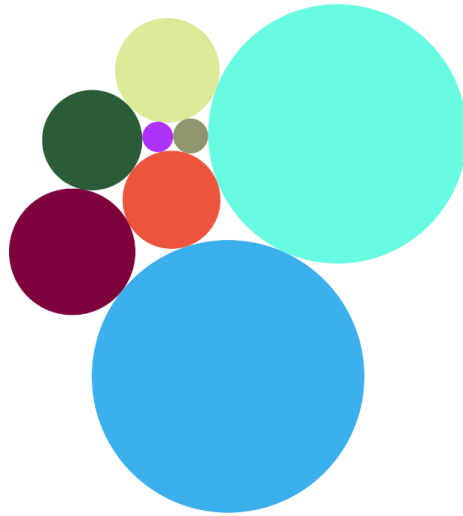
HITS
Heidelberg Institute for
Theoretical Studies

Planar graphs, circle packings, and conformal maps

Brice Loustau (HITS & Heidelberg University)



HITS Lab Meeting

07.09.2020

Planar graphs, circle packings, and conformal maps

Outline.

1. Planar graphs
2. Circle packings
3. Conformal maps
4. Beyond

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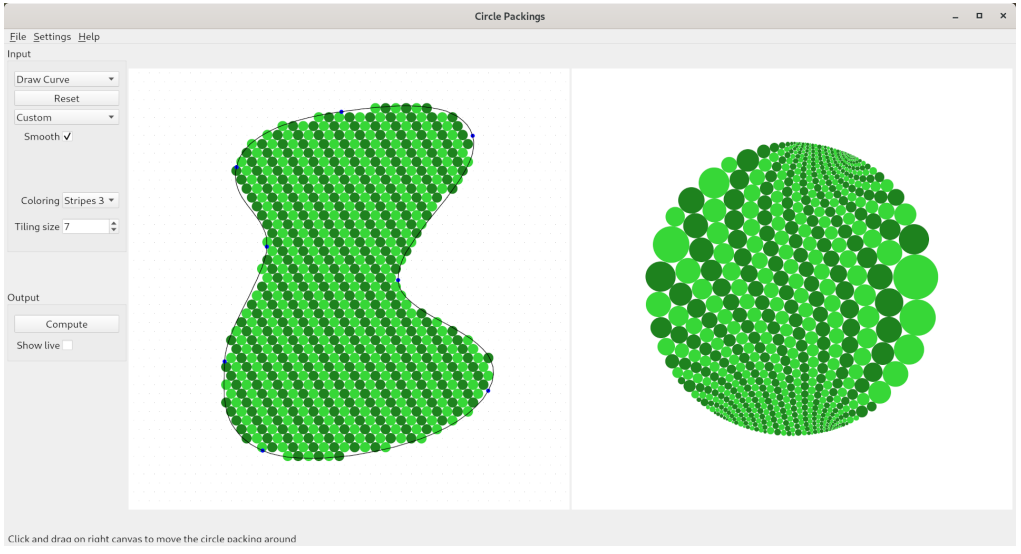

The software: *Circle Packings* (with B. Beeker)
brice.loustau.eu/circlepackings

A ***graph*** is a data structure consisting of:

- A set of *vertices*
- A set of *edges* = relation between vertices

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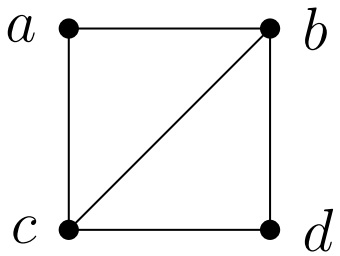

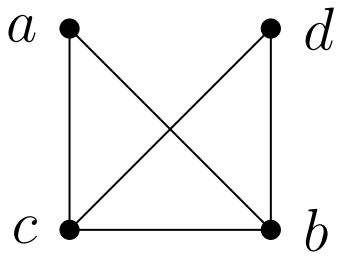

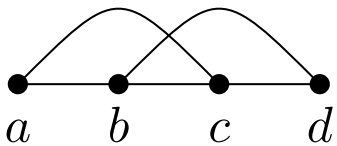

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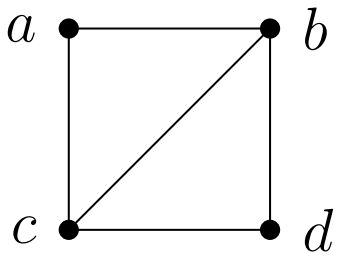

