* Cholesky's Method

It A is a positive-definite Symmetric matrix, then we can perform cholesky's ~~method~~. factoryzation.

$A \rightarrow$ symmetric $(A = A^T)$

$A \rightarrow$ positive symmetric

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix}$$

$\Delta 1 = a \longrightarrow$ leading Principal minors

$$\Delta 2 = \begin{vmatrix} a & b \\ d & e \end{vmatrix} = ae - bd$$

$$\Delta 3 = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

$$\boxed{\Delta 1 > 0 \ \&\& \ \Delta 2 > 0 \ \&\& \ \Delta 3 > 0}$$

$$L = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \approx U^T \quad \Bigg| \quad L^T = \begin{bmatrix} L_{11} & L_{21} & L_{31} \\ 0 & L_{22} & L_{32} \\ 0 & 0 & L_{33} \end{bmatrix} \approx U$$

$$\boxed{\begin{aligned} A &= L \cdot L^T \\ A &= U \cdot U^T \end{aligned}}$$

$$\boxed{\begin{aligned} &AX = B \\ &\Rightarrow LUX = B \end{aligned}}$$

$AX = B$

$LL^T X = B$ OR $U^T . U X = B$

Shortcut formula

$$l_{ii} = \sqrt{a_{ii} - \sum_{k=1}^{i-1} l_{ik}^2}$$

$$l_{ij} = \frac{1}{l_{jj}} \left[a_{ij} - \sum_{k=1}^{j-1} l_{ik} l_{jk} \right] \quad [\text{where } i > j]$$

Decompose a matrix A using cholesky's Method

$$A = \begin{bmatrix} 4 & 12 & -16 \\ 12 & 37 & -43 \\ -16 & -43 & 98 \end{bmatrix} \qquad l = \begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix}$$

$$L_{11} = \sqrt{a_{11} - \sum_{k=1}^{0} l_{1k}^2}$$

$$= \sqrt{4 - 0}$$

$$\boxed{= 2}$$

$$L_{21} = \boxed{\frac{1}{L_{11}} \left[a_{21} - \sum_{k=1}^{0} l_{2k} l_{1k} \right]} = \frac{1}{2} [12 - 0]$$

$$= 6$$

$$l_{22} = \sqrt{a_{22} - \sum_{k=1}^{2-1} l_{2k}^2}$$

$$= \sqrt{37 - (l_{21})^2}$$

$$= \sqrt{37 - 6^2}$$

$$= \sqrt{1}$$

$$= 1$$

$$l_{31} = \frac{1}{L_{11}} \left(a_{31} - \sum_{k=1}^{1-1} l_{3k} l_{1k} \right)$$

$$= \frac{1}{2} [-16 - 0]$$

$$= -8$$

$$l_{32} = \frac{1}{l_{22}} \left[a_{32} - \sum_{k=1}^{2-1} l_{3k} l_{2k} \right]$$

$$= \frac{1}{1} [-43 - (-8 \times 6)]$$

$$= -43 + 48$$

$$= 5$$

$$l_{33} = \sqrt{a_{33} - \sum_{k=1}^{3-1} l_{3k}^2}$$

$$= \sqrt{98 - \cancel{\sum}(l_{31}^2 + l_{32}^2)}$$

$$= \sqrt{98 - ((-8)^2 + (5)^2)}$$

$$= \sqrt{98 - 89}$$

$$= \sqrt{9}$$

$$= 3$$

$$\begin{array}{r} 64 \\ 25 \\ \hline 89 \end{array}$$

$$L = \begin{bmatrix} 2 & 0 & 0 \\ 6 & 1 & 0 \\ -8 & 5 & 3 \end{bmatrix}, \quad L^{T} = \begin{bmatrix} 2 & 6 & -8 \\ 0 & 1 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

$$4x + 12y - 16z = 4$$
$$12x + 37y - 43z = 7$$
$$-16x - 43y + 98 = 10$$

Basic Gauss Elimination Method

* use Gauss elimination Method to solve the equatio

$$2x + y + z = 10 \quad \text{--- (i)}$$
$$3x + 2y + 3z = 18 \quad \text{--- (ii)}$$
$$x + 4y + 9z = 16 \quad \text{--- (iii)}$$

