

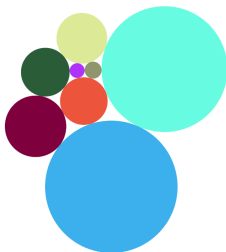


HITS

Heidelberg Institute for
Theoretical Studies

Planar graphs, circle packings, and conformal maps

Brice Loustau (HITS & Heidelberg University)



HITS Lab Meeting

07.09.2020

Planar graphs, circle packings, and conformal maps

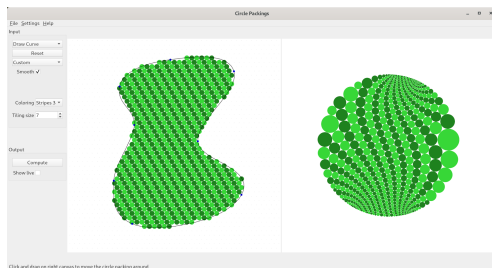
Outline.

1. Planar graphs
2. Circle packings
3. Conformal maps
4. Beyond

Planar graphs, circle packings, and conformal maps

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2. Circle packings
3. Conformal maps
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The software: *Circle Packings* (with B. Beeker)
brice.loustau.eu/circlepackings

1. Planar graphs

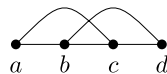
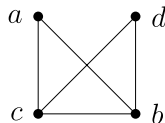
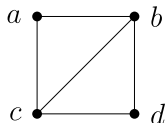
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- A set of *vertices*
- A set of *edges* = relation between vertices

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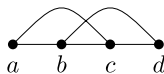
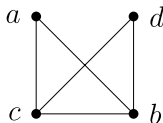
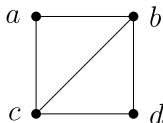
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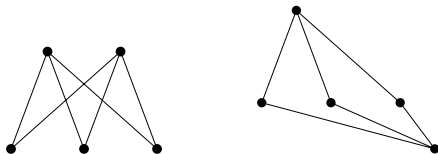
Applications of graph theory: Computer science (networks), linguistics, physics and chemistry, biology, social sciences, etc.

1. Planar graphs

A graph is called ***planar*** if it can be drawn on the plane with no edge crossings.

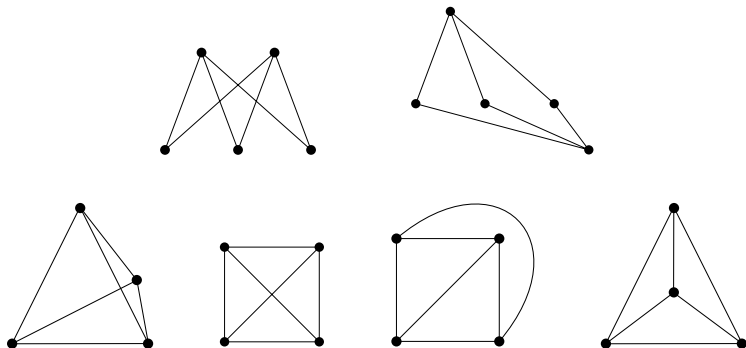
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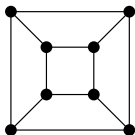
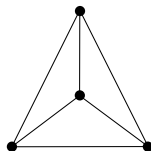
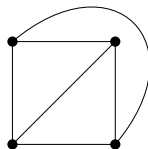
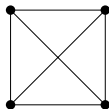
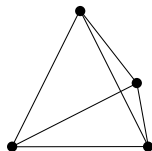
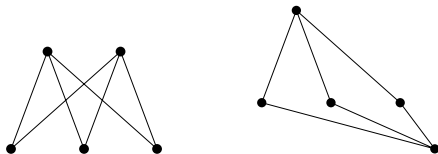
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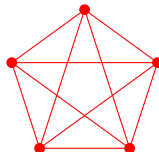
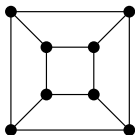
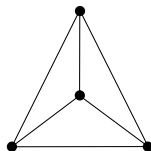
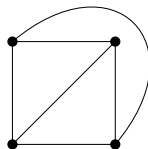
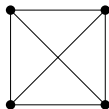
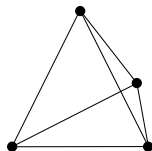
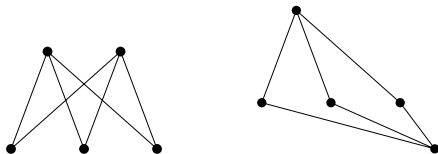
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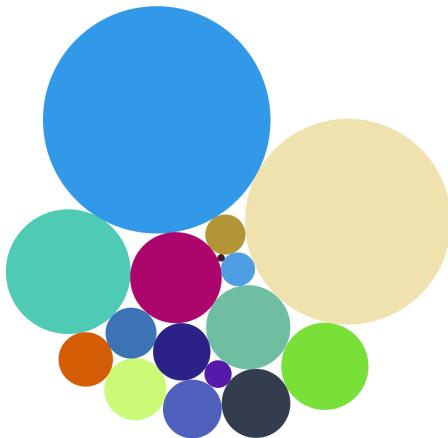
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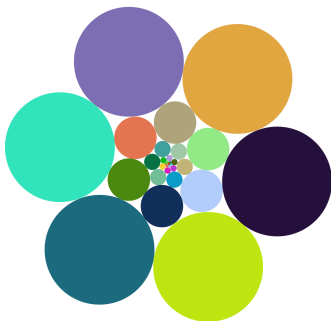
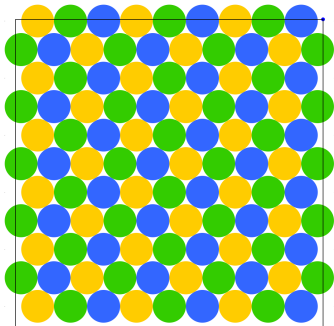


