particle exists somewhere in space, and so integral over all space must be finite.
  - And its derivatives $\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}, \frac{\partial \psi}{\partial z}$ must be continuous everywhere in the region where ψ is defined.

3.11 SCHRODINGER'S WAVE EQUATION

- Schrodinger started with De Broglie's idea of matter waves and developed it into a mathematical theory known as **Wave Mechanics**. Schrodinger's wave equation is the mathematical representation of matter waves associated with a moving particle. There are two types of Schrodinger's wave equations :
  1. Schrodinger's time independent wave equation
  2. Schrodinger's time dependent wave equation.

3.11.1 Schrodinger's Time Independent Wave Equation [Dec. 17, 18, May 19]

- According to De Broglie's theory, a particle of mass m moving with a velocity v has a wave system of some kind associated with it, and its wavelength is given by $\lambda = \frac{h}{mv}$. The waves are produced only when something oscillates. Though we do not know the quantity that vibrates to produce the matter waves, but we can indicate that quantity by ψ.
- The periodic changes in ψ produce the wave system associated with the particle, just as the periodic changes in the displacement y of a string produce a wave system along the string.

- In quantum mechanics, ψ corresponds to the displacement y of wave motion in a string. However, ψ, unlike y, is not itself a measurable quantity and it may be complex.
- Consider a system of stationary waves associated with a particle. Let (x, y, z) be the coordinates of the particle and let ψ denote the wave displacement of matter waves at time t.
- By analogy with the wave equation

$$\frac{d^2y}{dt^2} = v^2 \frac{d^2y}{dx^2}$$

of a two-dimensional wave (in xy plane), the wave equation for a three-dimensional wave with wave velocity u can be written as

$$\frac{\partial^2\psi}{\partial t^2} = u^2\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right)$$

$$\frac{\partial^2\psi}{\partial t^2} = u^2 \nabla^2 \psi \qquad \text{... (3.29)}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the **Laplacian Operator**.

The solution of equation (3.29) is

$$\psi(x, y, z, t) = \psi_0 (x, y, z)\, e^{-i\omega t} \qquad \text{... (3.30)}$$

where ψ_0 (x, y, z) represents the amplitude of the wave at the point considered.

- The position vector of a point whose Cartesian coordinates are (x, y, z) is given by

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$\hat{i}, \hat{j}, \hat{k}$ being unit vectors along the axes. So equation (3.30) can be written as

$$\psi(\vec{r}, t) = \psi_0(\vec{r})\, e^{-i\omega t} \qquad \text{... (3.31)}$$

Differentiating equation (3.31) twice with respect to time t, we get

$$\frac{\partial\psi}{\partial t} = -i\omega\psi_0(r)\, e^{-i\omega t}$$

and

$$\frac{\partial^2\psi}{\partial t^2} = (-i\omega)^2 \psi_0(\vec{r})\, e^{-i\omega t}$$

$$= -\omega^2\psi \qquad \text{... (3.32)}$$

From equations (3.29) and (3.32), we get

$$u^2 \nabla^2 \psi = -\omega^2 \psi$$

$$\therefore \nabla^2\psi + \frac{\omega^2}{u^2}\psi = 0 \qquad \text{... (3.33)}$$

But $\omega = 2\pi\upsilon$, and $u = \upsilon\lambda$

$\therefore$ Equation (3.33) becomes

$$\nabla^2\psi + \frac{4\pi^2}{\lambda^2}\psi = 0 \qquad \text{... (3.34)}$$

- The De Broglie wavelength of the waves associated with the particle is given by

$$\lambda = \frac{h}{mv} = \frac{h}{p} \qquad \text{... (3.35)}$$

Substituting equation (3.35) in (3.36), we get

$$\nabla^2\psi + \frac{4\pi^2 p^2}{h^2}\psi = 0 \qquad \text{... (3.36)}$$

- The total energy E of the particle is the sum of it's K.E. $= \frac{1}{2}mv^2$ and potential energy V.

$$\therefore \quad E = \frac{1}{2}mv^2 + V$$

$$E = \frac{p^2}{2m} + V$$

This gives $p^2 = 2m(E - V)$ ... (3.37)

Substituting equation (3.37) in (3.36), we get

$$\nabla^2\psi + \frac{8\pi^2 m(E - V)}{h^2}\psi = 0 \qquad \text{... (3.38)}$$

- Equation (3.38) is called 'Schrodinger's time independent wave equation'.

Taking $\hbar = \frac{h}{2\pi}$, equation (3.38) becomes

$$\nabla^2\psi + \frac{2m(E - V)}{\hbar^2}\psi = 0 \qquad \text{... (3.39)}$$

3.11.2 Schrodinger's Time Dependent Wave Equation

- Schrodinger's time independent wave equation is

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0 \qquad \text{... (3.40)}$$

- The time dependent wave equation is obtained by eliminating E from the time independent equation. Consider a system of stationary waves associated with a particle. Let (x, y, z) be the coordinates of the particle and let ψ denote the wave displacement of the matter waves at time t. If u be the wave velocity, then the equation for a three-dimensional wave motion can be written as,

$$\frac{\partial^2\psi}{\partial t^2} = u^2\left(\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2}\right) = u^2\nabla^2\psi \qquad \text{... (3.41)}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ is the '**Laplacian Operator**'

The solution of equation (3.41) is

$$\psi(x, y, z, t) = \psi_0(x, y, z)\, e^{-i\omega t} = \psi_0(\vec{r})\, e^{-i\omega t} \qquad \text{... (3.42)}$$

where ψ_0 (x, y, z) is the amplitude of the wave at the point considered.