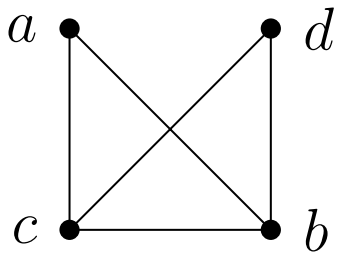

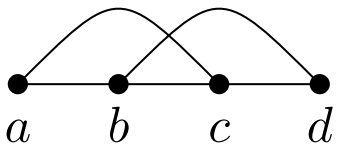

Applications of graph theory: Computer science (networks), linguistics, physics and chemistry, biology, social sciences, etc.

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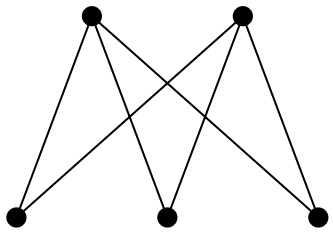

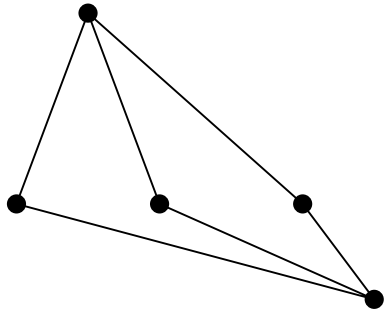

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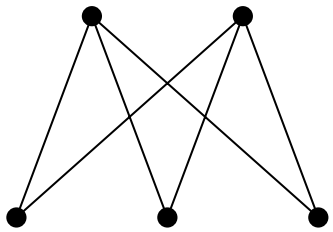

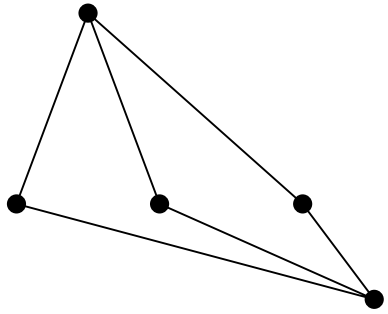

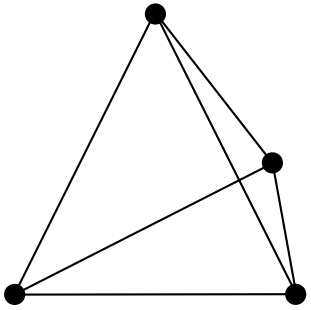

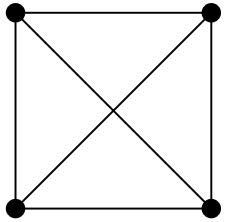

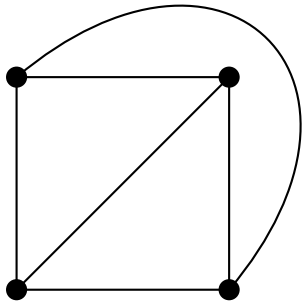

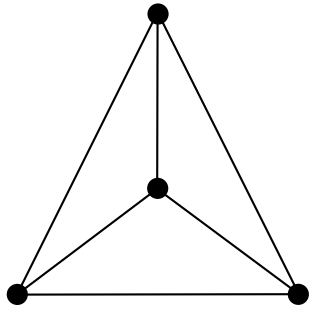

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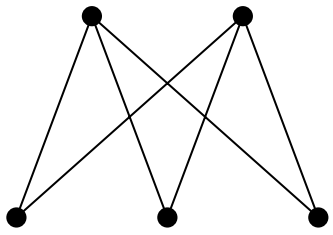

