Syntax - Introduction

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Week 5: Lecture 1

What is Syntax?

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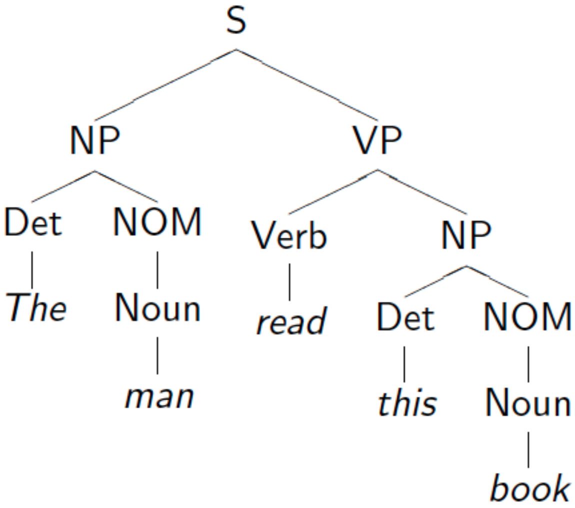
Syntax Tree: Example



Defining the notions: Constituency

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Constituency: Noun Phrase

- *Kermit the frog*
- *they*
- *December twenty-sixth*
- *the reason he is running for president*

Constituent Phrases

Usually named based on the word that heads the constituent:

the man from Amherst	is a Noun Phrase (NP) because the head man is a noun
extremely clever	is an Adjective Phrase (AP) because the head clever is an adjective
down the river	is a Prepositional Phrase (PP) because the head down is a preposition
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Joe appears in a place that a larger noun phrase could have been.

Evidence that constituency exists

They appear in similar environments

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Kermit the frog comes on stage
They come to Massachusetts every summer
December twenty-sixth comes after Christmas
The reason he is running for president comes out only now.
But not each individual word in the consituent
**The* comes our... **is* comes out... **for* comes out...

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Consituent = Prepositional phrase: *On December twenty-sixth*
On December twenty-sixth I'd like to fly to Florida.
I'd like to fly on December twenty-sixth to Florida.
I'd like to fly to Florida on December twenty-sixth.
But not split apart
**On December I'd like to fly twenty-sixth to Florida.*
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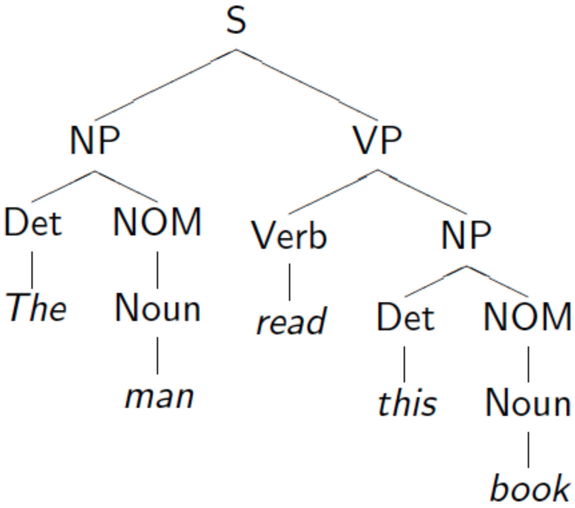
Modeling Constituency: what tool do we need?



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Consists of production Rules

These rules express the ways in which the symbols of the language can be grouped and ordered together

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$NP \rightarrow$ Det Nominal

$NP \rightarrow$ ProperNoun

$Nominal \rightarrow$ Noun | Noun Nominal

CFG for Languages

CFG for Languages

CFG: $G = (T, N, S, R)$

- T: set of terminals
- N: set of non-terminals
  - For NLP, we distinguish out a set $P \subset N$ of pre-terminals, which always rewrite as terminals
- S : start symbol
- R: Rules/productions of the form $X \rightarrow \gamma$, $X \in N$ and $\gamma \in (T \cup N)*$

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Terminals and pre-terminals

Terminals mainly correspond to words in the language while pre-terminals mainly correspond to POS categories

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Can you identify the terminal, non-terminals and preterminals?

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- Thus a CFG can be used to randomly generate a series of strings

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- Thus a CFG can be used to randomly generate a series of strings
- This sequence of rule expansions is called a derivation of the string of words, usually represented as a tree

CFGs and Grammaticality

A CFG defines a formal language = set of all sentences (string of words) that can be derived by the grammar

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- Sentences in this set are said to be **grammatical**
- Sentences outside this set are said to be **ungrammatical**

CFGs and Recursion

Recursive Definition

- PP → Prep NP
- NP → Noun PP

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- PP $\rightarrow$ Prep NP
- NP $\rightarrow$ Noun PP

Example Sentence

[$_S$The mailman ate his [$_{NP}$ lunch [$_{PP}$ with his friend [$_{PP}$ from the cleaning staff [$_{PP}$ of the building [$_{PP}$ at the intersection [$_{PP}$ on the north end [$_{PP}$ of town]]]]]]].

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$A \rightarrow BC$

- I can rewrite A as *B followed by C* regardless of the context in which A is found
- Or when I see a *B followed by a C*, I can infer an A regardless of the surrounding context

Syntax -Parsing I

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Week 5: Lecture 2

Grammar Rewrite Rules

S $\rightarrow$ NP VP
S $\rightarrow$ Aux NP VP
S $\rightarrow$ VP
NP $\rightarrow$ Det NOM
NOM $\rightarrow$ Noun
NOM $\rightarrow$ Noun NOM
VP $\rightarrow$ Verb
VP $\rightarrow$ Verb NP

Det $\rightarrow$ *that* | *this* | *a* | *the*
Noun $\rightarrow$ *book* | *flight* | *meal* | *man*
Verb $\rightarrow$ *book* | *include* | *read*
Aux $\rightarrow$ *does*

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S → NP VP
→ Det NOM VP
→ *The* NOM VP
→ *The* Noun VP
→ *The man* VP
→ *The man* Verb NP
→ *The man read* NP
→ *The man read* Det NOM
→ *The man read this* NOM
→ *The man read this* Noun
→ *The man read this book*

Parse Tree

S $\rightarrow$ NP VP
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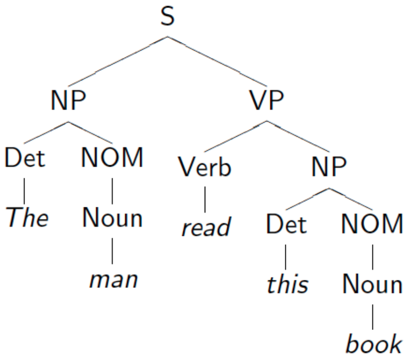

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What are the constraints? "book that flight"

- There must be three leaves, *book*, *that* and *flight*
- The tree must have one root, the start symbol S
- Give rise to two search strategies: *top-down* (goal-oriented) and *bottom-up* (data-directed)

