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1. Maximize $Z = 7x_1 + 10x_2$

$$x_1 \le 36$$

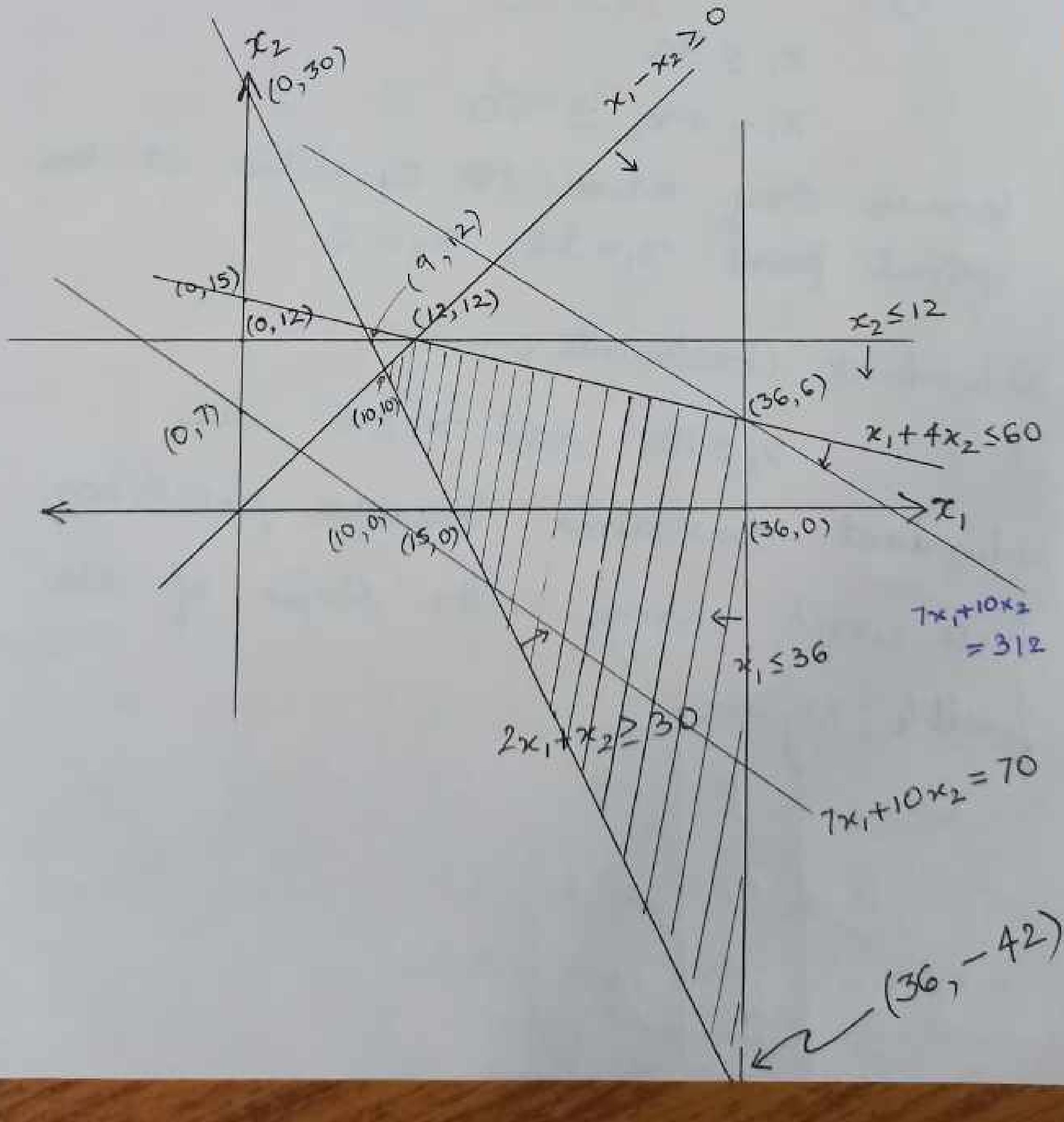
$$x_2 \le 12$$

$$x_1 + 4x_2 \le 60$$

$$2x_1 + x_2 \ge 30$$

$$x_1 - x_2 \ge 0$$

and $x_1 \ge 0$, x_2 unrestricted



From the graph, we find that z maximizes at (36, 6).

This can also be verified by finding value of z at all the corner points of the feasible region.