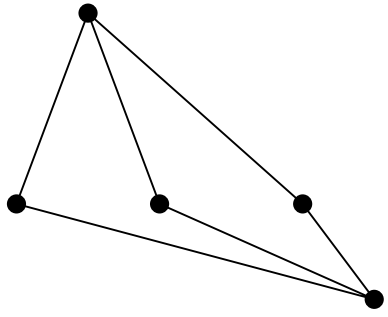

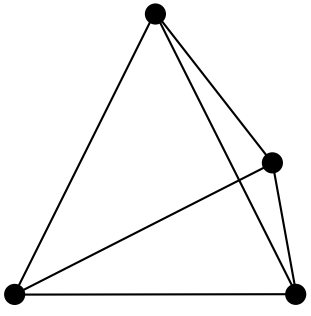

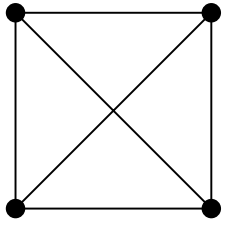

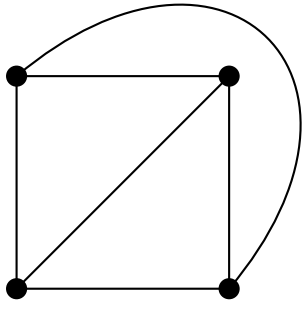

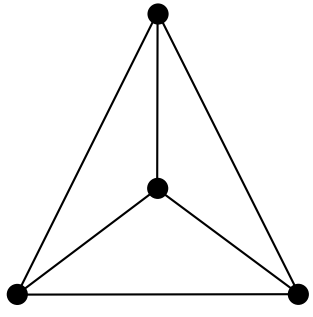

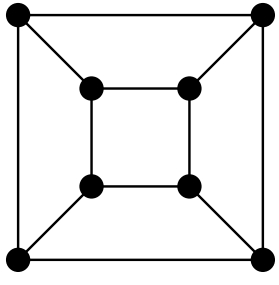

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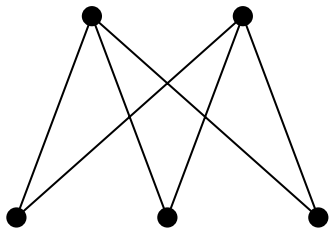
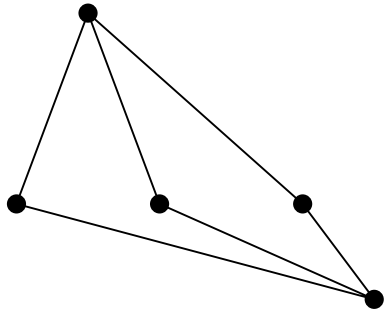
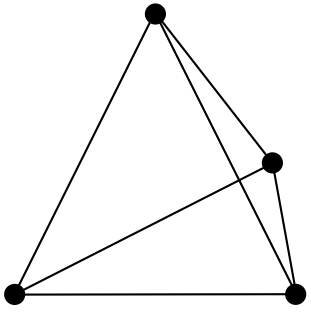
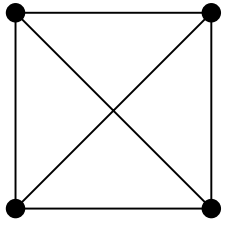
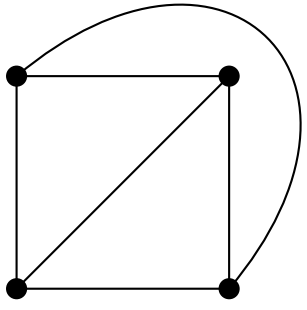
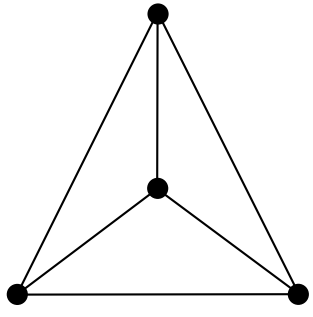