Parsing

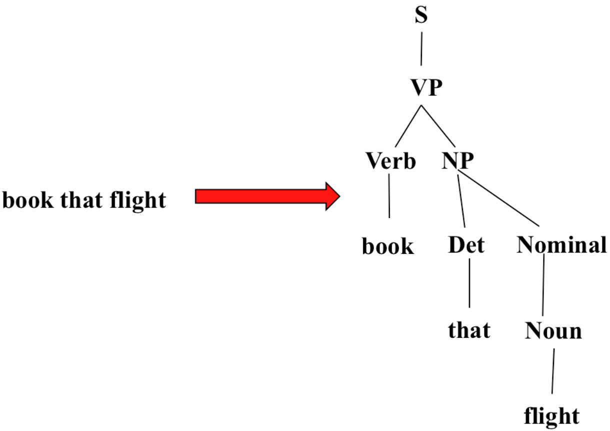
Grammar

S → NP VP
S → Aux NP VP
S → VP
NP → Pronoun
NP → Proper-Noun
NP → Det Nominal
Nominal → Noun
Nominal → Nominal Noun
Nominal → Nominal PP
VP → Verb
VP → Verb NP
VP → VP PP
PP → Prep NP

Lexicon

Det → the | a | that | this
Noun → book | flight | meal | money
Verb → book | include | prefer
Pronoun → I | he | she | me
Proper-Noun → Houston | NWA
Aux → does
Prep → from | to | on | near | through

Parsing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- Trees are grown downward until they eventually reach the POS categories at the bottom
- Trees whose leaves fail to match the words in the input can be rejected

Top-Down Parsing

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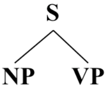

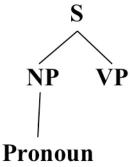
S
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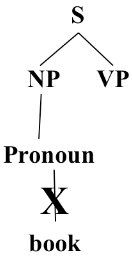
S
NP
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Pronoun
X
book

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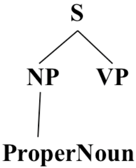

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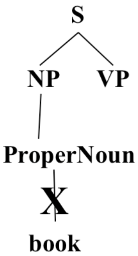

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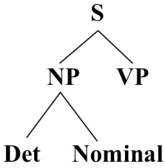

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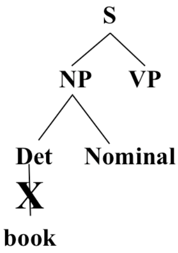

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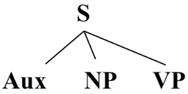

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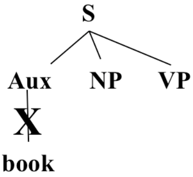

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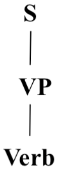

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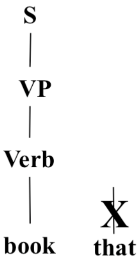

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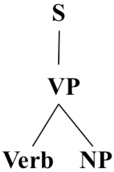

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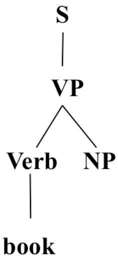

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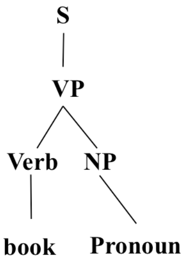

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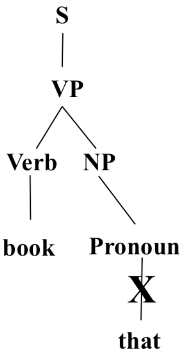

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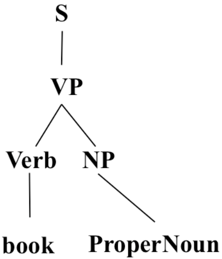

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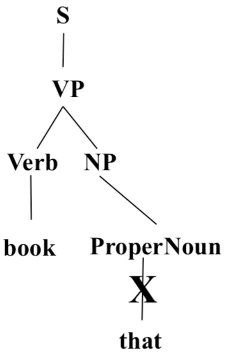

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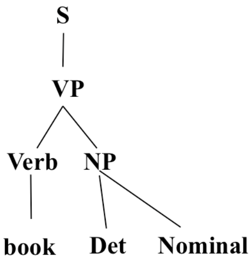

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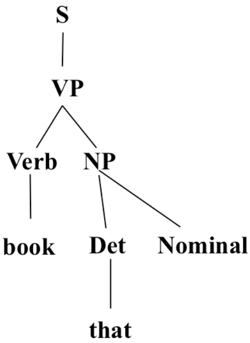

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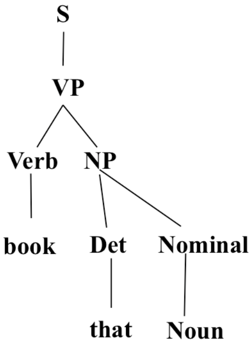
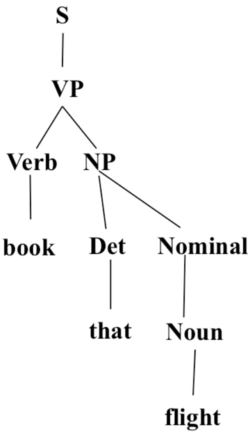


Top-Down Parsing



Bottom-Up Parsing

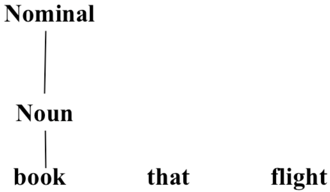
- The parser starts with the words of the input, and tries to build trees from the words up, by applying rules from the grammar one at a time
- Parser looks for the places in the parse-in-progress where the right-hand-side of some rule might fit.

Bottom-Up Parsing

book **that** **flight**


Noun
book
that
flight

Bottom-Up Parsing



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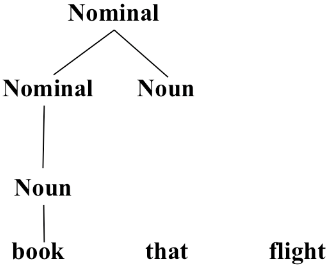

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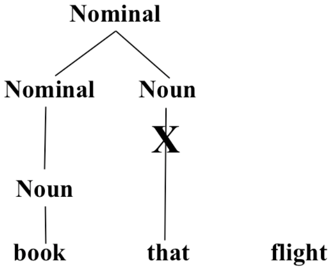

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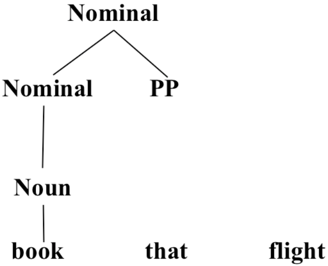

