CF969-7-SU: Machine Learning for Finance
Assignment 1

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Methodology

1. Linear Regression for expected returns and risk

I. To estimate the sensitivity of each stock's excess return to the market's excess return using the CAPM. We estimated it using the historical price data and the model is expressed by: Ri – Rf = $\alpha i + \beta i$ (Rm-Rf)+error term.
II. We then collected daily adjusted closing prices for the last 2 years for 10 individual stocks and S$P 500 index using yfinance library. We then computed daily returns as the difference of prices.
III. The risk free rate was set to 0.02 annually.
IV. For each stock we ran a linear regression of the excess stock return on the excess market return to estimate the values of intercept (αi) and slope (βi). We then computer the residual variance to measure the idiosyncratic risk.
V. All in all, the output of the linear regression included:
    a. Intercept (αi) and slope (βi)
    b. R-squared values
    c. Residual variance

2. Portfolio Optimization using Quadratic Programming

This was performed to find the optimal portfolio weights that minimizes portfolio variance while meeting expected return and investment constraints.
This was done using the CAPM output, we constructed a covariance matrix by using values of slope (βi), intercept (αi) and idiosyncratic risk.

a) We then formulated the portfolio optimization as quadratic programming(QP) problem using the formula: Minimize $\frac{1}{2} w^T \Sigma w$ subject to: $w^T \mu = \mu_p$, $w^T 1 = 1$, $w \geq 0$.
b) We solved this problem using the cvxpy library in python which supports convex optimization. We also added a regularization term to the diagno to ensure numerical stability.

3. Efficient Frontier

We constructed the efficient frontier by solving the optimization problem repeatedly across a range of target returns μ_p. For each μ_p, we minimized the variance and recorded the corresponding standard deviation and expected return. We then build a diagram representing the effiicent frontier.

Results

Linear Regression

Linear Regression for expected returns and risk:

Stock	Alpha	Beta	Expected Return(Annual)	Idiosyncratic Variance
BRK-A	0.00033	0.55134	0.10994	0.00008
AAPL	-0.00055	1.19787	0.21541	0.00015
MSFT	0.00018	1.02895	0.18786	0.00010
JPM	0.00100	0.90744	0.16803	0.00013
GOOG	0.00014	1.12389	0.20335	0.00022
BAC	0.00056	0.99130	0.18172	0.00017

INTC	-0.00141	1.70280	0.29779	0.00080
WFC	0.00089	0.98917	0.18137	0.00023
C	0.00066	1.19319	0.21465	0.00018
VZ	0.00049	0.14123	0.04304	0.00021

Interpretation of the above result:

Alpha:

- Mostly small in magnitude which states that market explains the variation in returns.
- Positive alpha values suggest slight outperformance not captured by market risk
- Negative alpha values suggest underperformance as compared to expectations.
- As the alpha value is small, this will not be statistically significant.

Beta:

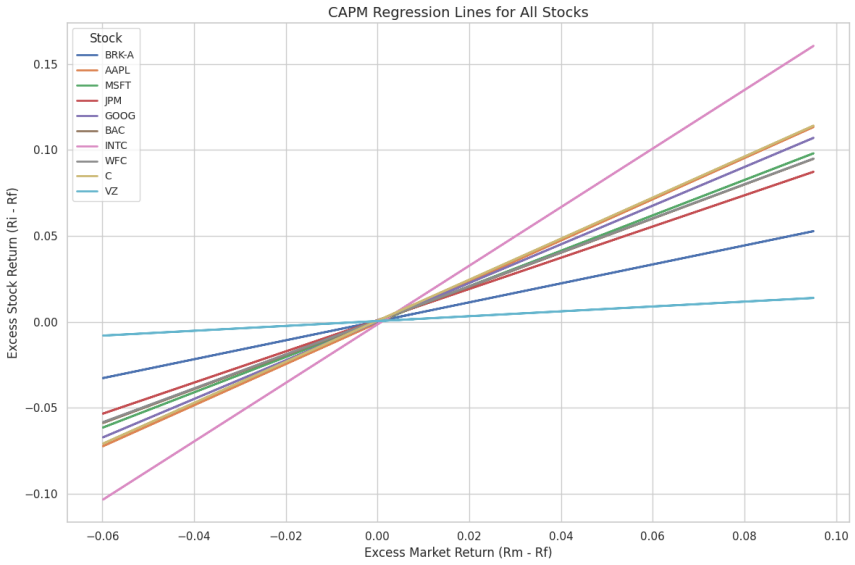
- Beta values are close to 1 that proves that stocks move in line with the market.
- Higher value of beta states greater senstivity to market movements.
- Low value of beta states that the market is less influenced by the market.
- Beta values are statistically significant as they exhibit expected behavior.

