HYPERFINE SPLITTING

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1 Laser Intensity and Microwave intensity

1.1 Value of Intensity for Ω=100 MHz

$$\Omega^2 = \frac{d^2 2c\mu_0 I}{\hbar^2}$$

where d is the transition dipole moment.

An alternate formula is

$$\Omega_{F_g,m_g,F_e,m_e} = \gamma\sqrt{\frac{I}{2I_{sat}}} d_{F_g,F_e}\langle F_e, m_e, 1, q|F_g m_g\rangle$$

Putting $I_{sat} = 1.66932$ mW/cm^2 and $\gamma = 6$ MHz we get

$I = 0.928$ W/cm^2 for $\Omega = 100$ MHz.

Since Ω^2 is proportional to I, so for $\Omega = 200$ MHz we get $I = \sqrt{2}0.927$ W/cm$^2 = 1.312$ W/cm^2.

1.2 What intensity is used for a microwave transition in hyperfine splitting of Rb-87?

The hyperfine splitting in Rb-87 occurs in the ground state of $5^2S_{1/2}$. The hyperfine hamiltonian resulting from coupling between total electronic angular momentum and nuclear angular momentum has the form in zero field

$$H_{hfs} = A_{hfs} I \cdot J = A_{hfs} I \cdot S$$

Here I is the nuclear angular momentum and we define $F = J + I$ and $J = L + S$. The hyperfine hamiltonian has eigen energies of $\frac{1}{2}A_{hfs}(F(F+1) - I(I+1) - S(S+1))$ which for $F = 1$ and $F = 2$ (the hyperfine split levels) comes to be $-1.25A_{hfs}$ and $0.75A_{hfs}$ which have an energy difference of $2A_{hfs}$. A_{hfs} is the hyperfine structure constant which equals $3.41734\hbar$ GHz hence showing that the levels differ by $6.83468\hbar$ GHz. The $F = 1$ state has three sub levels with $m_F = 0, \pm1$ and similarly $F = 2$ has $m_F = 0, \pm1, \pm2$. We can see that the it has essentially $2F + 1$ levels which are degenerate at zero field but are non degenerate at non zero field. There are transitions defined as $\sigma_\pm$ and π which is described by the figure 1.

For creating Rabi oscillations we are making use of the π transitions. Now once we do apply magnetic field it would be required to be calibrated and the reference [1] does it for a magnetic field bias of 200 milli gauss. A circular microwave resonant field was applied on the cold atoms and if θ was angle between plane of the microwave field and magnetic field bias, the clock transitions come to the following value for their frequency.

$$\Omega_{clock} = \frac{\mu_B g_s B'}{2\hbar}|\sin(\theta)|$$

For the given bias field the maximum possible Ω_{clock} ($\theta = \pi/2$) comes to be about 6kHz. Seeing the linear dependence relation we can extrapolate and claim that for driving at 1MHz we would need the $B \approx 33$ Gauss. This would however bring issues in re-calibration since being fairly large it would bring noticeable differences between the sub levels of $F = 1$ and $F = 2$.

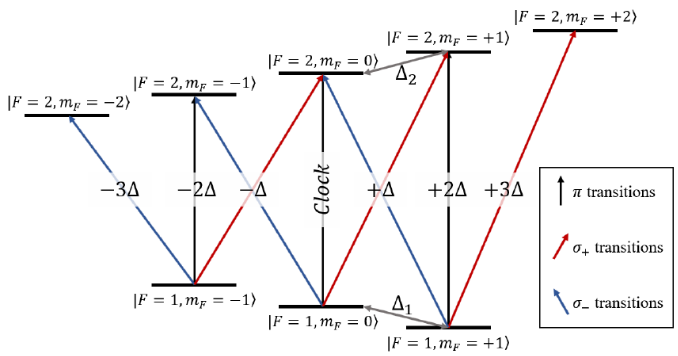

Figure 1: Transitions between $F = 1$ and $F = 2$

References

[1] All-microwave control of hyperfine states in ultracold spin-1 rubidium
[2] Two-Photon spectroscopy of rubidium in the vicinity of silicon devices