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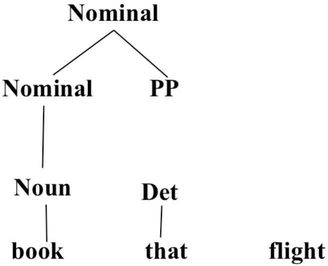

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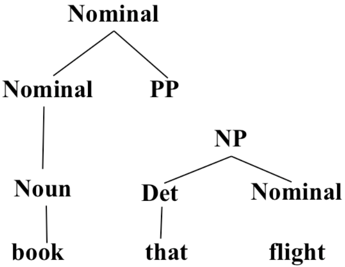

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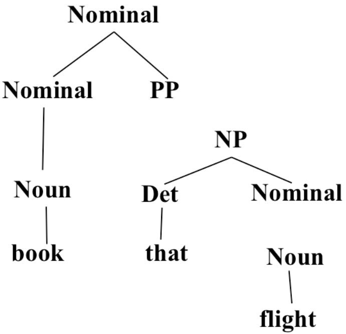

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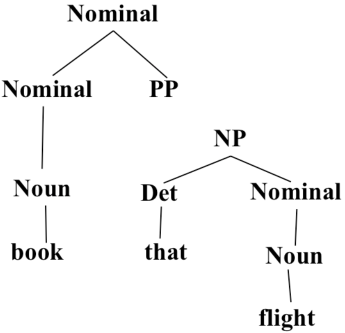

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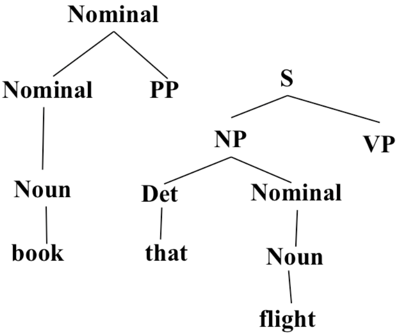

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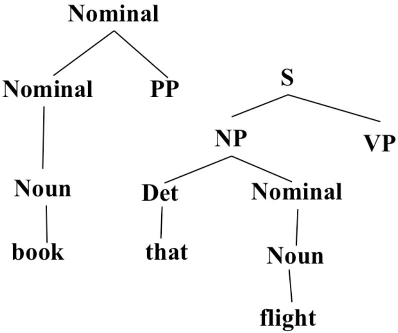

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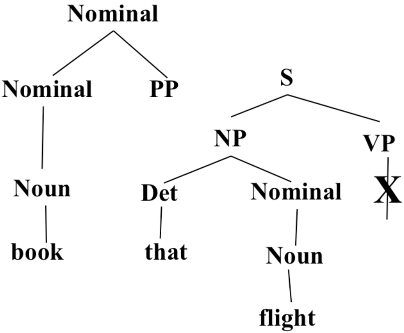

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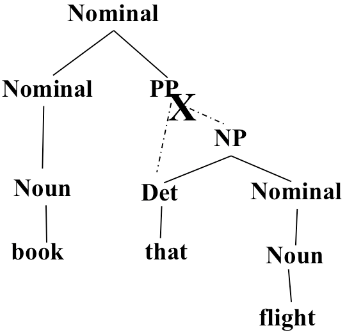

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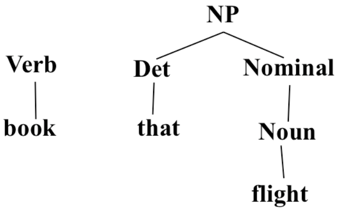

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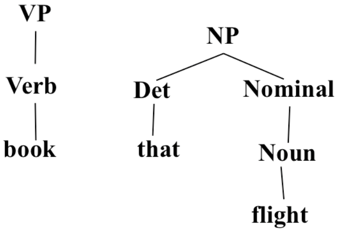

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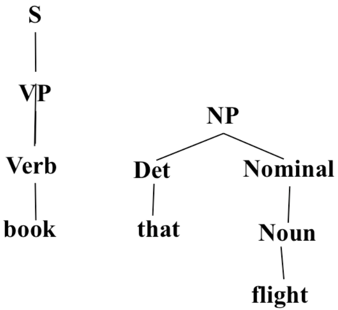

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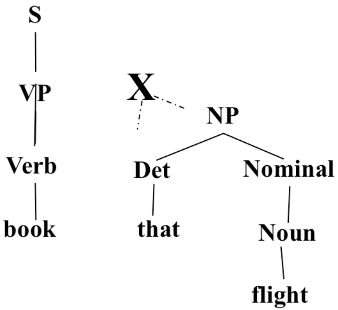

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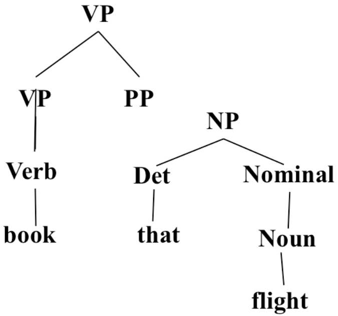

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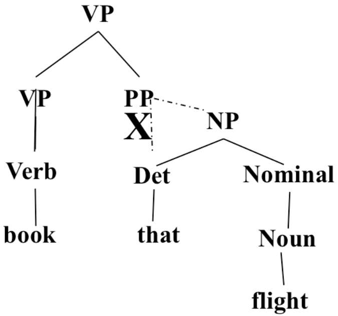

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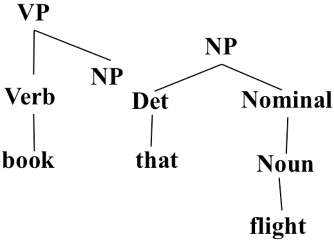

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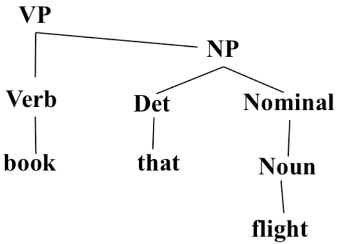

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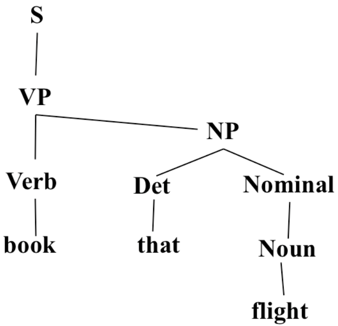

Top-Down vs. Bottom-Up

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Top-Down vs. Bottom-Up

- Top down never explores options that will not lead to a full parse, but can explore many options that never connect to the actual sentence.
- Bottom up never explores options that do not connect to the actual sentence but can explore options that can never lead to a full parse.
- Relative amounts of wasted search depend on how much the grammar branches in each direction.

Dynamic Programming Parsing

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- Caching (memoizing) critical to obtaining a polynomial time parsing (recognition) algorithm for CFGs.
- Dynamic programming algorithms based on both top-down and bottom-up search can achieve $O(n^3)$ recognition time where n is the length of the input string.

