Basic Variable	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
Z	1	0	0	14/3	5/3	2/3	0	190
x_2	0	0	1	5/3	2/3	-1/3	0	40
x_1	0	1	0	-1/3	-1/3	2/3	0	10
x_6	0	0	0	4/3	1/3	-5/3	1	10

$x_1 = 10$
$x_2 = 40$
$x_3 = 0$

is the optimal solution, for which $Z = 190$

5. Minimize $3x_1 + 2.5x_2$

Subject to

$$x_1 + 2x_2 \geq 20$$
$$5x_1 + 2x_2 \geq 50$$
$$x_1, x_2 \geq 0$$

Minimizing $Z = 3x_1 + 2.5x_2$

is equivalent to

maximizing $-Z = -3x_1 - 2.5x_2$

$$x_1 + 2x_2 \geq 20 \qquad\qquad x_1 + 2x_2 - x_3 + \bar{x}_4 = 20$$

$$5x_1 + 2x_2 \geq 50 \quad \Rightarrow \quad 5x_1 + 2x_2 - x_5 + \bar{x}_6 = 50$$

$$x_1, x_2 \geq 0 \qquad\qquad x_1, x_2, x_3, \bar{x}_4, x_5, \bar{x}_6 \geq 0$$

maximizing $-Z = -3x_1 - 2.5x_2 - M\bar{x}_4 - M\bar{x}_6$

Basic Variable	Z	x_1	x_2	x_3	$\bar{x}_4$	x_5	$\bar{x}_6$	RHS
Z	-1	3	2.5	0	m	0	m	0
$\bar{x}_4$	0	1	2	-1	1	0	0	20
$\bar{x}_6$	0	5	2	0	0	-1	1	50

entering variable

Basic Variable	Z	x_1	x_2	x_3	$\bar{x}_4$	x_5	$\bar{x}_6$	RHS
Z	-1	$-6m+3$	$-4m + 2.5$	m	0	m	0	$-70m$
$\bar{x}_4$	0	1	2	-1	1	0	0	20
$\bar{x}_6$	0	$\frac{5}{5} = 1$	$\frac{2}{5}$	0	0	$\frac{-1}{5}$	$\frac{1}{5}$	$\frac{\cancel{50}^{10}}{\cancel{5}}$

leaving

entering variable

Basic Variable	Z	x_1	x_2	x_3	$\bar{x}_4$	x_5	$\bar{x}_6$	RHS
Z	-1	0	$-8m/5 + 1.3$	m	0	$-m/5 + 3/5$	$6m/5 - 3/5$	$-10m - 30$
$\bar{x}_4$	0	0	$8/5$	-1	1	$1/5$	$-1/5$	10
x_1	0	1	$\frac{2}{5}$	0	0	$\frac{-1}{5}$	$\frac{1}{5}$	10

leaving

Basic Variable	Z	x_1	x_2	x_3	$\overline{x}_4$	x_5	$\overline{x}_6$	RHS
Z	-1	0	0	$13/16$	$m - \frac{13}{16}$	$\frac{7}{16}$	$m - 7/16$	$\frac{-305}{8}$
x_2	0	0	1	$-5/8$	$5/8$	$1/8$	$-1/8$	$25/4$
x_1	0	1	0	$1/4$	$-1/4$	$-1/4$	$1/4$	$15/2$

Solution:

$x_1 = \frac{15}{2}$ at which $Z = \frac{305}{8}$

$x_2 = \frac{25}{4}$

6. Maximize

$Z = 30x_1 + 20x_2$

Subject to

$-x_1 - x_2 \geq -8 \longrightarrow x_1 + x_2 \leq 8$

$-6x_1 - 4x_2 \leq -12 \longrightarrow 6x_1 + 4x_2 \geq 12$

$5x_1 + 8x_2 = 20$

$x_1, x_2 \geq 0$

$\Downarrow$

constraints:

$$\begin{cases} x_1 + x_2 + x_3 = 8 & \text{—— (1)} \\ 6x_1 + 4x_2 - x_4 + \overline{x}_5 = 12 & \text{—— (2)} \\ 5x_1 + 8x_2 + \overline{x}_6 = 20 & \text{—— (3)} \\ x_1, x_2, x_3, x_4, \overline{x}_5, \overline{x}_6 \geq 0 \end{cases}$$