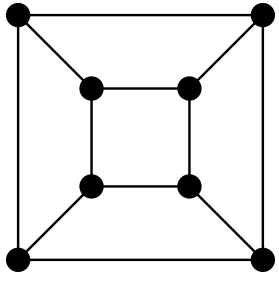


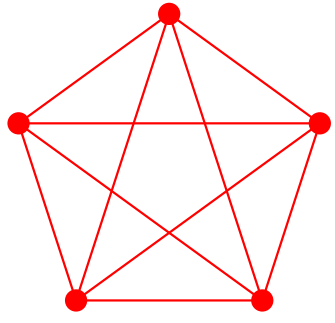




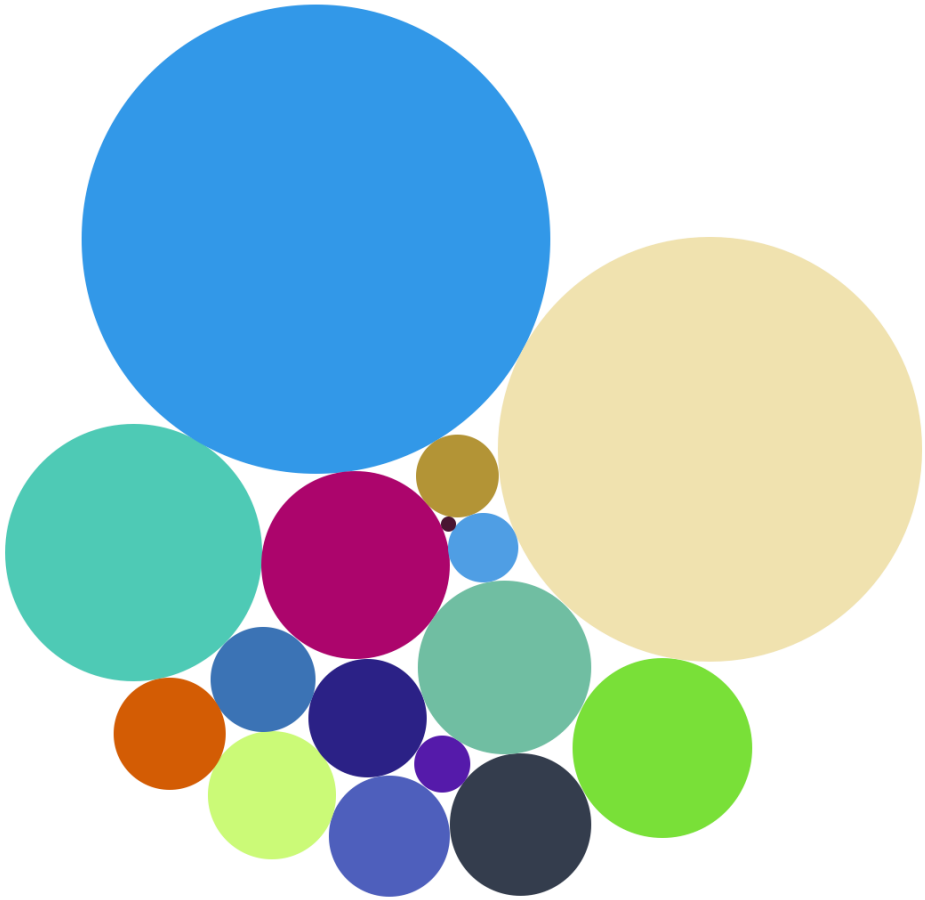




