HYPERFINE SPLITTING

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1 Laser Intensity and Microwave intensity

1.1 Value of Intensity for Ω=100 MHz

$$\Omega^2 = \frac{d^2 2c\mu_0 I}{\hbar^2} \tag{1}$$

where d is the transition dipole moment.

An alternate formula is

$$\Omega_{F_g,m_g,F_e,m_e} = \gamma\sqrt{\frac{I}{2I_{sat}}} d_{F_g,F_e}\langle F_e, m_e, 1, q|F_g m_g\rangle \tag{2}$$

Putting $I_{sat} = 1.66932$ mW/cm^2 and $\gamma = 6$ MHz we get

$I = 0.928$ W/cm^2 for $\Omega = 100$ MHz.

Since Ω^2 is proportional to I, so for $\Omega = 200$ MHz we get $I = \sqrt{2} \times 0.927$ W/cm$^2 = 1.312$ W/cm^2. Reference: [1].

1.2 What intensity is used for a microwave transition in hyperfine splitting of Rb-87?

The hyperfine splitting in Rb-87 occurs in the ground state of $5^2S_{1/2}$. The hyperfine hamiltonian resulting from coupling between total electronic angular momentum and nuclear angular momentum has the form in zero field

$$H_{hfs} = A_{hfs} I \cdot J = A_{hfs} I \cdot S$$

Here I is the nuclear angular momentum and we define $F = J + I$ and $J = L + S$. The hyperfine hamiltonian has eigen energies of $\frac{1}{2}A_{hfs}(F(F+1) - I(I+1) - S(S+1))$ which for $F = 1$ and $F = 2$ (the hyperfine split levels) comes to be $-1.25A_{hfs}$ and $0.75A_{hfs}$ which have an energy difference of $2A_{hfs}$. A_{hfs} is the hyperfine structure constant which equals $3.41734\hbar$ GHz hence showing that the levels differ by $6.83468\hbar$ GHz. The $F = 1$ state has three sub levels with $m_F = 0, \pm 1$ and similarly $F = 2$ has $m_F = 0, \pm 1, \pm 2$. We can see that the it has essentially $2F + 1$ levels which are degenerate at zero field but are non degenerate at non zero field. There are transitions defined as $\sigma_\pm$ and π which is described by the figure 1.

For creating Rabi oscillations we are making use of the π transitions which will be refereed to as clock transitions. Using RWA the matrix form of the hamiltonian for clock transition levels is written as follow

$$H_0 = \hbar\omega_0 \begin{pmatrix} \frac{3}{8} & 0 \\ 0 & -\frac{5}{8} \end{pmatrix} \tag{3}$$

In the interaction picture the hamiltonian in presence of a magnetic field will be written as follows

$$H_{B,I} = e^{\iota H_0 t/\hbar} \left(\sum_i B_i(t) \langle F, m_F | I_{nuc} \otimes \sigma_i | F', m'_F \rangle \right) e^{-\iota H_0 t/\hbar} \tag{4}$$

The evolution of state would be given by $\iota\hbar\dot{\psi} = H_{B,I}\psi$ Now once we do apply magnetic field it would be required to be calibrated to get rid of effects from background field present and the reference [2] does it for a magnetic field bias of $B_z = 200$ milli gauss. A circular microwave resonant field was applied on the cold atoms and if θ was angle

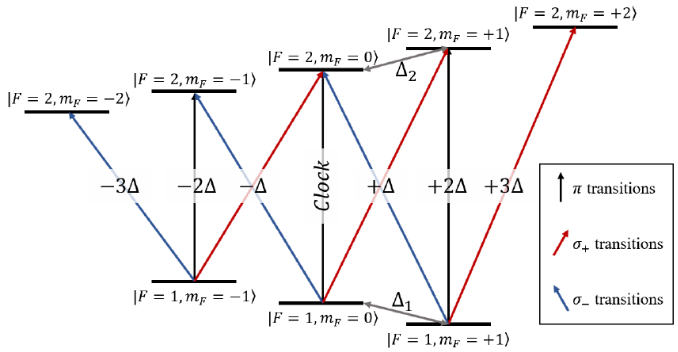

Figure 1: Transitions between $F = 1$ and $F = 2$

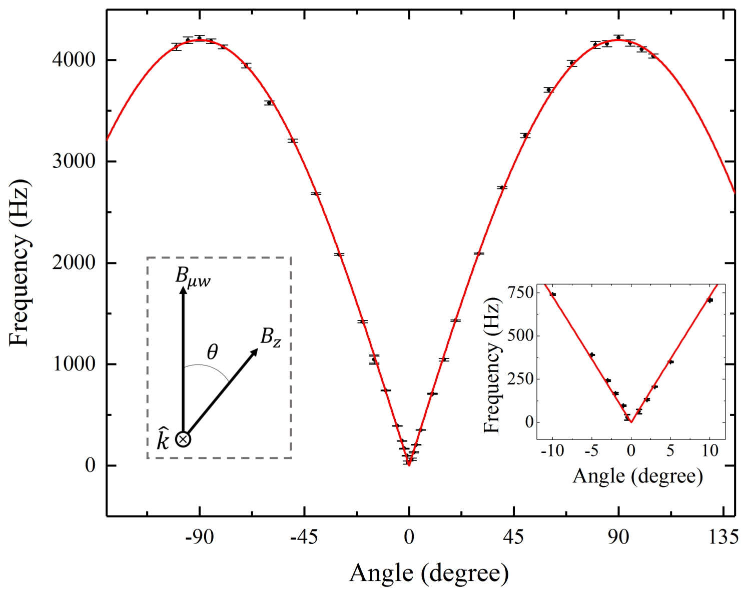

Figure 2: Variation of Ω_{clock} with θ

between plane of the microwave field and magnetic field bias, the clock transitions come to the following value for their frequency.