objective: maximize

$$Z = 30x_1 + 20x_2 - M\bar{x}_5 - M\bar{x}_6$$

Adding ② & ③,

$$11x_1 + 12x_2 - x_4 + \bar{x}_5 + \bar{x}_6 = 32$$

$$-11Mx_1 - 12Mx_2 + Mx_4 - M\bar{x}_5 - M\bar{x}_6 = -32M$$

$$\Rightarrow Z - 11Mx_1 - 12Mx_2 + Mx_4 = 30x_1 + 20x_2 - 32M$$

$$Z + x_1(-11M - 30) + x_2(-12M - 20) + x_4 M = -32M$$

Basic Variable	Z	x_1	x_2 (entering)	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	RHS
Z	1	$-11m-30$	$-12m-20$	0	m	0	0	$-32m$
x_3	0	1	1	1	0	0	0	8
$\bar{x}_5$	0	6	4	0	-1	1	0	12
$\bar{x}_6$ (leaving)	0	5	8	0	0	0	1	20

Basic Variable	Z	x_1 (entering)	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	RHS
Z	1	$-\frac{7m}{2}-\frac{35}{2}$	0	0	M	0	$\frac{3m}{2}+5/2$	$-2m+50$
x_3	0	$3/8$	0	1	0	0	$-1/8$	$11/2$
$\bar{x}_5$ (leaving)	0	$7/2$	0	0	-1	1	$-1/2$	2
x_2	0	$5/8$	1	0	0	0	$1/8$	$5/2$

entering

Basic Variable	z	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	RHS
z	1	0	0	0	-5	$m+5$	m	60
x_3	0	0	0	1	$3/28$	$-3/28$	$-1/14$	$37/7$
x_1	0	1	0	0	$-2/7$	$2/7$	$-1/7$	$4/7$
x_2	0	0	1	0	$5/28$	$-5/28$	$3/14$	$15/7$

leaving

Basic Variable	z	x_1	x_2	x_3	x_4	$\bar{x}_5$	$\bar{x}_6$	RHS
z	1	0	28	0	0	m	$m+6$	120
x_3	0	0	$-3/5$	1	0	0	$-1/5$	4
x_1	0	1	$8/5$	0	0	0	$1/5$	4
x_4	0	0	$28/5$	0	1	-1	$6/5$	12

Solution:

$x_1 = 4$ for which $z = 120$

$x_2 = 0$

7. Minimize $0.4x_1 + 0.5x_2$

Subject to $0.3x_1 + 0.1x_2 \le 1.8$

$x_1 + x_2 = 12$

$0.6x_1 + 0.4x_2 \ge 6$

$x_1, x_2 \le 0$

no feasible soln ← two negative nos can't add up to 12

Putting $y_1 = -x_1$, $y_2 = -x_2$

we get

Minimize $-0.4y_1 - 0.5y_2$

subject to

$-0.3y_1 - 0.1y_2 \le 1.8$

$-y_1 - y_2 = 12$

$-0.6y_1 - 0.4y_2 \ge 6$

$y_1, y_2 \ge 0$

$\Downarrow$

Minimize $-0.4y_1 - 0.5y_2 + M\bar{y}_4 + M\bar{y}_6$

$-0.3y_1 - 0.1y_2 + y_3 = 1.8$

$-y_1 - y_2 + \bar{y}_4 = 12$

$-0.6y_1 - 0.4y_2 - y_5 + \bar{y}_6 = 6$

$y_1, y_2, y_3, \bar{y}_4, y_5, \bar{y}_6$

is equivalent to

Phase 1: Minimize $\bar{y}_4 + \bar{y}_6$ (until $\bar{y}_4 = 0$, $\bar{y}_6 = 0$)

Phase 2: Minimize $-0.4y_1 - 0.5y_2$ (with $\bar{y}_4 = 0$, $\bar{y}_6 = 0$)

Phase 1: Minimize

$$Z = \bar{y}_4 + \bar{y}_6 \Rightarrow \text{Maximize } -Z = -\bar{y}_4 - \bar{y}_6$$

with
$$-0.3y_1 - 0.1y_2 + y_3 = 1.8$$
$$-y_1 - y_2 + \bar{y}_4 = 12$$
$$-0.6y_1 - 0.4y_2 - y_5 + \bar{y}_6 = 6$$
$$y_1, y_2, y_3, \bar{y}_4, y_5, \bar{y}_6 \geq 0$$

$$-Z = -\bar{y}_4 - \bar{y}_6 = -(12 + y_1 + y_2) - (6 + 0.6y_1 + 0.4y_2 + y_5)$$
$$= -12 - y_1 - y_2 - 6 - 0.6y_1 - 0.4y_2 - y_5$$
$$= -18 - 1.6y_1 - 1.4y_2 - y_5$$

