Department of Industrial & Systems Engineering
Indian Institute of Technology, Kharagpur

Mid-Semester Examination – Spring 2016

Operations Research (IM41082)

Time: 2 Hrs (Attempt ALL questions) **Full Mark: 30**

Question 1 (5 Marks)

A small refinery blends five raw gasoline types to produce two grades of motor fuel: regular and premium. The number of barrels per day of each raw gasoline type available, the performance rating, and cost per barrel are given in the following table:

Raw Gasoline Type	Performance Rating	Available(Barrels /Day)	Cost/Barrel (₹)
1	70	2000	80
2	80	4000	90
3	85	4000	95
4	90	5000	115
5	99	5000	200

Regular motor fuel must have a performance rating of at least 85 and premium at least 95. The refinery's contract requires that at least 8,000 barrels/day of premium be produced; at the same time, the refinery can sell its entire output of both premium and regular for ₹375/barrel and ₹285/barrel, respectively. Assume the performance rating of a blend is proportional, i.e., a 50–50 mixture of raw gasoline types 1 and 2 has a performance of 75. Formulate a linear program to maximize the refinery's profit. Be sure to define all of your variables and constraints.

Question 2 (5 Marks)

For the following LP

$$\text{Maximize } Z = x_1 + 3x_2$$

$$x_1 + x_2 \leq 2$$

$$-x_1 + x_2 \leq 4$$

x_1 unrestricted and $x_2 \geq 0$

a) Determine all the basic solutions of the problem, and classify them as feasible and infeasible.

b) Show the feasible region and find the optimal solution of the problem using graphical method.

Question 3 (5 Marks)

Consider the following problem, where the value of c_1 has not yet been ascertained.

$$\text{Maximize } Z = c_1x_1 + 2x_2,$$

s.t.

$$4x_1 + x_2 \leq 12$$

$$x_1 - x_2 \geq 2$$

$$x_1 \geq 0,\ x_2 \geq 0$$

Use graphical analysis to determine the optimal solution(s) for (x_1, x_2) for $c_1 \in (-\infty, \infty)$.

Question 4 (5 Marks)

The feasible region of a liner programming problem is defined by the following constraints

$3x_1 + x_2 + x_3 = 5$

$2x_1 + x_4 = 3$

$4x_1 + 3x_2 + x_5 = 6$

$x_1, x_2, x_3, x_4, x_5 \geq 0$

Identify with brief justification which of the solutions (0, 0, 5, 3, 6); (0, 5, 0, 3, -9); (1.5, 0, 0.5, 0, 0); (0.5, 1, 0, 1, 2); (1, 0.5, 1.5, 1, 0.5) are

a) Feasible solution

b) Basic solution

c) Basic feasible solution

d) Infeasible solution

Question 5 (5 Marks)

Solve the following LP using Simplex algorithm. In case of alternative/degenerate/unbounded solution, specify the nature of the solution with suitable reason.

$$\begin{aligned} &\text{Minimize } Z = -x_1 - 3x_2 \\ &\text{s.t.} \quad x_1 - 2x_2 \leq 4 \\ &\qquad -x_1 + x_2 \leq 3 \\ &\qquad x_1, x_2 \geq 0 \end{aligned}$$

Question 6 (5 Marks)

Consider the following problem

$$\begin{aligned} &\text{Minimize } Z = 4x_1 + x_2 + x_3 \\ &\text{s.t.} \quad 2x_1 + x_2 + 2x_3 = 4 \\ &\qquad 3x_1 + 3x_2 + x_3 = 3 \\ &\qquad x_1, x_2, x_3 \geq 0 \end{aligned}$$

Using the two-phase method

a) Construct the complete first simplex tableau for phase 1 and identify the corresponding initial basic feasible solution. Also identify the initial entering basic variable and the leaving basic variable.

b) Work through phase I step by step.

c) Construct the complete first simplex tableau for phase II and identify the corresponding initial basic feasible solution. Also identify the initial entering basic variable and the leaving basic variable.

d) Work through phase II step by step to solve the problem.

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