HYPERFINE SPLITTING

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1 Laser Intensity and Microwave intensity

1.1 Value of Intensity for Ω=100 MHz

$$\Omega^2 = \frac{d^2 2c\mu_0 I}{\hbar^2}$$

where d is the transition dipole moment.

An alternate formula is

$$\Omega_{F_g,m_g,F_e,m_e} = \gamma\sqrt{\frac{I}{2I_{sat}}} d_{F_g,F_e} \langle F_e, m_e, 1, q | F_g m_g\rangle$$

Putting $I_{sat} = 1.66932$ mW/cm^2 and $\gamma = 6$ MHz we get

$I = 0.928$ W/cm^2 for $\Omega = 100$ MHz.

Since Ω^2 is proportional to I, so for $\Omega = 200$ MHz we get $I = \sqrt{2}0.927$ W/cm$^2 = 1.312$ W/cm^2.

1.2 What intensity is used for a microwave transition in hyperfine splitting of Rb-87?

The hyperfine splitting in Rb-87 occurs in the ground state of $5^2S_{1/2}$. The hyperfine hamiltonian resulting from coupling between total electronic angular momentum and nuclear angular momentum has the form in zero field

$$H_{hfs} = A_{hfs} I \cdot J = A_{hfs} I \cdot S$$

Here I is the nuclear angular momentum and we define $F = J + I$ and $J = L + S$. The hyperfine hamiltonian has eigen energies of $\frac{1}{2}A_{hfs}(F(F+1) - I(I+1) - S(S+1))$ which for $F = 1$ and $F = 2$ (the hyperfine split levels) comes to be $-1.25A_{hfs}$ and $0.75A_{hfs}$ which have an energy difference of $2A_{hfs}$. A_{hfs} is the hyperfine structure constant which equals $3.41734\hbar$ GHz hence showing that the levels differ by $6.83468\hbar$ GHz. The $F = 1$ state has three sub levels with $m_F = 0, \pm 1$ and similarly $F = 2$ has $m_F = 0, \pm 1, \pm 2$. We can see that the it has essentially $2F + 1$ levels which are degenerate at zero field but are non degenerate at non zero field. There are transitions defined as $\sigma_\pm$ and π which is described by the figure 1.

For creating Rabi oscillations we are making use of the π transitions which will be refereed to as clock transitions. Using RWA the matrix form of the hamiltonian for clock transition levels is written as follow

$$H_0 = \hbar\omega_0 \begin{pmatrix} \frac{3}{8} & 0 \\ 0 & -\frac{5}{8} \end{pmatrix}$$

In the interaction picture the hamiltonian in presence of a magnetic field will be written as follows

$$H_{B,I} = e^{\iota H_0 t/\hbar} \left(\sum_i B_i(t) \langle F, m_F | I_{nuc} \otimes \sigma_i | F', m'_F \rangle \right) e^{-\iota H_0 t/\hbar}$$

The evolution of state would be given by $\iota\hbar\dot{\psi} = H_{B,I}\psi$ Now once we do apply magnetic field it would be required to be calibrated to get rid of effects from background field present and the reference [1] does it for a magnetic field

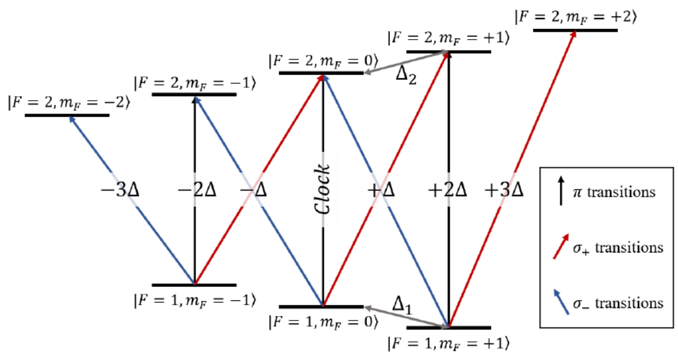

Figure 1: Transitions between $F = 1$ and $F = 2$

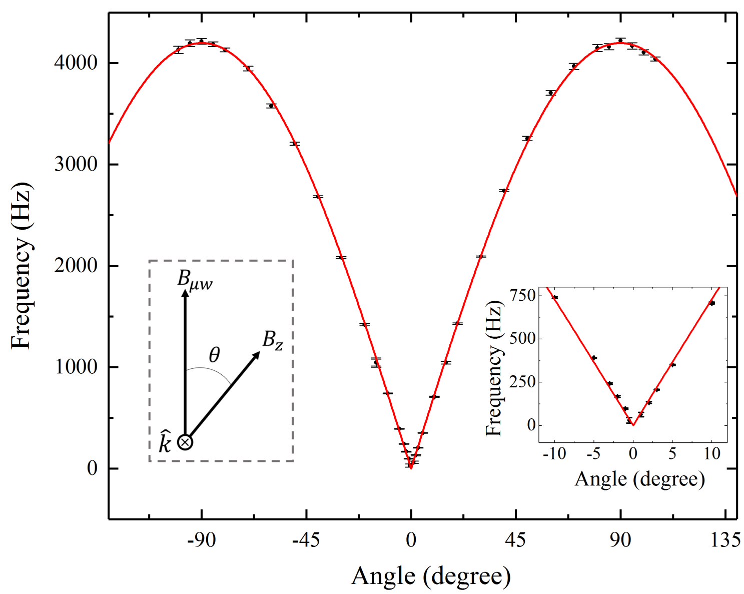

Figure 2: Variation of Ω_{clock} with θ

bias of $B_z = 200$ milli gauss. A circular microwave resonant field was applied on the cold atoms and if θ was angle between plane of the microwave field and magnetic field bias, the clock transitions come to the following value for their frequency.

$$\Omega_{clock} = \frac{\mu_B g_s B'}{2\hbar} |\sin(\theta)|$$

For the given bias field the maximum possible Ω_{clock} ($\theta = \pm\pi/2$) comes to be about 4.22 ± 0.03kHz. Seeing the linear dependence relation with the component of $B_{\mu w}$ on the bias direction (hence the $\sin(\theta)$ dependence) we can extrapolate and claim that for driving at 1MHz we would need the $B \approx 357$ milli gauss based on the fact that $g_F \mu_B B_z m_F = 700$ Hz/mG.

To convert magnetic field to intensity we simply can use the average intensity expression for electromagnetic waves

$$I = \frac{cB_0^2}{2\mu_0}$$

References

[1] All-microwave control of hyperfine states in ultracold spin-1 rubidium

[2] Two-Photon spectroscopy of rubidium in the vicinity of silicon devices