Step 01: Eliminate x

From (ii) $- \frac{3}{2}$ (i) we get :

$$3x + 2y + 3z - 3x - \frac{3}{2}y - \frac{3}{2}z = 18 - \frac{3}{2} \cdot 10$$

$$\Rightarrow 2y - \frac{3}{2}y + 3z - \frac{3}{2}z = 18 - \frac{30}{2}$$

$$\Rightarrow \frac{4y - 3y + 6z - 3z}{2} = \frac{36 - 30}{2}$$

$\Rightarrow y + 3z = 6$

from (iii) − (i) × $\frac{1}{2}$ we get :

$$x + 4y + 9z - x - \frac{1}{2}y - \frac{1}{2}z = 16 - \frac{1}{2}\times 10$$

$$\Rightarrow \frac{8y - y + 18z - z}{2} = 11$$

$$\Rightarrow 7y + 17z = 22$$

∴ New system of equation after Step-1

$$2x + y + z = 10 \quad \text{— (iv)}$$

$$y + 3z = 6 \quad \text{— (v)}$$

$$7y + 17z = 22 \quad \text{— (vi)}$$

Step-2

(vi) − 7 × (v) we get :

$$7y + 17z - 7y - 21z = 22 - 42$$

$$\Rightarrow -4z = -20$$

$$z = 5$$

New system of Equation after Step-2

$$2x + y + z = 10 \quad \text{— (vii)}$$

$$y + 3z = 6 \quad \text{— (viii)}$$

$$z = 5 \quad \text{— (ix)}$$

Ans:

$x = \cancel{3x}$.

$2x + y + z = 10$

$2x + (-9) + 5 = 10$

$\Rightarrow \boxed{x = 7}$

$y =$

$y + 3z = 6$

$y + 15 = 6$

$\boxed{y = -9}$

$\boxed{z = 5}$

Example-02

$$3x + 6y + z = 16 \quad \text{--- (i)}$$

$$2x + 4y + 3z = 13 \quad \text{--- (ii)}$$

$$x + 3y + 2z = 9 \quad \text{--- (iii)}$$

After all process → Black Box → as previous →

$x = 1$

$y = 2$

$z = 1$

* Gauss-Jordan Method

Solve the system of equation using Guass-Jordan method

$$2x_1 + 4x_2 - 6x_3 = -8 \quad \text{——— (I)}$$

$$x_1 + 3x_2 + x_3 = 10 \quad \text{——— (II)}$$

$$2x_1 - 4x_2 - 2x_3 = 12 \quad \text{——— (III)}$$

Normalizing eqn (i) by 2 we get,

$$x_1 + 2x_2 - 3x_3 = -4$$

$$x_1 + 3x_2 + x_3 = 10$$

$$2x_1 - 4x_2 - 2x_3 = 12$$

Eliminating x_1 from the second and third eqn

$$x_1 + 2x_2 - 3x_3 = -4$$

$$0 + x_2 + 4x_3 = 14$$

$$0 - 8x_2 + 4x_3 = 20$$

The second equation is already normalize.
Eliminating x_2 from the first & third equation we get, [First − 2×second]