$(10, 10) \Rightarrow \quad z = 7\times10 + 10\times10 = 170$

$(12, 12) \Rightarrow \quad z = 7\times12 + 10\times12 = 204$

$(36, 6) \Rightarrow \quad z = 7\times36 + 10\times6 = 312$ ✓

$(36, -42) \Rightarrow \quad z = 7\times36 - 10\times42 = -168$

Binding Constraints :—

$$x_1 \leq 36$$

$$x_1 + 4x_2 \leq 60$$

because they reduce to equalities at the optimal point $x_1 = 36$, $x_2 = 6$.

Redundant Constraints :—

$x_2 \leq 12$ and $x_1 \geq 0$ are

∴ redundant constraints because removing these won't change the shape of the feasible region.

Q. Maximize

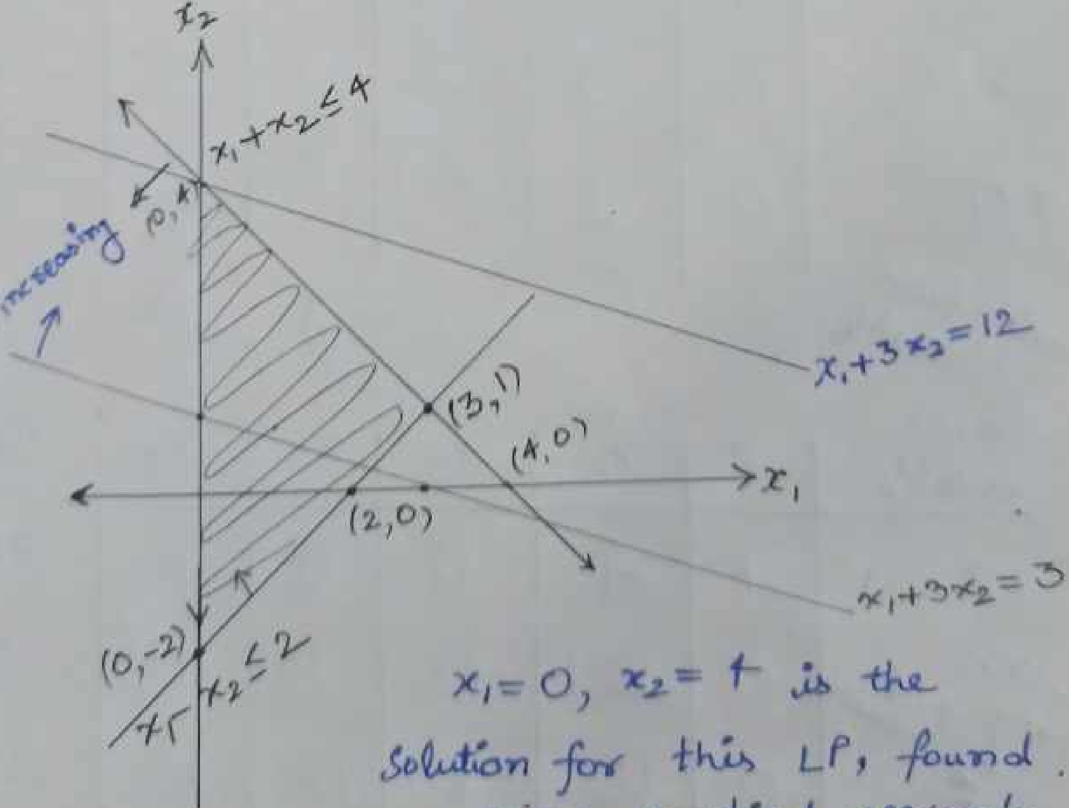
$$z = x_1 + 3x_2$$

$$\left.\begin{array}{l} x_1 + x_2 \le 4 \\ x_1 - x_2 \le 2 \\ x_1 \ge 0 \\ x_2 = \text{unrestricted} \end{array}\right\} \Rightarrow \begin{array}{l} x_1 + x_2 + x_3 = 4 \\ x_1 - x_2 + x_4 = 2 \\ x_1 \ge 0 \\ x_3 \ge 0 \\ x_4 \ge 0 \\ x_2 = \text{unrestricted} \end{array}$$

(i)

x_1	x_2	x_3	x_4	
0	0	4	2	Feasible
0	4	0	6	Feasible
0	−2	6	0	Feasible [$\because x_2 < 0$ is allowed]
4	0	0	−2	Infeasible
2	0	2	0	Feasible
3	1	0	0	Feasible

(ii)