Annual Expected Return:

This is close to beta values as high beta stocks have high expected returns like INTC has a beta vaue of 1.70280 and return rate of 29.78%. Similary, VZ has beta value of 14.12 and return rate of 4.30%. This trend is consistent with CAPM as it proves that high beta value means high expected return.

Idiosyncratic Variance:

- This states firm specifc risk and as the values are low, this states that market movements explain a large portion of stock returns.
- INTC has the highest value of idiosyncratic risk, which states that it has been the most influenced by companies's decisions.
- BRK-A has the lowest value of idiosyncratic risk, which states that barely any of the company's decision has effected the rate.



Interpretation of the above graph:

1. **Slope (βi)**

- The steepest line which is pink that represents INTC has the highest beta value which is 1.70 and this represents that it is quite sensitive to market movements
- The Flattest line(lowest line) which is light blue represnts VZ that has the lowest beta which is 0.14 which has low market senstivity and behaves independently.
- Stocks like AAPL, MSFT,GOOG & C have moderate to high slopes as this indicates that they have high level of correlation with the market.

2. Intercept (αi)

- All lines seem to converge near the origin with a slight vertical shift
- This proves that values are alpha are small and they have no significant abnormal return outside market performance.
- VZ show slight positive intercepts and INTC has a negative values as seen in the table and the graph.

3. Idiosyncratic Risk

- The tightness of lines proves that market has proved to have similar market experience and relatively low idiosyncratic variation.
- INTC has a high beta and high idiosyncratic variance as it has a higher sloper and is more dispersed,
- VZ has flat line, as it has low beta, which means market changes doesn't effect it.

Portfolio Optimization

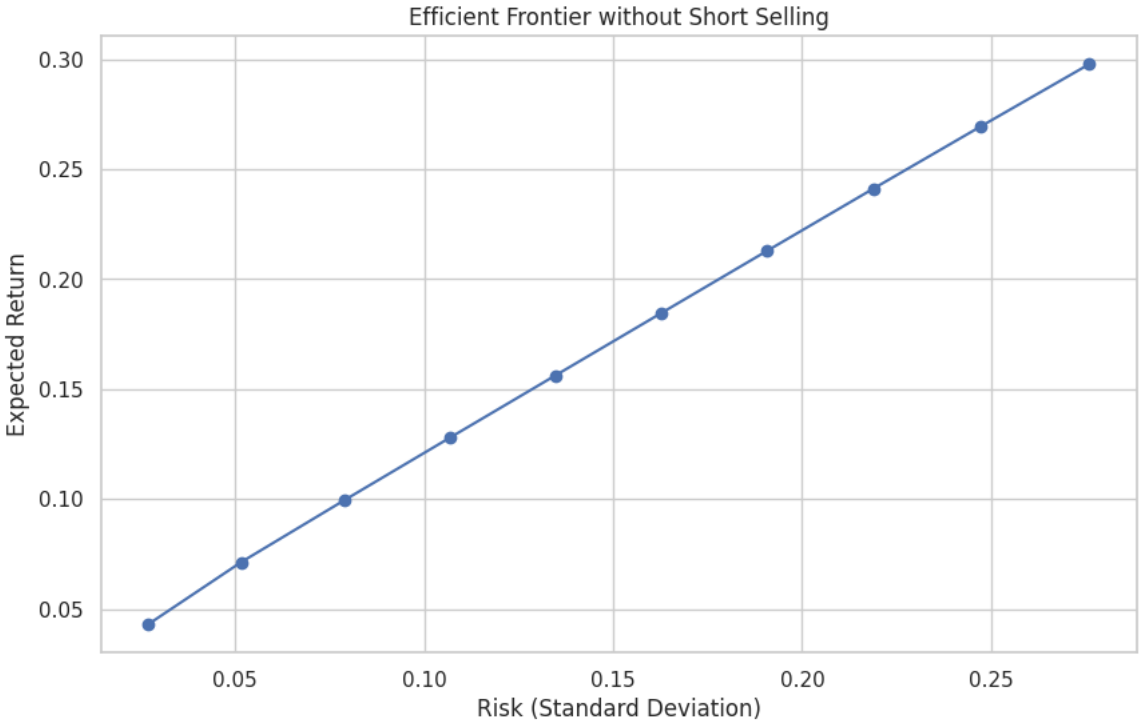
```
Expected returns: [0.10994271 0.21541396 0.18785806 0.16803499
0.20334566 0.18171652
 0.29778581 0.18136908 0.21465144 0.04303919]
--------------------------------------------------
Minimum: 0.04303919219674081 Maximum: 0.29778581414786676
--------------------------------------------------
Optimal Portfolio Weights:
BRK-A: 0.1441
AAPL: 0.1296
MSFT: 0.1744
JPM: 0.1205
GOOG: 0.0854
BAC: 0.0994
INTC: 0.0330
WFC: 0.0757
C: 0.1086

VZ: 0.0294
```