Dynamic Programming Parsing Methods

- CKY (Cocke-Kasami-Younger) algorithm: bottom-up, requires normalizing the grammar

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- CKY (Cocke-Kasami-Younger) algorithm: bottom-up, requires normalizing the grammar
- Earley Parser - top-down, does not require normalizing grammar, more complex
- More generally, *chart parsers* retain completed phrases in a chart and can combine top-down and bottom-up searches.

CKY Algorithm

- Grammar must be converted to Chomsky normal form (CNF) in which all productions must have
  - Either, exactly two non-terminals on the RHS
  - Or, 1 terminal symbol on the RHS

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  - Either, exactly two non-terminals on the RHS
  - Or, 1 terminal symbol on the RHS
- Parse bottom-up storing phrases formed from all substrings in a triangular table (chart)

Original Grammar

S → NP VP
S → Aux NP VP
S → VP
NP → Pronoun
NP → Proper-Noun
NP → Det Nominal
Nominal → Noun
Nominal → Nominal Noun
Nominal → Nominal PP
VP → Verb
VP → Verb NP
VP → VP PP
PP → Prep NP
Pronoun → I | he | she | me
Noun → book | flight | meal | money
Verb → book | include | prefer
Proper-Noun → Houston | NWA

Converting to CNF

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Chomsky Normal Form

S → NP VP
S → X1 VP
X1 → Aux NP
S → book | include | prefer
S → Verb NP
S → VP PP
NP → I | he | she | me
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CKY Algorithm

- Let n be the number of words in the input. Think about $n+1$ lines separating them, numbered 0 to n.
- x_{ij} will denote the words between line i and j
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Home Exercise

Use CKY algorithm to find the parse tree for "Book the flight through Houston" using the CNF form shown in the previous slide.

CKY for CFG

a 1	pilot 2	likes 3	flying 4	planes 5

$S \longrightarrow NP\ VP$
$VP \longrightarrow VBG\ NNS$
$VP \longrightarrow VBZ\ VP$
$VP \longrightarrow VBZ\ NP$
$NP \longrightarrow DT\ NN$
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CKY for CFG

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DT	NP	-	-	S S
	NN	-	-	-
		VBZ	-	VP VP
			JJ VBG	NP VP
				NNS

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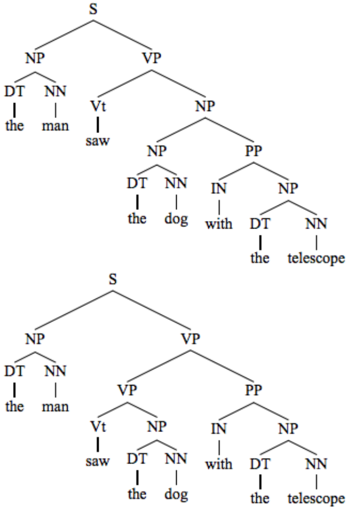
What about Ambiguities?



Probabilistic Context-free grammars (PCFGs)

PCFG: $G = (T, N, S, R, P)$

- T: set of terminals
- N: set of non-terminals
  - For NLP, we distinguish out a set $P \subset N$ of pre-terminals, which always rewrite as terminals
- S : start symbol
- R: Rules/productions of the form $X \rightarrow \gamma$, $X \in N$ and $\gamma \in (T \cup N)*$

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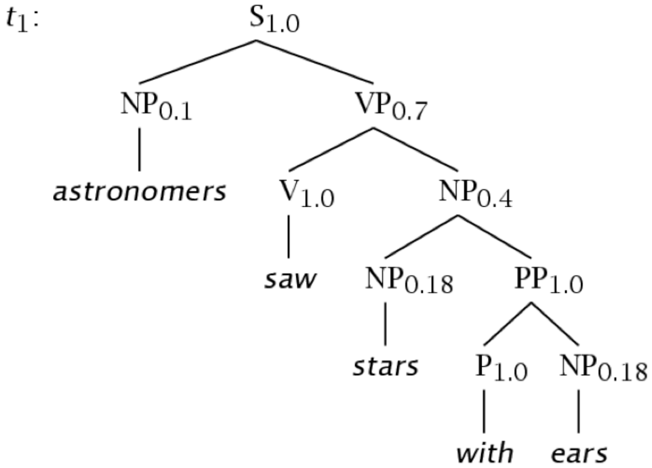
- R: Rules/productions of the form $X \rightarrow \gamma$, $X \in N$ and $\gamma \in (T \cup N)*$
- $P(R)$ gives the probability of each rule.

$$\forall X \in N, \sum_{X \rightarrow \gamma \in R} P(X \rightarrow \gamma) = 1$$

A Simple PCFG (in CNF)

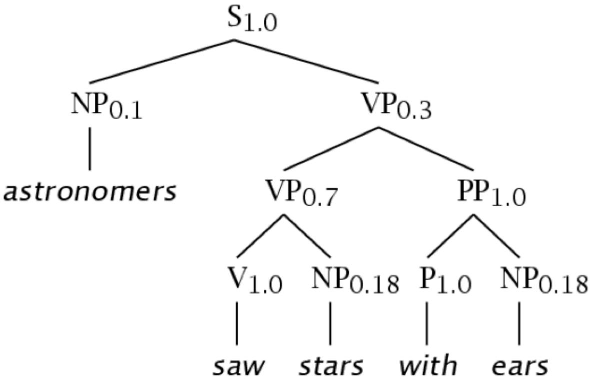
S	→	NP VP	1.0	NP	→	NP PP	0.4
VP	→	V NP	0.7	NP	→	*astronomers*	0.1
VP	→	VP PP	0.3	NP	→	*ears*	0.18
PP	→	P NP	1.0	NP	→	*saw*	0.04
P	→	*with*	1.0	NP	→	*stars*	0.18
V	→	*saw*	1.0	NP	→	*telescope*	0.1

Example Trees



Example Trees

t_2:



Probability of trees and strings

- $P(t)$: The probability of tree is the product of the probabilities of the rules used to generate it
- $P(w_{1n})$: The probability of the string is the sum of the probabilities of the trees which have that string as their yield

Tree and String probabilities

Tree and String probabilities

w_{15} = *astronomers saw stars with ears*

$$P(t_1) = 1.0 * 0.1 * 0.7 * 1.0 * 0.4 * 0.18 * 1.0 * 1.0 * 0.18 = 0.0009072$$

$$P(t_2) = 1.0 * 0.1 * 0.3 * 0.7 * 1.0 * 0.18 * 1.0 * 1.0 * 0.18 = 0.0006804$$

$$P(w_{15}) = P(t_1) + P(t_2) = 0.0009072 + 0.0006804 = 0.0015876$$

"Book the dinner flight"