- Differentiating equation (3.42) with respect to time t, we get

$$\frac{\partial\psi}{\partial t} = (-i\omega)\psi_0(\vec{r})\, e^{-i\omega t} = -i\omega\psi \qquad \text{... (3.43)}$$

Now, $\omega = 2\pi\upsilon$ and $E = h\upsilon$ or $\upsilon = \frac{E}{h}$

$$\therefore \quad \omega = \frac{2\pi E}{h}$$

Putting this value of ω in equation (3.43), we get

$$\frac{\partial \psi}{\partial t} = -i\frac{2\pi E}{h}\psi \quad \text{... (3.44)}$$

Multiplying both sides of equation (3.44) by i,

$$i\frac{\partial \psi}{\partial t} = i^2\frac{2\pi}{h}E\psi$$

$$\therefore \quad E\psi = i\frac{h}{2\pi}\frac{\partial \psi}{\partial t} = i\hbar\frac{\partial \psi}{\partial t} \quad \text{... (3.45)}$$

From equations (3.40) and (3.45), we get

$$\nabla^2\psi + \frac{8\pi^2 m}{h^2}\left(\frac{ih}{2\pi}\frac{\partial \psi}{\partial t} - V\psi\right) = 0$$

Multiplying both sides of this equation by $\frac{-h^2}{8\pi^2 m}$, we get

$$-\frac{h^2}{8\pi^2 m}\nabla^2\psi - \frac{ih}{2\pi}\frac{\partial \psi}{\partial t} + V\psi = 0$$

$$\left(-\frac{h^2}{8\pi^2 m}\nabla^2 + V\right)\psi = \frac{ih}{2\pi}\frac{\partial \psi}{\partial t} \quad \text{... (3.46)}$$

$$\text{or} \left(\frac{-\hbar^2}{2m}\nabla^2 + V\right)\psi = i\hbar\frac{\partial \psi}{\partial t} \quad \text{... (3.47)}$$

Equation (3.47) is called 'Schrodinger's time dependent wave equation'.

Taking $H = \left(-\frac{h^2}{8\pi^2 m}\nabla^2 + V\right) = \left(\frac{-\hbar^2}{2m}\nabla^2 + V\right)$ as **Hamiltonian Operator**

and $E = \frac{ih}{2\pi}\frac{\partial}{\partial t} = i\hbar\frac{\partial}{\partial t}$ as **Eigen Operator**,

equation (3.47) becomes

$$H\psi = E\psi \quad \text{... (3.48)}$$

SUMMARY

- e/m of an electron can be measured by Thomson's method.
- By using Millikan's method charge of an electron can be measured.
- **Bainbridge Mass Spectrograph** is a device used to (i) Detect the presence of isotopes, (ii) Find an accurate value of isotopic masses, (iii) Find their abundance.
- **Atomic Nucleus** occupies a very small volume with a diameter of 10^{-14} m. It consists of Z protons and A-Z neutrons. Proton and neutrons are collectively called as nucleons.
- **Nuclear Force :** Nucleons are held in the nucleus through short range nuclear forces.
- **A M U :** Masses of nucleons and nuclei are expressed in atomic mass units (amu)

  $$1 \text{ amu} = 1.67 \times 10^{-27} \text{ kg}$$
- **Mass Defect** of a nucleus is the difference between its theoretical mass and its actual mass.
- A GM counter can be used for detecting α, β and γ radiations.
- **Heisenberg's Uncertainty Principle :** It is impossible to determine precisely and simultaneously the values of both the members of a pair of physical variables which describe the motion of an atomic system. Such pairs of variables are called as canonically conjugate variables.

  $$\Delta x \cdot \Delta p \geq h$$

  (Heisenberg's uncertainty principle for position and momentum.)

  $$\Delta E \cdot \Delta t \geq h$$

  (Heisenberg's uncertainty principle for energy and time.)
- **Schrodinger's Wave Equation :** It is the mathematical representation of matter waves associated with a moving particle. They are of two types :

  (i) Schrodinger's time independent wave equation :

  $$\nabla^2\psi + \frac{8\pi^2 m}{h^2}(E - V)\psi = 0$$

  (ii) Schrodinger's time dependent wave equation :

  $$\left(\frac{-h^2}{8\pi^2 m}\nabla^2 + V\right)\psi = \frac{ih}{2\pi}\frac{\partial \psi}{\partial t}$$

  i.e. $H\psi = E\psi$
- **Wave Function :** The variable quantity characterizing De Broglie waves is denoted by ψ and it is called the wave function of the particle. This wave function contains all the information about the particle.
- **Probability Density :** The quantity $|\psi(x, y, z, t)|^2$, called the probability density or probability distribution function, determines the probability in unit volume of finding a particle at a given position at a given time.
- **The Probability** of a particle being present in a volume element dx·dy·dz is $|\psi|^2 \cdot dx\, dy\, dz$.