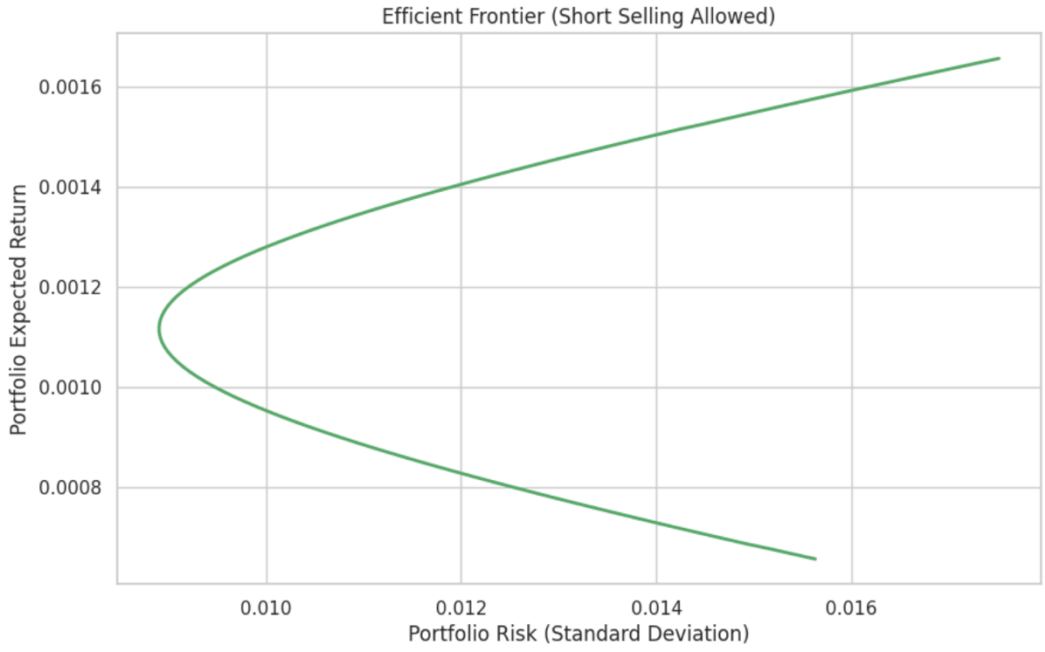
Interpretation of the above result:

- The results summarize the expected returns of the 10 stocks and their optimal portfolio weights which are based on mean-variance optimization which aims to maximize the return for a given level.

- INTC has the highest expected returns which is 29.77 but the portfolio weight is 3.3% because of high risk as beta is 1.70
- MSFT is heavily weighted which is 17.44% which means it has moderate risk and good return which is 18.78 whereas VZ has lowest weight which is 2.94% and low return as well.
- All in all, the weights are well-spread with not a single weight dominating the list.

EFFICIENT FRONTIER





Interpretation of the above graphs:

1. Efficient frontier without short selling

- The linear graph represents limited diversification
- We have the ability to combine assets with non-negative weights which limits the ability of offset risks
- The frontier starts at the minimum variance portfolio and moves upward by increasing allocations to higher return and higher risk assets
- All in allm without short selling the portfolio choices are limited and the trade-off between risk and return are limited.

2. Efficient frontier with short selling

- The concave shape has been created
- With short selling, the optimizer will use negative weights to adjust the position and shall construct the portfolio with lower variance for same return or high return for same risk.
- The left side of parabola represents portfolios with low risk but negative weights.
- All in all, short selling provides greater diversification and better risk return combination.