Basic Variable	Z	y_1	y_2	y_3	$\bar{y}_4$	y_5	$\bar{y}_6$	RHS
Z	-1	1.6	1.4	0	0	1	0	-18
y_3	0	-0.3	-0.1	1	0	0	0	1.8
$\bar{y}_4$	0	-1	-1	0	1	0	0	12
$\bar{y}_6$	0	-0.6	-0.4	0	0	-1	1	6

No basic feasible solution exists

(Looks like $x_1, x_2 \leq 0$ is a misprint in the question)

8. Maximize $2x_1 + 3x_2 + x_3$

subject to
$$x_1 + x_2 + x_3 \le 40$$
$$2x_1 + x_2 - x_3 \ge 10$$
$$-x_2 + x_3 \ge 10$$
$$x_1, x_2, x_3 \ge 0$$

$$\Downarrow$$

$$x_1 + x_2 + x_3 + x_4 = 40$$
$$2x_1 + x_2 - x_3 - x_5 + \bar{x}_6 = 10$$
$$-x_2 + x_3 - x_7 + \bar{x}_8 = 10$$

$$x_1, x_2, x_3, x_4, x_5, \bar{x}_6, x_7, \bar{x}_8 \ge 0$$

Phase-1 Maximize

$$-z = -\bar{x}_6 - \bar{x}_8$$
$$= -(10 - 2x_1 - \cancel{x_2} + \cancel{x_3} + x_5)$$
$$-(10 + \cancel{x_2} - \cancel{x_3} + x_7)$$

$$-z = -20 + 2x_1 - x_5 - x_7$$

entering

Basic Variable	z	x_1	x_2	x_3	x_4	x_5	$\bar{x}_6$	x_7	$\bar{x}_8$	RHS
z	-1	-2	0	0	0	1	0	1	0	-20
x_4	0	1	1	1	1	0	0	0	0	40
$\bar{x}_6$	0	2	1	-1	0	-1	1	0	0	10
$\bar{x}_8$	0	0	-1	1	0	0	0	-1	1	10

leaving

~~leaving~~ entering

Basic variable	Z	x_1	x_2	x_3	x_4	x_5	$\bar{x}_6$	x_7	$\bar{x}_8$	RHS
Z	-1	0	1	-1	0	0	1	1	0	-10
x_4	0	0	$1/2$	$3/2$	1	$1/2$	$-1/2$	0	0	35
x_1	0	1	$1/2$	$-1/2$	0	$-1/2$	$1/2$	0	0	5
$\bar{x}_8$	0	0	-1	1	0	0	0	-1	1	10

leaving

Basic Variable	Z	x_1	x_2	x_3	x_4	x_5	$\bar{x}_6$	x_7	$\bar{x}_8$	RHS
Z	-1	0	0	0	0	0	1	0	1	0
x_4	0	0	2	0	1	$1/2$	$-1/2$	$3/2$	$-3/2$	20
x_1	0	1	0	0	0	$-1/2$	$1/2$	$-1/2$	$1/2$	10
x_3	0	0	-1	1	0	0	0	-1	1	10

Initial Basic Feasible Solution:

$x_4 = 20 \quad x_2 = 0 \quad x_7 = 0$

$x_1 = 10 \quad x_5 = 0 \quad \bar{x}_8 = 0$

$x_3 = 10 \quad \bar{x}_6 = 0$

Phase-2

Maximize

$$Z = 2x_1 + 3x_2 + x_3$$

Basic Variable	Z	x_1	x_2	x_3	x_4	x_5	x_7	RHS
Z	1	-2	-3	-1	0	0	0	0
x_4	0	0	2	0	1	1/2	3/2	20
x_1	0	1	0	0	0	-1/2	-1/2	10
x_3	0	0	-1	1	0	0	-1	10

entering

Basic Variable	Z	x_1	x_2	x_3	x_4	x_5	x_7	RHS
Z	1	0	-4	0	0	-1	-2	30
x_4	0	0	2	0	1	1/2	3/2	20
x_1	0	1	0	0	0	-1/2	-1/2	10
x_3	0	0	-1	1	0	0	-1	10

leaving

Basic Variable	Z	x_1	x_2	x_3	x_4	x_5	x_7	RHS
Z	1	0	0	0	2	0	1	70
x_2	0	0	1	0	1/2	1/4	3/4	10
x_1	0	1	0	0	0	-1/2	-1/2	10
x_3	0	0	0	1	1/2	1/4	-1/4	20

