Measures of Dispersion :-

Measures of dispersion will give a measure how the whole data are distributed about the central value. When all the samples values are equal, the value of the dispersion will be zero.

Some important measures of dispersion

(i) Range
(ii) Quartile deviation VVI
(iii) Standard deviation VVI
(iv) # Mean deviation Imp
(v) Coefficient of variance.

(i) Range :- Range is defined as the difference b/w the greatest and the least sample values.

If x_{max} and x_{min} denote the greatest and the least sample values of a distribution, its range is

$$\boxed{\text{Range} = x_{max} - x_{min}}$$

Coefficient of range

$$= \frac{(x_{max} - x_{min})}{(x_{max} + x_{min})}$$

(ii) Quartile Deviation

Quartile Range is defined as the difference b/w first and 3rd quartiles

ie, $\boxed{\text{Quartile Range} = Q_3 - Q_1}$

$$\boxed{\text{Quartile Deviation} = \frac{Q_3 - Q_1}{2}}$$

Note Let us know 1st the quartiles Q_1, Q_2, Q_3

As median is the middle most value of arranged distribution, so by median the total data is partitioned into two parts. In a similar manner, if we devide the total distribution into four parts using three partitions, then

the value corresponding to 1st partion is 1st quantile
" " " " 2nd " " 2nd quantile
" " " " 3rd " " 3rd quantile

2nd Quartile $= Q_2$ = Median

1st quartile (Q_1) corresponds to $\left(\frac{N+1}{4}\right)$th term

2nd quartile (Q_2) " " $2\left(\frac{N+1}{4}\right)$th term

ie $\left(\frac{N+1}{2}\right)$ " "

3rd Quartile (Q_3) corresponds to $3\left(\frac{N+1}{4}\right)$ th term.

⋮

k-th quartile (Q_K) corresponds to $k\left(\frac{N+1}{4}\right)$ th term

Problem ① Find the Quartile Deviation ($Q.D$) for the following data distribution: (ungrouped data)

24, 7, 11, 9, 17, 3, 20, 14, 4, 22, 27

Soln: 1st re-arrange the data in ascending order —

3, 4, 7, 9, 11, 14, 17, 20, 22, 24, 27

$Q_1 = \left(\frac{N+1}{4}\right)$th term, Here $N = 11$

$= \frac{11+1}{4}$ th term $=$ 3 rd term

$\Rightarrow \boxed{Q_1 = 7}$

$Q_3 = 3\left(\frac{N+1}{4}\right)$ th term

$= 3 \times 3 = 9$ th term $= \boxed{22 = Q_3}$

$\therefore$ Quartile deviation $= \frac{Q_3 - Q_1}{2}$

$= \frac{22-7}{2} = 7.5$

Problem - (2) Find Q.D. for the following discrete data distribution (ungrouped datas)

5, 10, 15, 17, 18, 19, 20, 21, 25, 28

Soln :- Given datas are already in ascending order. Here $\boxed{N = 10}$

$Q_1 = \left(\frac{N+1}{4}\right)$ -th term

$= \frac{11}{4}$ th term $= 2.75$ th term

$= 2$ nd term $+ 0.75$ (3rd term $-$ 2nd term)

$= 10 + 0.75(15-10)$

$= 10 + 0.75 \times 5$

$\boxed{Q_1 = 13.75}$ Ans

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$$Q_3 = 3\left(\frac{N+1}{4}\right) \text{th term}$$

$$= \frac{33}{4} \text{th term}$$

$$= 8.25 \text{th term}$$

$$= 8 \text{th term} + 0.25(9\text{th term} - 8\text{th term})$$

$$= 21 + 0.25(25-21)$$

$$= 21 + 0.25 \times 4$$

$$\Rightarrow \boxed{Q_3 = 22}$$

Hence $QD = \dfrac{Q_3 - Q_1}{2}$

$$\Rightarrow QD = \frac{22 - 13.75}{2}$$

$$\Rightarrow QD = 4.125$$

PROBLEM - 3 Calculate the quartile deviation of the following distribution

x:	4	6	8	10	12	14
f:	9	12	15	16	5	5

Solⁿ :-

x	f	C.f (Less than type)
4	9	9
Q_1 → 6	12	12+9 = 21 ⇐ 15.75
8	15	15+21 = 36
Q_3 → 10	16	16+36 = 52 ⇐ 47.25
12	5	5+52 = 57
14	5	5+57 = 62

