Assignment 2

1. An airline has two-way flights between two cities A and B. The airline has four crews, and the crews can be stationed either at city A or at B. IATA rules require that the crew reaching any destination should not pick up the return flight within 3 hours of their arrival. In view of the excessive layover costs, the airline is interested in the minimization of the total layover time. If the airline operates between the cities as per the following schedule, formulate the problem as an assignment problem to find the best arrangement for the pairing of flights and the location of the crews. **(Construct the assignment problem table only, no need to solve it.)**

	From	To			From	To
Flight#	**A**	**B**		**Flight#**	**B**	**A**
1	6.00	8.30		**10**	7.30	9.30
2	8.15	10.45		**20**	9.15	11.15
3	13.30	16.00		**30**	16.30	18.30
4	15.00	17.30		**40**	20.00	22.00

2. A group of four boys and four girls are planning on a one day picnic. The extent of mutual happiness between a boy and a girl when they are together is given by the following matrix (data obtained from their previous experiences The problem is to decide the proper matching between the boys and the girls during the picnic that will maximize the sum of all the mutual happiness of all the couples. Formulate this as an assignment problem and solve using Hungarian algorithm.

		Girl			
		1	**2**	**3**	**4**
Boy	**1**	11	1	5	8
	2	9	9	8	1
	3	10	3	5	10
	4	1	13	12	11

3

a) How the following conditions can be represented as linear constraints using binary variables.

(i) Either $x_1 + x_2 \leq 2$ or $2x_1 + 3x_2 \geq 8$

(ii) Variable x_3 can assume values 0, 5, 9, and 12 only.

(iii) If $x_4 \leq 4$, then $x_5 \geq 6$. Otherwise $x_5 \leq 3$

b) Use branch-and-bound to solve the following problem. Clearly write the formulation of each subproblem and show the solution space.

$$\text{Min } Z = 50x_1 + 100x_2$$
$$\text{st } \quad 7x_1 + 2x_2 \geq 28$$
$$2x_1 + 12x_2 \geq 24$$
$$x_1, x_2 \geq 0, \text{ and integer}$$

4. At a hospital Vision, six types of surgical operations for eye are performed. The types of operations each surgeon is qualified to perform (indicated by an X) are given below. Suppose that surgeon 1 and surgeon 2 dislike each other and cannot be on duty at the same time. Formulate an IP to minimize the number of surgeons required so that the hospital can perform all types of surgery.

	Operation					
Surgeon	**1**	**2**	**3**	**4**	**5**	**6**
1	X	X		X		
2			X		X	X
3			X		X	
4	X					X
5		X				
6				X	X	

5. Each day, Sunco manufactures four types of gasoline: lead-free premium (LFP), lead-free regular (LFR), leaded premium (LP), and leaded regular (LR). Because of cleaning and resetting of machinery, the time required to produce a batch of gasoline depends on the type of gasoline last produced. For example, it takes longer to switch between a lead-free gasoline and a leaded gasoline than it does to switch between two lead-free gasolines. The time (in minutes) required to manufacture each day's gasoline requirements are shown in the Table given below. Use a branch-and bound approach to determine the order in which the gasolines should be produced each day (**DO NOT** present mathematical formulation). Clearly show all related calculations. Assume that the last gas produced yesterday precedes the first gas produced today.

Last-Produced Gasoline	**Gasoline to be next produced**			
	LFR	**LFP**	**LR**	**LP**
LFR	–	50	120	140
LFP	60	–	140	110
LR	90	130	–	60
LP	130	120	80	–

6

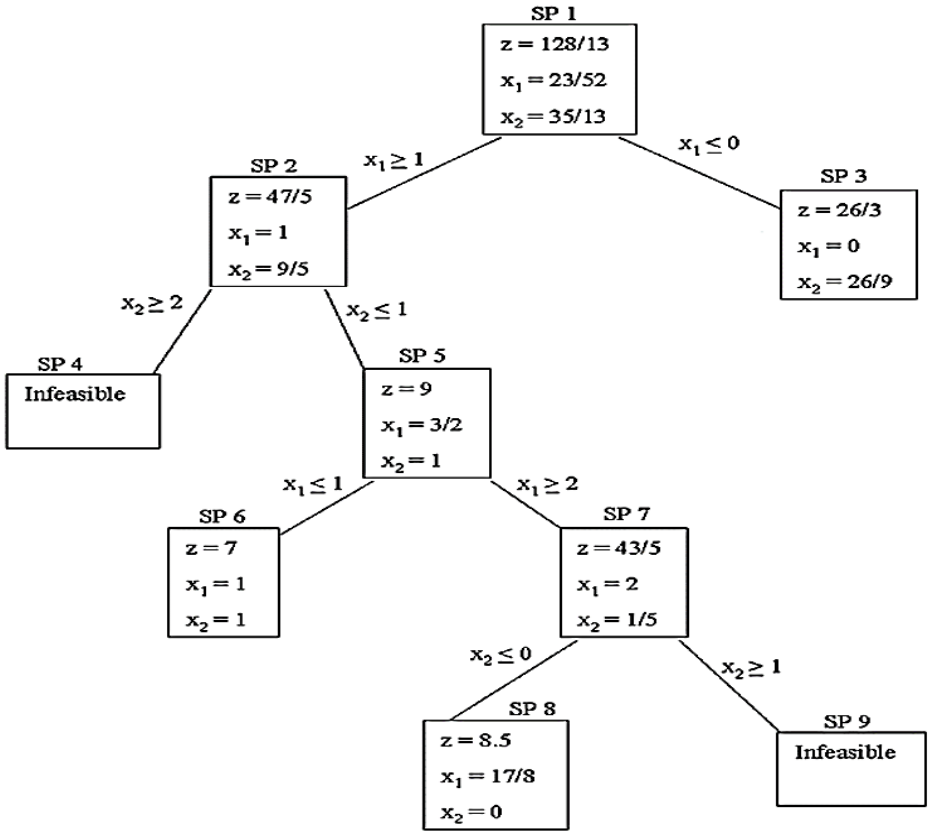
$$Z = 4x_1 + 3x_2$$
$$\text{s.t. } 4x_1 + 9x_2 \leq 26$$
$$8x_1 + 5x_2 \leq 17$$
$$x_1, x_2 \geq 0 \text{ and integer}$$

While solving above problem with branch and bound technique, we have a branch and bound solution tree at a certain stage as shown in the figure below:

(a) Is it a maximization or minimization problem? What is the incumbent (current best) solution?

(b) Indicate the node(s) that have been fathomed and explain why? Identify the node(s) that have not been fathomed and explain why not.

(c) Have we reached an optimal solution to the integer program? If yes, then explain why? If not, formulate the subsequent sub problem(s) and solve using graphical method to find the optimal solution.


SP 1
z = 128/13
x1 = 23/52
x2 = 35/13
x1 ≥ 1
x1 ≤ 0
SP 2
z = 47/5
x1 = 1
x2 = 9/5
SP 3
z = 26/3
x1 = 0
x2 = 26/9
x2 ≥ 2
x2 ≤ 1
SP 4
Infeasible
SP 5
z = 9
x1 = 3/2
x2 = 1
x1 ≤ 1
x1 ≥ 2
SP 6
z = 7
x1 = 1
x2 = 1
SP 7
z = 43/5
x1 = 2
x2 = 1/5
x2 ≤ 0
x2 ≥ 1
SP 8
z = 8.5
x1 = 17/8
x2 = 0
SP 9
Infeasible