$$x_1 + 0 - 11x_3 = -32$$

$$0 + x_2 + 4x_3 = 14$$

$$0 - 0 + 36x_3 = 132 \quad [\text{Third} + 8 \times 2\text{nd}]$$

Normalizing third equation

$$x_1 + 0 - 11x_3 = -32$$

$$0 + x_2 + 4x_3 = 14$$

$$0 - 0 + x_3 = \frac{11}{3}$$

Eliminating x_3 from first and 2nd equation

$$x_1 + 0 - 0 = -32 + \frac{121}{3} = \frac{25}{3} \quad [1st + 11\times 3rd]$$

$$0 + x_2 + 0 = 14 - \frac{44}{3} = \frac{-2}{3} \quad [2nd + -4\times 3rd]$$

$$0 - 0 + x_3 = \frac{11}{3}$$

$$x_1 = -\frac{25}{3}, \quad x_2 = -\frac{3}{2}, \quad x_3 = \frac{11}{3}$$

(4) Matrix দ্বারা → partial pivoting method

(X) Solve the system using Guass Jordan Method

$$3x + 2y + z = 10 \quad \text{---} \quad (I)$$

$$2x + 3y + 2z = 14 \quad \text{---} \quad (II)$$

$$x + 2y + 3z = 14 \quad \text{---} \quad (III)$$

Solⁿ

Normalizing equation (I) by 3

$$x + \frac{2}{3}y + \frac{1}{3}z = \frac{10}{3}$$

$$2x + 3y + 2z = 14$$

$$x + 2y + 3z = 14$$

Eliminating x from second and third Equation

$$x + \frac{2}{3}y + \frac{1}{3}z = \frac{10}{3}$$

$$\cancel{0 + 0 + \frac{3}{3}z = 14 - \frac{20}{3}}$$

$$0 + \frac{4}{}$$

$$x + \frac{2}{3}y + \frac{1}{3}z = \frac{10}{3}$$

$$0 + \frac{5}{3}y + \frac{4}{3}z =$$

(ii) partial pivoting Method (Gauss Jordan with) (100%)

$$2x + y - z = 8$$
$$-3x - y + 2z = -11$$
$$-2x + y + 2z = -3$$

সবসময়

এখানে যার (abs) value বেশি সে প্রথমে থাকবে

soln

$$\left[\begin{array}{ccc|c} 2 & 1 & -1 & 8 \\ -3 & -1 & 2 & -11 \\ -2 & 1 & 2 & -3 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} -3 & -1 & 2 & -11 \\ 2 & 1 & -1 & 8 \\ -2 & 1 & 2 & -3 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & \frac{1}{3} & -\frac{2}{3} & \frac{11}{3} \\ 2 & 1 & -1 & 8 \\ -2 & 1 & 2 & -3 \end{array}\right]$$

$\boxed{R_2 \to R_2 - 2 \times R_1}$

$$= \left[\begin{array}{ccc|c} 1 & \frac{1}{3} & -\frac{2}{3} & \frac{11}{3} \\ 0 & +\frac{1}{3} & \frac{1}{3} & \frac{2}{3} \\ 0 & \frac{5}{3} & \frac{2}{3} & \frac{13}{3} \end{array}\right] = \left[\begin{array}{ccc|c} 1 & \frac{1}{3} & -\frac{2}{3} & \frac{11}{3} \\ 0 & \frac{5}{3} & \frac{2}{3} & \frac{13}{3} \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{array}\right]$$

Again!

$$= \left[\begin{array}{ccc|c} 1 & \frac{1}{3} & -\frac{2}{3} & \frac{11}{3} \\ 0 & 1 & \frac{2}{5} & \frac{13}{5} \\ 0 & \frac{1}{3} & \frac{1}{3} & \frac{2}{3} \end{array}\right] = \left[\begin{array}{cccc} 1 & 0 & -\frac{4}{5} & \frac{14}{5} \\ 0 & 1 & \frac{2}{5} & \frac{13}{5} \\ 0 & 0 & \frac{1}{5} & -\frac{1}{5} \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & -\frac{4}{5} & \frac{14}{5} \\ 0 & 1 & \frac{2}{5} & \frac{13}{5} \\ 0 & 0 & 1 & -1 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right]$$

~~$x = 2$, $y = \frac{11}{5}$~~,

Now, $x = 2$, $y = 3$, $z = -1$ Ans