$N = \Sigma f = 62$

Now $Q_1 = \frac{N+1}{4} = \frac{63}{4} = 15.75$ th term

$= 6$ {the next higher term than 15.75 from c.f. column from the table and take corresponding x-value}

$Q_3 = 3\left(\frac{N+1}{4}\right) = 47.25$ th term

$= 10$

∴ Quartile Deviation,

$$QD = \frac{Q_3 - Q_1}{2} = \frac{10-6}{2} = 2$$

PROBLEM - ④ Calculate the quartile deviation for the following data:

Farm Size :	0-40	41-80	81-120	121-160	161-200	201-240	241 and over
No of Farms :	394	461	391	334	169	113	148

Soln :

class interval	class boundary	Frequency	C.F.
0-40	-0.5-40.5	394	394
41-80	40.5-80.5	461	855
81-120	80.5-120.5	391	1246
121-160	120.5-160.5	334	1580
161-200	160.5-200.5	169	1749
201-240	200.5-240.5	113	1862
241 —	240.5 -	148	2010
		$N=\Sigma f = 2010$	

Q_1 class

502.5

1507.5

Q_3 class

Formula for Quartile for grouped datas

$$\boxed{Q_i = l + \left(\frac{\frac{iN}{4} - F}{f} \times c\right)}$$

So, $Q_1 = l + \frac{\left(\frac{N}{4} - F\right)}{f} \times c$

where l = lower class boundary of i-th quartile class

~~Here~~ f = frequency of the ~~1st~~ i-th quartile class

F = cumulative frequency of the class just above the i-th ~~1st~~ quartile class.

c = class length of the i-th ~~1st~~ quartile class.

To recognise 1st quartile class :-

$= \frac{N}{4} = \frac{2010}{4} = 502.5$

So 1st quartile class ~~40.5-80~~. (41-80) and the corresponding row

3rd quartile class similarly

$= \frac{3N}{4} = 3 \times 502.5 = 1507.5$

So, $Q_1 = 40.5 + \frac{\cancel{502.5}\left(\frac{2010}{4} - 394\right) \times 40}{461}$

$= 40.5 + \cancel{9.64}\ 9.41$

$= \cancel{50.14}$

$= 49.91$

$80.5 - 40.5 = 40 = c$

$$\cancel{Q_3 = l + \left(\frac{3 \times \frac{2010}{4} - 1246}{334}\right) \times 40}$$

For Q_3, 1st of all find the 3rd quartile class by the formula written previously

$$Q_3 = l + \left(\frac{3 \cdot \frac{N}{4} - F}{f}\right) c$$

$$= 120.5 + \left(\frac{3 \times \frac{2010}{4} - 1246}{334}\right) \times 40$$

$$= 120.5 + 31.32 = 151.82$$

∴ Quartile Deviation

$$= \frac{Q_3 - Q_1}{2} = \frac{151.82 - 49.91}{2}$$

$$\boxed{\therefore QD = 50.96}$$

(iii) Mean Deviation :-

If there be n nos of observations, $(x_1, x_2, ---, x_n)$, then the mean deviation about any central value A is defined by

$$\text{Mean Deviation} = \frac{1}{n}\sum_{i=1}^{n}|x_i - A|$$

If $x_1 \to f_1$
$x_2 \to f_2$
$\vdots \quad \vdots$
$x_n \to f_n$

Then mean deviation about A is defined as,

$$\text{mean deviation} = \frac{\sum_{i=1}^{n} f_i|x_i - A|}{\sum_{i=1}^{n} f_i}$$

Problem ① Find the mean deviation from arithmetic mean (AM) of the following data: 11, 2, 9, 7, 4, 8, 6, 10, 3, 1

Soln $A.M = \frac{11+2+9+7+4+8+6+10+3+1}{10}$

i.e $A = 6.1$ (Arithmetic Mean)

Now Mean deviation from $A = \frac{1}{10}\sum_{i=1}^{10}|x_i - 6.1|$

$$= \frac{1}{10}\Big[|(11-6.1)| + |(2-6.1)| + |(9-6.1)| + |(7-6.1)| + |(4-6.1)| + |(8-6.1)| + |(6-6.1)| + |(10-6.1)| + |(3-6.1)| + |(1-6.1)|\Big]$$

$$= \frac{1}{10}\Big[|4.9| + |(-4.1)| + |2.9| + |0.9| + |(-2.1)| + |1.9| + |(-0.1)| + |3.9| + |(-3.1)| + |(-5.1)|\Big]$$

$$= \frac{1}{10}\Big[4.9 + 4.1 + 2.9 + 0.9 + 2.1 + 1.9 + 0.1 + 3.9 + 3.1 + 5.1\Big]$$

$$\boxed{M.D. = 2.9}$$

Problem - 2 Calculate the mean deviation from the mean of the following distribution:

H.W

Marks :	5	15	25	35	45	55	65
No of Students :	4	6	10	20	10	6	4

$\Downarrow$ f

(iv) Standard Deviation :-

The square root of deviations of scores from the mean is called the standard deviation.

