Department of Industrial & Systems Engineering
Indian Institute of Technology, Kharagpur
Mid-Semester Examination – Spring 2017
Operations Research (IM41082)

Time: 2 Hrs (Attempt ALL questions) **Full Mark: 30**

Question 1 (5 Marks)

Carco has a $150,000 advertising budget. To increase automobile sales, the firm is considering advertising in newspapers and on television. The more Carco uses a particular medium, the less effective is each additional advertisement. The following table shows the number of new customers reached by each advertisement.

	Number of advertisements	New Customers/advertisement
Newspaper	1 – 10	900
	11 –20	600
	21 –30	300
Television	1 – 5	10000
	6 –10	5000
	11 –15	2000

Each newspaper advertisement costs $1,000, and each television advertisement costs $10,000. At most, 30 newspaper advertisements and 15 television advertisements can be placed. Formulate an LP that can be used by Carco to maximize the number of new customers created by advertising. Be sure to define all of your variables and constraints.

Question 2 (2+4 Marks)

a) Consider the following LP:

maximize $Z = 10x_1 + x_2$

s.t. $x_1 \leq 1,\ 20x_1 + x_2 \leq 100,\ x_1, x_2 \geq 0$

Find all the basic solutions for this LP and identify the basic feasible solutions.

b) Consider the following problem, where the value of c has not yet been ascertained.

Maximize $Z = cx_1 + x_2$

s.t. $x_1 + x_2 \leq 6,\ x_1 + 2x_2 \leq 10$ and $x_1 \geq 0,\ x_2 \geq 0$

Use graphical analysis to determine the optimal solution(s) for (x_1, x_2) for the various possible values of $c(-\infty < c < \infty)$.

Question 3 (1+1+1+2 Marks)

Consider the following two variable linear program.

Maximize $z = x_1 + x_2$

subject to: $x_1 - x_2 \geq 1,\ x_1 + x_2 \leq 3,\ 2x_1 - x_2 \leq 3$ and $x_1 \geq 0,\ x_2 \geq 0$

a) Shade the feasible region.

b) Solve the problem graphically.

c) Show graphically that the optimal solution is degenerate.

d) On the figure, indicate which constraint can be dropped to obtain a nondegenerate optimal solution.

Question 4 (5 Marks)

Solve the following LP using Simplex algorithm. In case of alternative/degenerate/unbounded solution, specify the nature of the solution with suitable reason.

Maximize $z = 3x_1 + x_2 + 5x_3 + 4x_4$

subject to: $3x_1 - 3x_2 + 2x_3 + 8x_4 \leq 50$

$4x_1 + 6x_2 - 4x_3 - 4x_4 \leq 40$

$4x_1 - 2x_2 + x_3 + 3x_4 \leq 20$

and $x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0,\ x_4 \geq 0.$

Question 5 (5 Marks)

Using the Big-M / 2-phase method show that the following linear program has no feasible solution.

Maximize $z = 2x_1 - 3x_2 + 3x_3$

subject to: $4x_1 + x_2 + x_3 = 5$

$x_1 + x_2 + x_3 \geq 6$

$x_1 \geq 1$

$x_2 \geq 0,\ x3 \geq 0.$

Question 6 (4 Marks)

Answer the following:

a) A linear program has 5 decision variables and 3 less than equal to type constraints. What is the upper bound on the number of iterations of the simplex algorithm?

b) How do you detect infeasibility, alternative optima and unbounded solution for a problem while applying the simplex algorithm?

c) What is the logic behind minimum ratio rule?

d) In an LP with 4 decision variables (lower bound zero and upper bound infinity) and 2 less than equal to type constraints. Maximum how many variables can take non-zero value at optima?

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