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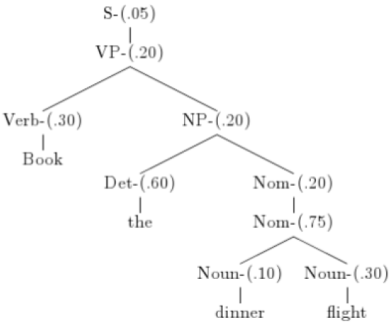

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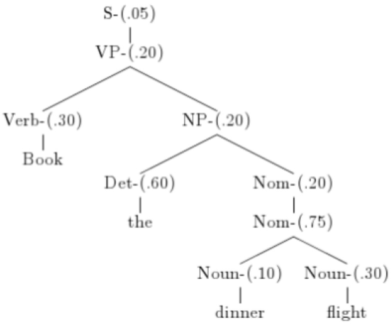

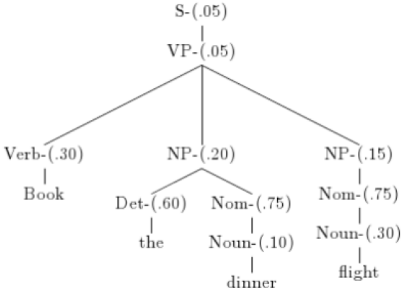

Probabilities

- Parse tree 1: $.05 \times .20 \times .30 \times .20 \times .60 \times .20 \times .75 \times .10 \times .30 = 1.62 \times 10^{-6}$
- Parse tree 2: $.05 \times .05 \times .30 \times .20 \times .60 \times .75 \times .10 \times .15 \times .75 \times .30 = 2.28 \times 10^{-7}$

Features of PCFGs

- As the number of possible trees for a given input grows, a PCFG gives some idea of the plausibility of a particular parse

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- Real text tends to have grammatical mistakes. PCFG avoids this problem by ruling out nothing, but by giving implausible sentences a low probability
- In practice, a PCFG is a worse language model for English than an n-gram model
- All else being equal, the probability of a smaller tree is greater than a larger tree

Important Questions?

Let W_{1m} be a sentence, G a grammar, t a parse tree

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$$P(w_{1m}|G)$$

- How to learn the rule probabilities in the grammar G?

PCFGs - Inside-outside probabilities

Niloy Ganguly

CSE, IIT Kharagpur

Week 5: Lecture 4

How to find the most likely parse?: CKY for PCFG

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a 1	pilot 2	likes 3	flying 4	planes 5

$S \longrightarrow NP\ VP$		[1.0]
$VP \longrightarrow VBG\ NNS$		[0.1]
$VP \longrightarrow VBZ\ VP$		[0.1]
$VP \longrightarrow VBZ\ NP$		[0.3]
$NP \longrightarrow DT\ NN$		[0.3]
$NP \longrightarrow JJ\ NNS$		[0.4]
$DT \longrightarrow a$		[0.3]
$NN \longrightarrow pilot$		[0.1]
$VBZ \longrightarrow likes$		[0.4]
$VBG \longrightarrow flying$		[0.5]
$JJ \longrightarrow flying$		[0.1]
$NNS \longrightarrow planes$		[.34]

CKY for PCFG

a 1	pilot 2	likes 3	flying 4	planes 5
DT [0.3]	NP [.009]	-	-	S [$1.4688 x 10^{-5}$] S [$6.12 x 10^{-6}$]
	NN [0.1]	-	-	-
		VBZ [0.4]	-	VP [.001632] VP [.00068]
			JJ [0.1] VBG [0.5]	NP [.0136] VP [.017]
				NNS [.34]

$S \longrightarrow NP\ VP$		[1.0]
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$VP \longrightarrow VBZ\ VP$		[0.1]
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$NP \longrightarrow JJ\ NNS$		[0.4]
$DT \longrightarrow a$		[0.3]
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$VBZ \longrightarrow likes$		[0.4]
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$0.009 \times 0.00068 \times 1.0 = 6.12 x 10^{-6}$

Probability of a String

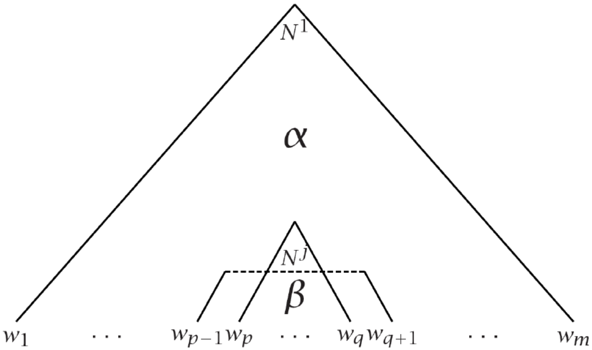
$$P(w_{1m}|G)$$

Probability of a String

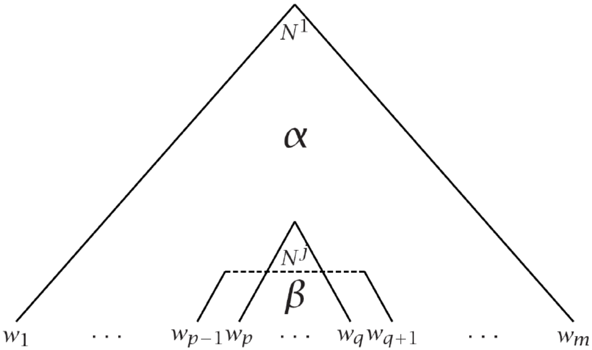
$$P(w_{1m}|G)$$

- In general, simply summing the probabilities of all possible parse trees is not an efficient way to calculate the string probability
- We use *inside algorithm*, a dynamic programming algorithm based on inside probabilities.

Inside and Outside Probabilities

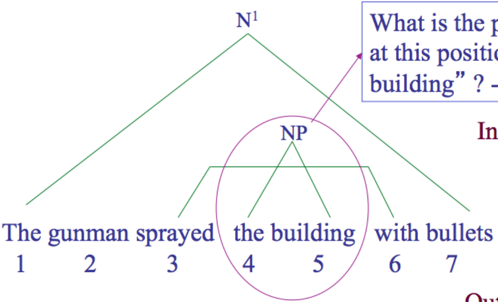


Inside and Outside Probabilities



Outside: $\alpha_j(p,q) = P(w_{1(p-1)}, N^j{}_{pq}, w_{(q+1)m}|G)$
Inside: $\beta_j(p,q) = P(w_{pq}|N^j{}_{pq}, G)$

Inside-outside probabilities



What is the probability that an NP at this position will derive "the building" ? - $\beta_{NP}(4,5)$