2. Circle packings

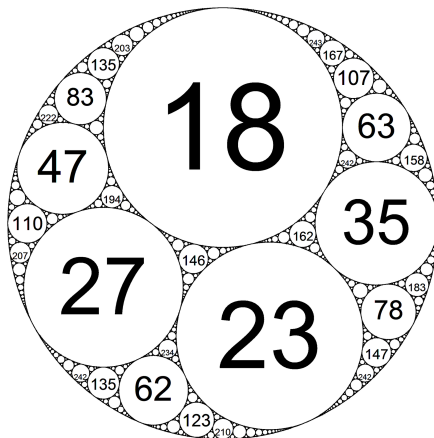
A ***circle packing*** is a collection of circles that are either disjoint or tangent.



2. Circle packings



Apollonian gasket:



The curvatures (inverse radii) of four mutually tangent circles satisfy:

$$(a + b + c + d)^2 = a^2 + b^2 + c^2 + d^2$$

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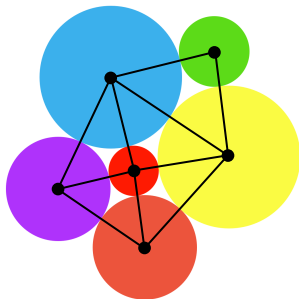
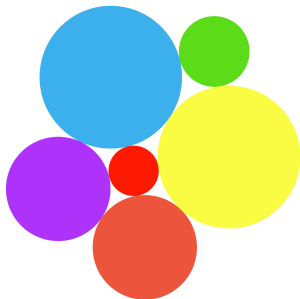
Key observation. A circle packing determines a graph :

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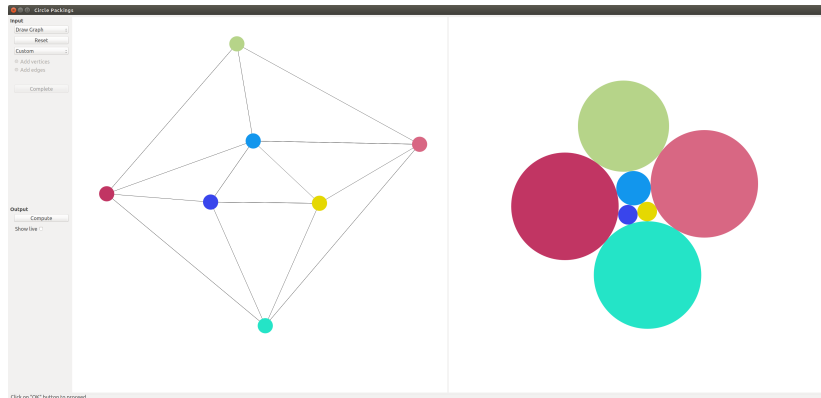
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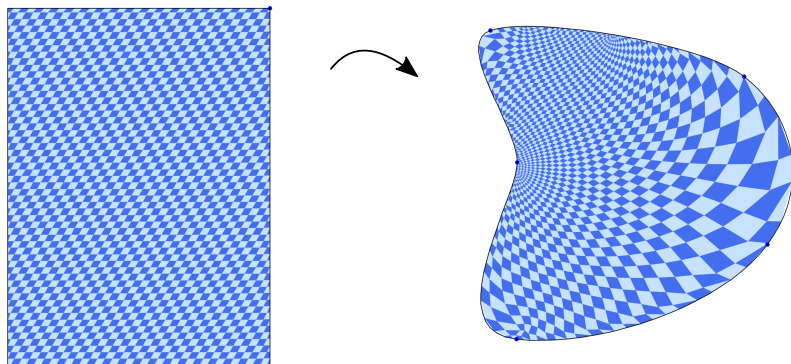


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A ***conformal map*** between two regions of the plane is a transformation that preserves shapes infinitesimally. More precisely: it preserves angles.

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