S.D. = Root mean square deviation about mean.

For ungrouped data

① If $x_1, x_2, \ldots, x_n$ be a set of n samples, then $\boxed{\text{deviation } d = x_i - \text{mean}}$

$$S.D. = \sqrt{\frac{\sum d^2}{n}} = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n}}$$

Another formula for

$$\boxed{SD = \sqrt{\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2}}$$

② Again, if $x_1, x_2, \ldots x_n$ be the samples having frequencies $f_1, f_2 \ldots f_n$ respectively.

$$S.D = \sqrt{\frac{\sum_{i=1}^{n} f_i (x_i - \bar{x})^2}{\sum_{i=1}^{n} f_i}}$$

Another formula for SD,

$$SD = \sqrt{\frac{\sum x_i^2 f_i}{\sum f_i} - \left(\frac{\sum x_i f_i}{\sum f_i}\right)^2}$$

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FOR Grouped data :-

$$S.D. = \sqrt{\frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f x}{\Sigma f}\right)^2} \times c$$

where C = length of the class boundary

x = deviation = $\dfrac{\text{Mid Point} - \text{A.M.}}{c}$ Assumed Mean

A.M = Mid value corresponding to the highest frequency.

P-1 Find the standard Deviation of the following distribution

x:	1	2	3	4	5	6
f:	4	3	2	5	6	5

Solⁿ:-

x_i	f_i	$x_i f_i$	x_i^2	$x_i^2 f_i$
1	4	4	1	4
2	3	6	4	12
3	2	6	9	18
4	5	20	16	80
5	6	30	25	150
6	5	30	36	180
	$\Sigma f_i = 25$	$\Sigma x_i f_i = 96$		$\Sigma x_i^2 f_i = 444$

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So $SD = \sqrt{\dfrac{\sum x_i^2 f_i}{\sum f_i} - \left(\dfrac{\sum x_i f_i}{\sum f_i}\right)^2}$

$= \sqrt{\dfrac{444}{25} - \left(\dfrac{96}{25}\right)^2}$

$= \sqrt{17.76 - 14.7456}$

$= 1.74$

Q-2 Find standard deviation of the following distribution:

Class:	10-19	20-29	30-39	40-49	50-59	60-69
Frequency:	3	5	7	9	4	2

Soln:

Class interval	Class Boundary	Mid Points (MP)	fre (f)	Deviation $x = \dfrac{MP - 44.5}{10 \approx c}$	xf	x^2	x^2f
10-19	9.5-19.5	14.5	3	-3	-9	9	27
20-29	19.5-29.5	24.5	5	-2	-10	4	20
30-39	29.5-39.5	34.5	7	-1	-7	1	7
40-49	39.5-49.5	44.5	9	0	0	0	0
50-59	49.5-59.5	54.5	4	1	4	1	4
60-69	59.5-69.5	64.5	2	2	4	4	8

Here $c = 19.5 - 9.5 = 10$, $AM = 44.5$

$\sum f = 30$, $\sum xf = -18$, $\sum x^2 f = 66$

$\therefore SD = \sqrt{\frac{\Sigma x^2 f}{\Sigma f} - \left(\frac{\Sigma x f}{\Sigma f}\right)^2} \times C$

$= \sqrt{\frac{66}{30} - \left(\frac{-18}{30}\right)^2} \times 10$

$= 1.356 \times 10 = 13.56$ Ans

(4) Relative measures of dispersion

(a) Coefficient of Variation

$= \frac{\text{Standard Deviation}}{\text{Mean}} \times 100$

(b) Coefficient of mean deviation

$= \frac{\text{Mean deviation about Mean (or Median)}}{\text{Mean (or Median)}} \times 100$

(c) Coefficient of Quartile deviation

$= \frac{Q_3 - Q_1}{Q_3 + Q_1} \times 100$