Inside Probabilities

Outside Probabilities

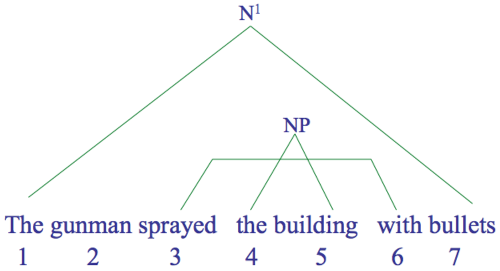
What is the probability of starting from N^1 and deriving "The gunman sprayed", a NP and "with bullets" ? - $\alpha_{NP}(4,5)$

Inside-outside probabilities

$\alpha_{NP}(4,5)$ for "the building"

$= P(\text{The gunman sprayed}, NP_{4,5}, \text{with bullets} \mid G)$

$\beta_{NP}(4,5)$ for "the building" $= P(\text{the building} \mid NP_{4,5}, G)$



Inside Probabilities: Base Step

$$\beta_j(p,q) = P(w_{pq}|N^j{}_{pq}, G)$$

Inside Probabilities: Base Step

$$\beta_j(p,q) = P(w_{pq}|N^j{}_{pq}, G)$$

Base case

$$\beta_j(k,k) = P(w_{kk}|N^j{}_{kk}, G)$$
$$= P(N^j \rightarrow w_k|G)$$

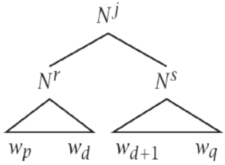
Base case for pre-terminals only

E.g., suppose $N^j = NN$ is being considered and $NN \rightarrow building$ is one of the rules with probability 0.5
$\beta_{NN}(5,5) = P(building|NN_{5,5}, G) = P(NN_{5,5} \rightarrow building|G)$

Inside Probabilities: Induction Step

Assuming Chomsky Normal Form, the first rule must be of the form $N^j \rightarrow N^r N^s$

$$\beta_j(p,q) = \sum_{r,s} \sum_{d=p}^{q-1} P(N^j \rightarrow N^r N^s) \beta_r(p,d) \beta_s(d+1,q)$$



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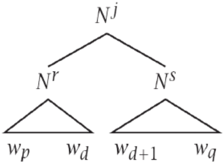
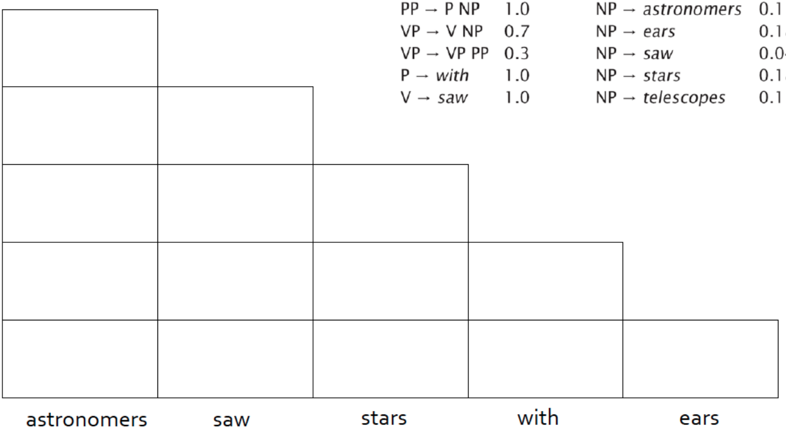


- Consider different splits of the words - indicated by d
  E.g., *the huge building*
- Consider different non-terminals to be used in the rule:
  E.g., $NP \rightarrow DT\ NN$, $NP \rightarrow DT\ NNS$

Calculation of inside probabilities

S → NP VP	1.0	NP → NP PP	0.4
PP → P NP	1.0	NP → *astronomers*	0.1
VP → V NP	0.7	NP → *ears*	0.18
VP → VP PP	0.3	NP → *saw*	0.04
P → *with*	1.0	NP → *stars*	0.18
V → *saw*	1.0	NP → *telescopes*	0.1



Calculation of inside probabilities

	1	2	3	4	5
1	$\beta_{NP} = 0.1$		$\beta_S = 0.0126$		$\beta_S = 0.0015876$
2		$\beta_{NP} = 0.04$ $\beta_V = 1.0$	$\beta_{VP} = 0.126$		$\beta_{VP} = 0.015876$
3			$\beta_{NP} = 0.18$		$\beta_{NP} = 0.01296$
4				$\beta_P = 1.0$	$\beta_{PP} = 0.18$
5					$\beta_{NP} = 0.18$
	astronomers	*saw*	*stars*	*with*	*ears*

Outside Probabilities

Compute top-down (after inside probabilities)

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Base Case

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$$\alpha_1(1, m) = 1$$

$$\alpha_j(1, m) = 0, j \neq 1$$

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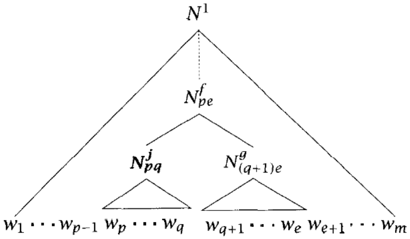
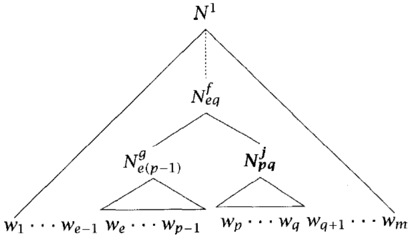
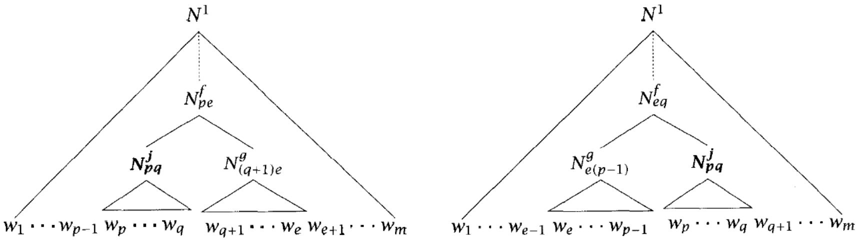




Outside Probabilities: Induction



$$\begin{aligned}\alpha_j(p,q) &= \sum_{f,g}\sum_{e=q+1}^{m}\alpha_f(p,e)P(N^f \to N^j N^g)\beta_g(q+1,e)\\ &\quad + \sum_{f,g}\sum_{e=1}^{p-1}\alpha_f(e,q)P(N^f \to N^g N^j)\beta_g(e,p-1)\end{aligned}$$

Product of inside-outside probabilities

$$\alpha_j(p,q)\beta_j(p,q) = P(w_{1(p-1)}, {N^j}_{pq}, w_{(q+1)m}|G)P(w_{pq}|{N^j}_{pq}, G) = P(w_{1m}, {N^j}_{pq}|G)$$