$x_1 = 0$, $x_2 = 4$ is the solution for this LP, found using graphical approach

3. (A) Maximize $3x_1 + 4x_2$ (B) Maximize $x_1 + 2x_2$

Subject to $3x_1 + 2x_2 \leq 30$

$x_1 + 2x_2 \leq 22$

$x_1, x_2 \geq 0$

Augmented constraint set:

$$3x_1 + 2x_2 + x_3 = 30$$

$$x_1 + 2x_2 + x_4 = 22$$

$$x_1, x_2, x_3, x_4 \geq 0$$

(A):

entering basic variable

Basic Variable	Z	x_1	x_2	x_3	x_4	RHS
Z	1	-3	-4	0	0	0
x_3	0	3	2	1	0	30
x_4	0	$\frac{1}{2}$	$\cancel{2}\ 1$	0	$\frac{1}{2}$	$\cancel{22}\ 11$

leaving basic variable

entering basic variable

Basic Variable	Z	x_1	x_2	x_3	x_4	RHS
Z	1	-1	0	0	2	44
x_3	0	$\cancel{2}\ 1$	0	$1/2$	$-1/2$	$\cancel{8}\ 4$
x_2	0	$1/2$	1	0	$1/2$	11

leaving basic variable

Basic Variable	Z	x_1	x_2	x_3	x_4	RHS
Z	1	0	0	1/2	3/2	48
x_1	0	1	0	1/2	-1/2	4
x_2	0	0	$\cancel{2}$ 1	$\cancel{1/2}$ -1/4	$\cancel{3/2}$ 3/4	$\cancel{18}$ 9

$x_1 = 4$ is the optimal Solution
$x_2 = 9$
and $z = 48$ for $(4, 9)$.

(B):

entering basic variable

Basic Variable	Z	x_1	x_2	x_3	x_4	RHS
Z	1	-1	-2	0	0	0
x_3	0	3	2	1	0	30
x_4	0	$\frac{1}{2}$	1	0	$\frac{1}{2}$	11

leaving basic variable

Basic Variable	Z	x_1	x_2	x_3	x_4	RHS
Z	1	0	0	0	1	22
x_3	0	2	0	1	-1	8
x_2	0	1/2	1	0	1/2	11

$x_1 = 0$
$x_2 = 11$ is the optimal Solution and $z = 22$ for this point

(B) has multiple optimal solutions as we can see in the last table that coefficient of the non-basic variable x_1 is zero in the first row.

Basic Variable	z	x_1	x_2	x_3	x_4	RHS
z	1	0	0	0	1	22
x_1	0	1	0	1/2	-1/2	4
x_2	0	0	1	-1/4	3/4	9

$x_1 = 4$
$x_2 = 9$ is another optimal soln for same $z = 22$.

4. Maximize

$$Z = 3x_1 + 4x_2 + x_3$$

Subject to

$$x_1 + 2x_2 + 3x_3 \le 90$$
$$2x_1 + x_2 + x_3 \le 60$$
$$3x_1 + x_2 + 2x_3 \le 80$$
$$x_1, x_2, x_3 \ge 0$$

$$\Downarrow$$

$$x_1 + 2x_2 + 3x_3 + x_4 = 90$$
$$2x_1 + x_2 + x_3 + x_5 = 60$$
$$3x_1 + x_2 + 2x_3 + x_6 = 80$$

$$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$$

entering

Basic Variable	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
Z	1	-3	-4	-1	0	0	0	0
x_4	0	1	2	3	1	0	0	90
x_5	0	2	1	1	0	1	0	60
x_6	0	3	1	2	0	0	1	80

leaving

entering

Basic Variable	Z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
Z	1	-1	0	5	2	0	0	180
x_2	0	1/2	1	3/2	1/2	0	0	45
leaving → x_5	0	3/2	0	-1/2	-1/2	1	0	15
x_6	0	5/2	0	1/2	-1/2	0	1	35

Basic Variable	z	x_1	x_2	x_3	x_4	x_5	x_6	RHS
z	1	0	0	14/3	5/3	2/3	0	190
x_2	0	0	1	5/3	2/3	-1/3	0	40
x_1	0	1	0	-1/3	-1/3	2/3	0	10
x_6	0	0	0	4/3	1/3	-5/3	1	10

$x_1 = 10$
$x_2 = 40$ is the optimal solution, for which $z = 190$
$x_3 = 0$

5. Minimize $3x_1 + 2.5x_2$

Subject to

$$x_1 + 2x_2 \geq 20$$
$$5x_1 + 2x_2 \geq 50$$
$$x_1, x_2 \geq 0$$