Planar graphs, circle packings, and conformal maps

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Planar graphs, circle packings, and conformal maps

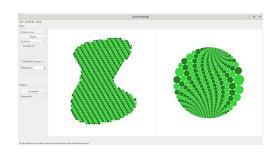
Outline.

- 1. Planar graphs
- 2. Circle packings
- 3. Conformal maps
- 4. Beyond

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The software: *Circle Packings* (with B. Beeker) brice.loustau.eu/circlepackings

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- A set of vertices
- A set of *edges* = relation between vertices

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Applications of graph theory: Computer science (networks), linguistics, physics and chemistry, biology, social sciences, etc.

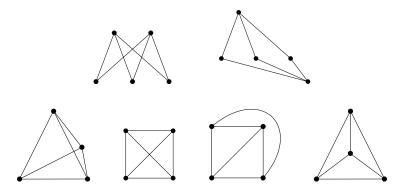
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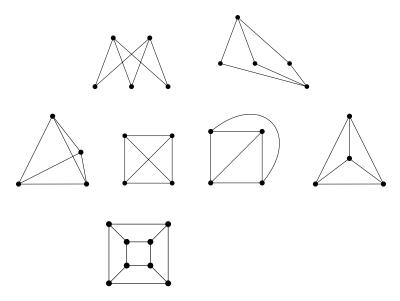




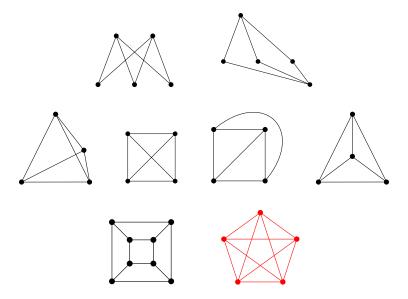
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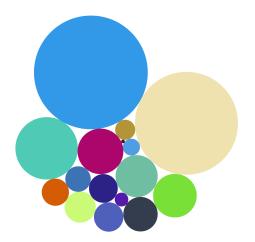


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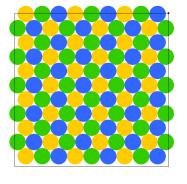


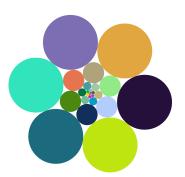
2. Circle packings

A $\emph{circle packing}$ is a collection of circles that are either disjoint or tangent.



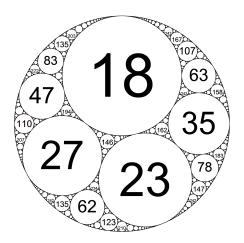
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Appollonian gasket:



The curvatures (inverse radii) of four mutually tangent circles satisfy: $(a+b+c+d)^2=a^2+b^2+c^2+d^2$