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The probability of the sentence and that there is some consistent spanning from word p to q is given by:

$$P(w_{1m}, N_{pq}|G) = \sum \alpha_j(p,q)\beta_j(p,q) = P(N_1 \rightarrow w_{1m}, N_{pq} \rightarrow w_{pq}|G)$$

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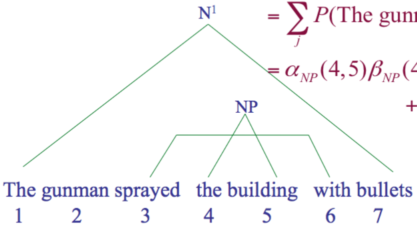
$$P(\text{The gunman....bullets}, N_{4,5} \mid G)$$
$$= \sum_j P(\text{The gunman...bullets} \mid N^j_{4,5}, G)$$
$$= \alpha_{NP}(4,5)\beta_{NP}(4,5)$$
$$+ \alpha_{VP}(4,5)\beta_{VP}(4,5) + \ldots$$



Inside-outside probabilities

Niloy Ganguly

CSE, IIT Kharagpur

Week 5: Lecture 5

How to get the rule probabilities

Parsed Training Data

You can count!

$$\hat{P}(N^j \to \delta) = \frac{C(N^j \to \delta)}{\sum_\gamma C(N^j \to \gamma)}$$

How to get the rule probabilities

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But what if the training data is not available?

i.e. gold standard parse is not known.

How to get the rule probabilities

Parsed Training Data

You can count!

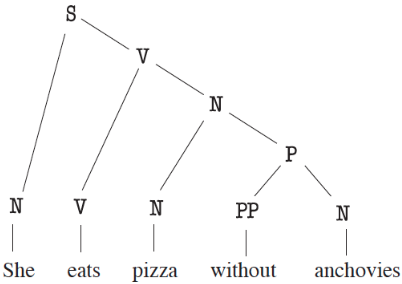
$$\hat{P}(N^j \to \delta) = \frac{C(N^j \to \delta)}{\sum_\gamma C(N^j \to \gamma)}$$

But what if the training data is not available?

i.e. gold standard parse is not known.

- Underlying CFG is known and we are given a set of sentences
- For each sentence, we can find out all the possible parses
- *Maximize the likelihood of the sentences in the data under the PCFG constraints*

Example data



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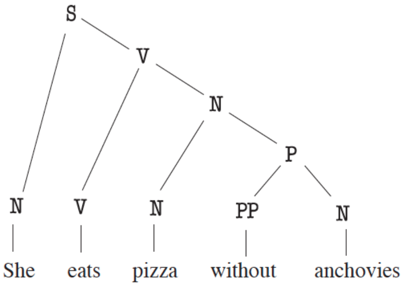


Rules of the form $A \rightarrow BC$

$\texttt{S} \rightarrow \texttt{N V}$

$\texttt{V} \rightarrow \texttt{V N}$

$\texttt{N} \rightarrow \texttt{N P}$

$\texttt{P} \rightarrow \texttt{PP N}$.

Rules of the form $A \rightarrow w$

$\texttt{N} \rightarrow$ She

$\texttt{V} \rightarrow$ eats

$\texttt{N} \rightarrow$ pizza

$\texttt{PP} \rightarrow$ without

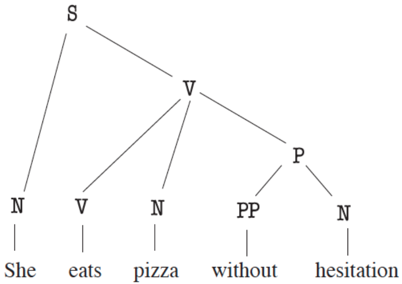
$\texttt{N} \rightarrow$ anchovies.

Example data

Is any other parse possible for *She eats pizza without anchovies* syntactically?

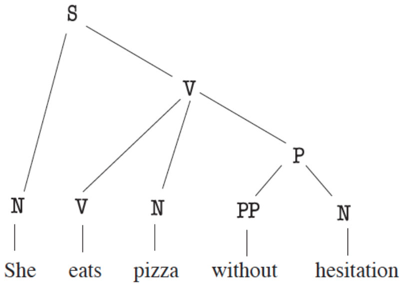
Example data

Is any other parse possible for *She eats pizza without anchovies* syntactically?
Consider *She eats pizza without hesitation*



Example data

Is any other parse possible for *She eats pizza without anchovies* syntactically?
Consider *She eats pizza without hesitation*



New Context-free rules:

$$\mathtt{V} \rightarrow \mathtt{V\ N\ P}$$

$$\mathtt{N} \rightarrow \text{hesitation}\ .$$

Estimating the model parameters

We need to find probabilities such as

- $\phi(S \rightarrow N\ V)$
- $\phi(N \rightarrow pizza)$

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Requirements

For each non-terminal A, the derivation probabilities sum up to 1

$$\sum_{\alpha} \phi(A \to \alpha) = 1$$

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- $\phi(N \rightarrow pizza)$

Requirements

For each non-terminal A, the derivation probabilities sum up to 1

$$\sum_{\alpha} \phi(A \rightarrow \alpha) = 1$$

For the example grammar:

$$\begin{aligned}
\phi(\mathtt{N} \rightarrow \mathtt{N\ P}) + \phi(\mathtt{N} \rightarrow \text{pizza}) + \phi(\mathtt{N} \rightarrow \text{anchovies}) &\quad + \\
+ \phi(\mathtt{N} \rightarrow \text{hesitation}) + \phi(\mathtt{N} \rightarrow \text{She}) &= 1 \\
\phi(\mathtt{V} \rightarrow \mathtt{V\ N}) + \phi(\mathtt{V} \rightarrow \mathtt{V\ N\ P}) + \phi(\mathtt{V} \rightarrow \text{eats}) &= 1
\end{aligned}$$

$$\begin{aligned}
\phi(\mathtt{S} \rightarrow \mathtt{N\ V}) &= 1 \\
\phi(\mathtt{P} \rightarrow \mathtt{PP\ N}) &= 1 \\
\phi(\mathtt{PP} \rightarrow \text{without}) &= 1
\end{aligned}$$

Likelihood computation

$W_1 =$ "She eats pizza without anchovies"

$W_2 =$ "She eats pizza without hesitation".

