

Department of Industrial & Systems Engineering
Indian Institute of Technology, Kharagpur
Mid-Semester Examination – Spring 2018
Operations Research (IM41082)
(Attempt ALL SIX questions)

Time: 2 Hrs

Full Mark: 30

Question 1 [4]

Each day, workers at a Toll Plaza work two 6-hour shifts chosen from 12 A.M. to 6 A.M., 6 A.M. to 12 P.M., 12 P.M. to 6 P.M., and 6 P.M. to 12 A.M. The following number of workers is needed during each shift: 12 A.M. to 6 A.M. — 15 workers; 6 A.M. to 12 P.M.—5 workers; 12 P.M. to 6 P.M.—12 workers; 6 P.M. to 12 A.M. — 6 workers. Workers whose two shifts are consecutive are paid \$12 per hour; workers whose shifts are not consecutive are paid \$18 per hour. Formulate an LP that can be used to minimize the cost of meeting the daily workforce demands of the Toll Plaza.

Question 2 [1+1+2+1]

Consider the following LP model

Maximize $Z = 5x_1 + 4x_2$

Subject to

$$6x_1 + 4x_2 \leq 24$$

$$6x_1 + 3x_2 \leq 22.5$$

$$x_1 + x_2 \leq 5$$

$$x_1 + 2x_2 \leq 6$$

$$-x_1 + x_2 \leq 1$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

- (a) Use graphical method to find the optimal solution of the problem
- (b) Identify the redundant constraint(s), if any.
- (c) Find the shadow price of constraints: $x_1 + 2x_2 \leq 6$ and $-x_1 + x_2 \leq 1$. Also, find the range of RHS (b_4 and b_5) of each of these two resources, in which their shadow prices remain valid.
- (d) Find the range of c_1 (profit coefficient of x_1) for which the current solution remains optimal.

Question 3 [1+2+1+1+1]

Consider the following LPP

Maximize $Z = 2x_1 - x_2 + x_3 + x_4$

Subject to $-x_1 + x_2 + x_3 + x_5 = 1$

$$x_1 + x_2 + x_4 = 2$$

$$2x_1 + x_2 + x_3 + x_6 = 6$$

$$x_1, x_2, \dots, x_6 \geq 0$$

- (a) Write down the initial basic feasible solution by inspection.
- (b) Find a feasible solution by increasing the nonbasic variable x_1 by one unit, while holding x_2 and x_3 as zero. What will be net change in the objective function?
- (c) What is the maximum increase in x_1 possible, subject to the constraints?
- (d) Find the new basic feasible solution when x_1 is increased to its maximum value found in (c).
- (e) Is the new basic feasible solution obtained in (d) optimal? Why or why not?

Question 4 [4]

Solve the following problem by the Big-M simplex method.

$$\text{Maximize } 3x_1 + x_2$$

$$\text{Subject to: } x_1 - x_2 \leq -1$$

$$-x_1 - x_2 \leq -3$$

$$2x_1 + x_2 \leq 4$$

$$x_1, x_2 \geq 0$$

Question 5 [2+1+2]

Consider the following linear program

$$\text{Minimize } 3x_1 + 6x_2 + 2x_3$$

$$\text{s.t. } x_1 + 3x_2 + 2x_3 \geq 6$$

$$2x_1 + x_2 + x_3 \geq 3$$

$$x_1, x_2, x_3 \geq 0$$

- (a) Construct and solve the phase I problem.
- (b) Identify the basic variables for the starting of Phase II
- (c) Find the optimal solution of phase II

Question 6 [5+1]

Consider the following linear program

$$\text{Maximize } 10x_1 + 12x_2 + 12x_3$$

$$\text{s.t. } x_1 + 2x_2 + 2x_3 \leq 20$$

$$2x_1 + x_2 + 2x_3 \leq 20$$

$$2x_1 + 2x_2 + x_3 \leq 20$$

$$x_1, x_2, x_3 \geq 0$$

- (a) Solve this LP using revised simplex or in matrix form of simplex
- (b) State the nature of the solution i.e. unique, infeasible, unbounded or multiple and justify it.