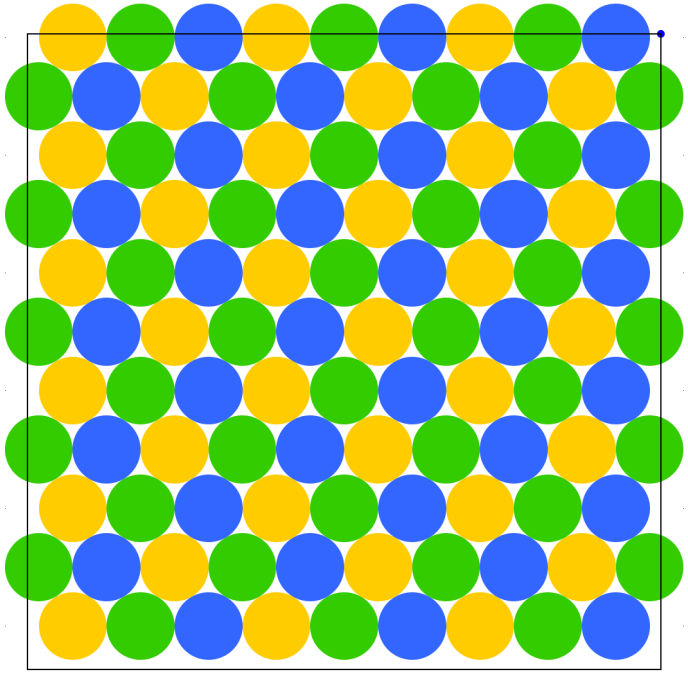


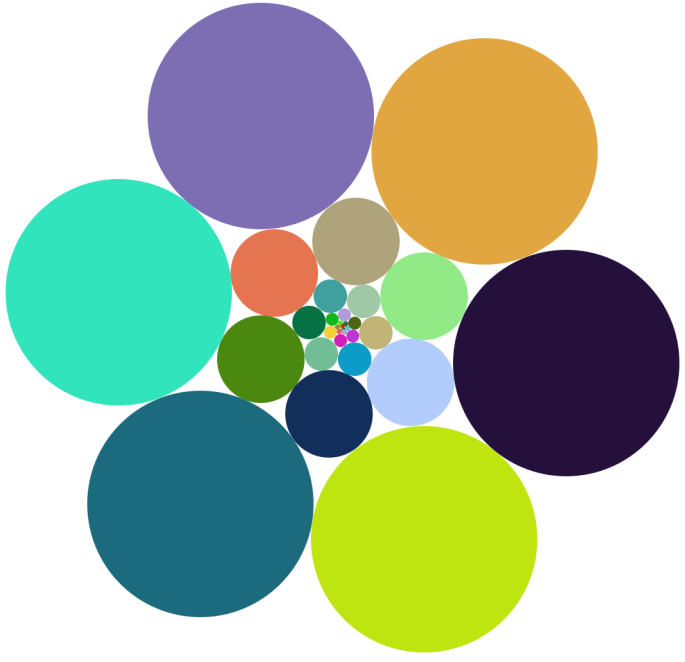


A ***circle packing*** is a collection of circles that are either disjoint or tangent.