Likelihood computation

$$W_1 = \text{“She eats pizza without anchovies”}$$

$$W_2 = \text{“She eats pizza without hesitation”}.$$

$$\begin{aligned} P_\phi(W_1, T_1) &= \phi(\texttt{S} \rightarrow \texttt{N V})\, \phi(\texttt{V} \rightarrow \texttt{V N})\, \phi(\texttt{N} \rightarrow \texttt{N P}) \times \\ &\times \phi(\texttt{P} \rightarrow \texttt{PP N})\, \phi(\texttt{N} \rightarrow \text{She})\, \phi(\texttt{V} \rightarrow \text{eats}) \times \\ &\times \phi(\texttt{N} \rightarrow \text{pizza})\, \phi(\texttt{PP} \rightarrow \text{without})\, \phi(\texttt{N} \rightarrow \text{anchovies}) \end{aligned}$$

$$\begin{aligned} P_\phi(W_2, T_1) &= \phi(\texttt{S} \rightarrow \texttt{N V})\, \phi(\texttt{V} \rightarrow \texttt{V N P})\, \phi(\texttt{P} \rightarrow \texttt{P PP}) \times \\ &\times \phi(\texttt{N} \rightarrow \text{She})\, \phi(\texttt{V} \rightarrow \text{eats})\, \phi(\texttt{N} \rightarrow \text{pizza}) \times \\ &\times \phi(\texttt{PP} \rightarrow \text{without})\, \phi(\texttt{N} \rightarrow \text{hesitation}) \end{aligned}$$

Likelihood computation

$$
\begin{aligned}
P_\phi(W_1, T_2) &= \phi(\texttt{S} \to \texttt{N V})\, \phi(\texttt{V} \to \texttt{V N P})\, \phi(\texttt{P} \to \texttt{P PP}) \times \\
&\times \quad \phi(\texttt{N} \to \text{She})\, \phi(\texttt{V} \to \text{eats})\, \phi(\texttt{N} \to \text{pizza}) \times \\
&\times \quad \phi(\texttt{PP} \to \text{without})\, \phi(\texttt{N} \to \text{anchovies})
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Likelihood of the corpus

Probability of a sentence W : $P_\phi(W) = \sum_T P_\phi(W, T)$

Likelihood computation

$$\begin{aligned} P_\phi(W_1, T_2) &= \phi(\mathtt{S} \to \mathtt{N\ V})\, \phi(\mathtt{V} \to \mathtt{V\ N\ P})\, \phi(\mathtt{P} \to \mathtt{P\ PP}) \times \\ &\times \phi(\mathtt{N} \to \text{She})\, \phi(\mathtt{V} \to \text{eats})\, \phi(\mathtt{N} \to \text{pizza}) \times \\ &\times \phi(\mathtt{PP} \to \text{without})\, \phi(\mathtt{N} \to \text{anchovies}) \end{aligned}$$

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Likelihood of the corpus

Probability of a sentence W : $P_\phi(W) = \sum_T P_\phi(W, T)$

If the training data comprises of sentences $W_1, W_2, \ldots, W_N$, then the likelihood is

$$L(\phi) = P_\phi(W_1) P_\phi(W_2) \cdots P_\phi(W_N)$$

Likelihood maximization

Approach

Starting at some initial parameters ϕ, re-estimate to obtain new parameters ϕ' for which $L(\phi') \geq L(\phi)$. *Repeat until convergence*

Parameter Estimation

Given some rule probabilities ϕ and training corpus $W_1, W_2 \ldots W_n$, the new parameters are obtained as:

$$\phi'(\mathtt{A} \rightarrow \mathtt{B}\ \mathtt{C}) = \frac{count(\mathtt{A} \rightarrow \mathtt{B}\ \mathtt{C})}{\sum_\alpha count(\mathtt{A} \rightarrow \alpha)}$$

$$\phi'(\mathtt{A} \rightarrow \mathrm{w}) = \frac{count(\mathtt{A} \rightarrow \mathrm{w})}{\sum_\alpha count(\mathtt{A} \rightarrow \alpha)}$$

What is $count(.)$?

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What is $count(.)$?

$$count(\texttt{A} \to \texttt{B C}) = \sum_{i=1}^{N} c_\phi(\texttt{A} \to \texttt{B C}, W_i)$$

$$count(\texttt{A} \to \mathrm{w}) = \sum_{i=1}^{N} c_\phi(\texttt{A} \to \mathrm{w}, W_i)$$

Parameter Estimation

Given some rule probabilities ϕ and training corpus $W_1, W_2 \ldots W_n$, the new parameters are obtained as:

$$\phi'(\mathtt{A} \rightarrow \mathtt{B}\ \mathtt{C}) = \frac{count(\mathtt{A} \rightarrow \mathtt{B}\ \mathtt{C})}{\sum_{\alpha} count(\mathtt{A} \rightarrow \alpha)}$$

$$\phi'(\mathtt{A} \rightarrow \mathrm{w}) = \frac{count(\mathtt{A} \rightarrow \mathrm{w})}{\sum_{\alpha} count(\mathtt{A} \rightarrow \alpha)}$$

What is $count(.)$?

$$count(\mathtt{A} \rightarrow \mathtt{B}\ \mathtt{C}) = \sum_{i=1}^{N} c_{\phi}(\mathtt{A} \rightarrow \mathtt{B}\ \mathtt{C}, W_i)$$

$$count(\mathtt{A} \rightarrow \mathrm{w}) = \sum_{i=1}^{N} c_{\phi}(\mathtt{A} \rightarrow \mathrm{w}, W_i)$$

$c_{\phi}(A \rightarrow \alpha, W_i)$ is the expected number of times $(A \rightarrow \alpha)$ is used in generating the sentence W_i, when the rule probabilities are given by ϕ

Computing Expected counts

Inside probabilities

The nonterminal A derives the string of words $w_i, \ldots w_j$ in the sentence :
$\beta_{ij}(A) = P_\phi(A \Rightarrow^* w_i \ldots w_j)$

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Outside probabilities

Beginning with the start symbol S we can derive the string
$w_1 \ldots w_{i-1} A w_{j+1} \ldots w_n : \alpha_{ij}(A) = P_\phi(S \Rightarrow^* w_1 \ldots w_{i-1} A w_{j+1} \ldots w_n)$

Computing Expected counts

Inside probabilities

The nonterminal A derives the string of words $w_i, \dots w_j$ in the sentence :
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Outside probabilities

Beginning with the start symbol S we can derive the string
$w_1 \dots w_{i-1} A w_{j+1} \dots w_n$: $\alpha_{ij}(A) = P_\phi(S \Rightarrow^* w_1 \dots w_{i-1} A w_{j+1} \dots w_n)$

Expected count

$$c_\phi(A \to BC, W) = \frac{\phi(A \to BC)}{P_\phi(W)} \sum_{1 \le i \le j \le k \le n} \alpha_{ik}(A) \beta_{ij}(B) \beta_{j+1,k}(C)$$

$$c_\phi(A \to w, W) = \frac{\phi(A \to w)}{P_\phi(W)} \sum_{1 \le i \le n} \alpha_{ii}(A)$$

And how to compute inside-outside probabilities

Inductively, as discussed earlier

$$\beta_{ii}(A) = \phi(A \rightarrow w_i)$$

$$\alpha_{1n}(S) = 1$$