Solution:

$x_1 = 10$
$x_2 = 10$ for which $z = 70$
$x_3 = 20$

9. Maximize $z = 3x_1 + x_2$

subject to
$$x_1 + 2x_2 \leq 5$$
$$x_1 + x_2 - x_3 \leq 2$$
$$7x_1 + 3x_2 - 5x_3 \leq 20$$
$$x_1, x_2, x_3 \geq 0$$
$$\Downarrow$$
$$x_1 + 2x_2 + x_4 = 5$$
$$x_1 + x_2 - x_3 + x_5 = 2$$
$$7x_1 + 3x_2 - 5x_3 + x_6 = 20$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$

entering

Basic variable	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
Z	1	-3	-1	0	0	0	0	0
x_4	0	1	2	0	1	0	0	5
x_5	0	1	1	-1	0	1	0	2
x_6	0	7	3	-5	0	0	1	20

leaving

entering

Basic Variable	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
Z	1	0	2	-3	0	3	0	6
x_4	0	0	1	1	1	-1	0	3
x_1	0	1	1	-1	0	1	0	2
x_6	0	0	-4	2	0	-7	1	6

leaving

Tie between leaving basic variables

$\Rightarrow$ Degenerate Solution

Basic Variable	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
Z	1	0	5	0	3	0	0	15
x_3	0	0	1	1	1	-1	0	3
x_1	0	1	2	0	1	0	0	5
x_6	0	0	-6	0	-2	-5	1	0

Solution:

$x_1 = 5$

$x_2 = 0$ at which $Z = 15$

$x_3 = 3$

10. Minimize $z = -x_1 + x_2$

Subject to $x_1 - 4x_2 \geq 5$

$x_1 - 3x_2 \leq 1$

$2x_1 - 5x_2 \geq 1$

$x_1, x_2 \geq 0$

$\Downarrow$

Maximize $-z = x_1 - x_2$

Subject to $x_1 - 4x_2 - x_3 + \bar{x}_4 = 5$

$x_1 - 3x_2 + x_5 = 1$

$2x_1 - 5x_2 - x_6 + \bar{x}_7 = 1$

$x_1, x_2, x_3, \bar{x}_4, x_5, x_6, \bar{x}_7$

Phase-1

Maximize $-z = -\bar{x}_4 - \bar{x}_7$

Basic Variable	z	x_1	x_2	x_3	$\bar{x}_4$	x_5	x_6	$\bar{x}_7$	RHS
z	-1	0	0	0	1	0	0	1	0
$\bar{x}_4$	0	1	-4	-1	1	0	0	0	5
x_5	0	1	-3	0	0	1	0	0	1
$\bar{x}_7$	0	2	-5	0	0	0	-1	1	1

entering

Basic Variable	Z	x_1	x_2	x_3	$\bar{x}_4$	x_5	x_6	$\bar{x}_7$	RHS
Z	-1	-3	9	1	0	0	1	0	-6
$\bar{x}_4$	0	1	-4	-1	1	0	0	0	5
x_5	0	1	-3	0	0	1	0	0	1
$\bar{x}_7$	0	2	-5	0	0	0	-1	1	1

leaving

entering

Basic Variable	Z	x_1	x_2	x_3	$\bar{x}_4$	x_5	x_6	$\bar{x}_7$	RHS
Z	-1	0	$3/2$	1	0	0	$-1/2$	$3/2$	$-9/2$
$\bar{x}_4$	0	0	$-3/2$	-1	1	0	$1/2$	$-1/2$	$9/2$
x_5	0	0	$-1/2$	0	0	1	$1/2$	$-1/2$	$1/2$
x_1	0	1	$\frac{-5}{2}$	0	0	0	$\frac{-1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$

leaving

Basic variable	Z	x_1	x_2	x_3	$\bar{x}_4$	x_5	x_6	$\bar{x}_7$	RHS
Z	-1	0	1	1	0	1	0	1	-4
$\bar{x}_4$	0	0	-1	-1	1	-1	0	0	4
x_6	0	0	-1	0	0	2	1	-1	1
x_1	0	1	-3	0	0	1	0	0	1

$\bar{x}_4 = 4 \neq 0$

Hence, feasible solution doesn't exist. Even if we change the objective to maximization type, there would not exist any optimal solution because there doesn't exist any feasible region satisfying all constraints.