$$\Omega_{clock} = \frac{\mu_B g_s B'}{2\hbar} |\sin(\theta)| \tag{5}$$

For the given bias field the maximum possible Ω_{clock} ($\theta = \pm\pi/2$) comes to be about 4.22 ± 0.03kHz. Seeing the linear dependence relation with the component of $B_{\mu w}$ on the bias direction (hence the $\sin(\theta)$ dependence) we can extrapolate and claim that for driving at 1MHz we would need the $B \approx 357$ milli gauss based on the fact that $g_F \mu_B B_z m_F = 700$ Hz/mG.
To convert magnetic field to intensity we simply can use the average intensity expression for electromagnetic waves

$$I = \frac{cB_0^2}{2\mu_0} \tag{6}$$

For a more general transfer from some state i to f we have

$$\Omega_{i,f} = \frac{\mu_B g_s B_P}{2\hbar} M_{IF,\{i,f\}} \tag{7}$$

2 Population transfer using STIRAP

We now aim to transfer population from $|F=1, m=1\rangle$ to $|F=3, m=3\rangle$ using $|F=2, m=2\rangle$ as an intermediate state. Here we use $\Omega_{\mu w}$ for transfer between $|F=1, m=1\rangle$ to $|F=2, m=2\rangle$ which is microwave induced and ω_O for transfer between $|F=2, m=2\rangle$ to $|F=3, m=3\rangle$ which is done by an optical laser.
For STIRAP [3] we need $\Delta_S = -\Delta_P$ for the two photon resonance in ladder configuration.

$$\hbar\Delta_P = E_2 - E_1 - \hbar\omega_P \tag{8}$$
$$\hbar\Delta_P = E_3 - E_2 - \hbar\omega_S \tag{9}$$

The effective hamiltonian in the basis of these three states can be written as

$$H = \frac{\hbar}{2}\begin{pmatrix} -2\Delta_P & \Omega_{\mu w}(t) & 0 \\ \Omega_{\mu w}(t) & 0 & \Omega_O(t) \\ 0 & \Omega_O(t) & -2\Delta_S \end{pmatrix} \tag{10}$$

Taking $|\psi\rangle = a_1|F=1, m=1\rangle + a_2|F=2, m=2\rangle + a_3|F=3, m=3\rangle$, at large detunings we can take $\dot{a}_2$ to be close to zero since the oscillations of Δ would average out to zero in the timescale of the pulses. This will reduce the problem to a two dimensional one where the rabi frequency is given by

$$\Omega_{eff}(t) = \frac{\Omega_{\mu w}(t)\Omega_O(t)}{2\Delta} \tag{11}$$
$$\Delta_{eff}(t) = \frac{\Omega_{\mu w}(t)^2 - \Omega_O(t)^2}{2\Delta} \tag{12}$$

To get an effective rabi frequency of 1 MHz where the detuning $\Delta = 1$ GHz we can use a microwave rabi frequency peaking at about 10 MHz and an optical rabi frequency peaking at about 100 MHz.

We know that Ω_O^2 is proportional to I, so for $I = 1$ W/cm^2, we get Ω_O = 100 MHz.

This gives us $\Omega_{\mu W}$ = 1 MHz for optical power $\approx$ 10 W/cm^2 and $\Omega_{\mu W}$ = 10 MHz for optical power $\approx$ 32 W/cm^2.

2.1 Some small scale simulations

This is a simulation for 10 GHz rabi frequencies with a $\Delta = 1$ GHz and over a time scale of 10^{-6} seconds. We tried with pulses of unequal peaks however they require a proper examining for their respective time scales to properly function and so this was just a small test.

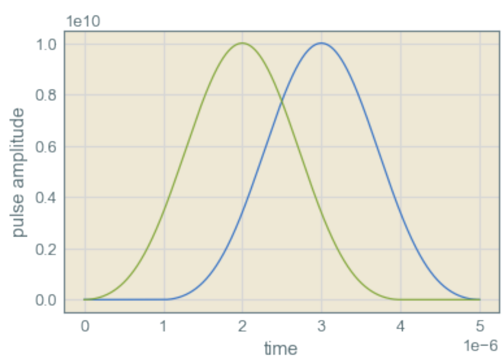

Figure 3: first figure

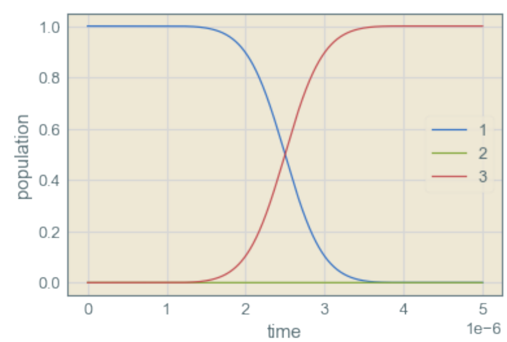

Figure 4: second figure

References

[1] Wenting Chen. *Two-Photon spectroscopy of rubidium in the vicinity of silicon devices*. Sept. 2019. URL: https://www.pi5.uni-stuttgart.de/documents/abgeschlossene-arbeiten/2019-Chen-Wenting-Two-Photon-spectroscopy-of-rubidium-in-the-vicinity-of-silicon-devices-BSC.pdf.

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