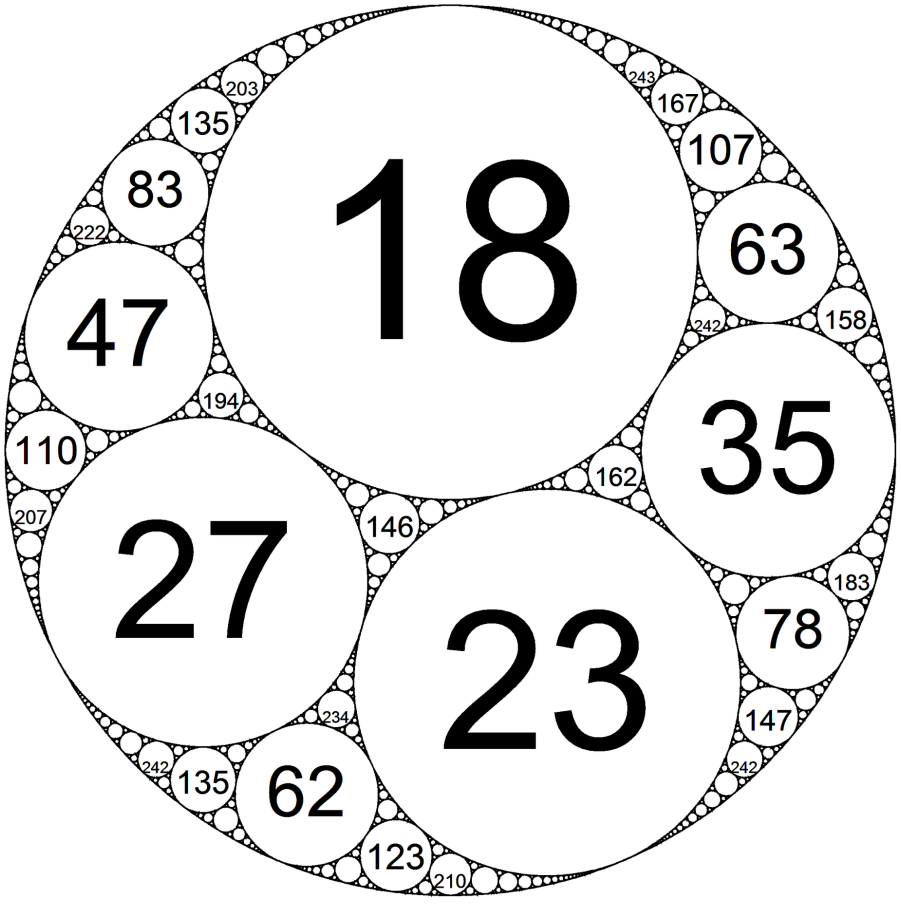


2. Circle packings





Apollonian gasket:



The curvatures (inverse radii) of four mutually tangent circles satisfy:
$(a+b+c+d)^2 = a^2+b^2+c^2+d^2$

Key observation. A circle packing determines a graph :

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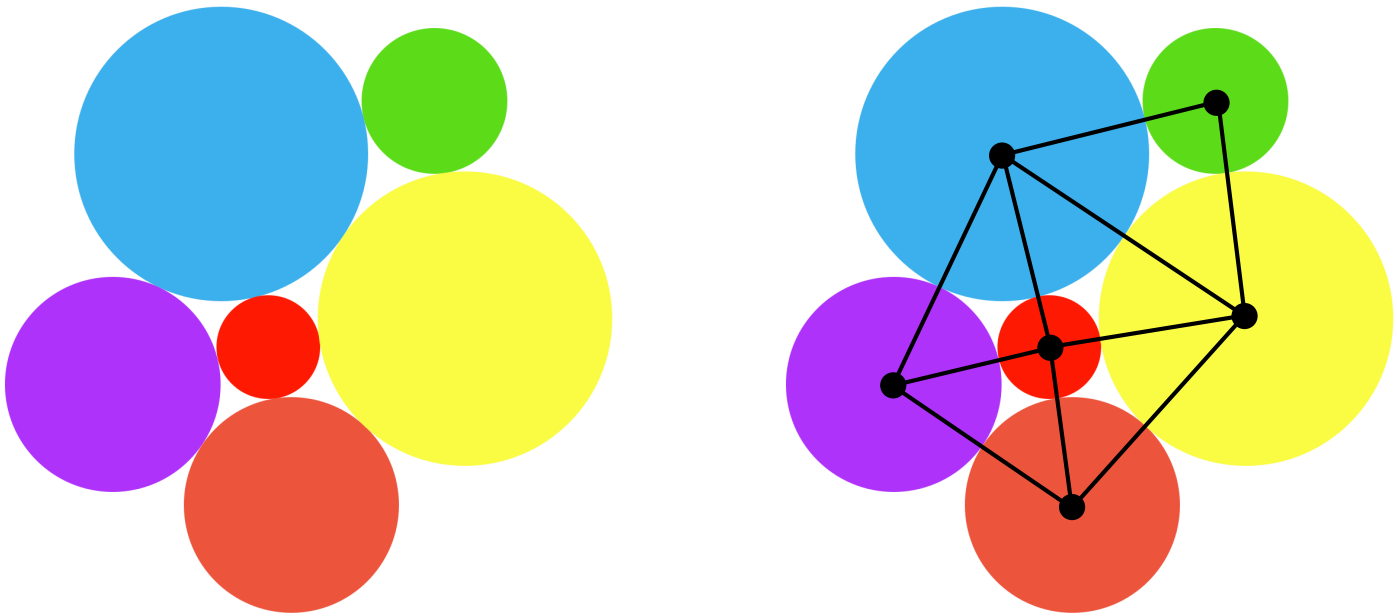