  Normalization condition

  The probability of finding the particle in all space is

  $$\iiint_{\text{all space}} |\psi|^2\, dx\, dy\, dz = 1.$$

  This is the normalization condition. A wave function ψ satisfying this relation is called a normalized wave function.
- **The Wave Function** should satisfy the following conditions :

  (i) It should be a normalized function.

  (ii) It should be a well behaved function i.e., single valued, finite and continuous.

IMPORTANT FORMULAE

- **e/m by Thomson's Method :**

  $$\frac{e}{m} = \frac{E}{B^2}\cdot\frac{d}{ED \times OG}$$
- **Millikan's Oil Drop Method :**

  $$q = \frac{\sigma\pi\eta}{E}\left(\frac{g\eta v1}{2g(\rho-\sigma)}\right)^{1/2}(v_1 - v_2)$$

- **Bainbridge Mass Spectrograph :** The charge to mass ratio is given by,

$$\frac{q}{m} = \frac{E}{B \cdot B'} \cdot \frac{1}{r}$$

- Heisenberg's Uncertainty Principle,
  For position and momentum
  $$\Delta x \cdot \Delta p \geq h,$$
  For energy and time,
  $$\Delta E \cdot \Delta t \geq h$$

- Normalization condition $\iiint_{-\infty}^{+\infty} \psi \psi^* \, dx\, dy\, dz = 1.$

- Schrodinger's wave equation,
  Time independent
  $$\nabla^2 \psi + \frac{8\pi^2 m}{h^2} (E - V) \psi = 0$$
  Time independent
  $$\left(-\frac{h^2}{8\pi^2 m} \nabla^2 + V\right) \psi = \frac{ih}{2\pi} \frac{\partial \psi}{\partial t} ; \quad H\psi = E\psi$$

UNSOLVED PROBLEMS

1. Two beams of U^{235} and U^{238} are separated by a 180° magnetic focussing (as in a mass spectrograph). Find the separation of the beams at the focus, if their velocities are equal and the U^{238} beam has a radius of 1 m in a field of 1 weber/sqm.
   [Ans. Beam separation = 0.0255 m]
2. The electric field between the plates of a velocity selector in a Bainbridge mass spectrograph is 100 kV/m and the magnetic induction in both magnetic fields is 0.6 wb/m^2. A stream of singly charged neons moves in a circular path of radius 6 cm in the magnetic field. Find the mass number of the neon isotope. **[Ans.** m = 21]
3. The average time that an atom retains excess excitation energy before emitting it as electromagnetic radiation is 10^{-8} sec. Calculate the limit of accuracy with which the excitation energy of the emitted radiation can be determined.
   [Ans. 6.63×10^{-26} J]

EXERCISE

1. Explain with a neat diagram the principle and working of a Bainbridge Mass Spectrograph.
2. Explain use of Bainbridge Mass Spectrograph for detection of isotopes.
3. Give principle construction and working of GM counter.
4. Write a short note on Heisenberg's uncertainty principle.
5. State Heisenberg's uncertainty principle and explain it using the concept of De-Broglie wave groups.
6. Explain Thomson's method for measurement of e/m.
7. How charge of an electron can be measured by Millikan's method.
8. Derive Schrodinger's time dependent and time independent wave equation.
9. Explain the physical significance of ψ, ψ^2.
10. State and explain the principle of uncertainty and illustrate it by an experiment on diffraction at a single slit.
11. State and explain Heisenberg's uncertainty principle. Illustrate it by the experiment for location of a particle by microscope.

REFERENCES

Animation of single slit diffraction pattern :
http://www.walter-fendt.de/ph14e/singleslit/htm
An interactive animation of diffraction pattern with a grating :
http://www.physics.uq.edu.au/people/mcintype/php/laboratories/index.php?e=14.
More information about resolving power :
http://www.astronomynotes.com/telescop/s6.htm
More information about X-ray diffraction :
http://www.eserc.stonybrook.edu/ProjectJava/Bragg/

UNIVERSITY QUESTIONS

December 2017

1. With neat diagram explain principle and working of Bainbridge Mass Spectrograph. **[6]**
2. Derive the time independent Schrodinger's wave equation. **[6]**

May 2018

1. State Heisenberg's Uncertainty Principle and prove that electron cannot exist in the nucleus. **[6]**

December 2018

1. Discuss Thomson's method of determination of e/m of an electron. **[6]**
2. Derive time independant Schrodingers wave equation. **[6]**

May 2019

1. Explain with diagram principle and working of Bainbridge mass spectrograph. **[6]**
2. Show that electron does not exists inside the nucleus, with the help of Heisenberg's uncertainty principle. **[3]**
3. Derive Schrodinger's time dependent wave equation. **[6]**