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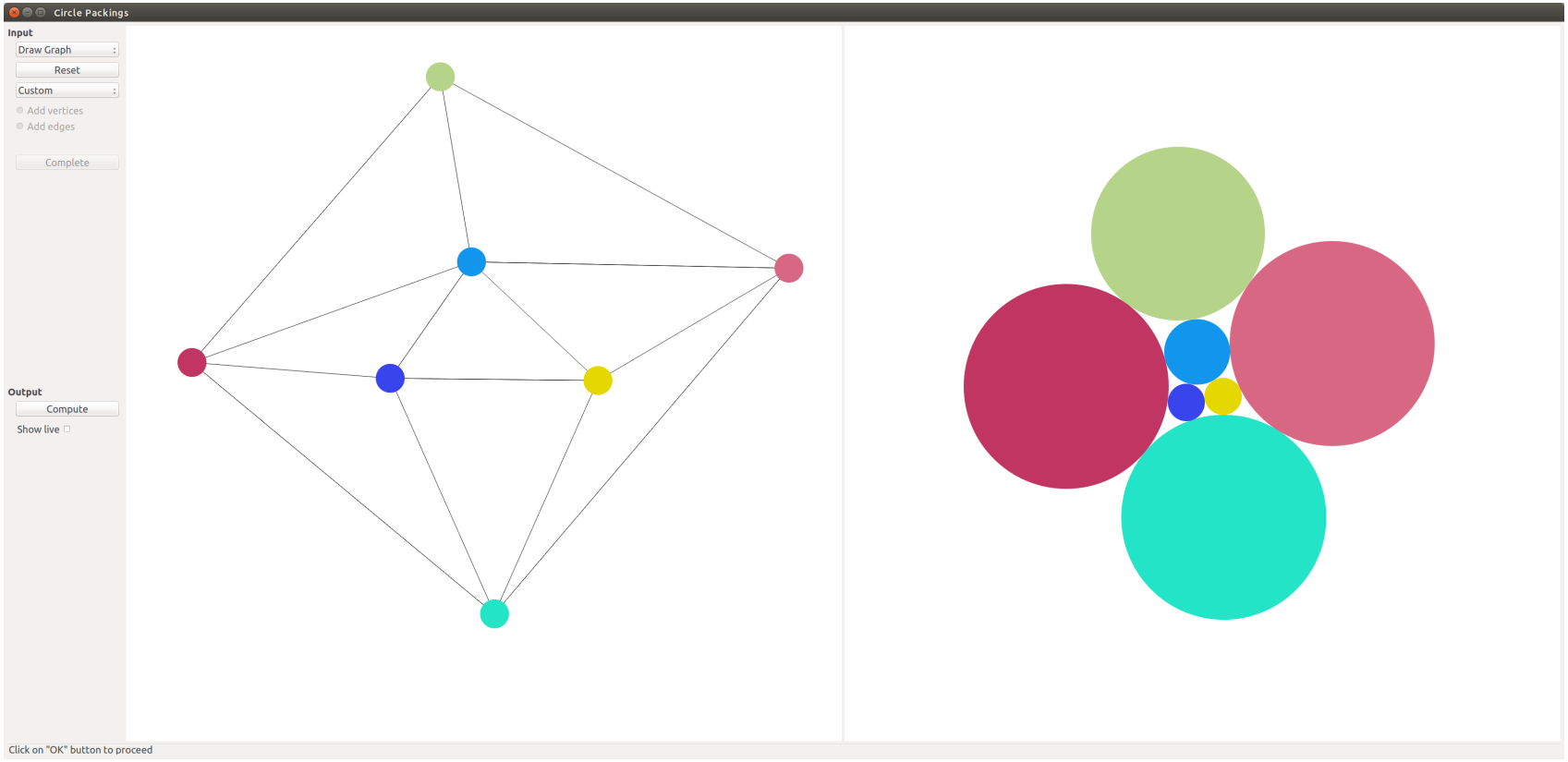

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